BASICS OF LASER COOLING THEORY

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OUTLINE

- **Motivation**
- **lightshifts and excitation rates**
- **the mean force**
- **Doppler cooling**
- **the magneto-optical trap**
- **Sisyphus cooling**
- **Below the recoil limit**

MOTIVATION

Why relevant for metrology:

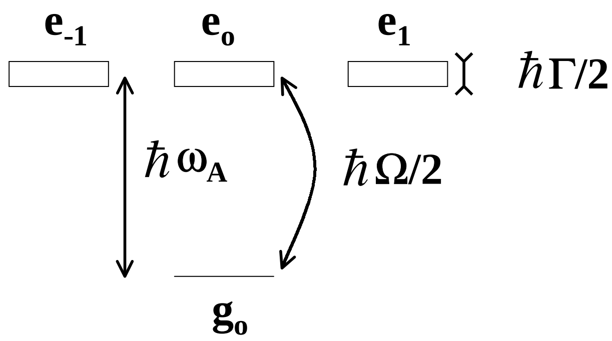
- $k\Delta v$ **reduced by** 10^4 **with respect to 300 K thermal beam**
- **suppression of a source of broadening for atomic and molecular spectroscopy (Hänsch, Wineland)**
- **very long interaction times are good for atomic clocks and inertial sensors**

Next lecture: Bose-Einstein condensation

- **ultimate control of atomic motion**
- **macroscopic atomic field coherence length**
- **atom laser is ideal source for interferometry**

LIGHTSHIFTS AND EXCITATION RATES

Generalized two-level model:



Hamiltonian:

$$H_0 = \hbar\omega_A \sum_m |e_m\rangle\langle e_m| \qquad V_{AL} = -\vec{D}\cdot\vec{E}_L(\vec{r},t)$$

$$\vec{E}_L(\vec{r},t) = \vec{\mathcal{E}}_L(\vec{r})e^{-i\omega_L t} + \text{c.c.}$$

$$\vec{D}\cdot\vec{e}_q = d|e_q\rangle$$

$$\vec{e}_\pm = \mp\frac{1}{\sqrt{2}}(\vec{e}_x \pm i\vec{e}_y) \qquad \vec{e}_0 = \vec{e}_z$$

Spontaneous emission: non hermitian Hamiltonian

$$|\Psi\rangle = |\psi_A^0\rangle \otimes |0\rangle + \sum_{\vec{k},\vec{\epsilon}} |\psi_A^{\vec{k},\vec{\epsilon}}\rangle \otimes |\vec{k},\vec{\epsilon}\rangle + \ldots$$

→ add $-i\hbar\Gamma/2 \sum_m |e_m\rangle\langle e_m|$ to Hamiltonian

Rotating wave approximation:

$$|\psi_A(t)\rangle = e^{-i\omega_L t \sum_m |e_m\rangle\langle e_m|}|\tilde{\psi}_A(t)\rangle \qquad V_{AL} \simeq -\vec{D}^{(+)} \cdot \vec{\mathcal{E}}_L + \text{h.c.}$$

$$H_{\text{eff}} = \begin{pmatrix} -\hbar\delta - i\frac{1}{2}\hbar\Gamma & \frac{1}{2}\hbar\Omega \\ \frac{1}{2}\hbar\Omega^* & 0 \end{pmatrix}$$

$$\delta = \omega_L - \omega_A \qquad \hbar\Omega/2 = d\mathcal{E}_L$$

Perturbative regime:

$$|\Omega/2| \ll |\delta + i\Gamma/2| \qquad s = \frac{|\Omega|^2/2}{\delta^2 + \Gamma^2/4} \ll 1$$

$$E_g \simeq 0 + \frac{|\hbar\Omega/2|^2}{\hbar(\delta + i\Gamma/2)} = \hbar\delta s/2 - i\hbar\Gamma s/4$$

→ lighshift, excitation rate

Optical trapping for $|\delta| \gg \Gamma$:

$$\Omega(\vec{r}) \longrightarrow E_g(\vec{r}) \longrightarrow \textbf{trapping}$$

- **in intensity maximum for** $\delta < 0$
- **in intensity minimum for** $\delta > 0$

Radiation pressure, beam slowing:

$$\vec{F} = \hbar\vec{k} \times \Gamma s/2$$

saturates at $\hbar\vec{k}\Gamma/2$ (**which corresponds to an acceleration as high as** 10^5 $\mathbf{m/s^2}$)

Optical Bloch equations:

$$\frac{d\sigma}{dt} = \frac{1}{i\hbar}[H_{\text{eff}}\sigma - \sigma H_{\text{eff}}^{\dagger}] + \Gamma \sum_{q=0,\pm1} \left(d^{-1}\vec{D}^{(+)} \cdot \vec{e}_q\right)^{\dagger} \sigma \left(d^{-1}\vec{D}^{(+)} \cdot \vec{e}_q\right)$$

THE MEAN FORCE

The result:

$$\vec{F} = -\langle \partial_{\vec{r}} V_{AL} \rangle (\vec{r}\,(t))$$

$$F_j = \langle \vec{D}^{(+)} \rangle (t) \cdot \partial_{r_j} \vec{\mathcal{E}}_L(\vec{r}\,(t)) + \text{c.c} \qquad j = x, y, z$$

Derivation in Heisenberg picture:

$$V_{\text{tot}} = V_{AL} + V_{AF} \qquad\qquad V_{AF} = -\vec{D} \cdot \vec{E}$$

$$\vec{E}^{(+)}(\vec{r}\,) = \sum_{\vec{k},\vec{\epsilon}} \mathcal{E}_k \, \vec{\epsilon} \, e^{i\vec{k}\cdot\vec{r}} \hat{a}_{\vec{k},\vec{\epsilon}}$$

$$\frac{d}{dt}\vec{p} = -\partial_{\vec{r}} H = -\partial_{\vec{r}} V_{AL} - \partial_{\vec{r}} V_{AF}$$

$$-\partial_{\vec{r}} V_{AF} = -\sum_{\vec{k},\vec{\epsilon}} \mathcal{E}_k \left(\vec{D}^{(+)} \cdot \vec{\epsilon} \right) i\vec{k} \, e^{i\vec{k}\cdot\vec{r}} \hat{a}_{\vec{k},\vec{\epsilon}}$$

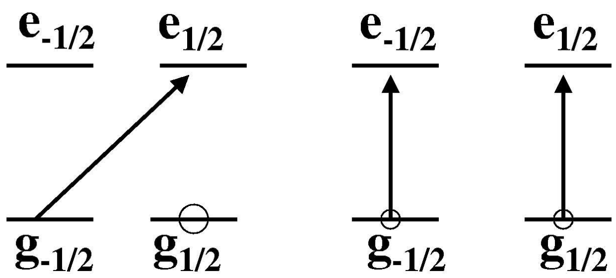
$$e^{i\vec{k}\cdot\vec{r}} \hat{a}_{\vec{k},\vec{\epsilon}}(t) = e^{-ickt} e^{i\vec{k}\cdot\vec{r}} \hat{a}_{\vec{k},\vec{\epsilon}}(0) - \frac{1}{i\hbar} \int_0^t d\tau \, e^{-ick(t-\tau)} \vec{D}^{(-)}(\tau) \cdot \vec{\epsilon}^{\,*} \, \mathcal{E}_k$$

i.e. field = free vacuum field plus emitted field

More details for isotropic atom at rest:

$$\langle \vec{D}^{(-)} \rangle = \epsilon_0 \alpha(\delta, s) \vec{\mathcal{E}}_L$$

true for $0 \to 1$, **not true in general:**



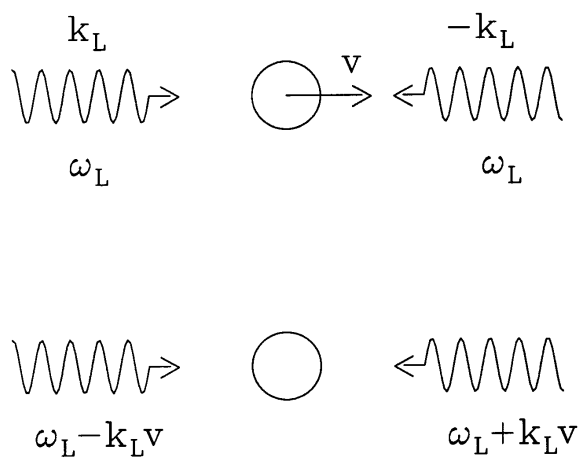
$$\alpha = -\frac{d^2}{\hbar \epsilon_0} \frac{1}{\delta + i\Gamma/2} \frac{1}{1+s} = \alpha_R + i\alpha_I$$

$F_j^R = \epsilon_0 \alpha_R \partial_{r_j} \left(\vec{\mathcal{E}}_L^* \cdot \vec{\mathcal{E}}_L \right)$ **sensitive to field intensity gradient**

$F_j^I = -i\epsilon_0 \alpha_I \left(\vec{\mathcal{E}}_L^* \cdot \partial_{r_j} \vec{\mathcal{E}}_L - \text{c.c.} \right)$ **sensitive to phase gradient and differs from Poynting vector by a vector of vanishing divergence**

$$\vec{\mathcal{E}}_L(\vec{r}) = \vec{\mathcal{E}}_0 e^{i\vec{k}\cdot\vec{r}} \qquad \to \qquad \vec{F} = \frac{1}{2}\hbar\vec{k}\Gamma\frac{s}{1+s}$$

DOPPLER COOLING



Add the two forces at low s:

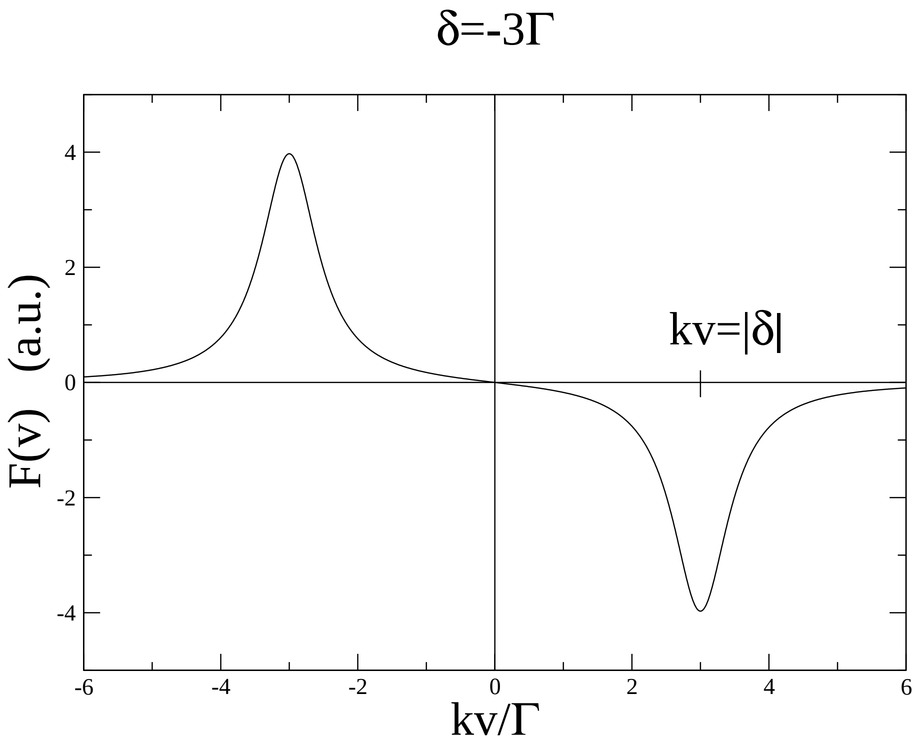
$$F = \frac{1}{2}\hbar k\Gamma\left[\frac{\Omega^2/2}{(\delta - kv)^2 + \Gamma^2/4} - \frac{\Omega^2/2}{(\delta + kv)^2 + \Gamma^2/4}\right]$$

At low velocity $kv \ll |\delta|, \Gamma$:

$$F \simeq -\alpha v$$

with friction coefficient

$$\alpha = -2\hbar k^2 s \frac{\Gamma\delta}{\delta^2 + \Gamma^2/4}$$



Equilibrium assuming $k\Delta v \ll \Gamma, |\delta|$:

$$\frac{d}{dt}\langle p \rangle = -\frac{\alpha}{m}\langle p \rangle \qquad \frac{d}{dt}\langle p^2 \rangle = -\frac{2\alpha}{m}\langle p^2 \rangle !?$$

wrong ! Heating due to fluctuations of force:

$$\vec{p} \rightarrow \vec{p} \pm \hbar\vec{k} - \hbar\vec{k}_s \qquad \langle \vec{k}_s \rangle = \vec{0}$$

Simple estimate of heating rate:

$$\frac{d}{dt}\langle p^2\rangle_{\text{heating}} \propto (\hbar k)^2 \Gamma s$$

so that in the absence of cooling

$$\langle p^2\rangle \simeq 2Dt$$

where momentum diffusion coefficient

$$D \propto (\hbar k)^2 \Gamma s$$

$$\frac{d}{dt}\langle p^2\rangle = -\frac{2\alpha}{m}\langle p^2\rangle + 2D$$

$$\frac{\langle p^2\rangle}{m} = \frac{D}{\alpha} = k_B T$$

Einstein's relation for brownian motion!

$$k_B T \propto \hbar \frac{\delta^2 + \Gamma^2/4}{-\delta}$$

$$k_B T_{\text{min}} \propto \hbar\Gamma \qquad \delta_{\text{opt}} = -\Gamma/2$$

Results of more precise calculations:

$$k_B T_{\min}^{1D} = \frac{7}{20}\hbar\Gamma \qquad k_B T_{\min}^{3D} = \frac{1}{2}\hbar\Gamma$$

Typical values:

$$\Gamma^{-1} \sim 30\text{ns} \qquad T_{\min} \sim 100\mu K$$

Validity condition of $k\Delta v \ll \Gamma$,

$$\left(\frac{\hbar k^2}{m\Gamma}\right)^{1/2} \ll 1$$

usually satisfied

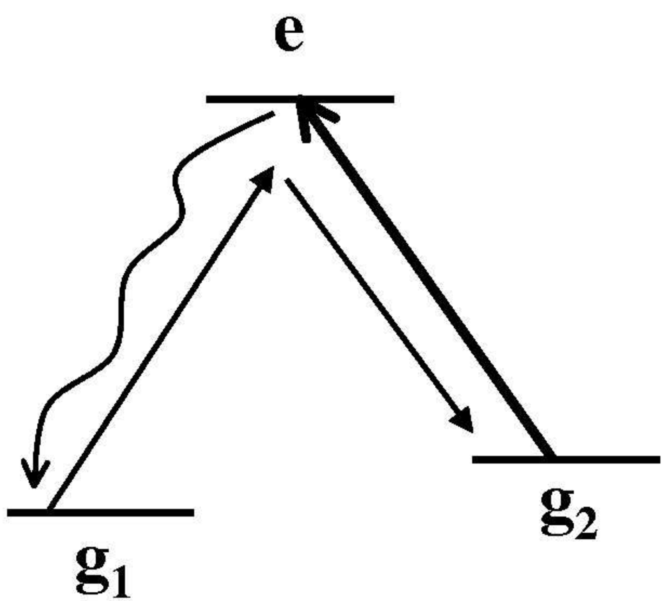
$$\frac{m\Gamma}{\hbar k^2} \sim 200(\text{Na}), 1200(\text{Cs}), 20(\text{He})$$

The opposite regime $\left(\frac{\hbar k^2}{m\Gamma}\right)^{1/2} \gg 1$ **leads to the cooling limit**

$$\langle p^2 \rangle^{1D}_{\min} = 0.53(\hbar k)^2$$

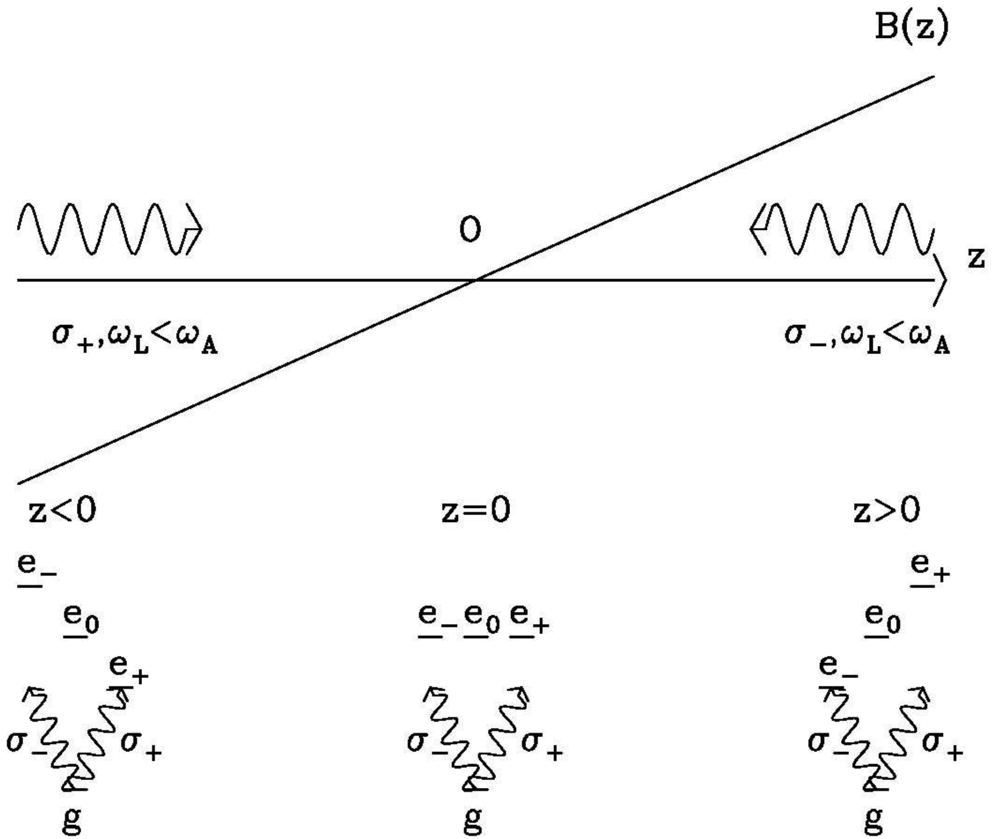
accessible on a narrow line or with an artificial two-level atom:

- **Raman coupling** $\leftrightarrow \Omega, \delta$
- **repumping** $\leftrightarrow \Gamma$



THE MAGNETO-OPTICAL TRAP

A magnetic field varying linearly along z:



Effective detunings: $\delta_{\pm} = \omega_L \mp kv - (\omega_A \pm \mu z) = \delta \mp (kv + \mu z)$

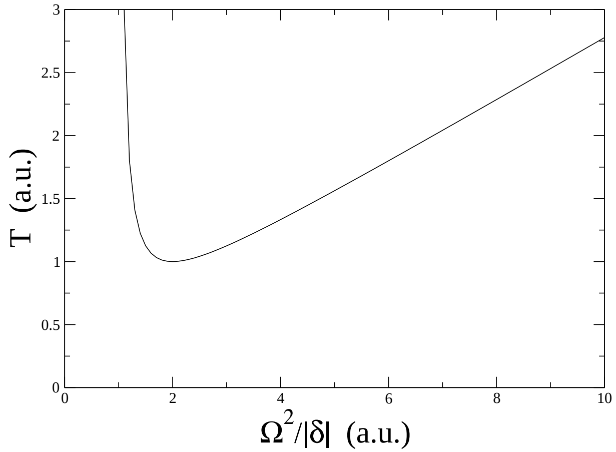
$$F_{\mathrm{MOT}}(z, v) = F_{\mathrm{mol}}(v + \mu z/k)$$

so that Doppler friction gives rise to restoring force $-\alpha\mu z/k$.

SISYPHUS COOLING

Comparison of Doppler theory with experiments is a complete failure:

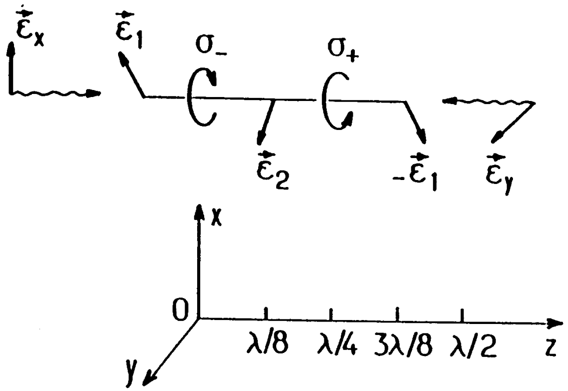
- T **depends on** Ω
  $\rightarrow$ **a threshold for laser intensity**
- T **universal function of** $\Omega^2/|\delta|$ **for** $|\delta| \gg \Gamma$
- **minimum temperatures are in the** μK **range, two orders of magnitude below the Doppler limit**
- **optimum cooling at** $|\delta| \gg \Gamma$

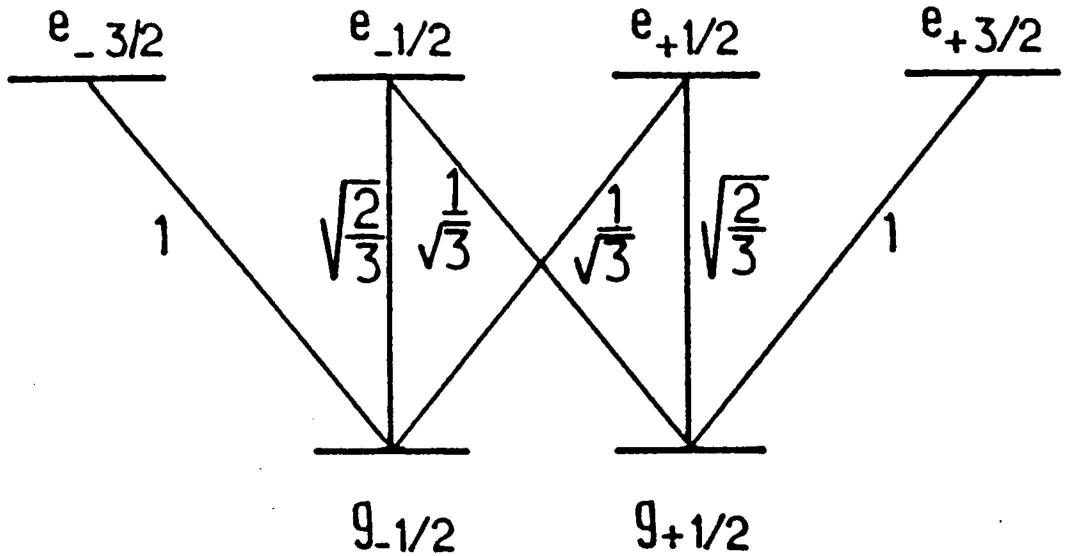


Two essential ingredients for sub-Doppler cooling:

- **several Zeeman sublevels in ground state**
- **spatially varying polarisation of laser field**

Simplest model:





$$\vec{\mathcal{E}}_L(z) = \mathcal{E}_+(z)\vec{e}_+ + \mathcal{E}_-(z)\vec{e}_-$$

$$\mathcal{E}_+(z) = -\sqrt{2}\mathcal{E}_0 i \sin kz \qquad \mathcal{E}_-(z) = \sqrt{2}\mathcal{E}_0 \cos kz$$

pure polarisation gradient, total intensity is uniform

Lightshifts:

$$U_+ = \mathrm{Re}\left(\frac{1}{-(\delta + i\Gamma/2)}\right)\left[|d\mathcal{E}_+|^2 + \frac{1}{3}|d\mathcal{E}_-|^2\right]$$

$$U_+ = -\frac{3}{2}U_0 + U_0 \cos^2 kz$$

$$U_+ = -\frac{3}{2}U_0 + U_0 \sin^2 kz$$

with $U_0 = -(2/3)\hbar\delta s_0$

Transition rates:

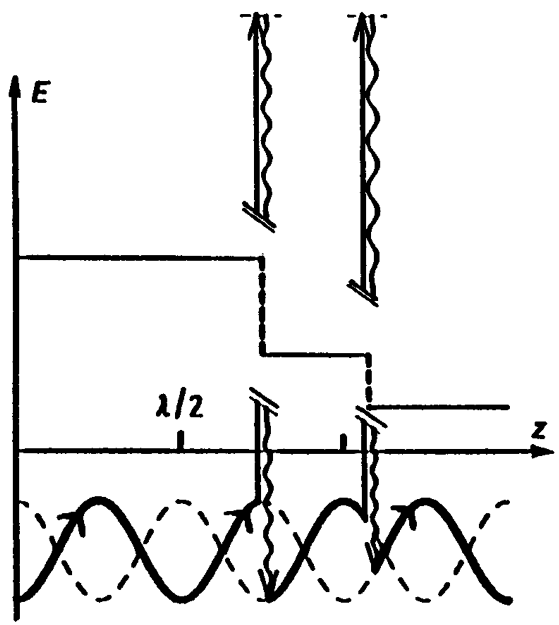
$$\gamma_{+\to-} = 2\mathrm{Im}\left(\frac{1}{-(\delta + i\Gamma/2)}\right)\frac{1}{3}|d\mathcal{E}_-|^2 \times \frac{2}{3}$$

$$\gamma_{+\to-} = \frac{2}{9}\Gamma s_0 \cos^2 kz$$

$$\gamma_{-\to+} = \frac{2}{9}\Gamma s_0 \sin^2 kz$$

Sisyphus mechanism:

If $\delta < 0$, $\gamma_{+\to-}$ **is maximum where** U_+ **is maximum**



Intuitive limit: $k_B T \sim U_0$

Existence of a threshold: For $E \gg U_0$ **but** $kv \ll \Gamma$,

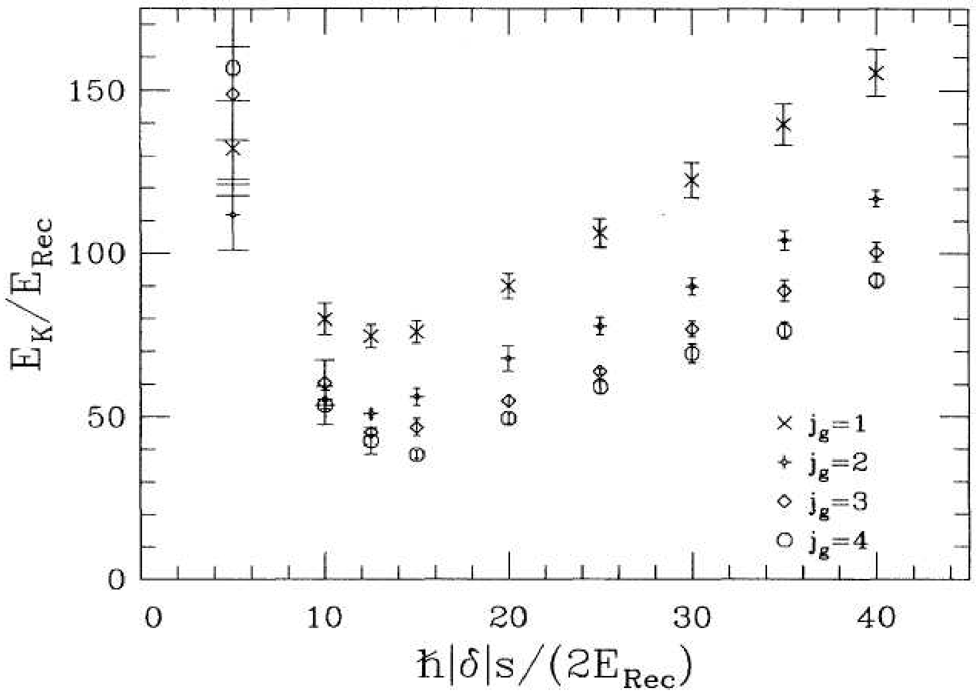
$$\frac{dE}{dt} = \frac{1}{9}\Gamma s_0 \left[-\frac{1}{2}U_0 + \frac{41}{5}\frac{\hbar^2 k^2}{2m} \right]$$

leads to a minimal temperature

$$k_B T_{\min} \propto \frac{\hbar^2 k^2}{m}$$

Comparison with experiments: the Monte Carlo wavefunction method:

- **deterministic evolution for σ replaced with random walk for $|\psi_A(t)\rangle$**
- **$|\psi_A(t)\rangle$ experiences quantum jumps (projection onto ground state) with rate $\Gamma \sum_m ||\langle e_m|\psi_A\rangle||^2$**
- **inbetween quantum jumps, evolution with H_{eff} plus renormalisation**



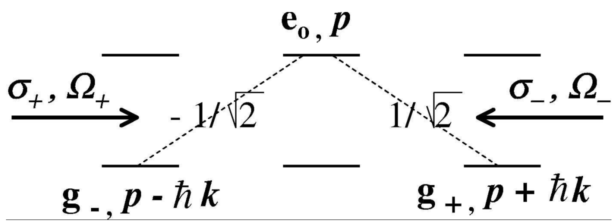
j_g	$\langle\vec{p}^2\rangle_{\text{MC}}^{\text{min}}[\frac{\hbar^2k^2}{m}]$	$3T_{\text{expt}}^{\text{min}}[\frac{\hbar^2k^2}{m}]$	slope$_{\text{MC}}$	slope$_{\text{expt}}$
1	74.7 ± 3.5	...	3.3 ± 0.5	...
2	51.0 ± 1.4	50 ± 5	2.6 ± 0.2	2.3 ± 0.2
3	45.1 ± 1.1	50 ± 5	2.5 ± 0.2	2.1 ± 0.1
4	38.3 ± 1.2	40 ± 10	2.1 ± 0.2	2.1 ± 0.5

BEYOND THE RECOIL LIMIT

Intuitive idea:

- **quasi-dark states with very low fluorescence rate**
- **these states should be populated in a narrow velocity class only**

Velocity Selective Coherent Population Trapping:



Closed families $\mathcal{F}(p)$ for $H_{\rm eff}$ coupled only by spontaneous emission
In each family, one state not coupled to laser:

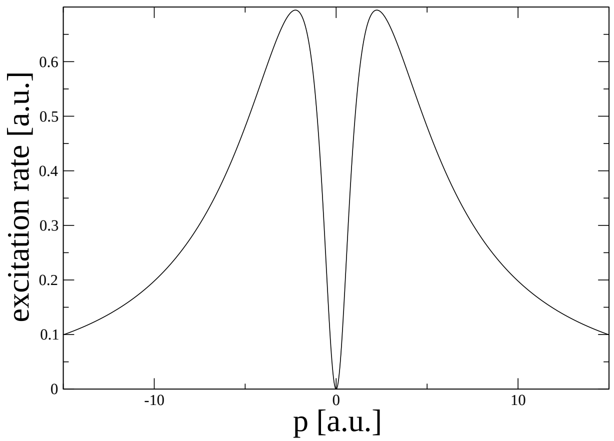
$$|NC(p)\rangle \propto \Omega_-|g_-, p-\hbar k\rangle + \Omega_+|g_+, p+\hbar k\rangle$$

When is the NC state a stationary state ?

$$(p-\hbar k)^2 = (p+\hbar k)^2 \longrightarrow p = 0$$

so that $|NC(0)\rangle$ **has infinite lifetime**.

Excitation rate of the non-coupled state:



No steady state, no intrinsic limit:

$$k_B T \propto \frac{1}{\text{interaction time}}$$

In 1D $k_B T \sim \frac{1}{800}\frac{\hbar^2 k^2}{m}$ **has been achieved**. **In 3D** $k_B T \sim \frac{1}{22}\frac{\hbar^2 k^2}{m}$.

Adiabatic transformation into a single peak:

- **slowly switch off all the laser plane waves but one**.