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LETTER TO THE EDITOR

An experimental check of higher order terms in the radiative shift of a coherence resonance

C Cohen-Tannoudji, J Dupont-Roc and C Fabre

Laboratoire de Spectroscopie Hertzienne de l'Ecole Normale Supérieure, associé au CNRS, 24 rue Lhomond, 75231 Paris Cedex 05

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Abstract. We present an experimental check of the method used in the preceding note for calculating higher order radiative shifts of magnetic resonances. Although indirect, this check is conclusive as regards the validity of the method.

The method described in the preceding note (later referred to as note I) for calculating the higher order terms of the Bloch–Siegert shift applies to all types of magnetic resonances. Among these, 'coherence' resonances are the most interesting for a precise check of the higher order calculations, as they are shifted without being broadened appreciably when the RF power is increased over a large range.

Coherence resonances appear in transverse optical pumping experiments and for various polarizations of the RF field (σ^+: Dodd *et al* 1963; π: Favre and Geneux 1964; σ: Cohen-Tannoudji and Haroche 1965). In the 'dressed atom' formalism (Cohen-Tannoudji 1968, Haroche 1971), they are interpreted as 'level crossing' resonances of the dressed atom. For example, the energy diagram represented in note I (figure 1) and corresponding to a σ polarization of the RF field (linear and perpendicular to the static field), exhibits a level crossing (point C) for $\omega_0 \approx 2\omega$; the abscissa of C gives the centre of the corresponding coherence resonance. We present in this note a calculation of the abscissa of C and an experimental test of these results.

The perturbed levels originating from the levels $|a\rangle = |+, n\rangle$ and $|c\rangle = |-, n+2\rangle$ (energies $E_a = \frac{1}{2}\omega_0 + n\omega$ and $E_c = -\frac{1}{2}\omega_0 + (n+2)\omega$) cross at point C. Since $|a\rangle$ and $|c\rangle$ are not coupled by H_1 to any order, the energy of the two levels is exactly given by two distinct implicit equations:

for $|a\rangle$:

$$E - E_a - R_{aa}'(E, \omega_0) = 0 \tag{1}$$

where

$$R' = P_a H_1 P_a + \sum_{n=1}^{\infty} P_a H_1 \left(\frac{Q_a}{E - H_0} H_1\right)^n P_a$$

with

$$P_a = |a\rangle\langle a| \text{ and } P_a + Q_a = 1$$

for $|c\rangle$:

$$E - E_c - R_{cc}''(E, \omega_0) = 0 \tag{2}$$

with similar notations.

Symmetry arguments show that the ordinate of the crossing point is independent of the coupling intensity, and is therefore equal to $(n+1)\omega$. Hence, the only unknown parameter is the abscissa ω_0 of the crossing, which is given by equation (1), with the value $(n+1)\omega$ for E. One gets:

$$\omega_0 = 2\omega - 2R_{aa}' \qquad (E = (n+1)\omega, \omega_0). \tag{3}$$

When the coupling parameter ω_1 (see equation (3) of note I) is small compared to ω, we can obtain an approximation of the shift by solving equation (3) by iteration. The solution up to sixth order is found to be:

$$\omega_0 = 2\omega - \frac{1}{6}\frac{\omega_1^2}{\omega} - \frac{7}{54\omega^3}(\tfrac{1}{2}\omega_1)^4 - \frac{103}{2430\omega^5}(\tfrac{1}{2}\omega_1)^6. \tag{4}$$

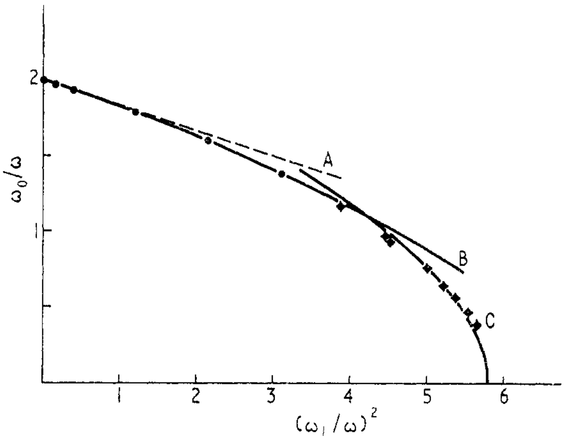

Figure 1. Position of the coherence resonance as a function of the RF intensity (measured by $(\omega_1/\omega)^2$). The experimental points have been obtained on ^{87}Rb. Curve A represents the lowest order approximation for the shift; curve B includes the next corrections up to sixth order; curve C is obtained from an approximation valid for $\omega_1/\omega \sim 2{\cdot}4$.

The corresponding curve is noted B on figure 1 of this paper (the straight line noted A is given by the lowest order calculation of the shift: $\omega_0 = 2\omega - \omega_1^2/6\omega$). When ω_1 increases, the centre of the resonance gets nearer and nearer to $\omega_0 = 0$. It can be shown (Haroche 1971) that the resonance disappears at $\omega_0 = 0$, when the g-factor of the dressed atoms cancels: (the energy levels starting from the point: $\omega_0 = 0$, $E = (n+1)\omega$ of figure 1 of note I become tangent at this point). This situation occurs when ω_1/ω reaches the value corresponding to the first zero of the zeroth order Bessel function J_0, ie when $\omega_1/\omega = 2{\cdot}405$. (The single photon resonance also vanishes for that value of ω_1/ω). In the neighbourhood of this value, we can treat the Zeeman term $\omega_0 J_z$ as a perturbation with respect to the remaining part of the hamiltonian:

$$\omega a^+ a + \tfrac{1}{2}\lambda(a+a^+)(J_+ + J_-).$$

The shape of the energy levels is then given by: (Landré 1970a).

$$E_{\pm} = (n+1)\omega \pm \frac{1}{2}\omega_0 J_0\left(\frac{\omega_1}{\omega}\right) \mp \frac{\omega_0^3}{2\omega^2} S\left(\frac{\omega_1}{\omega}\right) + \dots \tag{5}$$

where

$$S = \frac{1}{4} \sum_{p \text{ and } q \neq n} \left(-\frac{J_{n-p} J_{q-p} J_{q-n}}{(n-p)(n-q)} + \frac{J_0 J_p^2}{p^2} \right). \tag{6}$$

The position of the crossing is then given by:

$$\omega_0 = \omega \left(\frac{J_0(\omega_1/\omega)}{S(\omega_1/\omega)} \right)^{1/2} \tag{7}$$

which gives, at the lowest order in $\omega_1/\omega - 2405$:

$$\omega_0 = 1{\cdot}85\omega \left(-\frac{\omega_1}{\omega} + 2{\cdot}405 \right)^{1/2}. \tag{8}$$

The corresponding curve is noted C on the figure 1 of this note.

Hence, in addition to the sixth order expansion of the shift we have obtained an approximate expression for the position of the resonance when the shift reaches its maximum value.

Using the experimental method described by Cohen-Tannoudji and Haroche (1965), we have remeasured with great precision the position of the coherence resonance described above (we used ^{87}Rb atoms instead of ^{199}Hg; the RF field frequency $\omega/2\pi$ was equal to 399 Hz). The points of figure 1 represent the experimental results.

The deviation from the lowest order approximation (curve A) clearly appears. This deviation is correctly represented by the sixth order expansion (curve B) up to $\omega_1/\omega \approx 2$. For larger values of ω_1/ω, the higher order terms become important and the experimental points progressively meet the theoretical curve C. Because of the broadening of the resonance, the precision decreases in this region. Note however that it is possible to follow the resonance practically over all its domain of existence. For the ordinary one-quantum resonance (which has the same domain of existence), the Q-factor of the resonance decreases much more rapidly and becomes too small to allow a measurement of the position for ω_1/ω greater than 1.

We have shown how to compute quantum mechanically, to a given order, the shift of various types of magnetic resonances. In this formalism, the calculations appear to be quite simple, as they reduce to solving by iteration one or two implicit equations given by a time-independant perturbation theory.

Our quantum treatment of the interaction between a spin and a RF field gives results in complete agreement with the experimental data, even at high orders. Incidentally, it may be reminded that the eigen-frequencies of the system: 'atom + RF field' have already been checked with a great precision (Landré 1970b); these experiments represent another accurate test of the theory.

We must again insist on the fact that the results of a quantum theory will always be in agreement with those of a semi-classical theory in the RF domain.

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