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Potentialities of a new $\sigma_+-\sigma_-$ laser configuration for radiative cooling and trapping

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Abstract. In the process of cooling and trapping neutral atoms, we investigate a new laser configuration, which consists of two counterpropagating laser beams with orthogonal σ_+ and σ_- polarisations. We show that such a configuration looks more promising than an ordinary standing wave (where the two counterpropagating waves have the same polarisation), and we explain this result as being due to angular momentum conservation which prevents any coherent redistribution of photons between the two waves. Our conclusions are based on a quantitative calculation of the various parameters (potential depth, friction coefficient, diffusion coefficient) describing the mean value and the fluctuations of the radiative forces experienced, in such a laser configuration, by an atom with a $J = 0$ ground state and a $J = 1$ excited state.

1. Introduction

A great deal of interest has been recently devoted to the study of the motion of atoms in a quasi-resonant standing light wave (Minogin and Serimaa 1979, Gordon and Ashkin 1980, Letokhov and Minogin 1981, Cook 1980a, b, Minogin 1981a, b, Kazantsev *et al* 1981). One of the main reasons for this interest relies upon the possibility of using this standing wave as an optical trap (Letokhov *et al* 1977, Letokhov and Minogin 1978, Ashkin 1978): the radiation pressures of the two counterpropagating waves forming the standing wave compensate, while dipole forces add and may lead to a trapping of atoms. Unfortunately, such a scheme also leads to a diffusion of atomic momentum, and therefore to a heating of the atoms, which is prohibitive in view of trapping as soon as the atomic transition is saturated (Gordon and Ashkin 1980, Cook 1980a, b, 1981a, Kazantsev *et al* 1981).

We can understand the leading term of the atomic momentum diffusion coefficient, in the saturating case, in terms of coherent redistribution of photons between the two waves (Arimondo *et al* 1981, Bernhardt and Shore 1981): the atom, initially in its ground state, can first absorb a photon in one of the counterpropagating waves and then emit a stimulated photon in the other wave; it thus gains the momentum $2\hbar\boldsymbol{k}$, where $\hbar\boldsymbol{k}$ is the momentum of one photon. Since the number of such processes per unit of time is not limited, the resulting component of the momentum diffusion coefficient does not saturate when the light intensity increases.

The purpose of this paper is to present a new scheme which has the same trapping capabilities as the previous standing wave, but in which coherent redistribution processes cannot occur. It consists in two identical counterpropagating waves with

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orthogonal polarisations, respectively σ_+ and σ_- (figure 1(*a*)). For the sake of simplicity, we will choose a $J=0$ (ground level) to $J=1$ (excited level) atomic transition (figure 1(*b*)). It is then obvious that such a scheme does not allow coherent redistribution processes between the two waves: after the absorption of a σ_+ photon for instance, the atom is in the $(e, m=+1)$ level and, because of angular momentum conservation, it cannot emit a stimulated σ_- photon in the other wave.

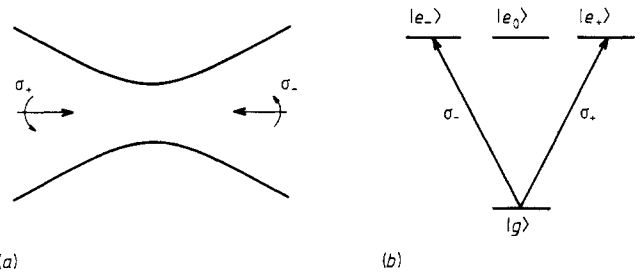

Figure 1. (*a*) Laser configuration studied in this paper: superposition of two focused σ_+ and σ_- counterpropagating waves (with eventually 'infinite' beam waists: plane waves). (*b*) Zeeman optical components of the $J=0$ to $J=1$ transition, excited by the σ_+ and σ_- waves.

In § 2 of this paper, we give our notations and our hypothesis. We then calculate the radiative forces acting, in the σ_+–σ_- configuration, on an atom at rest (§ 3.1), or moving with a velocity v (§§ 3.2 and 3.3). Section 4 is devoted to the study of the momentum diffusion coefficient. Finally, the implications of this work concerning the radiative trapping are discussed in § 5, where numerical estimations are given.

2. Notations and assumptions

The total Hamiltonian is the sum of four parts:

$$H = H_A + H_F + V_{A-L} + V_{A-F}. \tag{2.1}$$

H_A is the atomic Hamiltonian, H_F the quantised-field Hamiltonian. The laser field is supposed to be in a coherent state, so that we can treat it as a c-number field, and then split the atom–field coupling into two parts (Mollow 1975), the first one (V_{A-L}) describing the atom–laser coupling, and the second one (V_{A-F}) the atom–quantised-field coupling; the quantised field is taken in its ground state.

The atomic Hamiltonian is the sum of the kinetic energy of the atom and of its internal energy:

$$H_A = \frac{\boldsymbol{P}^2}{2m} + \hbar\omega_0 \sum_m Q_{e_m e_m} \tag{2.2}$$

where we have introduced the general notation

$$Q_{ab} = |a\rangle\langle b| \tag{2.3}$$

with $|a\rangle$, $|b\rangle$ internal atomic states. In equation (2.2), we have only taken into account the excited level $|e\rangle$ $(J=1)$ coupled to the ground level $|g\rangle$ $(J=0)$ by the laser light,

and the summation is taken over the three Zeeman sublevels $|e_m\rangle$ (with $m=+$, $m=0$, $m=-$) of $|e\rangle$; ω_0 is the atomic frequency.

The electromagnetic field is quantised on a complete set of orthonormal field distributions $\mathscr{E}_\lambda(\boldsymbol{R})$, for example the plane waves of wavevector $\boldsymbol{k}_\lambda$, frequency $\omega_\lambda = c|\boldsymbol{k}_\lambda|$ and polarisation $\boldsymbol{\varepsilon}_\lambda$. The Hamiltonian H_F of the quantised field is thus:

$$H_F = \sum_\lambda \hbar\omega_\lambda a_\lambda^+ a_\lambda \tag{2.4}$$

where a_λ and a_λ^+ are respectively the destruction and creation operators of a photon in the mode λ.

The atom–quantised-field coupling V_{A-F} can be written in the electric dipole approximation:

$$V_{A-F} = -\boldsymbol{D}\cdot\boldsymbol{E}(\boldsymbol{R}) \tag{2.5}$$

where $\boldsymbol{D}$ is the atomic dipole operator and $\boldsymbol{E}(\boldsymbol{R})$ the quantised electric field taken for the position operator $\boldsymbol{R}$ of the atom:

$$\boldsymbol{E}(\boldsymbol{R}) = \sum_\lambda (\mathscr{E}_\lambda(\boldsymbol{R})a_\lambda + \mathscr{E}_\lambda^*(\boldsymbol{R})a_\lambda^+). \tag{2.6}$$

The atom–laser coupling is also taken at the electric dipole approximation:

$$V_{A-L} = -\boldsymbol{D}\cdot\boldsymbol{\mathscr{E}}_{\text{laser}}(\boldsymbol{R}, t) \tag{2.7}$$

where $\boldsymbol{\mathscr{E}}_{\text{laser}}(\boldsymbol{r}, t)$ is the classical function describing the laser electric field. $\boldsymbol{\mathscr{E}}_{\text{laser}}(\boldsymbol{r}, t)$ results from the superposition of the two identical counterpropagating waves along the $0z$ axis, with σ_+ and σ_- polarisations:

$$\begin{aligned}
&\boldsymbol{\mathscr{E}}_{\text{laser}}(\boldsymbol{r}, t) = \boldsymbol{\mathscr{E}}_1(\boldsymbol{r}, t) + \boldsymbol{\mathscr{E}}_2(\boldsymbol{r}, t)\\
&\boldsymbol{\mathscr{E}}_1(\boldsymbol{r}, t) = -\frac{\mathscr{E}(\boldsymbol{r})}{2\sqrt{2}}\boldsymbol{\varepsilon}_+ \exp[-\mathrm{i}(\omega_L t - kz)] + \text{CC}\\
&\boldsymbol{\mathscr{E}}_2(\boldsymbol{r}, t) = -\frac{\mathscr{E}(\boldsymbol{r})}{2\sqrt{2}}\boldsymbol{\varepsilon}_- \exp[-\mathrm{i}(\omega_L t + kz)] + \text{CC}
\end{aligned} \tag{2.8}$$

where ω_L is the laser frequency.

The complex unit vectors $\boldsymbol{\varepsilon}_+$ and $\boldsymbol{\varepsilon}_-$ can be written in terms of $\boldsymbol{\varepsilon}_x$ and $\boldsymbol{\varepsilon}_y$ (unit vectors along $0x$ and $0y$):

$$\boldsymbol{\varepsilon}_+ = -2^{-1/2}(\boldsymbol{\varepsilon}_x + \mathrm{i}\boldsymbol{\varepsilon}_y) \qquad \boldsymbol{\varepsilon}_- = 2^{-1/2}(\boldsymbol{\varepsilon}_x - \mathrm{i}\boldsymbol{\varepsilon}_y). \tag{2.9}$$

Introducing now the r-dependent set

$$\begin{aligned}
&\boldsymbol{\varepsilon}' = \boldsymbol{\varepsilon}_x \sin kz + \boldsymbol{\varepsilon}_y \cos kz = 2^{-1/2}\mathrm{i}[\boldsymbol{\varepsilon}_+ \exp(\mathrm{i}kz) + \boldsymbol{\varepsilon}_- \exp(-\mathrm{i}kz)]\\
&\boldsymbol{\varepsilon}'' = -\boldsymbol{\varepsilon}_x \cos kz + \boldsymbol{\varepsilon}_y \sin kz = 2^{-1/2}[\boldsymbol{\varepsilon}_+ \exp(\mathrm{i}kz) - \boldsymbol{\varepsilon}_- \exp(-\mathrm{i}kz)]
\end{aligned} \tag{2.10}$$

the laser field can be written:

$$\boldsymbol{\mathscr{E}}_{\text{laser}}(\boldsymbol{r}, t) = \mathscr{E}(\boldsymbol{r})\boldsymbol{\varepsilon}' \sin(\omega_L t). \tag{2.11}$$

The resulting beam $\boldsymbol{\mathscr{E}}_{\text{laser}}$ then has at each point an amplitude $\sqrt{2}$ times larger than the one of each initial beam $\boldsymbol{\mathscr{E}}_1$ and $\boldsymbol{\mathscr{E}}_2$. Note in particular that this resulting amplitude does not present any important variation of the scale of a wavelength, as would be the case in a standing wave resulting from the superposition of $\boldsymbol{\mathscr{E}}_1$ and $\boldsymbol{\mathscr{E}}_2$ with the

same polarisation. On the other hand, the resulting polarisation $\boldsymbol{\varepsilon}'$ is linear and rotates along the $0z$ axis.

In order to describe the atom–laser coupling, it is convenient to introduce the following $\boldsymbol{r}$-dependent basis for the atomic excited state (compare with equation (2.10)):

$$\begin{aligned} |e'\rangle &= 2^{-1/2}\mathrm{i}[|e_+\rangle \exp(\mathrm{i}kz) + |e_-\rangle \exp(-\mathrm{i}kz)] \\ |e''\rangle &= 2^{-1/2}(|e_+\rangle \exp(\mathrm{i}kz) - |e_-\rangle \exp(-\mathrm{i}kz)] \\ |e_0\rangle & \end{aligned} \tag{2.12}$$

so that the atomic dipole reads:

$$\begin{aligned} \boldsymbol{D} &= d(Q_{ge_+}\boldsymbol{\varepsilon}_+ + Q_{ge_-}\boldsymbol{\varepsilon}_- + Q_{ge_0}\boldsymbol{\varepsilon}_0) + \mathrm{HC} \\ &= d(Q_{ge'}\boldsymbol{\varepsilon}' + Q_{ge''}\boldsymbol{\varepsilon}'' + Q_{ge_0}\boldsymbol{\varepsilon}_0) + \mathrm{HC} \end{aligned} \tag{2.13}$$

where d is the electric dipole moment of the transition.

The atom–laser coupling can finally be written with these notations:

$$V_{\mathrm{A-L}} = -d\mathscr{E}(\boldsymbol{R})(Q_{ge'} + Q_{e'g})\sin(\omega_{\mathrm{L}}t). \tag{2.14}$$

Let us mention at this point that, in some parts of this paper, we will be interested in the particular case of 'plane σ_+ and σ_- waves': this means that the waists of the two σ_+ and σ_- waves are very large so that we can neglect the dependence on $\boldsymbol{R}$ of $\mathscr{E}(\boldsymbol{R})$.

As usual, all the calculations will be done using the rotating-wave approximation, which consists in keeping only the resonant terms in the atom–laser coupling (2.14):

$$V_{\mathrm{A-L}} = \tfrac{1}{2}\mathrm{i}d\mathscr{E}(\boldsymbol{R})(\tilde{Q}_{ge'} - \tilde{Q}_{e'g}) \tag{2.15}$$

where we have put:

$$\begin{aligned} \tilde{Q}_{ge'} &= Q_{ge'}\exp(\mathrm{i}\omega_{\mathrm{L}}t) \\ \tilde{Q}_{e'g} &= Q_{e'g}\exp(-\mathrm{i}\omega_{\mathrm{L}}t). \end{aligned} \tag{2.16}$$

In this paper, we will also use the semiclassical approximation, which consists in replacing the atomic position operator R by its average value $\boldsymbol{r} = \langle \boldsymbol{R} \rangle$.

This is legitimate as soon as the spatial extension Δr of the atomic wavepacket is much smaller than the laser wavelength λ:

$$\Delta r \ll \lambda. \tag{2.17}$$

On the other hand, we will use the notion of atomic velocity and we will, in particular, consider atoms at rest, i.e. with a velocity spread Δv such that the Doppler effect $k\Delta v$ is very small compared with the natural linewidth of the excited level Γ:

$$k\Delta v \ll \Gamma. \tag{2.18}$$

One then immediately notes that the two conditions (2.17) and (2.18) are compatible with the Heisenberg inequality:

$$m\Delta r\,\Delta v \geqslant \hbar \tag{2.19}$$

only if the following relation holds:

$$\hbar^2 k^2/m \ll \hbar\Gamma. \tag{2.20}$$

This validity condition for the semiclassical calculation means that the recoil energy has to be very small compared with the natural linewidth, or, in other words, that the

atom is still in resonance with the laser light after a single photon absorption or emission. Note that when condition (2.20) is fulfilled, one can show (Cohen-Tannoudji 1983, Dalibard and Cohen-Tannoudji 1984) that the forces and the diffusion coefficient appearing in the Fokker-Planck equation describing the atomic motion, in a fully quantum treatment, are the same as the semiclassical ones, which we will calculate here.

3. Radiative forces in the σ_+–σ_- configuration

As usual in the semiclassical theory of radiative forces (see, for example, Cook 1980a, Gordon and Ashkin 1980), we will calculate these forces in the Heisenberg point of view. We first define a force operator as the time derivative of the atomic momentum:

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = \frac{\mathrm{i}}{\hbar}[H, \boldsymbol{P}] = -\boldsymbol{\nabla}_{\boldsymbol{R}} H. \tag{3.1}$$

This force can be expressed in terms of atomic and field operators; using (2.1), (2.5), (2.12) and (2.15), we get:

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{int}} + \boldsymbol{F}_{\mathrm{pol}} + \boldsymbol{F}_{\mathrm{vac}} \tag{3.2}$$

with

$$\boldsymbol{F}_{\mathrm{int}} = -\tfrac{1}{2}\mathrm{i}\, d\boldsymbol{\nabla}\mathscr{E}(\boldsymbol{R})(\tilde{Q}_{ge'} - \tilde{Q}_{e'g}) \tag{3.3a}$$

$$\boldsymbol{F}_{\mathrm{pol}} = \tfrac{1}{2}\mathrm{i}\, d\mathscr{E}(\boldsymbol{R})\boldsymbol{k}(\tilde{Q}_{ge''} - \tilde{Q}_{e''g}) \tag{3.3b}$$

$$\boldsymbol{F}_{\mathrm{vac}} = \sum_{i=x,y,z} D_i \boldsymbol{\nabla} E_i(\boldsymbol{R}). \tag{3.3c}$$

$\boldsymbol{F}_{\mathrm{vac}}$ results from the gradient of the quantised-field–atom coupling, while $\boldsymbol{F}_{\mathrm{int}}$ and $\boldsymbol{F}_{\mathrm{pol}}$ both come from the laser–atom coupling; more precisely, $\boldsymbol{F}_{\mathrm{int}}$ and $\boldsymbol{F}_{\mathrm{pol}}$ are respectively due to the $\boldsymbol{R}$ dependence of $V_{\mathrm{A-L}}$ via $\mathscr{E}(\boldsymbol{R})$ and via $Q_{ge'} - Q_{e'g}$.

As mentioned above, we will do a semiclassical treatment, and replace the position operator $\boldsymbol{R}$ by its average value $\boldsymbol{r}$. Furthermore, we will actually be interested in the average $\boldsymbol{f}$ of $\boldsymbol{F}$ over both field and internal atomic state:

$$\boldsymbol{f} = \langle \boldsymbol{F} \rangle = \boldsymbol{f}_{\mathrm{int}} + \boldsymbol{f}_{\mathrm{pol}} + \boldsymbol{f}_{\mathrm{vac}}. \tag{3.4}$$

3.1. Radiative forces on an atom at rest

If one considers an atom at rest at point $\boldsymbol{r}_0$, it is clear from the expression of the laser–atom coupling (2.15) that only the sublevel $|e'\rangle$ (at point $\boldsymbol{r}_0$) is coupled to the ground state. It follows that a $(J=0)$–$(J=1)$ atom at rest in the σ_+–σ_- configuration behaves as a two-level system. More precisely, the evolution of

$$\begin{aligned} \rho_{e'e'} &= \langle Q_{e'e'} \rangle \\ \rho_{ge'} &= \langle \tilde{Q}_{e'g} \rangle \\ \rho_{e'g} &= \langle \tilde{Q}_{ge'} \rangle \\ \rho_{gg} &= \langle Q_{gg} \rangle \end{aligned} \tag{3.5}$$

is given by the usual optical Bloch equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e'e'} = -\Gamma\rho_{e'e'} - \frac{d\mathscr{E}(\boldsymbol{r}_0)}{2\hbar}(\rho_{ge'} + \rho_{e'g}) \tag{3.6a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e'g} = (\mathrm{i}\delta - \tfrac{1}{2}\Gamma)\rho_{e'g} + \frac{d\mathscr{E}(\boldsymbol{r}_0)}{2\hbar}(\rho_{e'e'} - \rho_{gg}) \tag{3.6b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{ge'} = \left(\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e'g}\right)^* \tag{3.6c}$$

$$\rho_{e'e'} + \rho_{gg} = 1 \tag{3.6d}$$

where we have put $\delta = \omega_L - \omega_0$. Using the stationary solution of these equations, we get:

$$\boldsymbol{f}_{\mathrm{int}} = -\tfrac{1}{2}\hbar\delta\frac{\boldsymbol{\nabla} s}{1+s} \tag{3.7}$$

where we have introduced the $\boldsymbol{r}$-dependent saturation parameter:

$$s = \frac{1}{2}\left(\frac{d\mathscr{E}(\boldsymbol{r})}{\hbar}\right)^2(\delta^2 + \tfrac{1}{4}\Gamma^2)^{-1}. \tag{3.8}$$

It is important to note that this force is exactly equal to the usual dipole force (or gradient force) found for a two-level system.

We now turn to the calculation of $\boldsymbol{f}_{\mathrm{pol}}$ and $\boldsymbol{f}_{\mathrm{vac}}$. The result for $\boldsymbol{f}_{\mathrm{pol}}$ is immediate: since the level $|e''\rangle$ (at point $\boldsymbol{r}_0$) is not coupled to the laser field, the average value of $Q_{ge''}$ is zero, and using (3.3b):

$$\boldsymbol{f}_{\mathrm{pol}} = \boldsymbol{0}. \tag{3.9}$$

In order to calculate $\boldsymbol{f}_{\mathrm{vac}}$, we first remark that the quantised field $\boldsymbol{E}$ can be written, in the Heisenberg point of view, $\boldsymbol{E}_{\mathrm{free}} + \boldsymbol{E}_{\mathrm{source}}$, where $\boldsymbol{E}_{\mathrm{source}}$ is the field radiated by the atom, and $\boldsymbol{E}_{\mathrm{free}}$ the field that would exist in the absence of a source. Since the source field has no gradient at the atom position (Tanguy 1983) one gets:

$$\langle \boldsymbol{\nabla} E_i \rangle = \langle \boldsymbol{\nabla} E_{i\mathrm{free}} \rangle = \boldsymbol{0} \tag{3.10}$$

since the quantised field is taken initially in its ground state. It follows that:

$$\boldsymbol{f}_{\mathrm{vac}} = \boldsymbol{0} \tag{3.11}$$

so that

$$\boldsymbol{f} = \boldsymbol{f}_{\mathrm{int}} = -\tfrac{1}{2}\hbar\delta\frac{\boldsymbol{\nabla} s}{1+s}. \tag{3.12}$$

For an atom at rest, the only contribution to the average force is then $\boldsymbol{f}_{\mathrm{int}}$. In the perspective of using this force to trap atoms, it is useful to notice that this force derives from the potential:

$$U(\boldsymbol{r}) = \tfrac{1}{2}\hbar\delta\ln(1+s) \tag{3.13}$$

and to optimise the potential well depth. For a given laser power and waist, i.e. for a given maximal field amplitude $\mathscr{E}_m$, one finds the optimal values:

$$s_m \simeq 4 \qquad \text{or} \qquad \delta \simeq -0.35\, d\mathscr{E}_m/\hbar \tag{3.14}$$

leading to the optimal well depth:

$$U_0 \simeq 0.3\, d\mathscr{E}_m. \tag{3.15}$$

3.2. Radiative forces on slowly moving atoms

We now suppose that the atom is moving with a velocity $\boldsymbol{v}$, and we want to calculate in this section the first order velocity dependence of radiative forces, in the σ_+–σ_- configuration.

It is clear that f_{vac} is still zero (quantised field in its ground state), but the values of f_{int} and f_{pol} are now changed, since the atomic motion implies a time dependence of the laser–atom coupling via $\mathscr{E}(\boldsymbol{r}(t))$ and $Q_{ge'} - Q_{e'g}$. This motion will in particular cause a coupling of the level $|e''\rangle$ with the ground level, and the results will therefore be quite different from the two-level atom case. In order to calculate the average values of $Q_{e'g}$, $Q_{e''g}, \ldots$, appearing in the force, we shall use the general Heisenberg equation for a time-dependent observable:

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\partial A}{\partial t} + \frac{\mathrm{i}}{\hbar}[H, A]. \tag{3.16}$$

3.2.1. Calculation of f_{pol}. Applying (3.16) to the operator $Q_{e''g}$ and taking the average value, we get:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{ge''} = \boldsymbol{k}\cdot\boldsymbol{v}\rho_{ge'} - (\mathrm{i}\delta + \tfrac{1}{2}\Gamma)\rho_{ge''} + \frac{d\mathscr{E}(\boldsymbol{r})}{2\hbar}\rho_{e'e''} \tag{3.17}$$

where we have put, as in (3.5):

$$\rho_{ge''} = \langle \tilde{Q}_{e''g}\rangle \qquad \rho_{e'e''} = \langle Q_{e''e'}\rangle \tag{3.18}$$

and where we have used (see 2.12):

$$\frac{\partial}{\partial t}|e''\rangle = \boldsymbol{k}\cdot\boldsymbol{v}|e'\rangle. \tag{3.19}$$

We write the equivalent equation for $\rho_{e'e''}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e'e''} = \boldsymbol{k}\cdot\boldsymbol{v}(\rho_{e'e'} - \rho_{e''e''}) - \Gamma\rho_{e'e''} - \frac{d\mathscr{E}(\boldsymbol{r})}{2\hbar}\rho_{ge''}. \tag{3.20}$$

We now want to extract from equations (3.17) and (3.20), the stationary values of $\rho_{ge''}$ and $\rho_{e'e''}$, to order one in velocity. We first remark that these stationary values are at least of order one in v, since they are zero for an atom at rest. Consequently, the time derivatives of these stationary values are at least of order two in v: the time derivative of a stationary quantity A is indeed $(\boldsymbol{v}\cdot\boldsymbol{\nabla})A$. The first-order velocity dependences of the stationary values of $\rho_{ge''}$ and $\rho_{e'e''}$ can then be obtained from:

$$\begin{aligned}
&(\mathrm{i}\delta + \tfrac{1}{2}\Gamma)\rho_{ge''} - \frac{d\mathscr{E}(\boldsymbol{r})}{2\hbar}\rho_{e'e''} = -\boldsymbol{k}\cdot\boldsymbol{v}\frac{d\mathscr{E}/2\hbar}{\mathrm{i}\delta + \tfrac{1}{2}\Gamma}\frac{1}{1+s} \\
&\frac{d\mathscr{E}(\boldsymbol{r})}{2\hbar}\rho_{ge''} + \Gamma\rho_{e'e''} = \boldsymbol{k}\cdot\boldsymbol{v}\frac{s}{2(1+s)}.
\end{aligned} \tag{3.21}$$

We finally solve this system to get f_{pol}, to the first order in velocity:

$$f_{\text{pol}} = \hbar k(k \cdot v)\frac{s}{1+s}\frac{\delta/\Gamma}{(\delta^2/\Gamma^2)+\frac{1}{4}\{1+s[(\delta^2/\Gamma^2)+\frac{1}{4}]\}^2}. \tag{3.22}$$

3.2.2. Calculation of f_{int}. Applying (3.16) to $Q_{ge'}$, $Q_{e'g}$, $Q_{e'e'}$ and Q_{gg}, one gets the same result as (3.6), to the first order in v. The supplementary terms resulting from the component $\partial A/\partial t$ in (3.16) are indeed all equal to $k \cdot v$ times the average value of an operator where e'' appears†. These supplementary terms are then at least of order two in v, and do not contribute. It follows that the only v dependence of the system giving $\rho_{e'g}$, $\rho_{ge'}$, $\rho_{e'e'}$ and ρ_{gg} is due to $\mathscr{E}(r(t))$, and consequently, f_{int} at first order in velocity is the same as for the two-level atom case. The result is found to be (see also Gordon and Ashkin 1980)

$$f_{\text{int}} = -\tfrac{1}{2}\hbar\delta\frac{\nabla s}{1+s} - \frac{\hbar\delta}{4\Gamma}\frac{\nabla s(v \cdot \nabla s)}{s(1+s)^3}\left(2s^2+(s-1)\frac{\Gamma^2}{\delta^2+\frac{1}{4}\Gamma^2}\right). \tag{3.23}$$

3.2.3. Physical discussion. (i) Looking at the general structure of the results, one can note that there is no 'crossed' term as $(k \cdot v)\nabla s$ or $k(v \cdot \nabla s)$ in the expression of the total force. This is simply due to the physical invariance of the system by changing k in $-k$ (or, which is equivalent, by exchanging the polarisations for the waves σ_+ and σ_-).

(ii) We will not discuss here the result for f_{int} (3.23) since it is a well known expression, and we will rather focus on f_{pol}. We first note that f_{pol} can be a damping or an accelerating force depending on the sign of the detuning δ. If δ is negative, for example, the force is a damping one and this agrees with the following qualitative argument: an atom moving towards the right on figure 1 'sees', because of the Doppler effect, the σ_- wave closer to resonance than σ_+, and then 'experiences' a total force towards the left which damps its motion. In the case where the two waves σ_+ and σ_- are plane progressive waves, it is also interesting to note that there is no change of sign of the force $f = f_{\text{pol}}$ when the intensity of the waves varies. This has to be compared with the result obtained for a plane standing wave, where the sign of the force could change with the light intensity for a constant detuning (Minogin and Serimaa 1979, Minogin 1981a, b). In this way, the σ_+–σ_- configuration appears as a 'simpler' system than the standing-wave configuration.

(iii) In the perspective of cooling neutral atoms, it may be useful to optimise the damping qualities of f_{pol}. One gets the optimal values:

$$s \simeq 1 \qquad \delta \simeq -\tfrac{1}{2}\Gamma \tag{3.24}$$

giving a damping force:

$$f_{\text{pol}} \simeq -0.3\hbar k(k \cdot v). \tag{3.25}$$

(iv) Finally, one can ask for the validity of the expansion of the force in terms of the velocity. It appears from the calculations of this section that the following condition

† Equation (3.6*d*) should be replaced by $\rho_{e'e'}+\rho_{e''e''}+\rho_{gg}=1$, but one can easily show that $\rho_{e''e''}$ is at least of order two in v. To order one, one can then use $\rho_{e'e'}+\rho_{gg}=1$.

has to be fulfilled in order to do this expansion:

$$|\boldsymbol{k}\cdot\boldsymbol{v}|/\Gamma \ll 1. \tag{3.26}$$

This can be interpreted in two ways: as already mentioned, it first means that the Doppler effect has to be small compared with the natural width of the excited level; another formulation of (3.26) is the following

$$v\Gamma^{-1} \ll \lambda \tag{3.27}$$

which means that the atom travels, during the excited level lifetime Γ^{-1}, over a distance $v\Gamma^{-1}$ which is very small compared with the light wavelength λ, so that its internal state can follow the laser field nearly adiabatically.

3.3. Radiative forces in the case of plane waves

In the case of plane waves, it is possible to give the analytical expression of the radiative force acting on an atom for any velocity of this atom. Note that the amplitude $\mathscr{E}(\boldsymbol{r})$ is now independent of $\boldsymbol{r}$ so that only f_{pol} will contribute to the final result. In order to do the calculation in the simplest way, we will compute the force in the atomic frame, and we will use the $|e_m\rangle$ $(m = 0, \pm)$ basis for the excited level. We are then faced with the problem of computing the stationary state of a three-level atom (here $|g\rangle, |e_+\rangle, |e_-\rangle$) irradiated with two laser beams with the same intensities and two different frequencies $\omega_L + \boldsymbol{k}\cdot\boldsymbol{v}$ and $\omega_L - \boldsymbol{k}\cdot\boldsymbol{v}$ (figure 2). The radiative force is then given by:

$$\boldsymbol{f} = \frac{d\mathscr{E}}{2\sqrt{2}}\boldsymbol{k}\langle \mathrm{i}\tilde{Q}_{ge_+}\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}t) + \mathrm{HC}\rangle - \frac{d\mathscr{E}}{2\sqrt{2}}\boldsymbol{k}\langle \mathrm{i}\tilde{Q}_{ge_-}\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}t) + \mathrm{HC}\rangle. \tag{3.28}$$

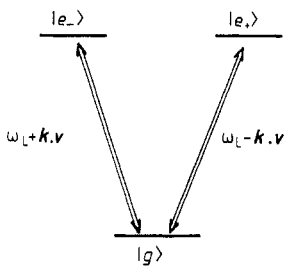

Figure 2. Atomic transitions and laser frequencies in the atom rest frame (atomic velocity v in laboratory frame).

(3.28) has been written by supposing the atom at the point $\boldsymbol{r} = 0$ of its frame. The stationary state of the atom can be deduced from the nine Bloch equations of the problem; putting

$$\begin{aligned}
&\rho_{++} = \langle Q_{e_+e_+}\rangle \\
&\rho_{+-} = \langle Q_{e_-e_+}\exp(-2\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}t)\rangle \\
&\rho_{+g} = \langle Q_{ge_+}\exp[\mathrm{i}(\omega_L - \boldsymbol{k}\cdot\boldsymbol{v})t]\rangle \\
&\text{etc}
\end{aligned} \tag{3.29}$$

one gets the following system, with time-independent coefficients:

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\rho_{++} = -\Gamma\rho_{++} + \frac{\mathrm{i}d\mathscr{E}}{2\sqrt{2}\hbar}(\rho_{+g} - \rho_{g+}) \\
&\frac{\mathrm{d}}{\mathrm{d}t}\rho_{+-} = -(\Gamma + 2\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v})\rho_{+-} + \frac{\mathrm{i}d\mathscr{E}}{2\sqrt{2}\hbar}(\rho_{+g} - \rho_{g-}) \\
&\frac{\mathrm{d}}{\mathrm{d}t}\rho_{+g} = [\mathrm{i}(\delta - \boldsymbol{k}\cdot\boldsymbol{v}) - \tfrac{1}{2}\Gamma]\rho_{+g} + \frac{\mathrm{i}d\mathscr{E}}{2\sqrt{2}\hbar}(\rho_{++} - \rho_{gg} + \rho_{+-}) \\
&\rho_{++} + \rho_{--} + \rho_{gg} = 1
\end{aligned} \tag{3.30}$$

and the five other equations deduced from (3.20) by taking complex conjugates and exchanging + and −. The analytical resolution of the linear system (3.20) in the stationary case is quite tedious but does not present any difficulty. One gets for the radiative force:

$$\boldsymbol{f} = \hbar\boldsymbol{k}\Gamma(\rho_{++} - \rho_{--}) = \hbar\boldsymbol{k}\frac{\mathscr{N}}{\mathscr{D}} \tag{3.31}$$

where we have put

$$\begin{aligned}
\mathscr{N} &= 0.5\,\Gamma\delta(\boldsymbol{k}\cdot\boldsymbol{v})[\Gamma^2 + 4(\boldsymbol{k}\cdot\boldsymbol{v})^2](d\mathscr{E}/\hbar)^2 \\
\mathscr{D} &= [\Gamma^2 + 4(\boldsymbol{k}\cdot\boldsymbol{v})^2]\{Q + \tfrac{1}{8}(d\mathscr{E}/\hbar)^2[4\Delta^2 + \tfrac{3}{8}(d\mathscr{E}/\hbar)^2 + 4(\boldsymbol{k}\cdot\boldsymbol{v})^2]\} \\
&\quad + \tfrac{1}{16}(d\mathscr{E}/\hbar)^4[\Delta^2 + \tfrac{3}{8}(d\mathscr{E}/\hbar)^2 - 3(\boldsymbol{k}\cdot\boldsymbol{v})^2] \\
\Delta^2 &= \delta^2 + \tfrac{1}{4}\Gamma^2 + \tfrac{1}{8}(d\mathscr{E}/\hbar)^2 \\
Q &= [\Delta^2 - (\boldsymbol{k}\cdot\boldsymbol{v})^2]^2 + (\boldsymbol{k}\cdot\boldsymbol{v})^2\Gamma^2.
\end{aligned} \tag{3.32}$$

The expression (3.31) appears as the ratio of two polynomials in $\boldsymbol{k}\cdot\boldsymbol{v}$: the numerator is of degree three, the denominator six. For small velocities, one can check that (3.31) is equal to the result of the previous section (3.22). One interest of formula (3.31) is to show that for a given detuning and a given light intensity, the radiative force has always the same sign as $\delta(\boldsymbol{k}\cdot\boldsymbol{v})$. Once more, this can be compared with the result for the plane standing wave case where a continuous fraction expansion† predicts changes of sign of the force with the velocity (Minogin and Serimaa 1979).

We have plotted in figure 3 the variation of the radiative force with the velocity, in the optimal case $\delta = -\frac{1}{2}\Gamma$ and $s = 1$. One sees that the force is linear until $\boldsymbol{k}\cdot\boldsymbol{v}$ becomes of the order of Γ, reaches a maximum of the order of $0.2\,\hbar k\Gamma$ and then decreases. This means that the linear term found in the previous section is dominant in the range $-\frac{1}{2}\Gamma \leqslant \boldsymbol{k}\cdot\boldsymbol{v} \leqslant \frac{1}{2}\Gamma$, as expected.

Note: In order to understand why the atom–laser system studied in this section appears simpler than the standing-wave case, (not only for the calculations but also for the results), it may be useful to consider the dressed-atom picture (Cohen-Tannoudji and Reynaud 1977, 1978, Reynaud 1983). Take first the case of an atom in a standing wave: the state $|g, n_1, n_2\rangle$, representing the atom in the presence of n_1 and n_2 photons in the two counterpropagating waves forming the standing wave, is coupled by atom–laser coupling to the two states $|e, n_1 - 1, n_2\rangle$ and $|e, n_1, n_2 - 1\rangle$. These two states are themselves coupled to $|g, n_1 - 1, n_2 + 1\rangle$ and $|g, n_1 + 1, n_2 - 1\rangle$ and so on (figure 4(a)).

† No analytical expression of the force could be obtained in the standing-wave case.

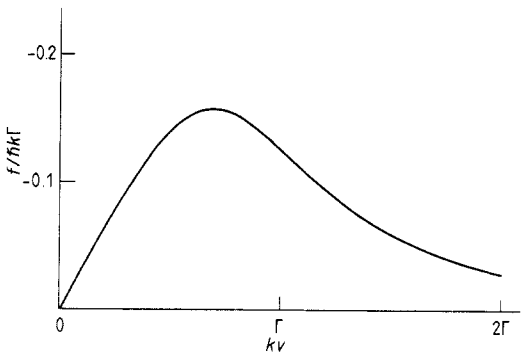

Figure 3. Variation of the intensity of the radiative force in the optimal cooling configuration ($\delta = -\frac{1}{2}\Gamma$ and $s = 1$).

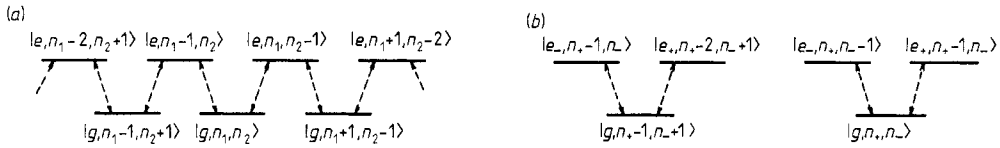

Figure 4. Dressed-atom picture. (*a*) Standing-wave case: all states corresponding to a given total number of excitations are coupled. Coherent redistribution can occur. (*b*) σ_+–σ_- case: manifolds involving only three coupled dressed states. No coherent redistribution can occur.

In particular, this coupling between all the states with a given value of $n_1 + n_2$ is responsible for the coherent redistribution described in the introduction of this paper. Consider now the case of a $(J = 0) - (J = 1)$ atom in two plane σ_+ and σ_- waves. The state $|g, n_+, n_-\rangle$ is coupled by atom–laser coupling to the two states $|e_+, n_+ - 1, n_-\rangle$ and $|e_-, n_+, n_- - 1\rangle$, and these two states are only coupled to the initial state $|g, n_+, n_-\rangle$ (figure 4(*b*)), due to the conservation of angular momentum. The 'dressed atom' then contains manifolds involving only three levels, and it is therefore much easier to compute; one can in particular calculate the stationary atomic state, when spontaneous emission is taken into account, for any values of the laser frequencies (or equivalently for any atomic velocity).

4. Atomic momentum diffusion in the σ_+–σ_- configuration

As indicated in the introduction, the momentum diffusion is a crucial feature of atomic motion in a light wave, since the corresponding heating limits the stability of any radiative trap. We will see in this section that, in the σ_+–σ_- configuration, one can control independently the depth of the trap and the diffusion coefficient, contrary to what happened in a standing-wave trap. Our calculation will be done for an atom at rest at point $\boldsymbol{r} = 0$, and we will actually be interested in the trace of the diffusion tensor (Gordon and Ashkin 1980, Cook 1980a)

$$D = \tfrac{1}{2}\langle \boldsymbol{F}\cdot\boldsymbol{P} + \boldsymbol{P}\cdot\boldsymbol{F}\rangle - \langle \boldsymbol{P}\rangle\cdot\langle \boldsymbol{F}\rangle$$

$$= \int_0^\infty d\tau \, (\tfrac{1}{2}\langle \boldsymbol{F}(0)\cdot\boldsymbol{F}(\tau) + \boldsymbol{F}(\tau)\cdot\boldsymbol{F}(0)\rangle - \langle \boldsymbol{F}\rangle^2). \tag{4.1}$$

4.1. Outline of the calculation

In order to compute D, we first replace the operators F in (4.1) by their expression (3.2):

$$\boldsymbol{F} = \boldsymbol{F}_{\text{int}} + \boldsymbol{F}_{\text{pol}} + \boldsymbol{F}_{\text{vac}}. \tag{4.2}$$

This will give *a priori* nine contributions to D. Actually, all the crossed terms vanish and one is left with only the three terms:

$$D = D_{\text{int}} + D_{\text{pol}} + D_{\text{vac}} \tag{4.3}$$

with

$$D_{\text{int}} = \int_0^\infty \mathrm{d}\tau (\tfrac{1}{2}\langle \boldsymbol{F}_{\text{int}}(0)\cdot\boldsymbol{F}_{\text{int}}(\tau) + \text{HC}\rangle - \boldsymbol{f}^2) \tag{4.4}$$

$$D_{\text{pol}} = \int_0^\infty \tfrac{1}{2}\mathrm{d}\tau \langle \boldsymbol{F}_{\text{pol}}(0)\cdot\boldsymbol{F}_{\text{pol}}(\tau) + \text{HC}\rangle \tag{4.5}$$

$$D_{\text{vac}} = \int_0^\infty \tfrac{1}{2}\,\mathrm{d}\tau \langle \boldsymbol{F}_{\text{vac}}(0)\cdot\boldsymbol{F}_{\text{vac}}(\tau) + \text{HC}\rangle. \tag{4.6}$$

There are various reasons for the cancellation of the crossed terms. Consider first the crossed terms involving $\boldsymbol{F}_{\text{vac}}$. One has then to take the average in the quantised-field ground state of the single operator $\nabla E_i(0)$ or $\nabla E_i(\tau)$. For $\nabla E_i(0)$, it is clear that the quantum average is zero since it is only a sum of creation and annihilation operators (see equation (2.6)). $\nabla E_i(\tau)$, on the other hand is equal to $\nabla E_{i\ \text{free}}(\tau)$ (see § 3.1), and one can note that $\nabla E_{i\ \text{free}}(\tau)$ commutes with any atomic operator taken at time 0, so that terms involving this single-field operator cannot contribute when averaged in the field vacuum state.

We are then left with crossed terms involving products of $\boldsymbol{F}_{\text{int}}$ and $\boldsymbol{F}_{\text{pol}}$. Physically, it is clear that such terms cannot contribute to D, since they would be proportional to $\boldsymbol{k}\cdot\nabla s$ (see § 3.2.3) and would not be invariant by exchanging the σ_+ and σ_- polarisations. Mathematically, they appear to be proportional to atomic correlation functions such as

$$\int_0^\infty \langle Q_{e'g}(0) Q_{e''g}(\tau)\rangle \,\mathrm{d}\tau \qquad \text{or} \qquad \int_0^\infty \langle Q_{e''g}(0) Q_{e'g}(\tau)\rangle \,\mathrm{d}\tau$$

and it is easy to show, using the equations of evolution of $Q_{e'g}(\tau)$ and $Q_{e''g}(\tau)$ given in § 3, and applying the quantum regression theorem that such correlation functions are always zero (see, for more details, § 4.3).

We have then to calculate three terms: D_{vac} (§ 4.2), D_{int} (§ 4.3) and D_{pol} (§ 4.4). Let us remark that the calculations of D_{vac} and D_{int} are very close to the corresponding calculations for a two-level system. We will then just briefly outline the explicit algebra for these two terms, referring for example to Gordon and Askin (1980) for more details.

4.2. Calculation of D_{vac}

We first calculate the average value of $(\boldsymbol{F}_{\text{vac}}(0)\cdot\boldsymbol{F}_{\text{vac}}(\tau) + \text{HC})$. (Note again that only $\boldsymbol{E}_{i\ \text{free}}(\tau)$ contributes to $\boldsymbol{F}_{\text{vac}}(\tau)$; § 3.1.) The quantised-field part of $\boldsymbol{F}_{\text{vac}}(0)\cdot\boldsymbol{F}_{\text{vac}}(\tau)$ consists of products of annihilation and creation operators. The only non-zero contribution will come from field operators in the antinormal order (a_λ on the left, a_λ^+ on the

right), so that D_{vac} can be written:

$$D_{\text{vac}} = \text{Re} \int_0^\infty d\tau \left\langle D_i(0) \sum_\lambda \nabla \mathscr{E}_{\lambda i}(\mathbf{0}) a_\lambda \cdot D_j(\tau) \sum_\mu \nabla \mathscr{E}^*_{\mu j}(\mathbf{0}) a^+_\mu \exp(i\omega_\mu \tau) \right\rangle$$

$$= \text{Re} \int_0^\infty d\tau \left(\sum_\lambda \nabla \mathscr{E}_{\lambda i}(\mathbf{0}) \cdot \nabla \mathscr{E}^*_{\lambda j}(\mathbf{0}) \exp(i\omega_\lambda \tau) \right) \langle D_i(0) D_j(\tau) \rangle. \tag{4.7}$$

We then decompose the dipole $D_j(\tau)$ into its positive and negative frequency parts.

$$D_j(\tau) = D_j^+(\tau) \exp(i\omega_0 \tau) + D_j^-(\tau) \exp(-i\omega_0 \tau) \tag{4.8}$$

where D_j^+ contains 'raising type' operators (Q_{eg}) and D_j^- 'lowering type' ones (Q_{ge}). We keep in the following only the resonant term, coming from $D_i(0) D_j^-(\tau)$, and we use the relation:

$$d^2 \sum_\lambda \nabla \mathscr{E}_{\lambda i}(\mathbf{0}) \cdot \nabla \mathscr{E}^*_{\lambda j}(\mathbf{0}) \exp[i(\omega_\lambda - \omega_0)\tau] = \hbar^2 k^2 \Gamma \delta_{ij} \delta(\tau). \tag{4.9}$$

We then get for D_{vac}:

$$D_{\text{vac}} = \tfrac{1}{2} \hbar^2 k^2 \Gamma \sum_m \langle Q_{e_m e_m} \rangle. \tag{4.10}$$

D_{vac} is then proportional to the population of the excited level, which is $s/[2(1+s)]$. This gives:

$$D_{\text{vac}} = \hbar^2 k^2 \Gamma \frac{s}{4(1+s)} = \tfrac{1}{2} \hbar^2 k^2 \mathscr{N} \tag{4.11}$$

where $\mathscr{N}$ is the rate of emission of fluorescence photons.

4.3. Calculation of D_{int}

Using the expression of F_{int} (3.3*a*), we get:

$$D_{\text{int}} = \tfrac{1}{4} d^2 (\nabla \mathscr{E})^2 \, \text{Re} \int_0^\infty d\tau \, (\langle A(0) A(\tau) \rangle - \langle A \rangle^2) \tag{4.12}$$

where $A(\tau)$ denotes the operator:

$$A(\tau) = i(\tilde{Q}_{ge'}(\tau) - \tilde{Q}_{e'g}(\tau)). \tag{4.13}$$

The calculation of the atomic correlation function $\langle A(0) A(\tau) \rangle$ is then performed using the quantum regression theorem (Lax 1968, Louisell 1973): if the average value of $A(\tau)$ evolves as:

$$\frac{d}{d\tau} \langle A(\tau) \rangle = \sum_i \alpha_i \langle A_i(\tau) \rangle \tag{4.14}$$

where the A_i are a set of internal atomic operators of the type Q_{ab}(ket-bra), then, for τ positive, the evolution of $A(0)A(\tau)$ is given by:

$$\frac{d}{d\tau} \langle A(0) A(\tau) \rangle = \sum_i \alpha_i \langle A(0) A_i(\tau) \rangle. \tag{4.15}$$

Writing the equivalent equations for each $\langle A(0) A_i(\tau) \rangle$, one then gets a differential system of coupled equations, from which it is possible to extract $\langle A(0) A(\tau) \rangle$.

Starting from the optical Bloch equations (3.6) giving the evolution of $\langle A(\tau)\rangle$, it first appears that one has to take into account the coupling between the four following correlation functions:

$$\langle A(0)Q_{e'e'}(\tau)\rangle \qquad \langle A(0)\tilde{Q}_{e'g}(\tau)\rangle \qquad \langle A(0)\tilde{Q}_{ge'}(\tau)\rangle \qquad \langle A(0)Q_{gg}(\tau)\rangle. \tag{4.16}$$

This set of coupled equations is the same as for the two-level case, but it is no longer closed in our problem, since the population of level $|g\rangle$ can be fed through the decay of levels $|e''\rangle$ and $|e_0\rangle$. This couples the last correlation function of (4.16) to $\langle A(0)Q_{e''e''}(\tau)\rangle$ and $\langle A(0)Q_{e_0e_0}(\tau)\rangle$. Fortunately, the evolution of these two quantities is very simple:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\tau}\langle A(0)Q_{e''e''}(\tau)\rangle &= -\Gamma\langle A(0)Q_{e''e''}(\tau)\rangle \\ \frac{\mathrm{d}}{\mathrm{d}\tau}\langle A(0)Q_{e_0e_0}(\tau)\rangle &= -\Gamma\langle A(0)Q_{e_0e_0}(\tau)\rangle\end{aligned} \tag{4.17}$$

and their initial values (for $\tau=0$) are both zero. It follows that these two quantities are zero for any time τ, so that the diffusion coefficient calculated through the resolution of the set (4.16), has the same mathematical expression as for the two-level case (Gordon and Ashkin 1980, Cook 1980a, b):

$$D_{\mathrm{int}}=\frac{\hbar^2(\nabla s)^2\Gamma}{4s(1+s)^3}\left[s^3\left(\frac{\delta^2}{\Gamma^2}+\frac{1}{4}\right)+\frac{3s^2}{4}+\frac{\frac{3}{4}\Gamma^2-\delta^2}{\Gamma^2+4\delta^2}s+\frac{1}{4}\right]. \tag{4.18}$$

It is, however, important to note that the order of magnitude of D_{int} is not at all the same for a $(J=0-J=1)$ atom in the σ_+-σ_- configuration and for a two-level system in a standing wave corresponding to the same value for the potential well depth U_0 imposed by a choice of δ and s (see (3.13)). In the standing-wave case $(\nabla s)^2$ is indeed of the order of $(ks)^2=4\pi^2s^2/\lambda^2$, while, in the σ_+-σ_- configuration, we have $(\nabla s)^2\simeq(s/w_0)^2$ where w_0 is the laser beam waist. The diffusion coefficient D_{int} in the σ_+-σ_- configuration is then $4\pi^2(w_0/\lambda)^2$ smaller than the one found in a standing wave. The corresponding reduction can be considerable for usual waists, which are much larger than the wavelength.

The physical interpretation for such a reduction has been given in the introduction of this paper: the use of the σ_+-σ_- configuration prevents the coherent redistribution of photons between the two beams, responsible for the huge diffusion coefficient found in the standing-wave case. Coherent redistribution of photons, in the σ_+-σ_- configuration, can then only occur inside a given beam, between the various plane waves forming this beam and which are at the origin of the gradient of s. In particular, if there is only one wavevector forming the σ_+ or σ_- beam (plane σ_+ and σ_- waves, $\nabla s=0$), no coherent redistribution can occur and D_{int} vanishes.

4.4. *Calculation of D_{pol}*

We first report the expression (3.3*b*) *of* $\boldsymbol{F}_{\mathrm{pol}}$ in the expression of D_{pol} and we get:

$$D_{\mathrm{pol}}=\frac{d^2k^2\mathscr{E}^2}{4}\,\mathrm{Re}\int_0^\infty \mathrm{d}\tau\,\langle B(0)B(\tau)\rangle \tag{4.19}$$

where we have put:

$$B(\tau) = \mathrm{i}(\tilde{Q}_{ge''}(\tau) - \tilde{Q}_{e''g}(\tau)). \tag{4.20}$$

As in § 4.3, we have to evaluate an atomic correlation function, using the quantum regression theorem. It appears that there are four correlations functions involved, which separate into two groups:

$$\langle B(0) Q_{ge''}(\tau)\rangle$$

$$\langle B(0) Q_{e'e''}(\tau)\rangle \qquad \text{for the first group}$$

and

$$\langle B(0) Q_{e''g}(\tau)\rangle$$

$$\langle B(0) Q_{e''e'}(\tau)\rangle \qquad \text{for the second group.}$$

The first group is easy to evaluate: the initial values ($\tau = 0$) for both functions are equal to zero so that these functions vanish for all τ. To evaluate the second set, let us put

$$\begin{aligned} \alpha(\tau) &= \langle B(0) Q_{e''g}(\tau)\rangle \\ \beta(\tau) &= \langle B(0) Q_{e''e'}(\tau)\rangle. \end{aligned} \tag{4.21}$$

The initial conditions are the following

$$\begin{aligned} \alpha(0) &= \mathrm{i}\langle Q_{gg}\rangle = \mathrm{i}\frac{2+s}{2(1+s)} \\ \beta(0) &= \mathrm{i}\langle Q_{ge'}\rangle = \frac{\mathrm{i}}{1+s}\,\frac{d\mathscr{E}/2\hbar}{\mathrm{i}\delta - \frac{1}{2}\Gamma}. \end{aligned} \tag{4.22}$$

The equations of motion for α and β are similar to (3.17) and (3.20) for $\boldsymbol{k}\cdot\boldsymbol{v} = 0$

$$\begin{aligned} \frac{\mathrm{d}\alpha}{\mathrm{d}\tau} &= -(\mathrm{i}\delta + \tfrac{1}{2}\Gamma)\alpha + \frac{d\mathscr{E}}{2\hbar}\beta \\ \frac{\mathrm{d}\beta}{\mathrm{d}\tau} &= -\frac{d\mathscr{E}}{2\hbar}\alpha - \Gamma\beta. \end{aligned} \tag{4.23}$$

We integrate these two equations from $\tau = 0$ to $\tau = \infty$ and find:

$$\int_0^\infty \alpha(\tau)\,\mathrm{d}\tau = \frac{\Gamma\alpha(0) + \beta(0) d\mathscr{E}/2\hbar}{(\mathrm{i}\delta + \frac{1}{2}\Gamma)\Gamma + (d\mathscr{E}/2\hbar)^2}. \tag{4.24}$$

We report the result in D_{pol} and finally get:

$$D_{\mathrm{pol}} = \hbar^2 k^2 \Gamma \frac{s}{4(1+s)}. \tag{4.25}$$

D_{pol}, which is a new term compared with the two-level atom case, is then found to be equal to D_{vac}; in particular, it saturates, when s increases, to the value $0.25\,\hbar^2 k^2 \Gamma$.

4.5. Corpuscular interpretation of the diffusion coefficients in the case of plane waves

In the case of plane σ_+ and σ_- waves, one can give a simple corpuscular interpretation of D_{vac} and D_{pol} (D_{int} vanishes). The atomic motion can in this case be described as

a pure random walk in $\boldsymbol{p}$ space. The variation $\Delta\boldsymbol{p}$ of the atomic momentum during a time ΔT can be written

$$\Delta\boldsymbol{p} = -\sum_{i=1}^{N_+ + N_-} \hbar\boldsymbol{k}_i + (N_+ - N_-)\hbar\boldsymbol{k} \tag{4.26}$$

where N_+ (N_- respectively) is the number of photons absorbed in the σ_+ (σ_- respectively) wave, and $\hbar\boldsymbol{k}_i$ the momentum of the ith fluorescence photon. The mean value of $\Delta\boldsymbol{p}$ is zero, since $\bar{N}_+ = \bar{N}_-$ and $\overline{\boldsymbol{k}_i} = 0$. The variance Δp^2 is:

$$\overline{\Delta p^2} = \hbar^2 k^2 \overline{(N_+ + N_-)} + \hbar^2 k^2 \overline{(N_+ - N_-)^2} \tag{4.27}$$

where we have used the fact that the momentum $\hbar\boldsymbol{k}_i$ of a given fluorescence photon is not correlated to the momentum of the other fluorescence photons, nor to the numbers N_+ and N_-. Expression (4.27) leads to the momentum diffusion coefficient D:

$$D = \frac{\overline{\Delta p^2}}{2\Delta T} = \tfrac{1}{2}\hbar^2 k^2 \frac{\overline{N_+ + N_-}}{\Delta T} + \tfrac{1}{2}\hbar^2 k^2 \frac{\overline{(N_+ - N_-)^2}}{\Delta T}. \tag{4.28}$$

The first contribution to D can be identified with D_{vac}: it describes the fluctuations of the momentum carried away by the fluorescence photons ($\overline{N_+ + N_-}/T$ is identical to the rate of emission $\mathcal{N}$ appearing in (4.11)).

The second term of (4.28), corresponding to D_{pol}, describes the fluctuations of the difference $N_+ - N_-$ between the number of photons absorbed in each wave. Remark now that there is no correlation between the polarisations (σ_+ or σ_-) associated with two successive absorption–emission cycles: after the emission of a σ_+ photon for example, the atom has the same probability of absorbing either a σ_+ photon or a σ_- one. It follows that the probability $P(N_+, N_-)$ for finding a realisation (N_+, N_-) is simply related to the probability $P(N_+ + N_-)$ for finding a total number $N_+ + N_-$ of emitted photons:

$$P(N_+, N_-) = \frac{(N_+ + N_-)!}{2^{N_+ + N_-} N_+! N_-!} P(N_+ + N_-). \tag{4.29}$$

One then deduces from (4.29):

$$\overline{(N_+ - N_-)^2} = \overline{N_+ + N_-}. \tag{4.30}$$

Consequently, D_{pol} is equal to D_{vac} (see (4.11) and (4.25)).

Note: The result (4.30) is the same as if N_+ and N_- were independent Poisson variables. Actually, this is not the case since:

$$\overline{\Delta(N_+ + N_-)^2} = (1 + Q)\overline{(N_+ + N_-)}. \tag{4.31}$$

The factor Q, which expresses the non-Poisson character of the distribution of the total number of fluorescence photons, has the same value as for the two-level atom (Mandel 1979, Smirnov and Troshin 1979, 1981, Cook 1981b, Reynaud 1983, Stenholm 1983).

5. Implications for optical trapping

We have presented in this paper a new laser configuration, which appears very well adapted for the trapping of neutral atoms. We have calculated the optical depth for

such a configuration (§ 3.1) and optimised it, the best parameters being $s_m = 4$, and a laser intensity as large as possible. We have also derived the expression of the total diffusion coefficient which appears to be, for these optimal values:

$$D \simeq 0.4\,\hbar^2 k^2 \Gamma + D_{\text{int}} \tag{5.1}$$

where D_{int}, using the laser waist w_0, can be overestimated after some algebra by:

$$D_{\text{int}} \lesssim \hbar^2 \frac{1}{w_0^2} \frac{\delta^2}{\Gamma}. \tag{5.2}$$

The first term of (5.1) appears as 'incompressible' since it only depends on k and Γ. On the contrary, D_{int} can be reduced as much as wanted, since we can control the intensity gradients in the trap. The optimal situation corresponds to the case where D_{int} is of the order of the first term of (5.1). Let us give a numerical example for ytterbium atoms, excited on the intercombination line ^1S–^3P at 556 nm. The spontaneous emission rate for this transition is $\Gamma/2\pi = 0.2$ MHz. Assuming a laser power of 1 W, focused on 250 μm, one then realises the condition $s_m = 4$ by choosing a detuning $\delta = 930\,\Gamma$, which leads to an optical well depth $U_0 = 750\,\hbar\Gamma$ (7 mK). The diffusion coefficient D_{int} is then $0.1\,\hbar^2 k^2 \Gamma$ so that the total coefficient is $D = 0.5\,\hbar^2 k^2 \Gamma$. (For a standing-wave case, the diffusion coefficient would have been more than 10^6 times larger.) Of course this single σ_+-σ_- configuration is not sufficient for achieving a stable trapping of neutral atoms, since the cooling properties of such beams optimised for trapping are very poor and will not compensate the heating due to diffusion (compare the present numerical values with the optimal ones for cooling (3.24)). However, we can finally mention that stable trapping can be achieved by alternating in time such trapping configurations with optimised cooling ones (Dalibard *et al* 1983a, b).

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