THREE-DIMENSIONAL IMPURITY PROBLEMS WITH COLD ATOMS

Yvan Castin, Christian Trefzger
LKB, Ecole normale supérieure (Paris, France)
Mauro Antezza
Lab. Charles Coulomb, Université de Montpellier (France)
David Hutchinson
University of Otaga, Dunedin, New Zealand

A RICH PROBLEM

An impurity atom A (mass m_A) interacting with another species (or spin state) B (mass m_B) [no interaction among B atoms]:

1. monomeron/dimeron pb: a Fermi sea of B atoms on a (narrow) Feshbach resonance [with C. Trefzger]
2. strong localisation of A: the B atoms are randomly pinned at the nodes of an optical lattice [with M. Antezza, D. Hutchinson, P. Massignan, U. Gavish]
3. photonic band gaps: A is a photon; one B atom per node of an optical lattice [with M. Antezza]

1 Monomeron-dimeron problem

1.1 Physical motivation

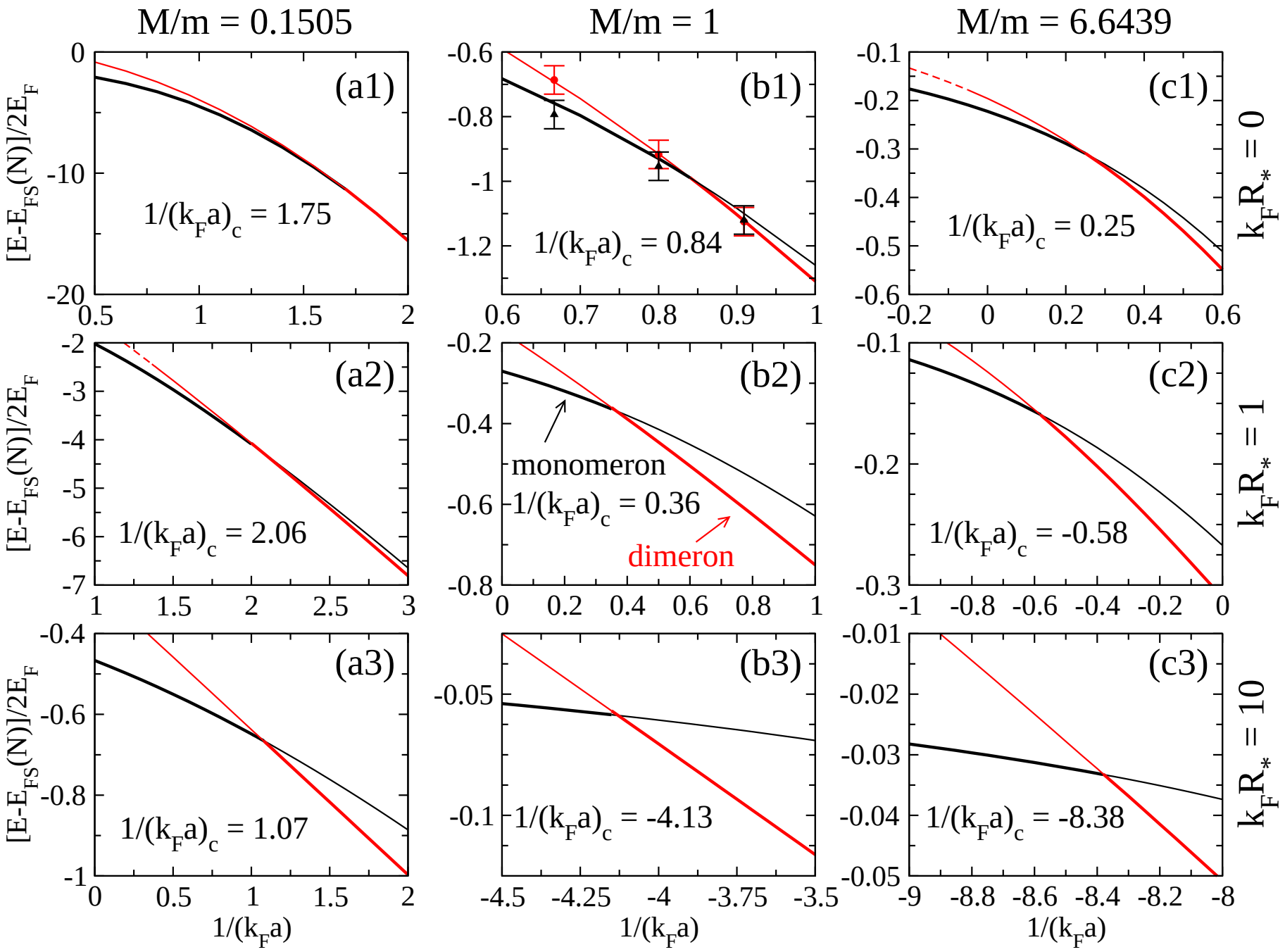
- Monomerons and dimerons are quasi-particles belonging to the general class of Fermi polarons [neutral objects dressed by the Fermi sea, rather than electrons dressed by phonons in solids]
- The ground state of strongly polarized spin 1/2 Fermi gas at unitarity [$N_A \ll N_B$, $A = \downarrow$, $B = \uparrow$] is a Fermi gas of monomerons [Chevy; Lobo, Recati, Giorgini, Stringari; Combescot, Giraud, Leyronas; Mora]
- monomeron = A dressed by particle-hole excitations of B Fermi sea. It is a quasi-particle of dispersion relation

$$\Delta E(\mathrm{P}) \underset{P \to 0}{=} \Delta E(0) + \frac{P^2}{2m_*} + O(P^4)$$

- At unitarity, measured equation of state of monomerons $\simeq$ an ideal Fermi gas [Navon, Nascimbène, Chevy, Salomon]

1.2 Monomeron vs dimeron

- The ground state of A, as the function of the AB scattering length a, has two branches with a cusp [Prokof'ev, Svistunov]
- Dimeron $=$ AB dimer dressed by particle-hole excitations of B Fermi sea
- transition from monomeron to dimeron at a_c
- $a_c > 0$ (broad Feshbach resonance) "intuitive": dimer exists in free space for $a > 0$ only.


M/m = 0.1505
M/m = 1
M/m = 6.6439
(a1)
(b1)
(c1)
(a2)
(b2)
(c2)
(a3)
(b3)
(c3)
1/(k_Fa)_c = 1.75
1/(k_Fa)_c = 0.84
1/(k_Fa)_c = 0.25
1/(k_Fa)_c = 2.06
monomeron
1/(k_Fa)_c = 0.36
dimeron
1/(k_Fa)_c = -0.58
1/(k_Fa)_c = 1.07
1/(k_Fa)_c = -4.13
1/(k_Fa)_c = -8.38
[E-E_FS(N)]/2E_F
1/(k_Fa)
k_FR_* = 0
k_FR_* = 1
k_FR_* = 10

1.3 Two-channel model

$$H = \sum_{\mathbf{k}} \frac{\hbar^2 \boldsymbol{k}^2}{2m_A} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{\hbar^2 \boldsymbol{k}^2}{2m_B} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + (E_{\mathrm{mol}} + \frac{\hbar^2 \boldsymbol{k}^2}{2(m_A + m_B)}) \gamma_{\mathbf{k}}^\dagger \gamma_{\mathbf{k}}$$
$$+ \frac{\Lambda}{L^{3/2}} \sum_{\mathbf{k}_A, \mathbf{k}_B} \chi(\mathbf{k}_{\mathrm{rel}}) [\gamma_{\mathbf{k}_A + \mathbf{k}_B}^\dagger a_{\mathbf{k}_A} b_{\mathbf{k}_B} + \mathrm{h.c.}]$$

Take cut-off to ∞ for a and Λ fixed ($E_{\mathrm{mol}} \to +\infty$). Feshbach length $R_* = \pi\hbar^4/(\mu^2\Lambda^2)$. Free-space dimer iff $a > 0$.

1.4 How to solve

For $\Lambda = 0$:

one A, zero molecule

$E_0 = E_{\mathrm{FS}}(N_B)$

$\Delta E_{\mathrm{pol}} = 0$

zero A, one molecule

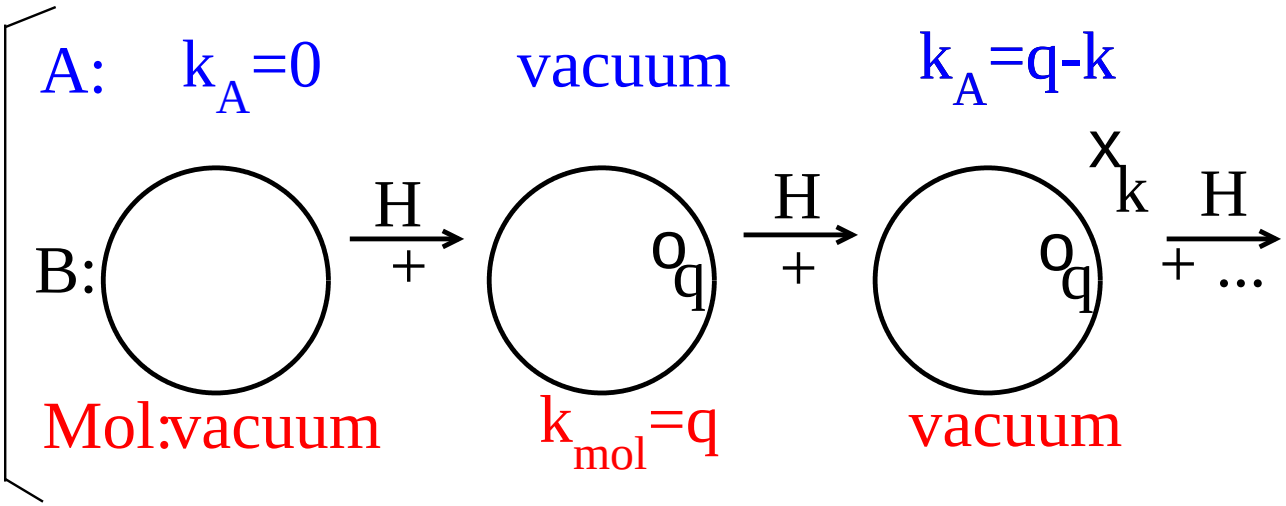
$E_0 = E_{\mathrm{FS}}(N_B - 1) + E_{\mathrm{mol}}$

$\Delta E_{\mathrm{dim}} = E_{\mathrm{mol}} - E_F$

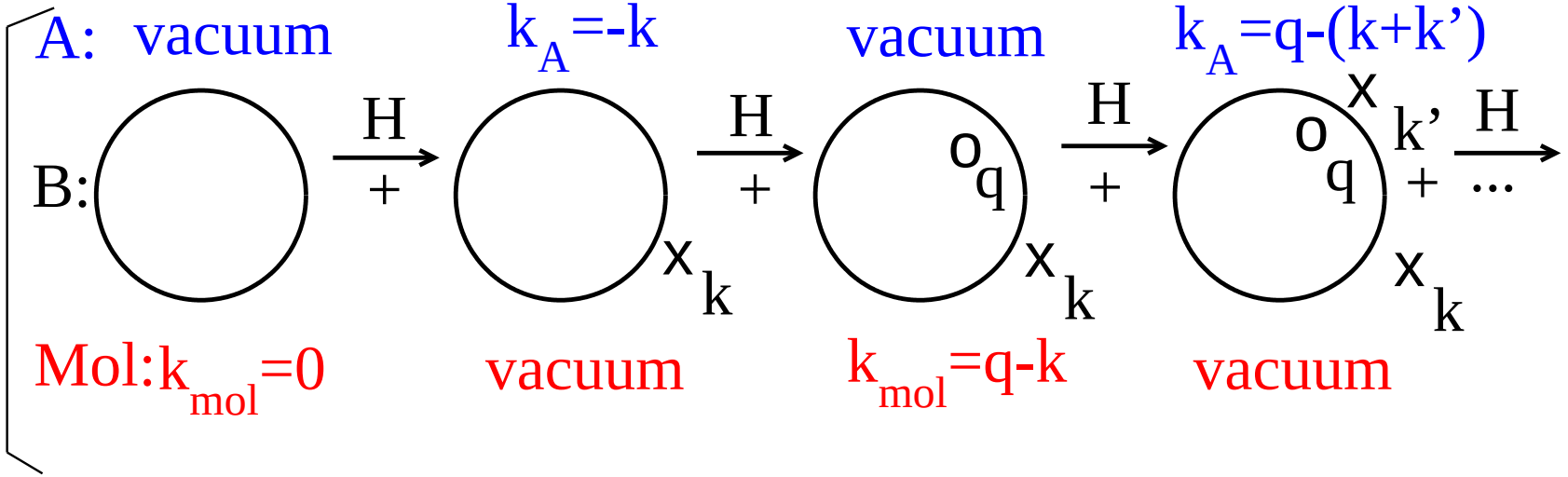
For $\Lambda > 0$: Expand in the number of particle-hole pairs [Chevy ; Combescot, Giraud], here up to one pair.

Generalized Chevy Ansätze

$|\psi_{mono}\rangle =$

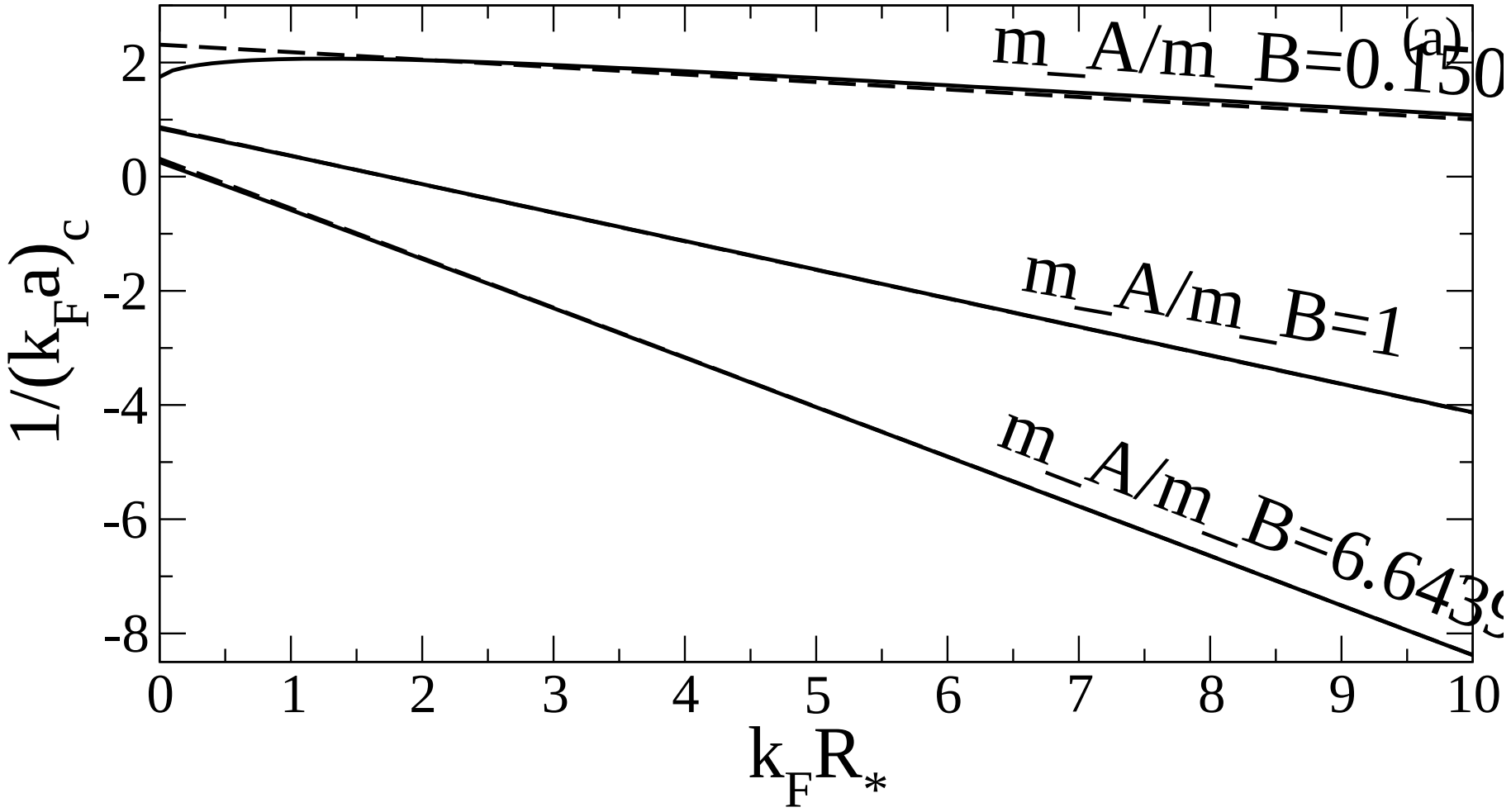


$|\psi_{dim}\rangle =$



1.5 Numerical result

$a_c < 0$ at large $k_F R_*$! Paradoxical. Stable dimeronic branch extends to a regime where no free space dimer.



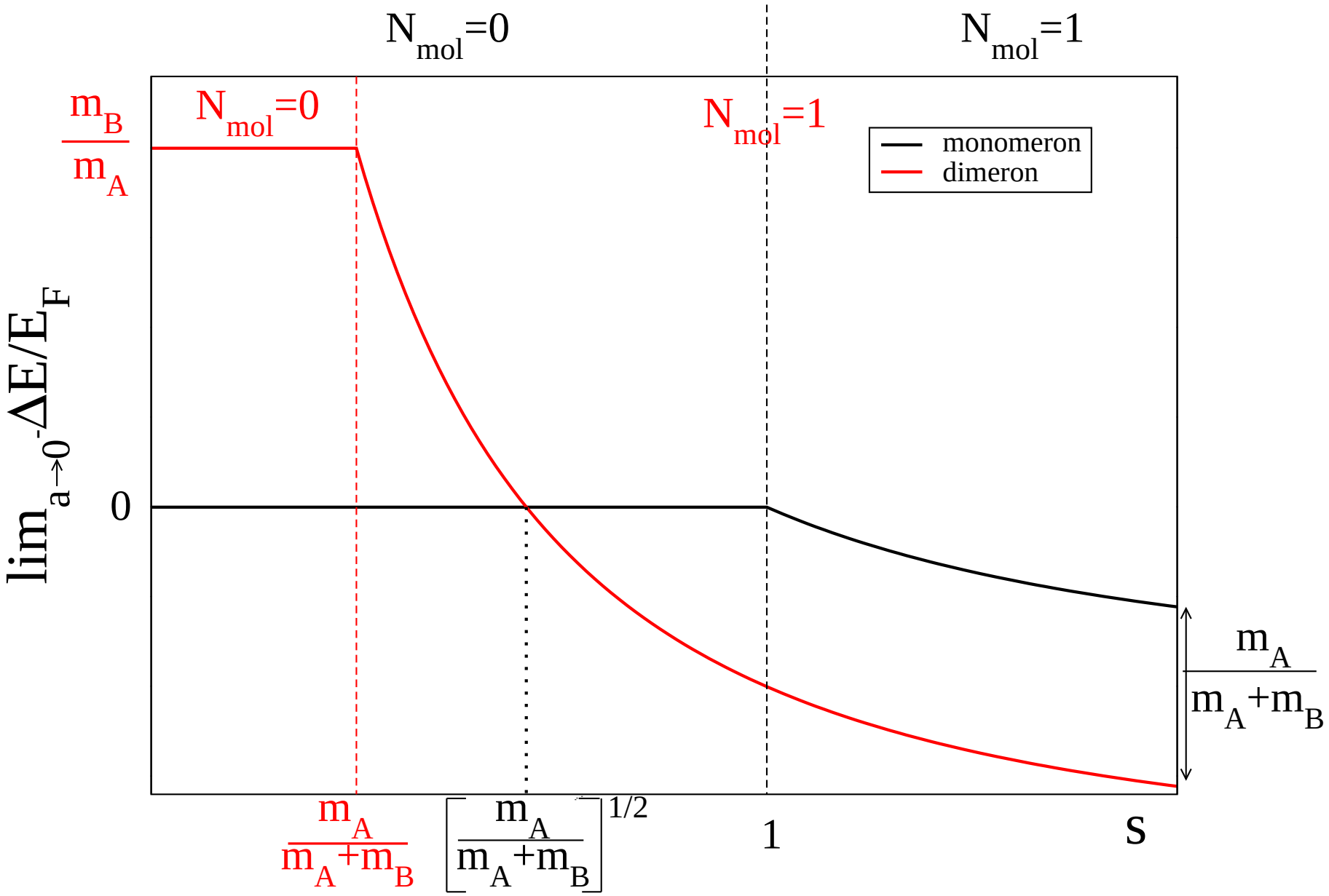
- Two-body scattering amplitude [Petrov]:

$$f_{k_{\text{rel}}} = -\frac{1}{\frac{1}{a} + ik_{\text{rel}} + k_{\text{rel}}^2 R_*}$$

- Usual weakly attractive limit: $a \to 0^-$, R_* fixed
- Appropriate weakly attractive limit: $a \to 0^-$, aR_* fixed
- Define $\alpha = \frac{m_A}{m_A + m_B}$ and

$$s \equiv \alpha k_F (-aR_*)^{1/2}$$

Note that $s \to 0$ in the weakly attractive limit $a \to 0^-$.


$N_{mol}=0$
$N_{mol}=1$
$\frac{m_B}{m_A}$
$N_{mol}=0$
$N_{mol}=1$
monomeron
dimeron
$\lim_{a\to 0^-}\Delta E/E_F$
0
$\frac{m_A}{m_A+m_B}$
$\left[\frac{m_A}{m_A+m_B}\right]^{1/2}$
1
S
$\frac{m_A}{m_A+m_B}$

Critical scattering length on a narrow Feshbach resonance:

$$\frac{1}{k_F a_c} \underset{k_F R_* \to \infty}{=} -\alpha k_F R_*$$

$$+\frac{2}{\pi}\left[1-\alpha^{-2}+\frac{1}{2}\left(\alpha^{-5/2}-\alpha^{1/2}\right)\ln\frac{1+\alpha^{1/2}}{1-\alpha^{1/2}}\right]+O(\frac{1}{k_F R_*})$$

1.7 Physical interpretation of $1/(k_F a)_c < 0$

- Stabilization of molecule by Fermi sea
- Molecule energy renormalized by Lambshift:

$$\tilde{E}_{\rm mol} = E_{\rm mol} + \int \frac{d^3k}{(2\pi)^3} \frac{\chi^2({\rm k})\Lambda^2}{0 - \hbar^2 k^2/2\mu} = -\frac{\Lambda^2 \mu}{2\pi\hbar^2 a}$$

- Free space: molecule stable iff $\tilde{E}_{\rm mol} < 0$ that is $a > 0$
- Fermi sea: molecule stable iff $\tilde{E}_{\rm mol} < E_F$ that is

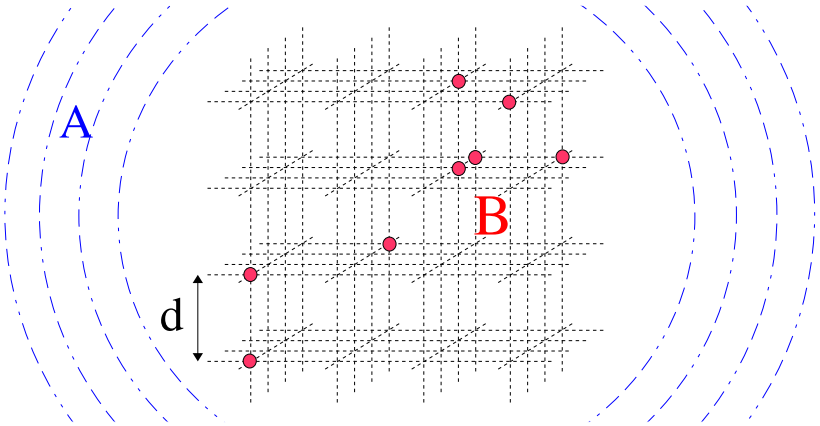
$$\frac{1}{k_F a} > -\alpha k_F R_*$$

- For $a < 0$ this imposes

$$s > \alpha^{1/2} = \left(\frac{m_A}{m_A + m_B}\right)^{1/2}$$

2 Strong (Anderson) localisation of A matterwave

2.1 Physical motivation and configuration



- B randomly filling the nodes of an optical lattice ($p_{\mathrm{occ}} \ll 1$) with no tunneling
- A does not see the lattice potential, it sees a disordered ensemble of scatterers
- $\hbar^2 k_A^2 / 2m_A \ll \hbar\omega_B$ so elastic AB scattering

- A solvable alternative to laser speckle (Aspect). Optimisation of localisation by tuning the $A-B$ scattering length
- One expects (in 3D) an Anderson transition [mobility edge] between extended states (continuous spectrum) and localized states (point-like spectrum).

2.2 Model

- Each trapped B atom replaced by Wigner-Bethe-Peierls contact conditions at lattice node on A wavefunction, with effective scattering length $a_{\rm eff}$:

$$\phi(\mathrm{r}_A) \underset{r_{AB}\to 0}{=} D(\mathrm{r}_B) \times \underbrace{\left(\frac{1}{r_{AB}} - \frac{1}{a_{\rm eff}}\right)}_{\text{zero energy scattering state for } r_{AB} \gg a_{\rm ho}} + O(r_{AB})$$

- Scatterers occupy a sphere of finite but large diameter

2.3 How to solve

- single particle Green's function in presence of a number N_B of B scatterers at energy $E = \hbar^2 k_A^2/2m_A$ exactly given by $N_B \times N_B$ matrix inversion

$$G(\mathrm{r}, \mathrm{r}_0) = g(\mathrm{r}-\mathrm{r}_0) + \frac{2\pi\hbar^2}{m_A} \sum_{i,j=1}^{N_B} g(\mathrm{r}-\mathrm{r}_i)[M^{-1}]_{ij} g(\mathrm{r}_j-\mathrm{r}_0)$$

$$M_{ij} = \begin{cases} -\dfrac{2\pi\hbar^2}{m} g(\mathrm{r}_i - \mathrm{r}_j) = e^{ik_A r_{ij}}/r_{ij} & \text{if } i \neq j, \\ ik_A + a_{\mathrm{eff}}^{-1} & \text{if } i = j. \end{cases}$$

where free-space Green's function $g(\mathrm{r} - \mathrm{r}_0)$ is translationally invariant.

- Gives access to localisation length ξ: decay length of field radiated by a source of A in the B medium:

$$G(\mathrm{r},\mathrm{r}_0) \underset{|\mathrm{r}-\mathrm{r}_0| \text{ large}}{\simeq} \mathcal{A}\frac{e^{-|\mathrm{r}-\mathrm{r}_0|/\xi}}{|\mathrm{r}-\mathrm{r}_0|^{\alpha}}$$

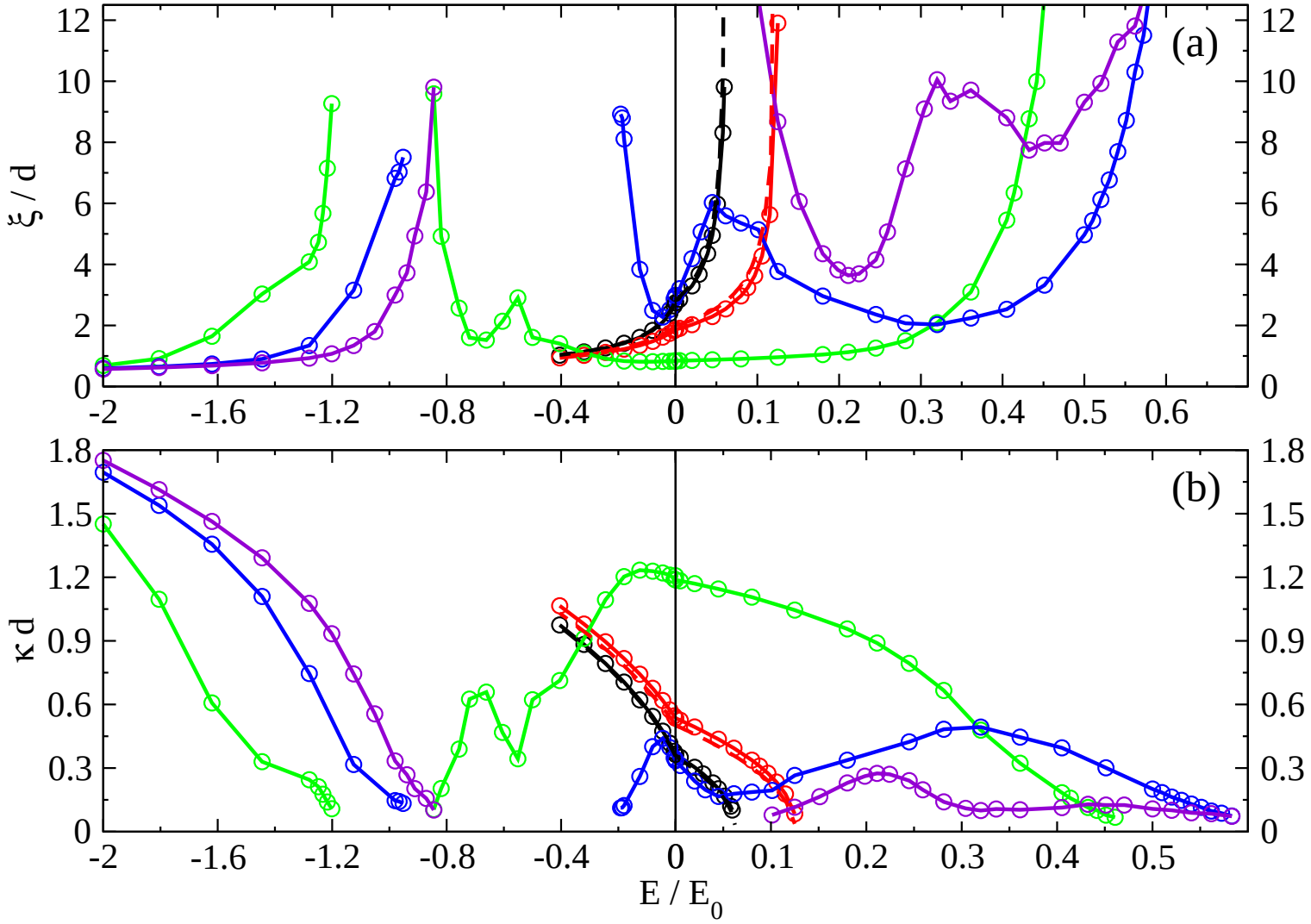
- Gives access to density ρ_E of states ($E<0$) [real poles of G] or resonances [complex poles of G] ($E{=}\mathrm{Re}\, z_0>0$, $\hbar\Gamma/2 = -\,\mathrm{Im}\, z_0 > 0$)

2.4 Matrix M is not an Anderson problem

- off-diagonal disorder only
- at $E>0$, M_{ij} decays slowly for $r_{ij}\to\infty$: more slowly than $1/r_{ij}^3$. So no localized states ? (Boris Altshuler, informal discussion)

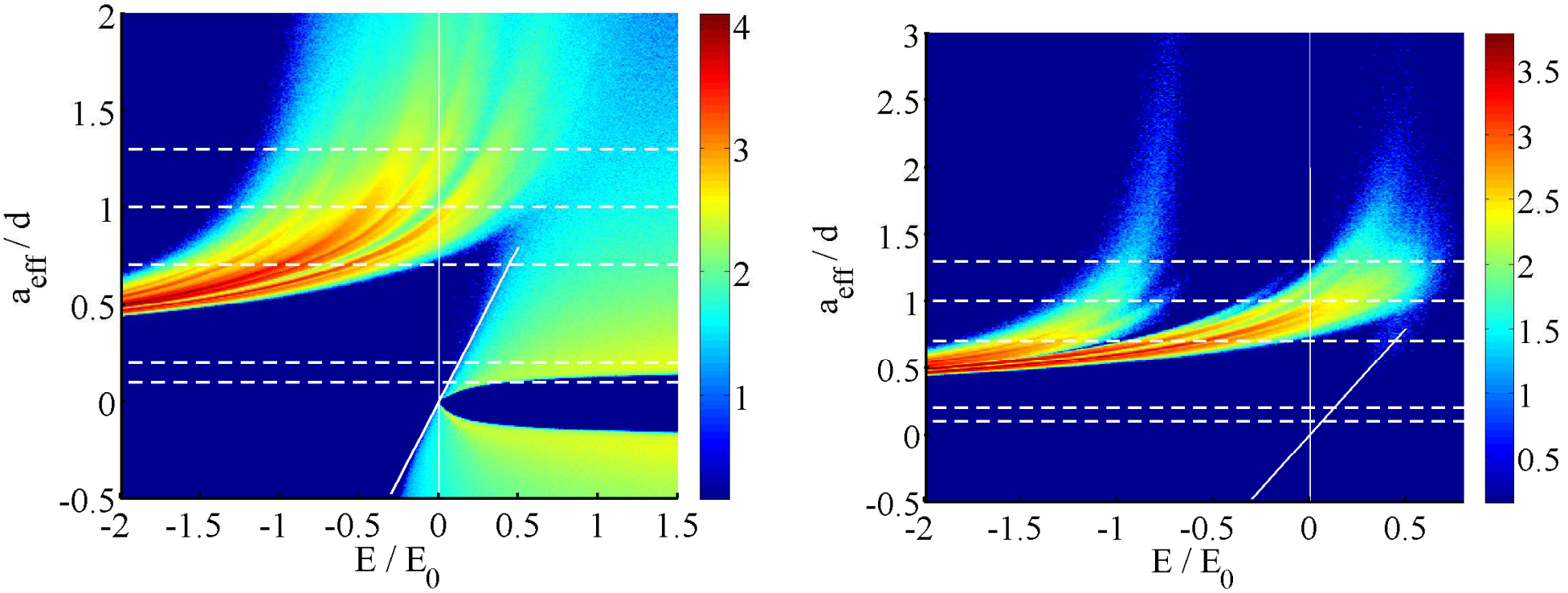
- $p_{\rm occ} = 1/10$, diameter$= 140d$; $\langle N_B \rangle \simeq 1.4 \times 10^5$
- for $a_{\rm eff} < 0$, for $a_{\rm eff} \to +\infty$, no evidence of localisation [van Tiggelen, Lagendijk]
- The best for localisation is to take $a_{\rm eff} \simeq$ half mean inter-B distance
- Rich situation because we consider also $E < 0$. E.g. for $a_{\rm eff} = d$ we shall find three mobility edges, one at positive energy and two at negative energy. Is certainly sensitive to the presence of the lattice.

LOCALISATION LENGTH

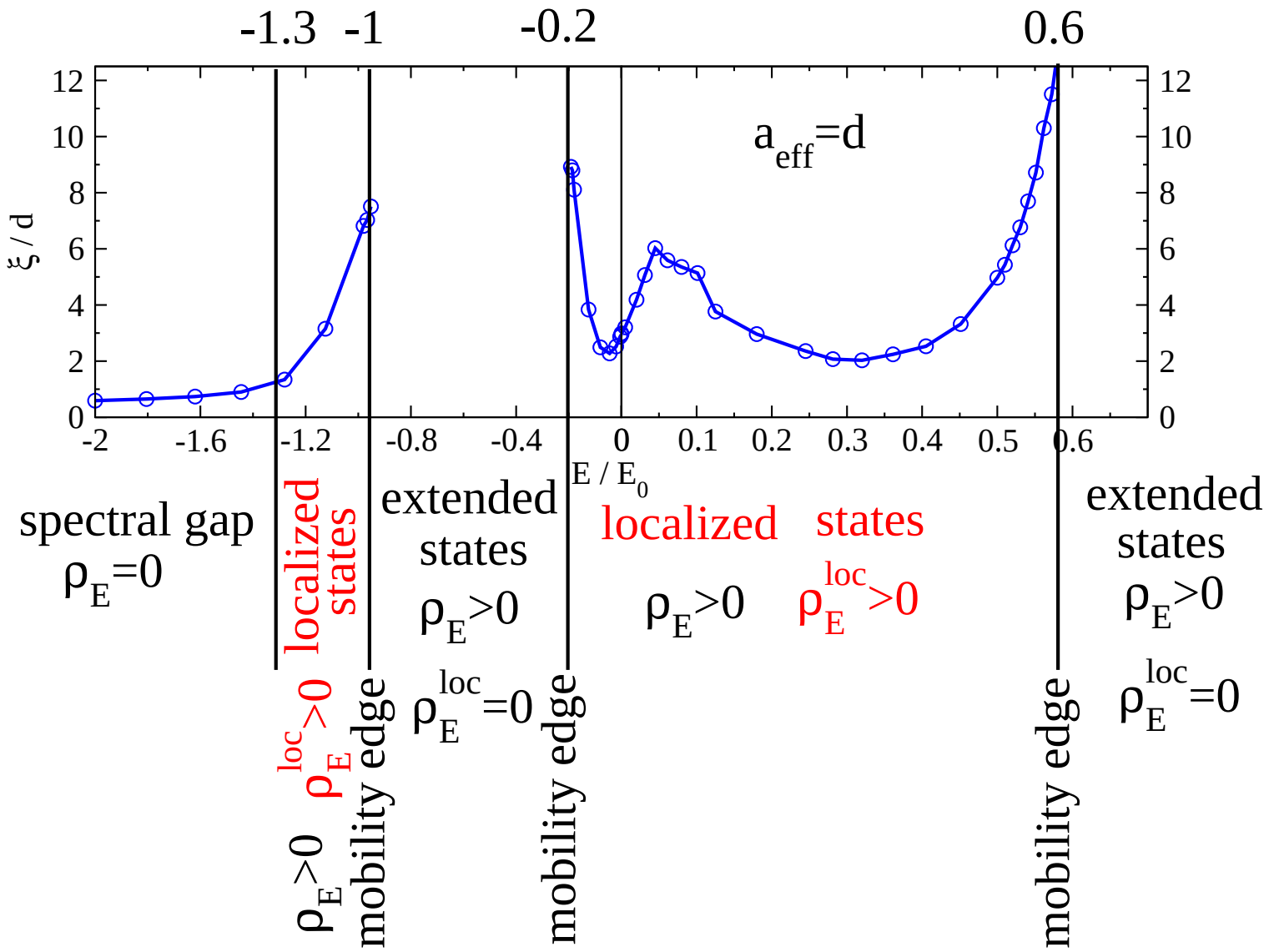


a_{eff}/d =0.1 (black), 0.2 (red), 0.7 (green), 1 (blue), 1.3 (violet)

DOS WITHOUT/WITH SPATIAL FILTER



Oblique line = mean-field bottom $E = \rho_B g_{\text{eff}}$


-1.3
-1
-0.2
0.6
ξ / d
E / E0
aeff=d
spectral gap
ρE=0
localized states
ρEloc>0
ρE>0
mobility edge
extended states
ρE>0
ρEloc=0
mobility edge
localized states
ρE>0
ρEloc>0
mobility edge
extended states
ρE>0
ρEloc=0

3 Photonic band gaps

3.1 Physical motivation

- find matter lattices leading to omnidirectional band gaps (OBG) for light
- first studies with extended objects (dielectric spheres) [Ho, Cha Soukoulis, 1990]
- for atoms: Lagendijk et al., 1996 predict OBG for fcc lattice
- Knoester et al. , 2006: Lagendijk's sum is divergent; addition by hand of a regularizing term; no OBG for fcc. But this no longer solves the original Hamiltonian problem.

- for a stationary state of the field at frequency ω, atom in R carries a mean electric dipole $\vec{D}\,(\mathrm{R})$:

$$\vec{D}\,(\mathrm{R}) = \epsilon_0 \alpha(\omega) \sum_{\mathrm{R}' \neq \mathrm{R}} g(\mathrm{R} - \mathrm{R}') \vec{D}\,(\mathrm{R}')$$

- $\alpha(\omega) \propto 1/(\omega - \omega_0 + i\Gamma/2)$ is atomic polarisability
- $g_{ij}(\mathrm{r})$ is component along i of electric field radiated in r by a unit dipole oscillating along j and located at the origin of coordinates:

$$g_{ij}(\mathrm{r}) \propto \underbrace{[(\omega/c)^2 \delta_{ij} + \partial_{r_i} \partial_{r_j}]}_{\text{ensures transversality}} \underbrace{\frac{e^{i(\omega/c)r}}{r}}_{\text{scalar field}}$$

- periodic case, Bloch theorem: $\hbar\omega = \epsilon_{\mathrm{q}}$ with

$$\vec{D}\,(\mathrm{R}) = \vec{\mathcal{D}}\; e^{i\mathrm{q}\cdot\mathrm{R}}$$

3.3 How to calculate the sum ?

- "formule sommatoire de Poisson" :

$$\sum_{\mathrm{R}\in DL} f(\mathrm{R}) = \frac{1}{\mathcal{V}} \sum_{\mathrm{K}\in RL} \tilde{f}(\mathrm{K})$$

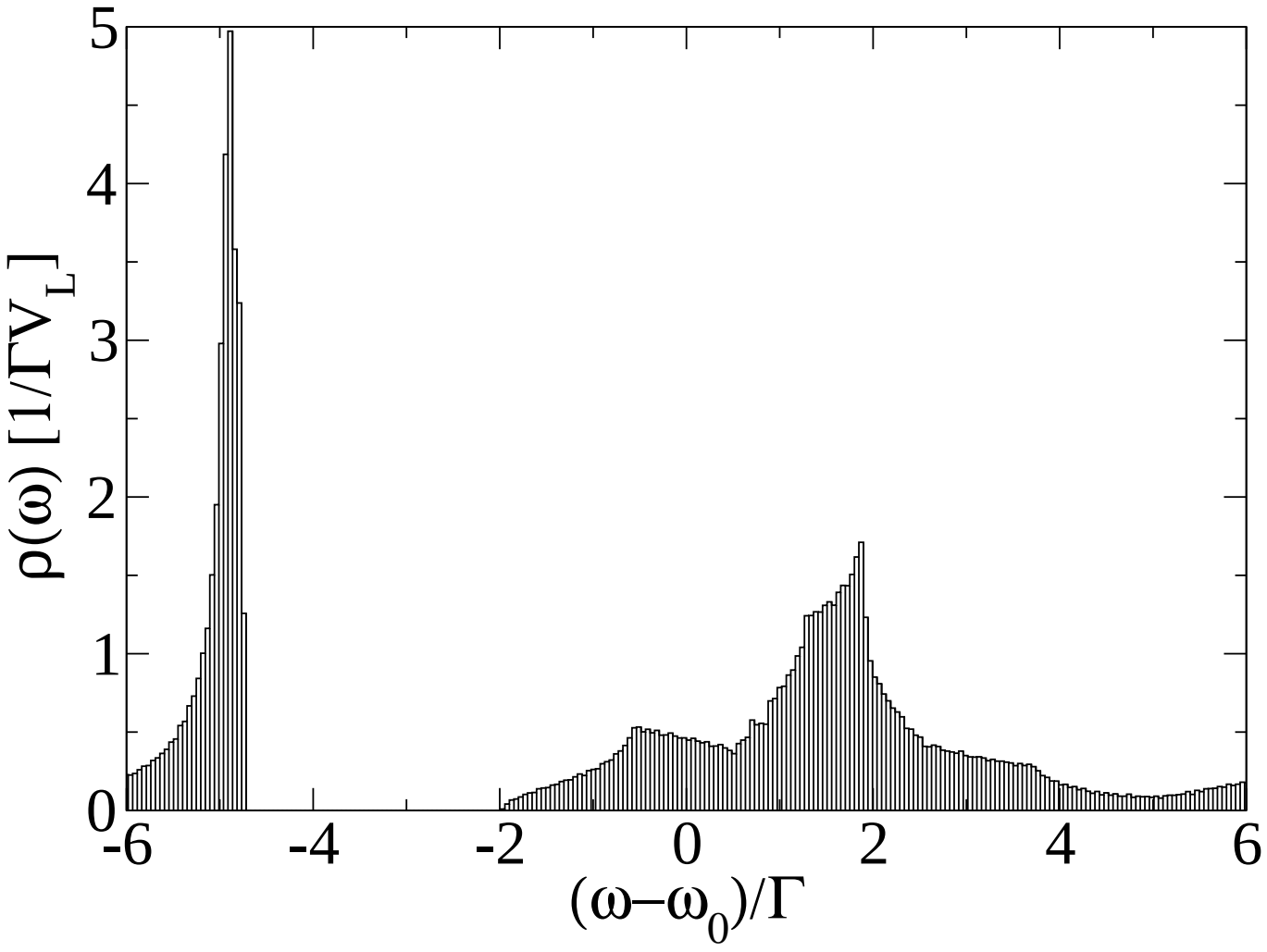
- but $g(0) = \infty$ and $\sum_{\mathrm{K}} K_i K_j / K^2$ not absolutely convergent
- Physical regularising effect: atomic positions delocalized over a_{ho}, provides a Gaussian cut-off in Fourier space

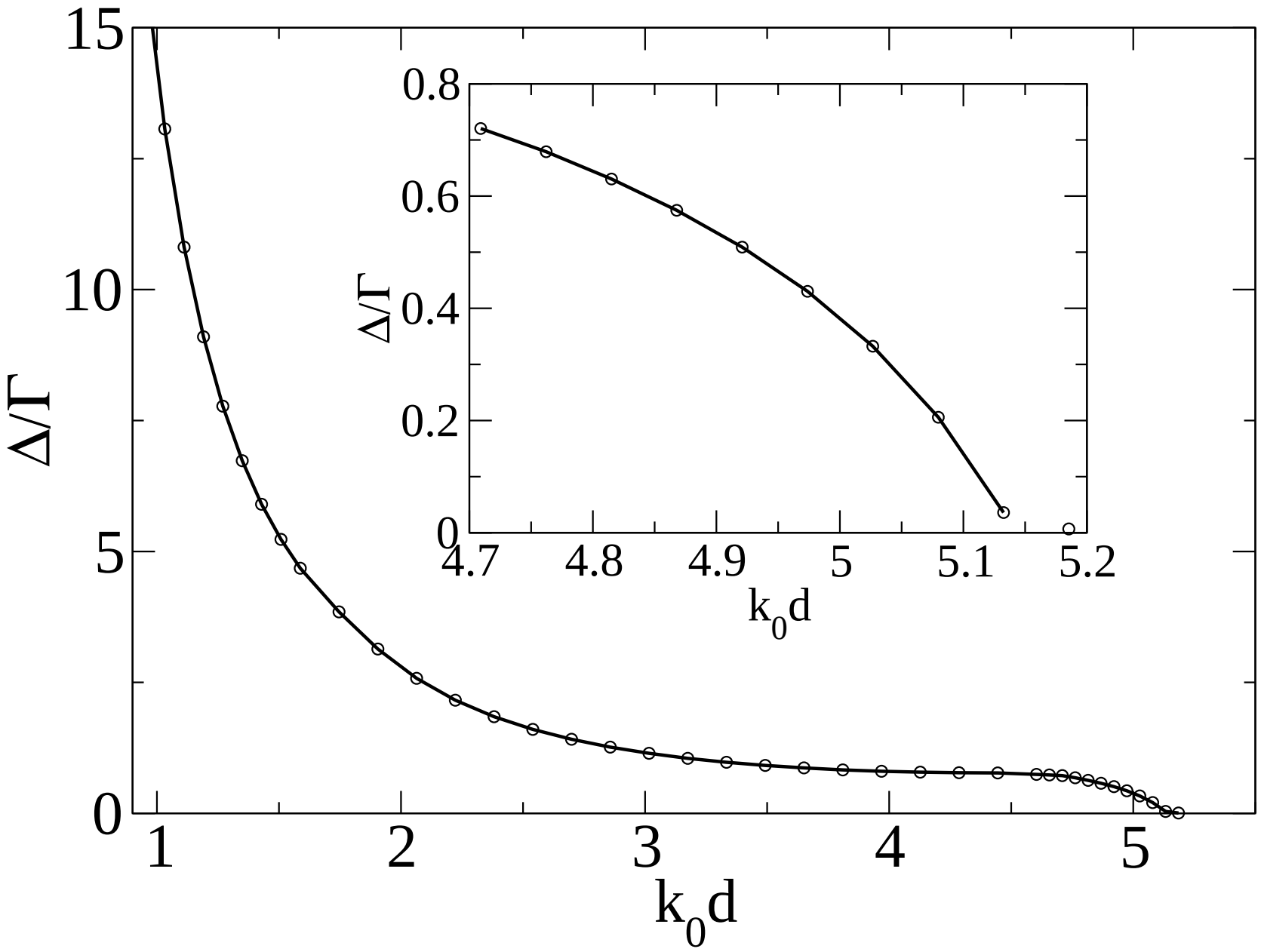
$$g(\mathrm{R}-\mathrm{R}') \to \langle g(\mathrm{R}+\mathrm{u}-\mathrm{R}'-\mathrm{u}')\rangle_{\mathrm{u},\mathrm{u}'} = \bar{g}(\mathrm{R}-\mathrm{R}')$$

- none for Bravais lattices (Knoester was right)
- the historical work of Soukoulis predicted OBG for a diamond lattice of dielectric spheres (fcc+translated copy by $(\frac{d}{4}, \frac{d}{4}, \frac{d}{4})$). May be it also works with atoms ?
- we indeed predict a gap for an atomic diamond lattice
- important to check for the absence of free wave (field vanishes on all lattice sites) using

$$\frac{\omega_{\text{free}}}{c} \geq \inf_{\text{K}\neq 0} \frac{K}{2}$$

Photonic density of states for diamond, $k_0 d = 2$ ($k_0 = \omega_0/c$)




15
10
5
0
Δ/Γ
1
2
3
4
5
k0d
0.8
0.6
0.4
0.2
0
Δ/Γ
4.7
4.8
4.9
5
5.1
5.2
k0d