Thermoelectricity with cold atoms ?

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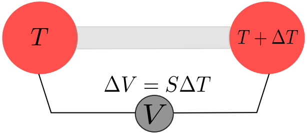
Introduction to Thermoelectricity

• **Seebeck effect :** A difference of temperature ΔT creates a voltage ΔV



• **Peltier effect :** An electric current I generates a heat current

$$I_Q = \Pi \cdot I = (T \cdot S) \cdot I$$

$\Rightarrow$ Seebeck coefficient S : Entropy per carrier

$\Rightarrow$ Stationnary effects[1] : permanent currents/differences

[1] L. Onsager, Phys. Rev. 38, 2265 (1931)& Phys. Rev. 37, 405 (1931)
H.B. Callen, Phys. Rev. 73, 1349-1358 (1948)

Thermoelectricity and materials

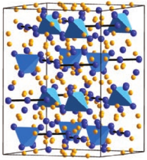
Good thermoelectrics : a recurrent interest in material physics

→ Good Peltier cooling

→ Energy saving purposes

Idea : increase figure of merit

$$ZT = \frac{TS^2}{\sigma_{th}}$$

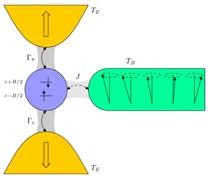


G. Snyder & E. Toberer, Nat. Mat. 7 105 (2008)

More fundamental : access high temperature transport properties, without phonons...

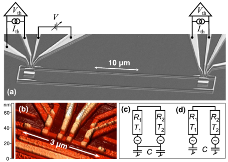
Thermoelectricity and mesoscopic physics

GOALS :
- Extract energy from fluctuating environment
- Optimize energy to electricity conversion (cooling purposes)



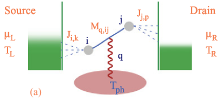
Heat engines with magnons

B. Sothmann & M. Büttiker, EPL (2012)



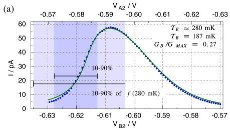
Quantum limited refrigerator

Timofeev et al PRL (2009)



Three terminal thermoelectricity

J.-H. Jiang et al Phys. Rev. B (2012)



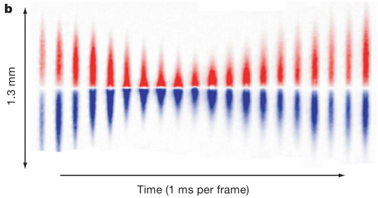
Quantum dot refrigerator

Prance et al PRL (2009)

Introduction - Transport and cold atoms

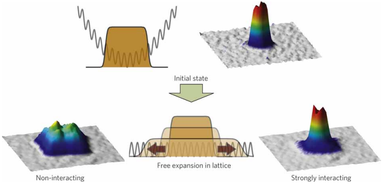
Spin transport (MIT, 2011)

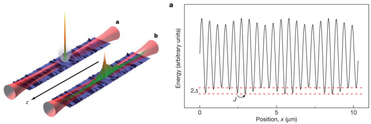
A. Sommer et al., Nature



Interactions (LMU, 2012)

U. Schneider et al., Nat. Phys.





Disorder (Inst. d'optique - LENS, 2008)

J. Billy et al.-G. Roati et al., Nature

Also:

H. Ott et al, Phys. Rev. Lett. 92, 160601 (2004)

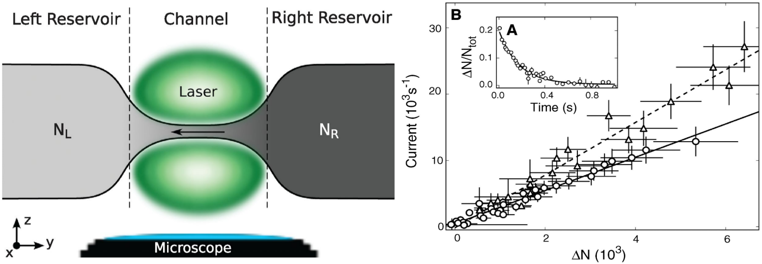
S. Palzer et al, Phys. Rev. Lett. 103, 150601 (2009)

J. Catani et al, Phys. Rev. A 85, 023623 (2012)

K.K. Das et al, Phys. Rev. Lett. 103, 123007 (2009)

And many others ...

Introduction - Experimental motivation



ETH, **2012** (Brantut et al., Science)

$\rightarrow$ Realization of a two terminal transport setup

Discharge of a mesoscopic capacitor (reservoirs) in a resistor (the conduction channel)

$\Rightarrow$ Simulation of mesoscopic physics with cold atoms

Question : Can this setup demonstrate offdiagonal transport ?

Outline

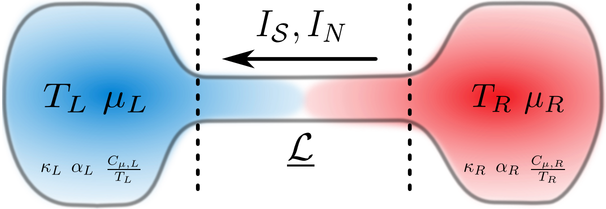
1. General framework

2. Proposal for thermoelectricity

3. Experimental signal estimates

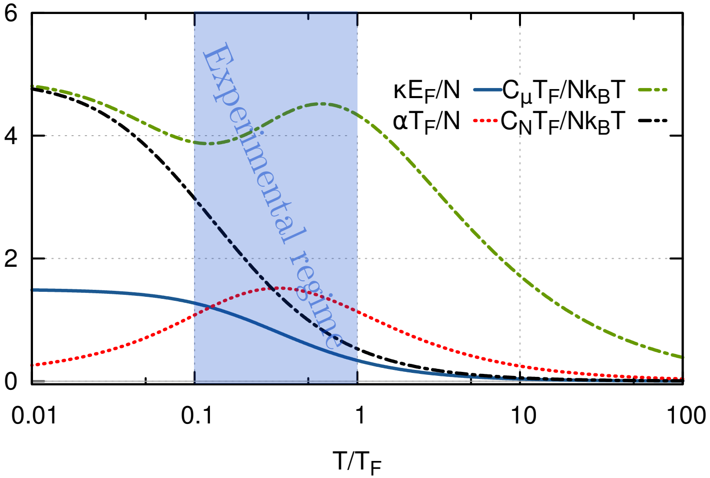
Outline

Transport setup



- Two terminal configuration
- Grand canonical ensemble : flow of entropy and particles between reservoirs
- Linear response coefficients in $\underline{\mathcal{L}}$

Thermodynamic coefficients



Transport in linear response

Our approach : Constriction $\leftrightarrow$ Black box responding linearly
$\equiv$ **Linear circuit picture**

Linear response :

$$\begin{pmatrix} I_N \\ I_S \end{pmatrix} = -\underline{\mathcal{L}} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix}, \quad \underline{\mathcal{L}} = \begin{pmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} \\ \mathcal{L}_{12} & \mathcal{L}_{22} \end{pmatrix}$$

And thermodynamics :

$$\begin{pmatrix} \Delta N \\ \Delta S \end{pmatrix} = \underline{\mathcal{M}} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix}, \quad \underline{\mathcal{M}} = \begin{pmatrix} \kappa & \alpha \\ \alpha & \frac{C_\mu}{T} \end{pmatrix}$$

$\Rightarrow$ Equations for chemical potential and temperature difference

$$\frac{d}{dt} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix} = -\underline{\mathcal{M}}^{-1} \underline{\mathcal{L}} \begin{pmatrix} \Delta\mu \\ \Delta T \end{pmatrix}$$

Transport equations and coefficients

Equations for particle number and temperature difference :

$$\boxed{\tau_0 \frac{d}{dt}\begin{pmatrix} \Delta N/\kappa \\ \Delta T \end{pmatrix} = -\underline{\Lambda}\begin{pmatrix} \Delta N/\kappa \\ \Delta T \end{pmatrix}, \underline{\Lambda} = \begin{pmatrix} 1 & -S \\ -\frac{S}{\ell} & \frac{L+S^2}{\ell} \end{pmatrix}.}$$

$\Rightarrow$ Discharge of a capacitor through a resistor, including thermal properties

Global timescale $\tau_0 = \frac{\mathcal{L}_{11}}{\kappa} \sim RC$

Effective transport coefficients :

$L \equiv \mathcal{L}_{22}/\mathcal{L}_{11} - (\mathcal{L}_{12}/\mathcal{L}_{11})^2 \sim R/TR_T \rightarrow$ Lorenz number

$\ell \equiv C_\mu/\kappa T - (\alpha/\kappa)^2 = C_N/\kappa T \rightarrow$ Charac. of reservoirs, analogue to L

$S \equiv \alpha/\kappa - \mathcal{L}_{12}/\mathcal{L}_{11} \rightarrow$ Total Seebeck coefficient

Transport coefficients

Both reservoirs and constriction participate to transport

•$S = 0$:

-Only pure thermal and mass transport

-At low T: $L/\ell \rightarrow 1$ for a free Fermi gas

$\Rightarrow$W-F law $\leftrightarrow$ ΔT and ΔN relaxation timescales are identical

•$\mathcal{L}_{12} = 0$:

-Total Seebeck $S \neq 0$

$\Rightarrow$ Intrinsic thermoelectric effect driven by finite α

Questions:

i. How to reveal thermoelectric effects ?

ii. 'Smoking gun' protocol ?

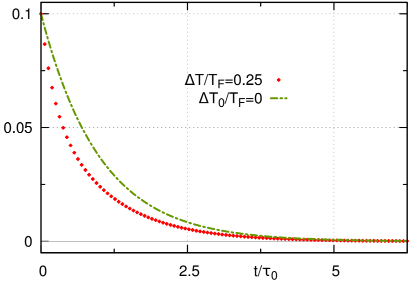
Outline

1. General framework
2. Proposal for thermoelectricity
3. Experimental signal estimates

Solution to transport equations

$$\Delta N(t) = \underbrace{\left\{ \frac{1}{2}\left[e^{-t/\tau_-} + e^{-t/\tau_+} \right] + \left[1 - \frac{L + S^2}{\ell} \right] \frac{e^{-t/\tau_-} - e^{-t/\tau_+}}{2(\lambda_+ - \lambda_-)} \right\} \Delta N_0}_{\text{Diagonal transport: exponential decrease}} +$$

$$\underbrace{\boxed{\frac{S\kappa}{\lambda_+ - \lambda_-}\left[e^{-t/\tau_-} - e^{-t/\tau_+} \right] \Delta T_0}}_{\text{Thermoelectric effect !}}, \tau_\pm^{-1} = \tau_0^{-1}\lambda_\pm$$



Particle imbalance with and without initial temperature difference

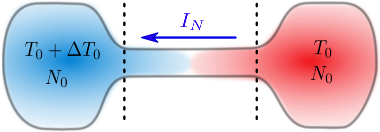
With $\frac{\Delta N_0}{N_0} = 10\%$

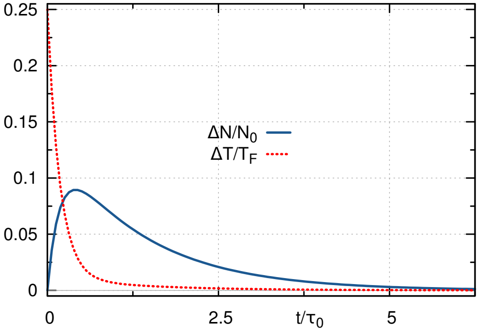
$\Rightarrow$ Hard to isolate thermoelectric contribution when exp. decrease is switched on

An appropriate setup

Two steps

i. Prepare reservoirs with equal particle number and different temperatures, with closed constriction
ii. Open the constriction and monitor particle number





Particles flow in both ways during temperature equilibration :
$\rightarrow$ Transient analogue of the Seebeck effect

What happens ?

$$\Delta N(t) = \frac{S\kappa}{\lambda_+ - \lambda_-}\left[e^{-t/\tau_-} - e^{-t/\tau_+}\right] \Delta T_0 \,, \tau_\pm^{-1} = \tau_0^{-1}\lambda_\pm$$

with $\lambda_\pm = \frac{1}{2}\left(1 + \frac{L+S^2}{\ell}\right) \pm \sqrt{\frac{S^2}{\ell} + \left(\frac{1}{2} - \frac{L+S^2}{2\ell}\right)^2}$ and $S = \alpha/\kappa - \mathcal{L}_{12}/\mathcal{L}_{11}$.

i. Particle imbalance and current proportional to ΔT_0 and S

ii. Reservoir properties participate to the effect

iii. Sign of ΔN (and I_N) given by the sign of S

$$I_{\mathcal{S}} = SI_N - \underbrace{\sigma_{th}\Delta T}_{\text{Dominant}}$$

Entropy flows from hot to cold : 2nd principle ✓

Protocol-Summary

- Simple protocol to observe thermoelectric effects

Very general up to now ... Need model for :

$\rightarrow$ Reservoirs

$\rightarrow$ Constriction

- For practical purposes:

$\leadsto$ How to measure the efficiency of the protocol ?

$\leadsto$ Estimates ?

$\leadsto$ Which configuration(s) favor the observation of the effects ?

$\leadsto$ Change of sign observable ?

Outline

1. General framework
2. Proposal for thermoelectricity
3. **Experimental signal estimates**

How to compute transport coefficients ?

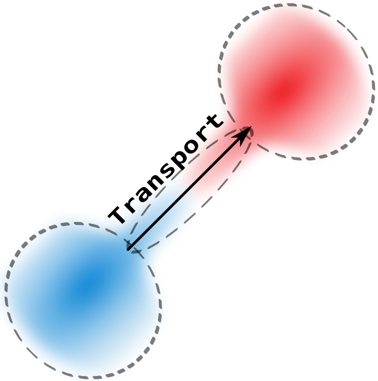
Need to take care of channel **and** reservoirs
$\rightarrow$ Reservoirs and constriction are treated separately

Reservoirs

i. Free Fermi gas $\simeq$ metallic reservoir
ii. Trapped Fermi gas
iii. Noninteracting fermions

Constriction

i. Geometry: trap, no trap, (1-2-3) D
ii. Conduction regime : Ballistic or diffusive
iii. Response coefficients : Landauer-Büttiker formalism
iv. Noninteracting fermions



Computing coefficients

- Thermodynamic coefficients $\Rightarrow$ Proportional to moments of DOS $\cdot\, \partial f/\partial\epsilon$

$$\mathcal{R}_n = \int_0^{+\infty} d\epsilon g(\epsilon)(-\frac{\partial f}{\partial \epsilon})(\epsilon-\mu)^n$$

$$\kappa \rightarrow n=0,\ \alpha \rightarrow n=1\ \frac{C_\mu}{T} \rightarrow n=2$$

- Transport coefficients $\Rightarrow$ Proportional to moments of $\Phi \cdot \partial f/\partial\epsilon$

$$\mathcal{T}_n = \int_0^{+\infty} d\epsilon \Phi(\epsilon)(-\frac{\partial f}{\partial \epsilon})(\epsilon-\mu)^n$$

$$\mathcal{L}_{11} \rightarrow n=0,\ \mathcal{L}_{12} \rightarrow n=1\ \mathcal{L}_{22} \rightarrow n=2$$

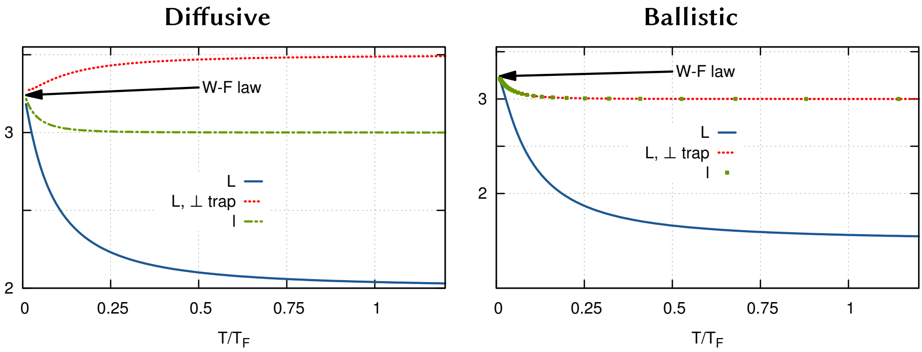
Φ : transport function $\simeq$ # modes $\cdot$ velocity $\cdot$ transmission

$\propto$ Differential conductance[2]

[2] R. Kim et al Appl. Phys. Lett. 105, 034506 (2009)
G. D. Mahan & J. O. Sofo, PNAS 93, 7436 (1996)

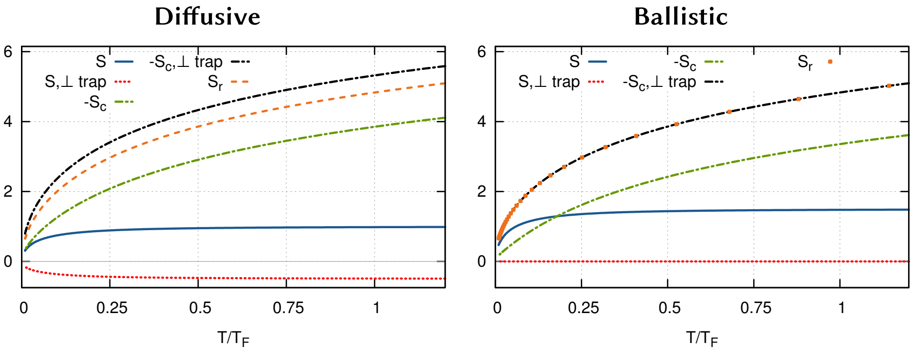
Results : Transport coefficients 1

Lorenz number L and ℓ



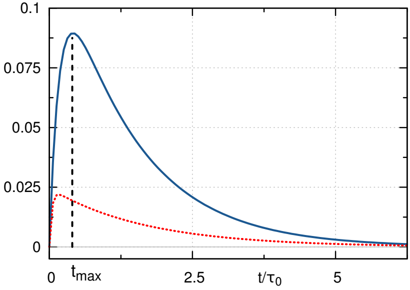
Results : Transport coefficients 2

Reservoirs, constriction contributions and total Seebeck



Thermoelectric efficiency

How to measure the setup's efficiency ?
GOAL : Find appropriate parameter range



Recall:

$$\frac{\Delta N(t)}{N_0} = \frac{S\kappa T_F}{N_0(\lambda_+ - \lambda_-)}\left[e^{-t/\tau_-} - e^{-t/\tau_+}\right]\frac{\Delta T_0}{T_F}$$

$\rightarrow$ Maximum at t_{max}

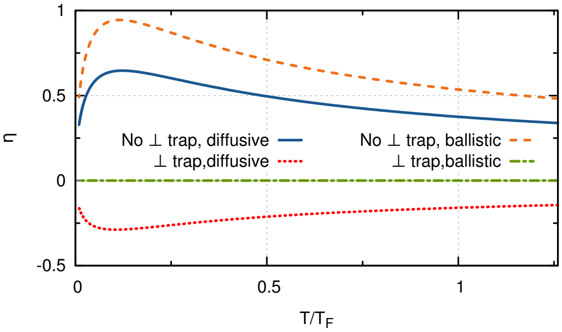
Efficiency defined as

$$\eta = \frac{\Delta N(t_{max})}{N_0} \Big/ \frac{\Delta T_0}{T_F}$$

Efficiency for Peltier-like protocol : $\eta_{\text{Peltier}} = \frac{\eta}{\ell} < \eta$

Results : Thermoelectric efficiency

Comparison of different regimes and geometries



Conclusions-Perspectives

- Thermoelectricity with cold atoms !
- Transport : combination of reservoir and channel properties
- Intrinsic thermoelectricity
- Protocols to reveal offdiagonal transport with cold atoms
  $\rightarrow$ Sizeable (10 to 20%) effects ☺
- High-T transport without phonons

What's next ?

- Interactions : improvement of thermopower
- Lattice in the constriction
- Pumping reservoirs

CG, C. Kollath, A. Georges arxiv:1209.3942

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Aknowledgements

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D. Stadler

S. Krinner

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Thank you for your attention

Solution of transport equations

$$\Delta N(t) = \left\{ \frac{1}{2}\left[e^{-t/\tau_-} + e^{-t/\tau_+}\right] + \left[1 - \frac{L+S^2}{\ell}\right] \frac{e^{-t/\tau_-} - e^{-t/\tau_+}}{2(\lambda_+ - \lambda_-)} \right\} \Delta N_0 + \frac{S\kappa}{\lambda_+ - \lambda_-}\left[e^{-t/\tau_-} - e^{-t/\tau_+}\right] \Delta T_0$$

$$\Delta T(t) = \left\{ \frac{1}{2}\left[e^{-t/\tau_-} + e^{-t/\tau_+}\right] + \left[\frac{L+S^2}{\ell} - 1\right] \frac{e^{-t/\tau_-} - e^{-t/\tau_+}}{2(\lambda_+ - \lambda_-)} \right\} \Delta T_0 + \frac{S}{\ell\kappa(\lambda_+ - \lambda_-)}\left[e^{-t/\tau_-} - e^{-t/\tau_+}\right] \Delta N_0$$

with $\tau_\pm^{-1} = \tau_0^{-1}\lambda_\pm$ given by

$$\lambda_\pm = \frac{1}{2}\left(1 + \frac{L+S^2}{\ell}\right) \pm \sqrt{\frac{S^2}{\ell} + \left(\frac{1}{2} - \frac{L+S^2}{2\ell}\right)^2}\,.$$

Transport and thermodynamic coefficients

Thermodynamic coefficient	Onsager coefficient
$\kappa = \int_0^\infty d\epsilon g(\epsilon) \left(-\frac{\partial f}{\partial \epsilon}\right)$	$\mathcal{L}_{11} = \int_0^\infty d\epsilon \Phi(\epsilon) \left(-\frac{\partial f}{\partial \epsilon}\right)$
$\alpha = k_B \beta \int_0^\infty d\epsilon g(\epsilon)(\epsilon - \mu) \left(-\frac{\partial f}{\partial \epsilon}\right)$	$\mathcal{L}_{12} = \int_0^\infty d\epsilon \Phi(\epsilon)(\epsilon - \mu) \left(-\frac{\partial f}{\partial \epsilon}\right)$
$\frac{C_\mu}{T} = k_B^2 \beta^2 \int_0^\infty d\epsilon g(\epsilon)(\epsilon - \mu)^2 \left(-\frac{\partial f}{\partial \epsilon}\right)$	$\mathcal{L}_{22} = \int_0^\infty d\epsilon \Phi(\epsilon)(\epsilon - \mu)^2 \left(-\frac{\partial f}{\partial \epsilon}\right)$

Table:

Comparison between the integral expressions for thermodynamic and Onsager coefficients. In each case, f is the equilibrium Fermi distribution. The energy dependence of the transport function Φ depends on the dimensionality and on the transport regime.