Scattering and colliding bright quantum solitons

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€€ EU, Studienstiftung
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0. Outline

1. Bright solitons

2. Interference: detecting quantum superpostions

3. Gross-Pitaevskii solitons

4. Schrödinger kittens

5. Bell states

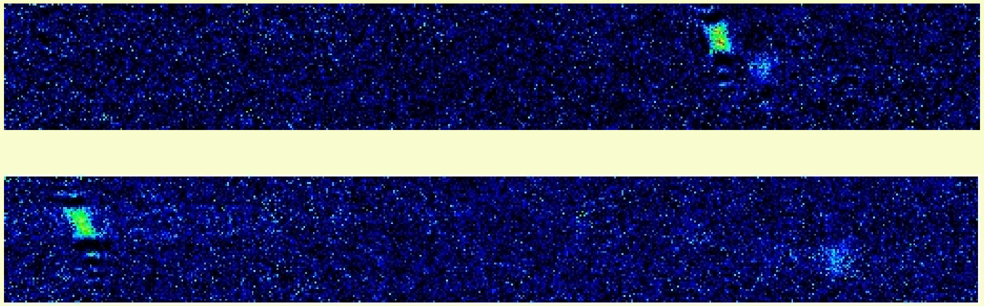
6. Conclusion

1. Bright solitons

- water: Heriot-Watt University, Edinburgh, Scotland
- nonlinear optics: nonlinear Schrödinger equation
- attractive BECs: Gross-Pitaevskii equation $\leftrightarrow$ nonlinear Schrödinger equation

Experiments: bright solitons from BECs

- single solitons:



L. Khaykovich *et al.*, Science **296**, 1290 (2002).

- soliton trains

K.E. Strecker *et al.*, Nature, **417**, 150 (2002).

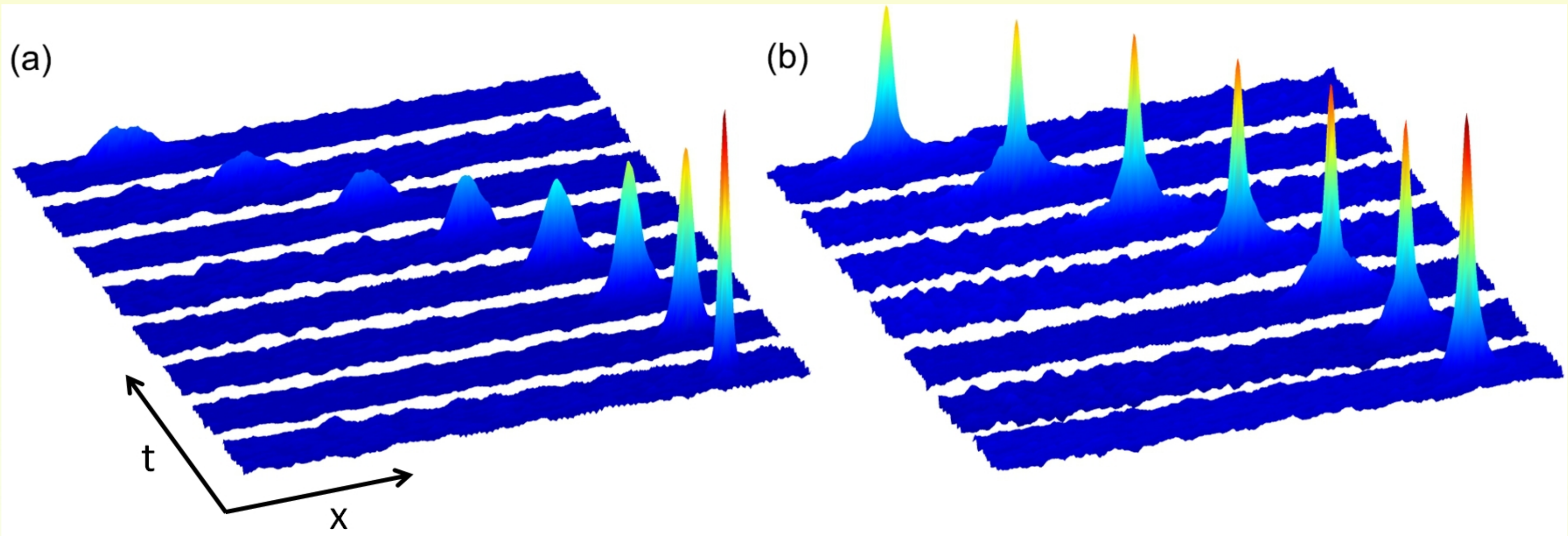
- in (more) 3D trap

S.L. Cornish, S.T. Thompson, C.E. Wieman
Phys. Rev. Lett. **96**, 170401 (2006).

Ongoing experiments: bright solitons from BECs

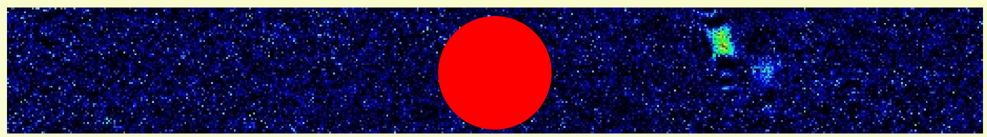
- P. Dyke, S. Lei, R.G. Hulet http://adsabs.harvard.edu/abs/2012APS..DMP.U6001D

- T.P. Billam, A.L. Marchant, S.L. Cornish *et al.* arXiv:1209.0560 (2012).

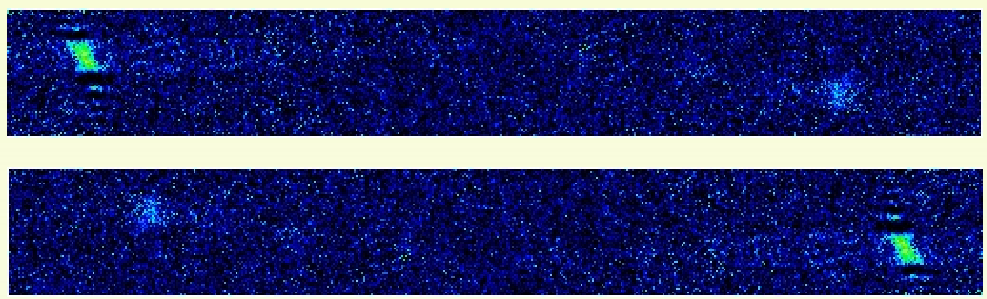


Aim: Schrödinger-cat states

- Scattering potential: laser focus in 1D geometry



spatial superposition



C. Weiss and Y. Castin, Phys. Rev. Lett. **102**, 010403 (2009);

A.I. Streltsov, O.E. Alon, L.S. Cederbaum,
Phys. Rev. A **80**, 043616 (2009).

Model

- Lieb-Liniger-McGuire model with scattering-potential

$$\widehat{H} = \sum_{i=1}^{N} \frac{p_i^2}{2m} + g_{1\mathrm{D}} \sum_{i<j} \delta(x_i - x_j) + U, \; g_{1\mathrm{D}} < 0$$

- $U = 0$: exact solutions ($\widetilde{\beta} = -m g_{1\mathrm{D}}/(2\hbar^2) > 0$)

$$\psi_{N,k}(\underline{x}) = \exp\left(-\widetilde{\beta} \sum_{1\leq\nu<\mu\leq N} |x_\nu - x_\mu| + \mathrm{i}k \sum_{\nu=1}^{N} x_\nu\right)$$

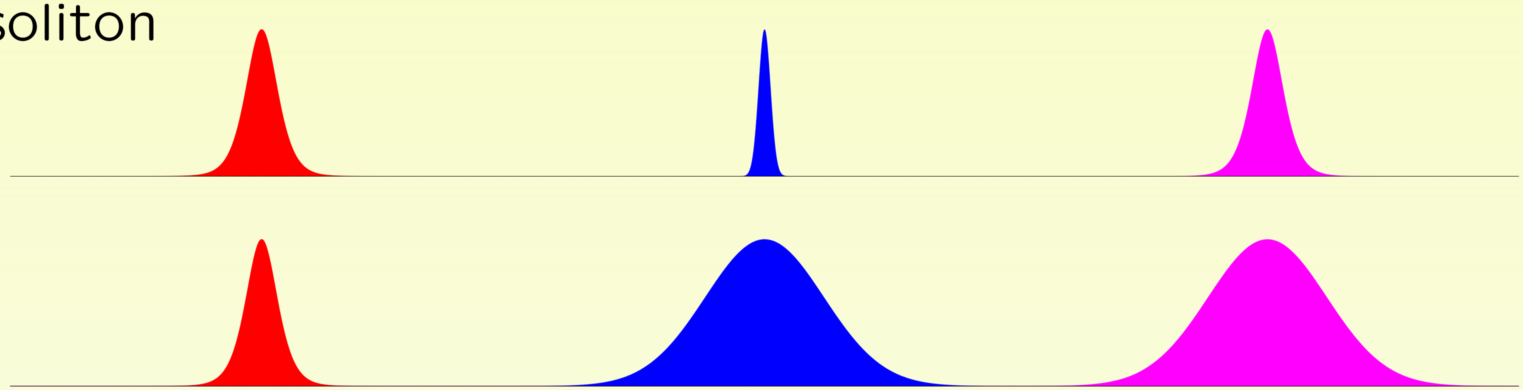
- $U = U\left(\frac{1}{N}\sum_\nu x_\nu\right)$: center of mass and relative coordinates seperate. Here: $U = \sum_\nu \widetilde{U}\left(x_\nu\right)$

- “unbreakable” soliton of $N \approx 100$ particles $E_1 - E_0 > \dfrac{\hbar^2 K^2}{2M}$

C. Weiss and Y. Castin, PRL **102**, 010403 (2009).

Our method: effective potential approach

unbreakable soliton — scattering potential — effective potential/N



$$\int \mathrm{d}^N x |C\psi_{N,k}(\underline{x})|^2 V(\underline{x}) \delta\left(X_{\mathrm{C}} - \frac{1}{N}\sum_{\nu=1}^{N} x_\nu\right) = V_{\mathrm{eff}}(X_{\mathrm{C}})$$

slow solitons, perfect superpositions, $N \approx 100$

C. Weiss and Y. Castin, Phys. Rev. Lett. **102**, 010403 (2009),
J. Phys. A **45**, 455306 (2012);
K. Sacha *et al.*, Phys. Rev. Lett. **103**, 210402 (2009).

Proving the effective potential approach

- incoming soliton, CoM-wave function plane wave
- low kinetic energies prevent soliton from breaking
- scattering of a short-range potential
- bounds on transmission/reflection coefficients

C. Weiss and Y. Castin, J. Phys. A **45**, 455306 (2012).

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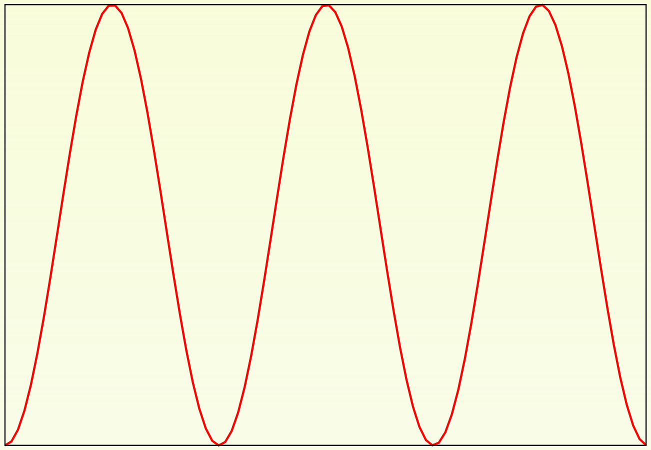
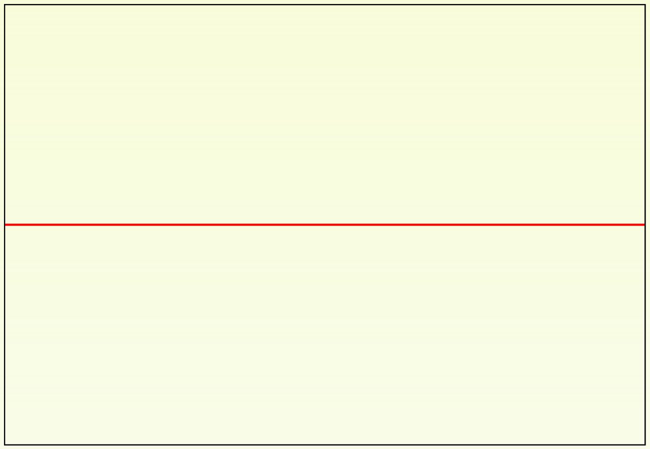
2. Interference: detecting quantum superpostions

Single particle either in exp(ikx) or in exp$(-ikx)$

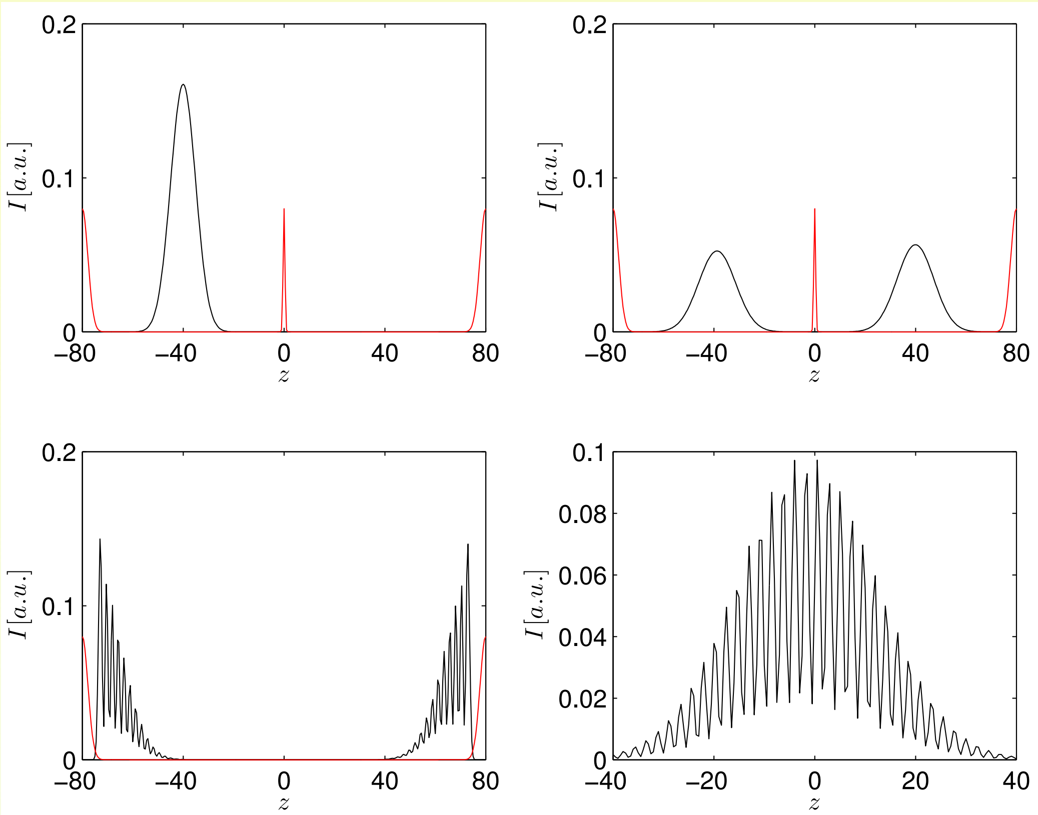
Single-particle density

quantum superposition   statistical mixture





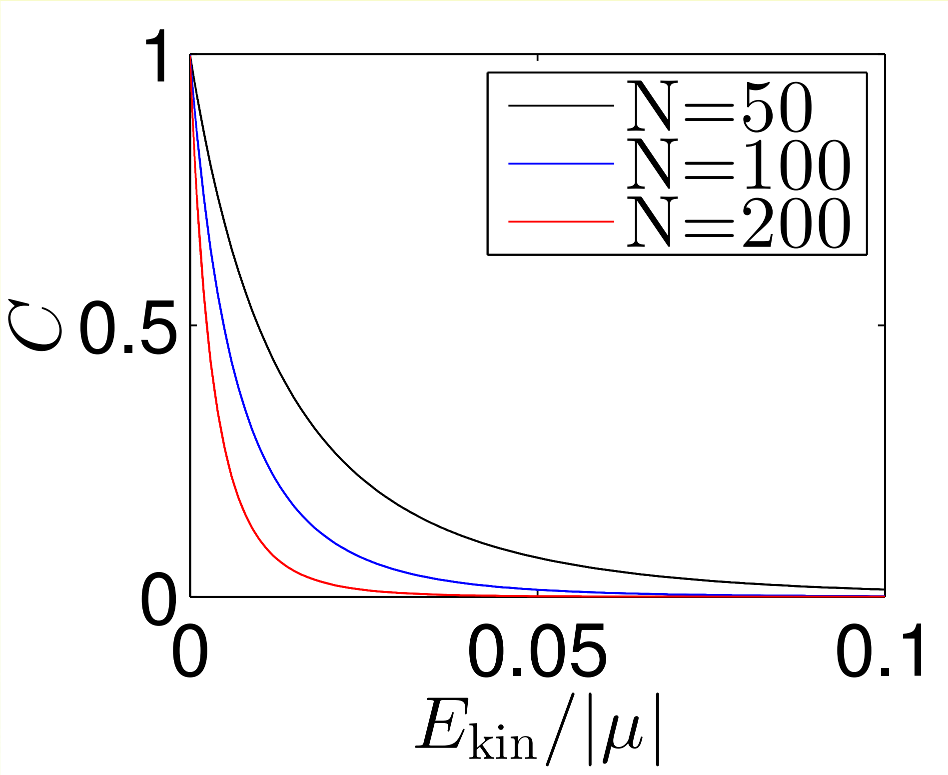
$N = 2$, single-particle density



B. Gerjerenken and C. Weiss, J Phys. B **45**, 165301 (2012).

Soliton, quantum superposition of

$\exp(ikx_{\mathsf{c}})$ and $\exp(-ikx_{\mathsf{c}})$



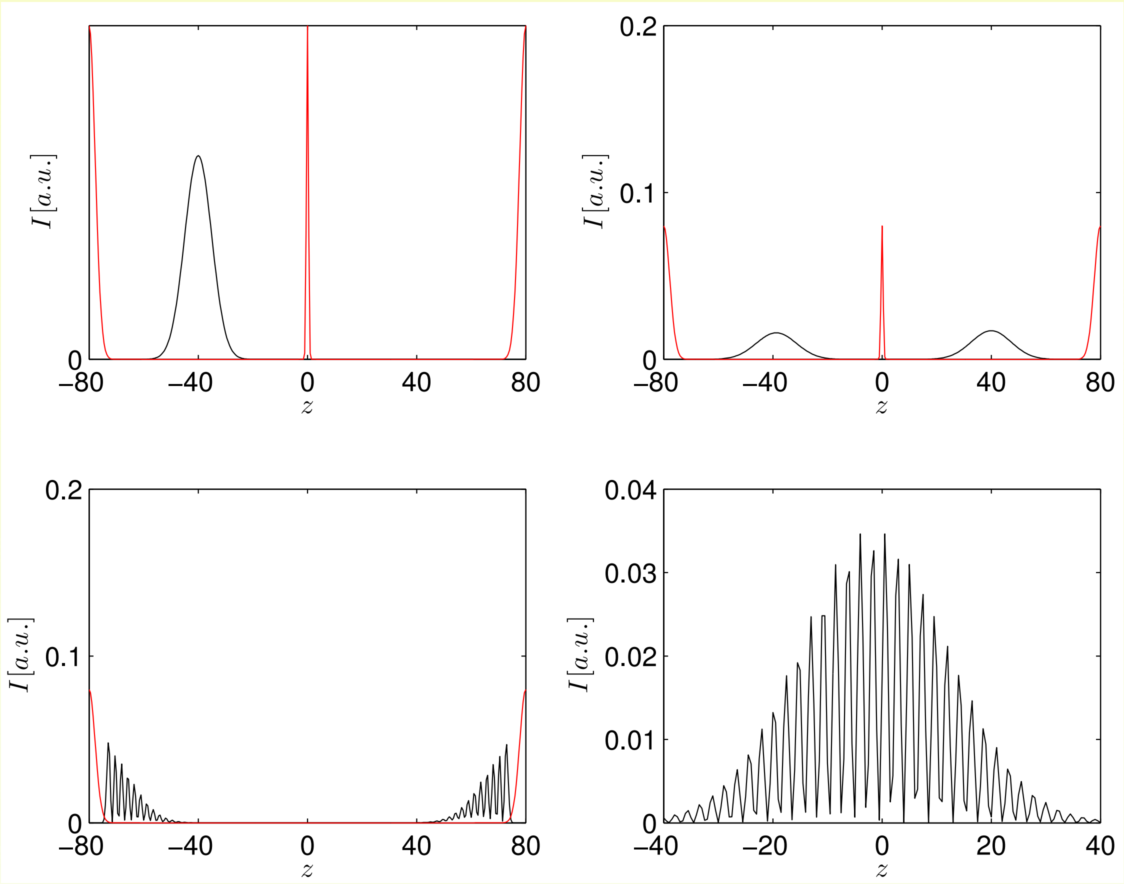
Experiment:

$$E_{\mathsf{kin}} \approx 0.5|\mu|$$

$$\mu \equiv E_1 - E_0$$

B. Gerjerenken and C. Weiss, J. Phys. B **45**, 165301 (2012).

$N = 2$, centre-of-mass density

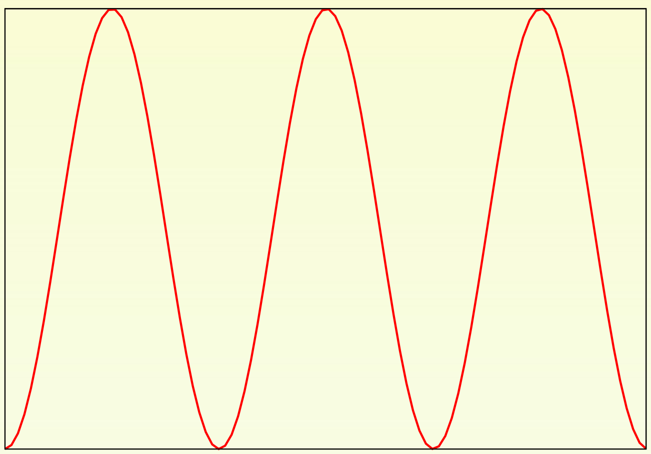


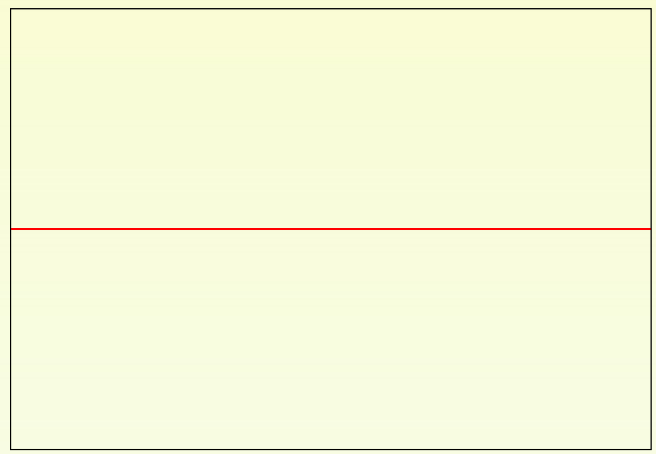
B. Gerjerenken and C. Weiss, J. Phys. B **45**, 165301 (2012).

Soliton ($N \approx 100$) quantum superposition of $\exp(ikx_{\mathrm{c}})$ and $\exp(-ikx_{\mathrm{c}})$

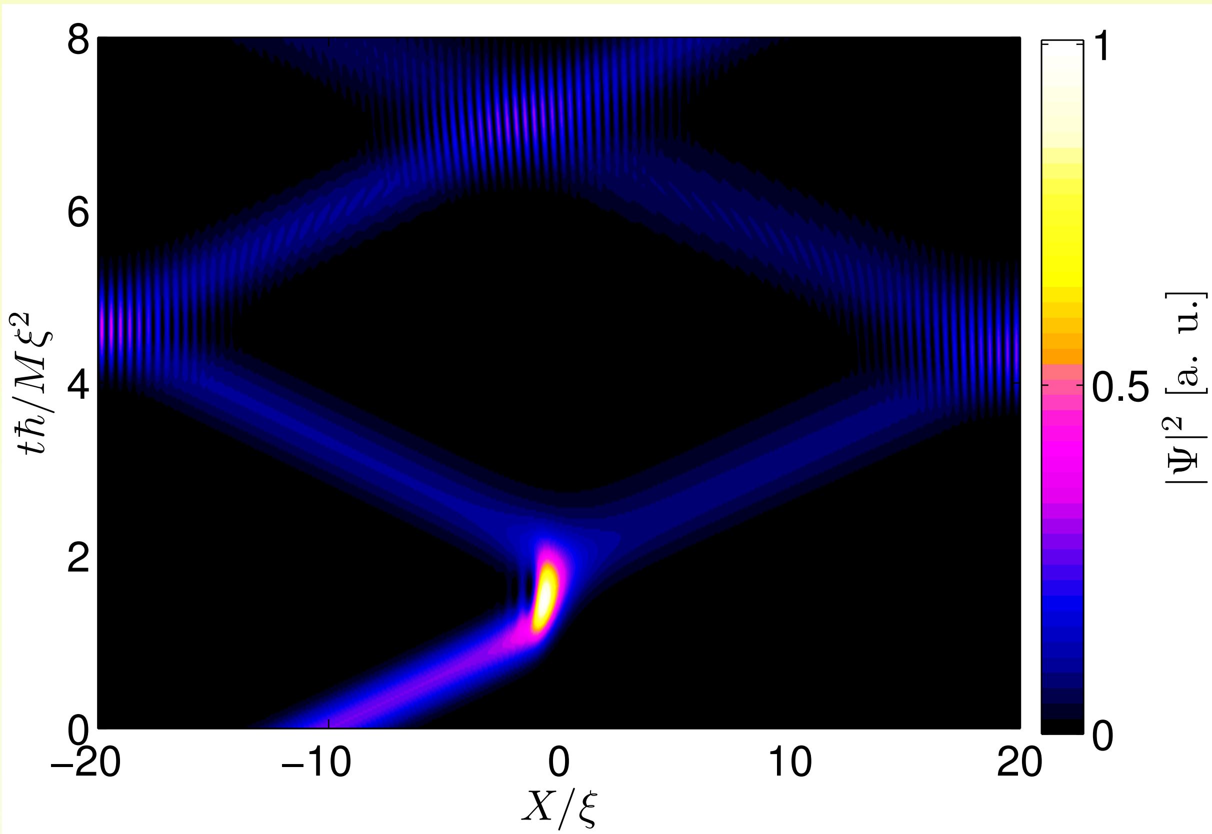
Centre-of-mass density

single-particle density





Centre-of-mass density, $N = 100$



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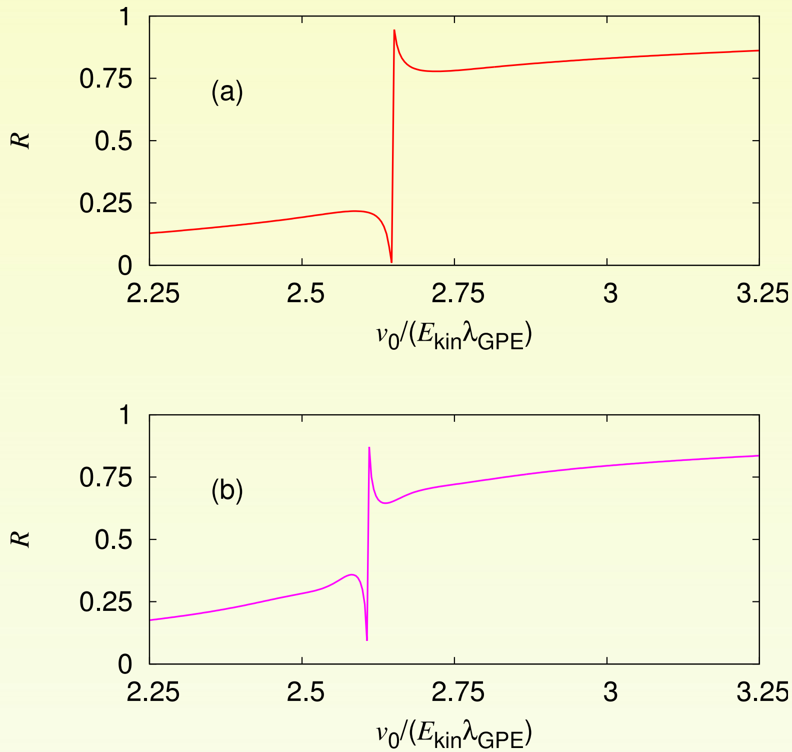
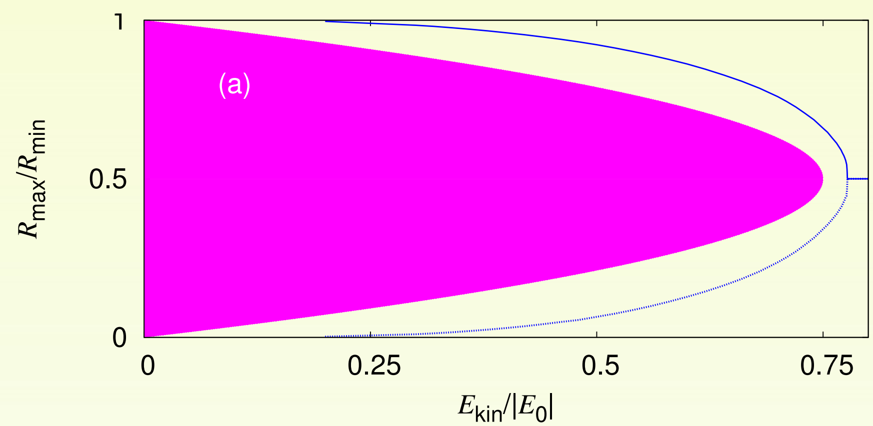
3. Gross-Pitaevskii solitons

$$i\hbar\partial_t\varphi(x,t) = -\frac{\hbar^2}{2m}\partial_x^2\varphi(x,t) + V_{\text{ext}}(x)\varphi(x,t) + (N-1)g_{1\text{D}}|\varphi(x,t)|^2\varphi(x,t)$$



Jumps



B. Gertjerenken, T. P. Billam, L. Khaykovich, C.W., PRA **86**, 033608 (2012).

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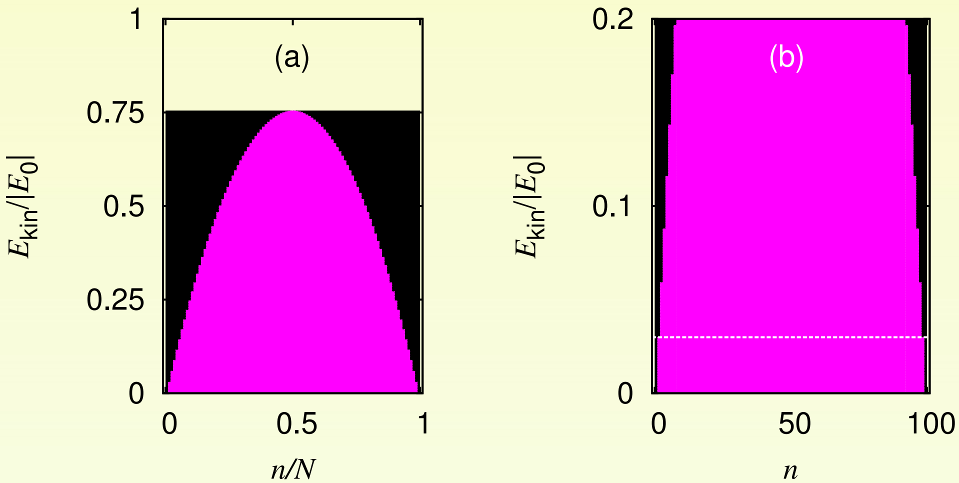
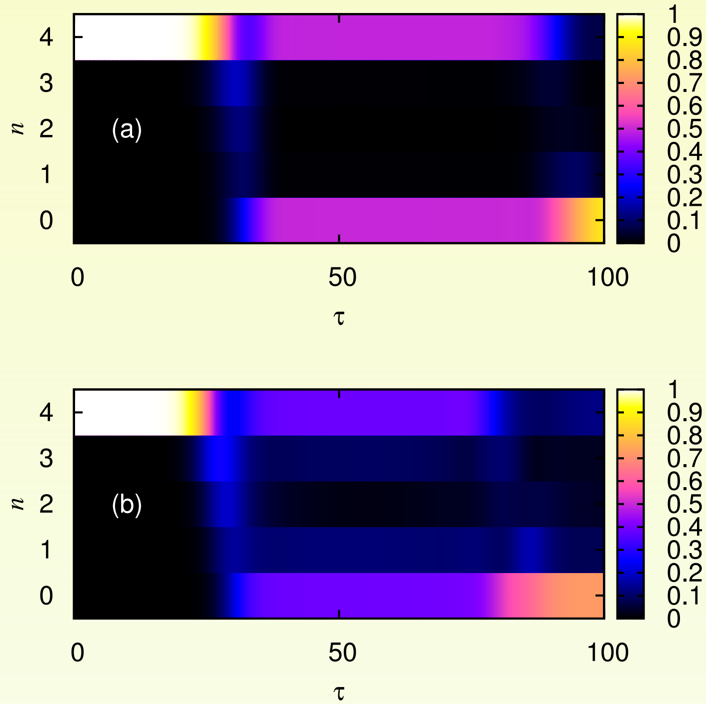
4. Schrödinger kittens

Energetically allowed particle number left/right of the barrier:



$N = 4$



B. Gertjerenken, T. P. Billam, L. Khaykovich, C.W., PRA **86**, 033608 (2012).

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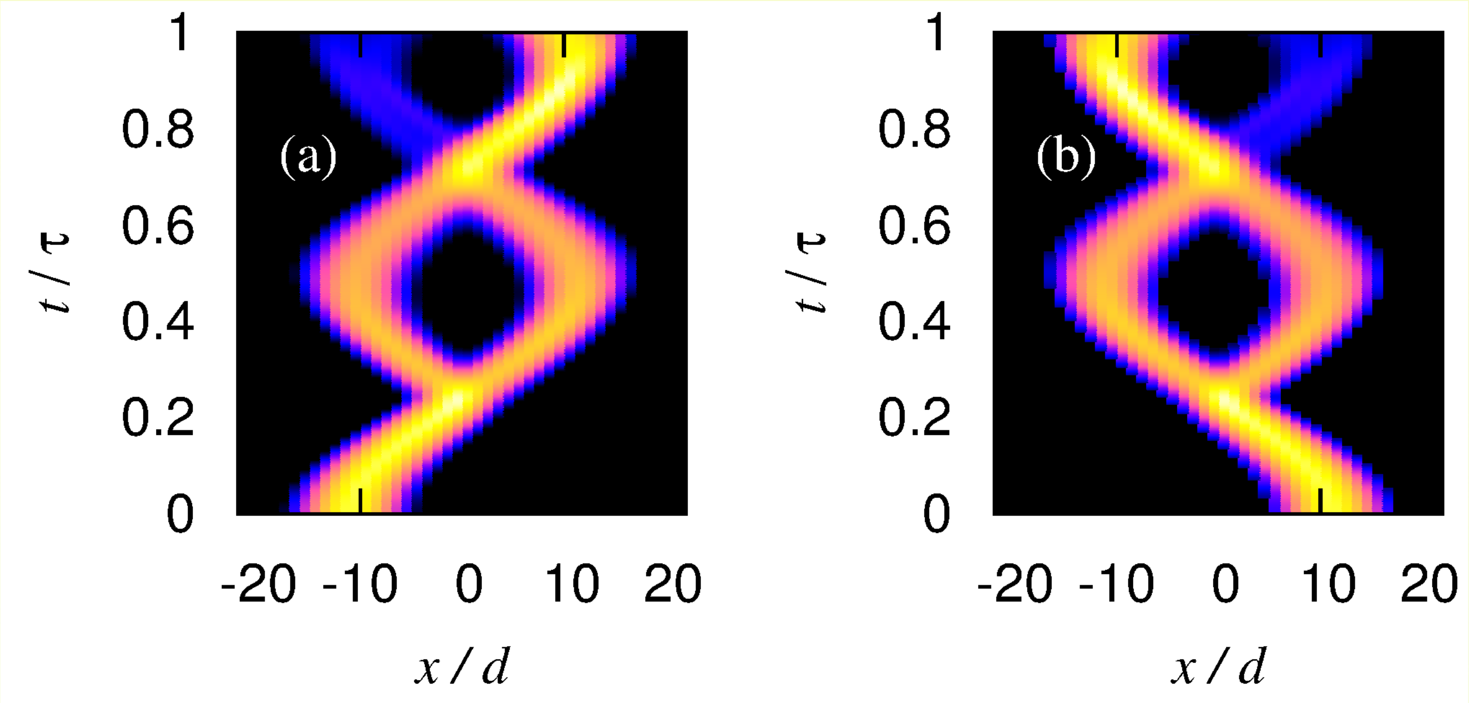
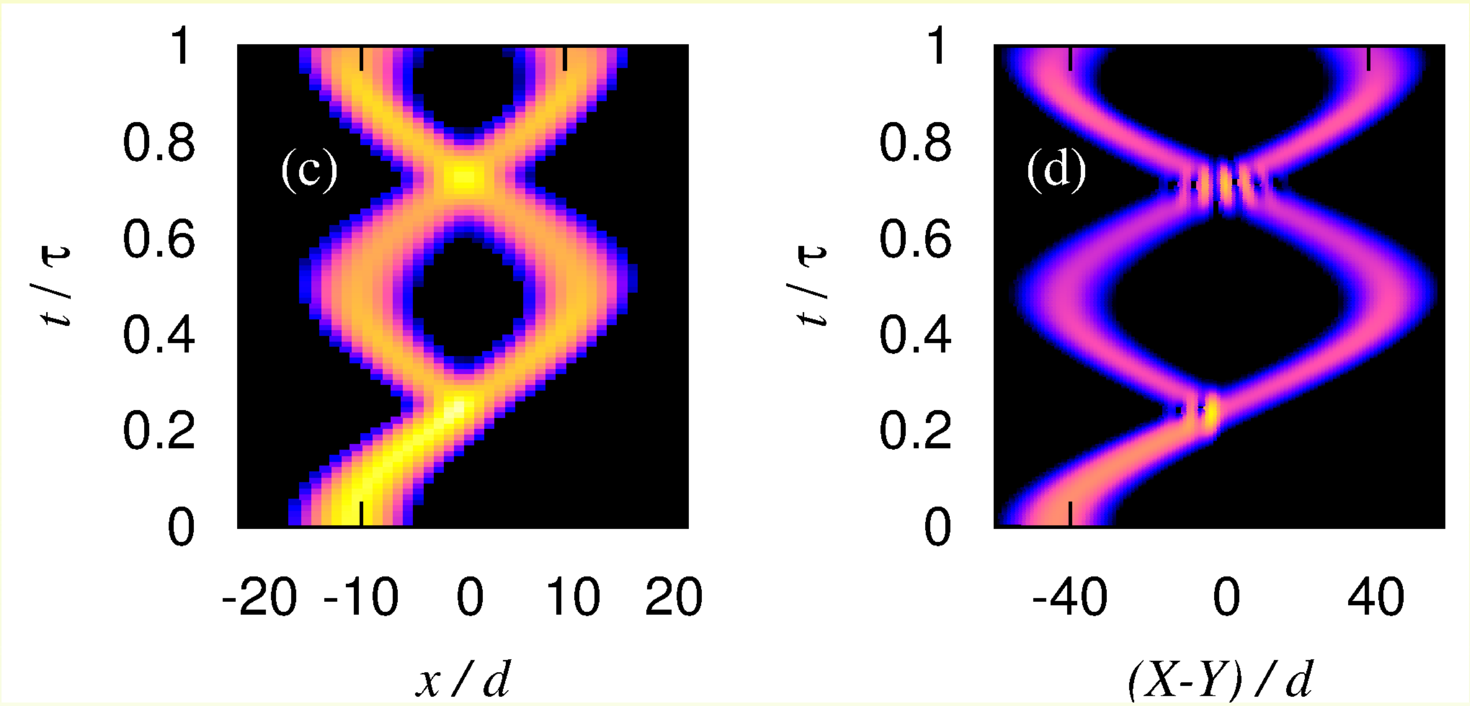
Two distinguishable bright two-particle solitons



B. Gertjerenken, T.P. Billam, C.L. Blackley, C.R. Le Sueur, L. Khaykovich, S.L. Cornish, C. Weiss, arXiv:1301.0718 (2012).

Two distinguishable dimers: detecting Bell states

density for statistical mixture interference pattern CoM

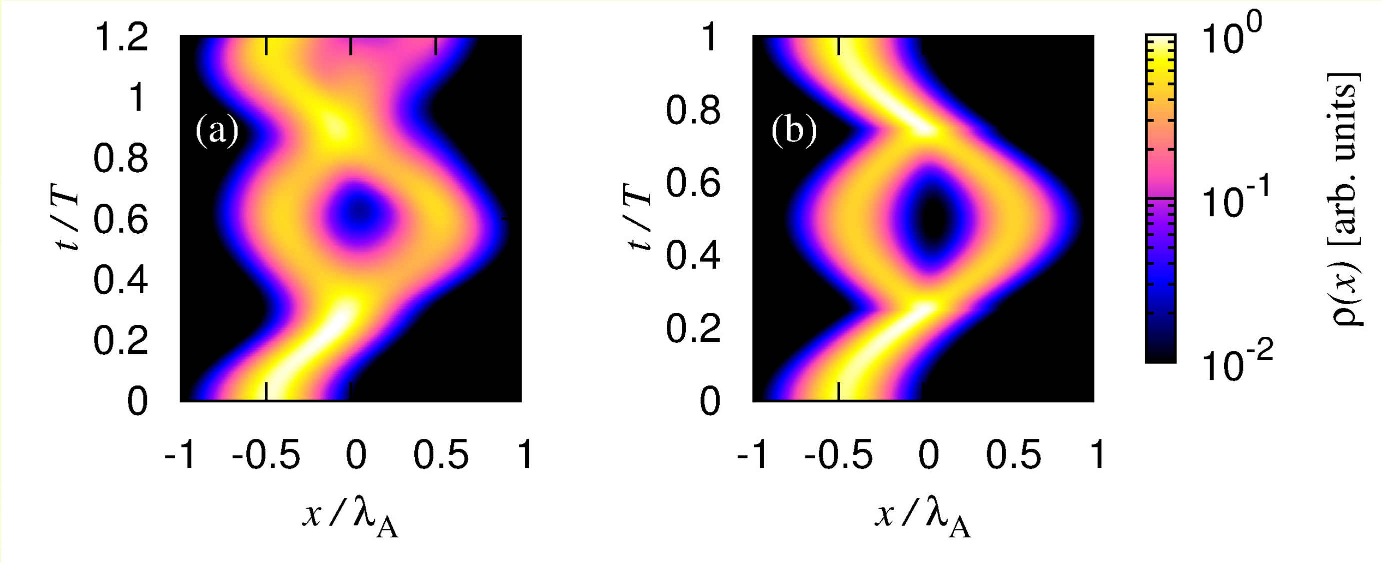


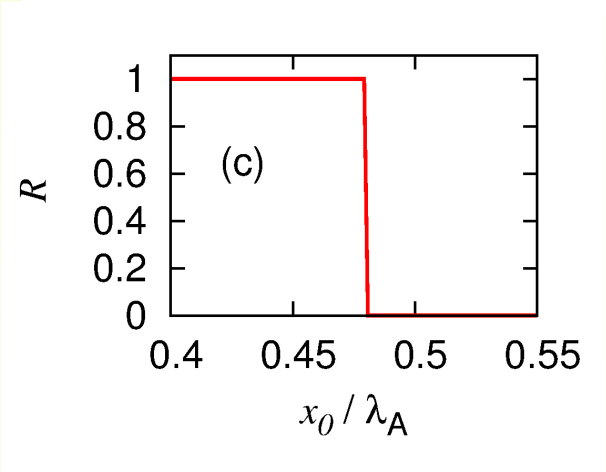
B. Gertjerenken *et al.*, arXiv:1301.0718 (2012).

Distinguishable solitons: classical field method

effective Potential method

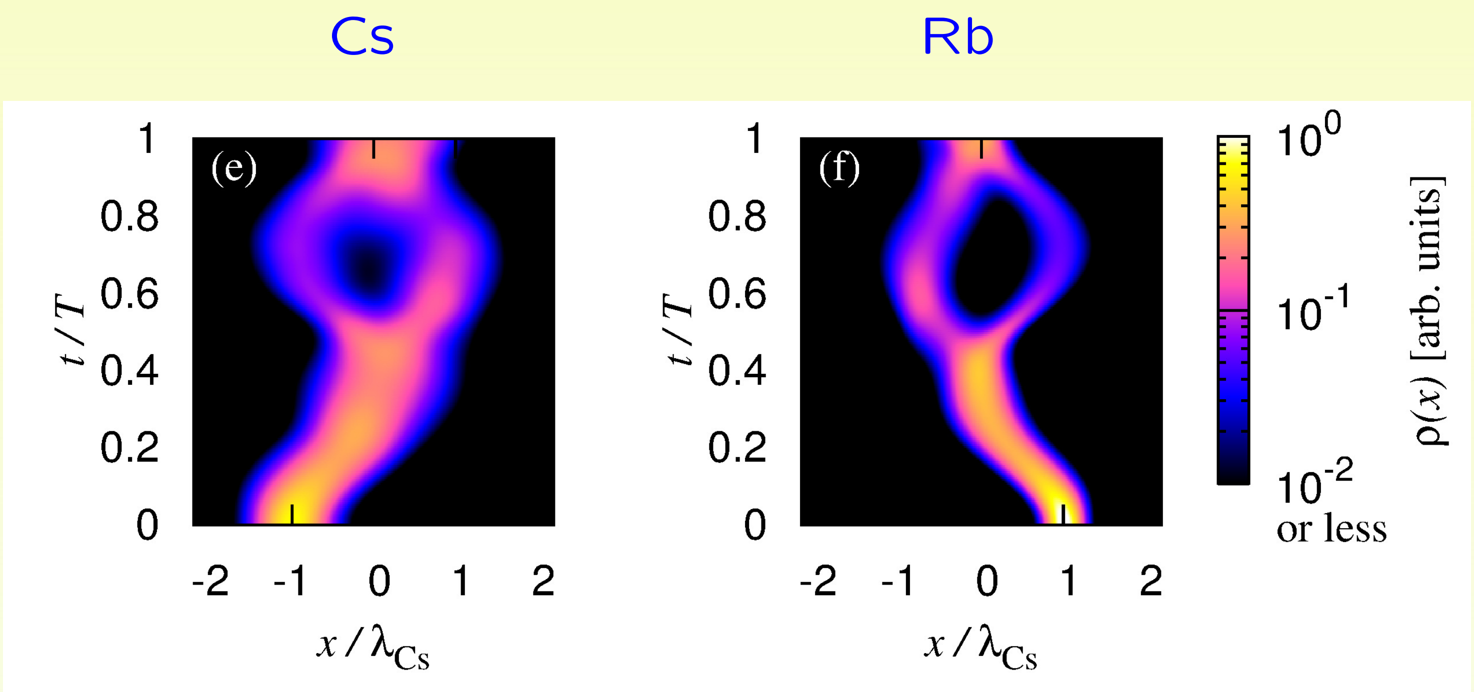
truncated Wigner CoM





B. Gertjerenken *et al.*,
arXiv:1301.0718 (2012).

Two distinguishable solitons: Bell states



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6. Conclusion

- Scattering bright solitons off potentials
  - N-particle quantum physics Schrödinger cat states
  - mean-field (GPE): jumps
  - detection via interference pattern: centre-of-mass density
- Colliding two distinguishable solitons
  - Bell states
- Outlook
  - Colliding indistinguishable solitons

with D. Holdaway and S. Gardiner

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