
Spin-injection Spectroscopy of a Spin-orbit coupled Fermi Gas

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July 20, 2012 ENS



Why spin-orbit coupling ?



A little bit of History

- Last century : classification of quantum states in terms of spontaneous symmetry breaking [[Anderson 1997](#)]
- In 1980 : Quantum Hall state. First topological state characterized by a topological invariant [[von Klitzing *et al.* 1980](#)]
- 2008-2010 : a new topological class predicted and discovered, where time-reversal symmetry is preserved. Spin-Orbit coupling plays a crucial role [[M.Z. Hasan and C.L. Kane, RMP , 2010](#)]

Also : Modified interactions, unconventional pairing, Majorana fermions

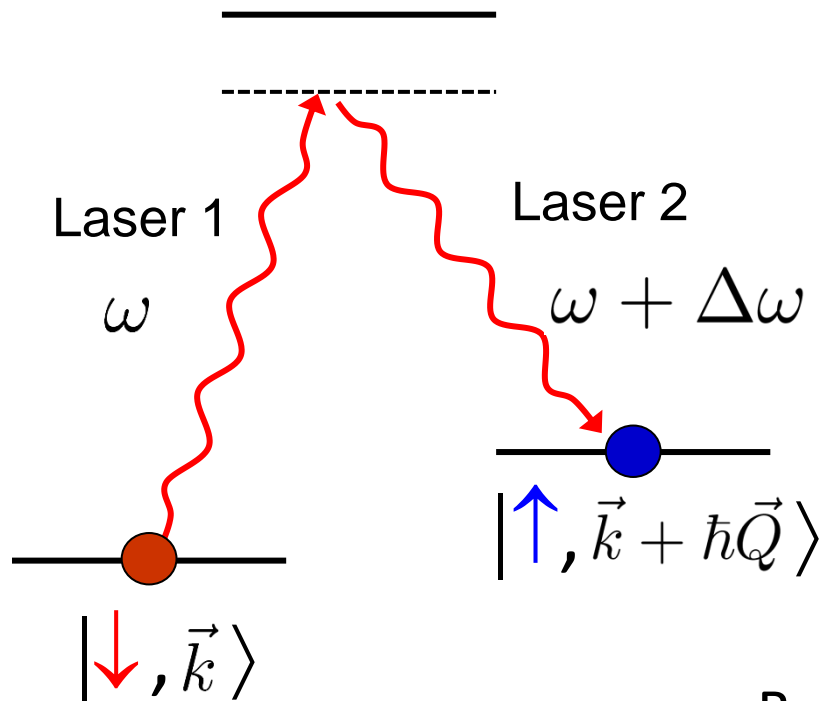
Cold atoms : (very often) constitute optimal system thanks to purity and controle

- Electron moving in an electric field creates a momentum-dependent magnetic fields in the moving frame
- In 2D semiconductor electric field can arises from structure

$$\mathcal{H} = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbb{I} - \mu \cdot [\mathbf{B} + \mathbf{B}_{\text{SO}}(\mathbf{k})]$$

$$-\mu \cdot \mathbf{B}_{\text{SO}}(\mathbf{k}) \propto \begin{cases} \sigma_x k_y - \sigma_y k_x & \text{Rashba} \\ -\sigma_x k_y - \sigma_y k_x & \text{Dresselhaus} \end{cases}$$

Provides a good description of 2D SOC in solids



Raman Beams



Flip spin + imparts momentum

How does the Hamiltonian look like ?

Reminder

2-level system + electric field

e —

g —

$$\vec{E} = E_0 \vec{\epsilon} \cos(\omega t + \phi)$$

RWA approx. :

$$-\vec{d} \cdot \vec{E} = \frac{\hbar\Omega}{2} (\sigma_x \cos \phi - \sigma_y \sin \phi)$$

By adiabatic elimination of the excited state
the Raman process can be described as
the interaction of a 2-level system with
a field $E_0 \vec{\epsilon} \cos(\Delta\omega t + Qx)$

$$\longrightarrow -\vec{d} \cdot \vec{E} = \frac{\hbar\Omega_R}{2} (\sigma_x \cos Qx - \sigma_y \sin Qx)$$

$$\mathcal{H} = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbb{I} + \frac{\hbar \Omega_R}{2} (\sigma_x \cos Qx - \sigma_y \sin Qx) + \frac{\delta}{2} \sigma_z$$

Local pseudo-spin rotation of angle Qx around the z-axis

$$\mathcal{H} = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbb{I} + \frac{\hbar^2 Q}{2m} \sigma_z k_x + \frac{\hbar \Omega_R}{2} \sigma_x + \frac{\delta}{2} \sigma_z + \frac{E_R}{4} \mathbb{I}$$

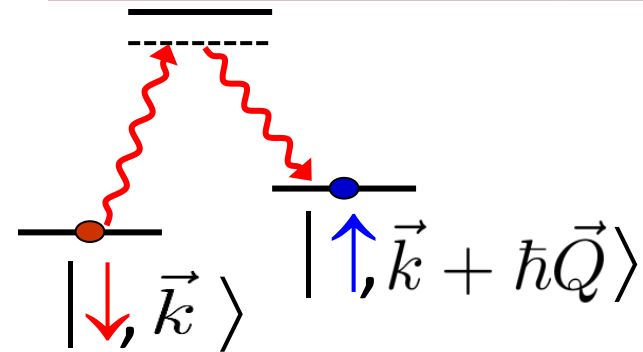
Global rotation $\sigma_z \rightarrow \sigma_y, \sigma_y \rightarrow \sigma_x, \sigma_x \rightarrow \sigma_z$

$$\mathcal{H} = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbb{I} + \underbrace{\frac{\hbar^2 Q}{2m} \sigma_y k_x}_{-\mu \cdot \mathbf{B}_{\text{SO}}(\mathbf{k})} + \underbrace{\frac{\hbar \Omega_R}{2} \sigma_z + \frac{\delta}{2} \sigma_y}_{-\mu \cdot \mathbf{B}} + \frac{E_R}{4} \mathbb{I}$$

Momentum dependent Zeeman field

“equal Rashba and Dresselhaus contributions”

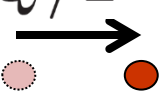
Y. J. Lin *et al.* *Nature* **471**, 83-86 (2011)

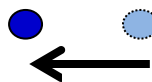


Define quasi-momentum q

$$|\downarrow\rangle \quad q = k + Q/2$$

$$|\uparrow\rangle \quad q = k - Q/2$$

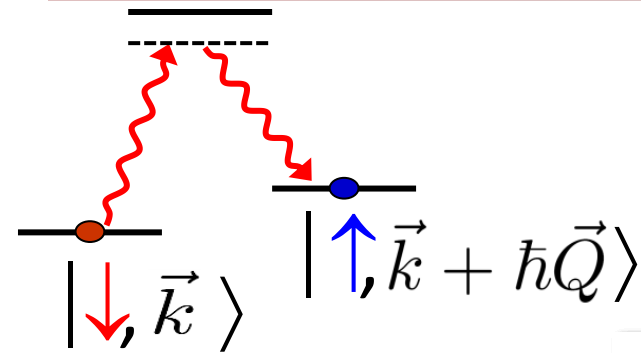
$$+ Q/2$$




$$- Q/2$$

quasi-momentum space

Engineering SO coupling



$$\mathcal{H}_{SO} = \begin{pmatrix} \frac{\hbar^2(q - Q/2)^2}{2m} + \frac{d}{2} & \\ & \frac{\hbar^2(q + Q/2)^2}{2m} - \frac{d}{2} \end{pmatrix}$$

Define quasi-momentum q

$$|\downarrow\rangle \quad q = k + Q/2$$

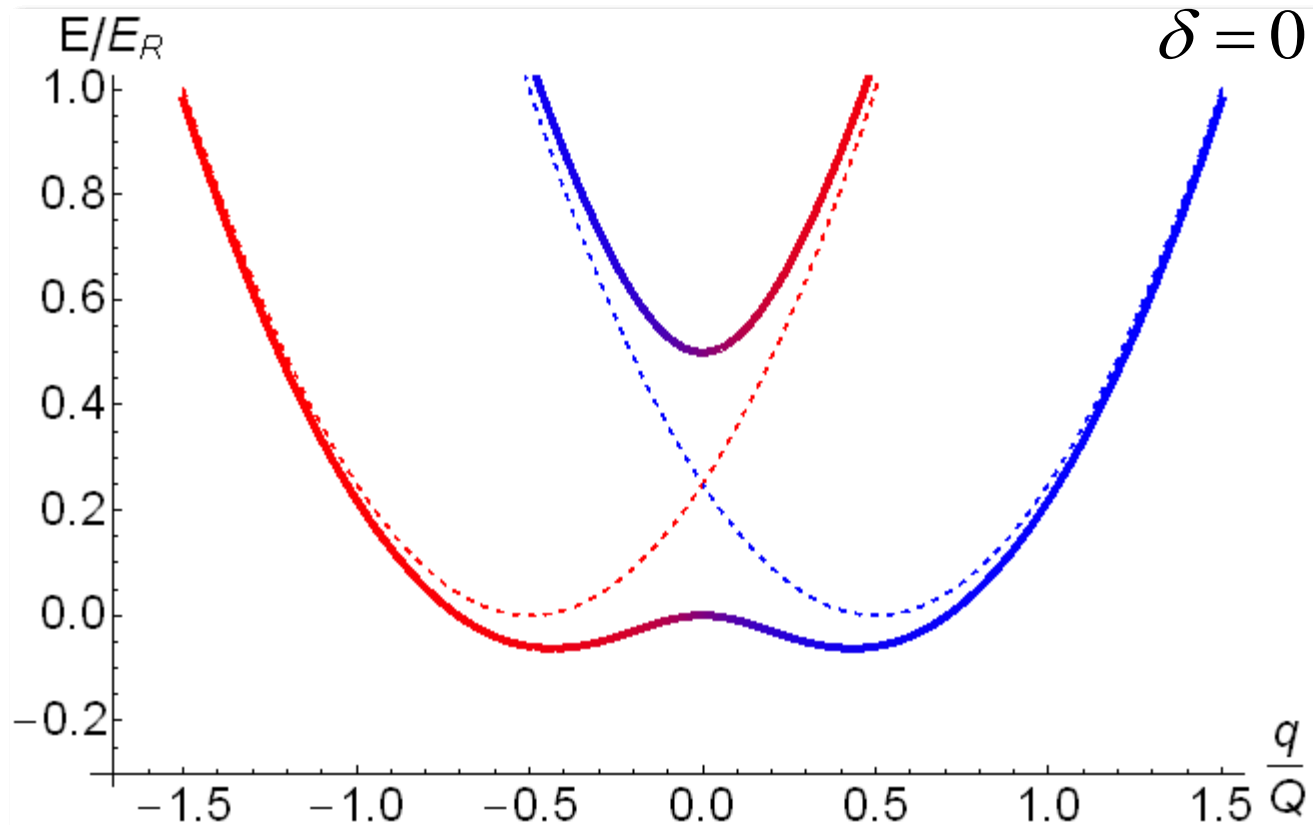
$$|\uparrow\rangle \quad q = k - Q/2$$

$$+ Q/2$$



$$- Q/2$$

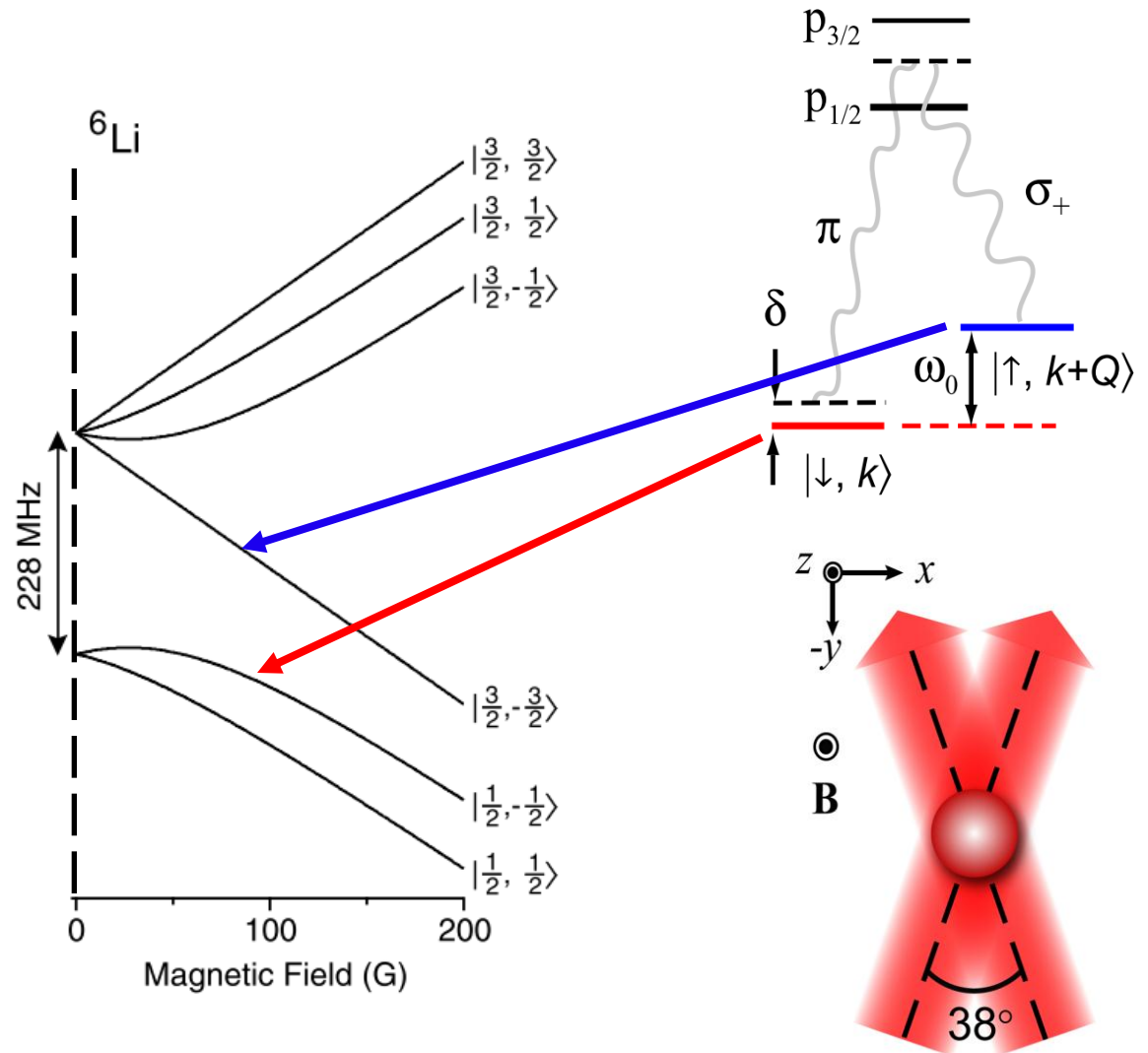
quasi-momentum space



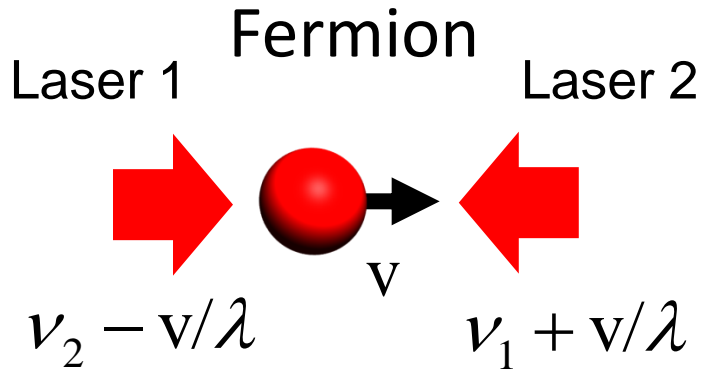
Experimental Setup



- Fermionic ${}^6\text{Li}$ atoms sympathetically cooled by ${}^{23}\text{Na}$
- Relevant states are 2nd and 3rd lowest states at 11G
- Interactions are negligible ($20a_0$)

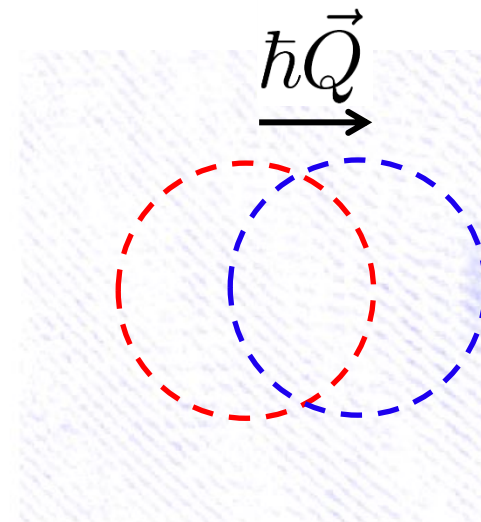
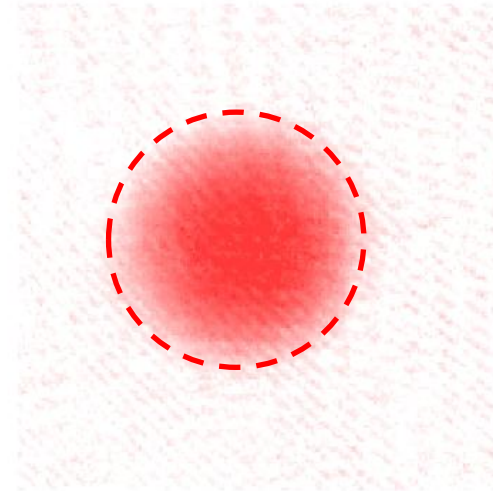


Coupling spin and momentum via Raman

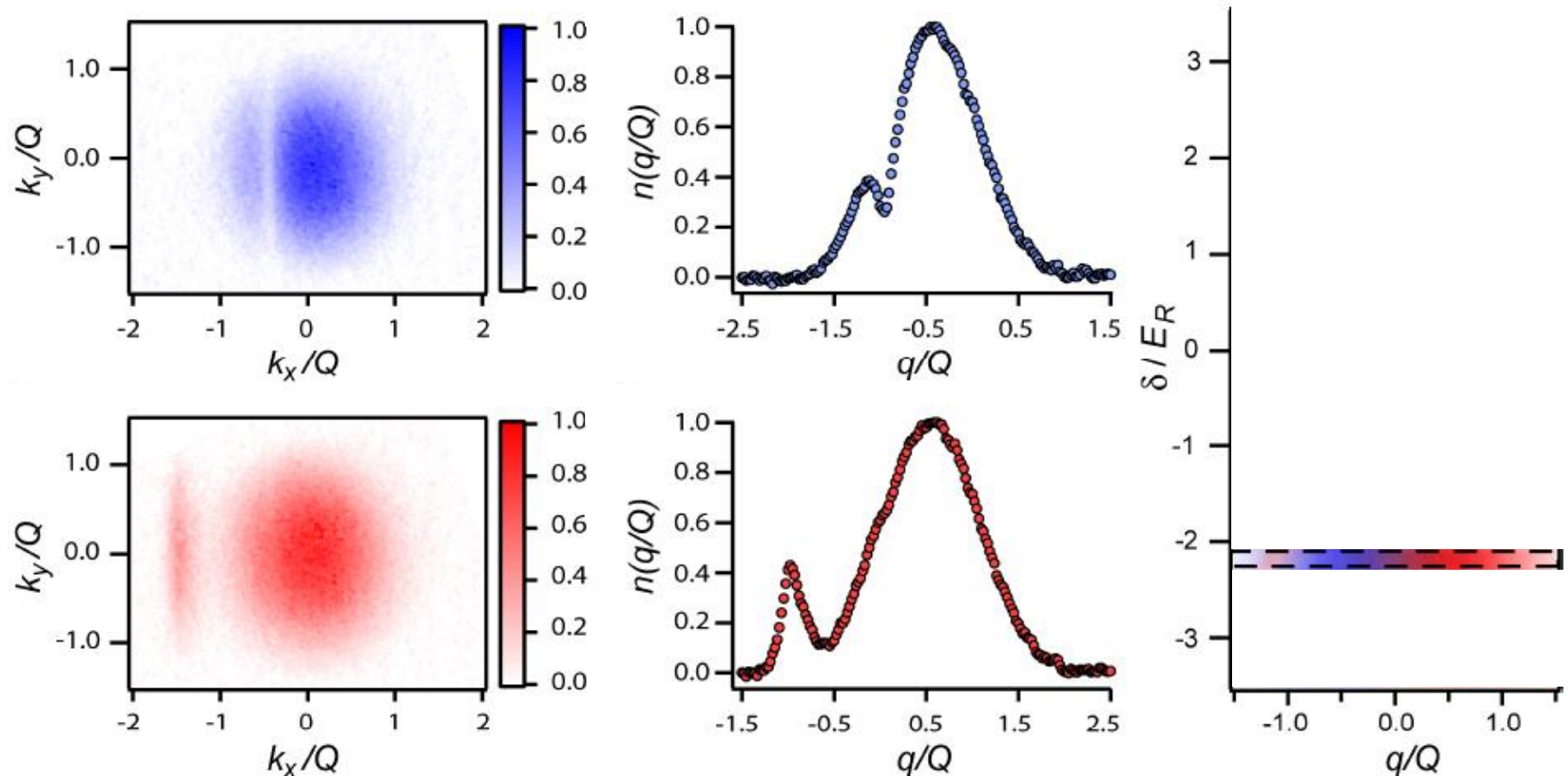


Vary detuning
Short pulse

State-selective imaging after TOF
provides
spin and momentum information

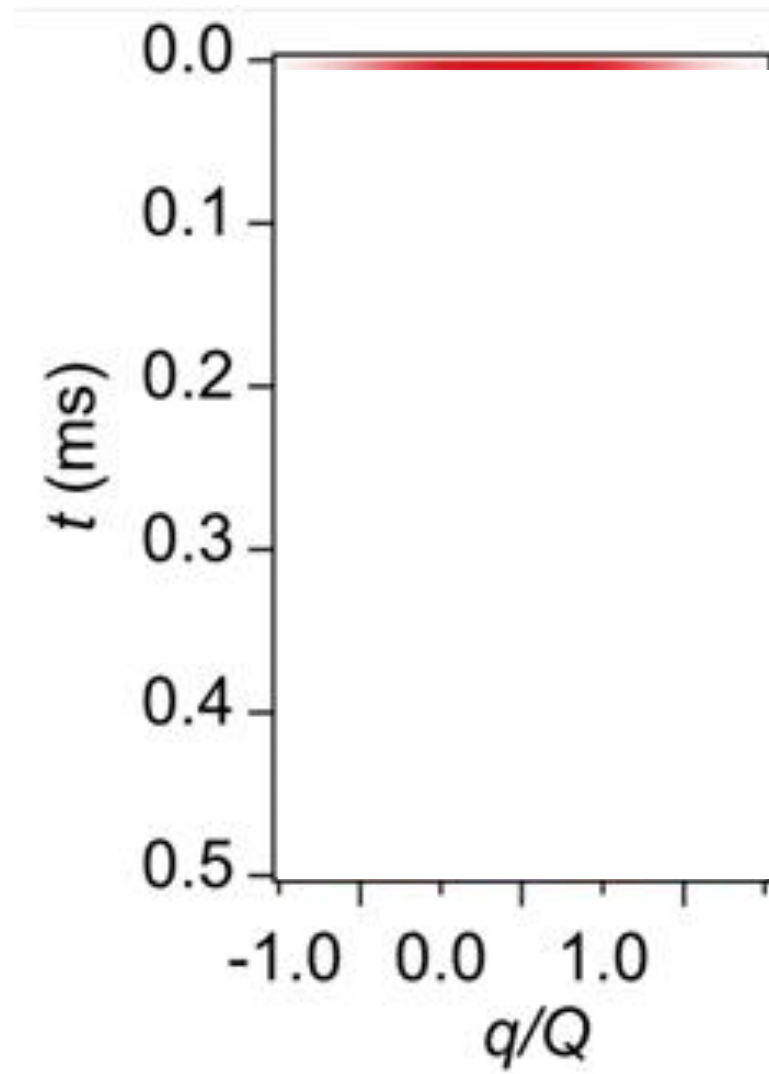


- Start with a mixture of $\left| \downarrow \right\rangle$ and $\left| \uparrow \right\rangle$, and apply a Raman pulse for a given δ

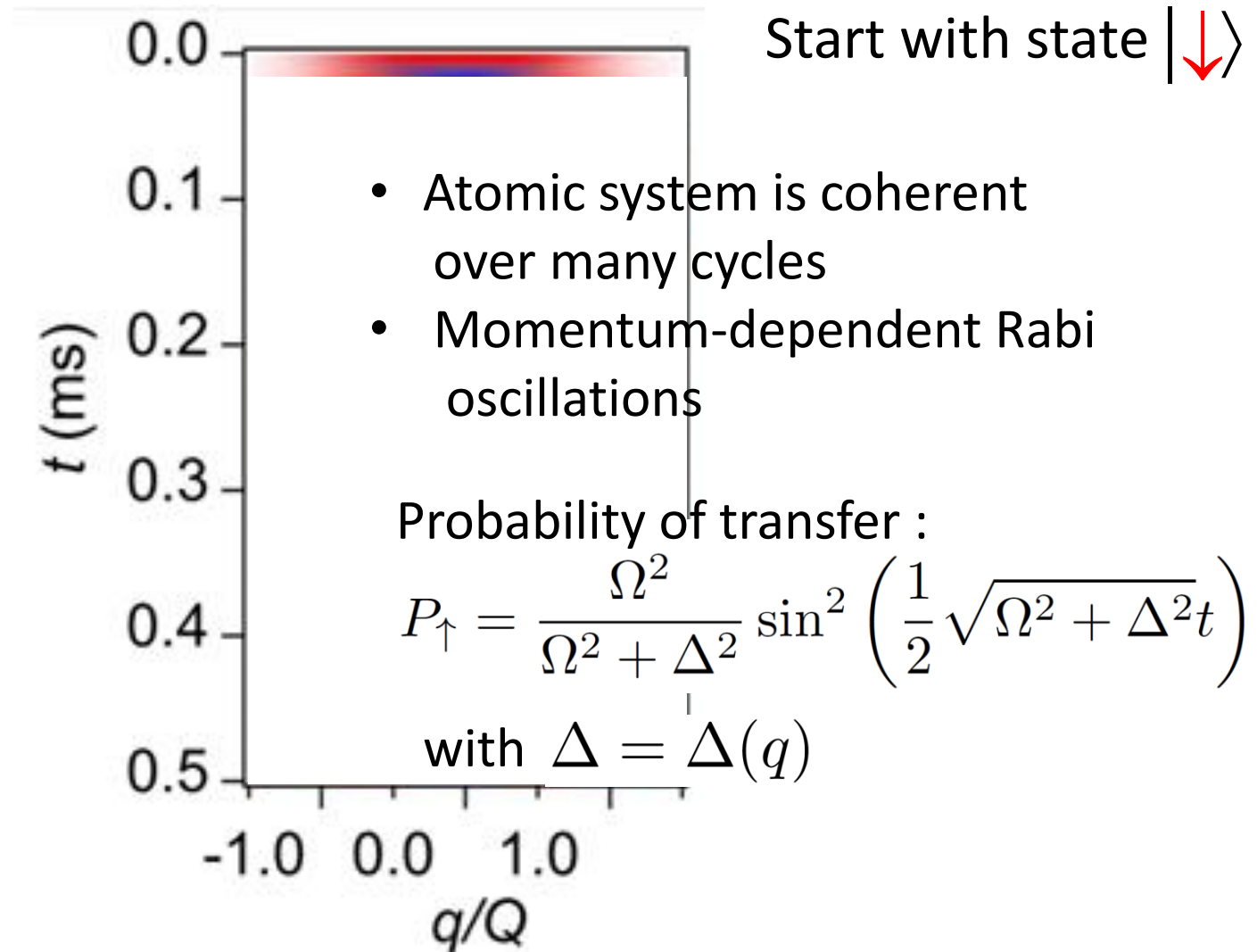


Check for the linear dependence of the transfer with momentum q
(Doppler shift $\propto k_x Q$)

Pulsing on Raman Beams



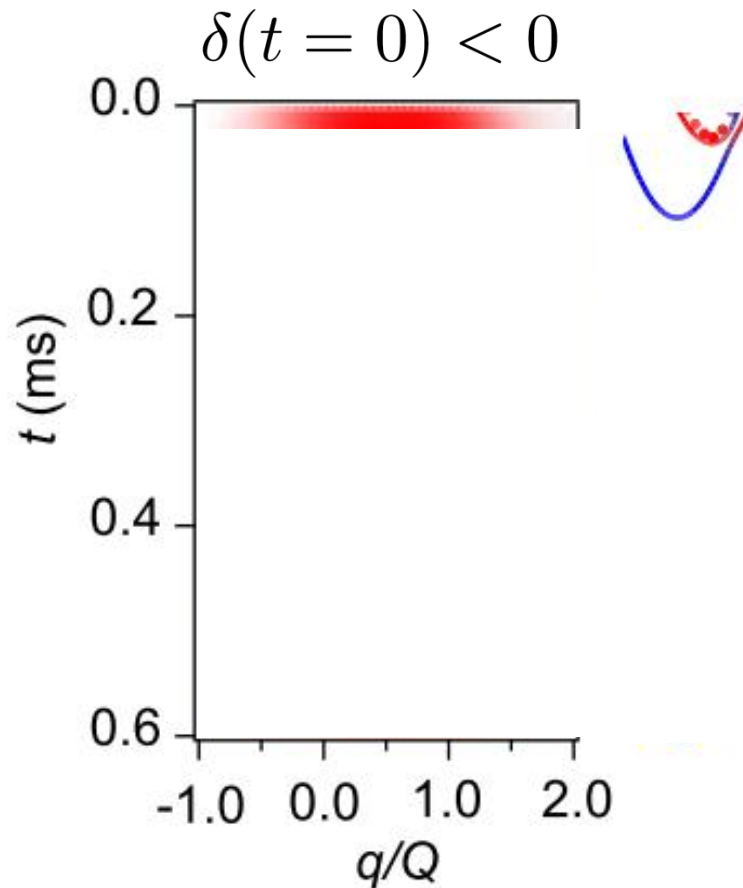
Start with state $|\downarrow\rangle$



Adiabatic Sweep



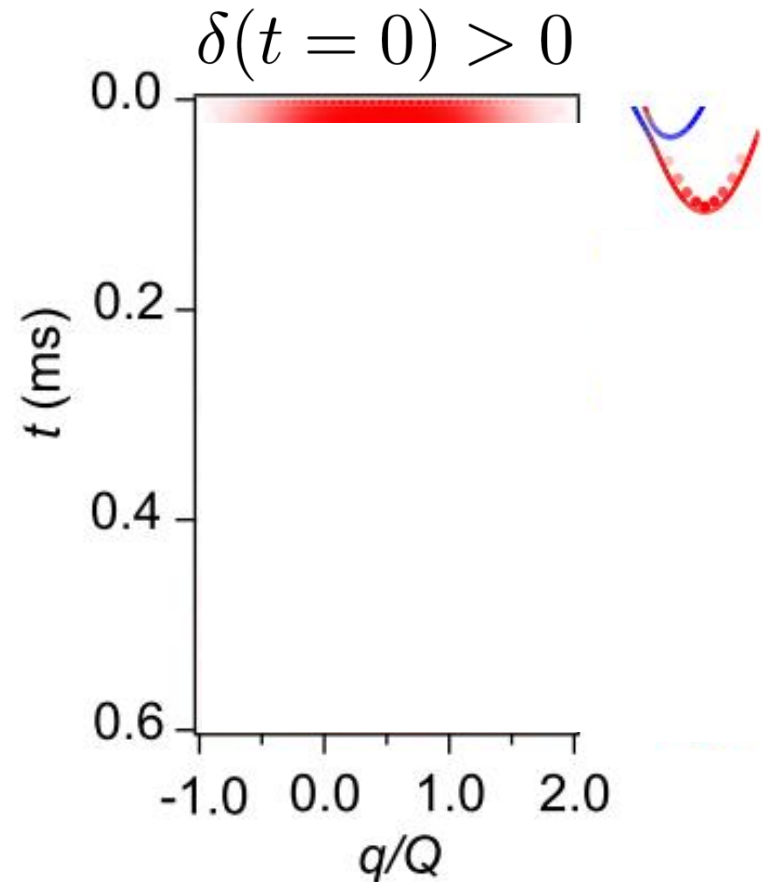
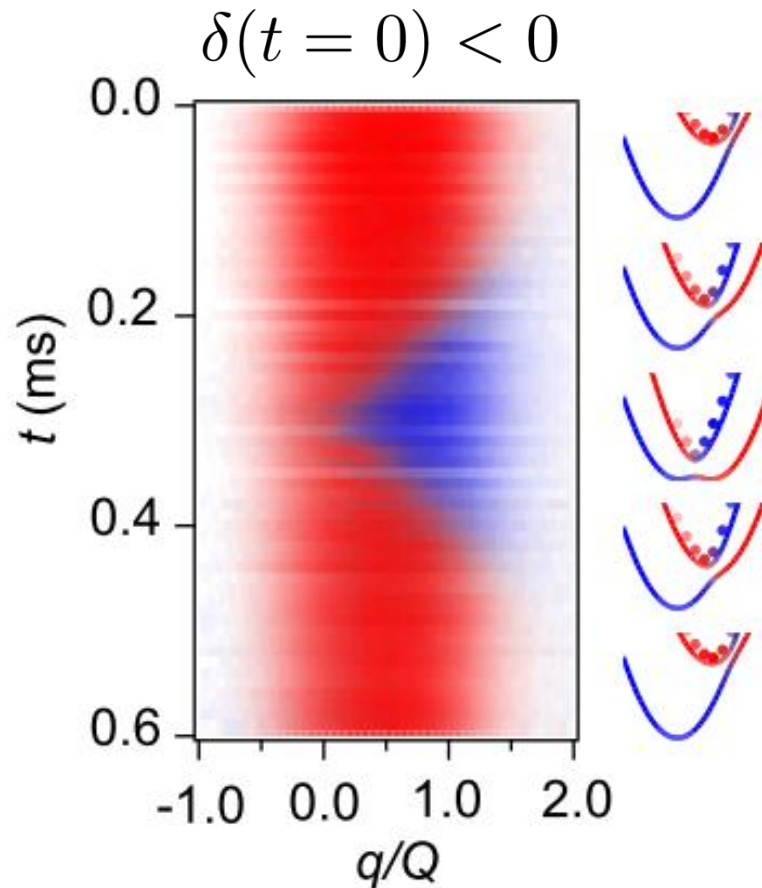
- Start with state $|\downarrow\rangle$
- Set large initial detuning ($|\delta| \gg E_R$) and then sweep



Adiabatic Sweep



- Start with state $|\downarrow\rangle$
- Set large initial detuning ($|\delta| \gg E_R$) and then sweep



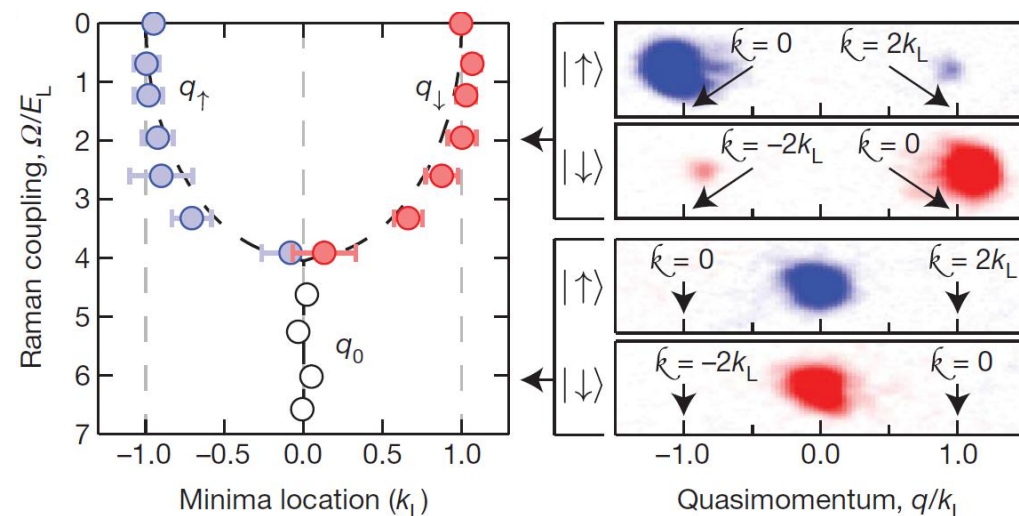
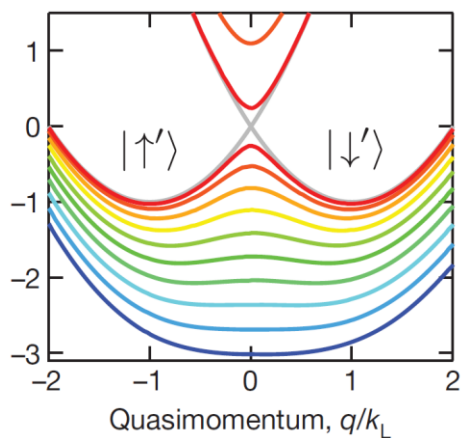
- How to characterize Hamiltonian?
 - Can topology be measured?
- Condensed matter: transport, (spin-)ARPES, STM ...
- Cold atom analog:
momentum resolved RF (Jin, Koehl)
(=photoemission spectroscopy)
- Photoemission Spectroscopy probes dispersion $E(k)$

What has been done so far



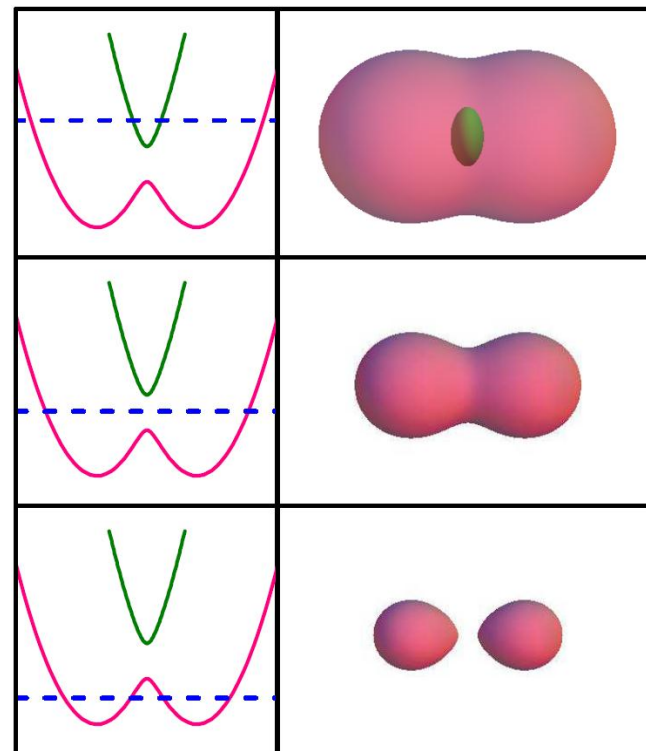
Y. J. Lin *et al.* *Nature* **471**, 83-86 (2011)

Ian Spielman's group



P. Wang *et al.* *arXiv:1204.1887*

(Jing Zhang's group)



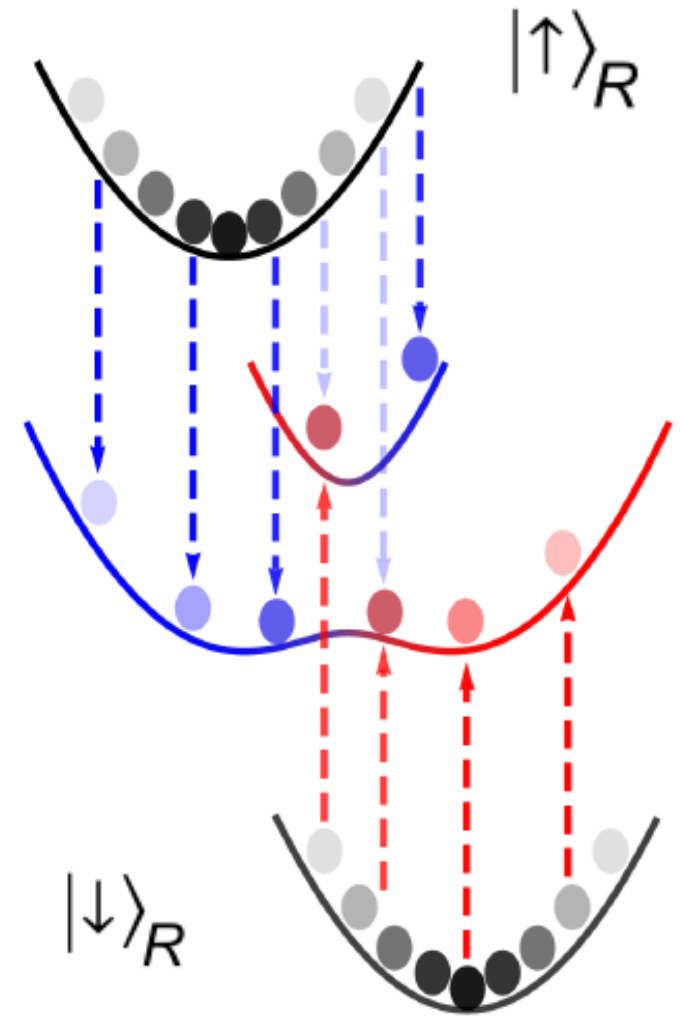
Can Topology be measured?

Spin-injection spectroscopy:

Measures spin, energy, momentum

1. Inject atoms from “reservoir”
2. Project into free space
3. Spin-selective imaging

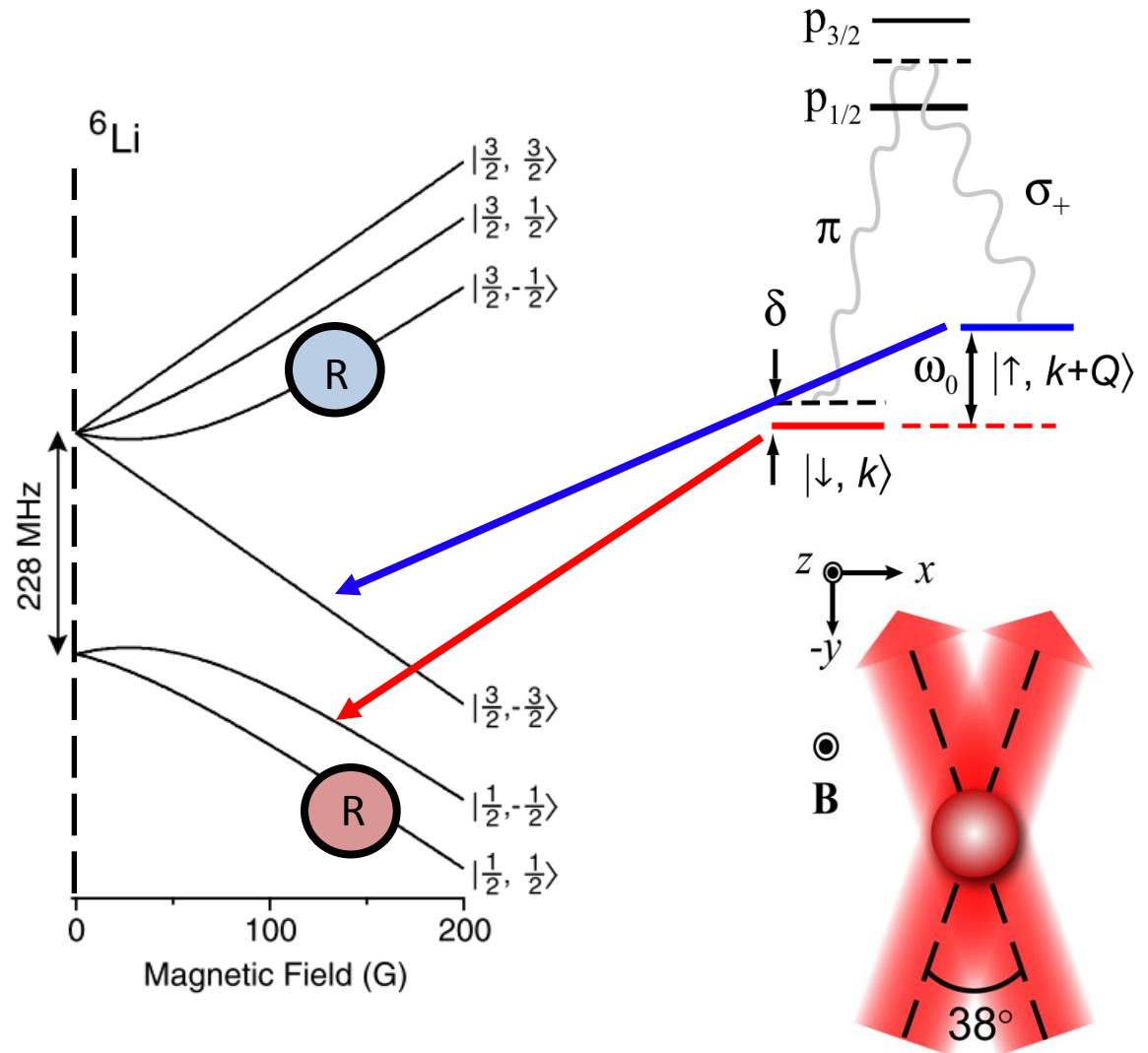
→ Reconstruct $E(k)$ along with
“color” of band

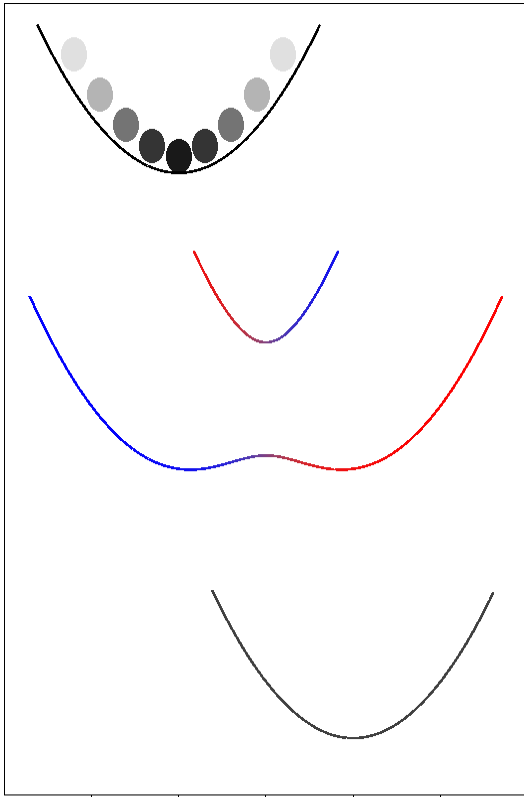


Experimental Setup

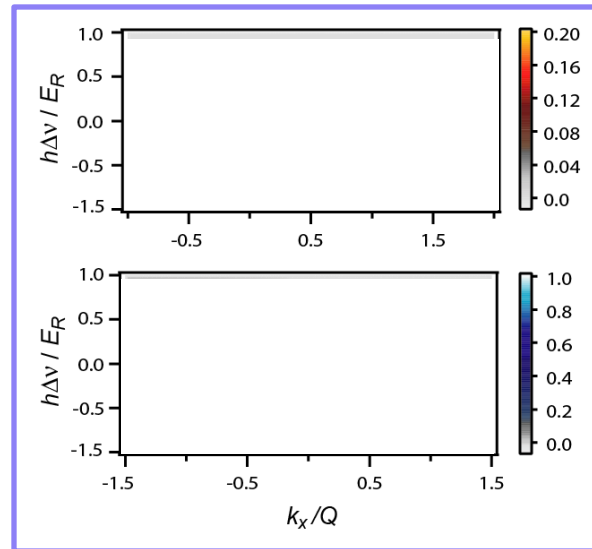
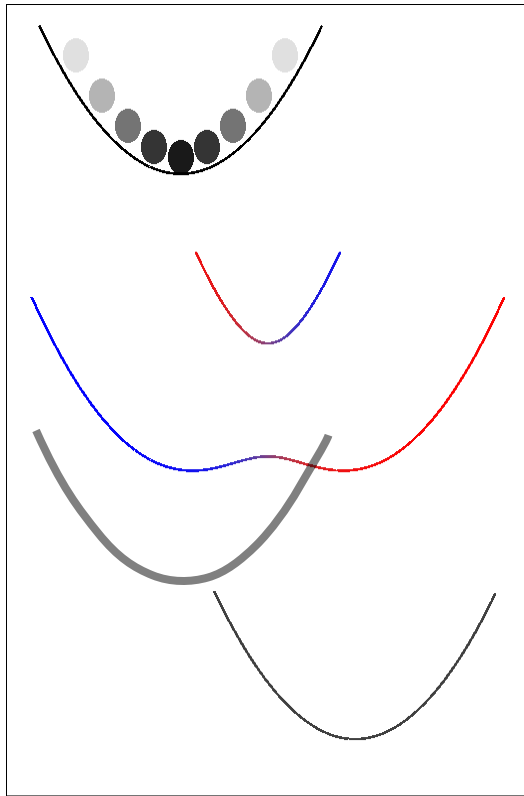


- 1st and 4th states used as reservoir states

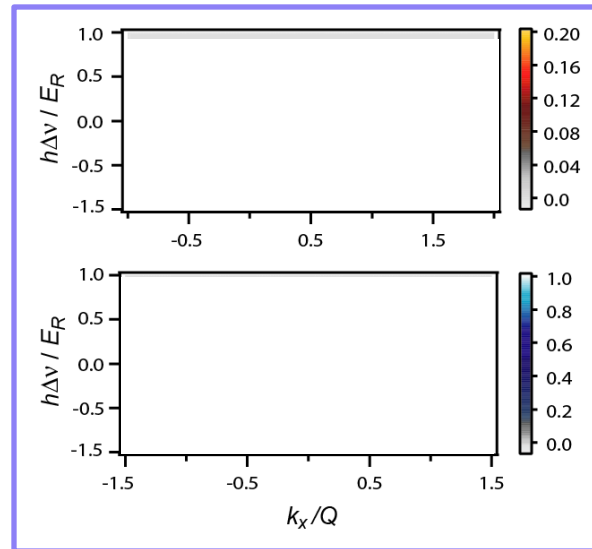
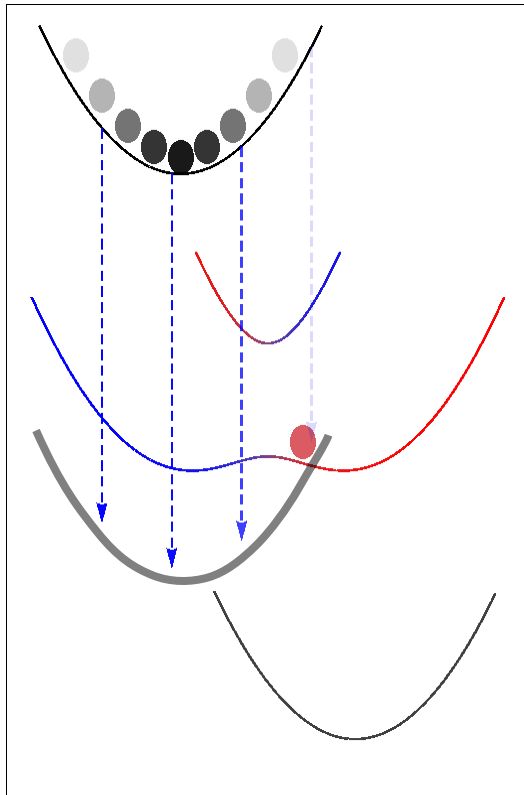




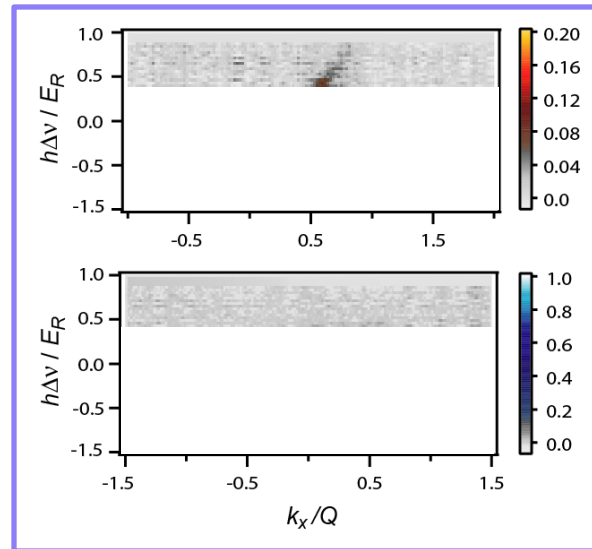
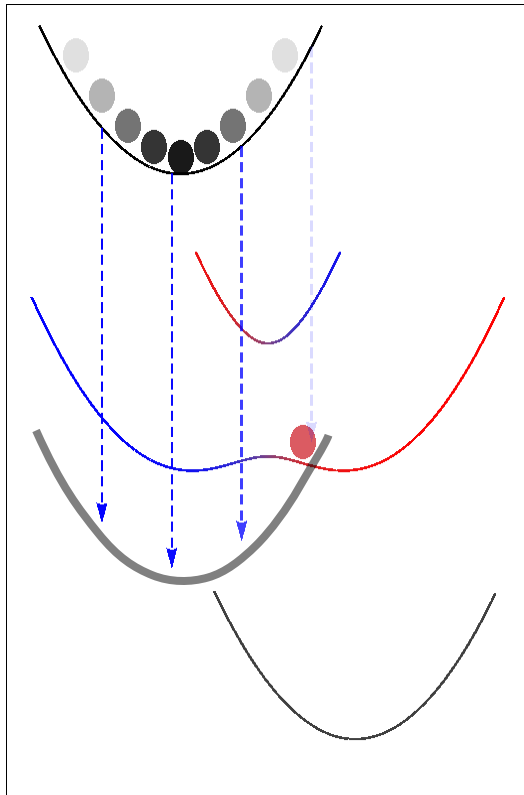
Spin-injection spectroscopy



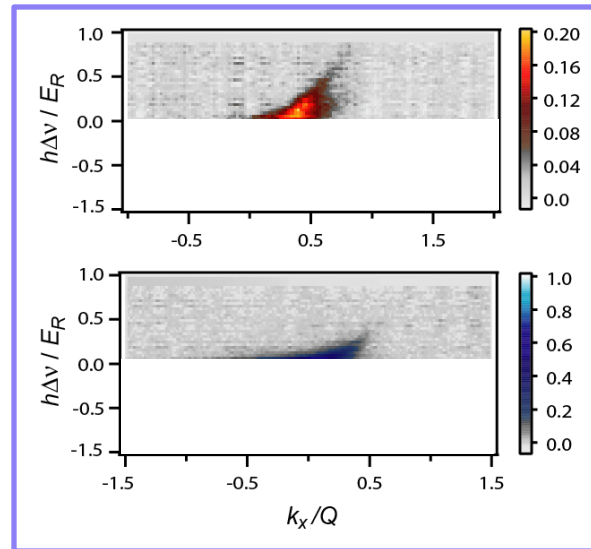
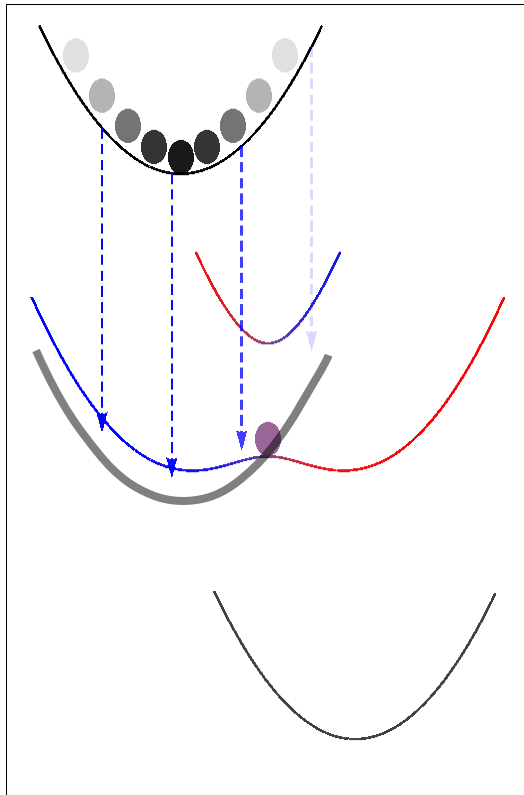
Spin-injection spectroscopy



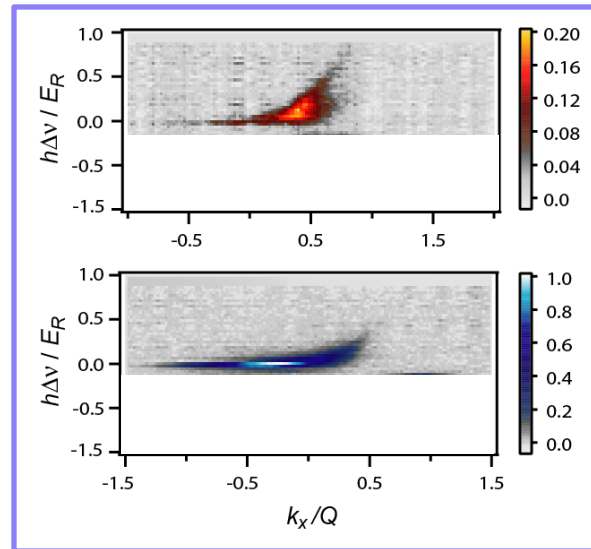
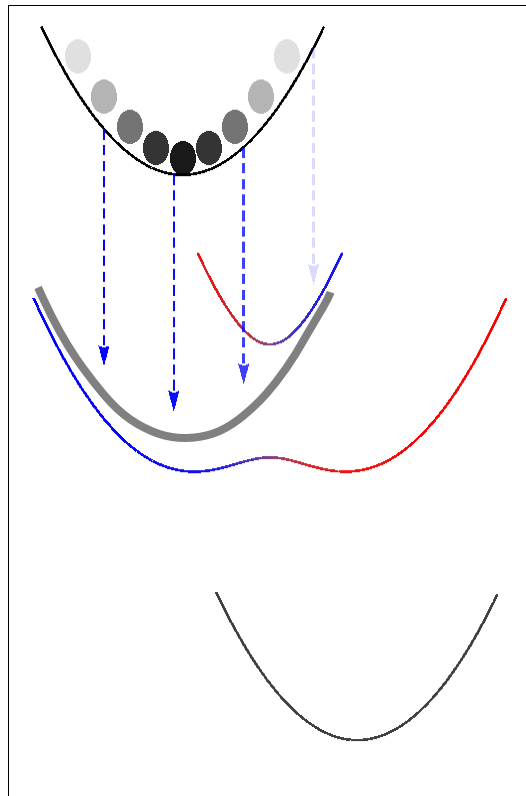
Spin-injection spectroscopy



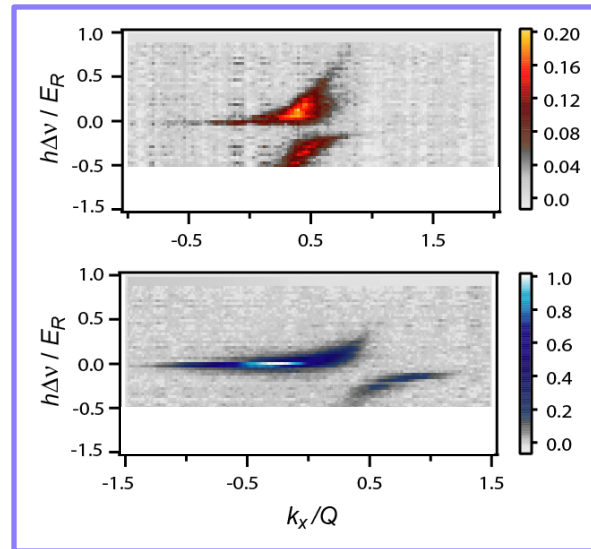
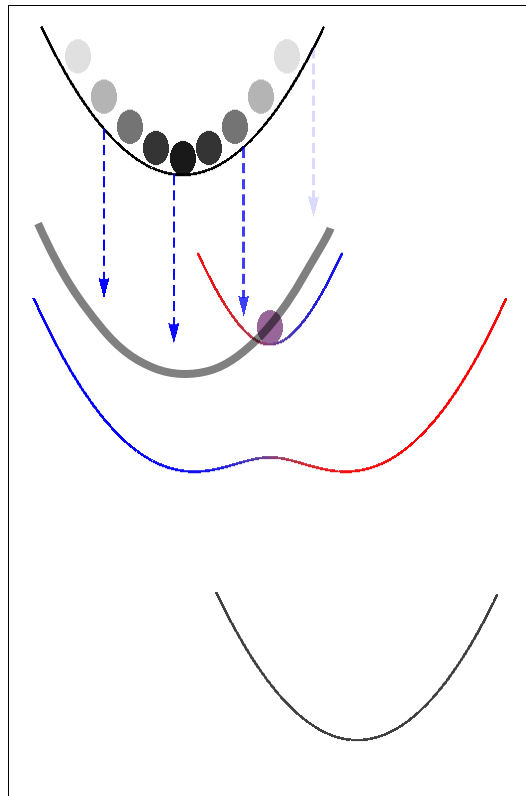
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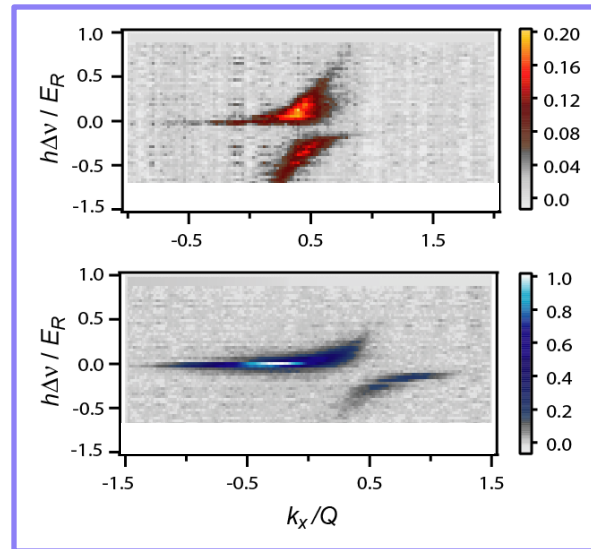
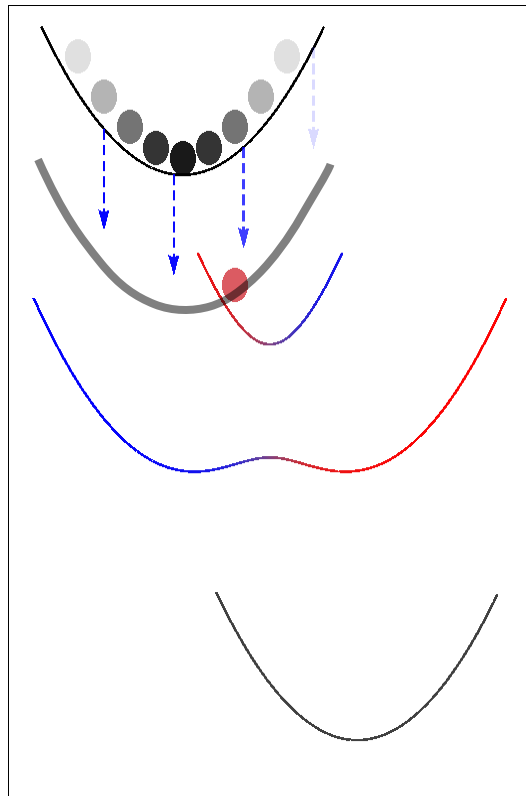
Spin-injection spectroscopy



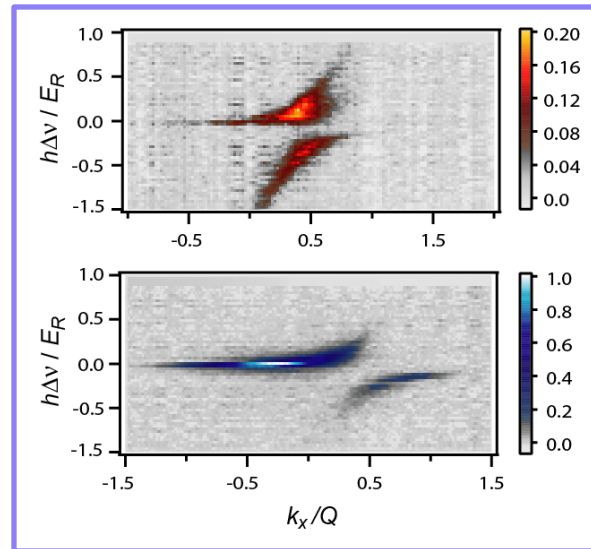
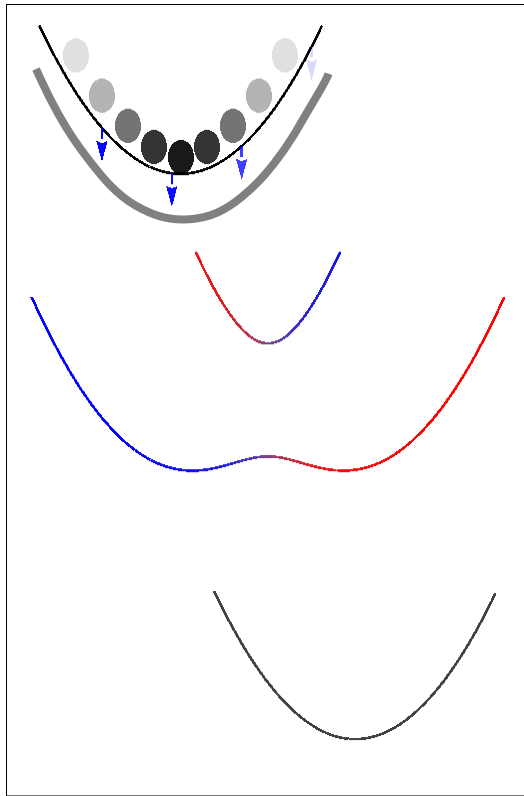
Spin-injection spectroscopy



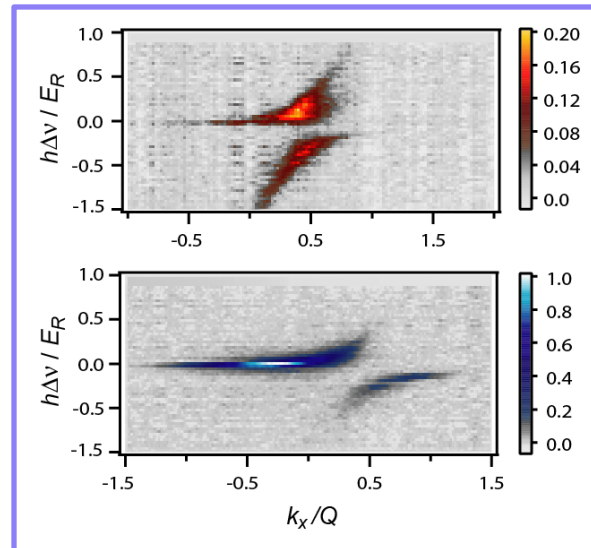
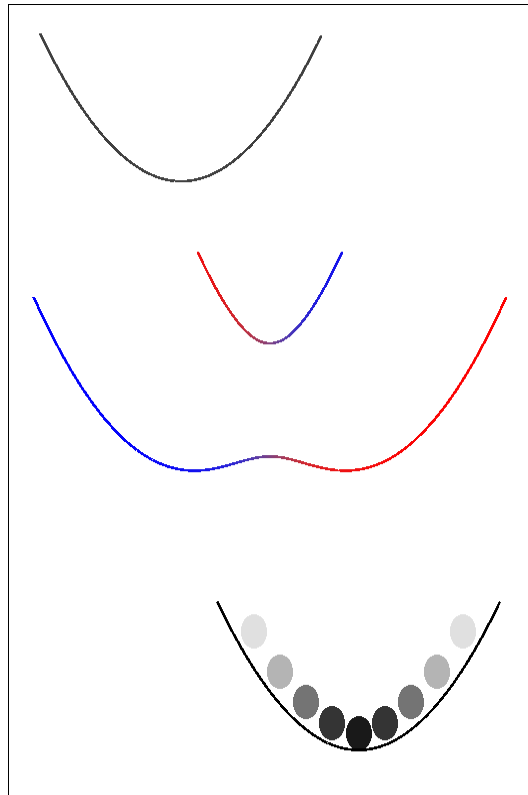
Spin-injection spectroscopy



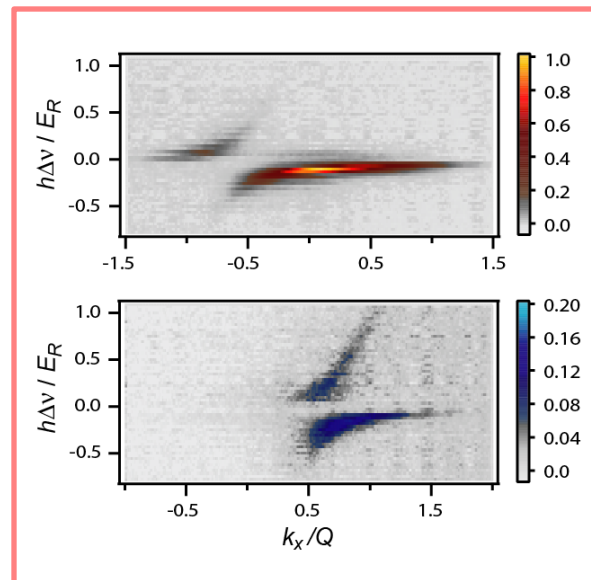
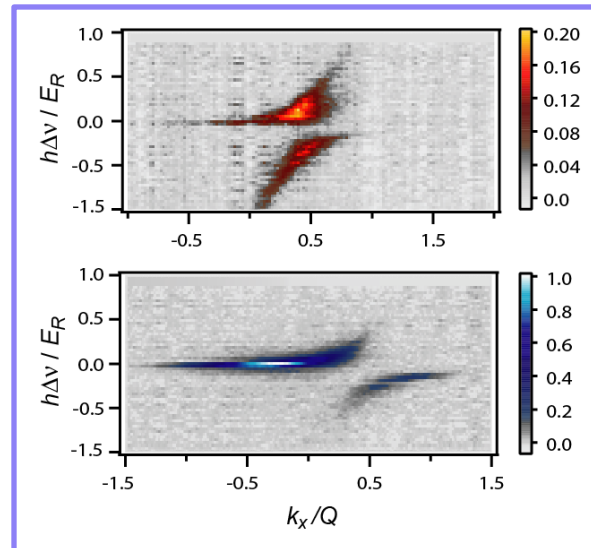
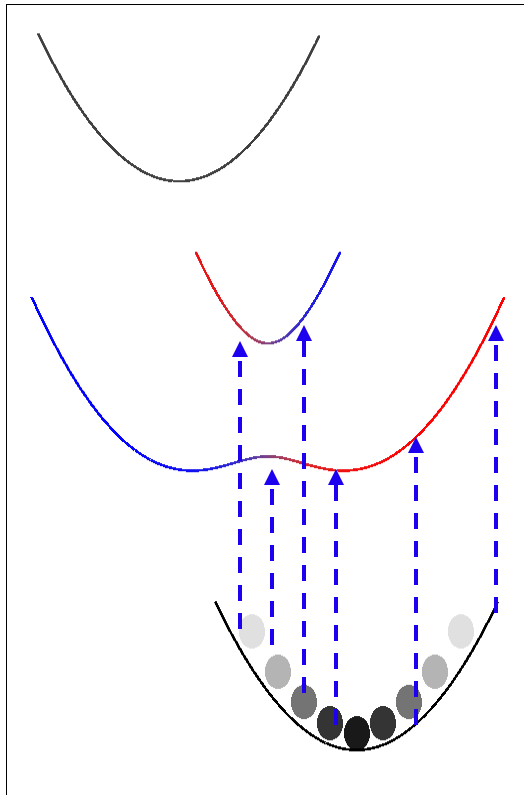
Spin-injection spectroscopy



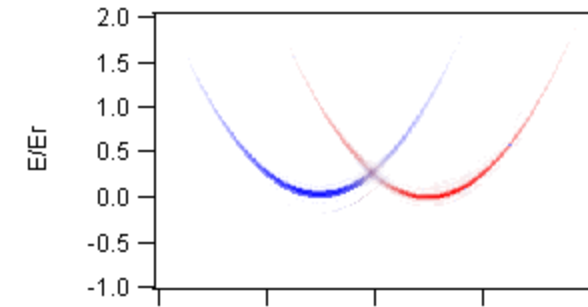
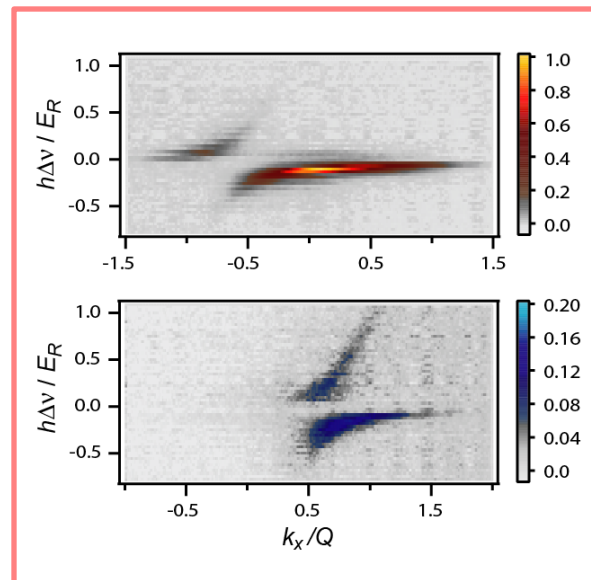
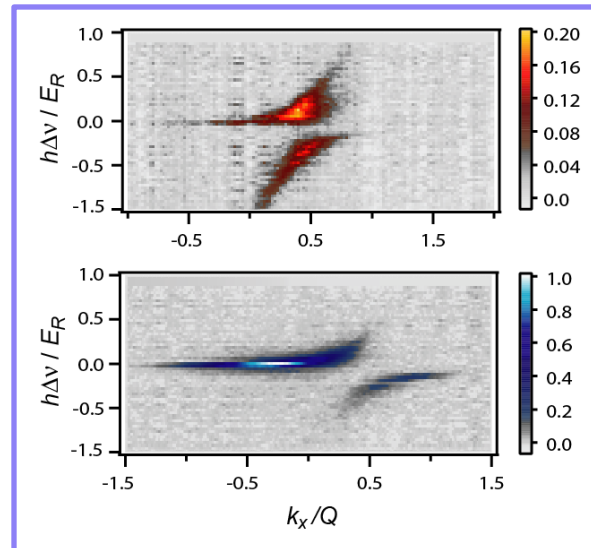
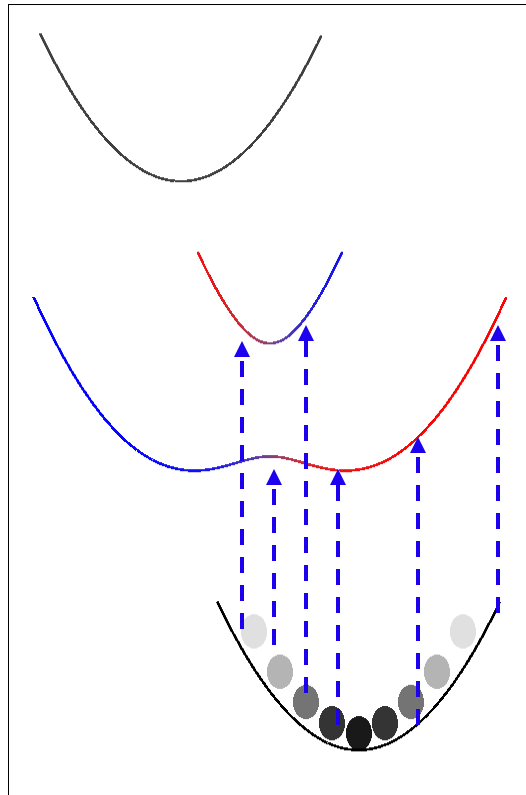
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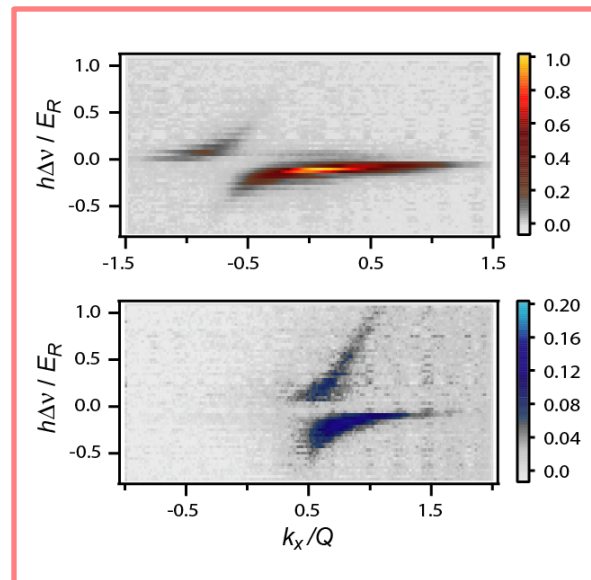
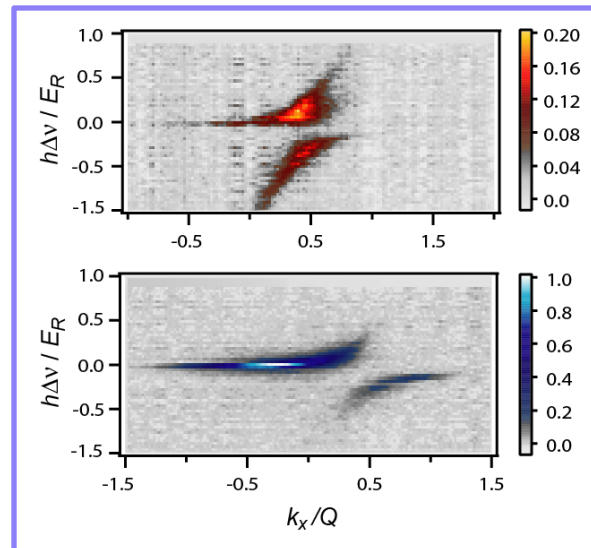
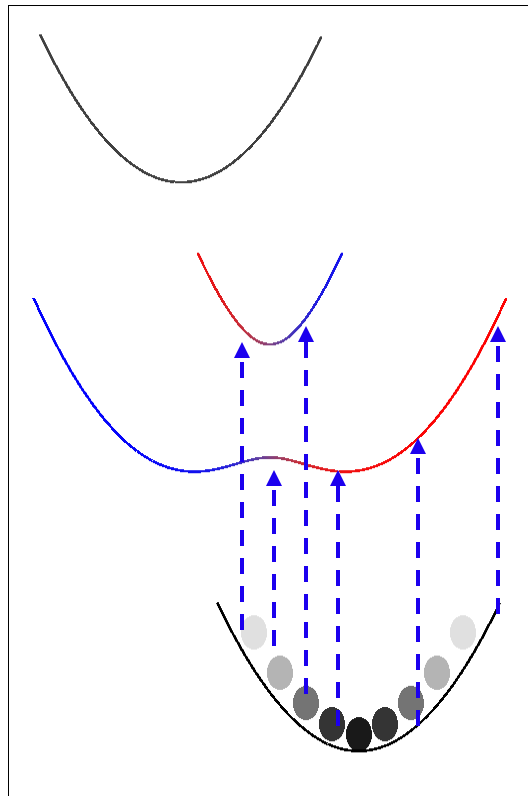
Spin-injection spectroscopy



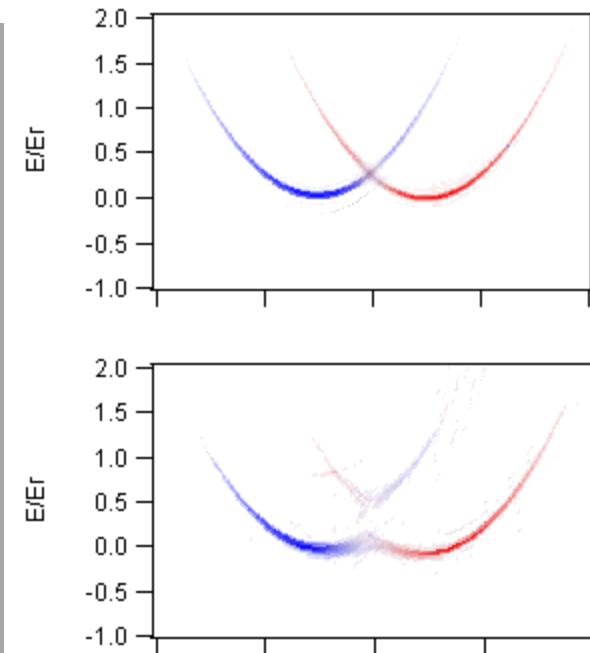
Spin-injection spectroscopy



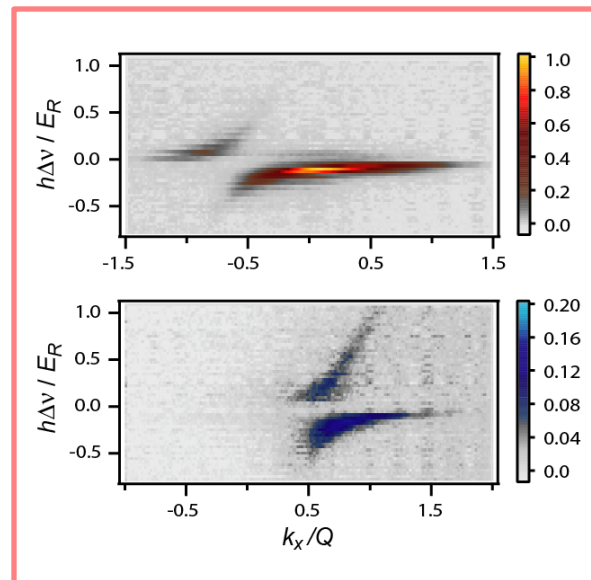
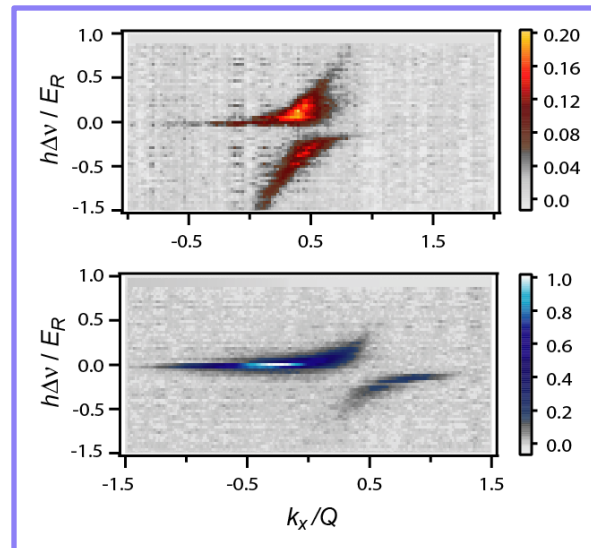
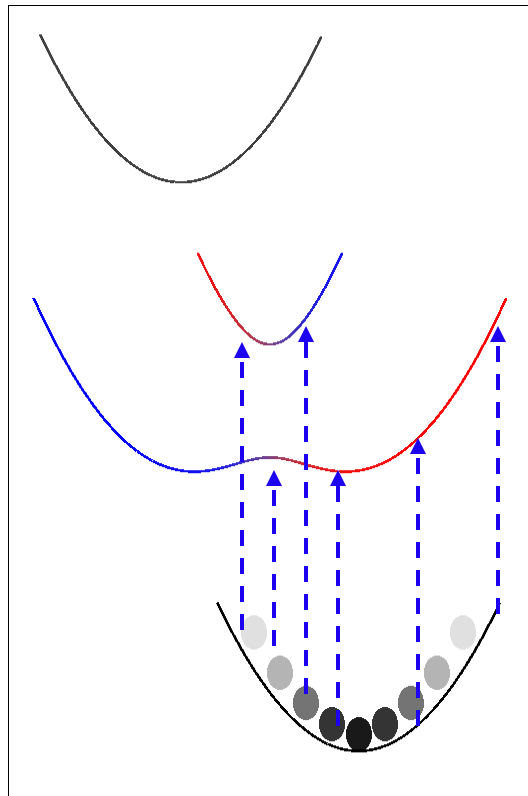
Spin-injection spectroscopy



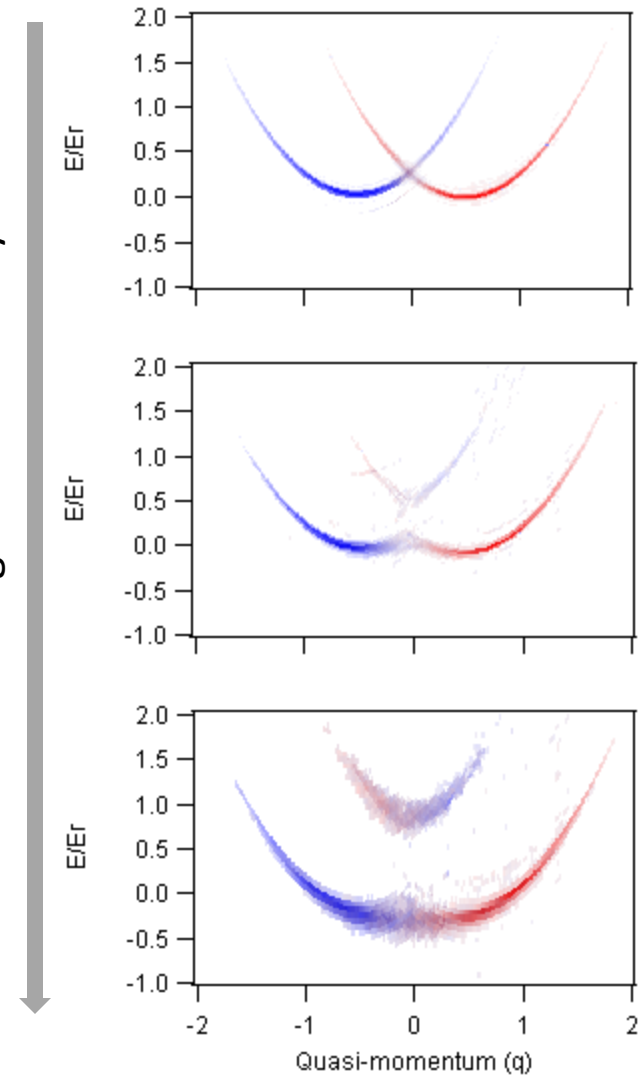
Increasing Raman Intensity



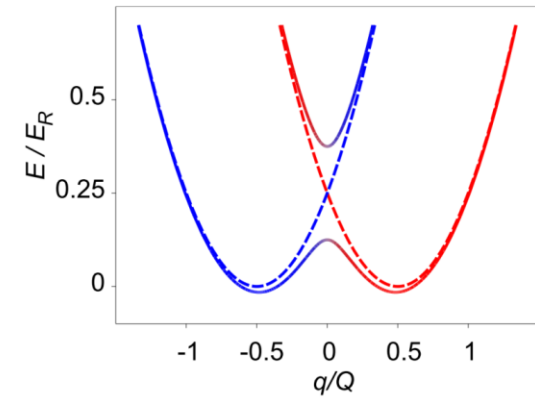
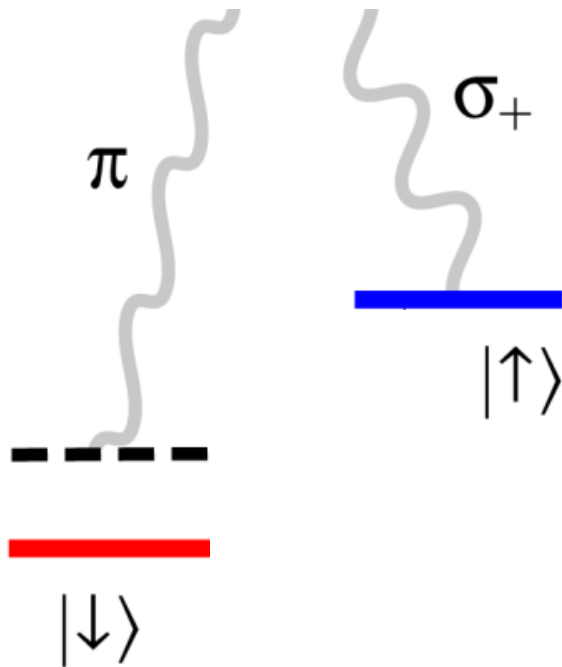
Spin-injection spectroscopy



Increasing Raman Intensity

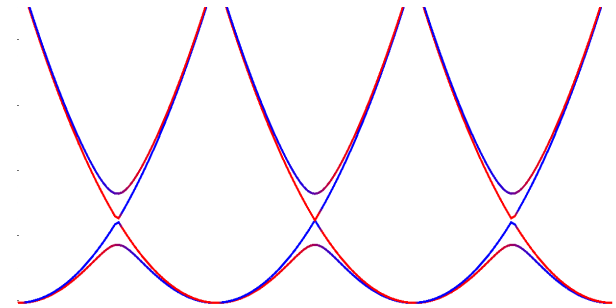
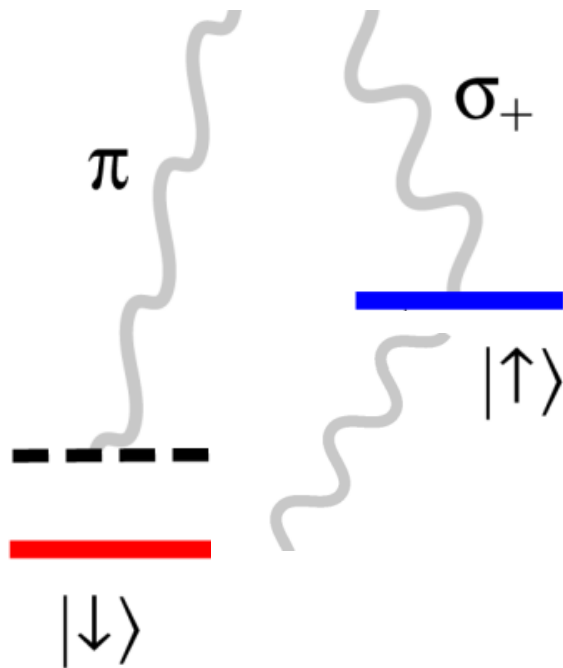


- Add RF coupling \rightarrow lattice system with full bandgaps and spinful bands



K. Jimenez-Garcia et al **PRL** **108**, 225303 (2012)

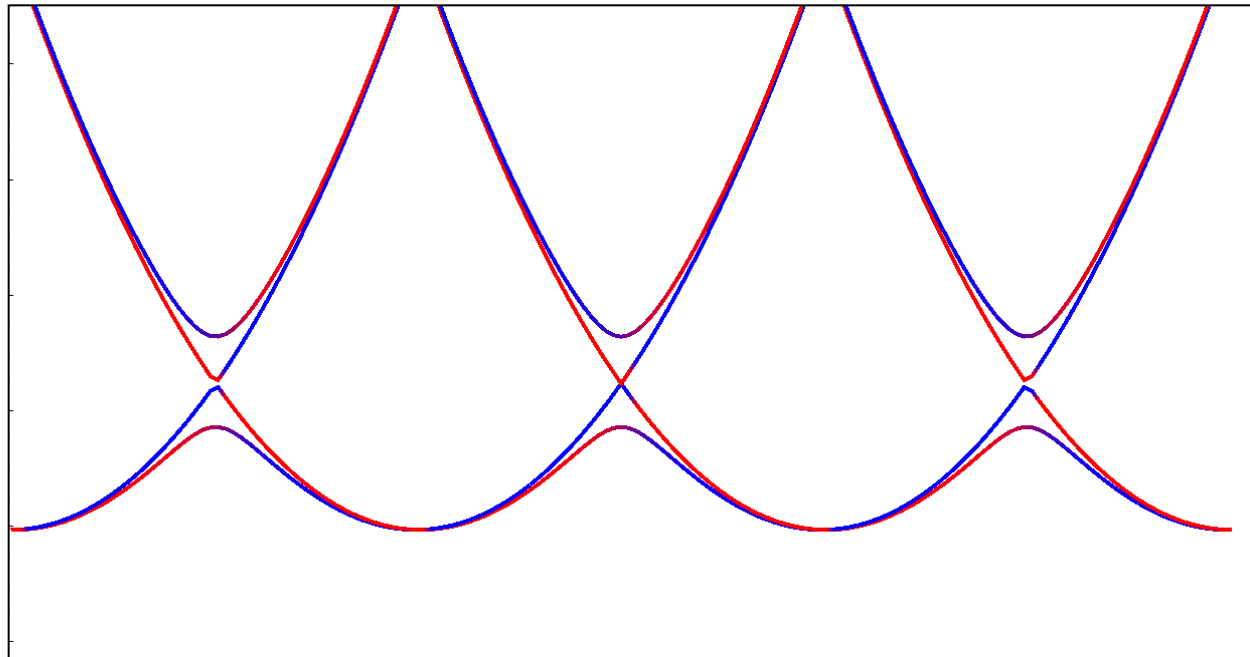
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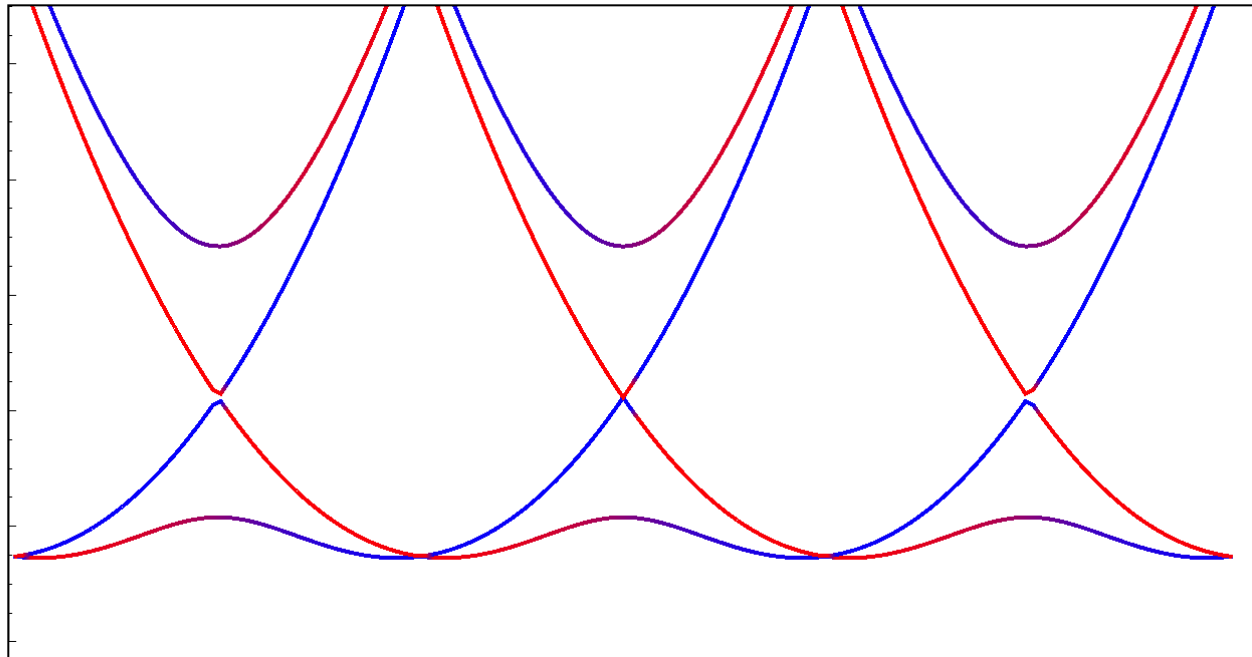
The Spin-Orbit band structure is periodically repeated

K. Jimenez-Garcia et al **PRL** **108**, 225303 (2012)

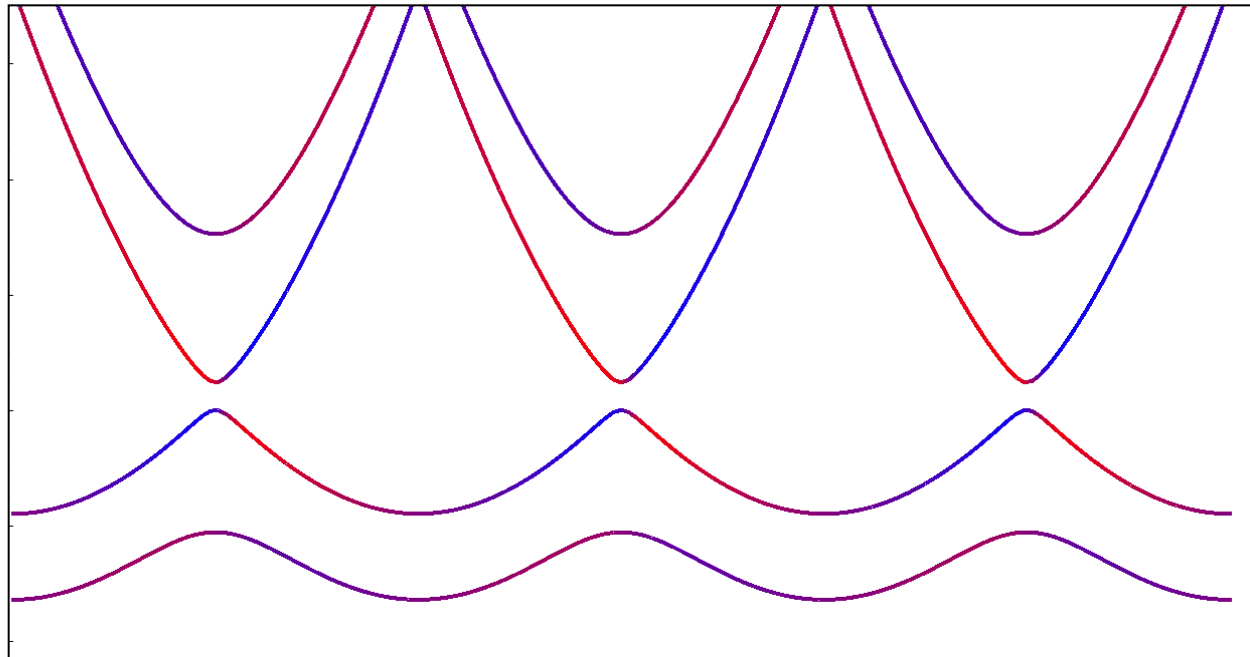
- In repeated scheme



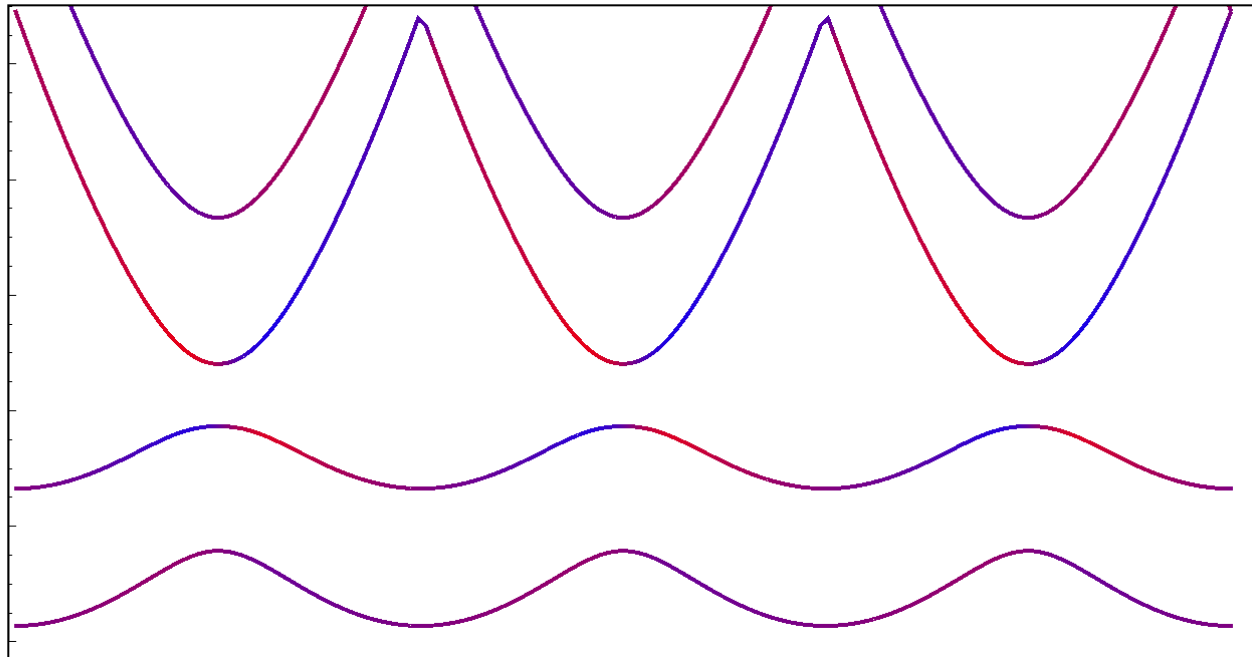
- Degenerate point inside spin orbit gap



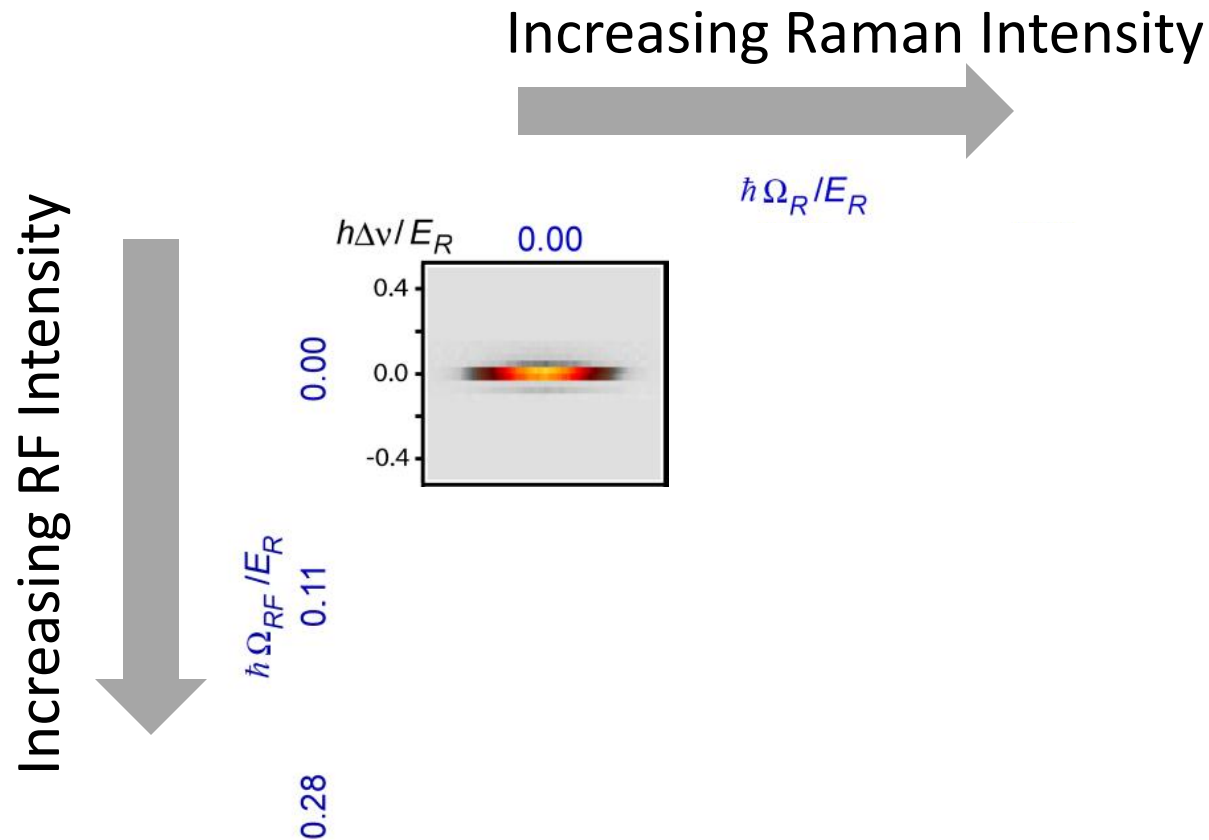
- Bandgap opens between 2nd and 3rd band



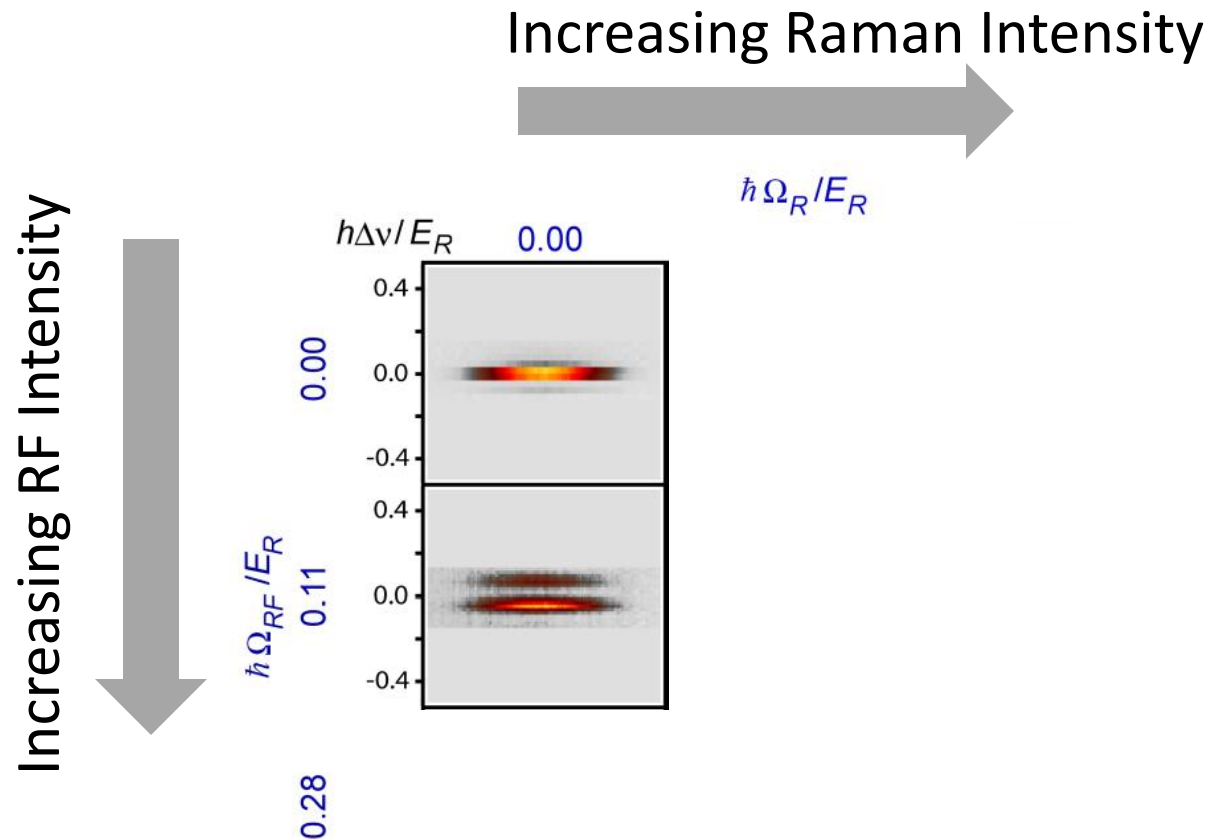
- Larger RF, gap between lowest bands



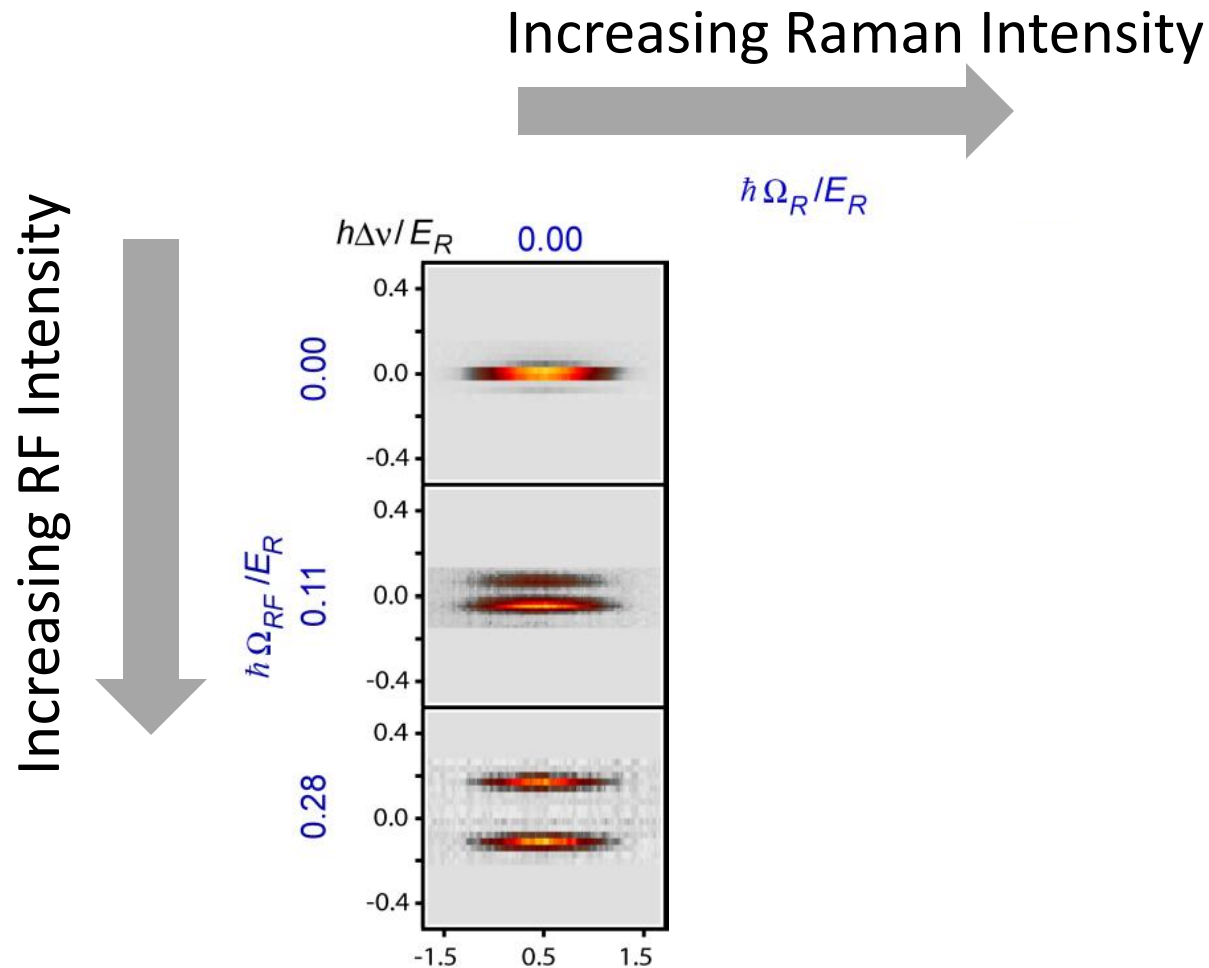
Spin-injection Spectra



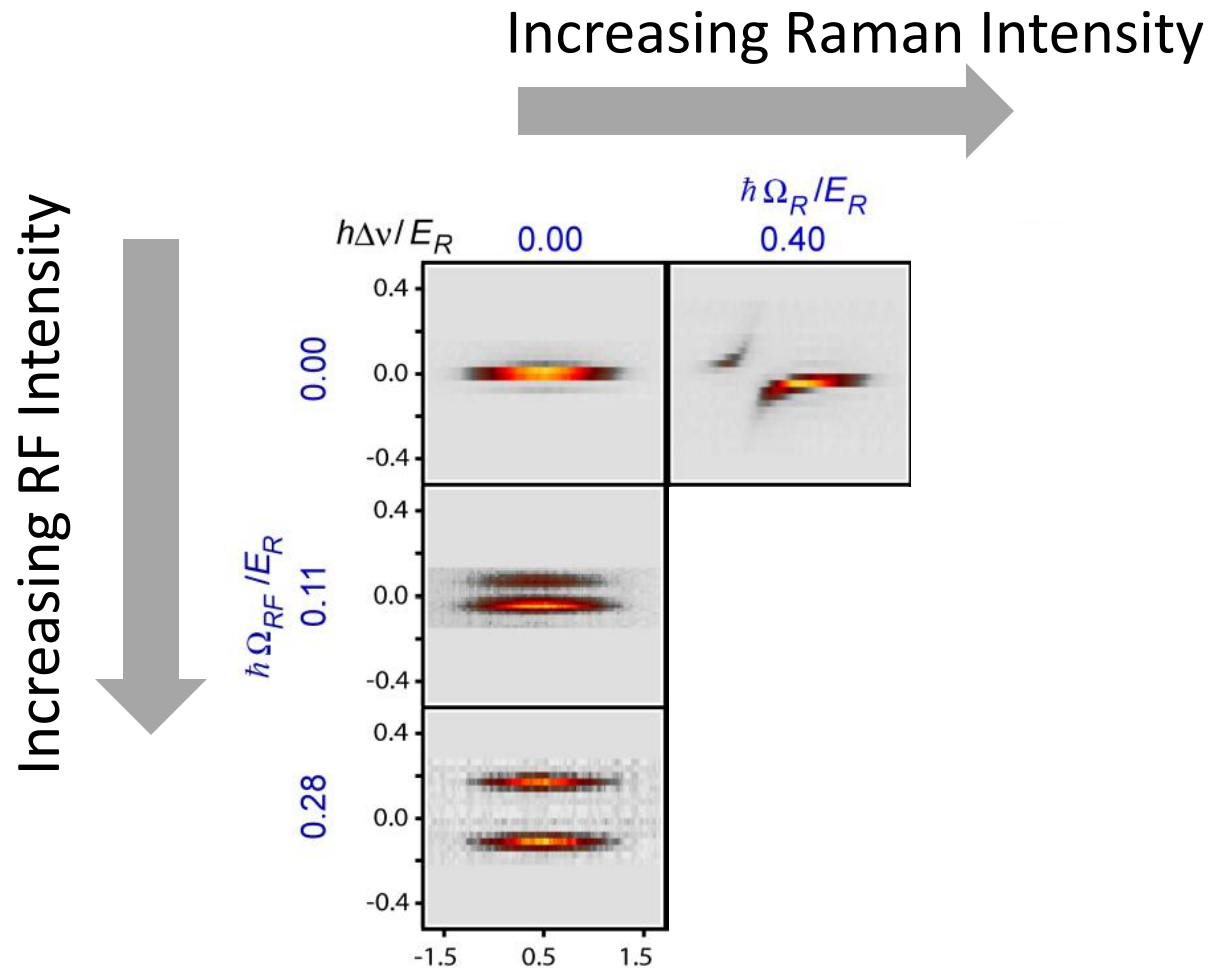
Spin-injection Spectra



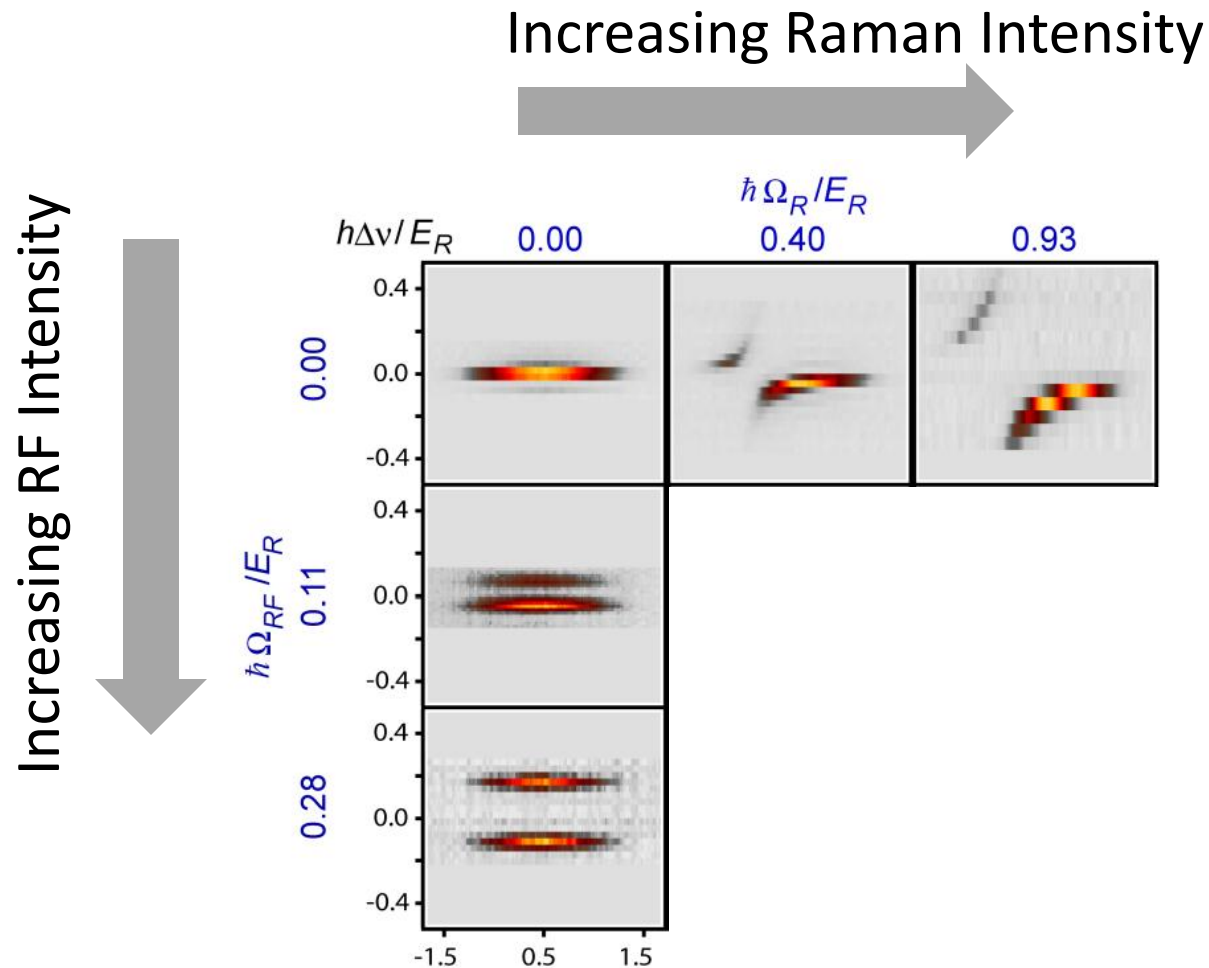
Spin-injection Spectra



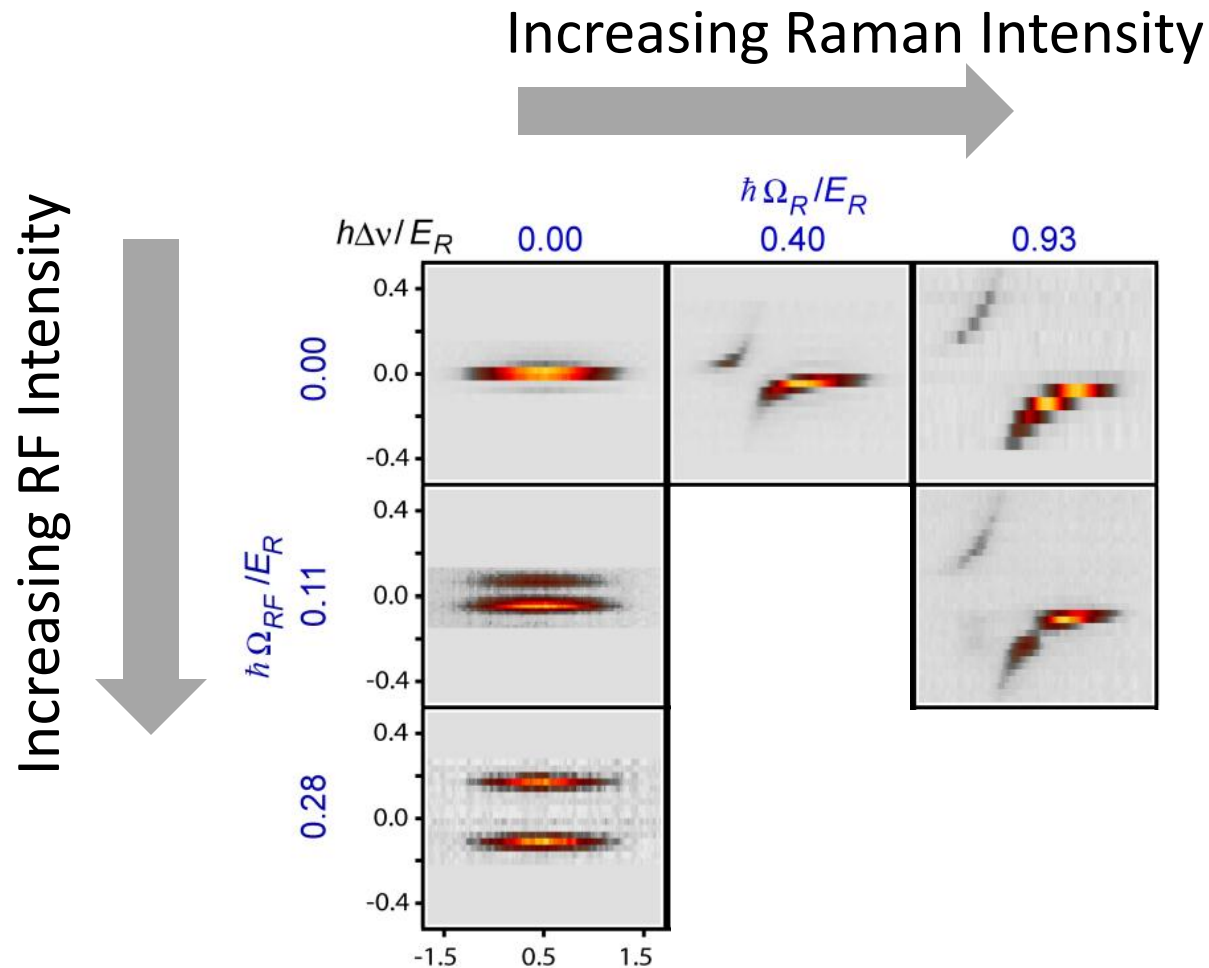
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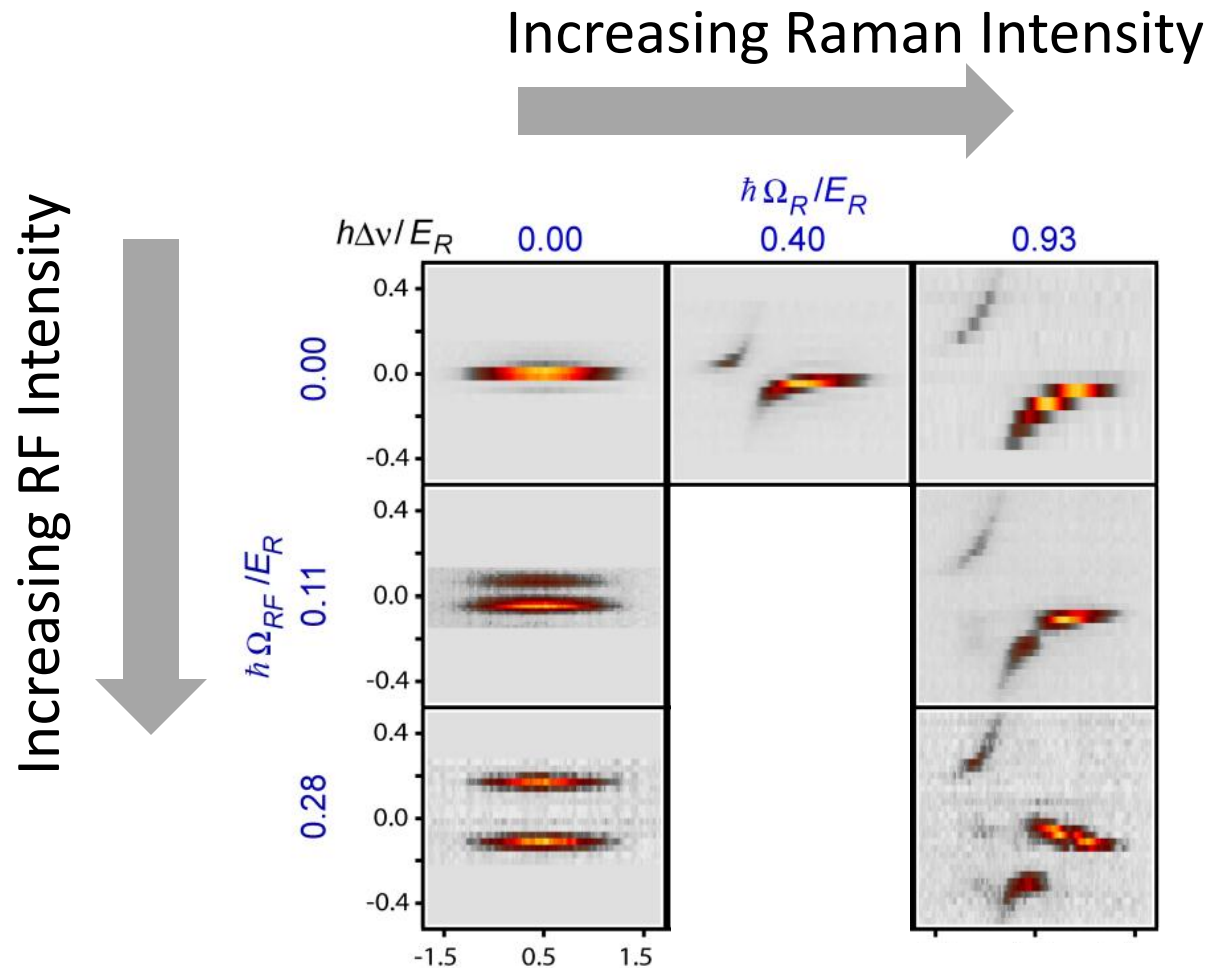
Spin-injection Spectra



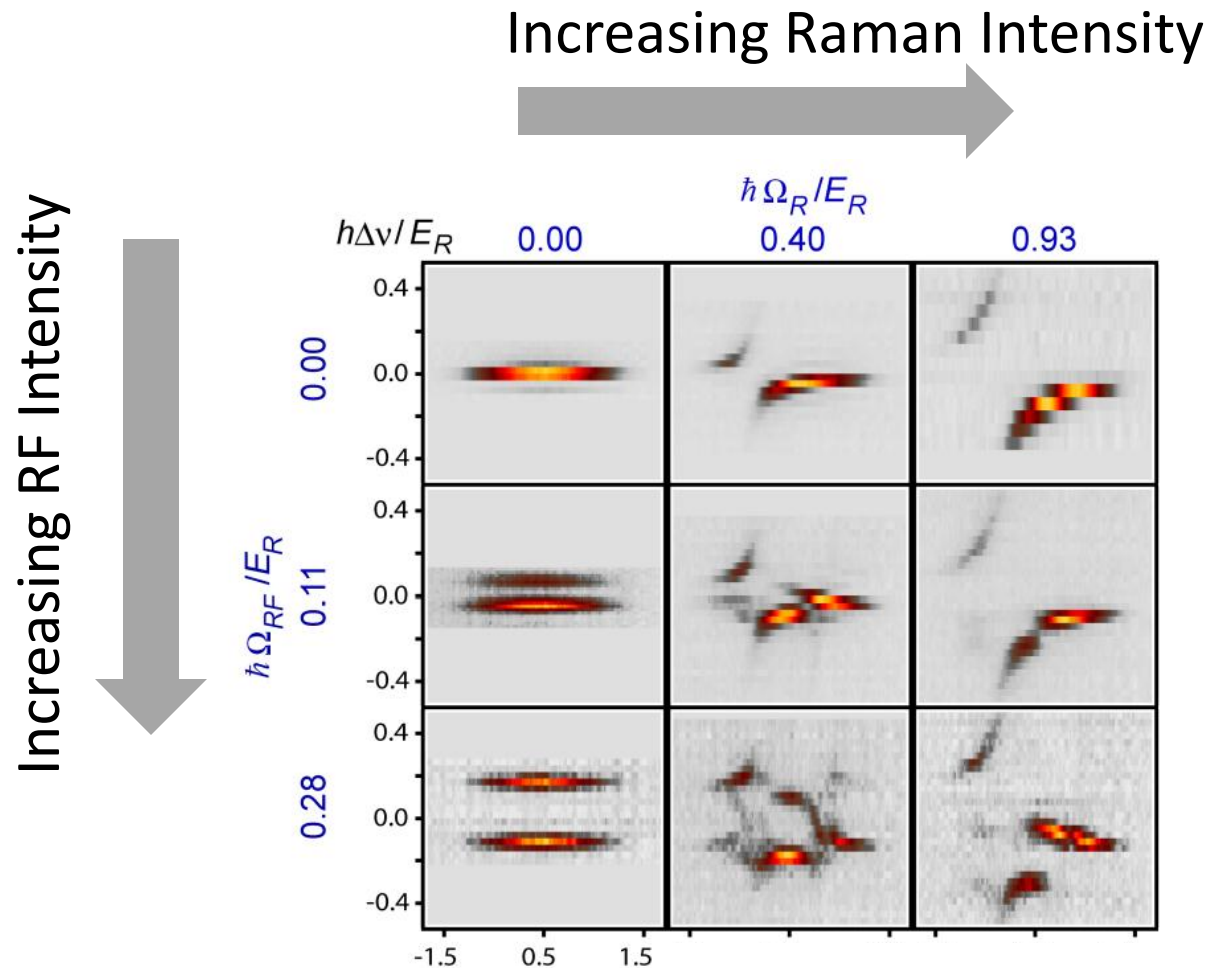
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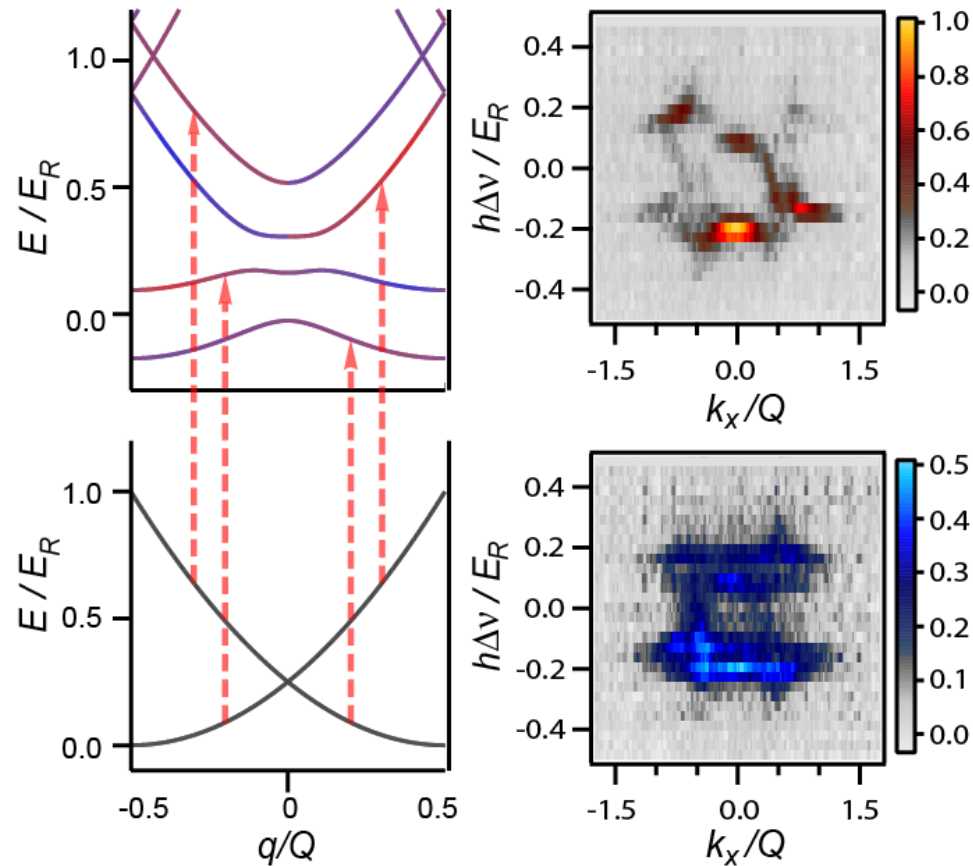
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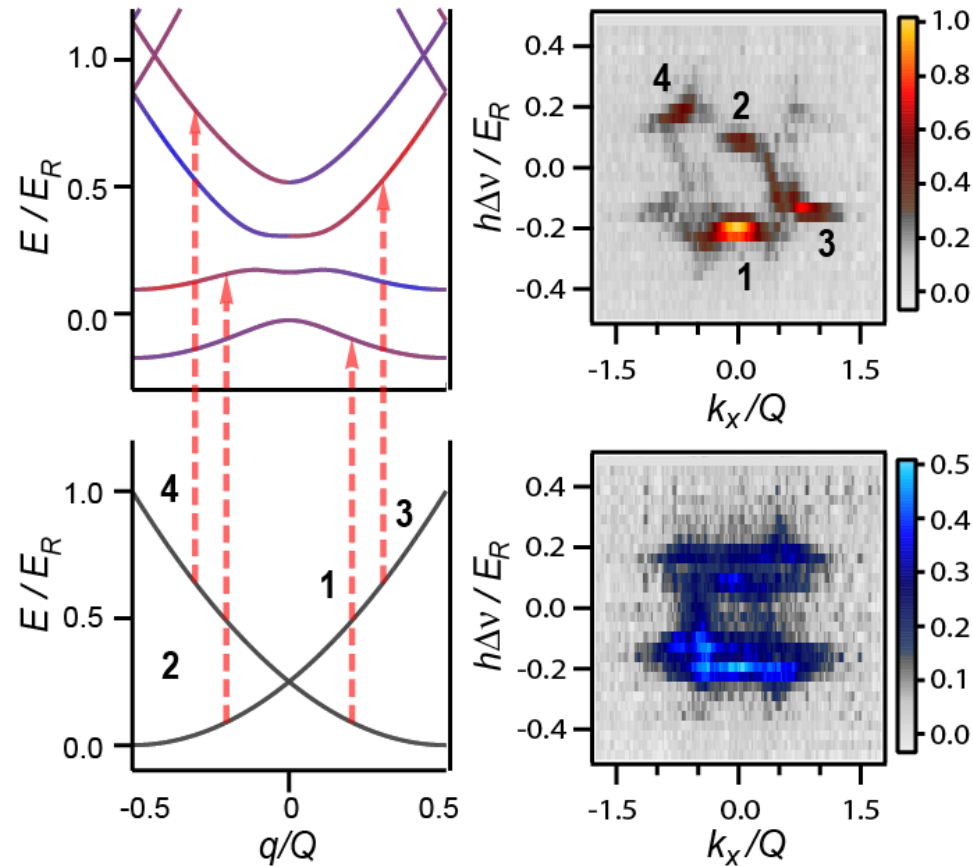
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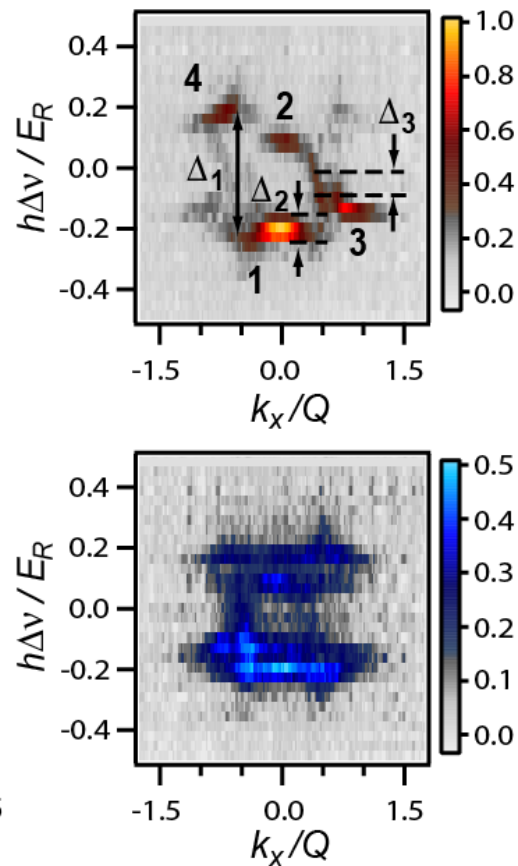
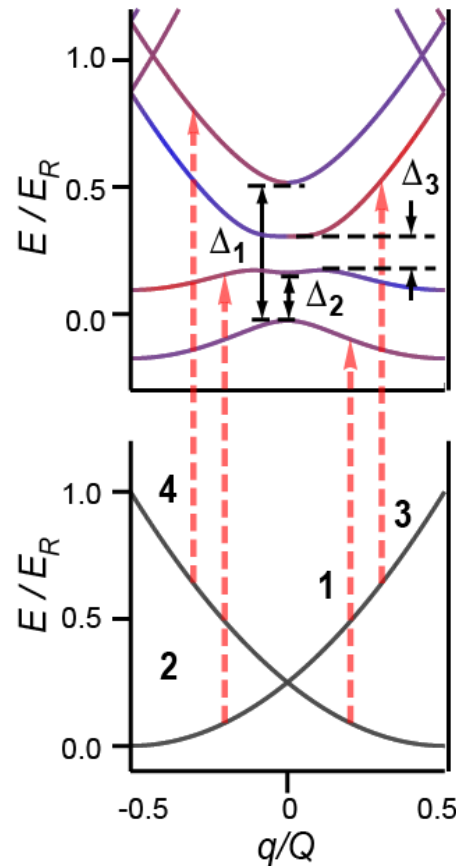
Spin-injection Spectra



Spin-injection Spectra



Spin-injection Spectra



$\Delta_1 \rightarrow$ Raman gap

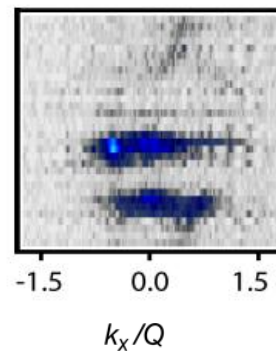
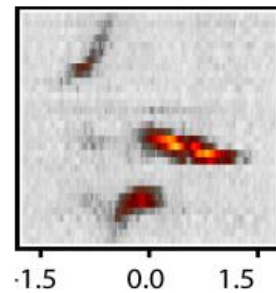
$\Delta_2 \rightarrow$ RF gap

$\Delta_3 \rightarrow$ Raman + RF gap

Reconstructing the Bandstructure



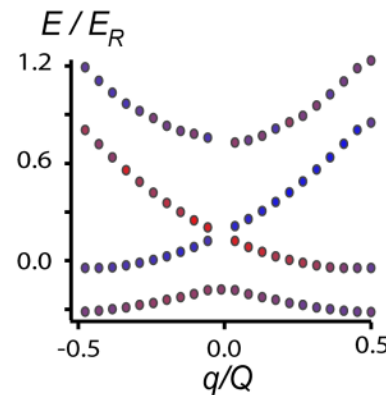
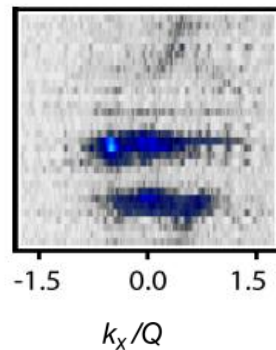
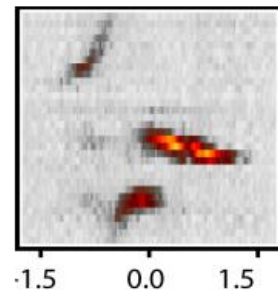
- In addition to dispersion, can reconstruct eigenstates
- TOF gives eigenstate in the basis of free space spin/momentum states



Reconstructing the Bandstructure



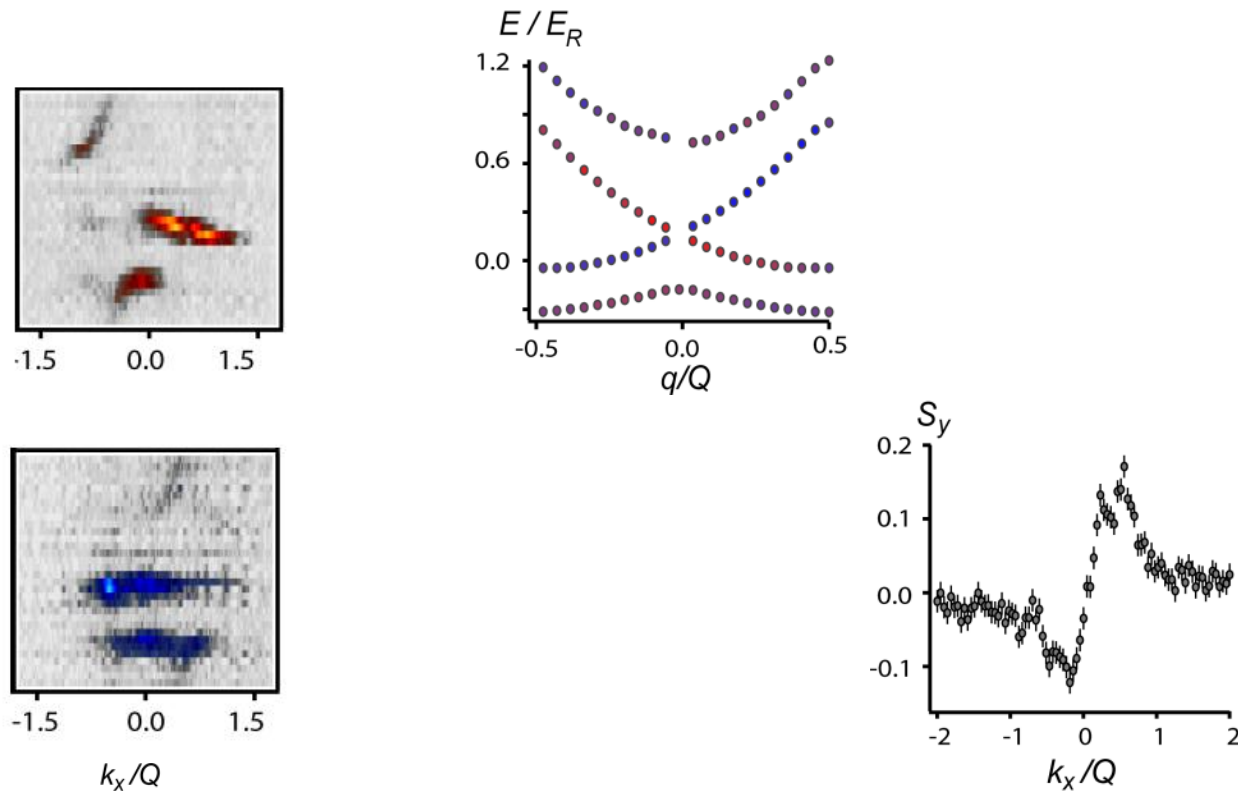
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Reconstructing the Bandstructure



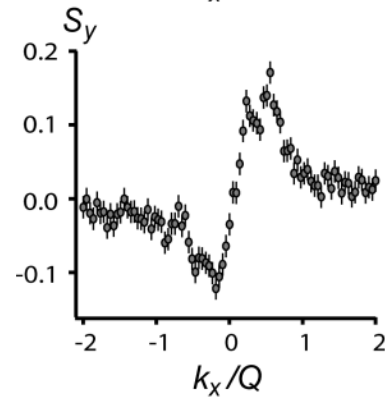
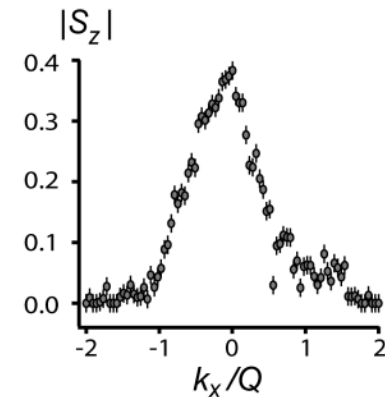
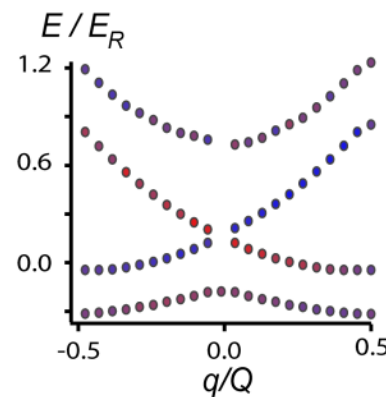
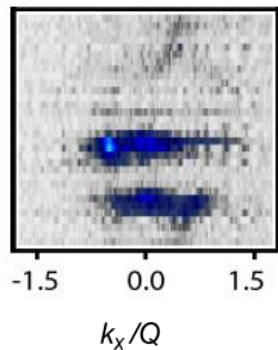
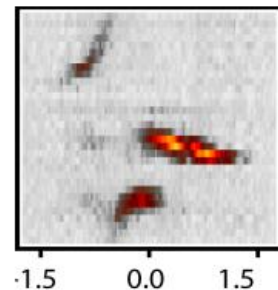
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Reconstructing the Bandstructure



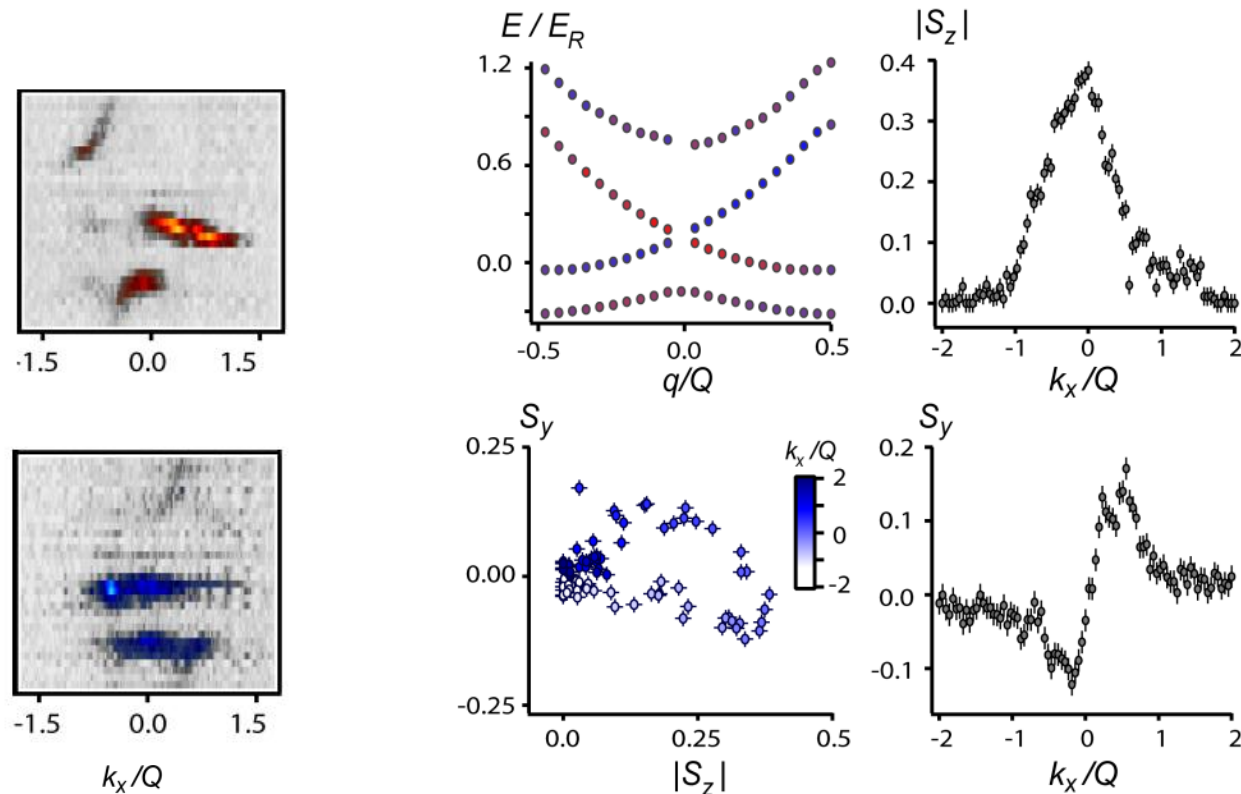
- In addition to dispersion, can reconstruct eigenstates
- TOF gives eigenstate in the basis of free space spin/momentum states



Reconstructing the Bandstructure



- In addition to dispersion, can reconstruct eigenstates
- TOF gives eigenstate in the basis of free space spin/momentum states



- Summary:
 - SO-coupled Fermi gas
 - Spinful lattice
 - Spin-injection spectroscopy
 - Band and eigenstate reconstruction
- Future:
 - Interactions : p-wave
 - Pairing in 1D tubes : Majorana edge mode ?

For details see:

L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Y. W. Bakr, M. W. Zwierlein, **arXiv:1205.3483**

Collaborators



Lawrence Cheuk



Ariel Sommer



Zoran Hadzibabic



Waseem Bakr

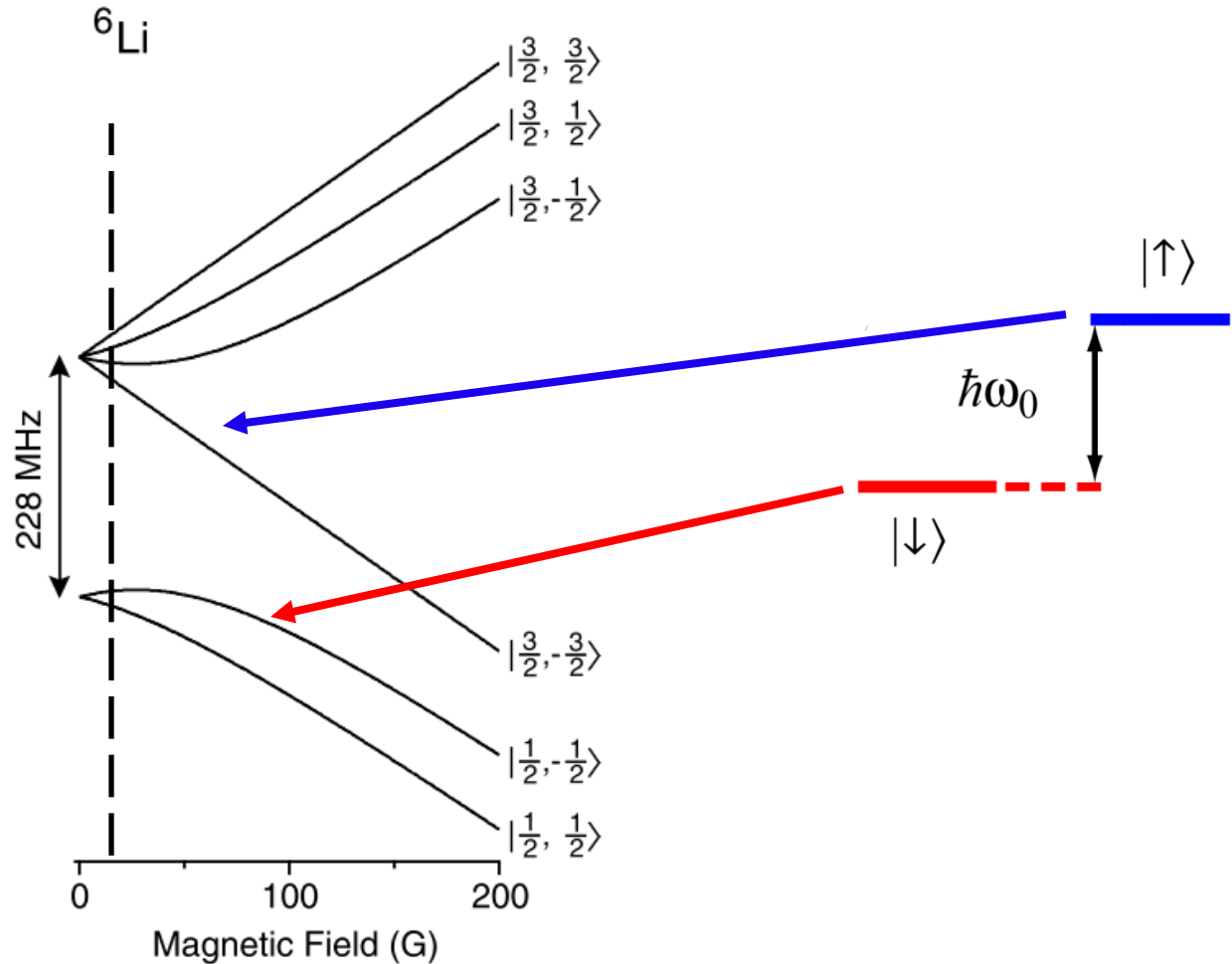


Martin Zwierlein

We thank these organizations for their support: DARPA, NSF, ONR, AFOSR, Sloan Foundation



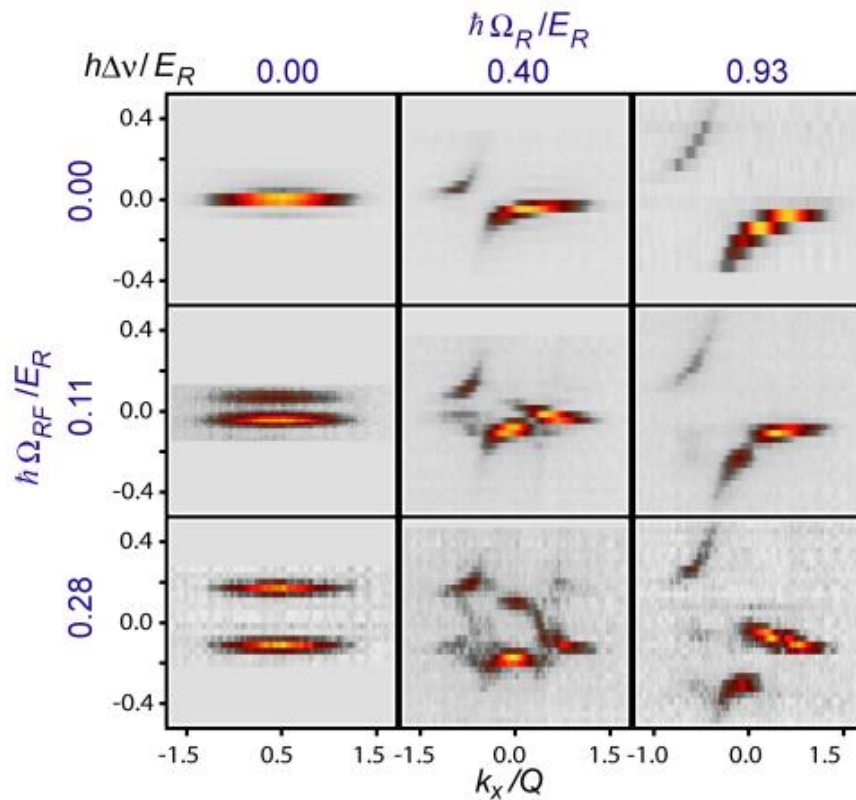
- Fermionic ${}^6\text{Li}$ atoms sympathetically cooled by ${}^{23}\text{Na}$
- Relevant states are 2nd and 3rd lowest states at 11G



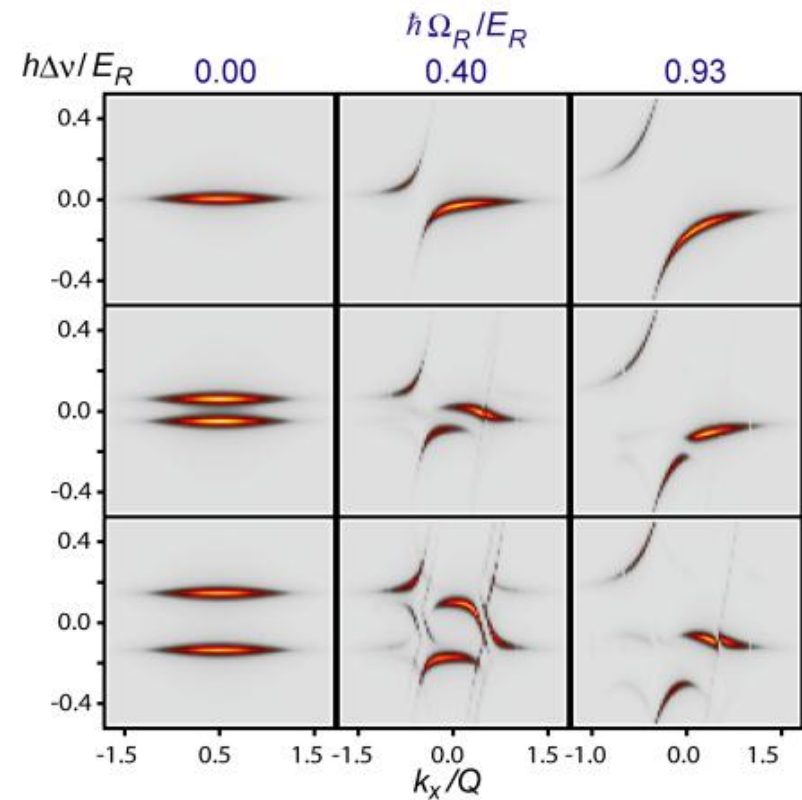
Spin-injection spectroscopy



Experiment



Theory



- Direct demonstration of SO-coupling through Rabi oscillations
- Controlled adiabatic loading of SO-coupled bands.
- Reversibility of loading shows adiabaticity.

- The SO Hamiltonian

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} - \frac{g\mu_B}{\hbar} \mathbf{S} \cdot (\mathbf{B}^{(D)} + \mathbf{B}^{(R)} + \mathbf{B}^{(Z)})$$

$$\mathbf{B}^{(R)} = \alpha(-k_y, k_x, 0) \quad \mathbf{B}^{(D)} = \beta(k_y, k_x, 0)$$

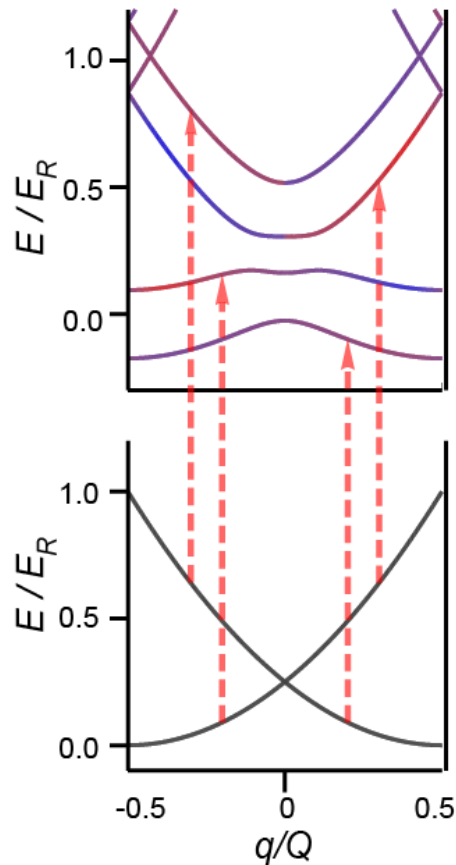
- Raman Coupling Hamiltonian

$$\mathcal{H}_{SO} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} + \frac{\delta}{2} & \frac{\hbar\Omega_R}{2} \\ \frac{\hbar\Omega_R}{2} & \frac{\hbar^2 (k+Q)^2}{2m} - \frac{\delta}{2} \end{pmatrix}$$

maps to 1D spin-orbit Hamiltonian with

$$\alpha = \beta = \frac{\hbar^2 Q}{2mg\mu_B} \quad B_z^{(Z)} = \hbar\Omega_R/g\mu_B \quad B_y^{(Z)} = \hbar\delta/g\mu_B$$

- Spin-injection spectroscopy on a spinful lattice



The spin-orbit Hamiltonian



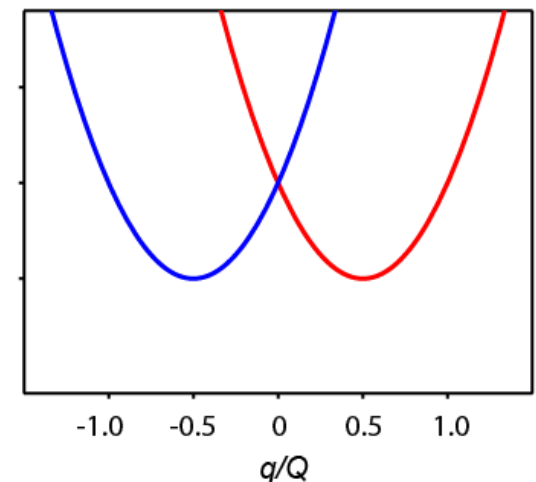
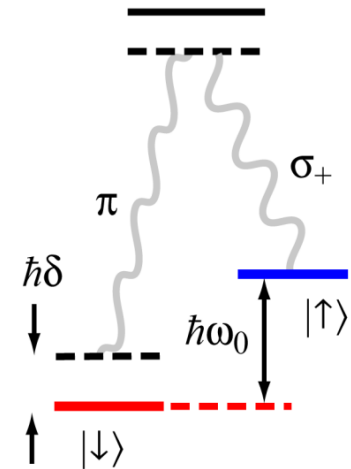
- Raman coupled atomic system maps to SO Hamiltonian.
- Rotating-Frame approximation:

$$\mathcal{H}_{SO} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} + \frac{\delta}{2} & \frac{\hbar\Omega_R}{2} \\ \frac{\hbar\Omega_R}{2} & \frac{\hbar^2 (k+Q)^2}{2m} - \frac{\delta}{2} \end{pmatrix}$$

- Write in terms of COM momentum q (spin-dependent transformation):

$$\uparrow \quad q = k + Q/2$$

$$\downarrow \quad q = k - Q/2$$

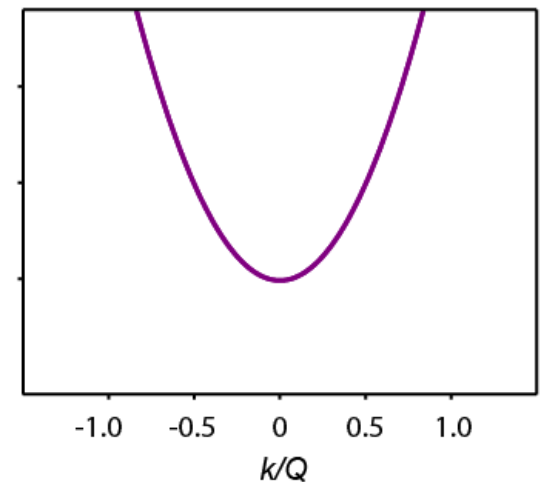
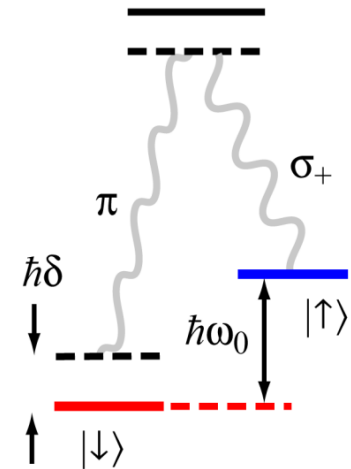


The spin-orbit Hamiltonian



- Raman coupled atomic system maps to SO Hamiltonian.
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The spin-orbit Hamiltonian



- Raman coupled atomic system maps to SO Hamiltonian.
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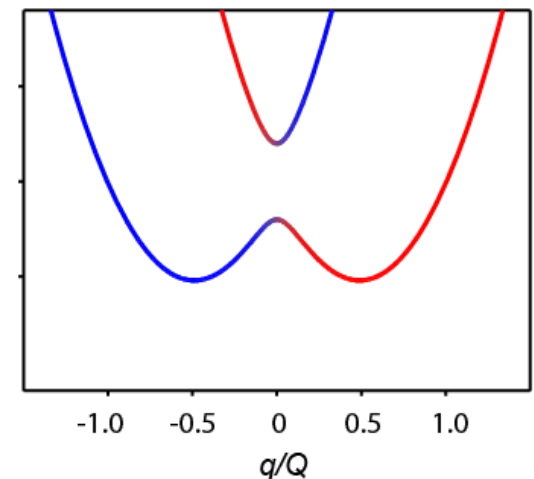
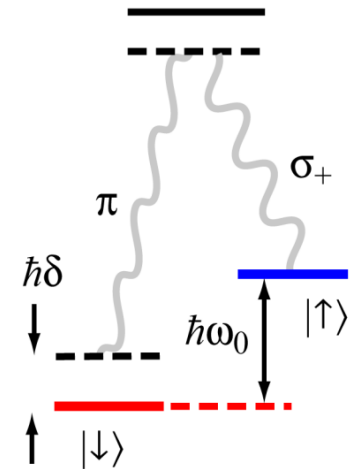
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- Amplitude of Raman beams give splitting
- Detuning imbalances the two wells



The spin-orbit Hamiltonian



- Raman coupled atomic system maps to SO Hamiltonian.
- Rotating-Frame approximation:

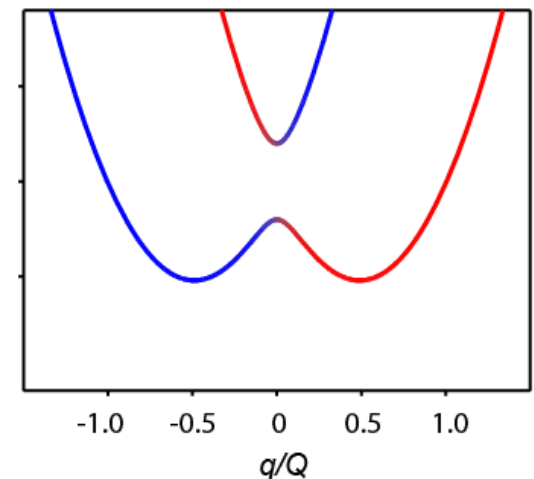
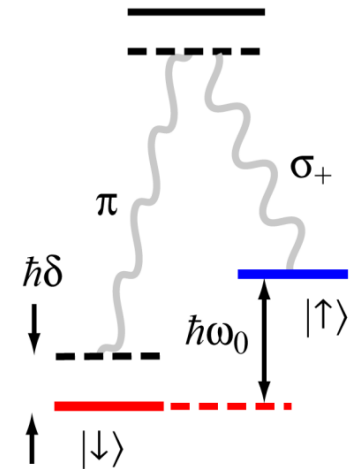
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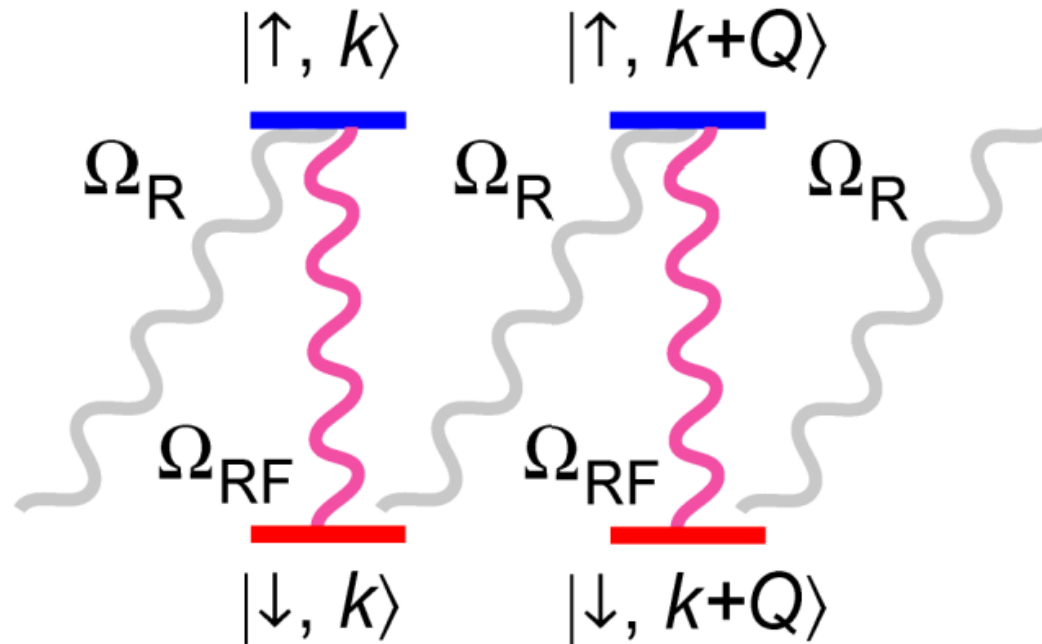
- Amplitude of Raman beams give splitting
- Detuning imbalances the two wells



- When SO coupling is ramped slowly:
 - Spin composition follows effective magnetic field
 - Process is reversible
 - By changing detuning, either upper band or lower band

- How to characterize Hamiltonian?
 - Can we measure topology?
- Condensed matter: transport, ARPES, STM ...
- Cold atom analog: photoemission spectroscopy (PES) has been
- PES probes $E(k)$
 - Transfer to hyperfine states outside system with RF
 - Measure momentum in TOF
 - Use RF frequency, free particle dispersion and momentum to reconstruct $E(k)$

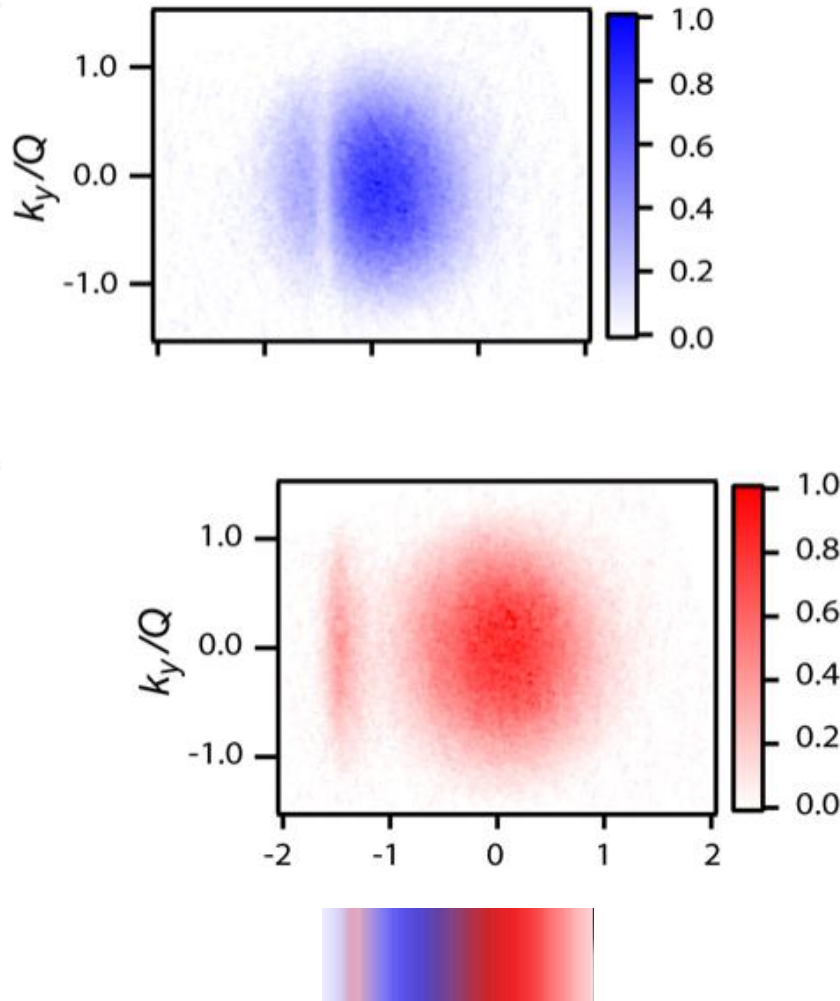
- Add RF coupling -> lattice system with full bandgaps and spinful bands



K. Jimenez-Garcia et al **PRL** **108**, 225303 (2012)

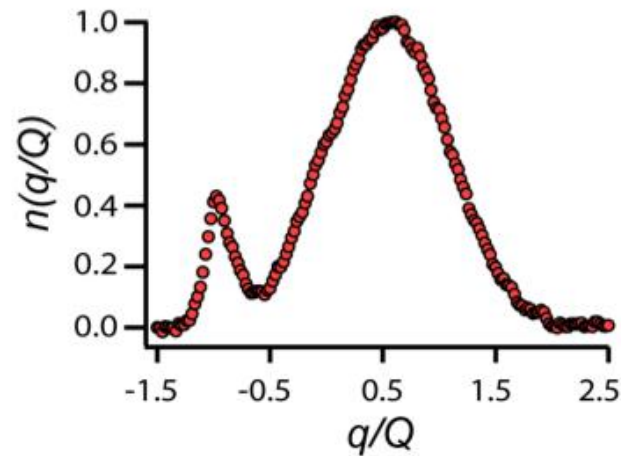
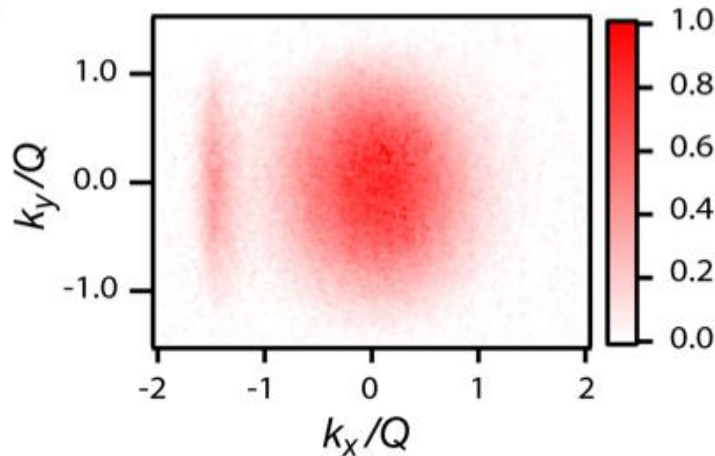
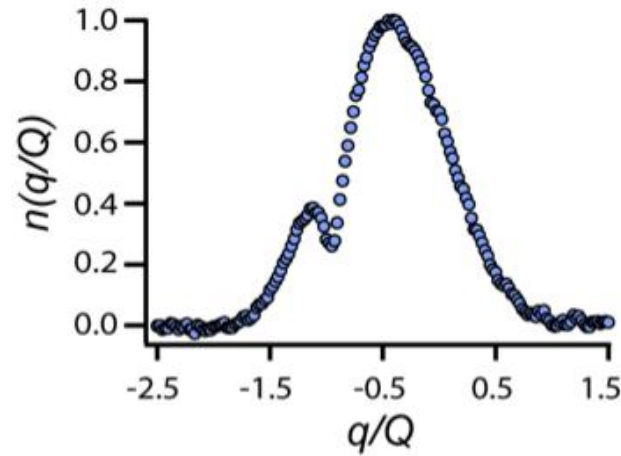
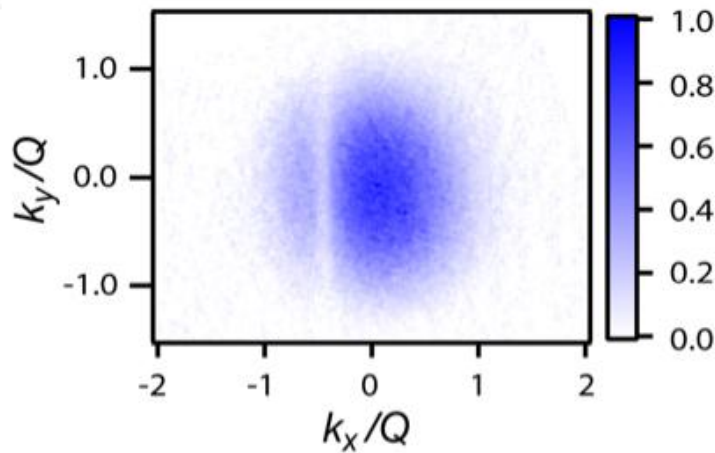
Detecting Spin Texture

- Image Sequence: TOF + state-selective imaging



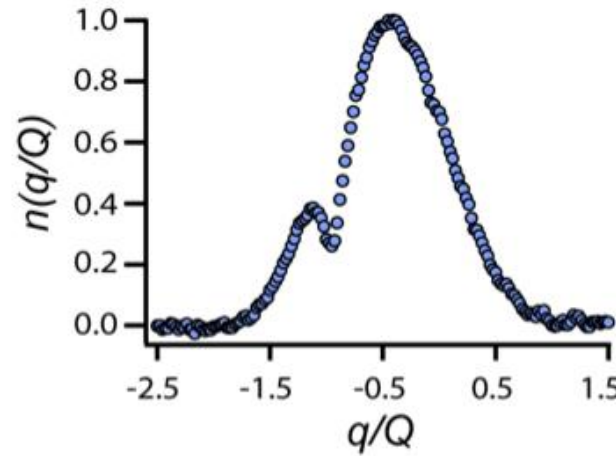
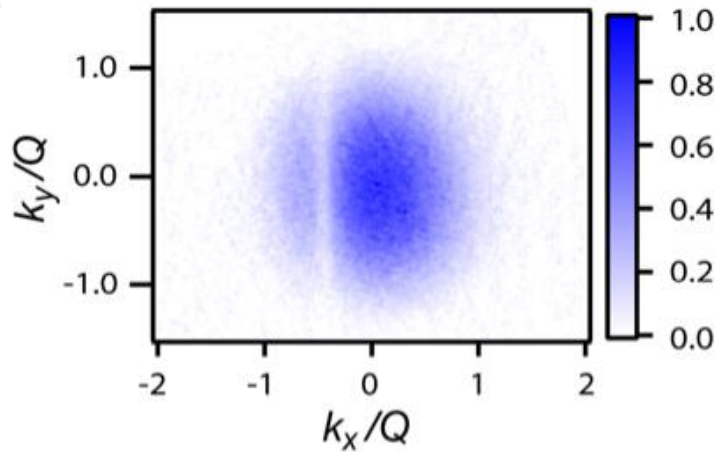
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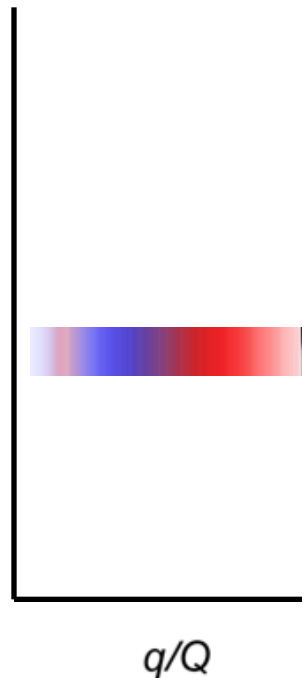
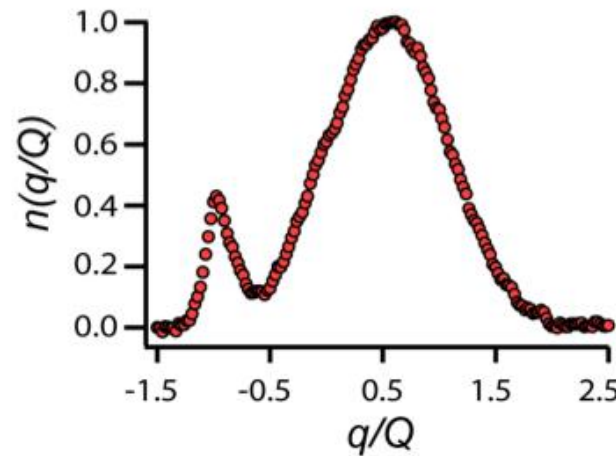
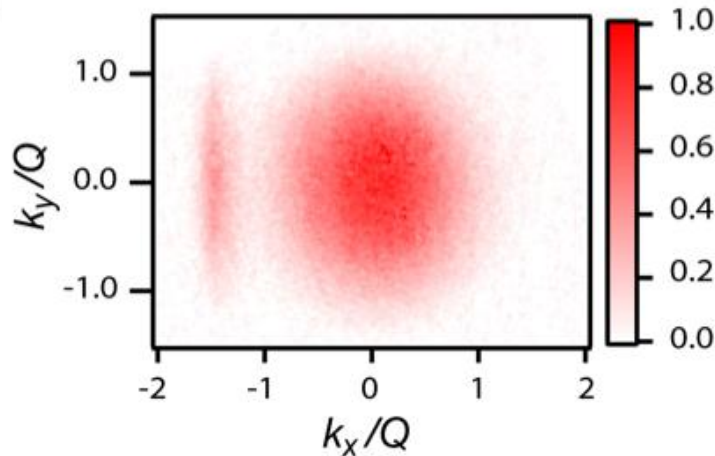


Detecting Spin Texture

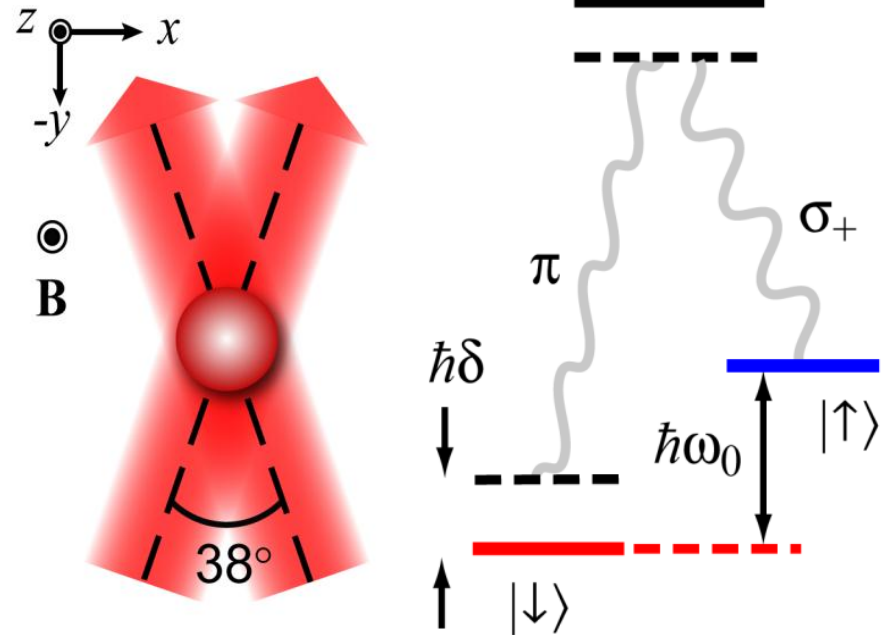
- Image Sequence: TOF + state-selective imaging



Some
parameter



- Raman Beams couple two hyperfine states
- SO coupling along one direction
- Recoil momentum: Q
- Recoil energy : $E_R = \frac{\hbar^2 Q^2}{2m}$



- Realized in bosons:
 - Modified dispersion
 - Synthetic higher-order partial waves
 - Synthetic magnetic field

Y. J. Lin *et al.* *Nature* **471**, 83-86 (2011)

R. A. Williams *et al.* *Science* **335**, 314-317 (2011)

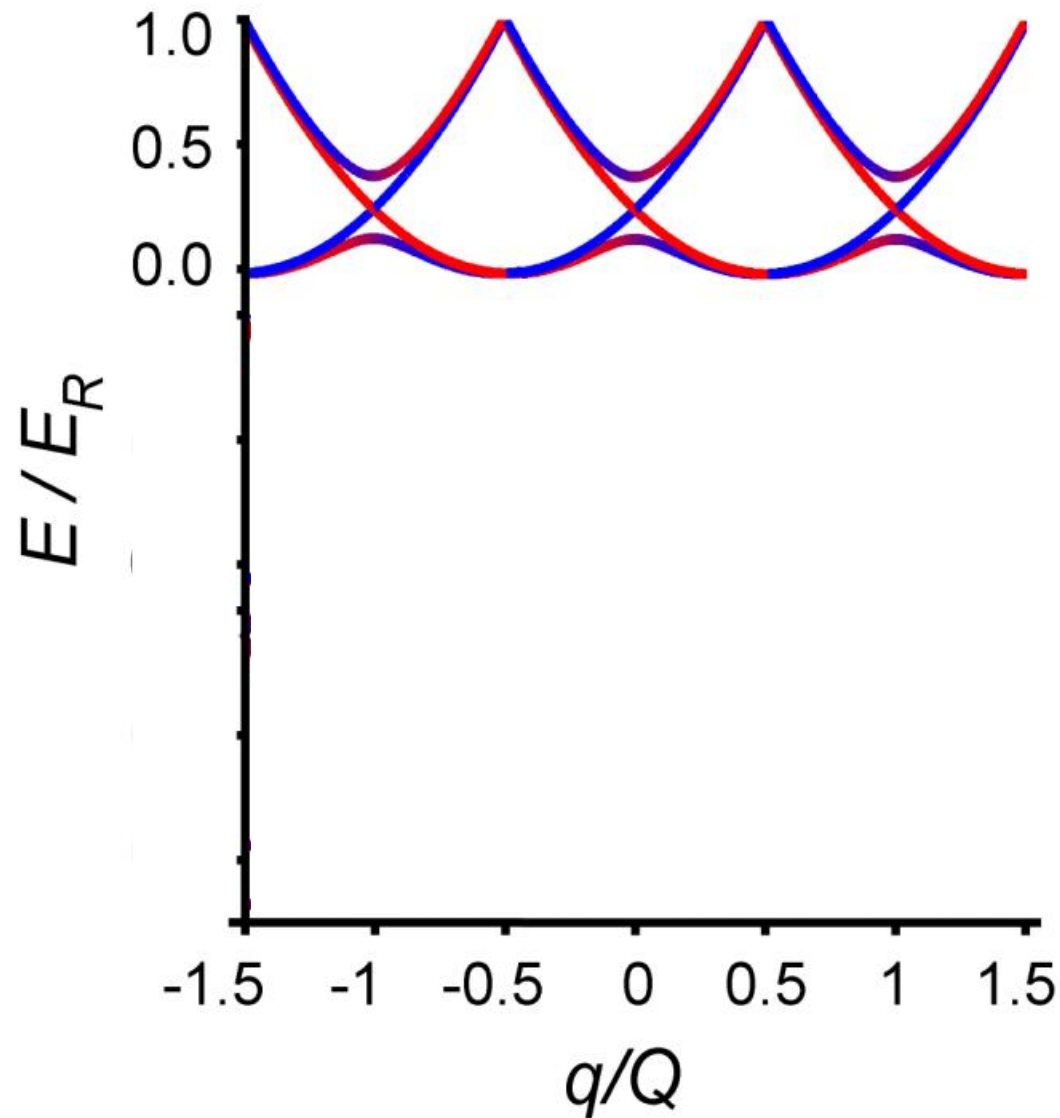
Y. J. Lin *et al* *Nature* **462** 628-632 (2009).

- Recently realized in fermions

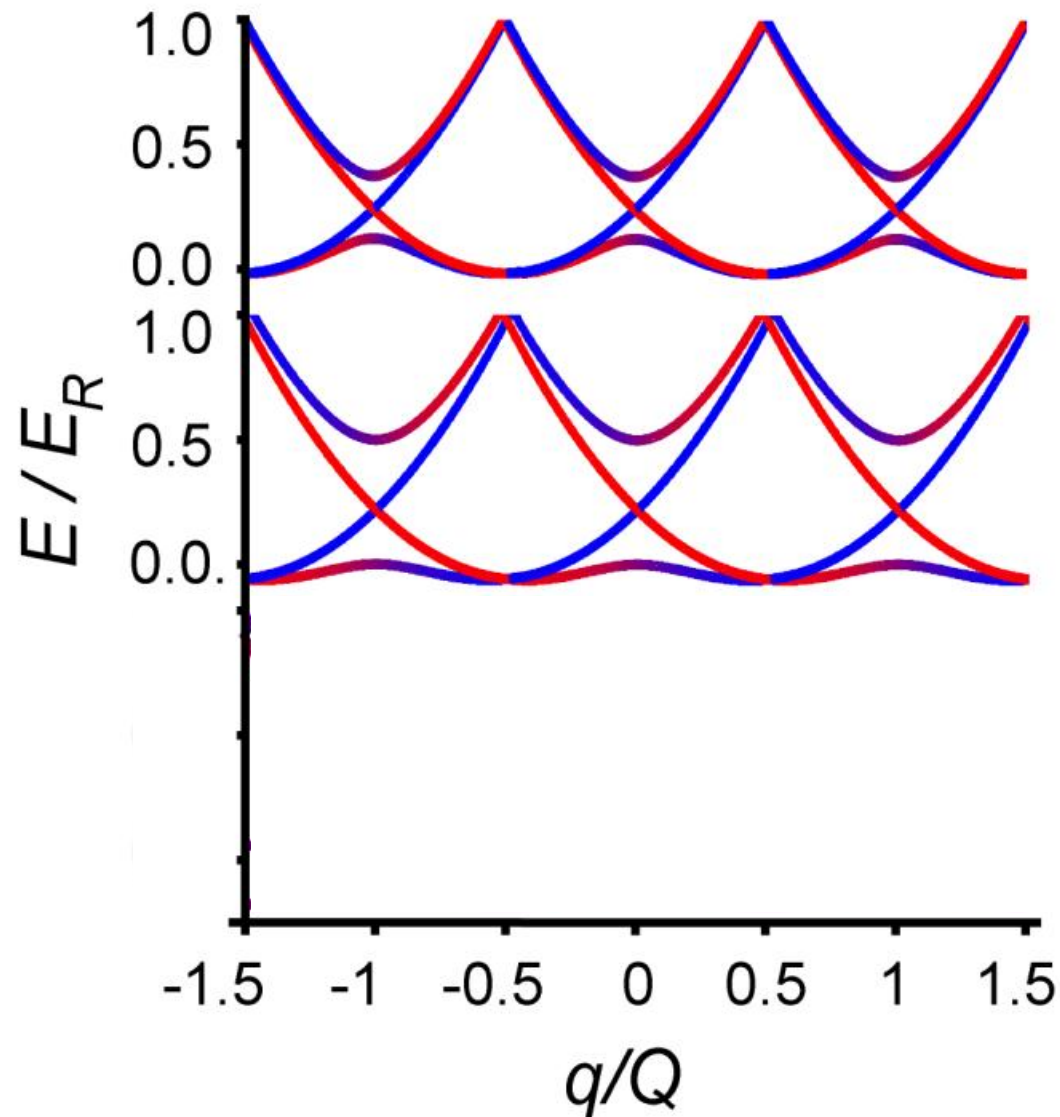
P. Wang et al **arXiv:1204.1887**

L. W. Cheuk et al **arXiv:1205.3483**

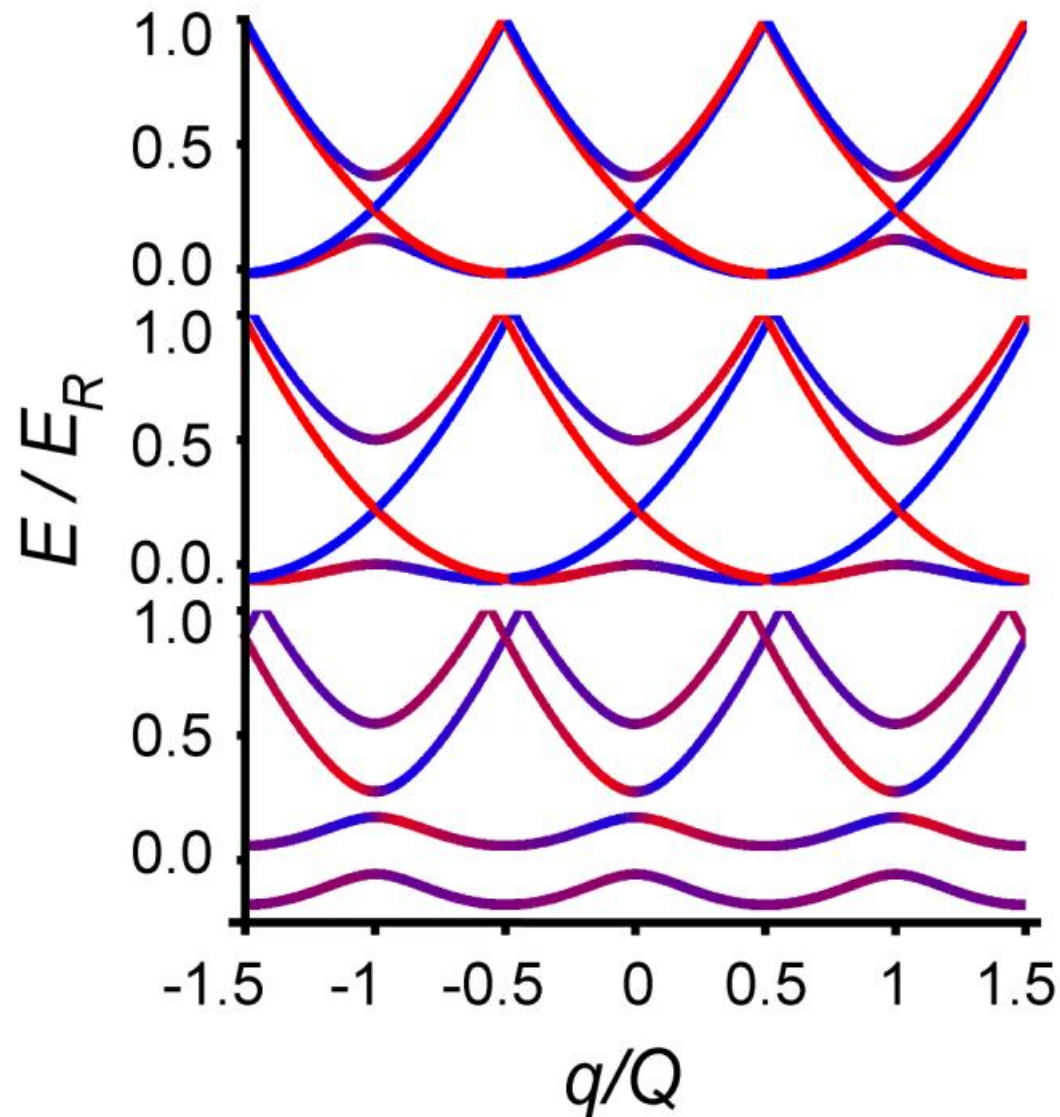
Bandstructure of Raman + RF lattice



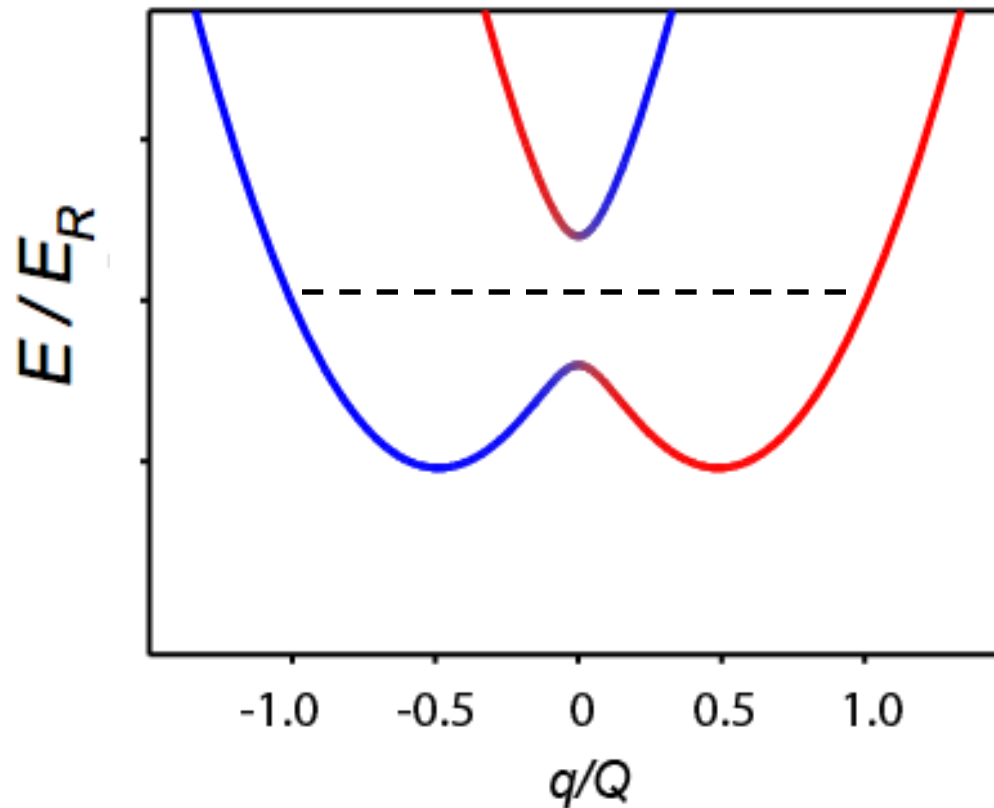
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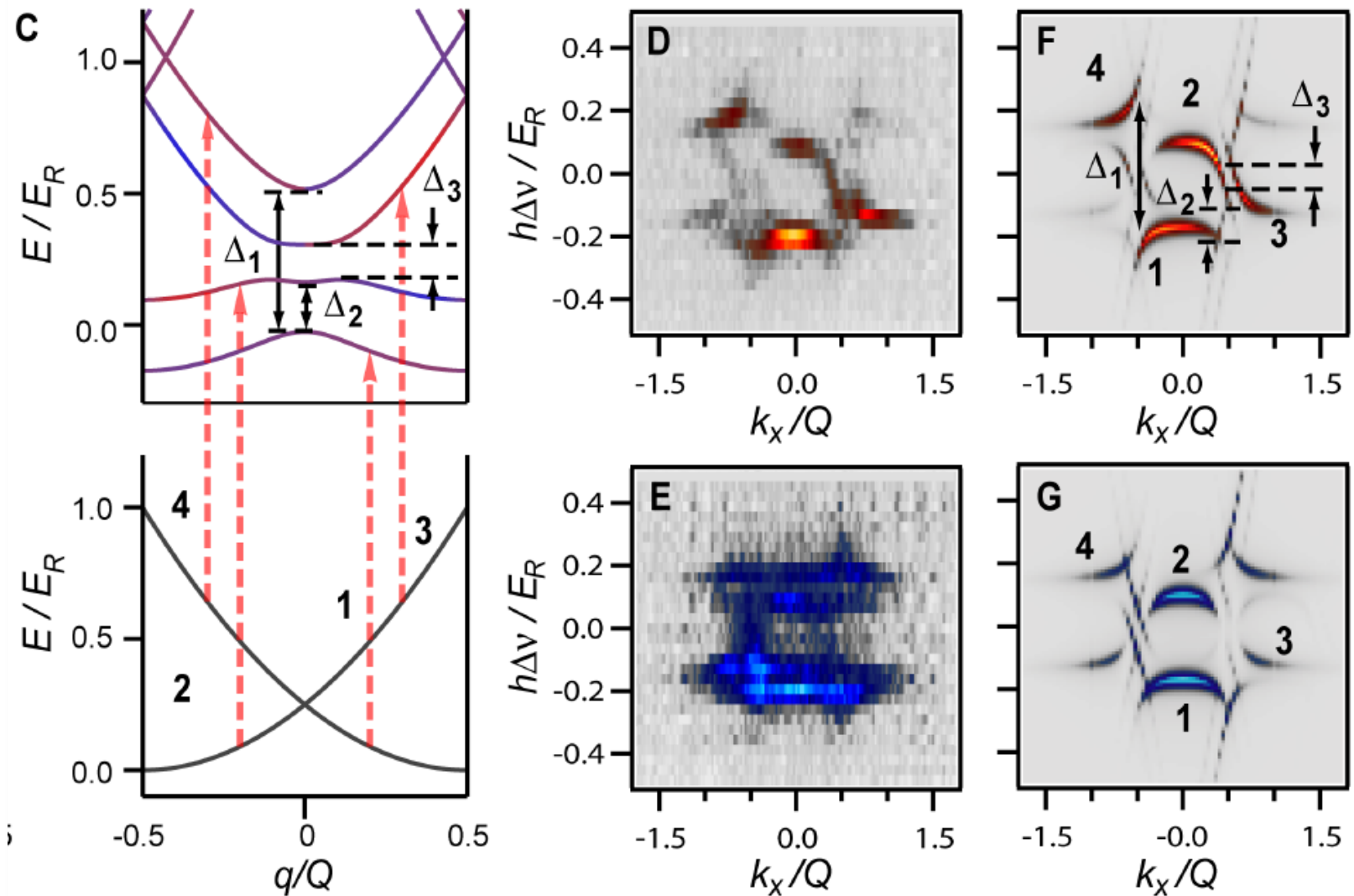
Bandstructure of Raman + RF lattice



- Spin diode when the Fermi level is inside the spin gap



Experiment vs Simulation



Why spin-orbit coupling?

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,^{1*} K. Zuo,^{1*} S. M. Frolov,¹ S. R. Plissard,² E. P. A. M. Bakkers,^{1,2} L. P. Kouwenhoven^{1†}

