# Les états de bord d'un

# isolant de Hall atomique (2)

séminaire Atomes Froids

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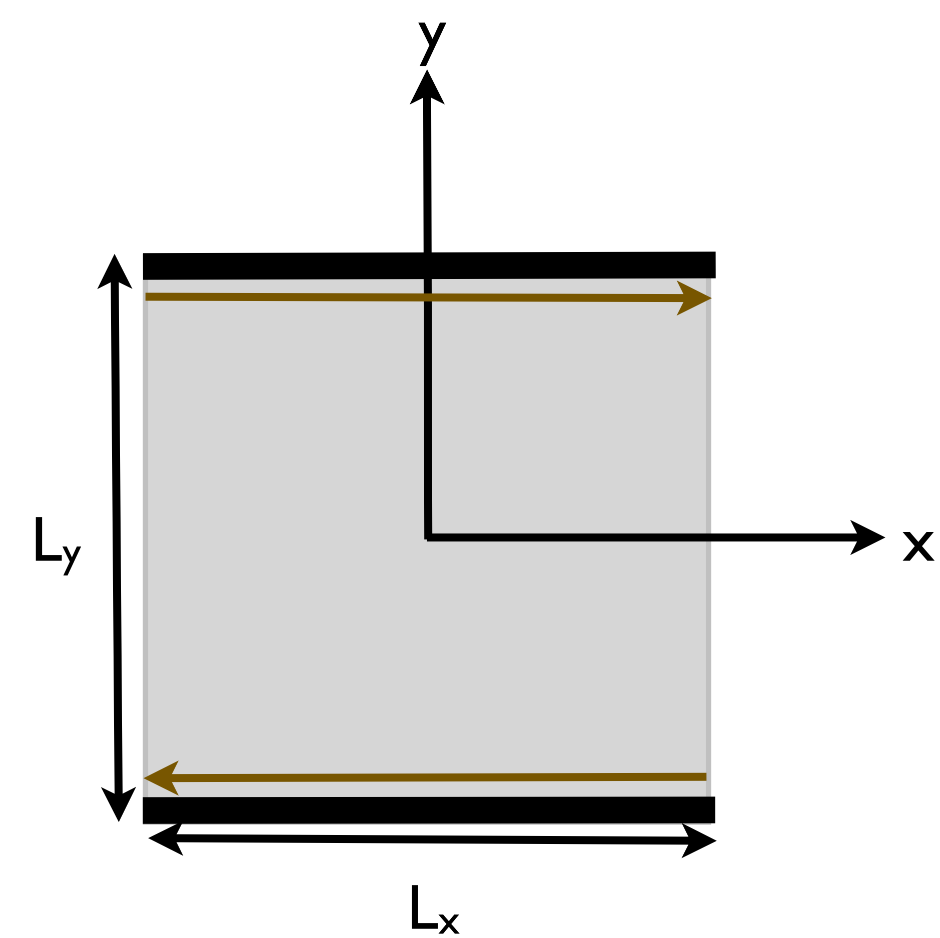
# Outline

- Quantum Hall effect : bulk Landau levels and edge states

- Quantum Hall effect on a lattice : Hofstadter model
  - Refresher on tight-binding lattice
  - Hofstadter model
  - Hall conductivity as topological invariant
  - Bulk-edge correspondence
- Realization with cold atoms and detection of edge states
  - angular momentum spectroscopy of edge states
  - experimental scheme to realize the Hofstadter model
  - shelving technique to detect edge states

# Properties of edge states in quantum Hall systems

- Hall current carried by chiral edge states: group velocity $v_g$ non zero
- Edge states and bulk Hall conductivity are connected.
- states on opposite edges at the same energy carry opposite currents in equilibrium
  - same chirality $v_g/k$
  - *explains robustness of edge currents : no state available for backscattering in presence, e.g., of impurities*




red: current flow

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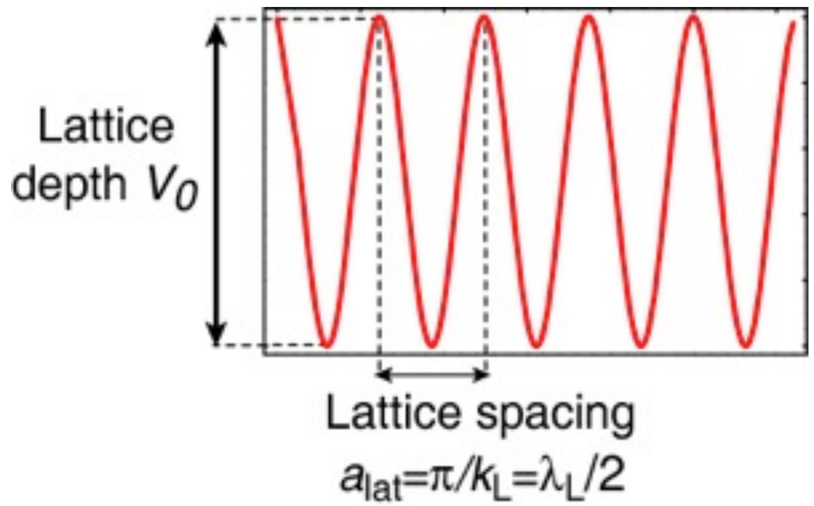
# Reminder : quantum particle in periodic potential

invariance by translation of a basis vector : Bloch theorem

- allowed and forbidden energy bands

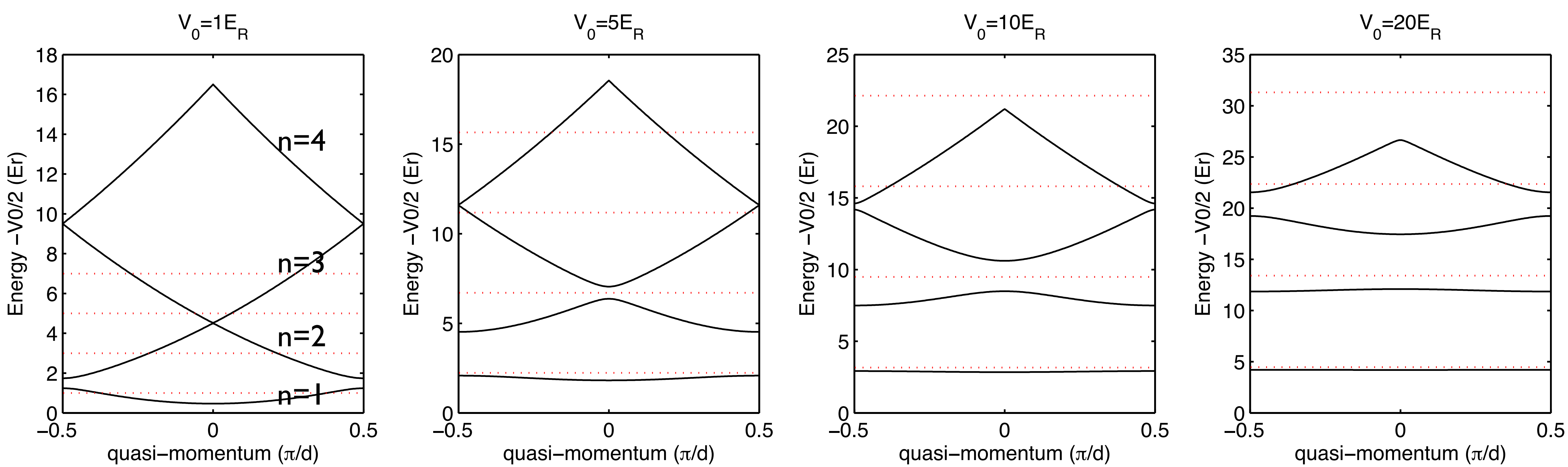
- energy eigenstates are Bloch states $u_{n,\mathbf{k}}(\mathbf{r})$ labeled by a band index n and a crystal momentum k restricted to the first Brillouin zone

Ex: 1D lattice




$$V(x) = V_0 \sin^2(k_L x)$$

Solid : energy bands
Dots: harmonic oscillator approximation



# Wannier functions

Instead of working in the Bloch basis $u_{n,\mathbf{k}}(\mathbf{r})$, it is often convenient to use the so-called Wannier functions defined as

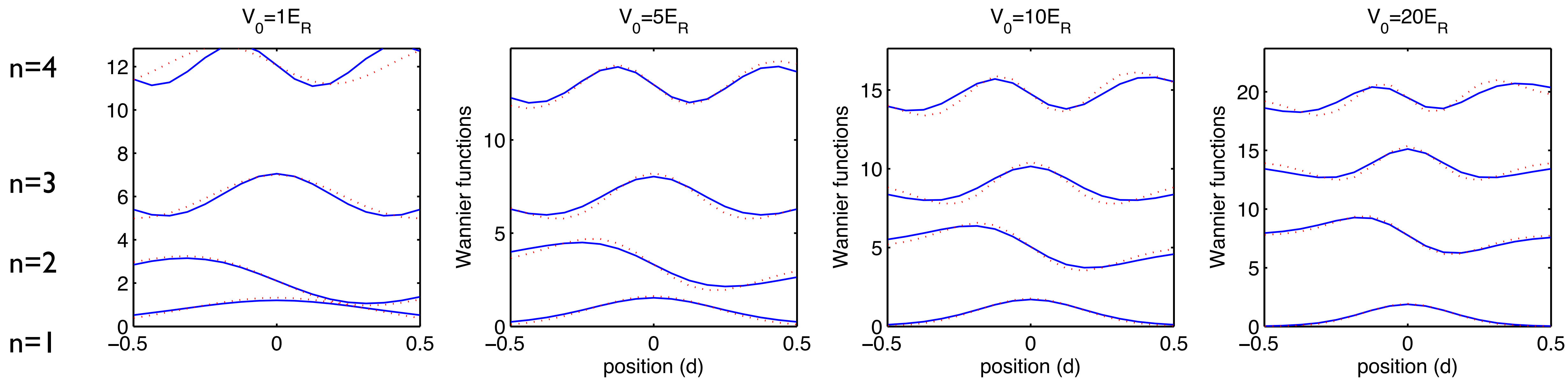
$$w_n(\mathbf{r}-\mathbf{r}_i) = \frac{1}{\sqrt{N_s}} \sum_{\mathbf{k}} u_{n,\mathbf{k}}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}_i}$$

(some subtleties for higher band, see Luttinger Phys. Rev. 1962)

Wannier functions form an orthogonal basis

For large lattice depths, they become more and more localized around site $r_i$

Well-suited to described the tight-binding limit




Solid: Wannier functions

Dots: harmonic oscillator approximation

# Tight-binding limit

In the Wannier basis, we can express the hamiltonian as $$H = -\sum_{n,\mathbf{r}_i,\mathbf{r}_j} J_{ij}^{(n)} c^\dagger_{n,\mathbf{r}_i} c_{n,\mathbf{r}_j}$$

Tight-binding approximation :

When the lattice depth is large (roughly V0>10 ER) Jij(n) decreases very fast with site distance : one can neglect all terms but the ones connecting nearest neighbors.

Single-band approximation :

Moreover, at low temperatures/chemical potential, only the lowest energy band n=0 is occupied appreciably : higher energy bands can be neglected.

This leads to the simplest non-interacting lattice model describing particles in the ground band tunneling from sites to sites:

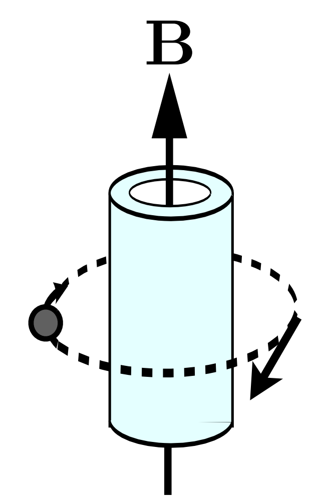
$$H = -J \sum_{\langle \mathbf{r}_i,\mathbf{r}_j \rangle} c^\dagger_i c_j$$

# Aharonov-Bohm effect

- Key concept: Aharonov-Bohm phase

  Phase accumulated by a charged particle revolving around a magnetic flux tube

$$\Psi(\theta + 2\pi) = e^{i\frac{e}{\hbar}\oint \mathbf{A}\cdot\mathbf{dl}}\Psi(\theta) = e^{i\frac{e}{\hbar}\int\int \mathbf{B}\cdot\mathbf{dS}}\Psi(\theta)$$




Simulating a magnetic field is equivalent to changing the phase of the wavefunction

Condition: finite "flux" on a closed surface

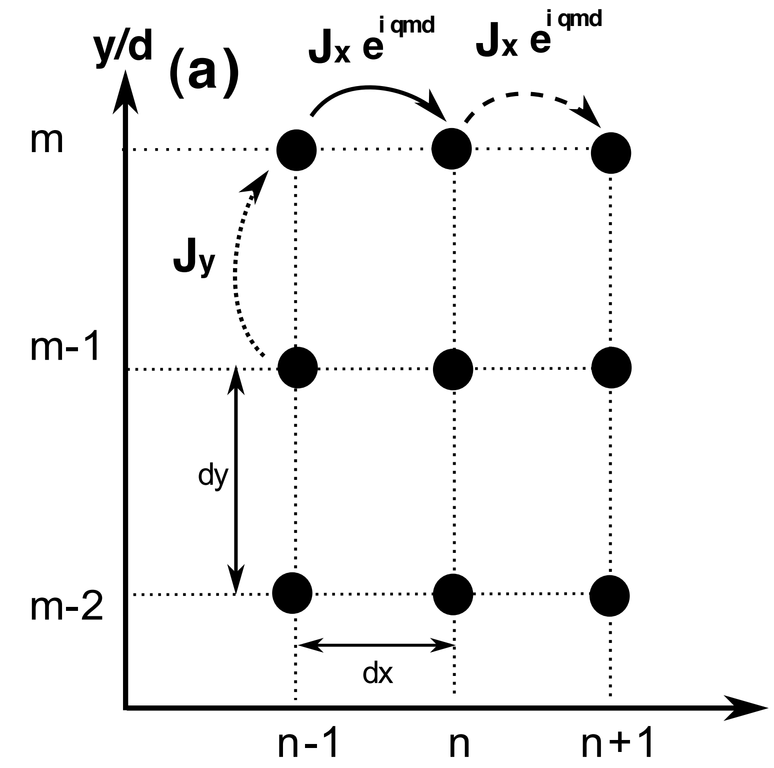
= non-zero phase around any closed contour: equivalent to Berry's geometric phase

# Harper hamiltonian : lattice and magnetic field

Harper hamiltonian on a tight-binding lattice :

$$H = -J \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} e^{i\phi_{\mathbf{r},\mathbf{r}'}} \hat{c}^\dagger_{\mathbf{r}'} \hat{c}_{\mathbf{r}} + \text{h.c.}$$

Harper, 1956; Azbel 1964; Hofstadter, 1976; Thouless et al., 1983; Kohmoto; Osadchy-Avron 2001...




- Aharonov-Bohm phase: $\phi_{\mathbf{r},\mathbf{r}'} = \frac{-e}{\hbar} \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A} \cdot \mathbf{dl}$

- Landau gauge: $\mathbf{A} = \begin{pmatrix} -By \\ 0 \\ 0 \end{pmatrix}$

$$\int_x^{x+d} A_x dx = B d_x y$$

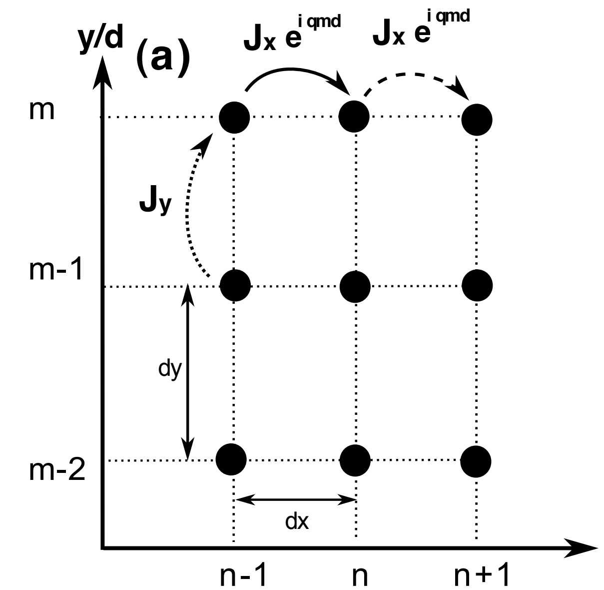
$$\phi_{\mathbf{r},\mathbf{r}+d\mathbf{e}_x} = 2\pi \frac{eBd_x y}{h} = 2\pi\alpha \frac{y}{d_y}$$

- Finite flux: $\int_{\square} \mathbf{A} \cdot \mathbf{dl} = B d_x d_y$

$$\sum_{\square} \phi_{\mathbf{r},\mathbf{r}'} = 2\pi\alpha$$

# Hofstadter's butterfly:
# interplay between lattice and vector potentials

$$H = -J\sum_{n,m} e^{i2\pi\alpha m}\hat{c}^\dagger_{n+1,m}\hat{c}_{n,m} + \text{h.c.}$$
$$-J\sum_{n,m} \hat{c}^\dagger_{n,m+1}\hat{c}_{n,m} + \text{h.c.}$$




- If α=0, we have a single Bloch band of width *8J*.
- Bloch theorem does not apply in general for arbitrary α.
- When α=p/q rational:
    - Translation by q sites reproduces the same model : we recover lattice translational invariance, but with an enlarged unit cell of size q dx*dy
    - Crystal momentum in the «magnetic Brillouin zone» ]-π/q,π/q] * ]-π,π]
    - Eigenstates along y («magnetic Bloch functions») have q components
    - the Bloch band splits in q subbands contained within the original one for α=0

# Hofstadter's butterfly: interplay between lattice and vector potentials

$$H = -J\sum_{n,m} e^{i2\pi\alpha m}\hat{c}^{\dagger}_{n+1,m}\hat{c}_{n,m} + \text{h.c.}$$

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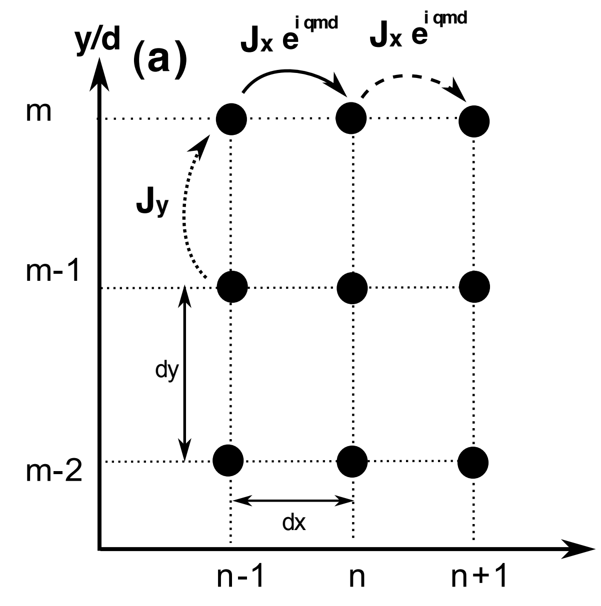
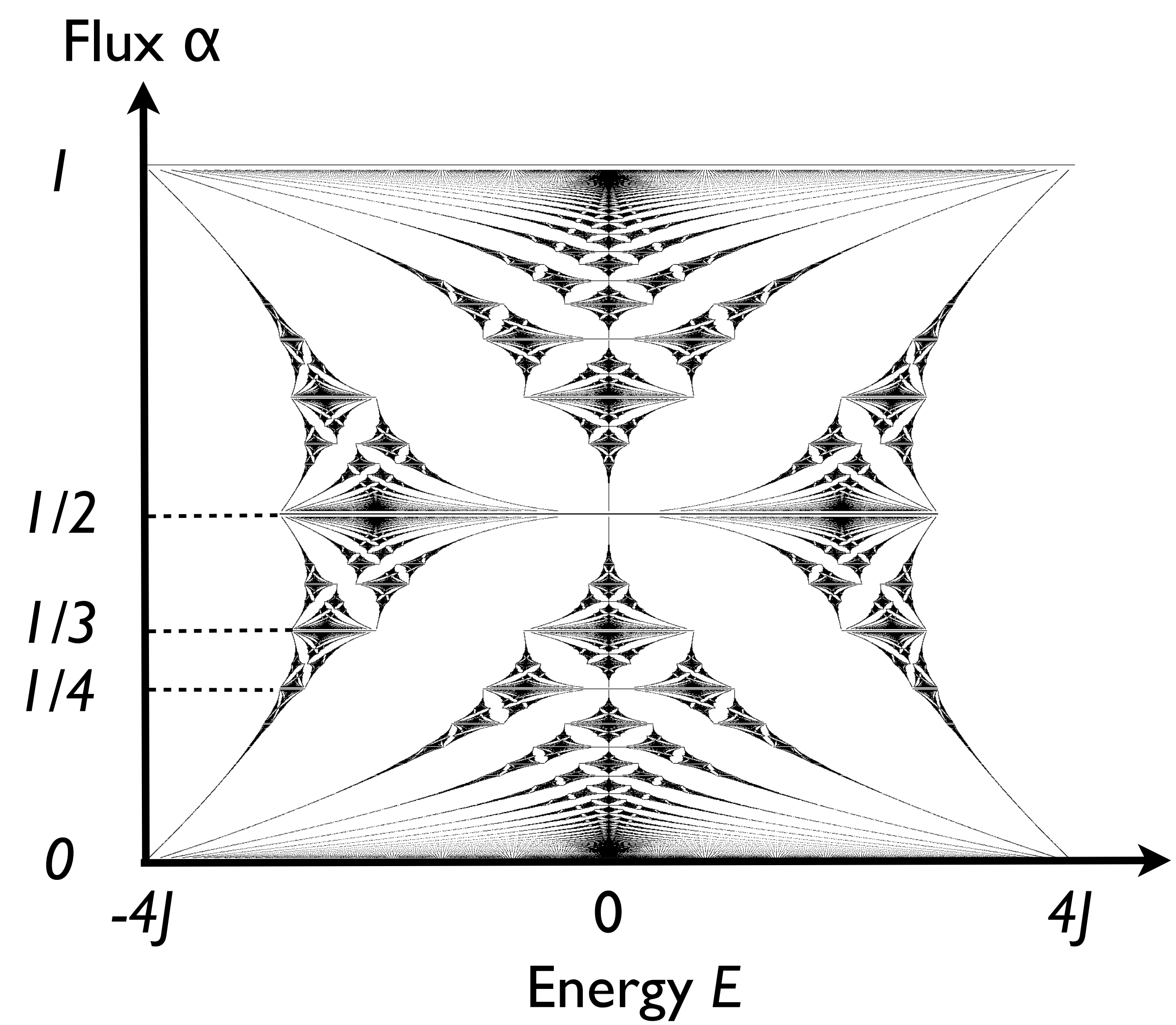


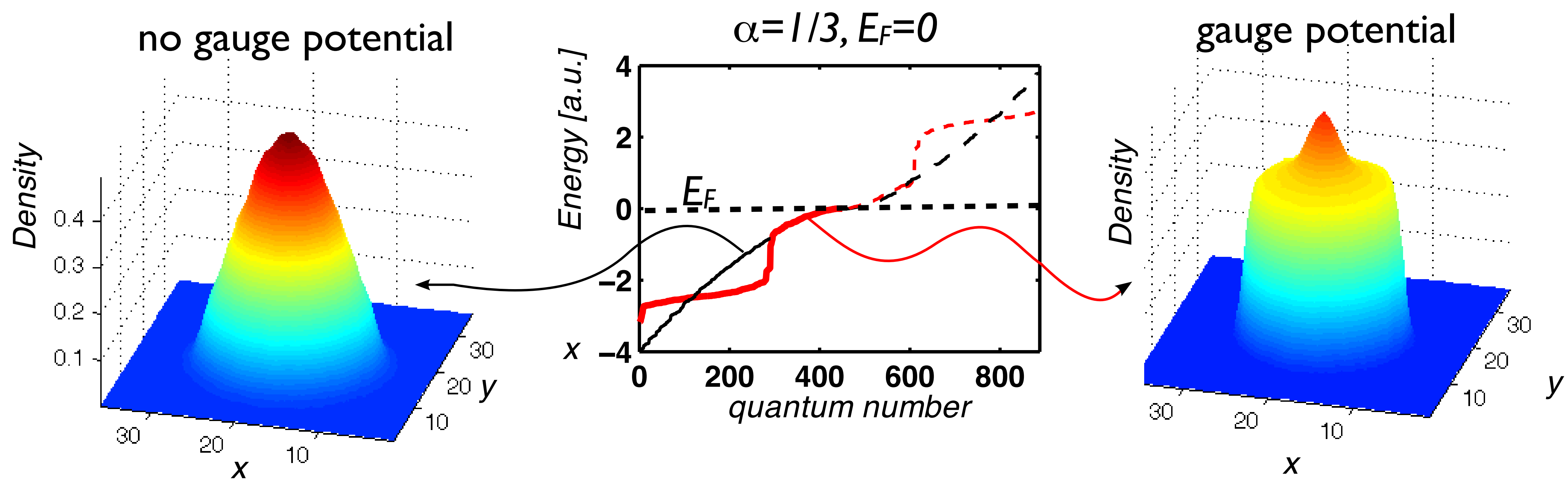

When α=*p*/*q* rational:

- *q* sub-bands, width <<8*J*
- Recursive structure, discovered by Azbel and Hofstadter
- Very different from Landau levels (which appear for α<<1)



# Hofstadter model with weak harmonic potential

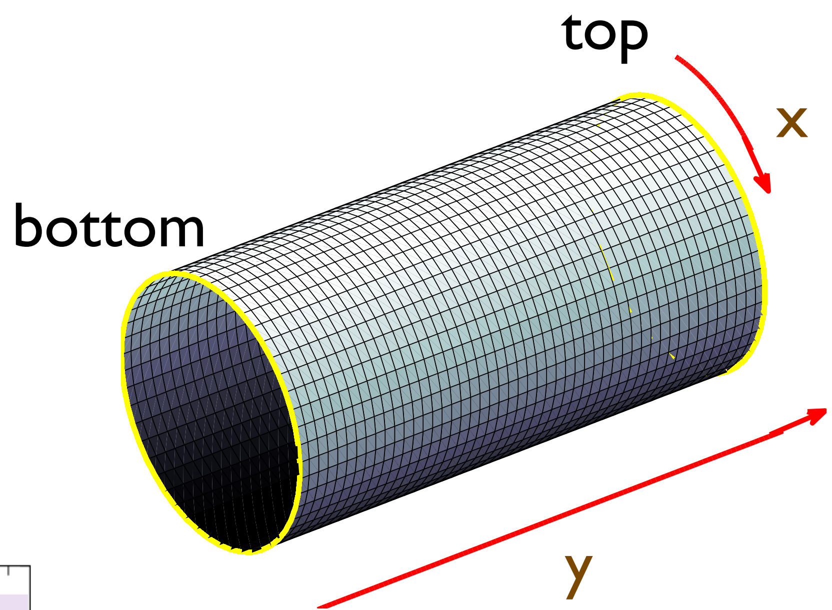
Non-interacting fermions: spatial distribution



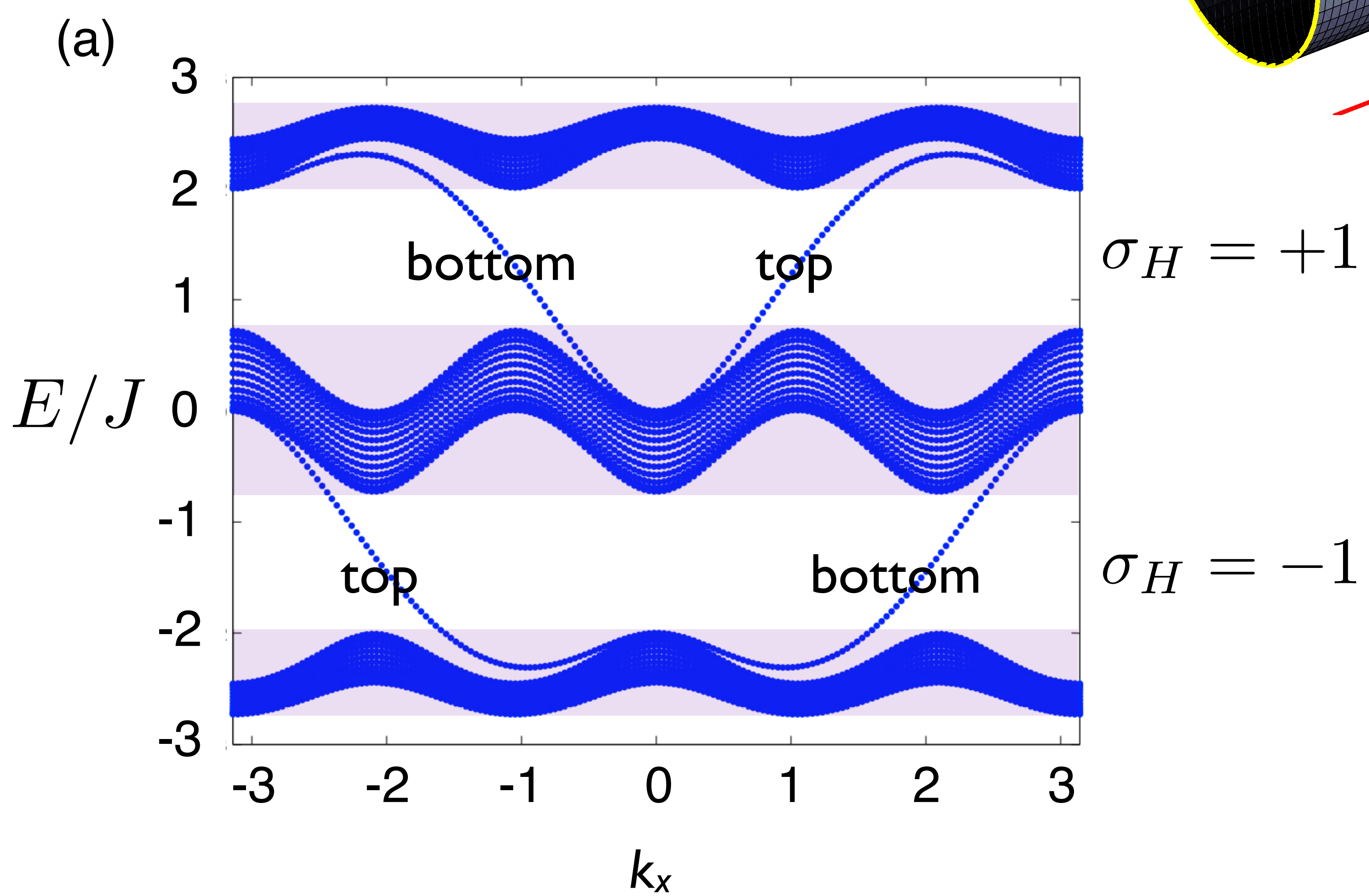

In the local density approximation, the density profile reflects the density of states of the uniform system.

Gap opens within the fondamental Bloch band : density plateau corresponding to a filled band

# Closer look at α=1/3




Energy spectrum
on a cylinder
($k_x$=0)




Similar structure as in the «bulk» :

chiral edge states are present in the bandgaps, with opposite currents flowing at the edges

# Hall conductivity as a topological invariant :

# TKNN formula

Key result from Thouless, Kohmoto, Nightingale, den Niijs (TKNN) :

$$\sigma_H = \sigma_{xy} = \frac{e^2}{h} \sum_n \nu_n$$

provided the Fermi energy is in a gap

$$\nu_n = \frac{i}{2\pi} \int_{BZ} \underline{\left( \langle \partial_{k_x} u_n(\mathbf{k}) | \partial_{k_y} u_n(\mathbf{k}) \rangle - (k_x \leftrightarrow k_y) \right)} \, d^2\mathbf{k}$$

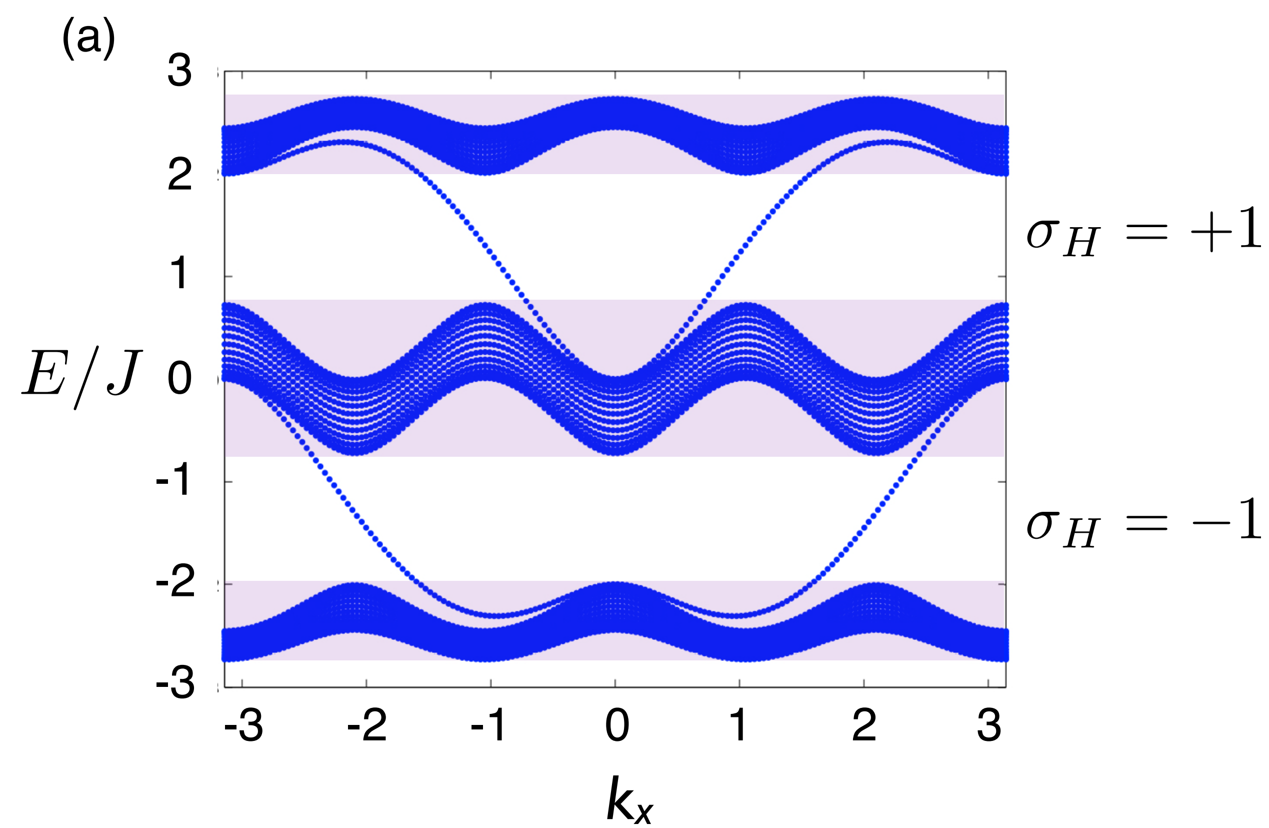
Berry curvature associated with the band eigenstates

- Chern number = topological index (integer-valued)
- characterizes the topology of the subspace associated with each energy band : non zero only if the Bloch eigenstates have a vortex structure in k space
- always defined, but related to Hall conductivity only if the Fermi energy lies in the gap between two bands

# Bulk-edge correspondence

There is a deep relation between the Chern number characterizing the bulk material and the edge states appearing at its edge :

- number of edge states branches (per physical edge) : $|\sigma_H|$
- chirality of edge states : $\mathrm{sign}(\sigma_H)$




Although the Chern number does not always exist in the present form (e.g., spin Hall topological insulators or interacting systems), other topological invariants can be defined in these situations.

When non-zero, chiral edge states always appear at the boundaries.

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# The Jaksch & Zoller scheme

Jaksch & Zoller, NJP (2003)

also: Mueller, PRA 2004; Sorensen *et al.*, PRL 2005

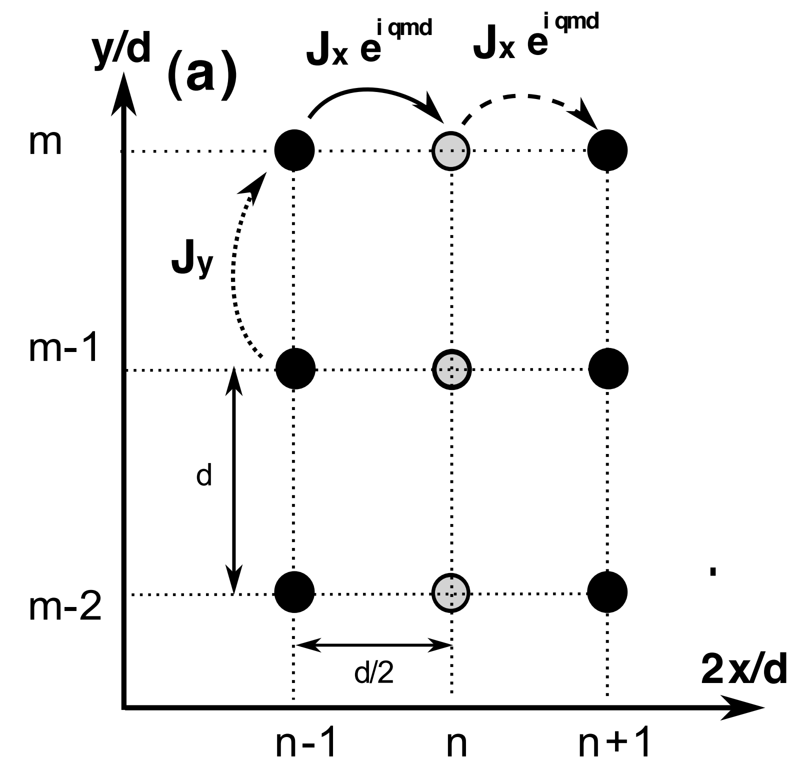
optical flux lattices : Cooper, PRL 2010

Mimic the Aharonov-Bohm effect using laser-induced tunneling in an optical lattice

Spin-dependent lattice :

- Atoms with two internal states *a* and *b*

- Spin-dependent 2D lattice:
  - Lattice potential along *y* state-independent
  - Lattice potential along *x* state dependent:
    - *a* trapped at potential minima
    - *b* trapped at potential maxima
    - no free tunneling

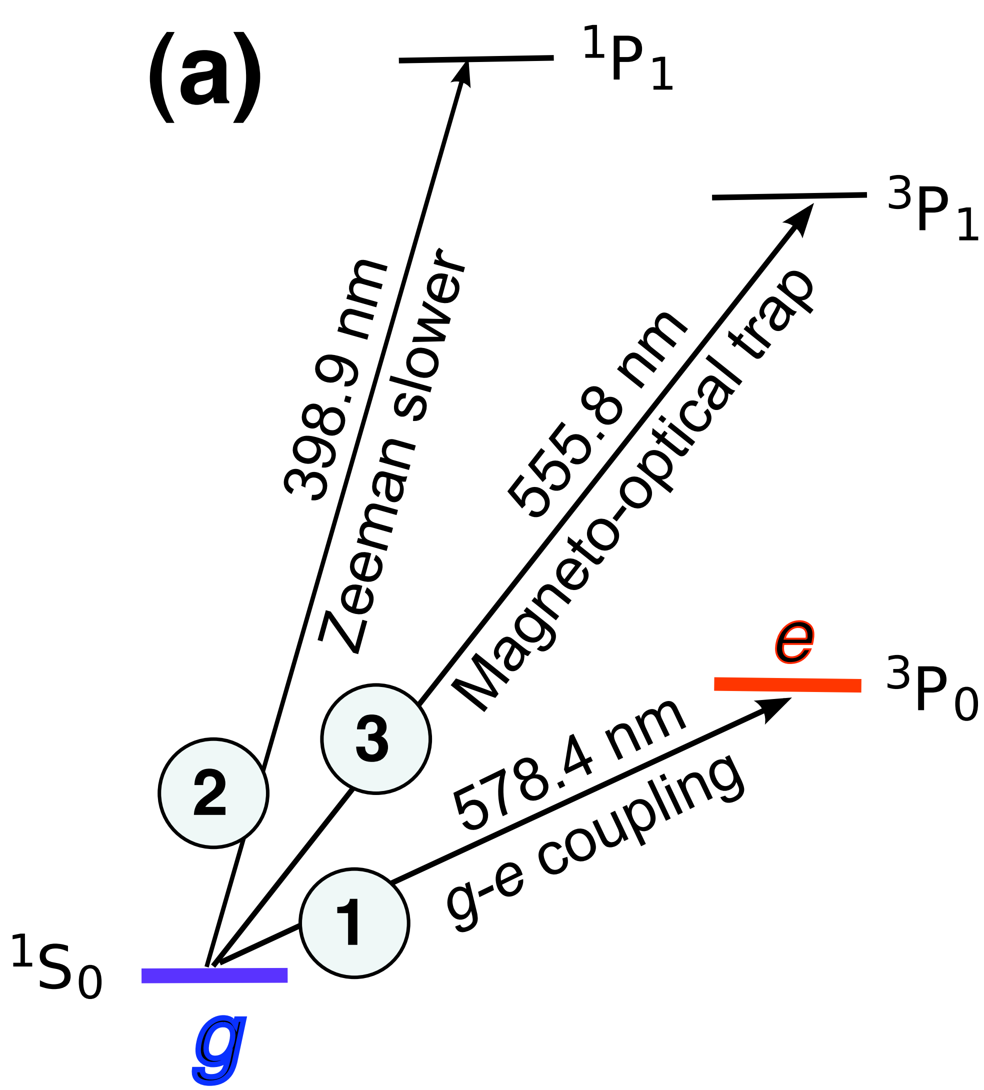



Laser-induced tunnel matrix element

$$\propto \hbar\Omega_L e^{iq_L y}$$

F. Gerbier & J. Dalibard, NJP (2010)

# Ytterbium




1 Internal state manipulation using ultra-narrow (1S0-3P0) transition

- "doubly forbidden" *lifetime >20 s !*
- weak coupling in presence of hyperfine or Zeeman interactions
- optical atomic clocks

2 broad (1S0-1P1) and narrow (1S0-3P1) transitions for laser cooling

3 weak sensitivity to magnetic fields (nuclear magneton)

Bosons (spin 0): 170 Yb, 172 Yb, 174 Yb, 176 Yb

Fermions: (spin 1/2) 171 Yb, (spin 5/2) 173 Yb

quantum degeneracy reached at Kyoto University for all isotopes

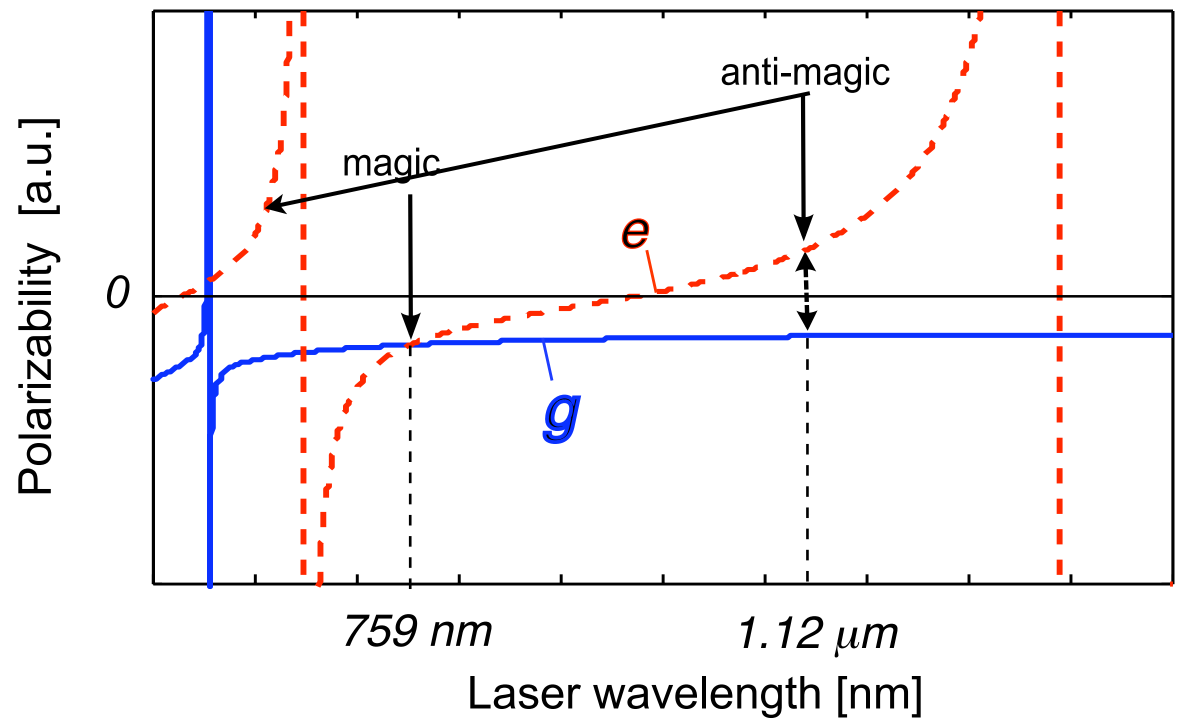
Y. Takahashi & coworkers

# State-dependent lattices

- Optical trap potential:

$$V_{\mathrm{dip}} = -\frac{1}{2}\alpha(\lambda_L)|\mathbf{E}|^2$$

dynamic polarisability

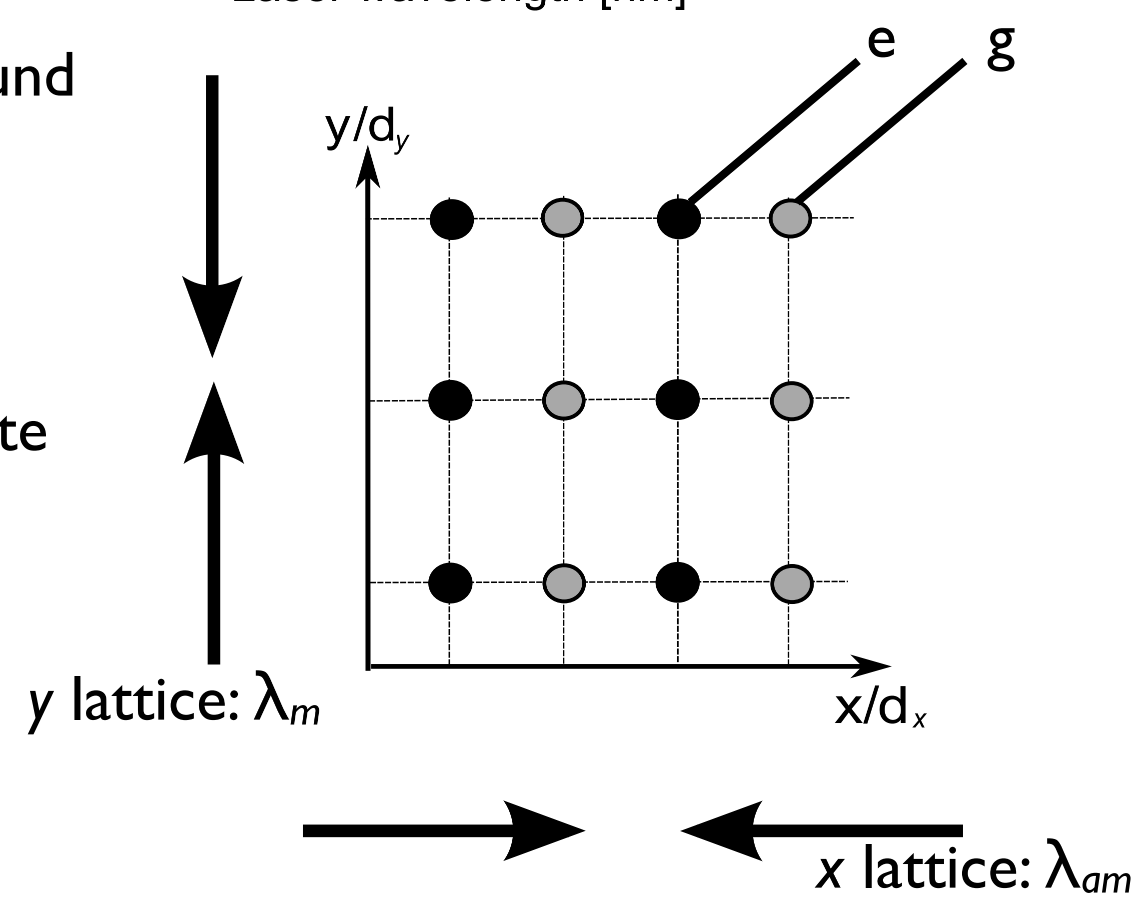



- At magic wavelength (~760 nm):

  optical potentials attractive, identical for ground and excited states (atomic clocks)

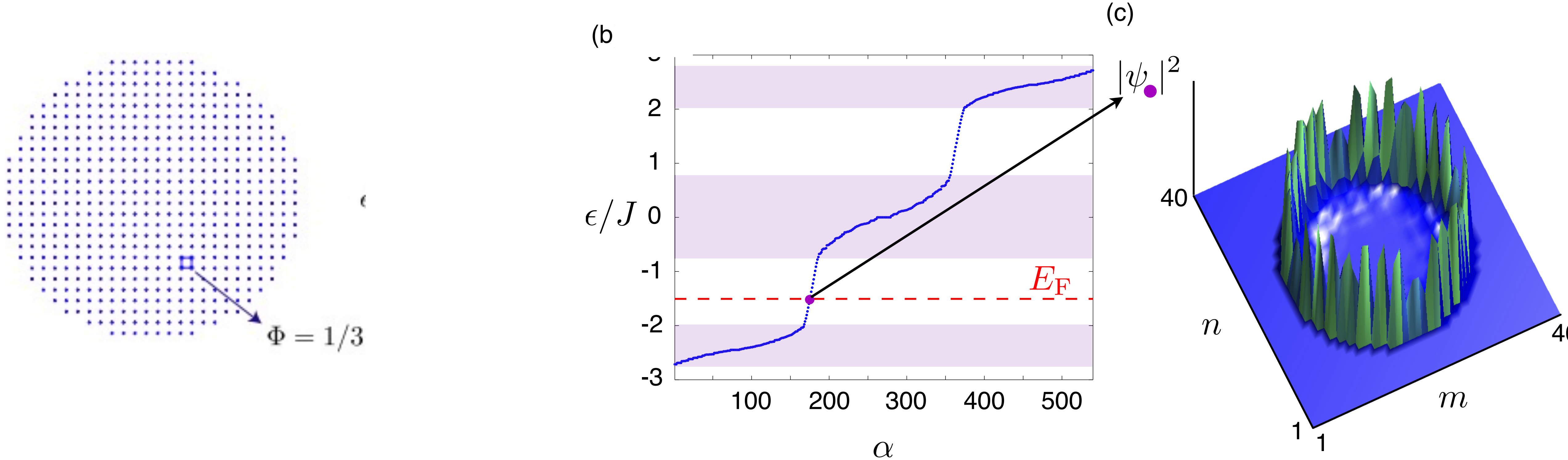
- At anti-magic wavelength (1.1 µm & 620 nm):

  optical potentials is attractive for ground state atoms, repulsive for excited states atoms



# Edge states with an external potential

Planar geometry with additional potential : edge states still present

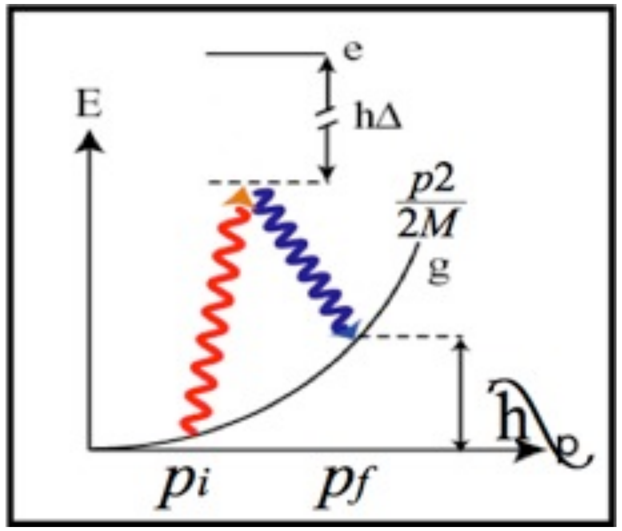


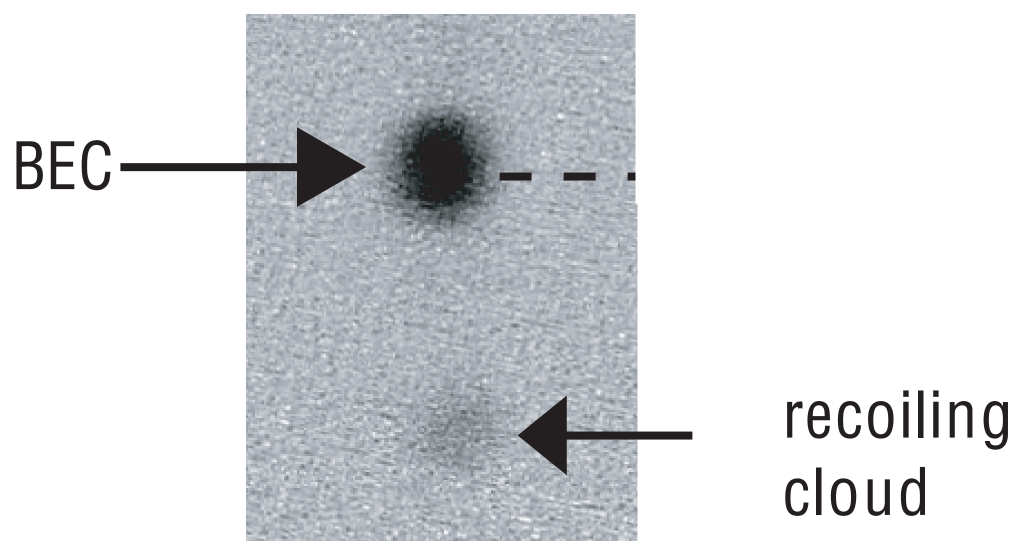

Theoretically, edge states are easy to isolate.

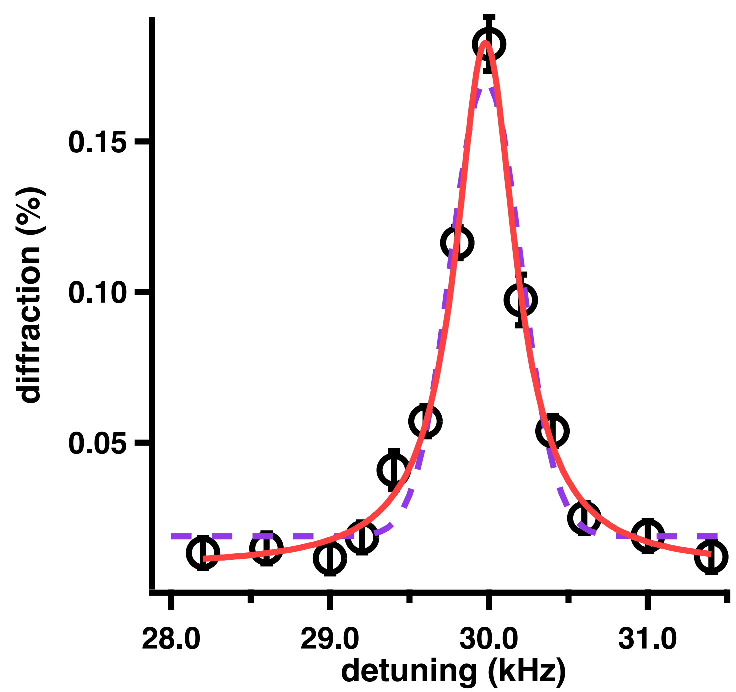
Their chirality reveal the topological nature of the bulk phase.

Is it possible to distinguish the edge states from the bulk ones experimentally, and to prove their chirality ?

# Bragg spectroscopy

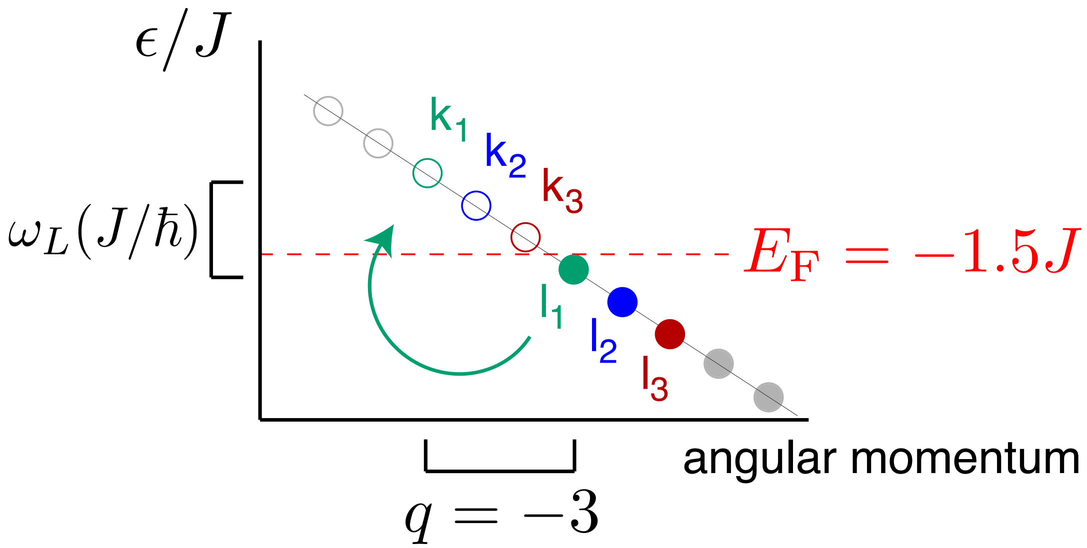

Resonance condition :

$$\hbar\delta = \frac{\hbar^2(k_i + 2k_L)^2}{2m} - \frac{\hbar^2 k_i^2}{2m} = 4\frac{\hbar^2 k_L^2}{2m} + 2k_L\frac{\hbar k_i}{m}$$

Recoil shift    Doppler effect

# «Angular momentum» spectroscopy

N. Goldman, J. Beugnon, F. Gerbier, arxiv 12031246

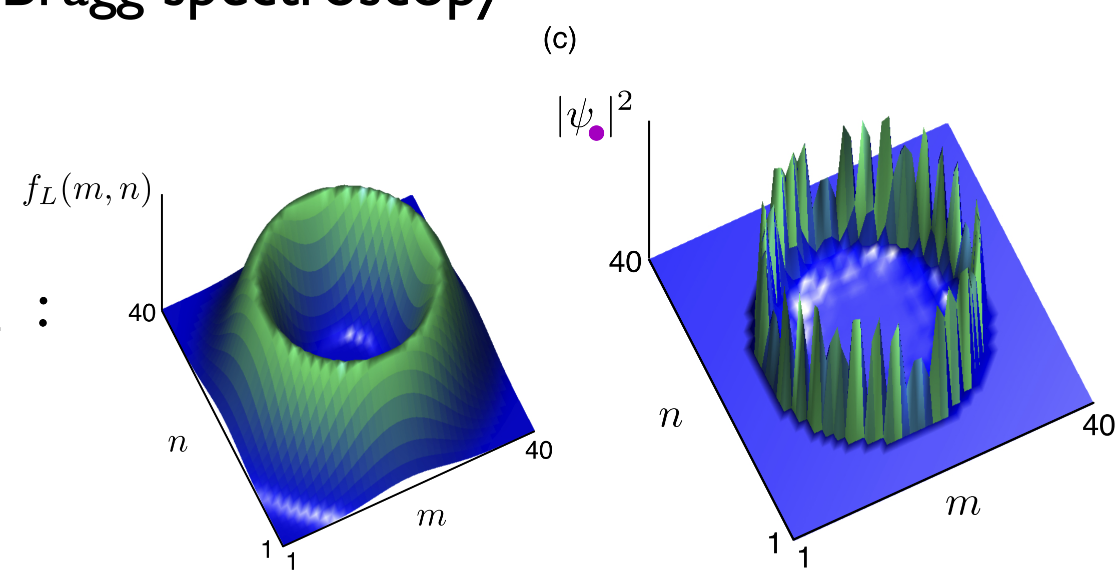



Probe field : $E(\mathbf{r},t) \sim f_L(r)e^{i(q\theta-\omega_L t)}$

Angular-momentum sensitive light scattering by interfering lasers carrying orbital angular momentum

Similar to Bragg spectroscopy

Probe spatial profile $f_L$ :

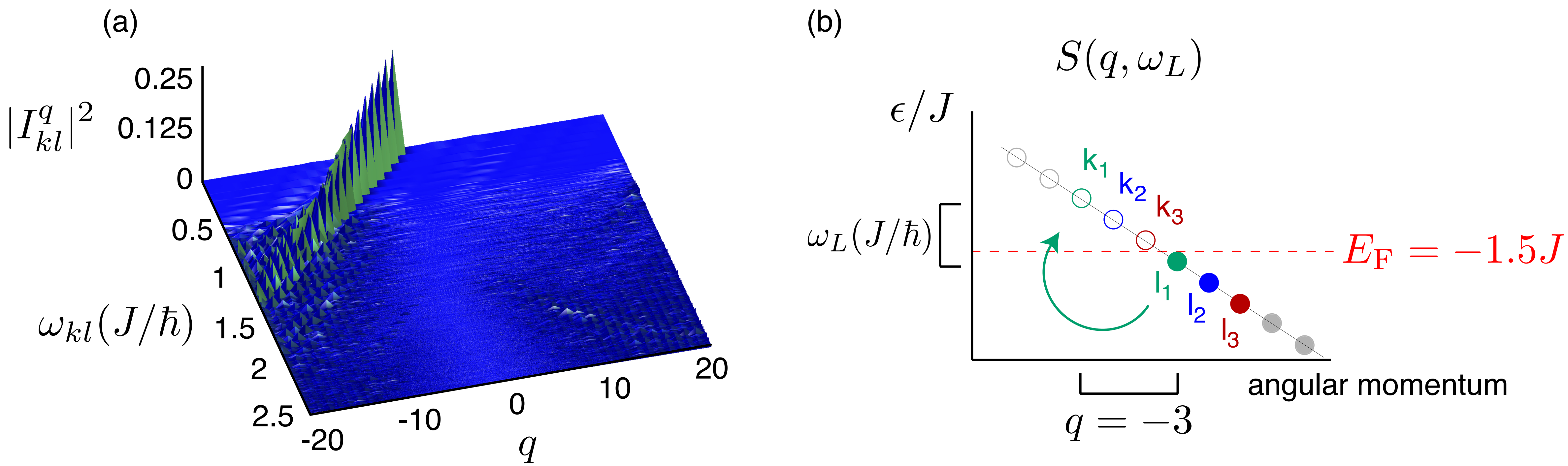



Match mode radius to edge state geometry to optimize coupling

# «Angular momentum» spectroscopy probes chirality

$N(q,\omega_L)$ : number of atoms transferred by the excitation pulse

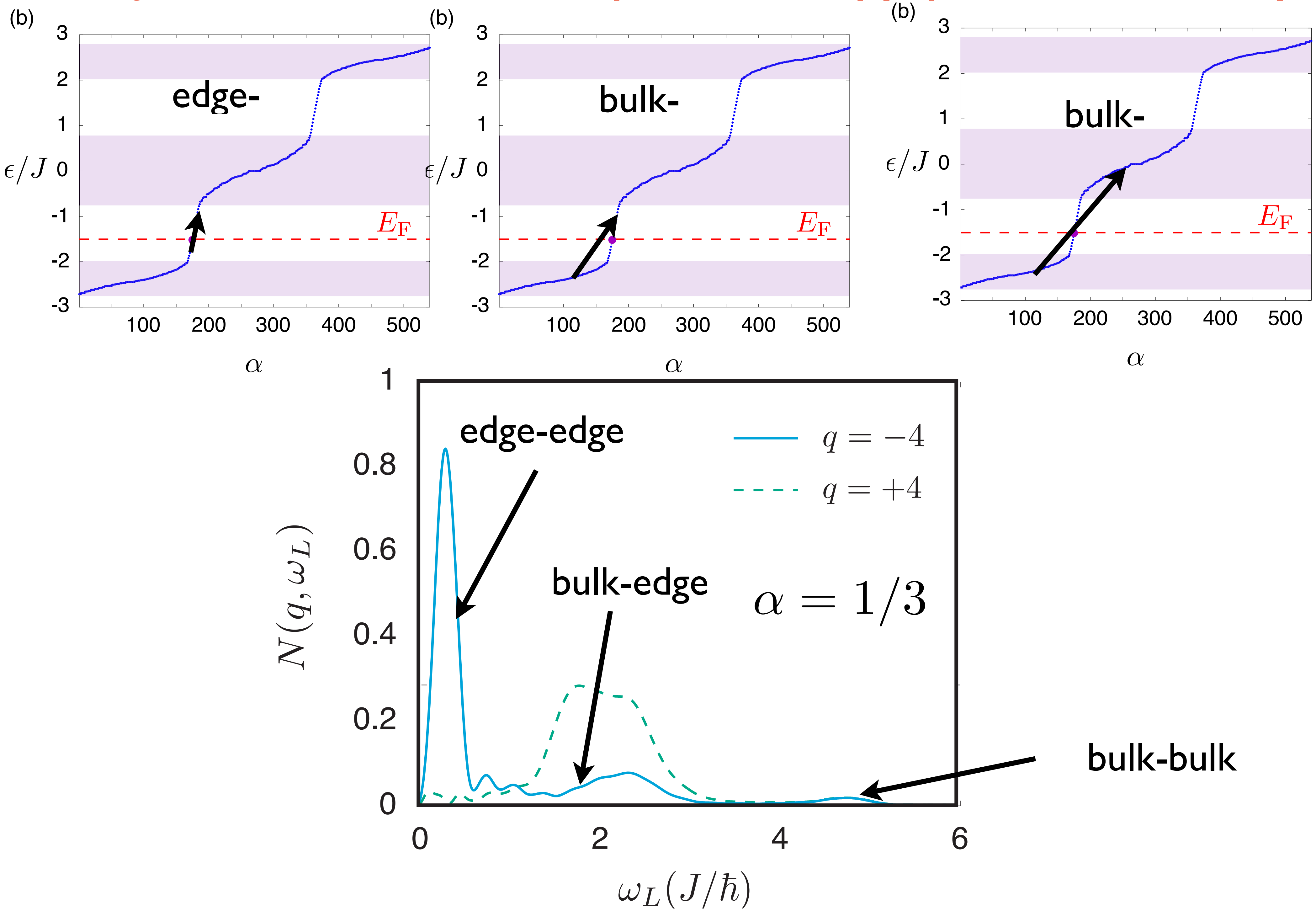
$$N(q,\omega_L) = 2\pi\Omega^2 T \sum_{k>E_F, l\leq E_F} |I^q_{kl}|^2 \delta^{(t)}(\omega_{kl}-\omega_L)$$ Fermi Golden Rule




Dispersion relation approximately linear : $E_{\text{edge}} = \hbar\dot{\theta}_e m$ *m*~angular momentum

Resonance : $k \approx l + q$ $\hbar\omega_L \approx E_F + \hbar\dot{\theta}_e q$

# «Angular momentum» spectroscopy probes chirality

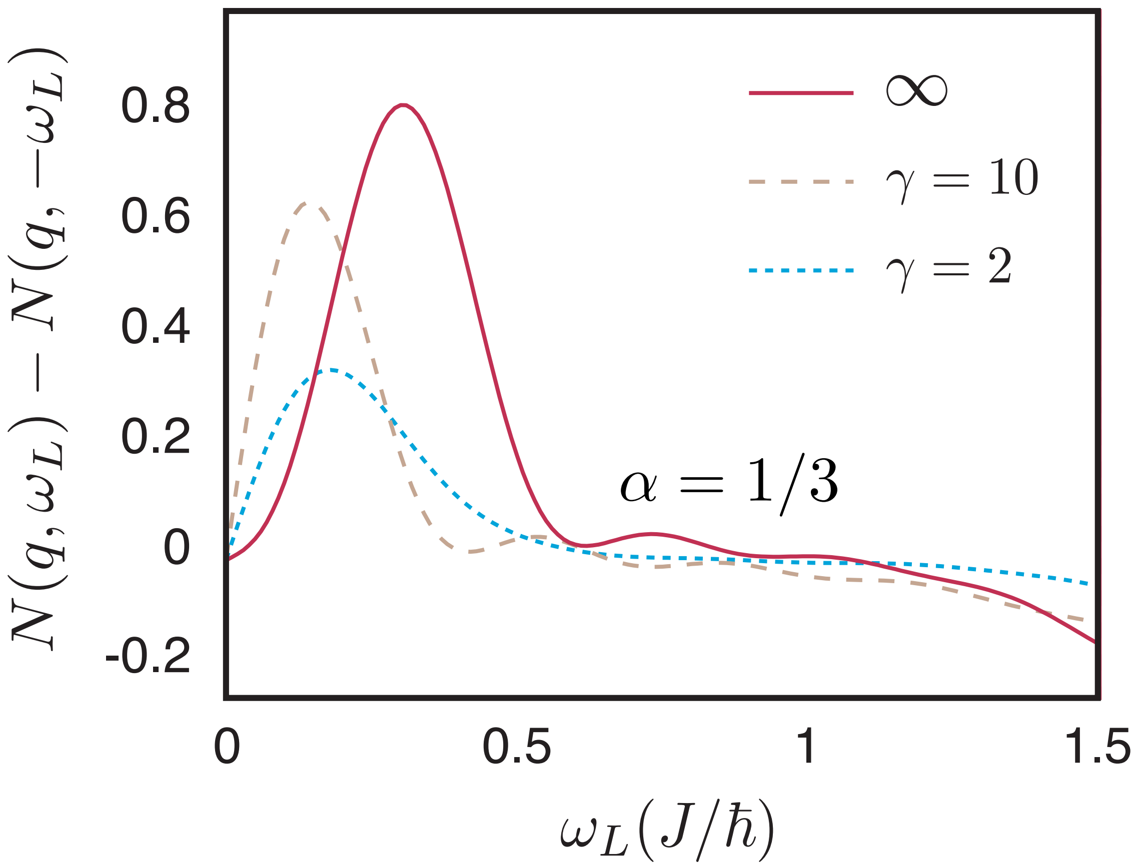



Clear signature found in the spectra :

- sensitive to chirality
- peak position allows to measure the angular velocity

# «Angular momentum» spectroscopy for different potential shapes

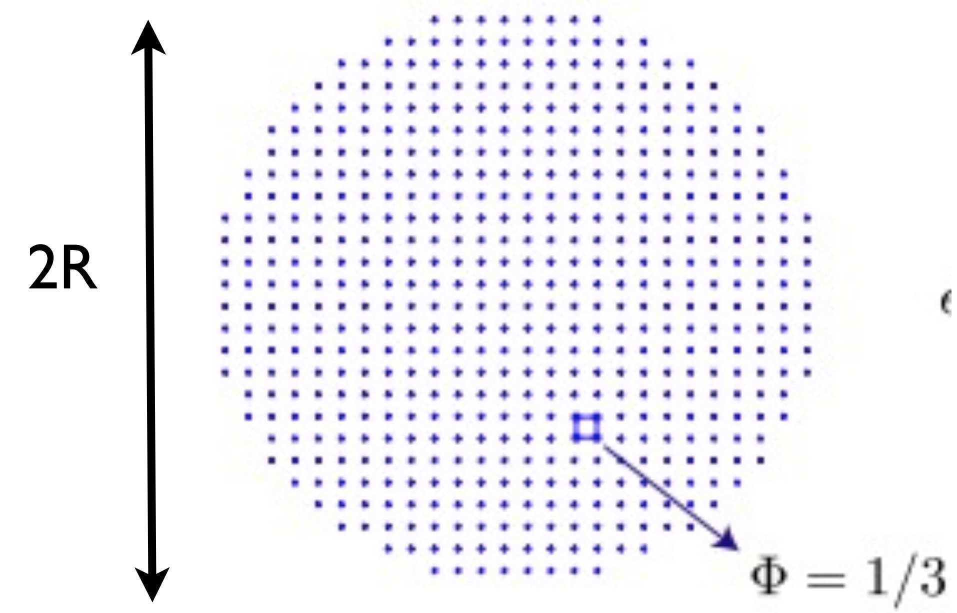
$$V_{\rm trap} \propto (r/r_0)^\gamma$$




- weak dependence on the presence/shape of an external trap
- still present in harmonic traps, although «flat-bottom» traps are better

# Detection issue : how to resolve edge states on top of the Fermi sea ?

# of edge states : $N_{\rm edge} \approx \dfrac{R}{d}$




$$N_{\rm tot} \approx \pi R^2/d^2$$

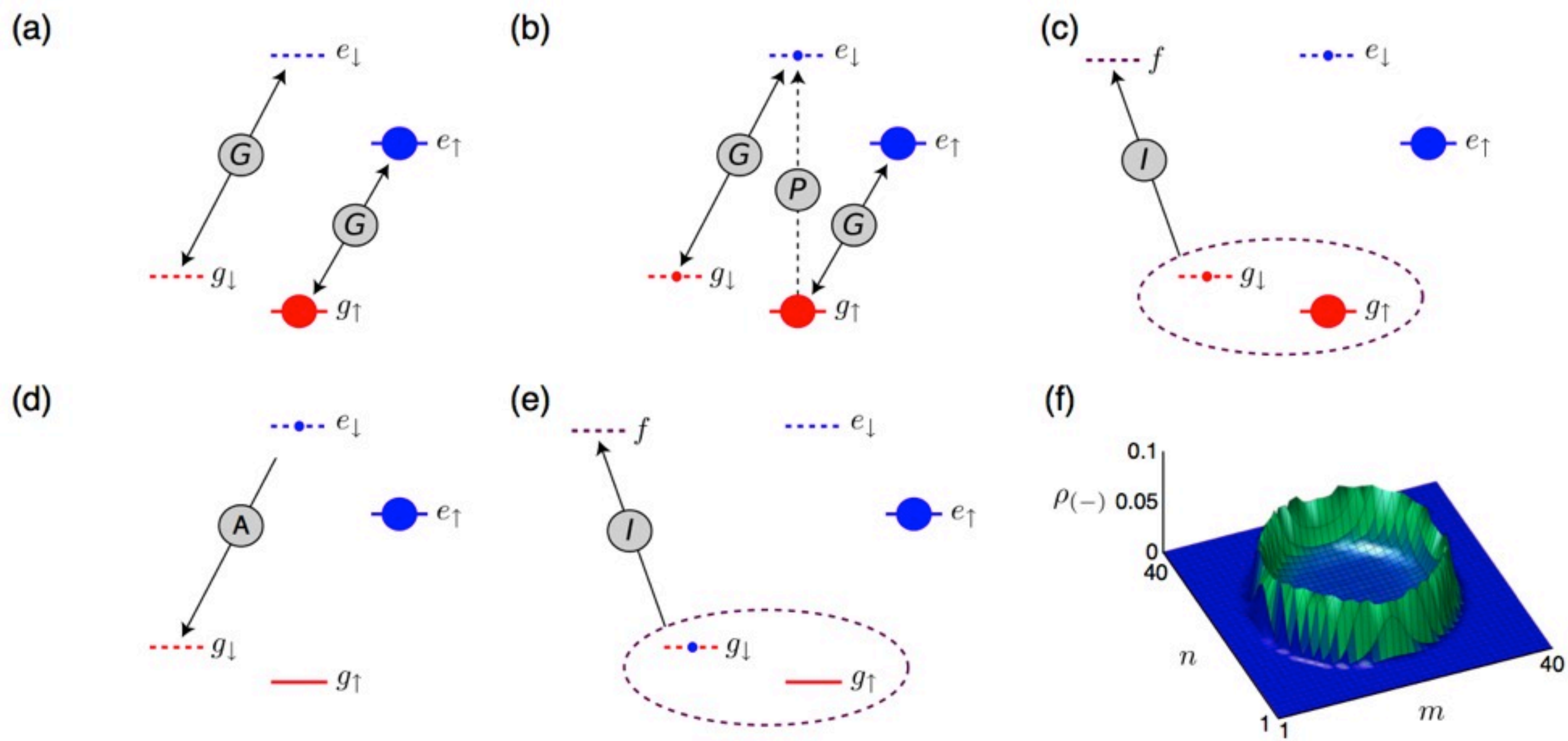
$$N_{\rm edge}/N_{\rm tot} \approx d/R \ll 1$$

Since the spectroscopy method will adress a small fraction of the edge states population, we have to detect a few tens of atoms on top of ~$10^4$ : this is <u>**hard**</u>

# Proposal : shelving technique for $^{171}$Yb

ground and metastable states with electronic spin 0, nuclear spin 1/2 coupled by near-resonant laser

All transitions can be resolved separately by applying a bias field ~100G (nuclear Zeeman shifts)



# Shelving spectroscopy

Raman excitation (no Pauli blocking to any final state)

Signature of chirality still present in the asymmetry of the spectra :