Artificial gauge potentials with optical flux lattices

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Work done in collaboration with
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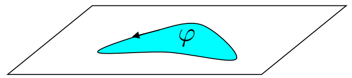
Simulation of orbital magnetism with neutral particles

Effective single particle Hamiltonian : $H_{\text{orbital}} = \frac{(\vec{p} - q\vec{A})^2}{2m}$

Many particles: Quantum Hall physics

When a charged particle is placed in a magnetic field and follows a closed contour, it acquires a geometric phase (Aharonov-Bohm)



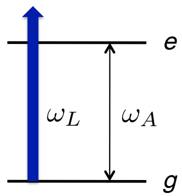
Any other process that creates a 'geometrical' phase can simulate magnetism
for example : Sagnac (rotation), Berry (adiabatic following)

Dalibard, Gerbier, Juzeliūnas & Öhberg, Rev. Mod. Phys. **83**, 1523 (2011)

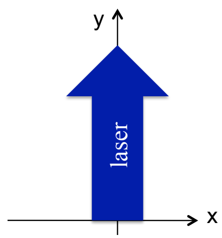
Outline of the talk

1. A toy model : Berry's phase in a bulk system
2. Optical lattices in the tight-binding regime
3. Flux lattices

Simulating orbital magnetism without rotation: toy model



Neglect spontaneous emission from *e* (intercombination line of alkaline-earth or Ytterbium atoms)



Using the Rotating-Wave Approximation, the coupling reads in the basis $|g\rangle,\ |e\rangle$:

$$V = \frac{\hbar}{2}\begin{pmatrix} \Delta & \kappa\, e^{-iky} \\ \kappa\, e^{iky} & -\Delta \end{pmatrix}$$

$\Delta = \omega_L - \omega_A$: detuning
κ : Rabi frequency

$$= \frac{\hbar\tilde{\kappa}}{2}\begin{pmatrix} \cos\theta & \sin\theta\; e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}$$

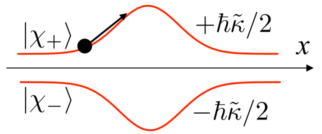
$\tilde{\kappa} = \sqrt{\kappa^2 + \Delta^2}$

Dressed states (energy $\pm\hbar\tilde{\kappa}/2$): $|\chi_\pm\rangle$

$$|\chi_+\rangle = \cos\frac{\theta}{2}\,|g\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|e\rangle$$

$\tan\theta = \kappa/\Delta \qquad \phi = ky$

A toy model (II): adiabatic following of a dressed state



$$\Psi(\vec{r},t) = \sum_\pm \psi_\pm(\vec{r},t)|\chi_\pm(\vec{r})\rangle$$

The atom follows adiabatically the dressed state $\chi+$

Adiabatic elimination of the dressed state $\chi-$ ⟶ $i\hbar\frac{d\psi_+}{dt} = H'\psi_+$

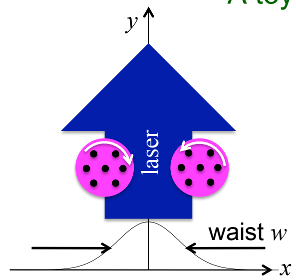
$$H' = \frac{(\vec{P}-\vec{A})^2}{2M} + \frac{\hbar\tilde{\kappa}}{2} + W \qquad \vec{A} = i\hbar\langle\chi_+|\vec{\nabla}\chi_+\rangle$$

vector potential, Berry's phase

Magnetism ?
$$\begin{cases} \vec{A} = \frac{\hbar}{2}(1-\cos\theta)\vec{\nabla}\phi & \textit{Berry connection} \\ \vec{B} = \vec{\nabla}\times\vec{A} = \frac{\hbar}{2}\vec{\nabla}\phi\times\vec{\nabla}(\cos\theta) & \textit{Berry curvature} \end{cases}$$

One needs both a gradient of the mixing angle θ and the phase ϕ

A toy model (III): effective magnetic field

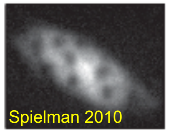


$$\phi = ky$$

Gaussian beam: $\tan\theta = \frac{\kappa}{\Delta} = \frac{\kappa_0}{\Delta}e^{-x^2/w^2}$

$$\vec{B} = \frac{\hbar}{2}\vec{\nabla}\phi\times\vec{\nabla}(\cos\theta) \qquad \vec{\nabla}\theta \sim 1/w$$

⟶ typical scale: $B \sim \hbar k/w$



Number of vortices in area w^2: $N_v \approx \frac{B_0 w^2}{2\pi\hbar} \approx \frac{w}{\lambda}$ possibly $\gg 1$

For reaching quantum Hall states, one needs filling factors of order unity:

$$\frac{N_{\text{atoms}}}{N_v} \sim \frac{\rho_{\text{at}}w^2}{w/\lambda} \sim \rho_{\text{at}}w\lambda$$

For degenerate atomic gases, one typically has : $\rho_{\text{at}}\lambda^2 \gtrsim 1$

Need to go to small w: optical lattice-like configuration

Outline of the talk

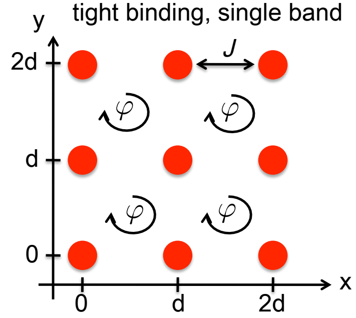
1. A toy model : Berry's phase in a bulk system

⟶ 2. Optical lattices in the tight-binding regime

3. Flux lattices

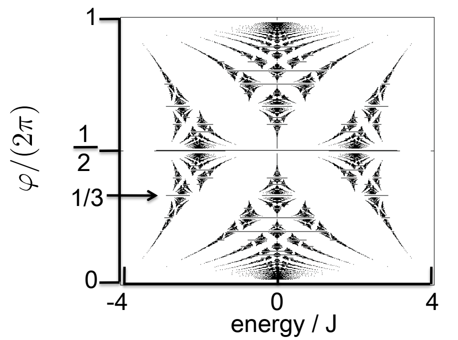
Magnetism in a 2D square lattice

electron in a square lattice
tight binding, single band



$$\varphi = qBd^2/\hbar$$

single particle energy spectrum



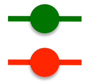
Hofstadter butterfly

Implementation in an optical lattice

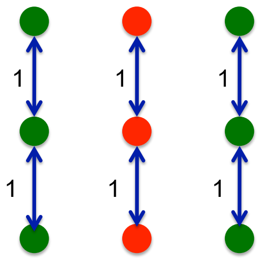
Ruostekoski
Sorensen-Demler-Lukin
Muller, Dudarev *et al.*
Lim-Smith-Hemmerich
Zhang *et al.*
Goldman *et al.*
Gerbier-Dalibard

Seminal paper: Jaksch-Zoller

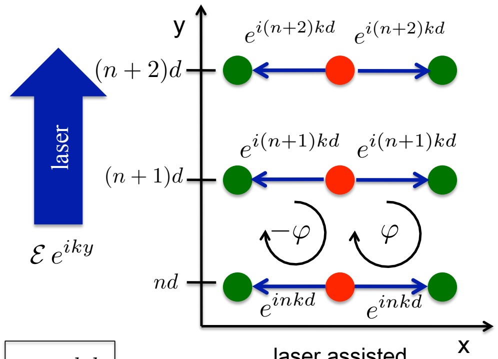
For simplicity, consider again a two-level system *e-g*



With properly chosen laser standing waves, achieve a state-dependent lattice



standard tunnelling along y



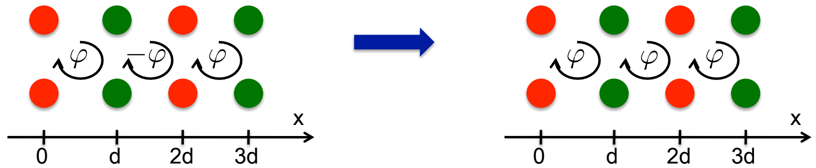
$\varphi = kd$

laser assisted tunnelling along x

Gauge fields in the tight-binding regime

Staggered field: I.Bloch (2011)
Time dependent potentials: K. Sengstock (2012)
Use of radiofrequency fields: Spielman (2012)

Rectification is needed to reach the Hofstadter butterfly



Among other proposals: Gerbier & Dalibard, use of a superlattice

For large filling factors (≈ 1), these lattices in the tight-binding regime are quite different from the bulk Lowest Landau Level.

The are interesting in their own right, but one can look for systems closer to bulk Quantum Hall systems.

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1. A toy model : Berry's phase in a bulk system

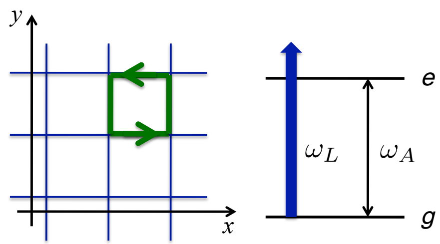
2. Optical lattices in the tight-binding regime

→ 3. Flux lattices

Cooper, PRL **106**, 175301 (2011)
Cooper & Dalibard, EPL **95**, 66004 (2011)
Cooper & Moessner, PRL **109**, 215302 (2012)
Cooper & Dalibard, arXiv: 1212.3552

See also Juzeliunas & Spielman, NJP **14** 123022 (2012)

Artificial magnetic field in a periodic optical lattice?



$$V = \frac{\hbar\tilde{\kappa}}{2}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}$$

$\tilde{\kappa}, \theta, \phi$: periodic functions of x,y

$\vec{A} = \frac{\hbar}{2}(1-\cos\theta)\vec{\nabla}\phi$ is also periodic, hence $\oint_C \vec{A}\cdot d\vec{l} = 0$

$$\iint B_z(x,y)\; dx\, dy = 0$$

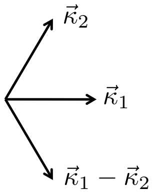
?

Correct only if $\vec{A}$ has no singularity in the plaquette

Singularities of $\vec{A}$ can occur at any point where $\sin\theta$ vanishes: ϕ undefined

An example of flux lattice (Cooper 2011)

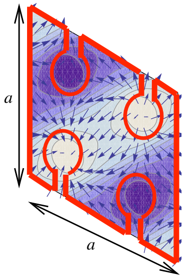
$$V = \frac{\hbar\tilde{\kappa}}{2}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}$$



Consider a triangular geometry where :

$$\cos\theta = \cos\left[(\vec{\kappa}_1 - \vec{\kappa}_2)\cdot\vec{r}\right]$$
$$\sin\theta\ e^{i\phi} = \cos(\vec{\kappa}_1\cdot\vec{r}) - i\cos(\vec{\kappa}_2\cdot\vec{r})$$

Vortex-type singularities in points where : $\cos(\vec{\kappa}_1\cdot\vec{r}) = 0 \qquad \cos(\vec{\kappa}_2\cdot\vec{r}) = 0$



$$\iint B_z(x,y)\ dx\,dy = -\pi\hbar \sum_{\text{singular } j} \text{sign}(j)$$
$$= 4\pi\hbar$$

How to operate a flux lattice?

Tight binding limit ? Because of the relation $\iint B_z(x,y)\ dx\,dy = -\pi\hbar \sum_{\text{singular } j} \text{sign}(j)$

there is an integer number of flux quanta per plaquette: not so interesting

Leaving the tight binding limit: $\hbar\tilde{\kappa} \sim E_{\text{recoil}}$

The adiabatic approximation becomes questionable and one has to calculate the band structure of $\dfrac{p^2}{2m} + \dfrac{\hbar\tilde{\kappa}}{2}\begin{pmatrix} \cos\theta & \sin\theta\ e^{-i\phi} \\ \sin\theta\ e^{i\phi} & -\cos\theta \end{pmatrix}$

How to operate a flux lattice (II) ?

Look e.g. at the lowest band $n = 0$:

$|u_{\vec{k}}\rangle$: periodic part of the Bloch functions of this band

- The band should be as flat as possible:

  Mimics the degeneracy of Landau levels, increase the role of interactions

- The band should possess "magnetic properties", characterized by the Chern index c_0

  Thouless et al. 1982: Hall conductivity (filled band) $\sigma_{xy}^{\text{Hall}} = -\dfrac{e^2}{h}c_0$

$$\mathcal{A}(\vec{k}) = i\langle u_{\vec{k}}|\vec{\nabla}_{\vec{k}} u_{\vec{k}}\rangle$$
$$\mathcal{B}(\vec{k}) = \vec{\nabla}\times\mathcal{A}(\vec{k})$$

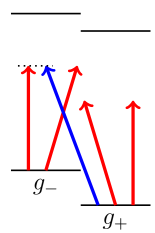
"vector potential" and "magnetic field" in momentum space

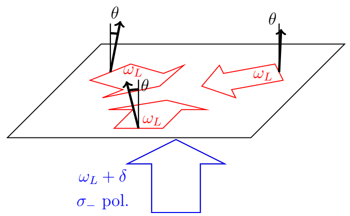
$$c_0 = \frac{1}{2\pi}\int_{\text{BZ}} \mathcal{B}(\vec{k})\ d^2k$$

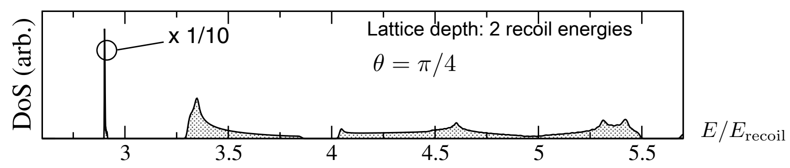
Realistic example for a spin1/2 atom

Cooper & Dalibard 2011

Raman coupling between two ground states







Lowest band : Chern index=+1, width=0.01 recoil energy

Gap above the lowest band: 0.4 recoil energy

Which ground state in the presence of interactions? Quantum Hall physics?

A guide line to progress

Cooper & Moessner

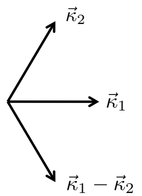
View *optical lattices* as a *tight-binding model* *in momentum space*

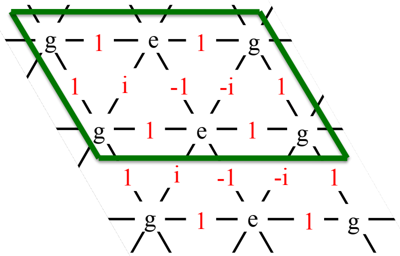
Consider again the two-state model $\{|g\rangle, |e\rangle\}$ in a triangular configuration:

$$V_{A-L} = \frac{\hbar\tilde{\kappa}}{2}\begin{pmatrix}\cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta\end{pmatrix}$$

$$\cos\theta = \cos\left[(\vec{\kappa}_1 - \vec{\kappa}_2)\cdot\vec{r}\right]$$

$$\sin\theta\; e^{i\phi} = \cos(\vec{\kappa}_1\cdot\vec{r}) - i\cos(\vec{\kappa}_2\cdot\vec{r})$$





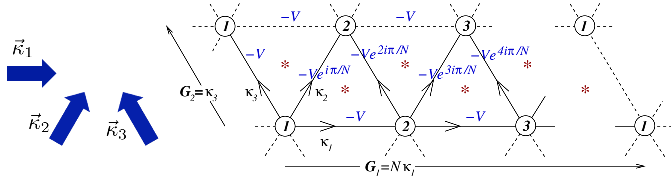
phase π/2 for each of the four triangles of the unit cell

A guide line to progress

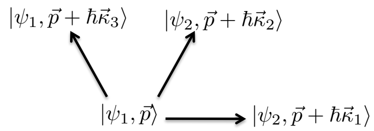
Cooper & Moessner

View *optical lattices* as a *tight-binding model* *in momentum space*

Consider an atom with $N >> 1$ internal states $|\psi_1\rangle, \ldots, |\psi_N\rangle$ and assume a laser scheme (non trivial!) that provides the following coupling in momentum state:



$2N$ triangles, with flux π/N per triangle

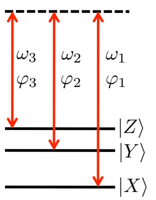


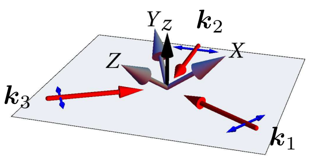
For $N \gg 1$ one recovers magnetic properties identical to those of the lowest Landau level with a uniform Berry curvature

How to implement this idea in practice

Cooper & Dalibard
arXiv: 1212.3552

Three-level atom (typically [87]Rb, J=1 ground state) + 3 beams at 120° in the xy-plane

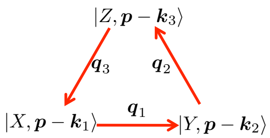




$$|X\rangle = |J = 1, m_X = 0\rangle, \ldots$$

The degeneracy between $|X\rangle,\ |Y\rangle,\ |Z\rangle$ is lifted e.g. by micro-wave dressing

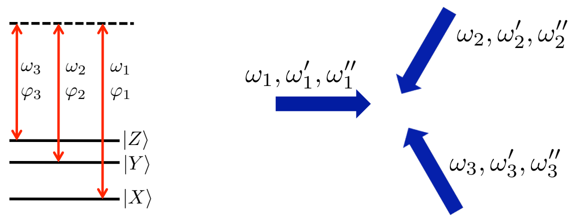
Momentum space:

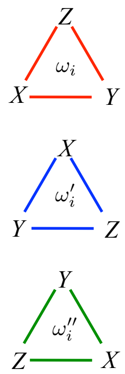


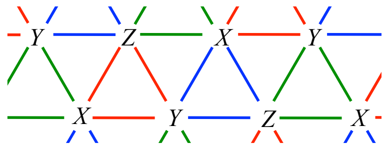
$$\boldsymbol{q}_1 = \boldsymbol{k}_1 - \boldsymbol{k}_2$$

How to implement this idea in practice (II)

Use three triplets of waves (same optical modes) generated by acousto-optic modulators







The control of the nine phases $\varphi_j, \varphi_j', \varphi_j''$ allows one to adjust the phases of the triangles in **momentum space**

Quasi-flat band in **position space**: 0.015 recoil energy, Gap/50

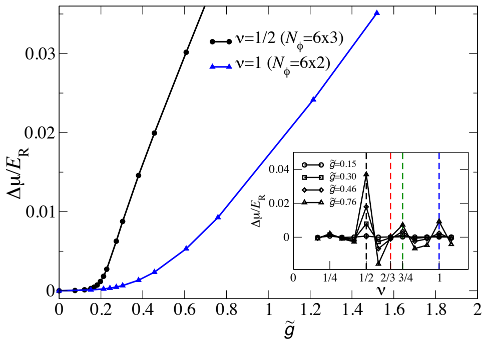
Is it good for reaching quantum Hall state?

Cooper & Dalibard
arXiv: 1212.3552

Exact diagonalization for various filling factors; calculate the gap :

$$\Delta\mu = \frac{1}{N}\left[E(N+1) + E(N-1) - 2E(N)\right]$$

A non-zero gap signals an incompressible, strongly correlated ground state



Filling factors ½ and 1

Very good news!

Incompressible states even for moderate interaction strength

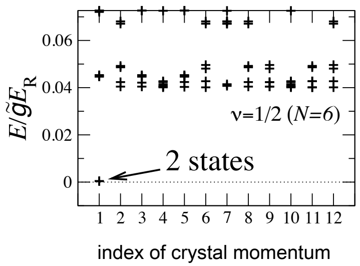
What are these incompressible states?

Cooper & Dalibard
arXiv: 1212.3552

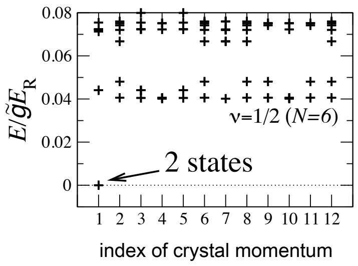
Low energy spectra + adiabatic continuity confirm that these states are

- → Laughlin-like state for filling factor ½
- → Non-Abelian Moore-Read phase (Pfaffian) for filling factor 1

Our 3-level atom



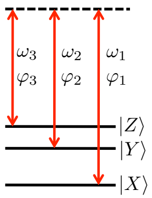
LLL (fictitious 12-level atom)



Summary

Optical flux lattices offer the possibility to simulate bulk fractional quantum Hall effect, with large filling factors.

They do not operate in the tight-binding regime (relatively weak laser intensity is needed)



The more internal states are included, the better...
Here a realistic 3-level configuration, where all needed frequencies (9) can be derived from the same laser source with programmable devices

Characterization of many-body groundstates of bosons in an OFL

Robust quantum Hall states, including Laughlin and non-Abelian Moore-Read, for relatively weak interaction strength