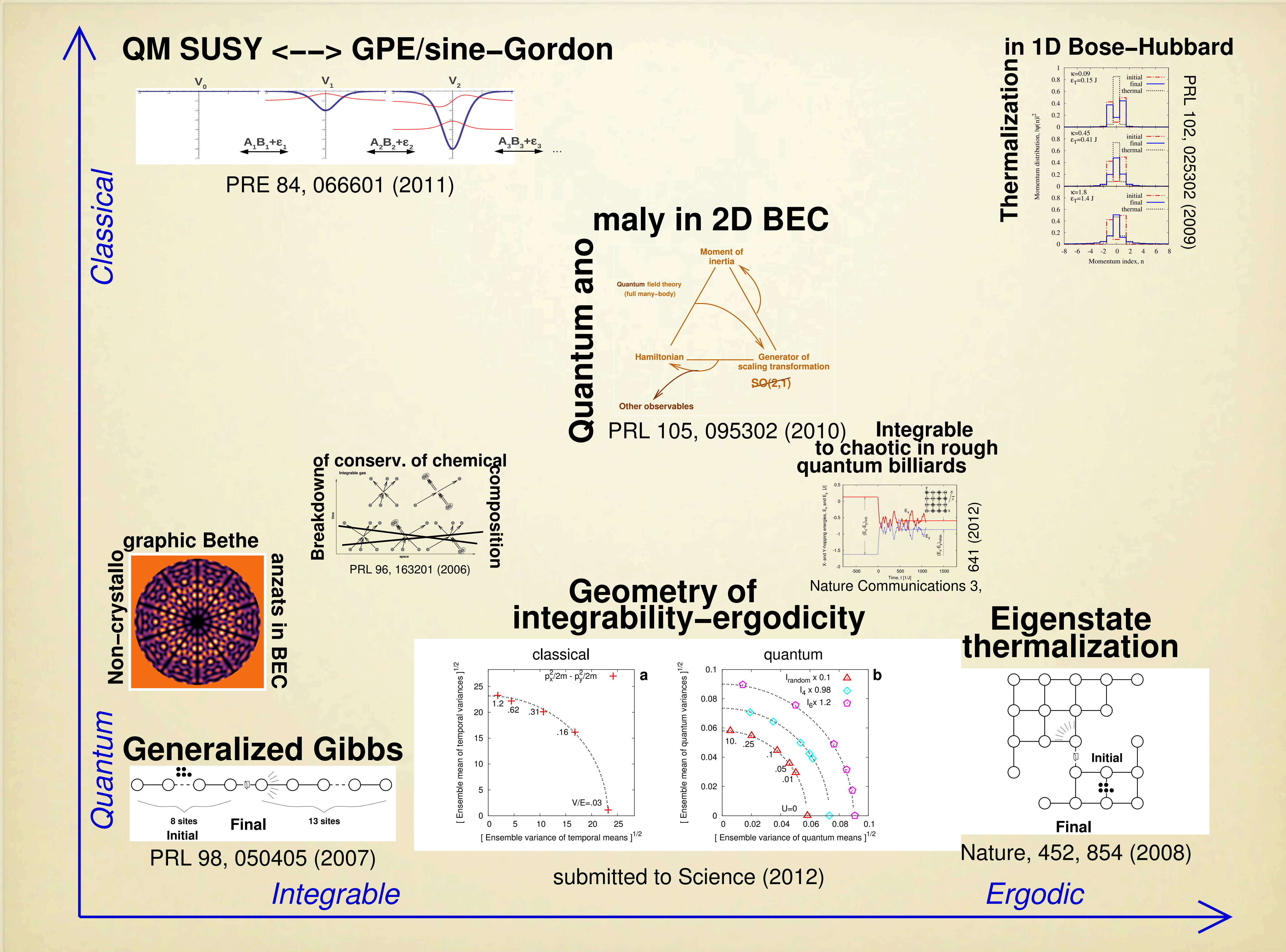
Geometry of Quantum Observables, Integrability-Thermalizability Transition, and Extended Thermodynamics of Integrable and/or Mesoscopic Systems

UMASS BOSTON

Maxim Olshanii (Olchanyi)
University of Massachusetts
Boston



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Preview

- Proposition:

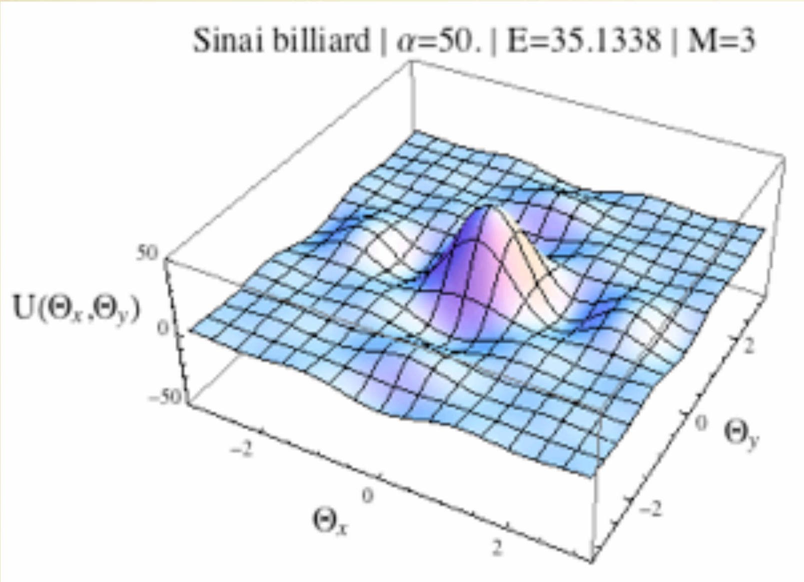
  The sum of the ensemble variance of the temporal means and the ensemble mean of the temporal variances remains approximately constant across the integrability-to-ergodicity transition

- Example of a Sinai-type billiard
- Example of long-range interacting hard-core bosons
- Ensemble variance of temporal means as a cos^2 of the Hilbert-Schmidt (HS) angle between the observable and integrals of motion

  HS geometry of density matrices for Quantum Information: 164 arXiv articles
  HS geometry of observables: 0 arXiv, hints in Suzuki's quantum extension of Mazur's theorem (Physica 51 (1971))

- IPR as an angle between the original and perturbed integrals of motion
- An application of the HS geometry: Optimal integrals of motion for GGE
- Mesoscopicity and integrability on the same footing $\longrightarrow$ "nano-meso"

Trajectories in a soft Sinai billiard



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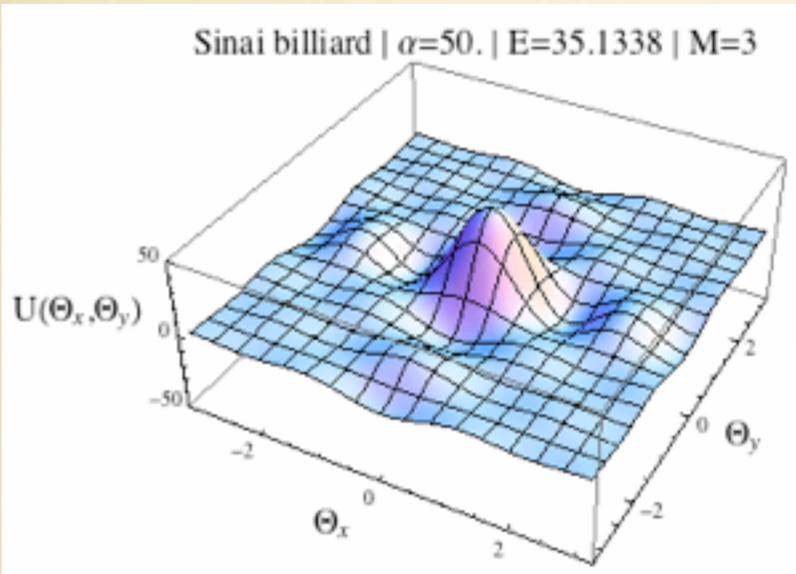

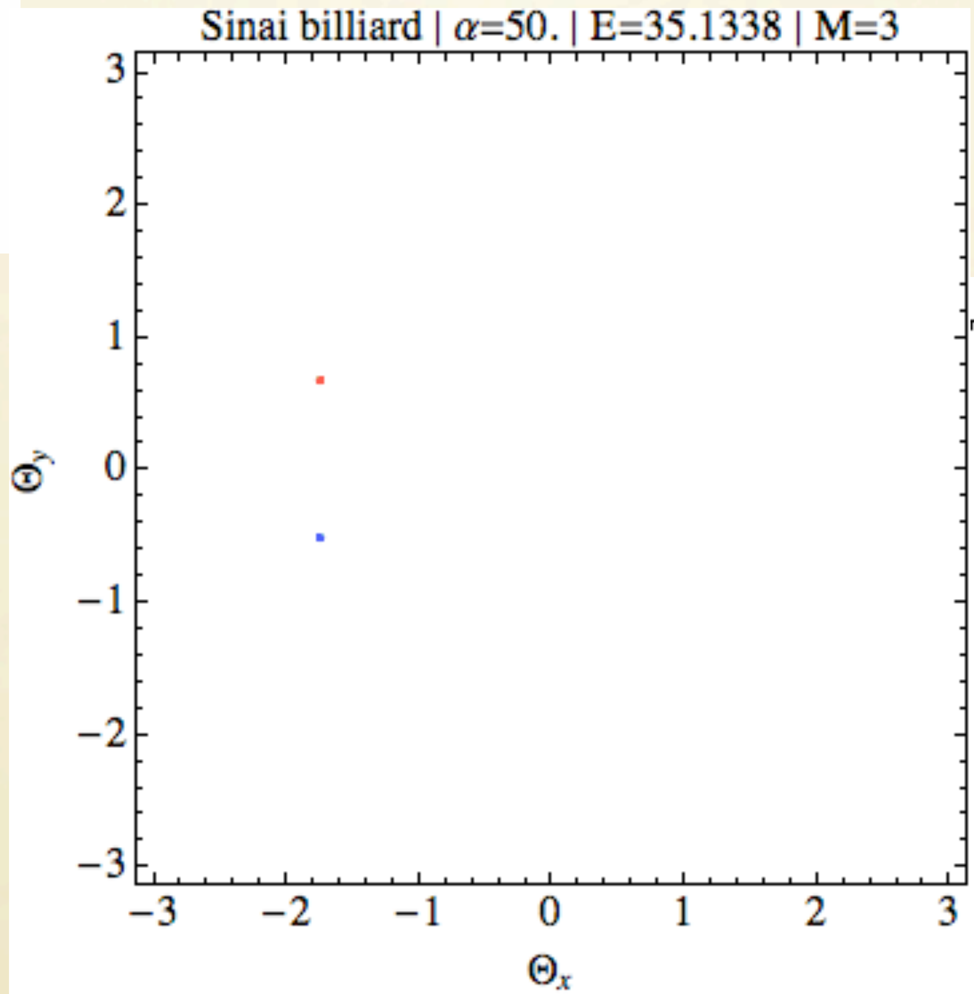

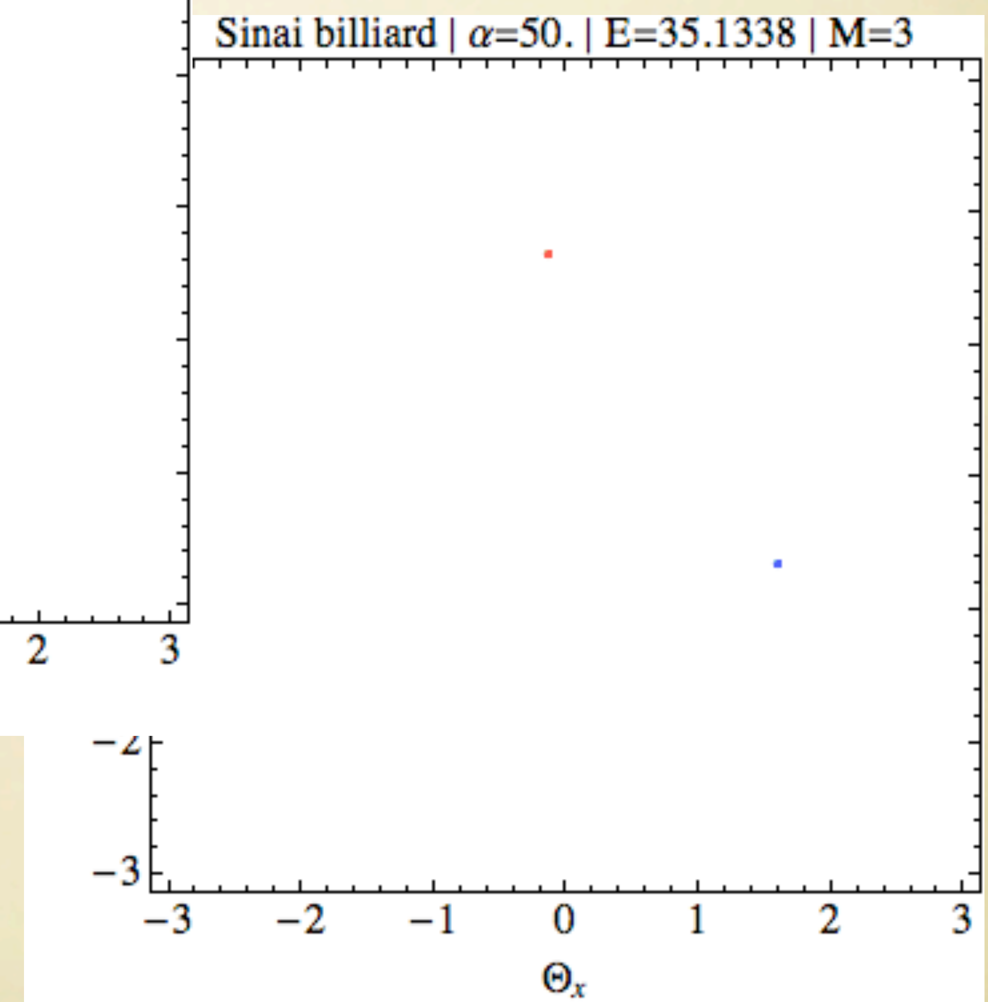

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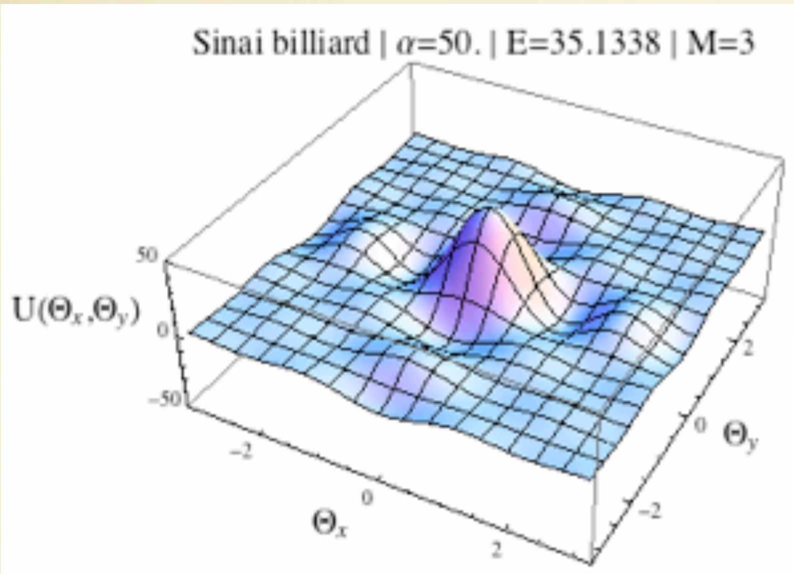
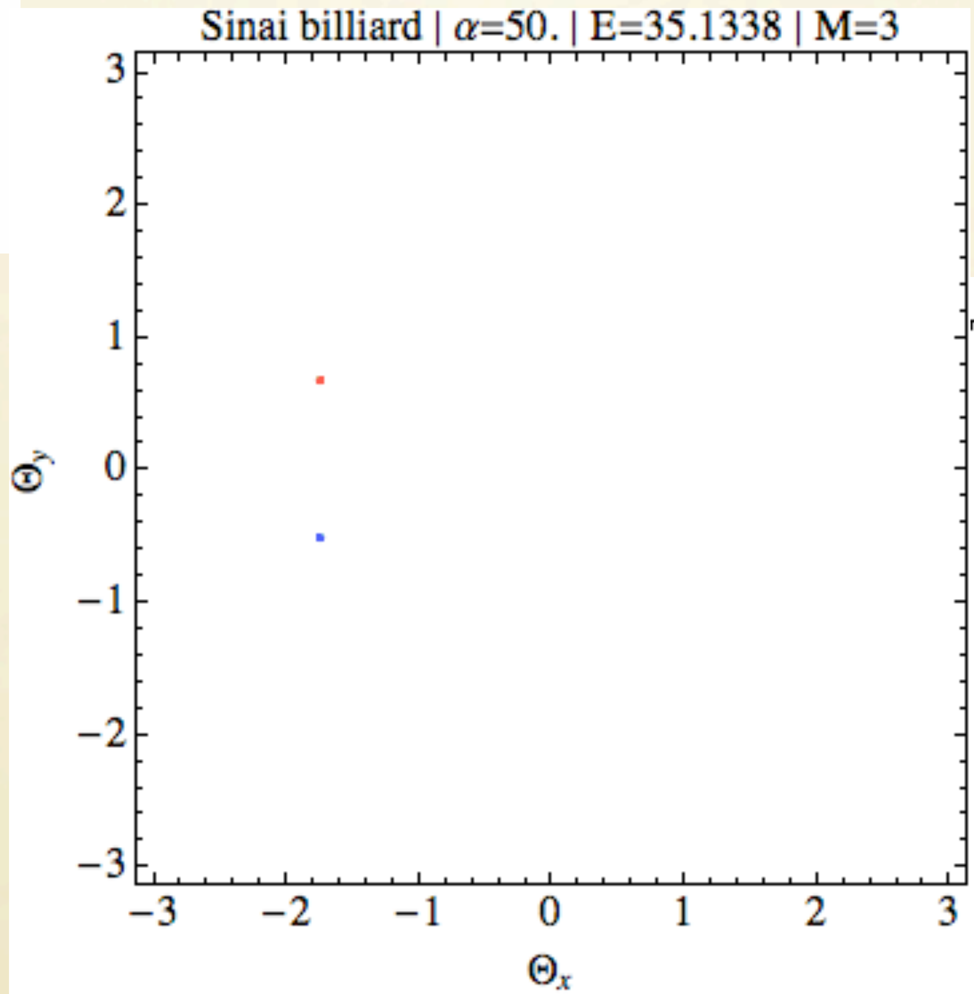
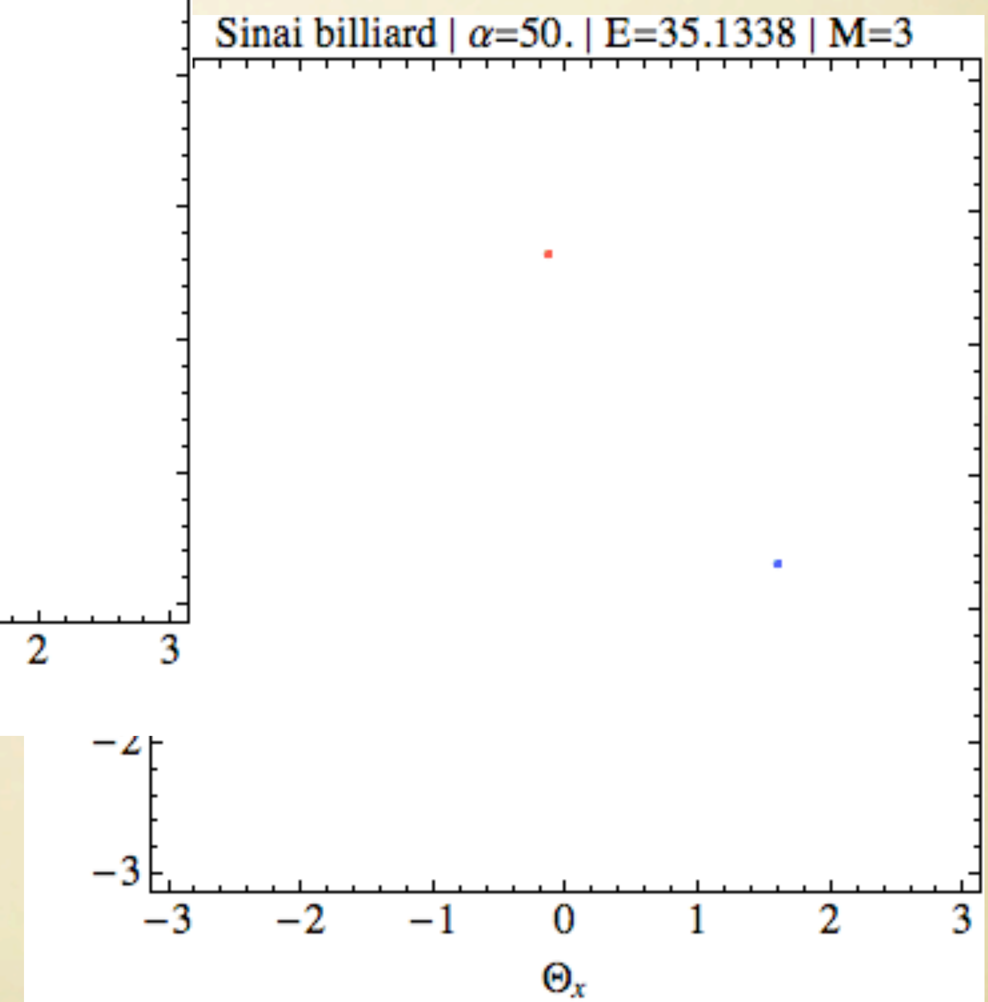
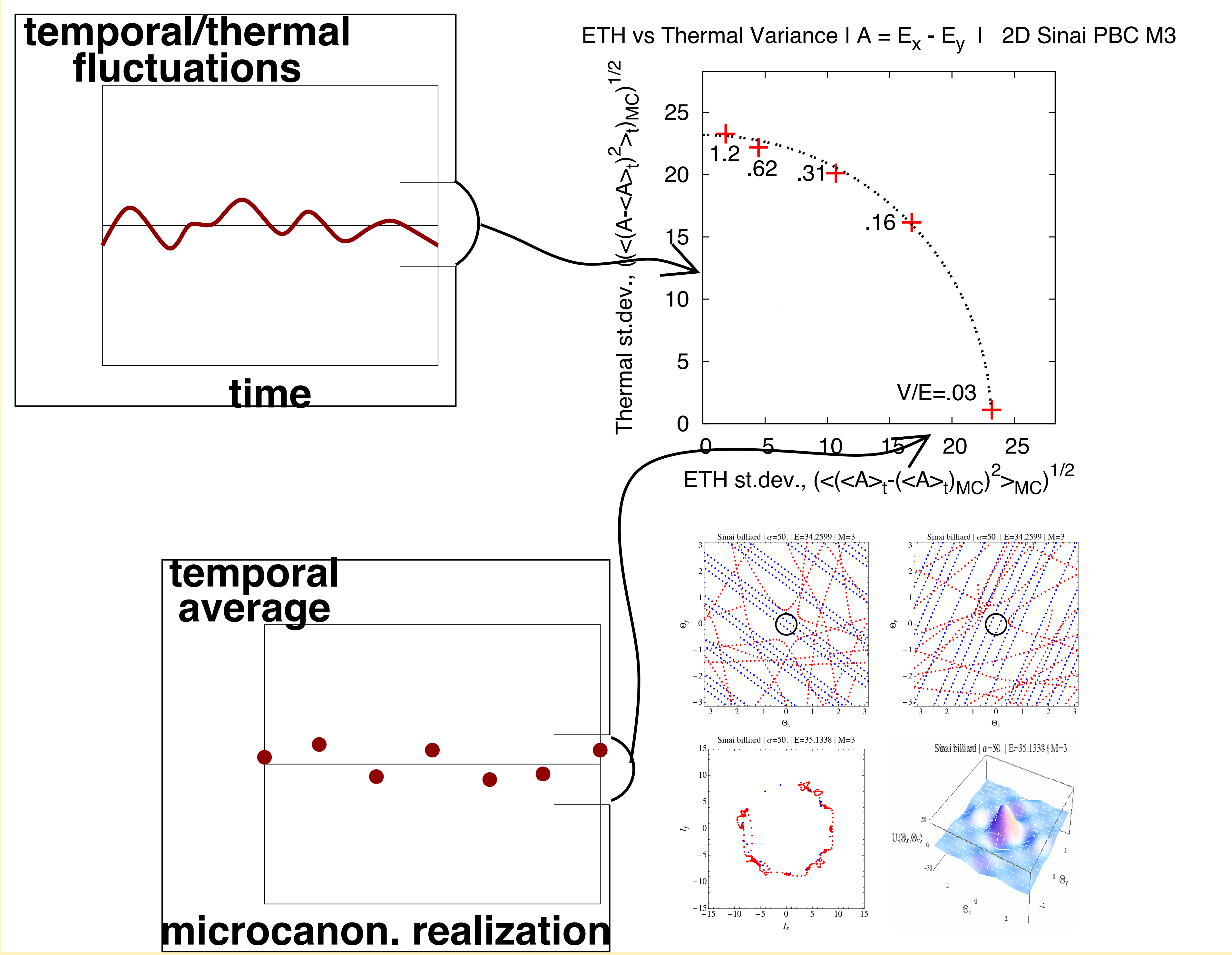






A Sinai-type billiard: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



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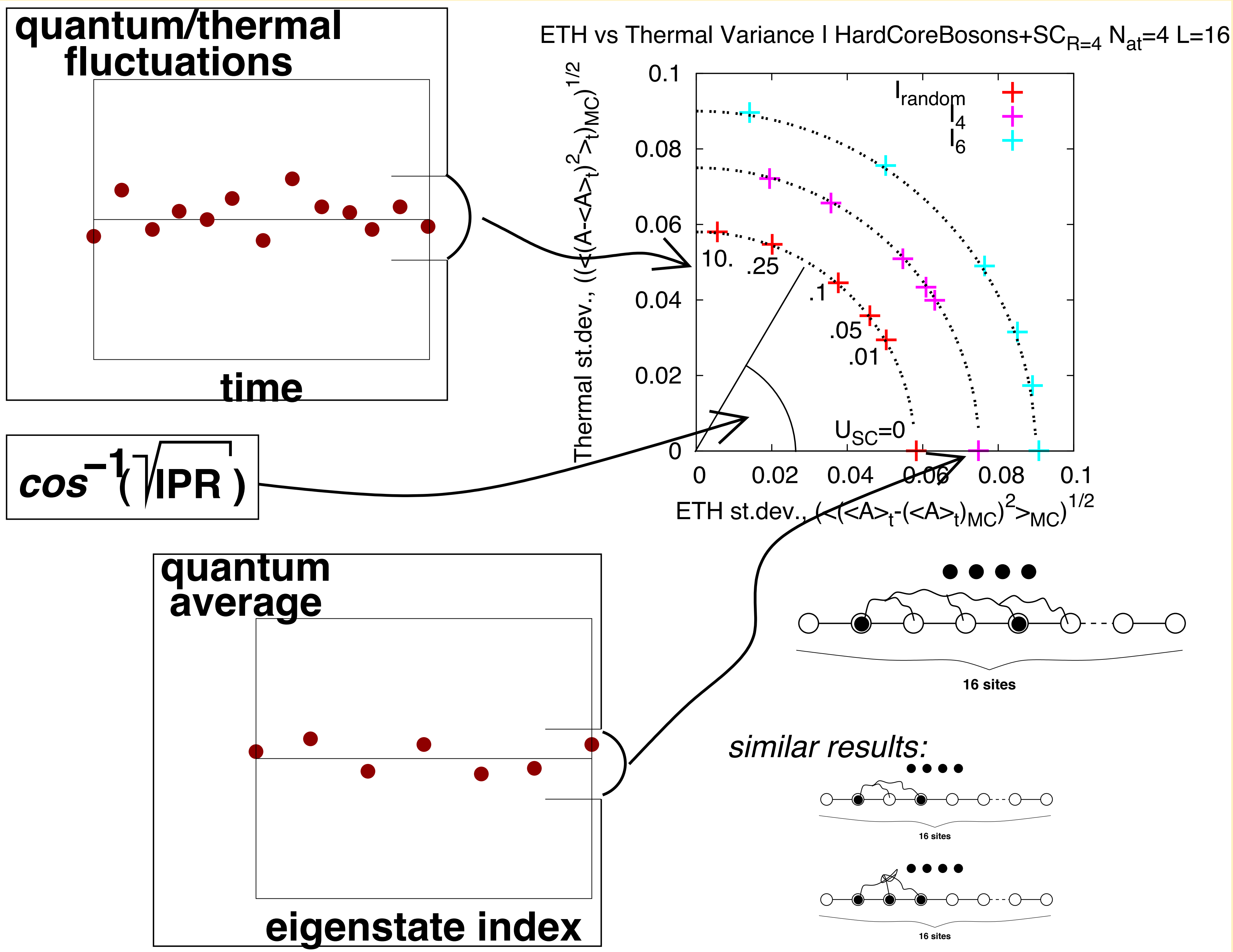
HCB+NNNNN: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



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Hilbert-Schmidt inner product

"[T]he angel of geometry and the devil of algebra share the stage, illustrating the difficulties of both."

Hermann Well

The Hilbert-Schmidt (HS) inner product between two matrices:

$$(\hat{A}|\hat{B}) \equiv Tr[\hat{A}^{\dagger}\hat{B}] \quad .$$

HS product is invariant under unitary transformations:

$$(\hat{U}\hat{A}\hat{U}^{-1}|\hat{U}\hat{B}\hat{U}^{-1}) = (\hat{A}|\hat{B}) \quad .$$

The unitary transformations form a (small) subgroup of the group of HS rotations: the latter preserve the HS norm, defined as

$$||\hat{A}|| \equiv \sqrt{Tr[\hat{A}^2]} \quad .$$

Introduce:

(a) a microcanonical window: $\mathcal{W}_{MC} \equiv \{|\alpha\rangle \quad | \quad E_\alpha \in [E_{min}, E_{max}]\}$, where $|\alpha\rangle$ and E_α are the eigenstates and eigenenergies of a Hamiltonian $\hat{H}$;

(b) a HS normalized identity operator: $\hat{\mathcal{I}} = (N_{MC})^{-1/2} I$;

(c) a space of the diagonal (w.r.t. $\hat{H}$) observables: $\mathcal{L}_{d,\,\hat{H}} \equiv Span[\{|\alpha\rangle\langle\alpha| \quad | \quad |\alpha\rangle \in \mathcal{W}_{MC}\}]$;

(d) a space of the off-diagonal (w.r.t. $\hat{H}$) observables: $\mathcal{L}_{o-d,\,\hat{H}} \equiv Span[\{2^{-1/2}(|\alpha\rangle\langle\beta| + h.c.)\} \cup i2^{-1/2}(|\alpha\rangle\langle\beta| - h.c.)\} \quad | \quad |\alpha\rangle \in \mathcal{W}_{MC}; |\beta\rangle \in \mathcal{W}_{MC}; \quad \beta > \alpha \quad \}]$.

The integrability-ergodicity-to-HS dictionary

Then:

ensemble variance of the temporal means (ETH variance) $\equiv$

$$\left(N_{MC}^{-1}\sum_{\alpha}^{\mathcal{W}_{MC}} A_{\alpha,\,\alpha}^2 - \langle\hat{A}\rangle_{MC}\right) / \left(\langle\hat{A}^2\rangle_{MC} - \langle\hat{A}\rangle_{MC}^2\right) = \cos^2(\hat{A}\widehat{\ }\mathcal{L}_{d,\,\hat{H}}) - \cos^2(\hat{A}\widehat{\ }\hat{\mathcal{I}})$$

ensemble mean of the temporal (quantum) variance $\equiv$

$$N_{MC}^{-1}\left(\sum_{\alpha,\,\beta\neq\alpha}^{\mathcal{W}_{MC}} A_{\alpha,\,\beta}^2\right) / \left(\langle\hat{A}^2\rangle_{MC} - \langle\hat{A}\rangle_{MC}^2\right) = \cos^2(\hat{A}\widehat{\ }\mathcal{L}_{o-d,\,\hat{H}})$$

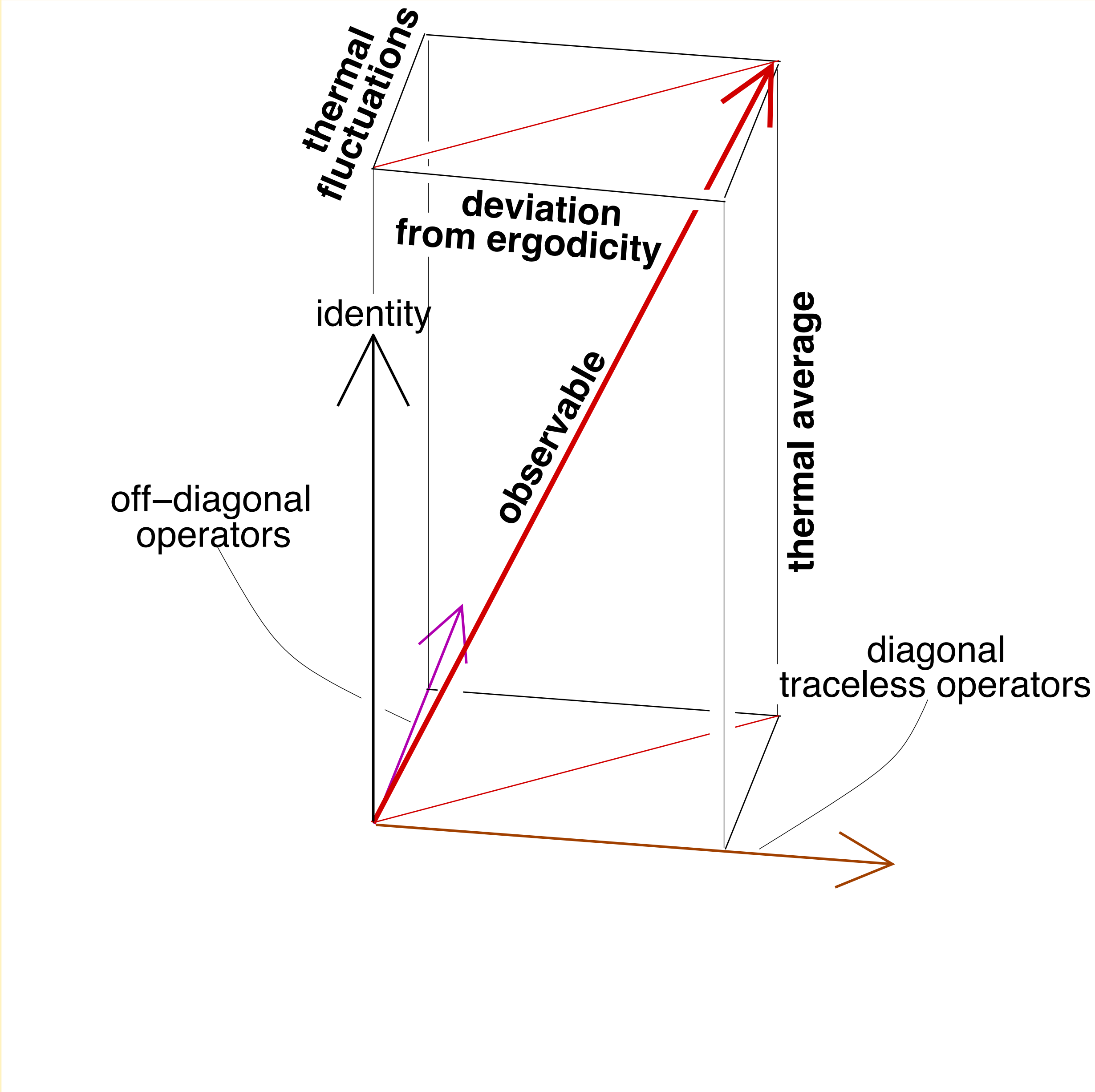
ensemble variance $\equiv$

$$\langle\hat{A}^2\rangle_{MC} - \langle\hat{A}\rangle_{MC}^2 = N_{MC}^{-1}\left(||\hat{A}^2|| - N_{MC}^{-1}||\hat{A}||^2\right)$$

inverse participation ratio (ITR) between the eigenstates of an integrable ($\hat{H}_0$)

$$N_{MC}^{-1}\sum_{\alpha,\,\alpha_o}^{\mathcal{W}_{MC}} |\langle\alpha_0|\alpha\rangle|^4 = \cos^2(\mathcal{L}_{d,\,\hat{H}_0}\widehat{\ }\mathcal{L}_{d,\,\hat{H}})$$

HS Geometry of the Integrability-Thermalizability transition



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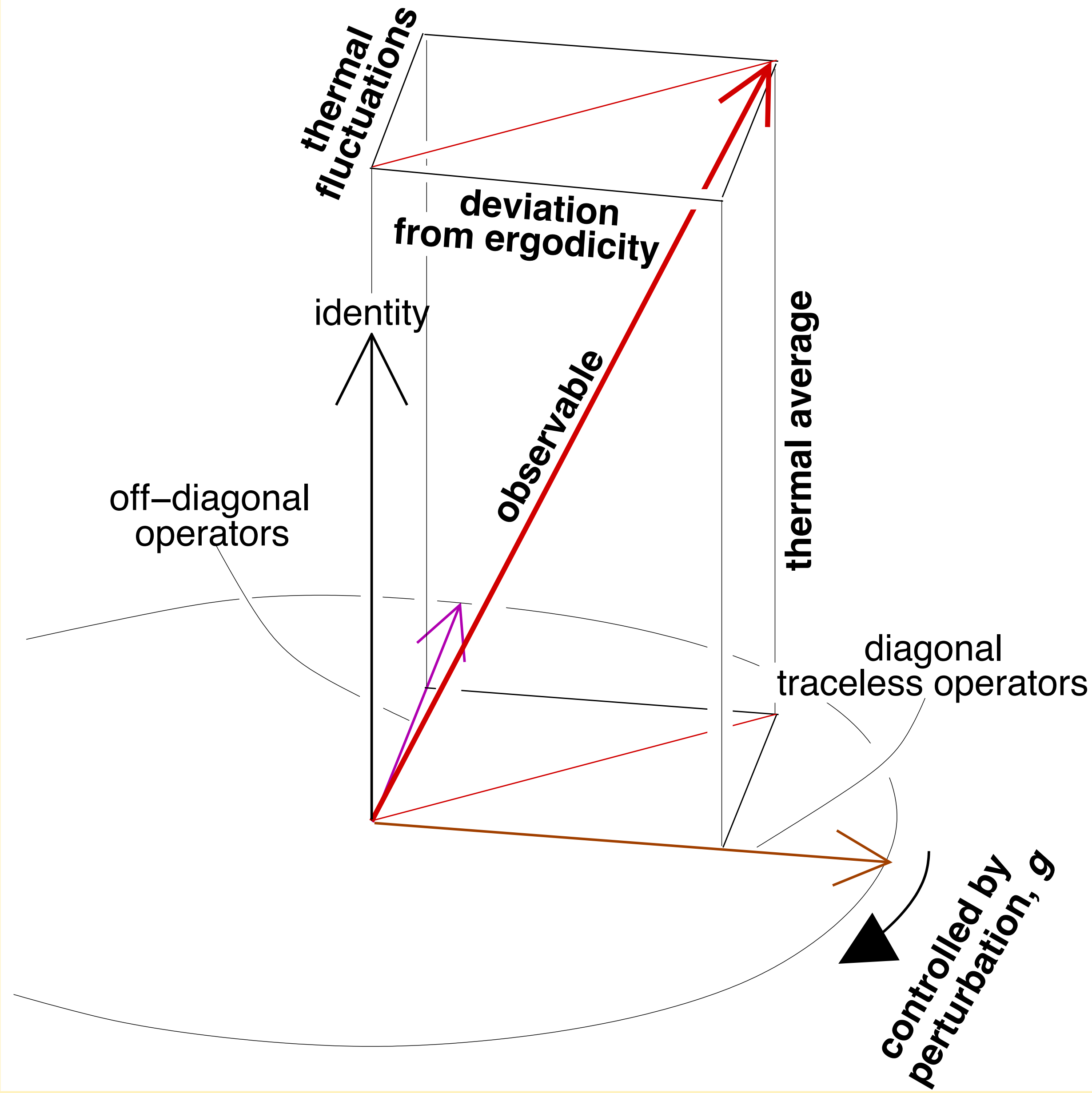
HS Geometry of the Integrability-Thermalizability transition



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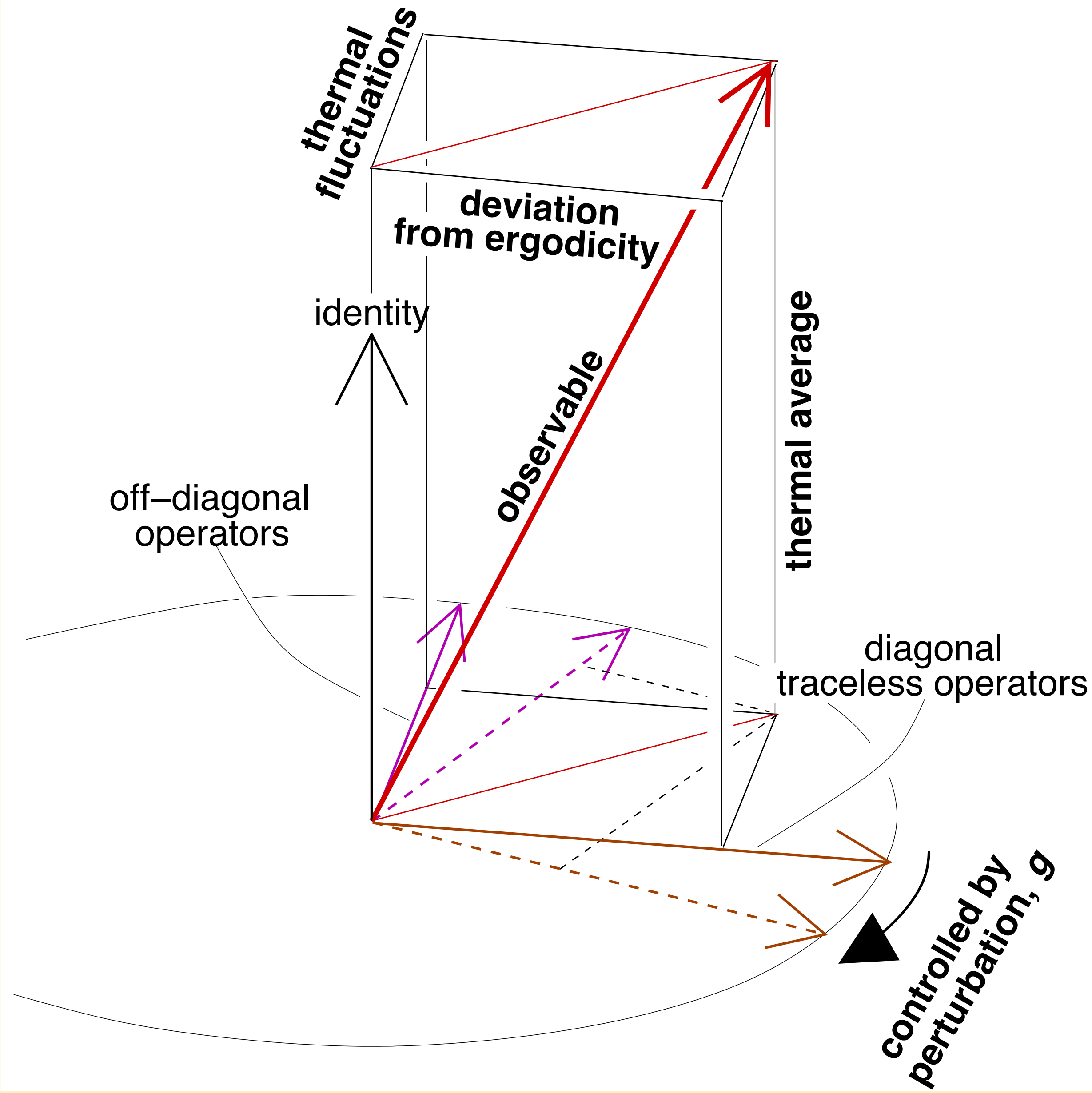

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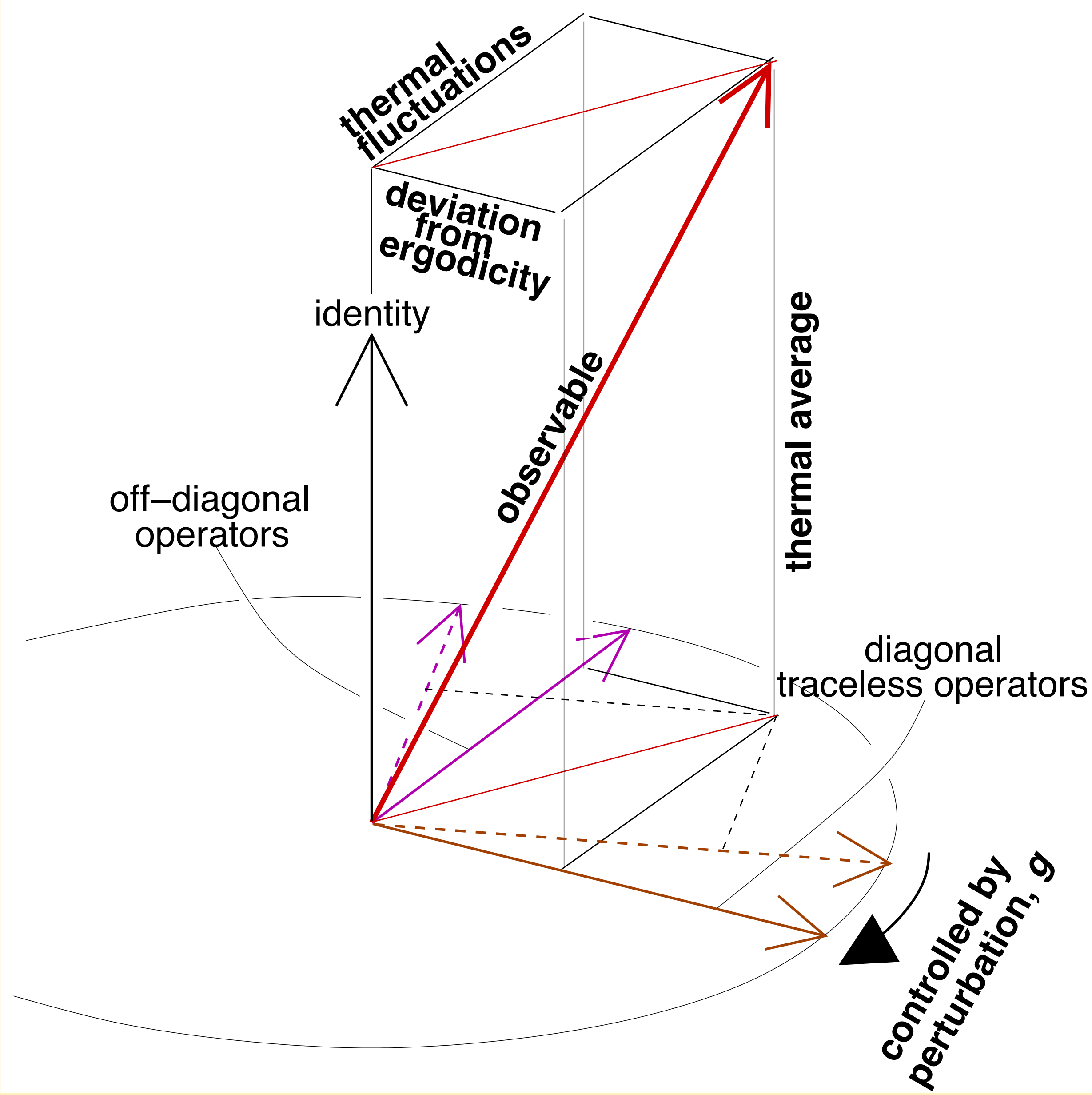
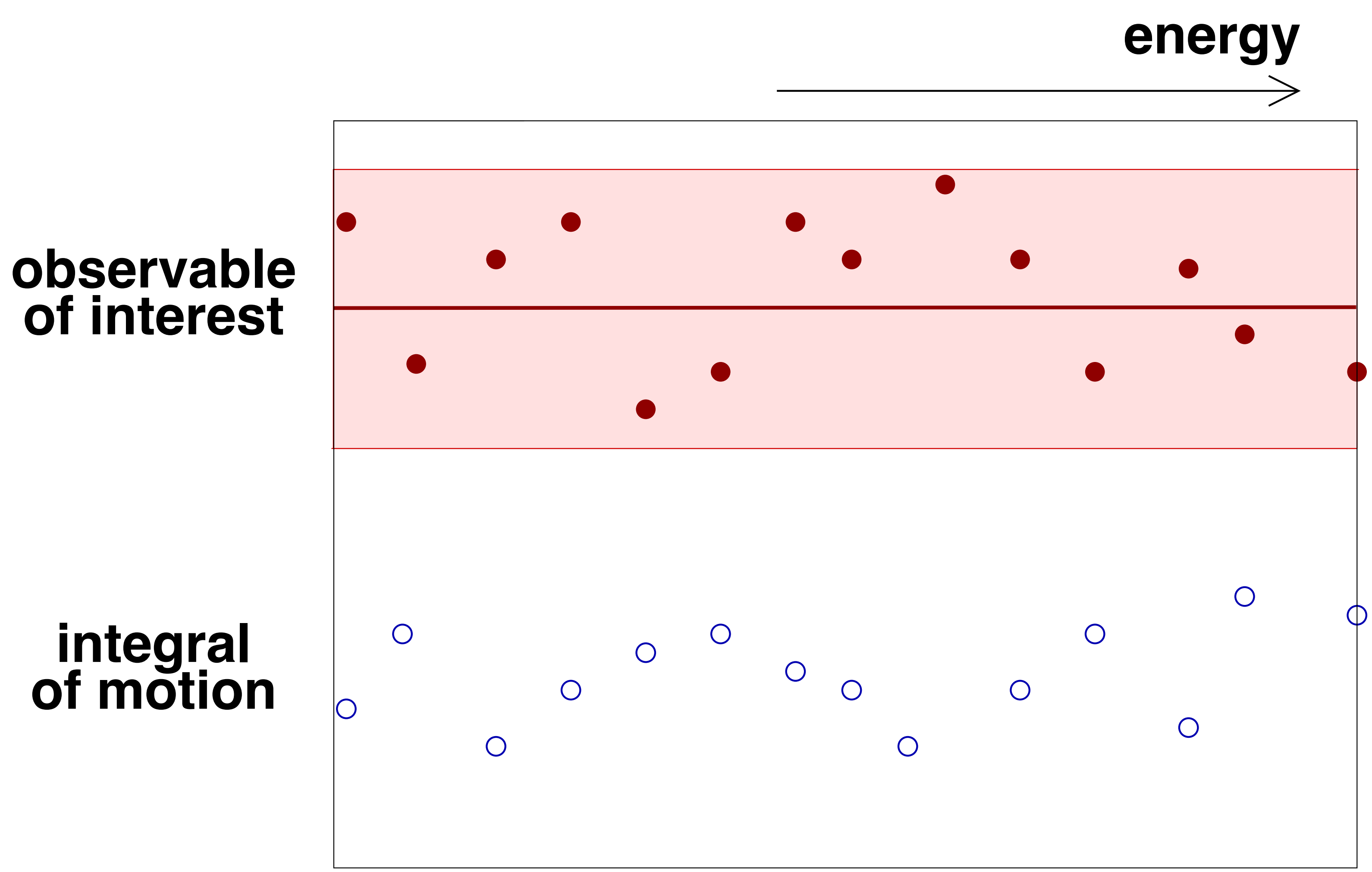


Conventional microcanonical vs generalized Gibbs microcanonical ensemble



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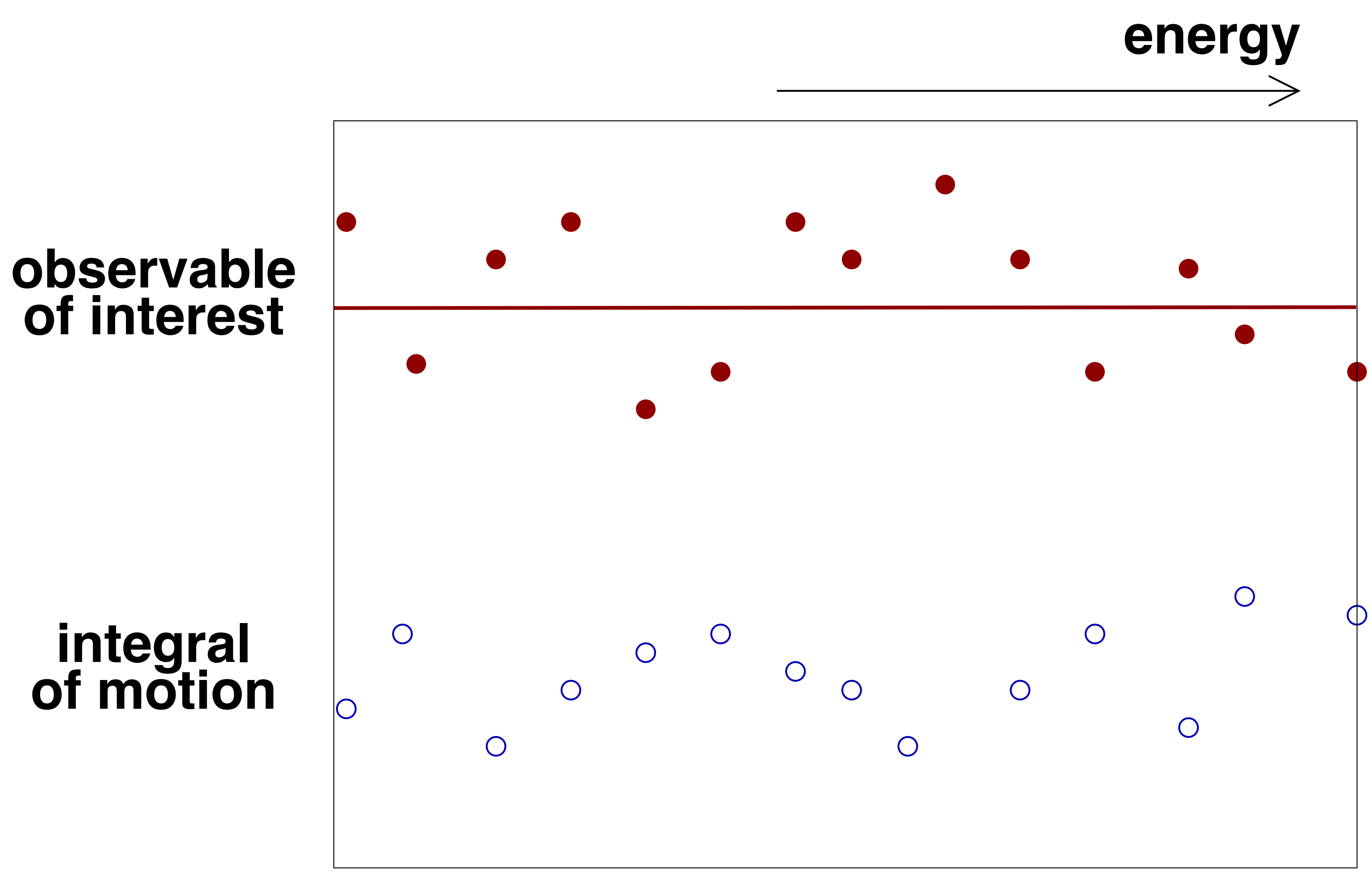

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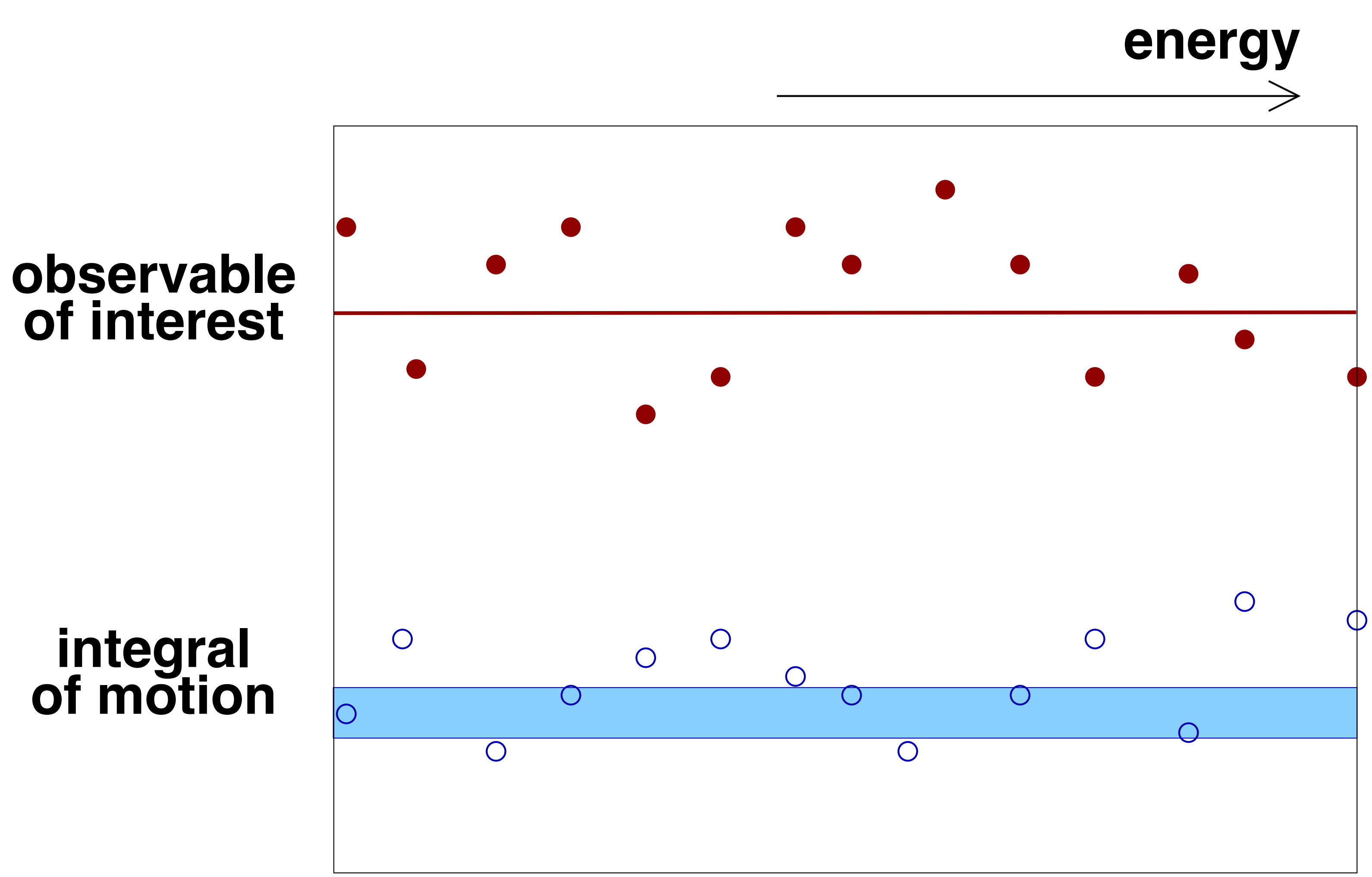

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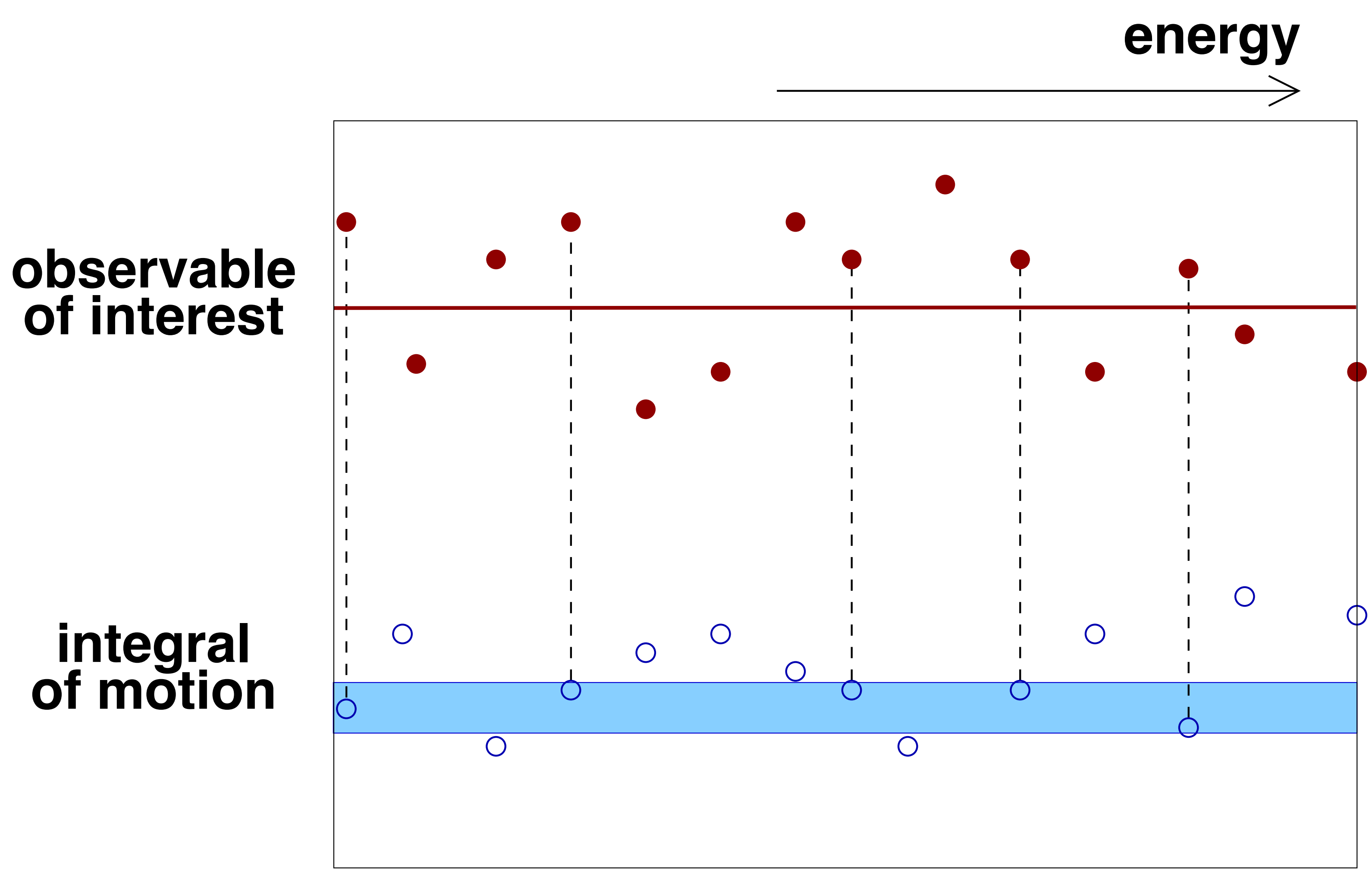

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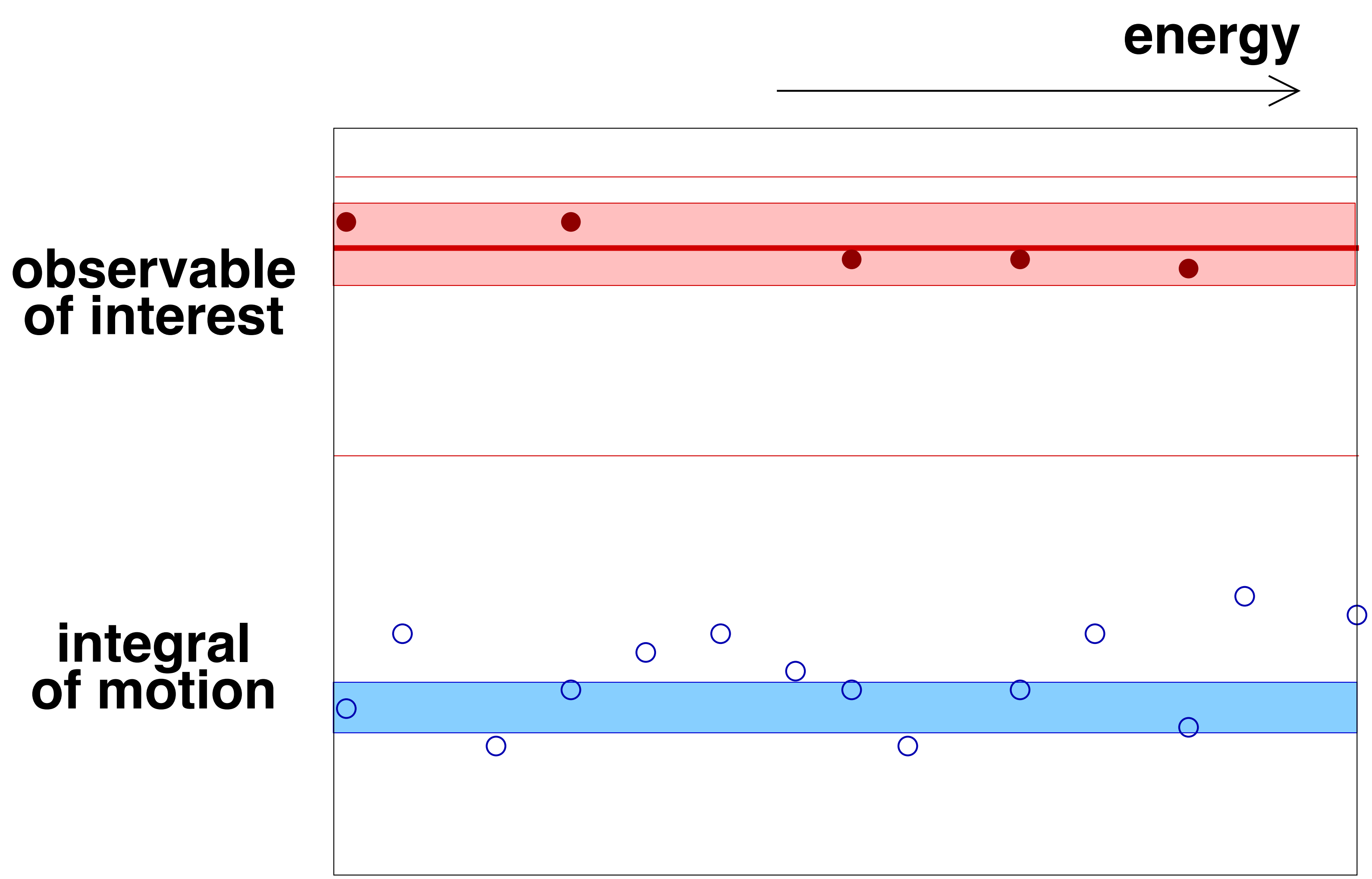
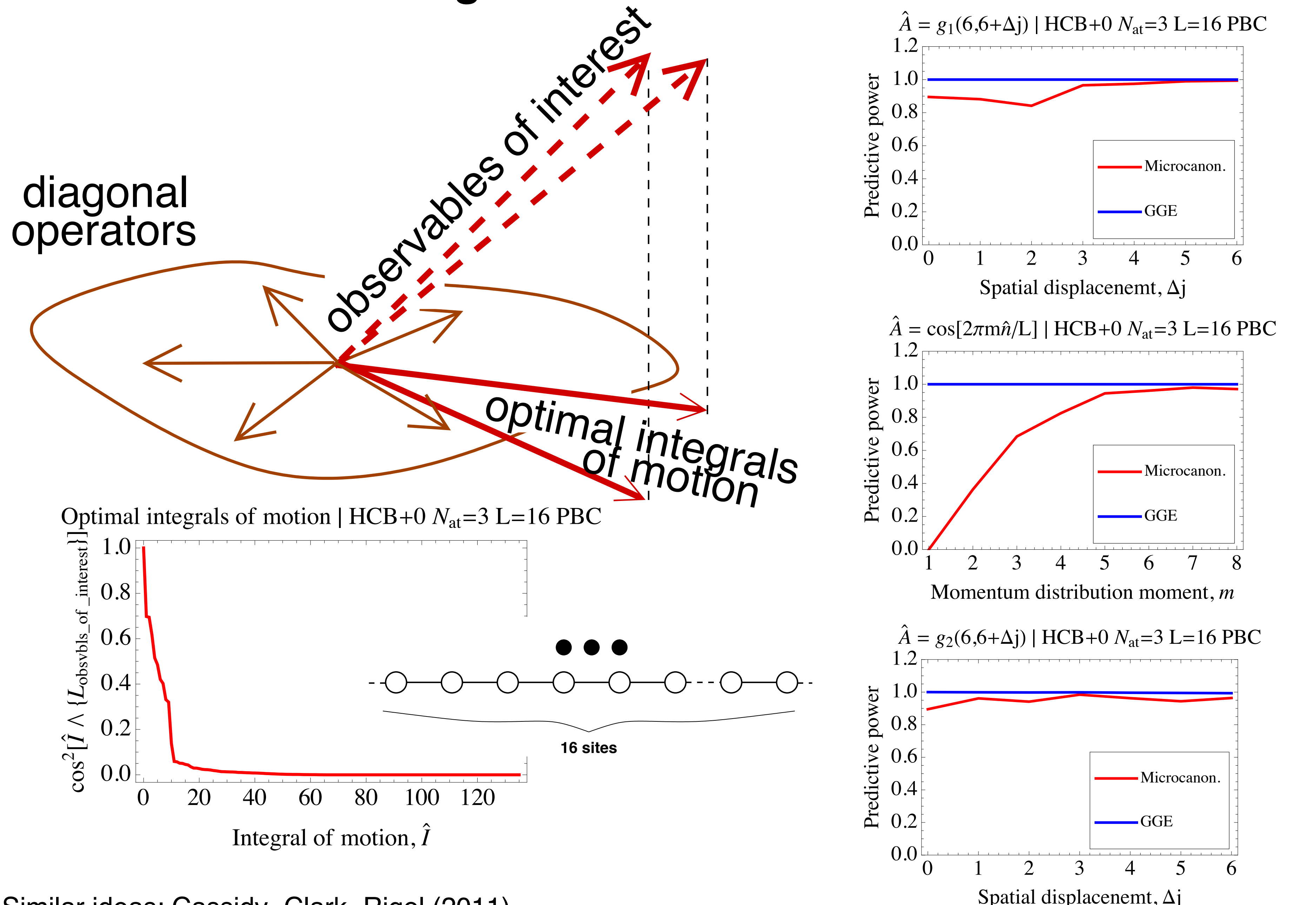


Optimizing the GGE: underlying exact inequality

$$\frac{Var_{\mathsf{GGE}}[\langle\alpha|\hat{A}|\alpha\rangle]}{Var_{\mathsf{MC}}[\langle\alpha|\hat{A}|\alpha\rangle]} \leq \sin^2[\hat{I}_{tl,d} \wedge \hat{A}_{tl,d}] + 2|\cos[\hat{I}_{tl,d} \wedge \hat{A}_{tl,d}]| \underbrace{\sqrt{\frac{Var_{\mathsf{GGE}}[\langle\alpha|\hat{I}|\alpha\rangle]}{Var_{\mathsf{MC}}[\langle\alpha|\hat{I}|\alpha\rangle]}}}_{\mathcal{O}\left(\frac{\Delta I}{\sqrt{Var_{\mathsf{MC}}[\langle\alpha|\hat{I}|\alpha\rangle]}}\right)},$$

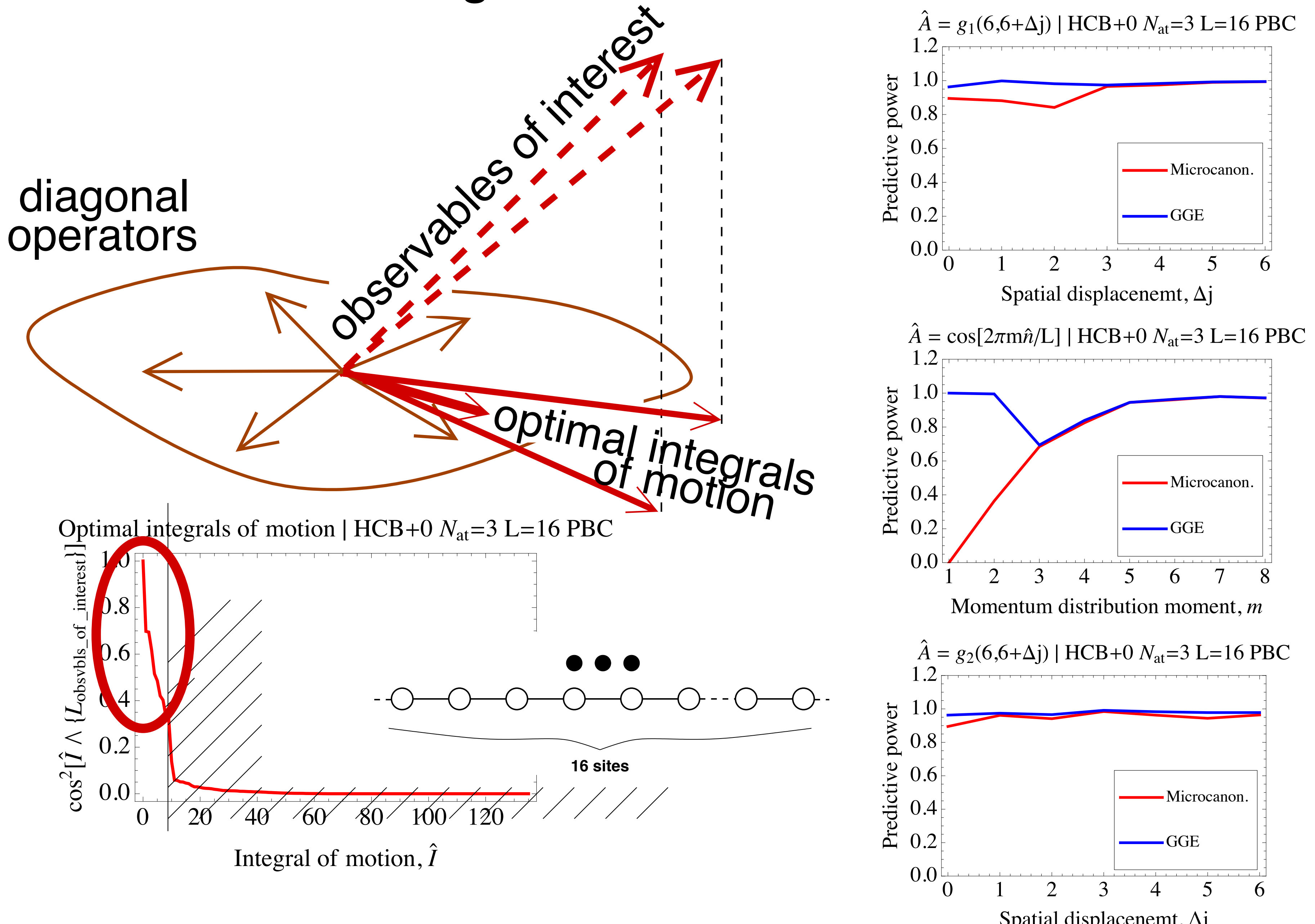
where $\Delta I \equiv \max_j (I_{j+1} - I_j) =$ maximal GGE interval for $\hat{I}$,
$\hat{B}_{tl,d} \equiv \sum_\alpha (\langle\alpha|\hat{B}|\alpha\rangle - Mean_{\mathsf{MC}}[\langle\alpha|\hat{B}|\alpha\rangle])|\alpha\rangle\langle\alpha|$

How to choose the optimal set of integrals of motion for GGE



Similar ideas: Cassidy–Clark–Rigol (2011)

How to choose the optimal set of integrals of motion for GGE



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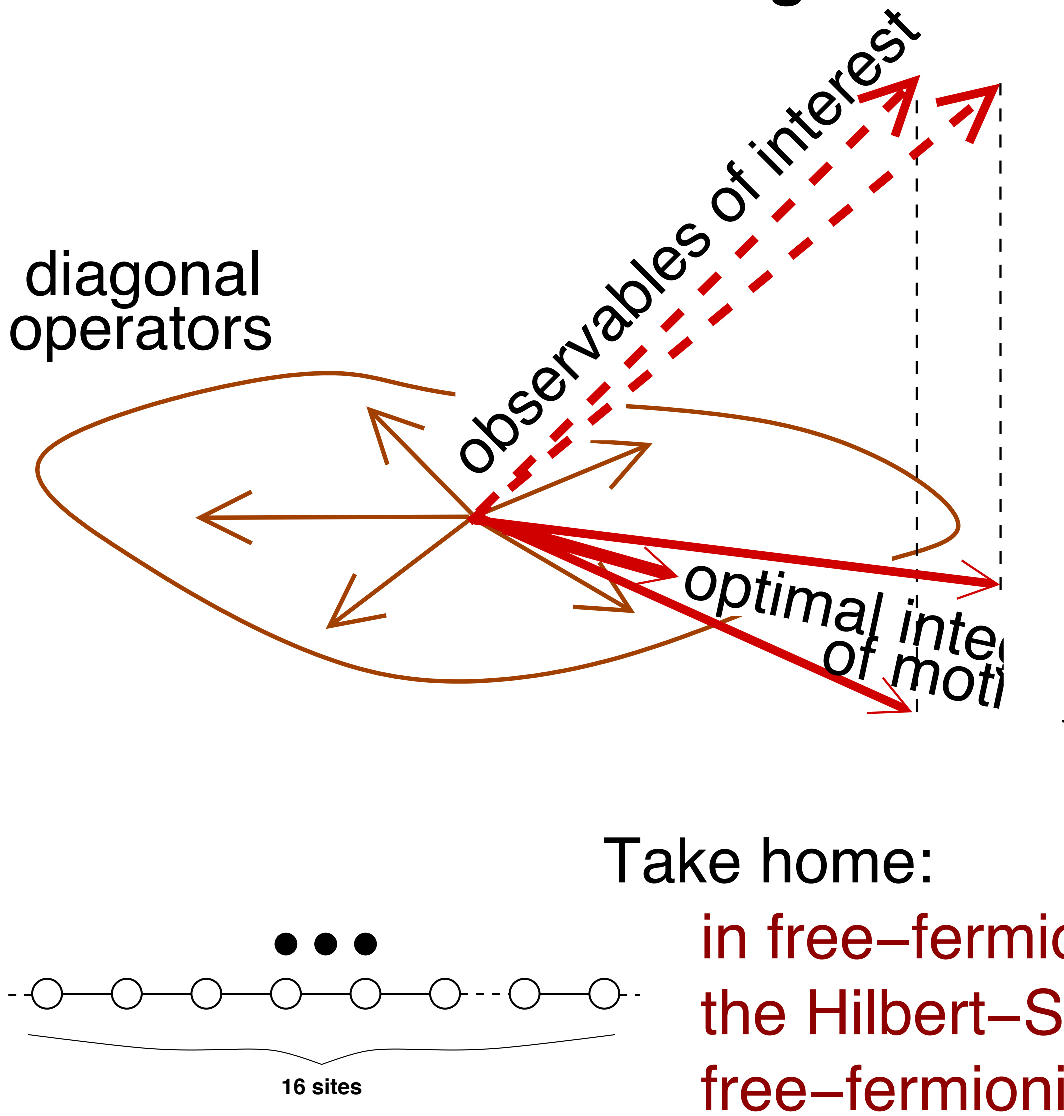
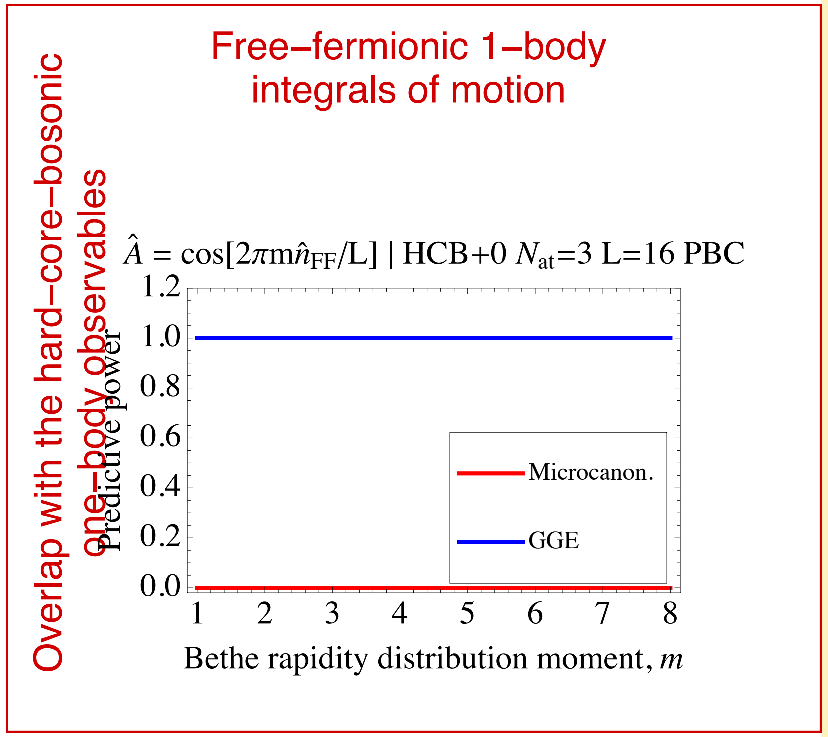
How to choose the optimal set of integrals of motion





Take home:

in free–fermionic integrable systems, the Hilbert–Schmidt angle between the free–fermionic and "system proper" one–body observables is close to zero

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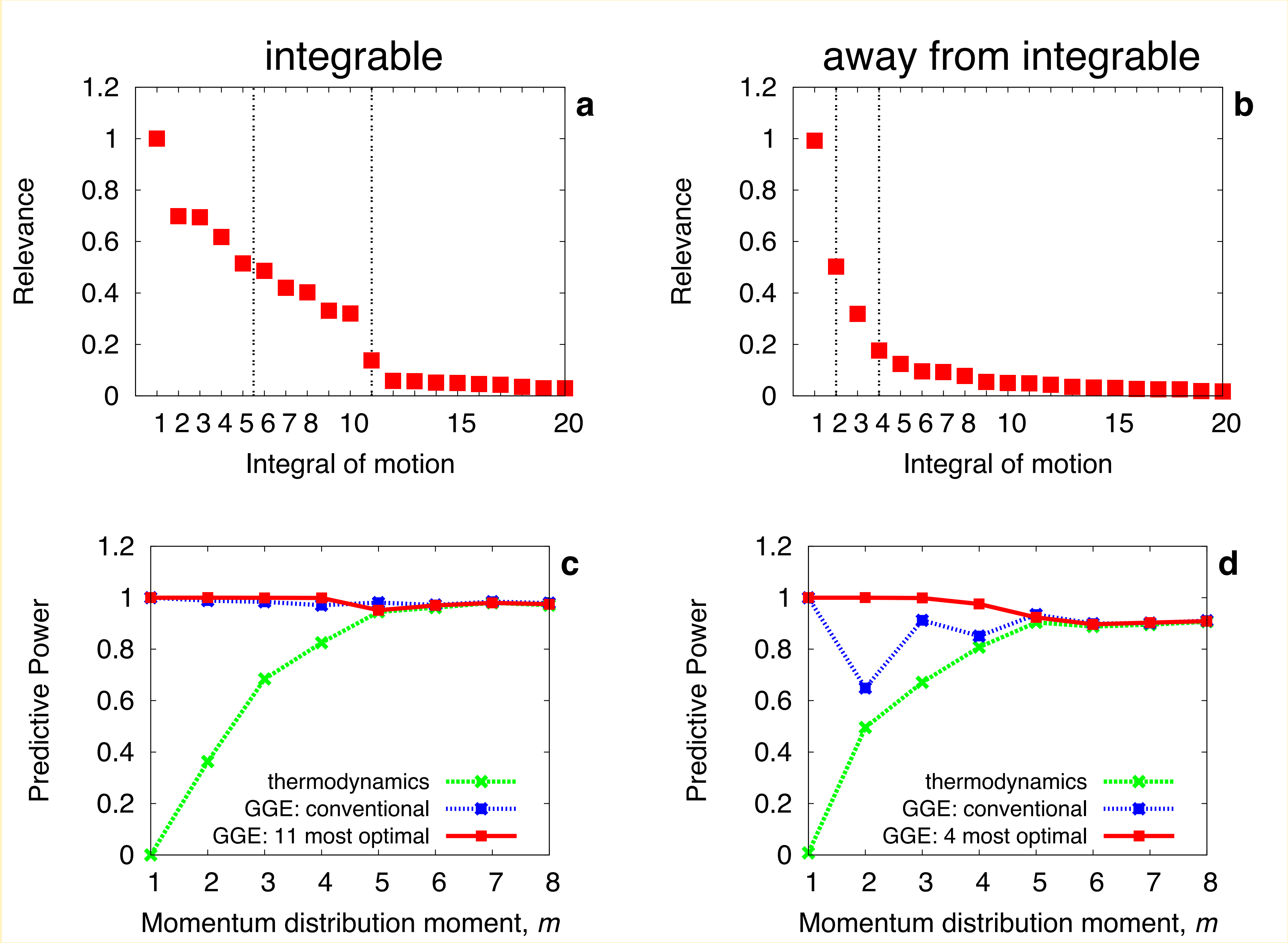
Optimizing the GGE: beyond integrability

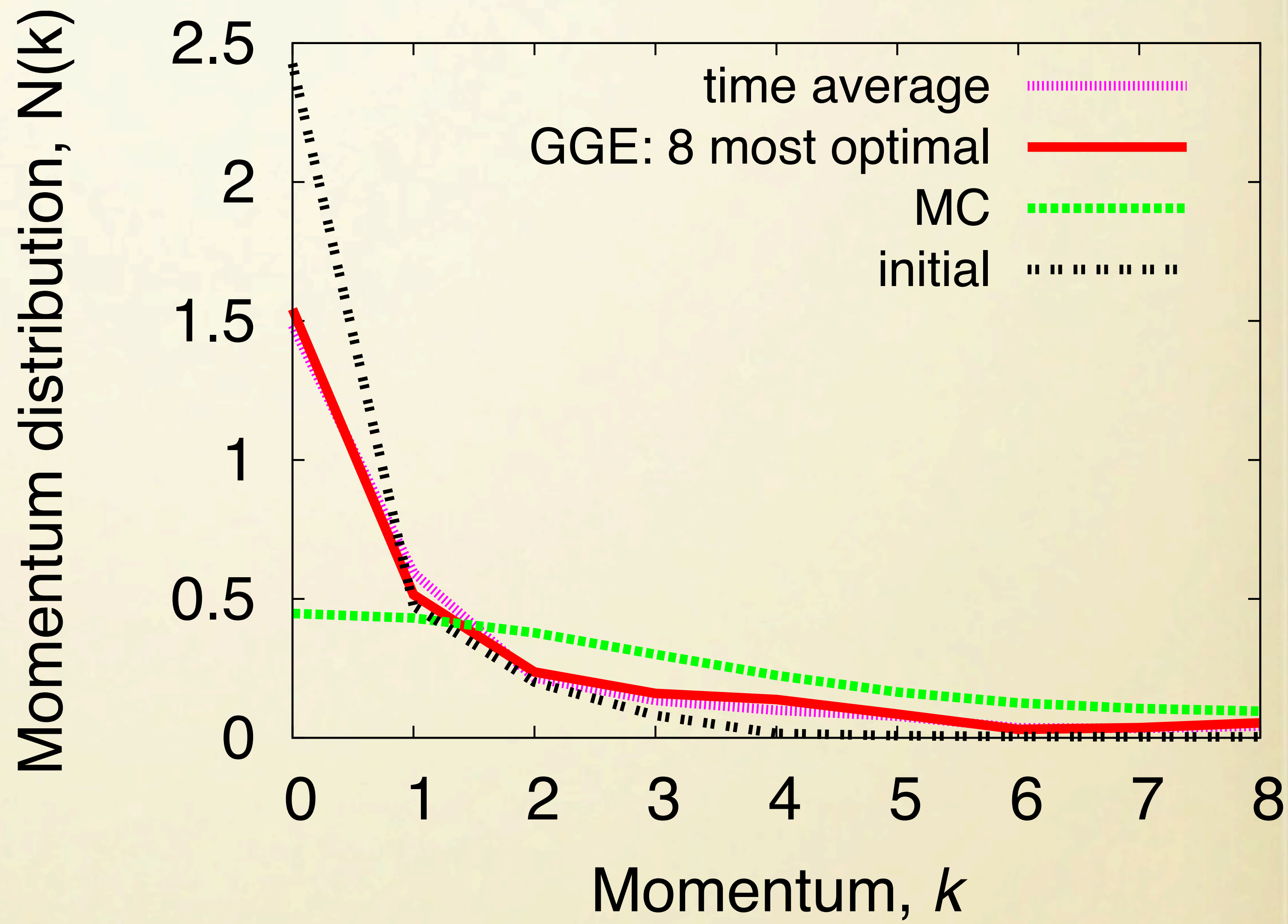


Generalized thermodynamics: 4 interacting bosons on 16 sites

- N = 4 | L = 16 | PBC

- Interactions: soft-core repulsion of radius 4 and strength U=3 + hard-core

- Initial state: ground state with no soft-core

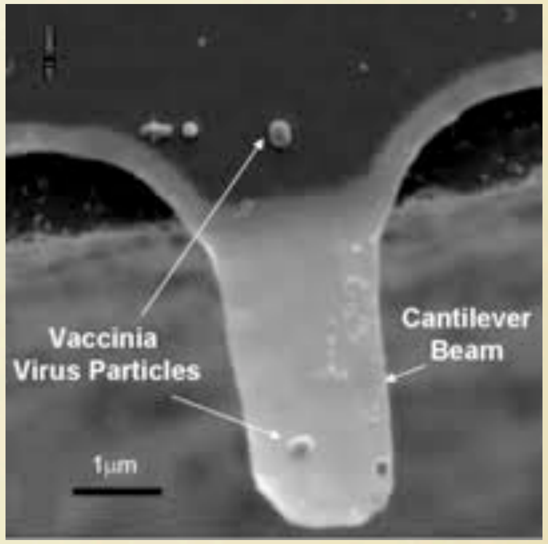


- # of additional integrals ofmotion= 7

- MC windows = 1/10 of state-to-state variance

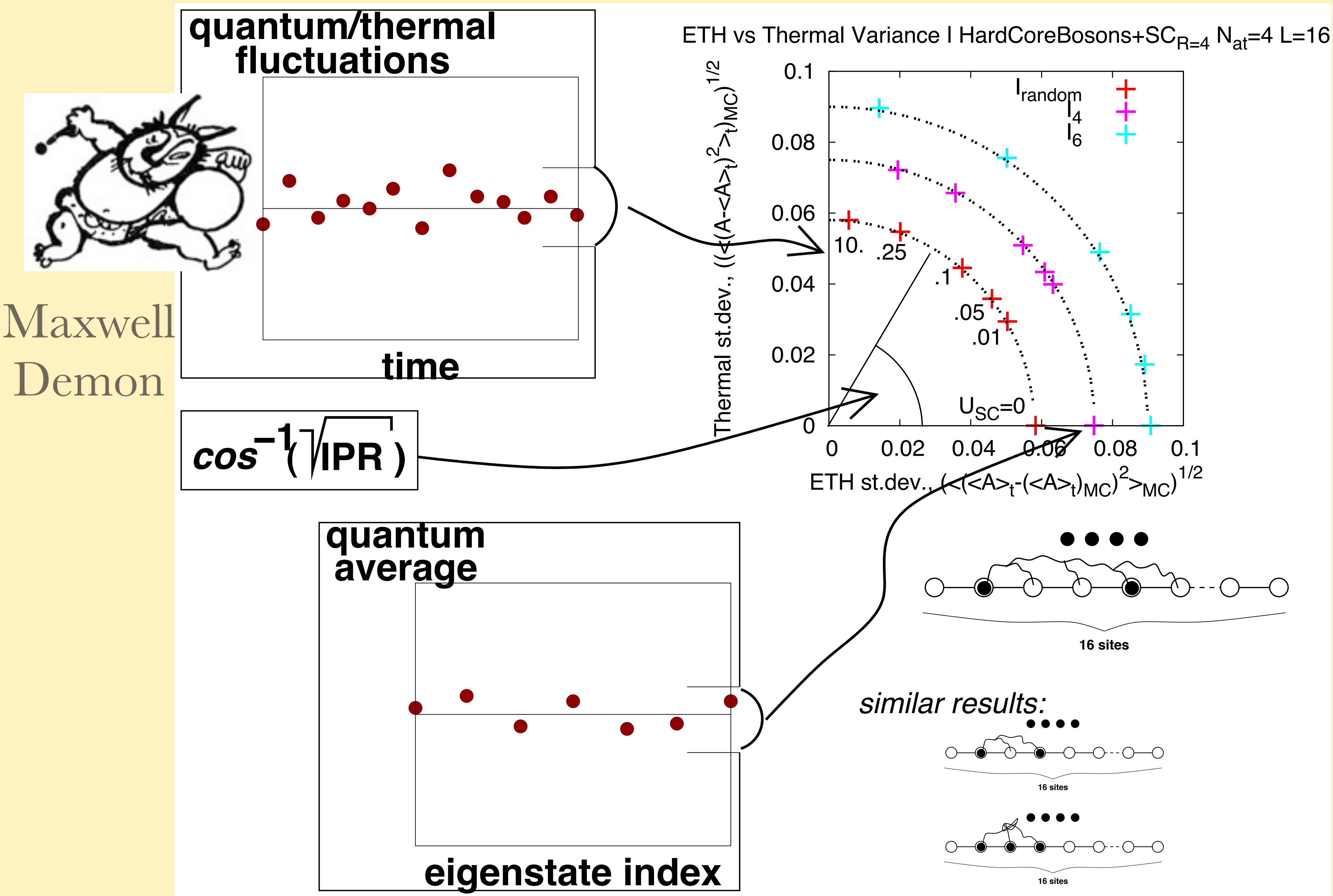
Outlook: Maxwell demon of non-ergodicity

Idea: in mesoscopic systems, use finite-size-induced deviations from ergodicity to control observables of interest



Credits: Zack Hilt*, Amit Gupta, Prof. Nick Peppas*, Prof. Rashid Bashir,
School of Electrical and Computer Engineering, Purdue University
*School of Chemical Engineering
UT Austin, Austin, TX

HCB+NNNNN: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



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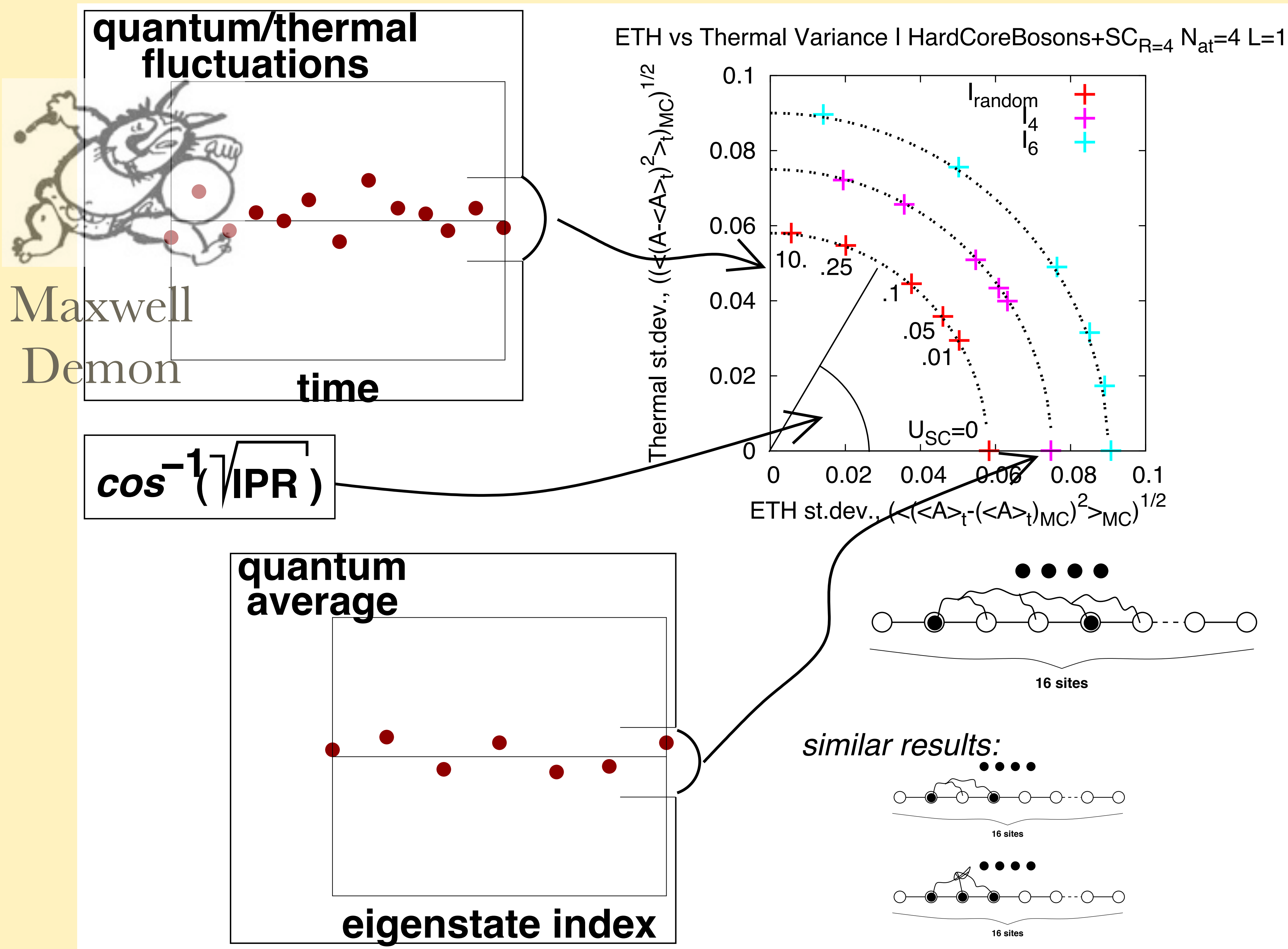

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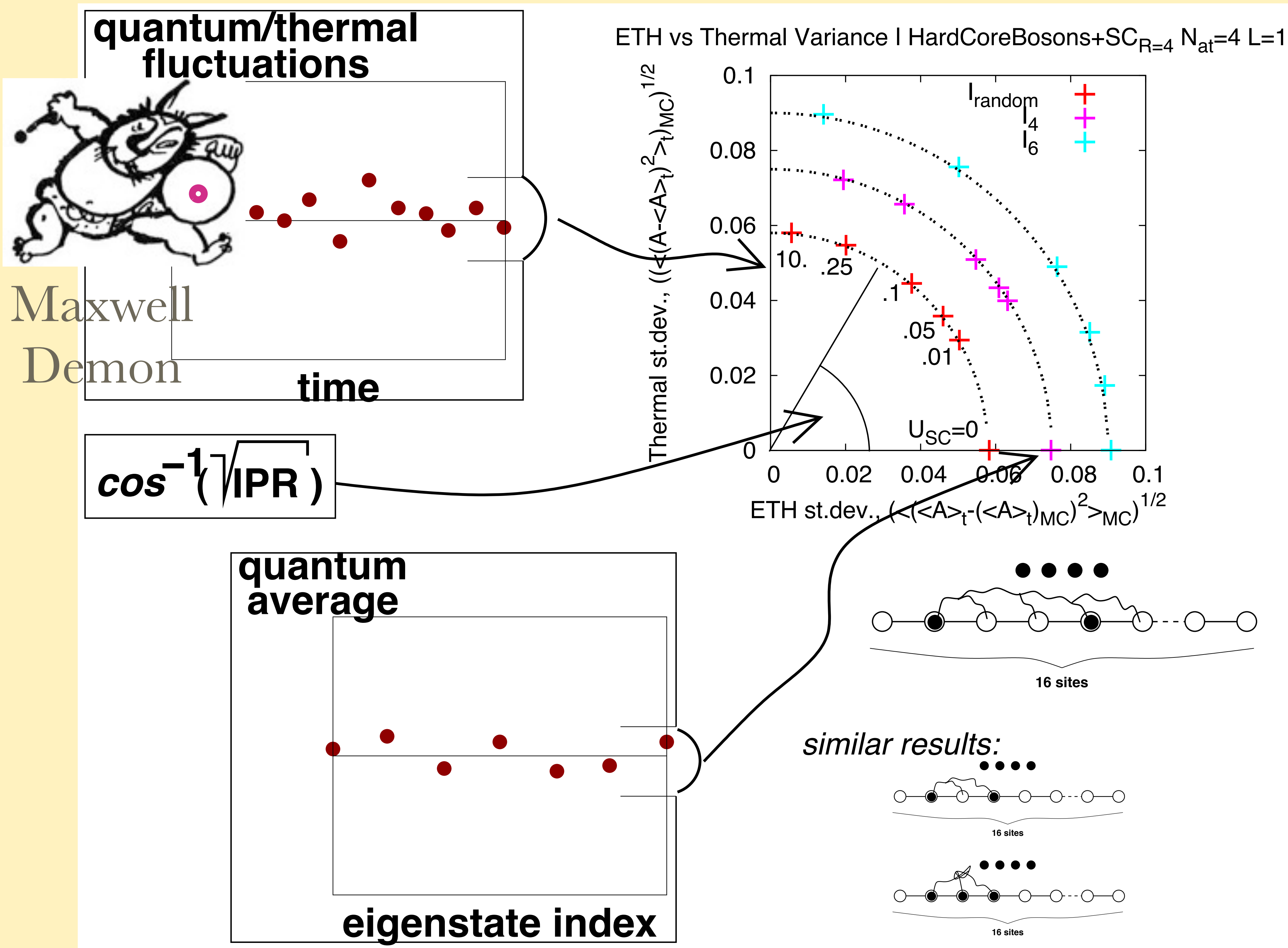

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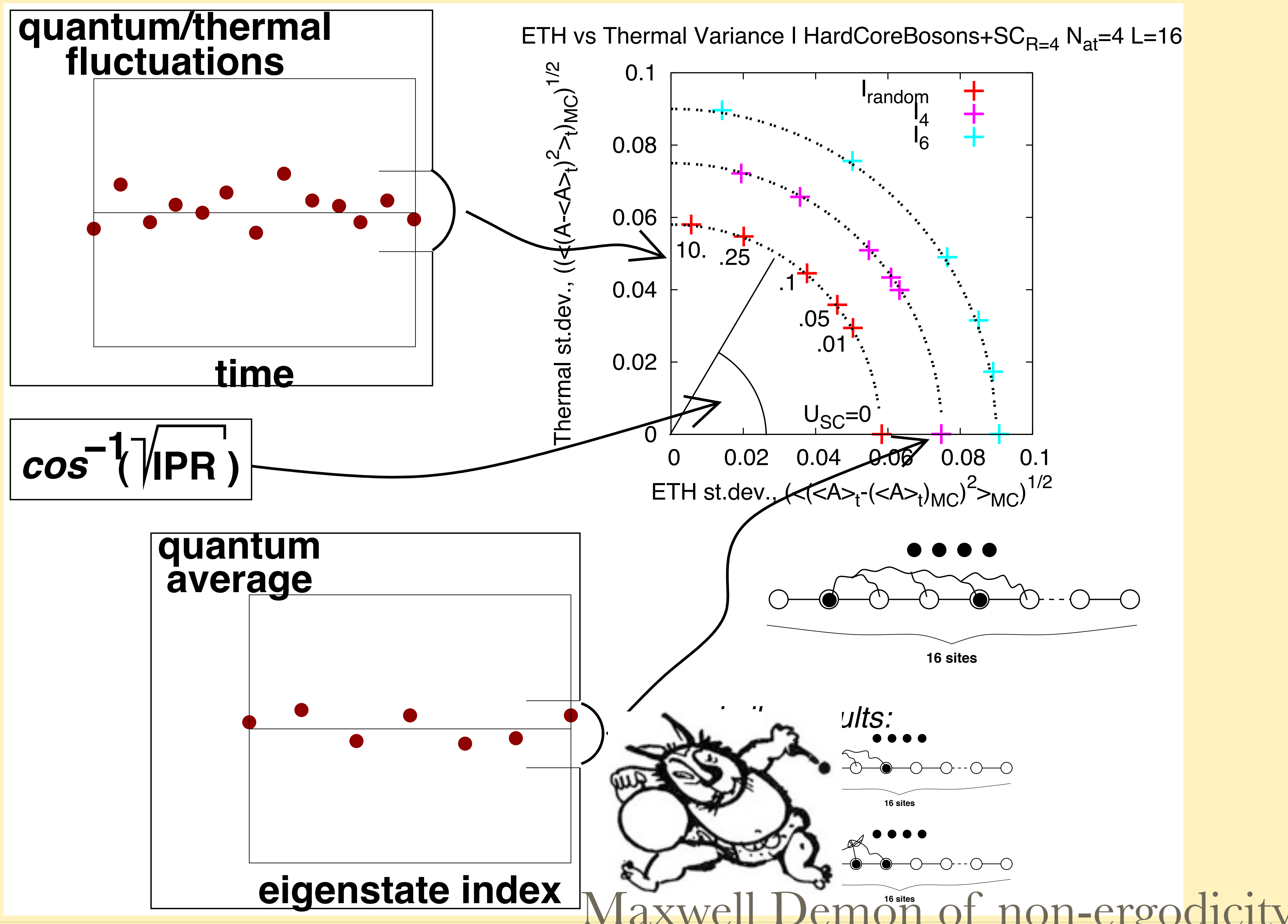

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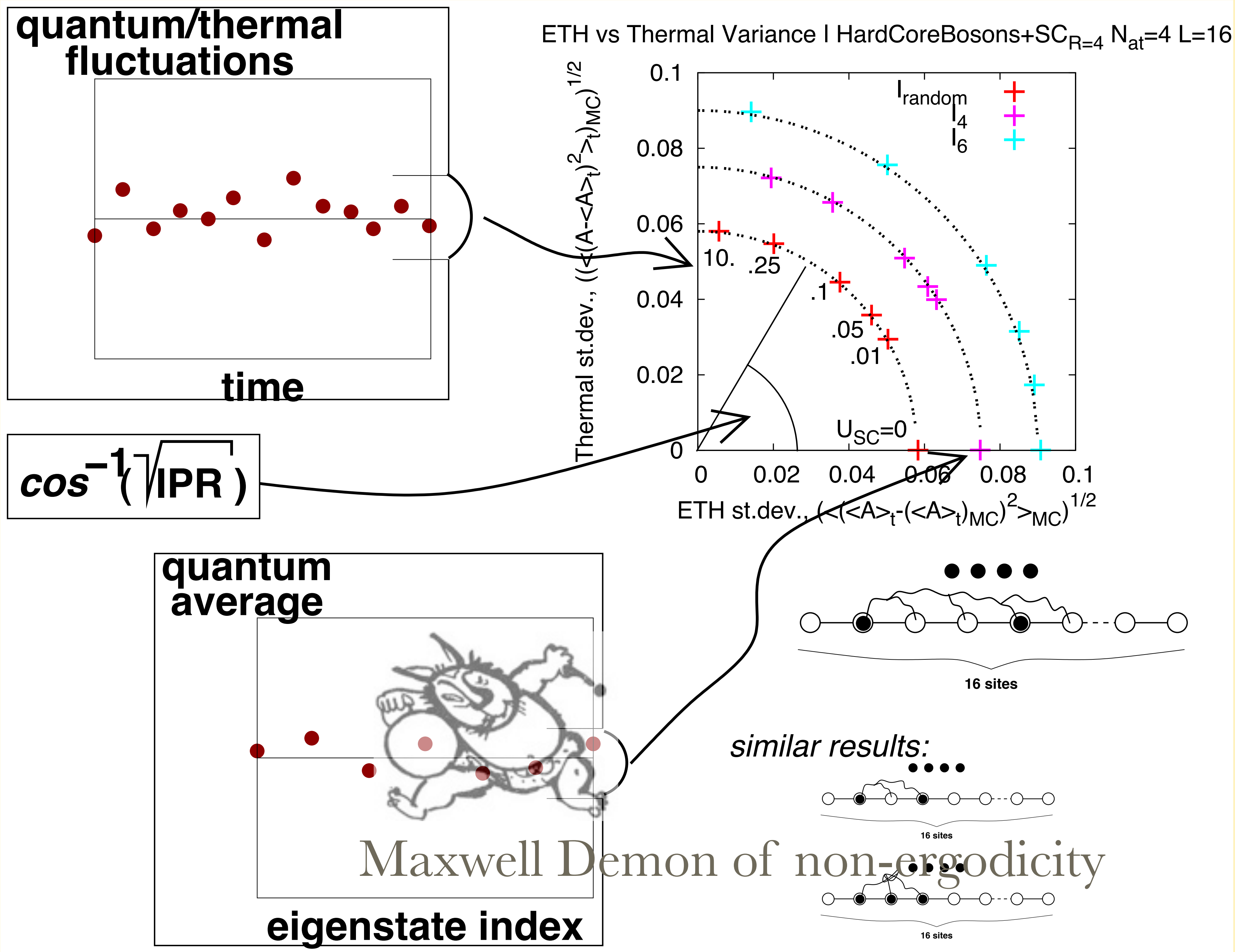

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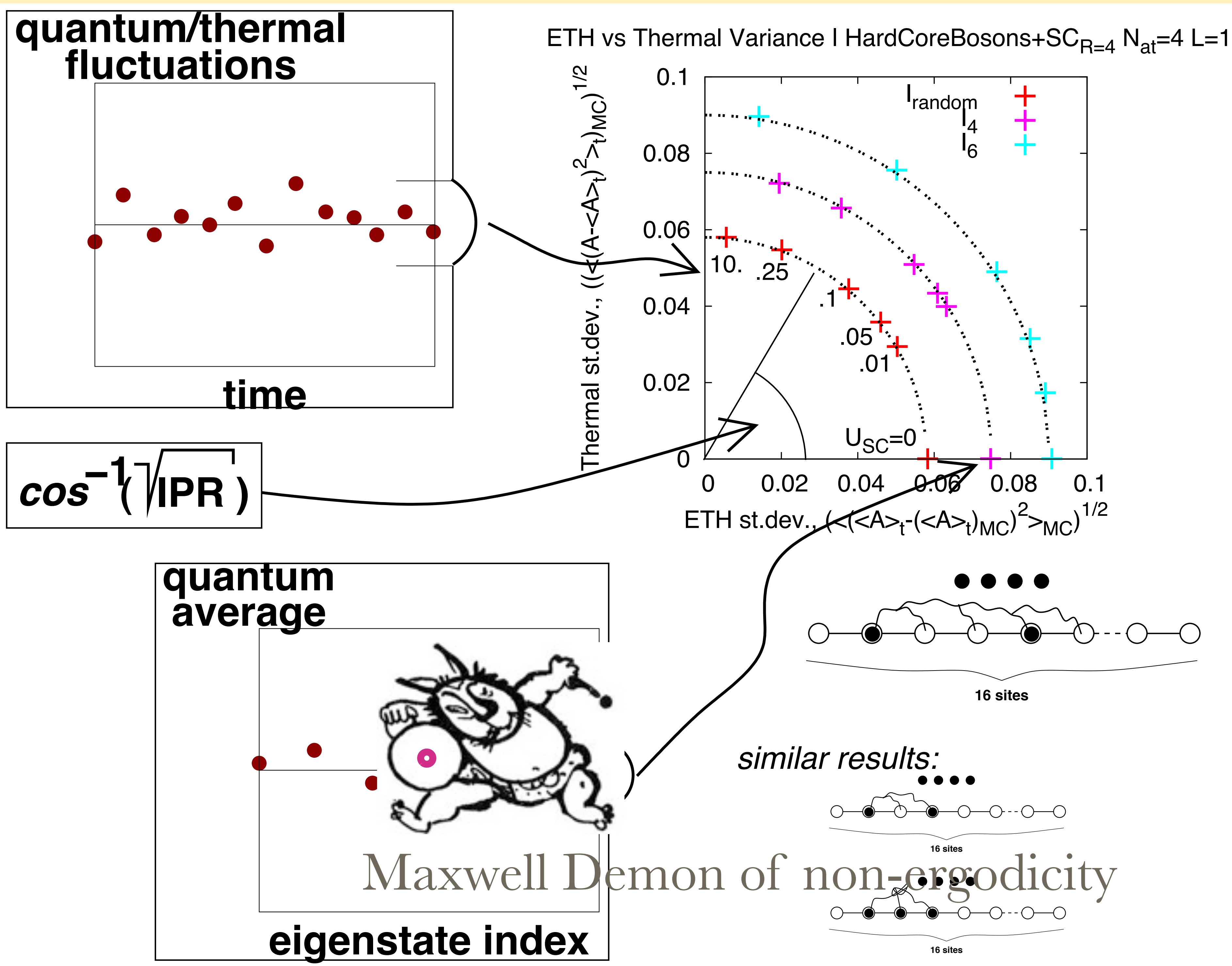
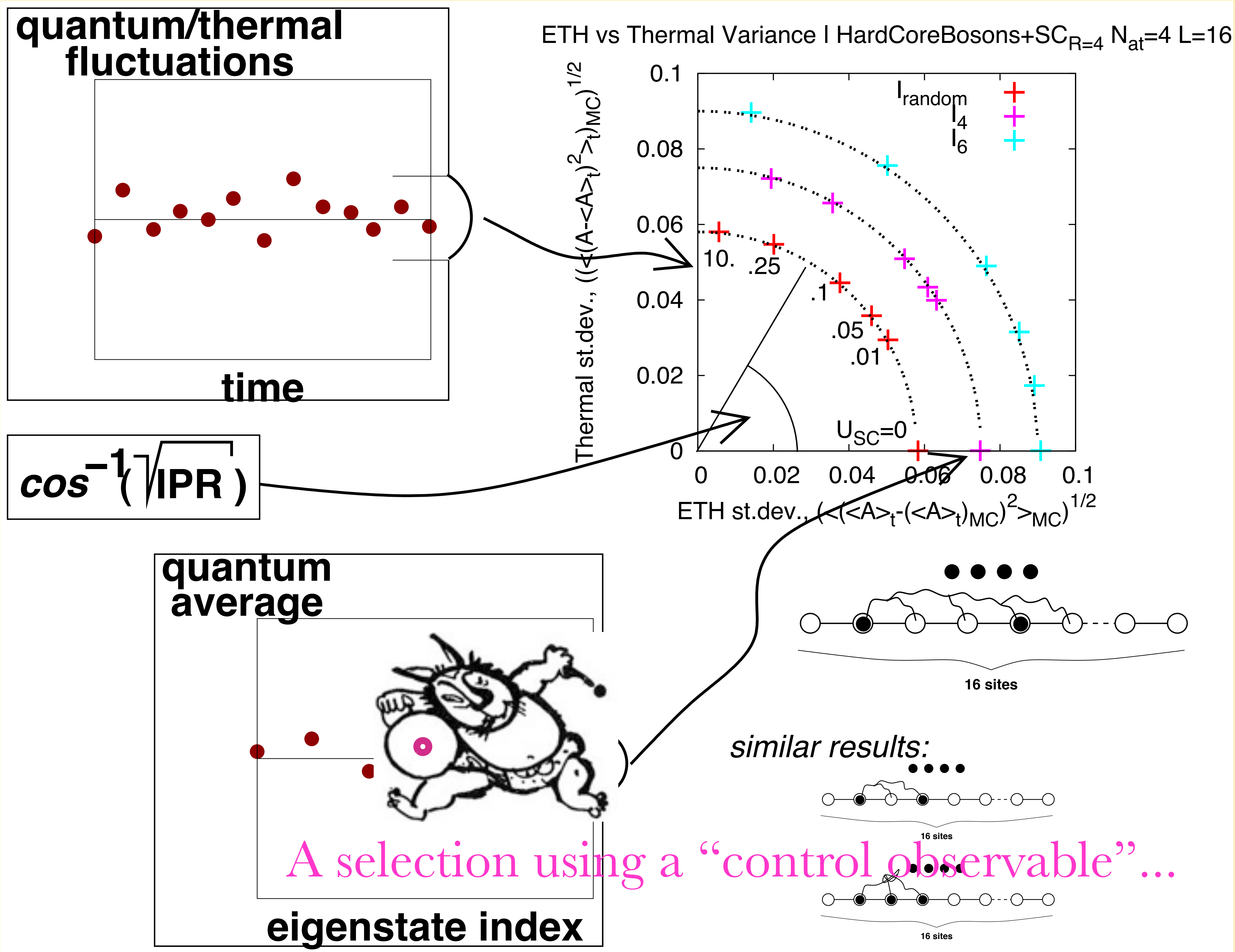


Maxwell Demon of non-ergodicity

HCB+NNNNN: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



A selection using a "control observable"...

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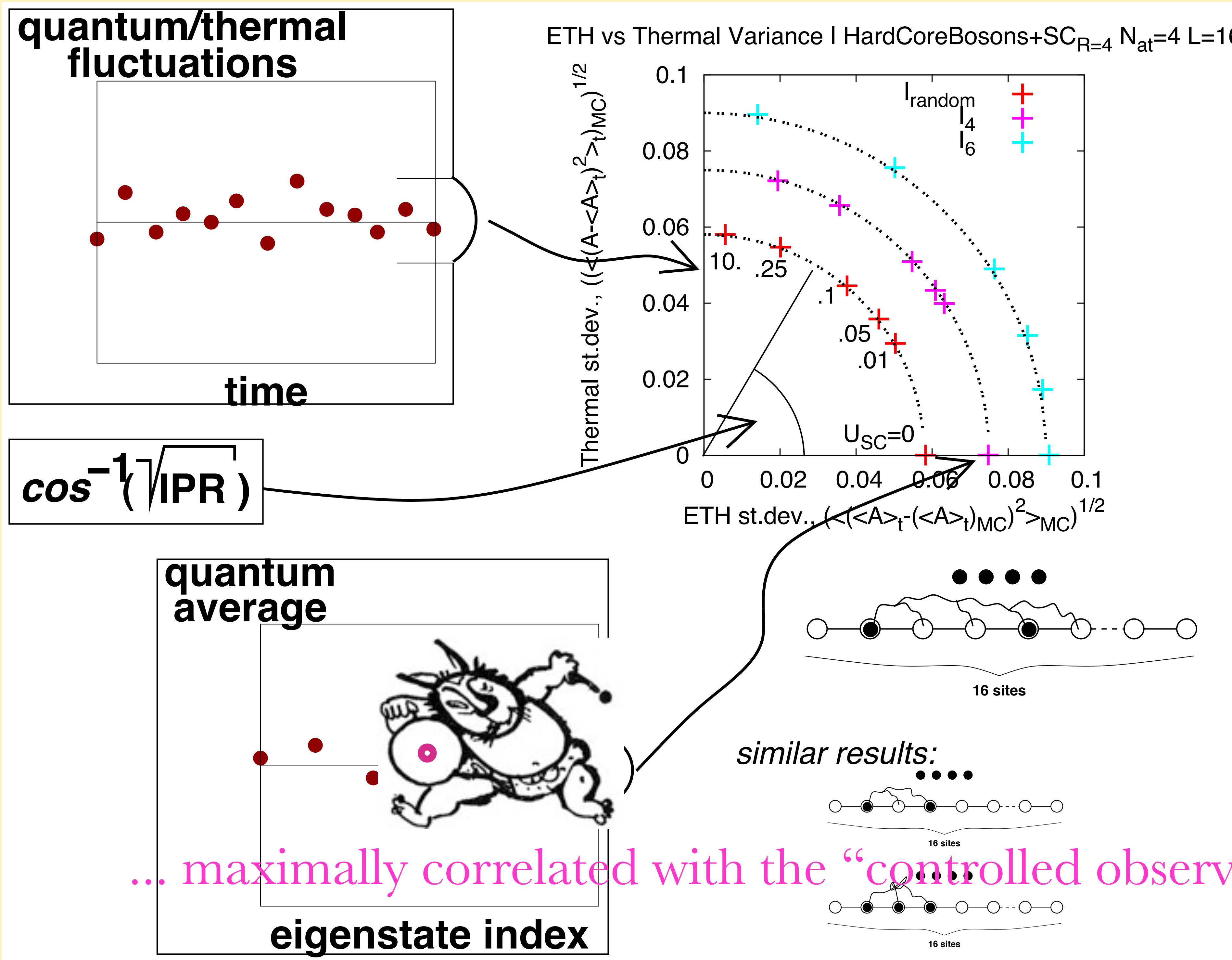
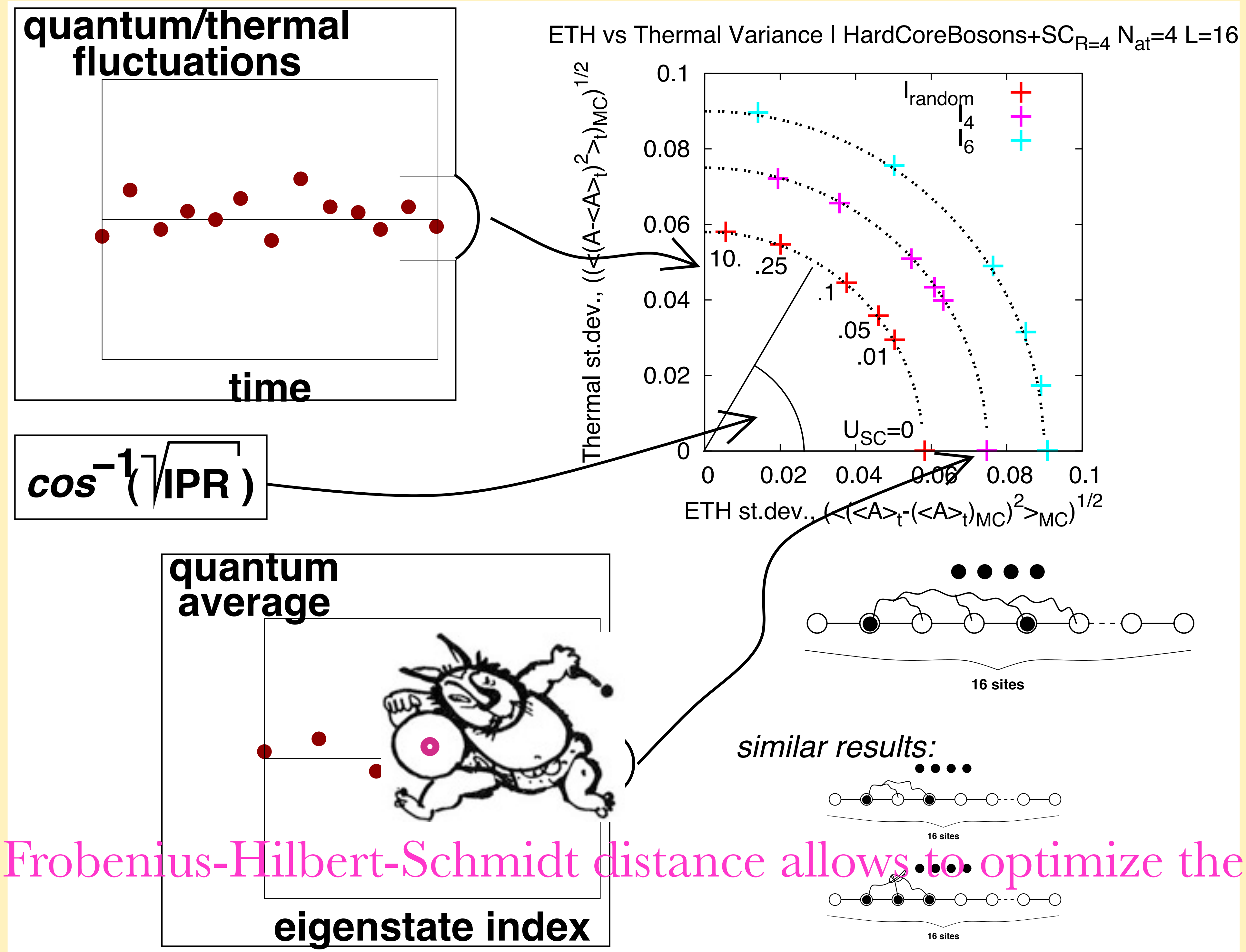
HCB+NNNNN: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



... maximally correlated with the "controlled observable"

HCB+NNNNN: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



Frobenius-Hilbert-Schmidt distance allows to optimize the "controls"

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Summary

In this presentation we

- suggested the following proposition:

  The sum of the ensemble variance of the temporal means and the ensemble mean of the temporal variances remains approximately constant across the integrability-to-ergodicity transition

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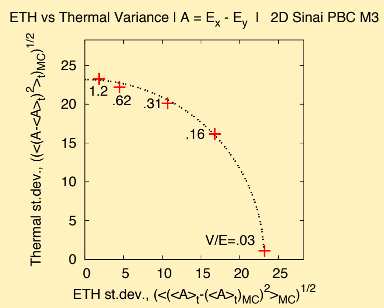

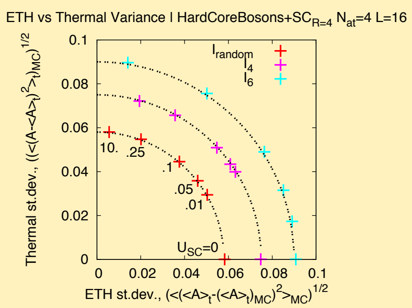

- linked the proposition to the Hilbert-Schmidt (HS) geometry of the observables: ETH variance $= cos^2$(HS angle between the observable and integrals of motion); IPR $= cos^2$(HS angle between the original and perturbed integrals of motion);
- found a way to identify the optimal integrals of motion for GGE;
- found a way to treat the integrability and mesoscopicity under the same umbrella, with possible applications in nano-systems

Credits

Preview

We suggest

$$\tan^2[\Theta_A] \equiv \frac{Var_{MC}[Mean_t[A]]}{Mean_{MC}[Var_t[A]]}$$

as a measure of the position of an observable A on the (Integral of Motion)-(Thermalizable Observable) continuum.