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REPLY

Reply to comment on: 'Does an atom interferometer test the gravitational redshift at the Compton frequency?'

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Abstract
We reply to a comment by Hohensee *et al* on: 'Does an atom interferometer test the gravitational redshift at the Compton frequency' and show that the main result of that paper, namely that atom interferometric gravimeters do not test the gravitational redshift at the Compton frequency, remains valid.

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In their comment, Hohensee *et al* re-affirm their earlier interpretation, based on an analogy with classical clock experiments, of atom interferometric gravimeters as testing the gravitational redshift. They repeat previous arguments, in particular the cancellation between the time dilation and laser phase contributions to the phase shift, and arguments centered around Schiff's conjecture. Here we re-affirm that these arguments are incorrect or irrelevant, as shown below, and that the main result of our paper [1], namely that atom interferometric gravimeters do not test the gravitational redshift at Compton's frequency, remains valid.

First Hohensee *et al* argue that the absence of the Compton frequency (or equivalently the atom's mass) in the final phase shift does not prevent the experiment from being a redshift test at that frequency. They claim that this is also the case for redshift tests using standard clocks, like the Pound–Rebka experiment, where the redshift effect is cancelled by a controlled first-order Doppler effect, thus effectively leaving a null signal independent of the 14.4 keV transition frequency. This analogy is fallacious: the Pound–Rebka experiment is based on a

resonance between atoms sitting at the top and bottom of a tower. The controlled Doppler shift is just a clever experimental method for scanning the resonance which is affected by the redshift (cf figure 1 in [2]). The resulting signal is thus null on the exact center of the resonance curve but not at other frequencies. In the atom gravimeter in contrast, the cancellation of the Compton frequency is inherent to the interferometer: the action phase shift involving the Compton frequency is always zero, as shown in [1], and the measurement of that frequency (and of the associated redshift) is intrinsically impossible. Furthermore, it is manifestly incorrect to state as Hohensee *et al* do that the cancellation of the clock frequency is a generic feature of any redshift measurement. Most clock experiments [3, 4] measure frequencies of two electromagnetic signals delivered by two clocks. The redshift effect is thus associated with a relation such as $\Delta f = f_0 \Delta U/c^2$, where Δf is the measured frequency difference and f_0 is the common proper frequency of the clocks. This could also be done in principle for a Pound–Rebka-like experiment if experimental means (say frequency combs) were available for accurate measurements of frequencies in the x-ray domain. By contrast, in the atomic interferometer considered by Hohensee *et al*, there is no real emission of a photon at the Compton frequency, and no measurement of the frequency of this photon by a detector.

The interferometer considered by Hohensee *et al* is, in fact, not an atomic clock. The two states g and g' appear symmetrically in the two arms of the interferometer, and no resonance is obtained when one varies the laser frequency. As explained in [1], there are other non-symmetric atomic interferometers where the output signal exhibits resonances when the laser frequency is varied. They provide a measurement of the atomic frequency $\omega_{gg'}$ (see discussion after (2.21) in [1]), so that these interferometers can also be considered as atomic clocks oscillating at $\omega_{gg'}$. To get an oscillation at the Compton frequency, one would need to have in the two arms of the interferometer two states, whose energies differ by mc^2, which is far beyond present-day technology. Furthermore, to test the redshift at the Compton frequency, two different clocks of this type located at two different altitudes would have to be built and their frequencies be compared.

In a related argument (equation (1) and text below in their comment), Hohensee *et al* restate their previous claim that in the total phase shift the time dilation phase ϕ_t cancels the laser phase ϕ_{laser} and not the gravitational phase ϕ_r, whereas we have demonstrated in [1] that the free evolution phase $\phi_S = \phi_r + \phi_t$ is necessarily zero for all closed paths that follow trajectories obtained from the same Lagrangian as the one used for the phase integral. The latter fact implies that the difference of action integrals is zero and that the Compton frequency and redshift at that frequency are unmeasurable, no matter what modified Lagrangian is used (i.e. independent of any redshift violating parameter β). We re-affirm that the fundamental cancellation is between ϕ_r and ϕ_t, one reason being that $\phi_S = \phi_r + \phi_t = 0$ is an invariant statement, since ϕ_S is the contour integral of the proper time (see the first term on the right-hand side of (1.3) in [1]) which is invariant under coordinate transformations. By contrast the statement that $\phi_t + \phi_{\text{laser}} = 0$ is not invariant and is only true in the rest frame of the lasers. To see this in a simple example, analyze the atom gravimeter in a frame that is freely falling in the Earth's gravitational field. An elementary calculation shows that ϕ_r and ϕ_t are separately zero in that frame, $\phi_r = \phi_t = 0$, but that $\phi_{\text{laser}} = k\, g\, T^2$, where k is the laser wavevector and T the interrogation time, because the lasers are accelerated upward in that frame. Thus there is no cancellation between ϕ_t and ϕ_{laser} in that frame, but we still have $\phi_S = \phi_r + \phi_t = 0$.

Concerning Schiff's conjecture, we do not agree with the analysis of Hohensee *et al*. Schiff's conjecture states that any violation of the universality of free fall (UFF) also implies a violation of the universality of clock rates (UCR) and vice versa. Does this mean that it makes no sense to distinguish between the two types of tests? Our answer is 'No', because the relation between UFF and UCR tests depends on the experiments under consideration, on the

nuclear or atomic models used for the analysis of the clocks and the test masses, and on the particular model used for the violation of UFF and UCR.

The next question is then whether the atom gravimeter measurement is more directly comparable to UFF or UCR tests. As shown in [1] and also e.g. in [5] the answer is clearly UFF, as it constrains the same parameters as other UFF tests, whereas it requires nuclear and atomic models to relate it to parameters measured in UCR tests. To make this point more explicit, let us quote the SME analysis presented in [6]: the UFF test of [7] sets a limit on $\beta^{\mathrm{Be}} - \beta^{\mathrm{Al}}$ (to leading order), the atom gravimeter test on $\beta^{\mathrm{Cs}} - \beta^{\mathrm{grav}}$ (where 'grav' stands for the test mass in the classical gravimeter) and the Pound–Rebka experiment on $\xi^{\mathrm{Mossb}}_{^{57}\mathrm{Fe}} - \beta^{\mathrm{grav}}$. To relate the former two, one only requires the atomic composition of Be, Al and grav (equation (8) of [6]) but to relate them to the Pound–Rebka measurement one additionally requires a nuclear model for the 14.4 keV transition in ^{57}Fe (see the text before (11) in [6]).

To conclude, let us briefly recall the compelling argument (see e.g. [5]) that atom interferometers test the UFF. This argument is agreed on since the realization of the first atomic gravimeters two decades ago [8]. The atoms with the inertial mass m_i and gravitational mass m_g obey the Lagrangian $L = \frac{1}{2} m_i \dot{z}^2 - m_g g z$ in the gravitational field of the Earth. The phase shift is the sum of the free evolution phase given by the difference of action integrals ϕ_S along the two paths and of the contribution ϕ_{laser} coming from the interaction with the lasers. The free evolution phase ϕ_S is exactly zero for a closed total path (see e.g. [1]), and the phase shift reduces to $\phi_{\mathrm{laser}} = (m_g/m_i)\, g\, k\, T^2$ using the position of the atoms deduced from the Lagrangian. Comparing with the measurement by a nearby classical gravimeter of the gravitational acceleration of a freely falling macroscopic mass (corner cube), $g_{\mathrm{meas}} = (M_g/M_i)g$, one obtains to first order $\phi_{\mathrm{laser}} = (1 + \eta)\, g_{\mathrm{meas}}\, k\, T^2$, where $\eta = m_g/m_i - M_g/M_i$ is the Eötvos parameter between the atoms and the corner cube, showing that the measurement of the phase shift gives a test on possible violations of the UFF between atoms and macroscopic masses.

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