Les états de bord d'un isolant de Hall atomique

séminaire Atomes Froids

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Outline

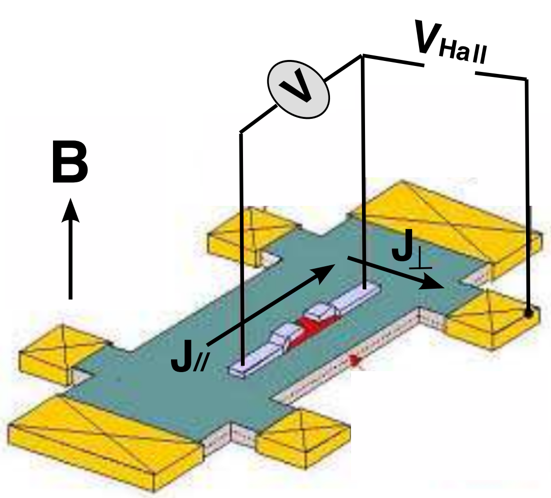
- Quantum Hall effect : bulk Landau levels and edge states

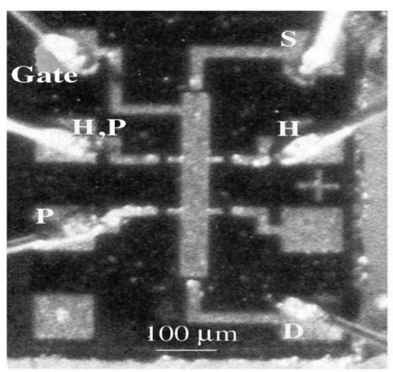
- Quantum Hall effect on a lattice : Hofstadter model
  - Hall conductivity as topological invariant
  - Bulk-edge correspondence

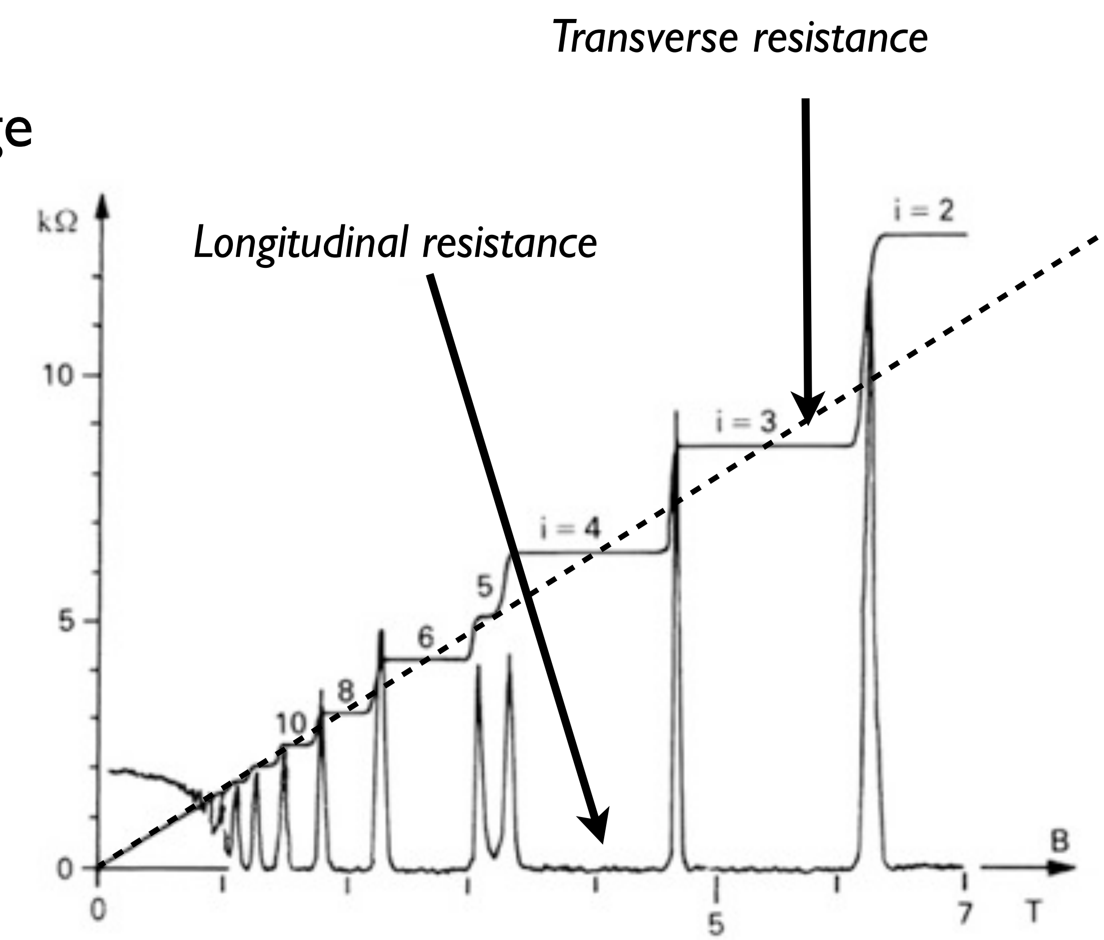
- Realization with cold atoms and detection of edge states
  - angular momentum spectroscopy of edge states
  - experimental scheme to realize the Hofstadter model
  - shelving technique to detect edge states

Quantum Hall effect

- 2D electron "gases" (very pure semiconductors) in Hall geometry:
  - large perpendicular magnetic field
  - Hall current perpendicular to applied voltage





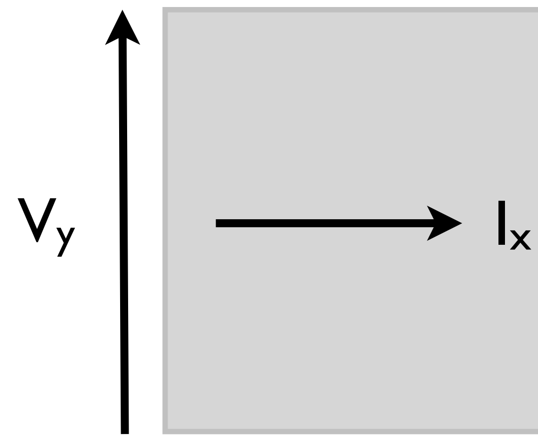


- Around certain "magic" values of magnetic field (QH plateaux) :
  - Longitudinal resistance vanishes
  - Hall conductance assumes quantized values $\sigma_{\rm H} = n\frac{e^2}{h}$

Hall conductivity

Hall geometry :

- current flows along x
- Hall voltage develops along y



Ideal system
without impurities

Resistivity tensor : $\mathbf{E} = \overline{\rho}\mathbf{j}$

$$\overline{\rho} = \begin{pmatrix} 0 & -\frac{B}{en_{el}} \\ \frac{B}{en_{el}} & 0 \end{pmatrix} \sim \frac{\text{E field}}{\text{current density}} \sim \frac{\text{voltage}}{\text{current}}$$

Conductivity tensor : $\overline{\sigma} = \overline{\rho}^{-1}$

$$\overline{\sigma} = \begin{pmatrix} 0 & \frac{en_{el}}{B} \\ -\frac{en_{el}}{B} & 0 \end{pmatrix}$$

$$\sigma_{xx} = 0 \qquad\qquad \sigma_{xy} = \frac{e}{B} n_{\mathrm{el}}$$

Landau levels and integer quantum Hall effect

Hamiltonian for a 2D electron in a uniform magnetic field :

$$H = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m_e}$$

Landau gauge : $\mathbf{A} = \begin{pmatrix} -By \\ 0 \end{pmatrix}$

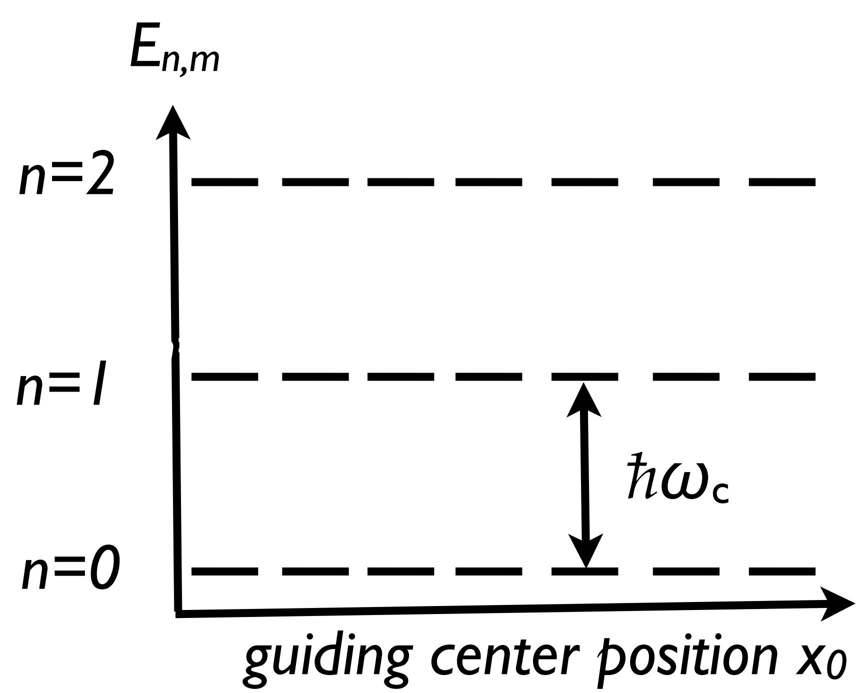
Energy eigenstates of the form : $\phi_n(\mathbf{r}) = e^{ik_x x}\chi_n(y)$

- Plane wave along x, wavevector x
- Harmonic oscillator wavefunction along y, centered at the guiding center position $y_0 = \frac{\hbar k_x}{eB}$

cyclotron frequency $\omega_c = \frac{eB}{m}$

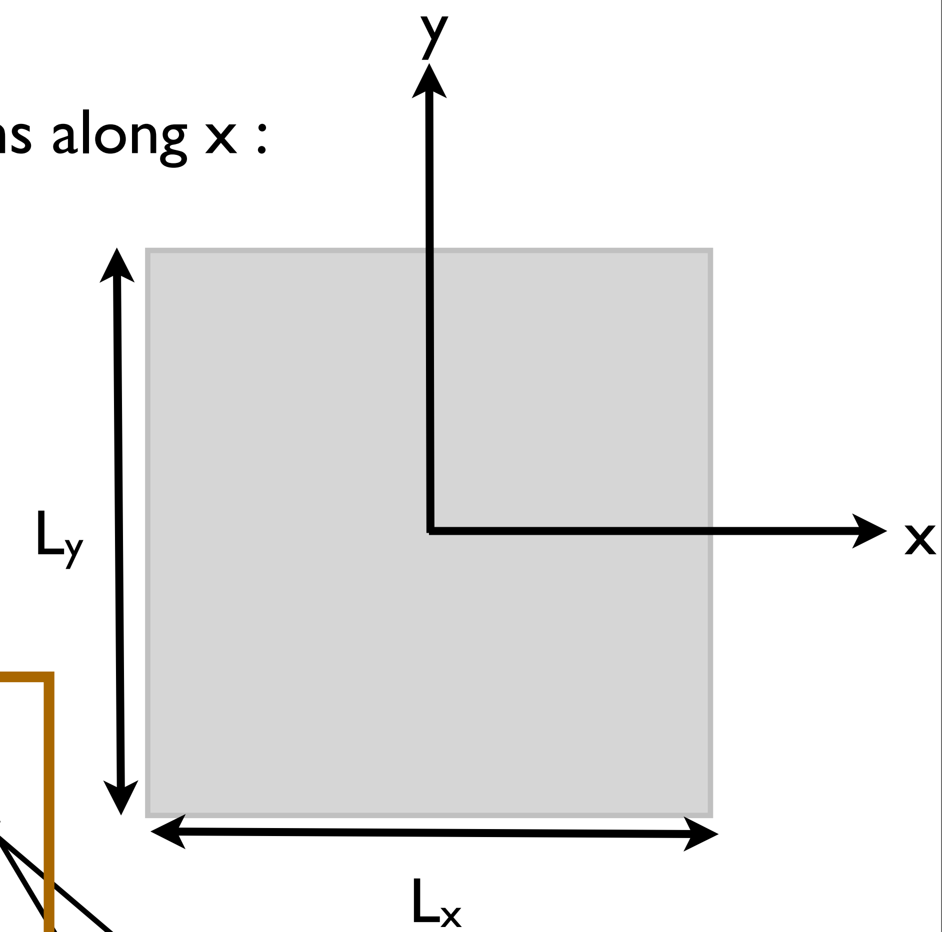
magnetic length $l_B = \sqrt{\frac{\hbar}{eB}}$

Sequence of Landau levels
with high orbital degeneracy



Bulk system : Hall conductivity

Consider one Landau level (LL), with periodic boundary conditions along x :



$$y_0 = \frac{\hbar k_x}{eB}$$ is between $-L_y/2$ and $L_y/2$

$$N_{\text{states}} = \left(\frac{eB}{\hbar} L_y\right) / \left(\frac{2\pi}{L}\right) = \frac{eB}{h} L_x L_y$$

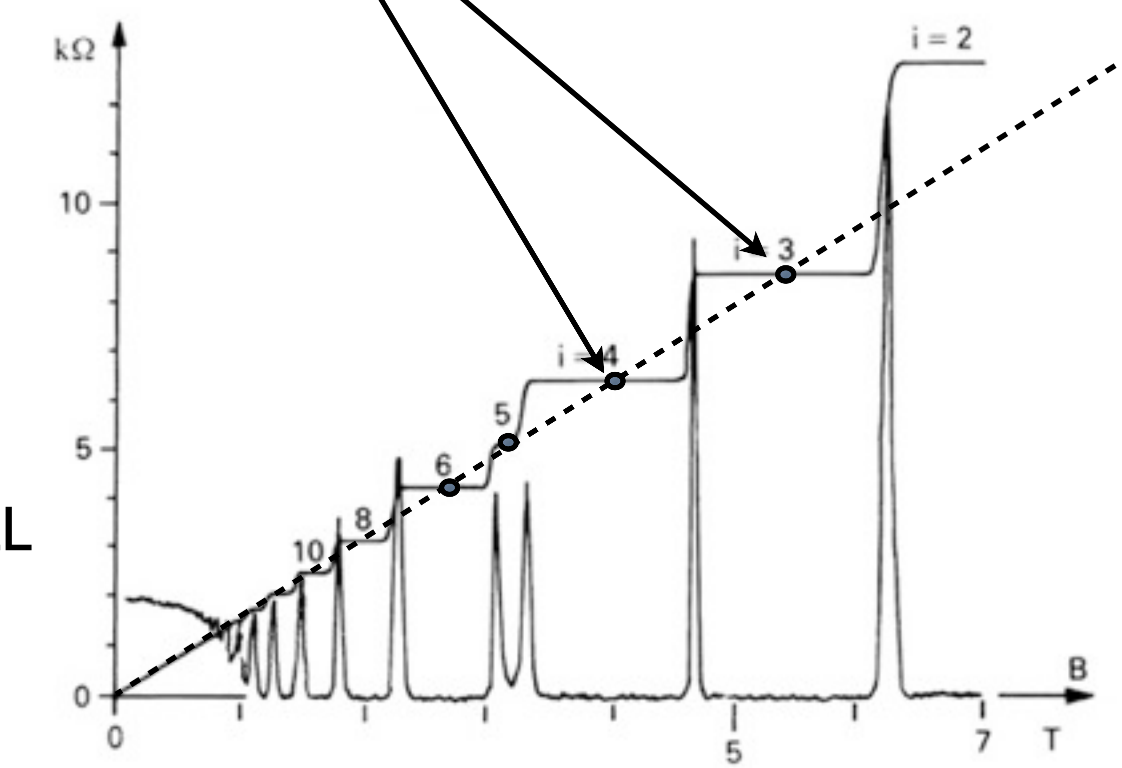
Electron density for one filled LL :

$$n_{\text{el}} = \frac{eB}{h} \qquad \sigma_{xy} = \frac{e}{B} n_{\text{el}}$$

This alone is not sufficient to explain the existence of conductance plateaus :

One must invoke disorder :

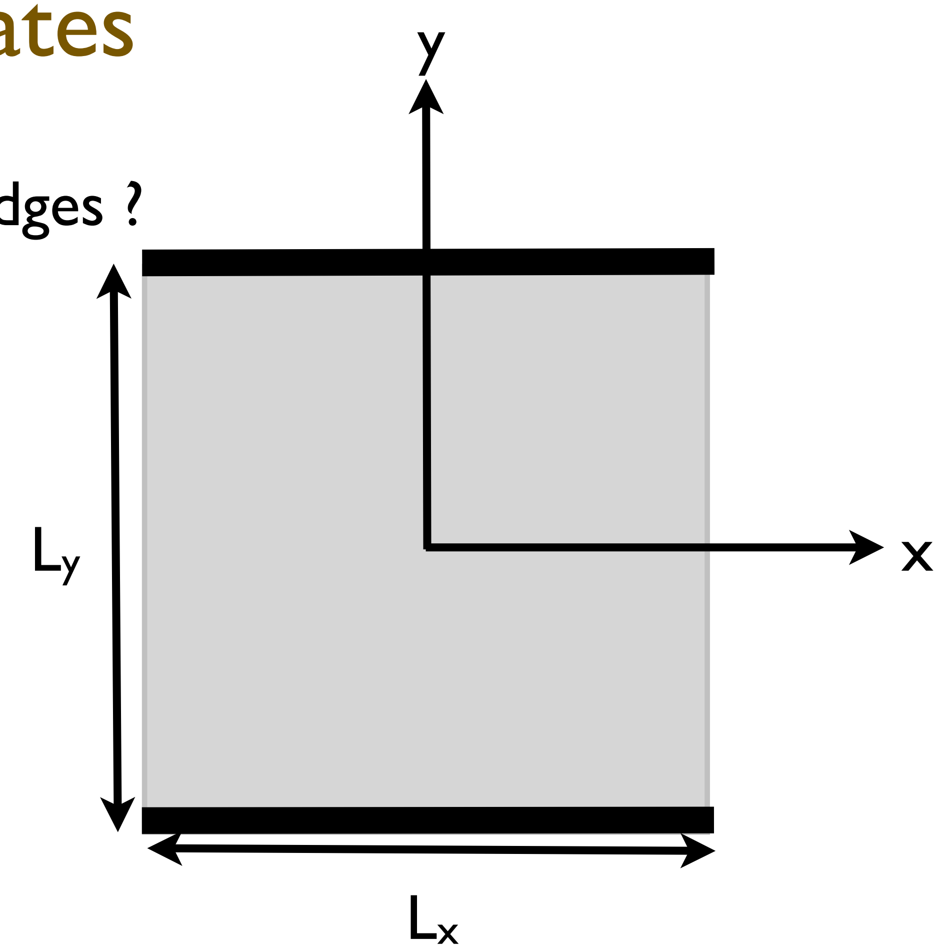
- localized states between each LL
- The localized states tend to be denser between two LL
- They do not contribute to transport

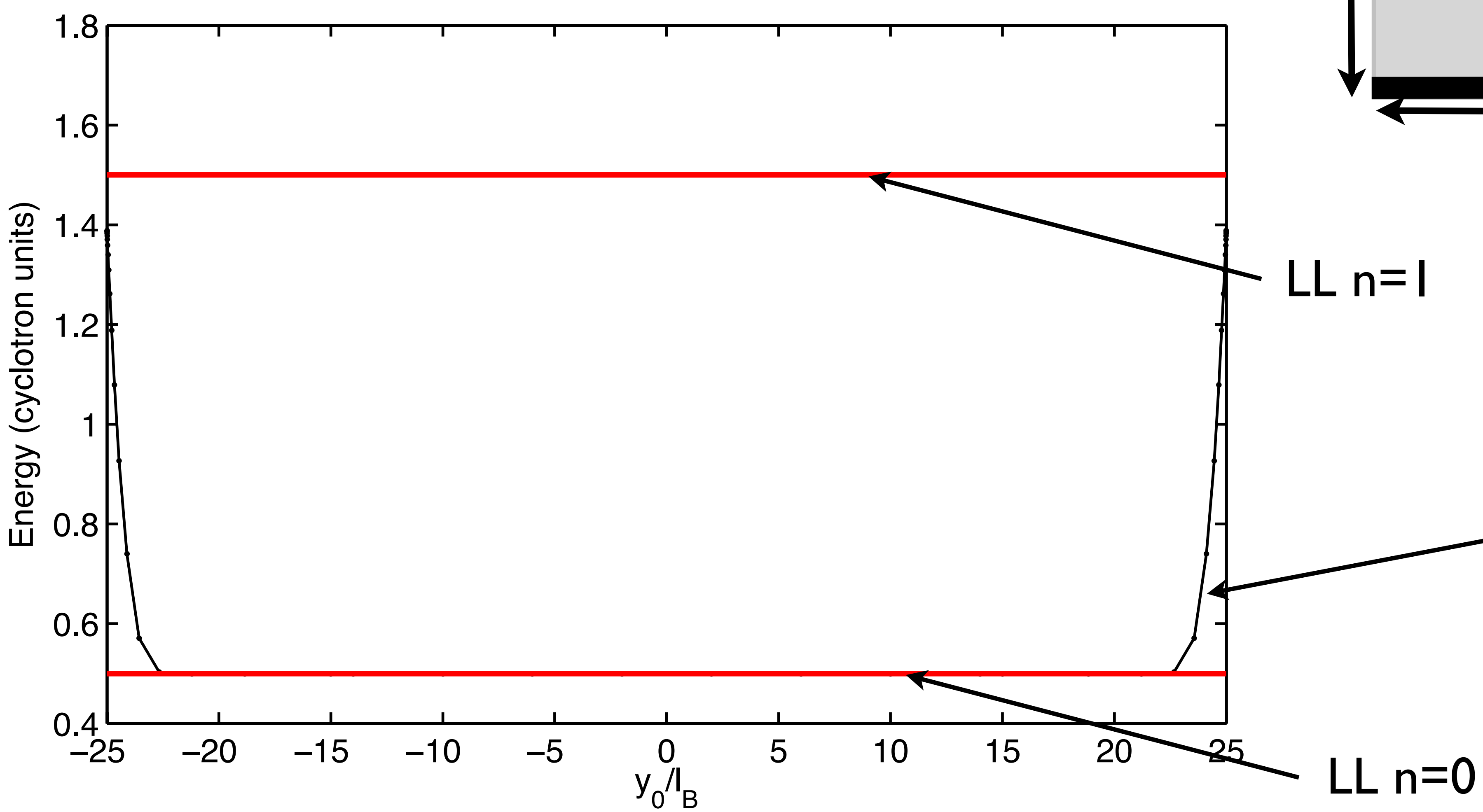


Strip geometry : edge states

Assume hard wall potential along y : what happens near the edges ?

Recall $y_0 = \dfrac{\hbar k_x}{eB}$

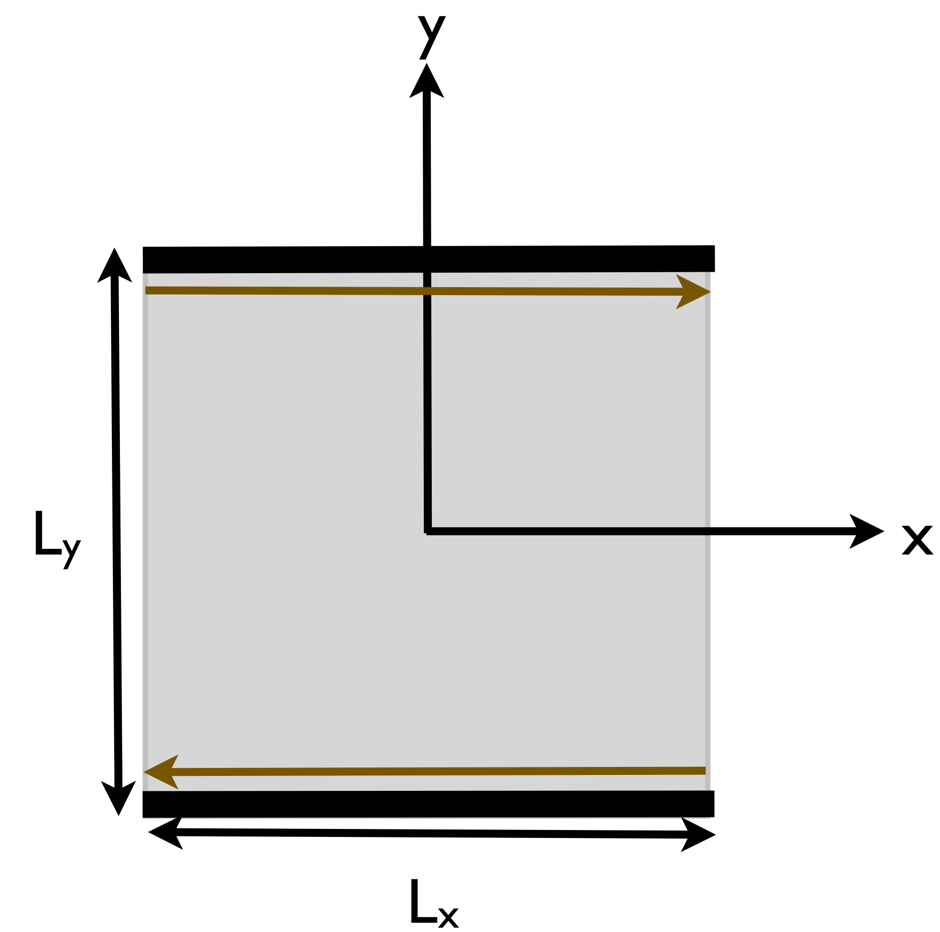




Properties of edge states

- They carry current : group velocity v_g non zero

- states on opposite edges at the same energy carry opposite currents in equilibrium
  - same chirality v_g/k
  - *explains robustness of edge currents : no state available for backscattering in presence, e.g., of impurities*



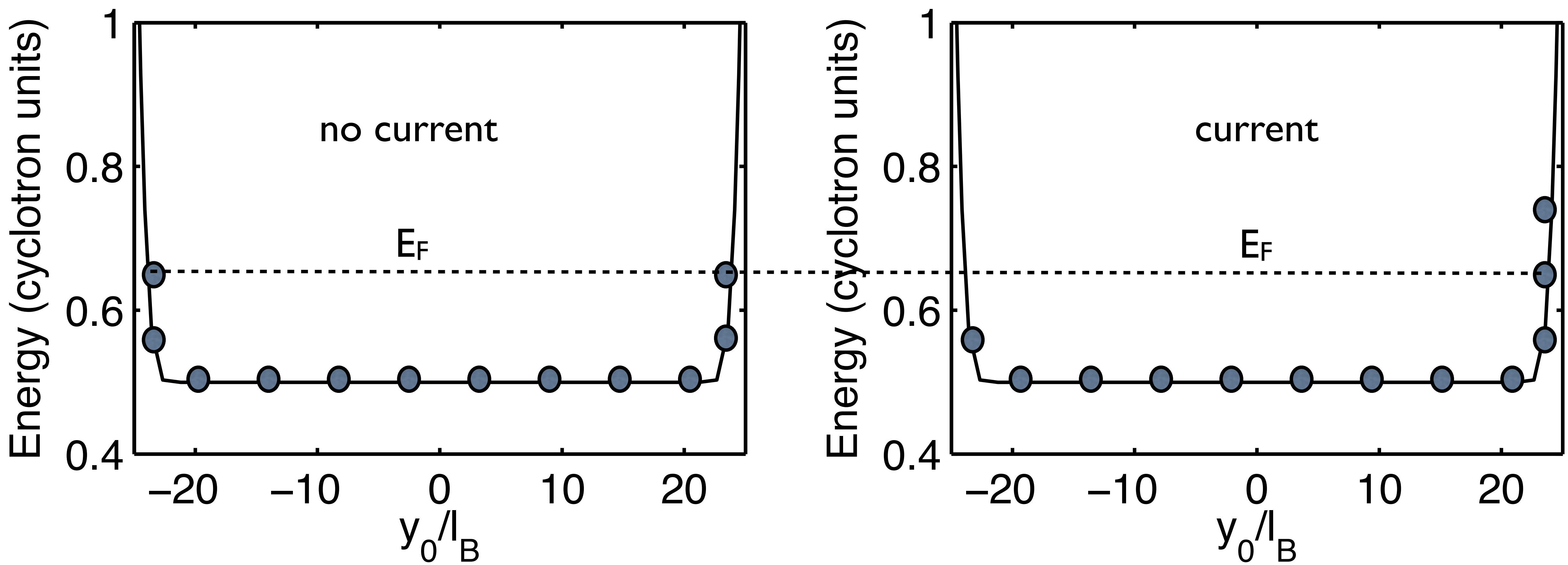
red: current flow

Properties of edge states

- Add weak electric field along y, producing an electrostatic potential $-e\mathcal{E}y$

$$v_{n,k_x} = \frac{1}{\hbar}\frac{\partial E_n(k_x)}{\partial k_x} \approx \frac{L_x}{h}\left(E_n(k_x + 2\pi/L) - E_n(k_x)\right)$$

$$I_x^{(n)} = -\frac{e}{L_x}\sum v_{n,k_x} \approx -\frac{e}{h}\left(\underbrace{E_n(k_{\max}) - E_n(k_{\min})}_{-e\Delta V_y}\right)$$



- This reproduces the bulk result, with the added information that the Hall current is flowing at the edge of the sample only (bulk is an insulator)

Bulk-edge correspondence

Hall current carried by chiral edge states.

Edge states and bulk Hall conductivity are connected.

This connexion between bulk properties and current-carrying edge states is a general (one could even say defining) feature of topological phases.

Topological phase : state of matter defined by an integer-valued topological invariant, usually physically related to transport properties, characterizing the topology of the Hilbert space.

To make this connection more precise, we consider now the same situation on a lattice

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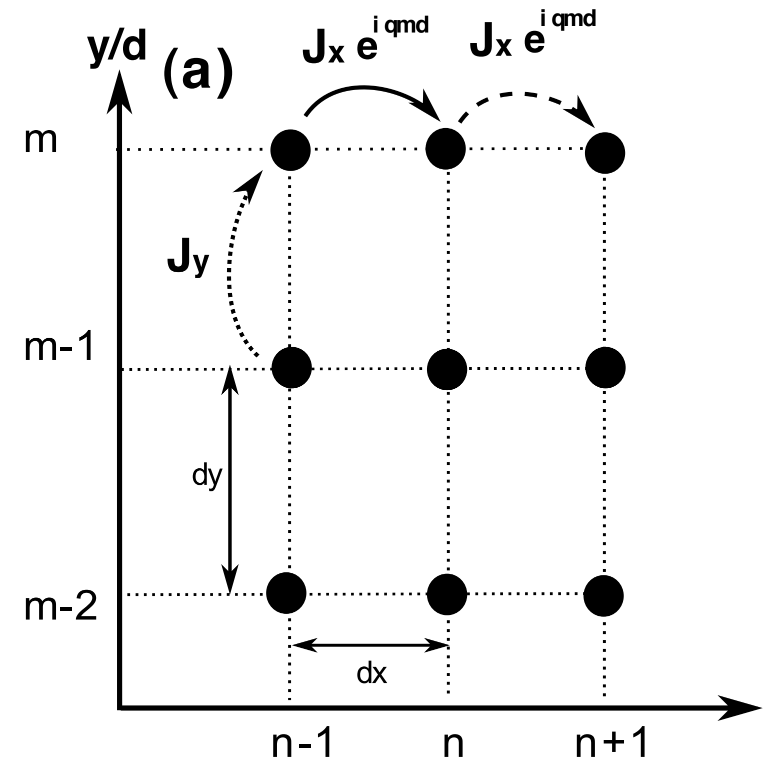
Harper hamiltonian : lattice and magnetic field

Harper hamiltonian on a tight-binding lattice :

$$H = -J \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} e^{i\phi_{\mathbf{r},\mathbf{r}'}} \hat{c}^\dagger_{\mathbf{r}'} \hat{c}_{\mathbf{r}} + \text{h.c.}$$

Harper, 1956; Azbel 1964; Hofstadter, 1976; Thouless et al., 1983; Kohmoto; Osadchy-Avron 2001...



- Aharonov-Bohm phase: $\phi_{\mathbf{r},\mathbf{r}'} = \frac{-e}{\hbar} \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A} \cdot \mathbf{dl}$

- Landau gauge: $\mathbf{A} = \begin{pmatrix} -By \\ 0 \\ 0 \end{pmatrix}$

$$\int_x^{x+d} A_x dx = B d_x y$$

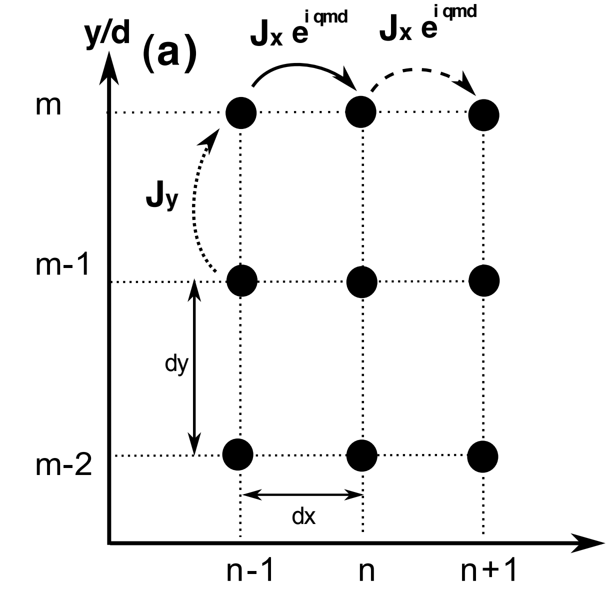
$$\phi_{\mathbf{r},\mathbf{r}+d\mathbf{e}_x} = 2\pi \frac{eBd_x y}{h} = 2\pi\alpha \frac{y}{d_y}$$

- Finite flux: $\int_{\square} \mathbf{A} \cdot \mathbf{dl} = B d_x d_y$

$$\sum_{\square} \phi_{\mathbf{r},\mathbf{r}'} = 2\pi\alpha$$

Hofstadter's butterfly:
interplay between lattice and vector potentials

$$H = -J\sum_{n,m} e^{i2\pi\alpha m}\hat{c}^{\dagger}_{n+1,m}\hat{c}_{n,m} + \text{h.c.}$$
$$-J\sum_{n,m} \hat{c}^{\dagger}_{n,m+1}\hat{c}_{n,m} + \text{h.c.}$$



- If α=0, we have a single Bloch band of width *8J*.

- Bloch theorem does not apply in general for arbitrary α.

- When α=p/q rational:

  - Translation by q sites reproduces the same model : we recover lattice translational invariance, but with an enlarged unit cell of size q dx*dy

  - Crystal momentum in the «magnetic Brillouin zone»]-π/q,π/q] *]-π,π]

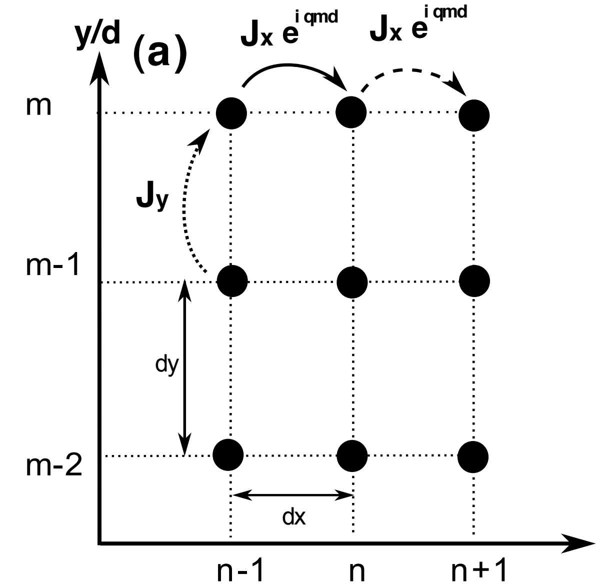
  - Eigenstates along y («magnetic Bloch functions») have q components

  - the Bloch band splits in q subbands contained within the original one for α=0

Hofstadter's butterfly: interplay between lattice and vector potentials

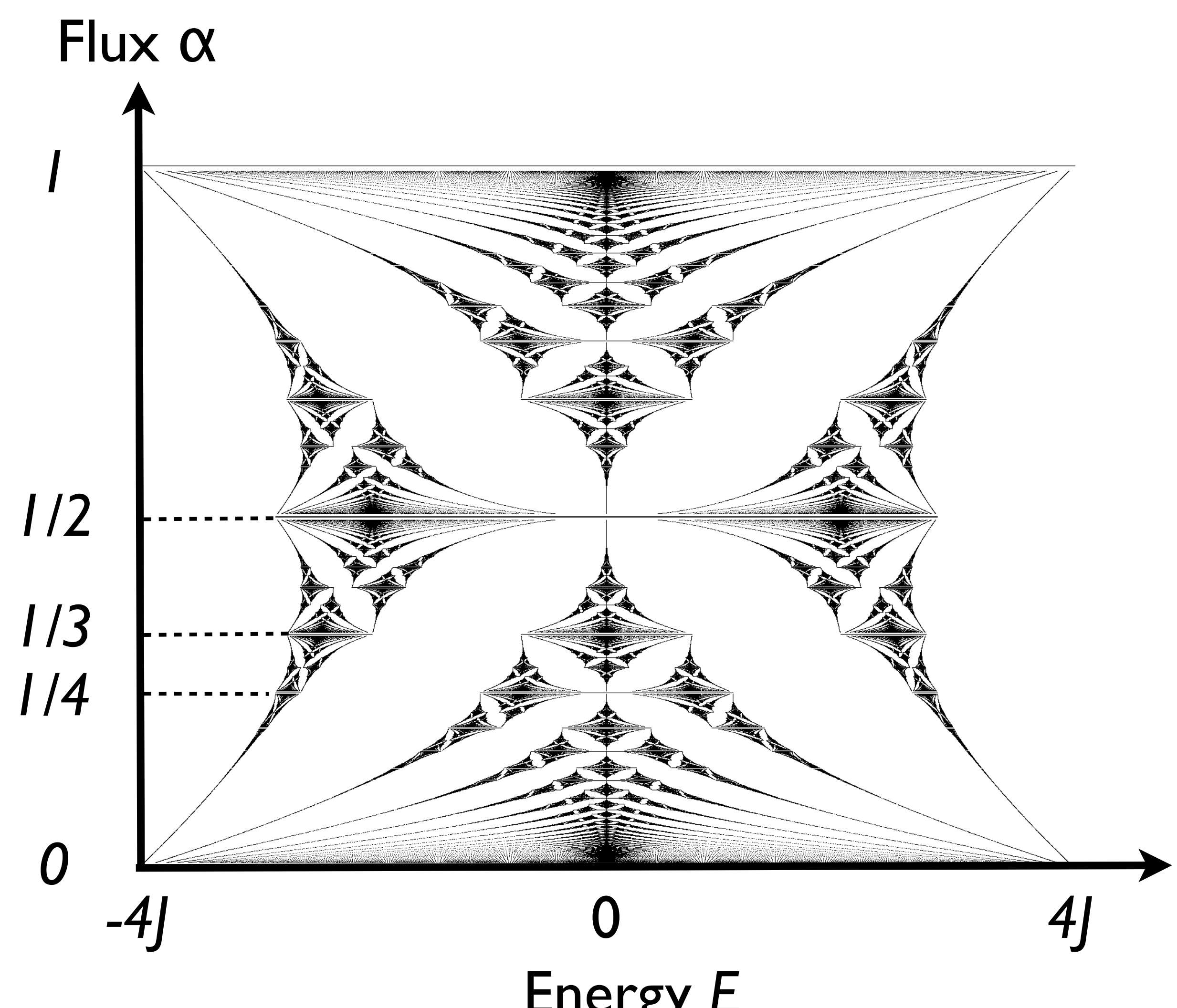
$$H = -J \sum_{n,m} e^{i2\pi\alpha m} \hat{c}^{\dagger}_{n+1,m} \hat{c}_{n,m} + \text{h.c.}$$

$$- J \sum_{n,m} \hat{c}^{\dagger}_{n,m+1} \hat{c}_{n,m} + \text{h.c.}$$

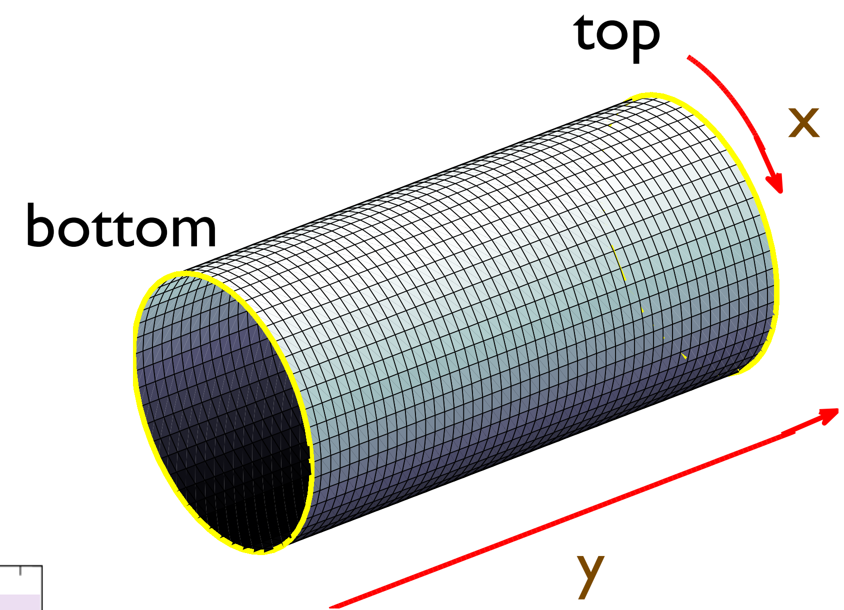


When α=*p*/*q* rational:

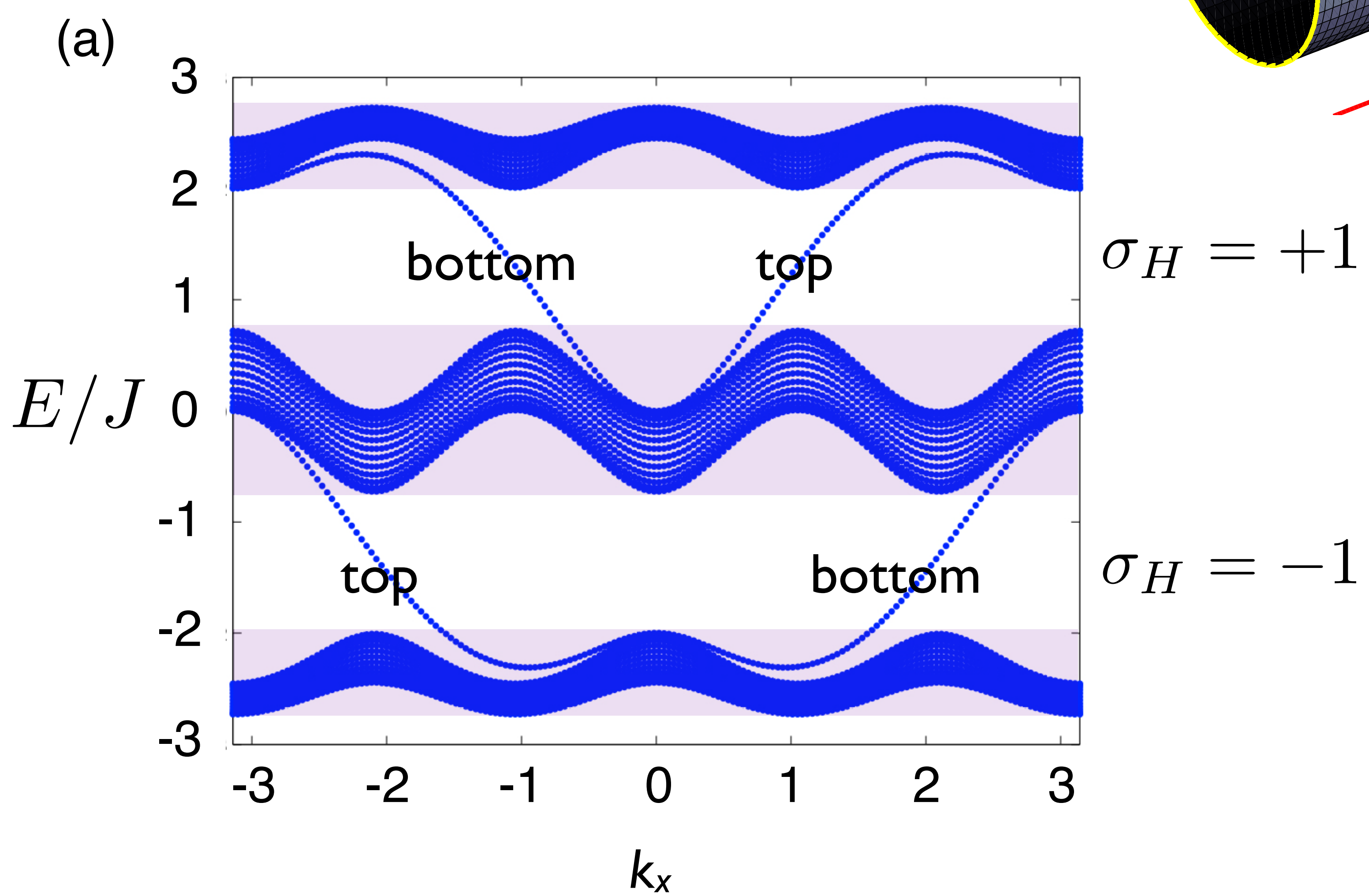
- *q* sub-bands, width <<8*J*
- Recursive structure, discovered by Azbel and Hofstadter
- Very different from Landau levels (which appear for α<<1)



Closer look at α=1/3



Energy spectrum on a cylinder (k_x=0)



Similar structure as in the «bulk» :

chiral edge states are present in the bandgaps, with opposite currents flowing at the edges

Hall conductivity as a topological invariant :

TKNN formula

Key result from Thouless, Kohmoto, Nightingale, den Niijs (TKNN) :

$$\sigma_H = \sigma_{xy} = \frac{e^2}{h} \sum_n \nu_n$$

provided the Fermi energy is in a gap

$$\nu_n = \frac{i}{2\pi} \int_{BZ} \underline{\left(\langle \partial_{k_x} u_n(\mathbf{k}) | \partial_{k_y} u_n(\mathbf{k}) \rangle - (k_x \leftrightarrow k_y) \right)} \, d^2\mathbf{k}$$

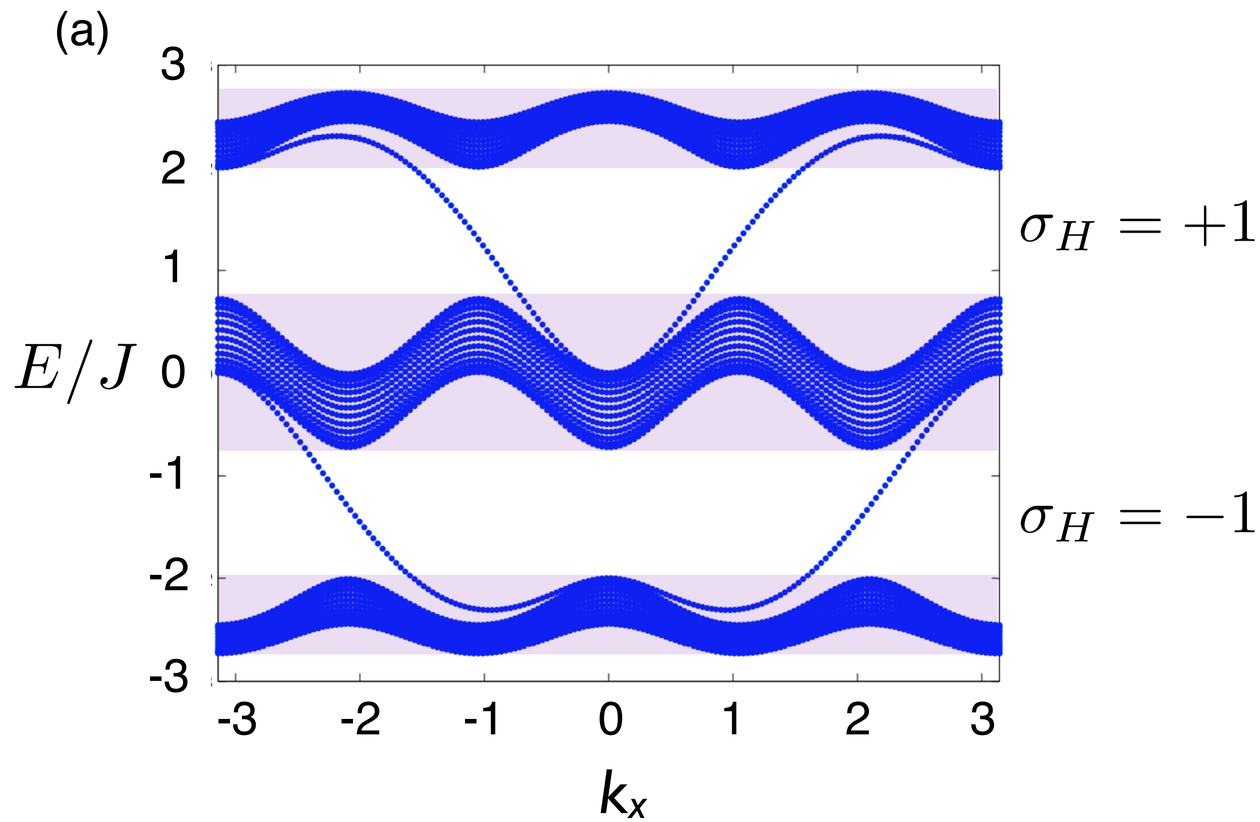
Berry curvature associated with the band eigenstates

- Chern number = topological index (integer-valued)
- characterizes the topology of the Hilbert space (more precisely, of the subspace associated with each energy band) : non zero only if the Bloch eigenstates have a vortex structure in k space
- always defined, but related to Hall conductivity only if the Fermi energy lies in the gap between two bands

Bulk-edge correspondence

There is a deep relation between the Chern number characterizing the bulk material and the edge states appearing at its edge :

- number of edge states branches (per physical edge) : $|\sigma_H|$
- chirality of edge states : $\mathrm{sign}(\sigma_H)$



Although the Chern number does not always exist in the present form (e.g., spin Hall topological insulators or interacting systems), other topological invariants can be defined in these situations.

When non-zero, chiral edge states always appear at the boundaries.