

Direct Measurement of The Spatial Correlation Function of Ultracold Atoms

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We demonstrate a new method to directly measure, from the overlap of two atomic wave packets, the spatial correlation function of metastable helium atoms cooled by velocity selective coherent population trapping. This approach allows us to determine precisely the effective temperature of the atoms and to make quantitative comparison with theory. Temperatures as low as $1/800$ of the recoil temperature are found. Moreover, we extend this method to realize a temporal interferometer with de Broglie matter waves in the subrecoil regime. [S0031-9007(97)04377-9]

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Laser manipulation of atoms, sometimes combined with evaporative cooling, provides ultracold atomic samples and opens the way to many applications ranging from ultrahigh resolution spectroscopy to atomic interferometry and quantum degenerate gases [1]. Among the purely optical cooling methods, velocity selective coherent population trapping (VSCPT) [2] and Raman cooling [3] allow the atomic momentum spread δp to be reduced below the value $\hbar k$ of the photon momentum (subrecoil cooling). The implementation of methods to measure such a small momentum width or the associated large coherence length of the atomic wave packets is therefore a main issue. Two methods have been used to measure δp in subrecoil cooling experiments: time-of-flight techniques (TOF) [4] for VSCPT and velocity selective Raman transitions [5] for Raman cooling. We present, in this Letter, a new approach which is based on a direct measurement of the atomic spatial correlation function and fits in the current effort to investigate first and higher order correlation functions in ultracold atomic samples [6]. We apply our method to metastable helium atoms cooled by one-dimensional VSCPT. We deduce from our measurements effective temperatures as low as $T_R/800$, where T_R is the recoil limit [7]. This is well below the lowest temperatures we can measure by TOF. The high resolution of this method allows quantitative tests to be made of theoretical predictions based on Lévy statistics. In addition, we can recombine the atomic wave packets after they have flown apart, achieving in this way a Mach-Zender type interferometer with subrecoil cooled atoms (for a review on atomic interferometry see [8,9]; see also [10]).

The idea followed here is that the dispersion of a variable, which is too narrow to be precisely measured, can be inferred from the correlation function of the conjugate variable. In classical optics, for example, the width $\delta\omega$ of a very narrow spectral line can be more easily obtained from the time correlation function $G(\tau) = \int dt E^*(t + \tau)E(t)$ of the light field which is the Fourier transform of the spectral intensity distribution $I(\omega)$. Here we measure a signal proportional to the overlap integral $G(a) = \int dx \varphi^*(x + a)\varphi(x)$ between two identical atomic wave packets separated by a distance

a . This integral is the atomic spatial correlation function and can also be written as $\int dp |\phi(p)|^2 e^{iap/\hbar}$ where $\phi(p)$ is the Fourier transform of $\varphi(x)$.

The principle of subrecoil cooling is to make the atoms perform a random walk in momentum space with a jump rate which vanishes for $p = 0$, so that atoms falling in a small zone around $p = 0$ remain trapped there for a long time and accumulate. Although VSCPT has also been demonstrated in two and three dimensions [2], we will restrict ourselves here to a simple 1D configuration [11]. A $J_g = 1 \rightarrow J_e = 1$ transition is driven by two counterpropagating laser waves with the same complex amplitude and orthogonal $\sigma_{\pm}$ polarizations [Fig. 1(a)]. The only nonzero matrix elements of the atom-laser interaction Hamiltonian V_{AL} are $\langle e_0, p | V_{AL} | g_{\pm}, p \pm \hbar k \rangle$ where $|i, q\rangle$ is the state of an atom in internal state $|i\rangle$ with momentum q along the z axis. Since the two transitions starting from $|g_{\pm}, p \pm \hbar k\rangle$ have the same

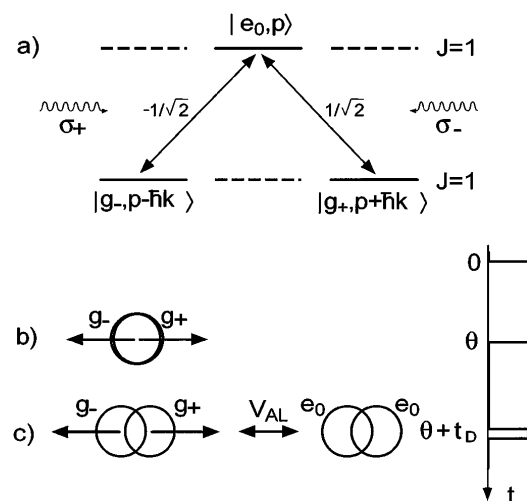


FIG. 1. (a) Three level Λ configuration on the 2^3S_1 - 2^3P_1 transition of ^4He . The Clebsch-Gordan coefficients are equal to $\pm 1/\sqrt{2}$. (b) Sketch of the atomic wave packets at the beginning of the dark period. (c) Moving ground state wave packets coupled to excited state wave packets at rest by the probe pulse (V_{AL}) at the end of the dark period. The temporal sequence is shown on the right.

final state $|e_0, p\rangle$, the two absorption amplitudes can interfere destructively (or constructively) resulting in a decoupling (or enhanced coupling) between the atoms and the laser field. Taking into account the opposite values of the Clebsch-Gordan coefficients, the corresponding noncoupled (or coupled) states are found to be

$$|\Psi_{\text{NC}}^{\pm}(p)\rangle = \frac{1}{\sqrt{2}}(|g_{+}, p + \hbar k\rangle \pm |g_{-}, p - \hbar k\rangle). \quad (1)$$

The only eigenstate of the total Hamiltonian, including the kinetic energy operator $\mathbf{P}^2/2M$, is $|\Psi_{\text{NC}}(0)\rangle$ which is therefore completely stable (perfect trapping state). If $p \neq 0$, off-diagonal elements of $\mathbf{P}^2/2M$, proportional to p , contaminate $|\Psi_{\text{NC}}(p)\rangle$ by $|\Psi_{\text{C}}(p)\rangle$. The departure rate from the noncoupled states therefore varies as p^2 , rendering the random walk velocity selective. After an interaction time θ , the atoms end up in a statistical mixture of states $|\Psi\rangle$, each state $|\Psi\rangle$ being of the form

$$|\Psi\rangle = \int dp \phi(p) |\Psi_{\text{NC}}(p)\rangle, \quad (2)$$

with p distributed about $p = 0$, over a narrow range δp which decreases when θ increases. This state corresponds to a superposition of two wave packets with orthogonal internal states $|g_{\pm}\rangle$ and a momentum spread δp around the mean values $\pm \hbar k$. This is at the origin of the two narrow peaks characteristic of the momentum distribution created by 1D-VSCPT cooling.

To measure the spatial correlation function of the atomic wave packets, we use the time sequence shown in the right part of Fig. 1. After the cooling stage of duration θ , we switch off the VSCPT beams for a “dark period” of duration t_D during which the two wave packets fly apart freely. We then switch on a short probe pulse of VSCPT light to measure their overlap as explained now. At the beginning of the dark period ($t = \theta$), the atomic state is given by Eq. (2) which, in position representation, is a linear superposition of two wave packets $\varphi(x)e^{\pm ikx}|g_{\pm}\rangle$ [Fig. 1(b)]. Just before the probe pulse, at $t = \theta + t_D$, the atomic state $|\Psi(t_D)\rangle$ is described by a superposition of two wave packets $\varphi(x \mp a/2)e^{\pm ikx}|g_{\pm}\rangle$ whose centers are separated by a distance $a = 2\hbar kt_D/M$ [Fig. 1(c)]. As soon as the light is turned on again, V_{AL} couples $|\Psi(t_D)\rangle$ to an excited state corresponding to a linear superposition of two wave packets $\varphi(x \mp a/2)|e_0\rangle$ [Fig. 1(c)]. The two $\sigma_{\pm}$ absorption amplitudes lead therefore to two different final states, having the same internal state $|e_0\rangle$ but two different external states described by two wave packets at rest separated by the distance a . The destructive interference between the two absorption amplitudes can be complete only if the two final states are identical. This only occurs for $a = 0$, i.e., for $t_D = 0$. As soon as $a \neq 0$, the destructive interference can only take place partially so that atoms can absorb light. More precisely, the inhibition of the absorption process, maximum at $t_D = 0$, decreases in proportion to the scalar product of the two

final states, i.e., the overlap integral $G(a) = \int dx \varphi^*(x + a/2)\varphi(x - a/2)$ between two identical atomic wave packets [12].

The previous discussion can be made more quantitative by using the $\{|\Psi_{\text{C}}(p)\rangle, |\Psi_{\text{NC}}(p)\rangle\}$ basis. After the free evolution of duration t_D , $|\Psi\rangle$ is transformed into

$$|\Psi(t_D)\rangle = \int dp \phi(p) e^{-i\omega t_D} \left[\cos \frac{kp t_D}{M} |\Psi_{\text{NC}}(p)\rangle - i \sin \frac{kp t_D}{M} |\Psi_{\text{C}}(p)\rangle \right], \quad (3)$$

with $\omega = (p^2 + \hbar^2 k^2)/2M\hbar$. When the light is turned on again, only the projection of $|\Psi(t_D)\rangle$ onto the subspace of states $|\Psi_{\text{C}}(p)\rangle$ absorbs light, a principle also used in [10]. The corresponding atoms have their momenta dispersed over a wide momentum range, and the resulting background is easily distinguishable from the sharp peaks associated with the atoms remaining in the states $|\Psi_{\text{NC}}(p)\rangle$. This allows us to measure the probability $\Pi_{\text{NC}}(t_D)$ for an atom to be found in a noncoupled state at the time t_D . From Eq. (3) a simple calculation gives

$$\Pi_{\text{NC}}(t_D) = \frac{1}{2} + \frac{1}{2} \int dp |\phi(p)|^2 \cos \frac{2kp t_D}{M}. \quad (4)$$

As expected, $\Pi_{\text{NC}}(t_D)$ is related to the Fourier transform of the momentum distribution $\mathcal{P}(p) = |\phi(p)|^2$. Theoretical treatments of VSCPT [13] predict $\mathcal{P}(p)$ to have tails varying as $1/p^2$. One can therefore approximate $\mathcal{P}(p)$ by a Lorentzian of HWHM $\tilde{\delta} p$ [14] which yields

$$\Pi_{\text{NC}}(t_D) = \frac{1}{2} + \frac{1}{2} e^{-t_D/\tau_c}, \quad (5)$$

where $\tau_c = M/2k\tilde{\delta} p$ is the coherence time. The coherence length is thus given by $\xi = 2(\hbar k/M)\tau_c = \hbar/\tilde{\delta} p$. Notice that Π_{NC} does not vanish but tends to 1/2 when $t_D \gg \tau_c$, or equivalently when $a \gg \xi$. This nonzero asymptotic limit can be easily understood by inverting Eq. (1) which shows that each component $|g_{\pm}, p \pm \hbar k\rangle$ of the two wave packets has a probability 1/2 to be projected onto a noncoupled state by the probe pulse.

Our experimental apparatus has been described previously [11]. We release about 10^5 metastable helium atoms from a magneto-optical trap where they are pre-cooled to $\sim 100 \mu\text{K}$ by means of laser beams tuned to the 2^3S_1 - 2^3P_2 transition. We then perform the VSCPT cooling by turning on two horizontally counterpropagating laser beams tuned on the 2^3S_1 - 2^3P_1 transition, with the same Rabi frequency Ω , orthogonal polarizations $\sigma_{\pm}$, and detuning $\delta \simeq 0$. After a given interaction time θ , we shut off the VSCPT laser beams for a dark period of duration t_D and subsequently apply a very short pulse of VSCPT light as discussed above. After a free flight the atoms hit a detector located 6.8 cm below, which records

the time and position of arrival of each atom. As we do in the usual TOF method, we infer from these coordinates the characteristic double-peak momentum distribution of the atoms. However, we no longer deduce the temperature from the width of these peaks but rather deduce Π_{NC} from their area after subtracting the background.

In all experiments, the duration of the probe pulse is $\sim 10 \mu\text{s}$, short enough to avoid any detectable further cooling and sufficiently long to expel all the atoms in the states $|\Psi_{\text{C}}(p)\rangle$ out of the two sharp peaks. Special care is taken to control residual magnetic fields. Any magnetic field perpendicular to the laser direction induces Larmor oscillation between the states $|\Psi_{\text{C}}(p)\rangle$ and $|\Psi_{\text{NC}}(p)\rangle$ in the absence of light. To avoid the resulting oscillation of Π_{NC} , we compensate transverse magnetic fields within 0.5 mG, at the position of the atomic cloud, by means of the mechanical Hanle effect [15]. Moreover, we apply a comparatively strong (~ 8 mG) magnetic field along the laser axis [16] to keep the spins aligned along their initial orientation during the dark period.

A typical measurement of Π_{NC} as a function of t_D is shown in Fig. 2. As expected from Eq. (5), Π_{NC} decreases from 1.0 to 0.5. Data are in good agreement with an exponential fit (solid line) which directly confirms that the momentum distribution of atoms cooled by VSCPT is close to a Lorentzian. The time constant of this exponential is found to be $\tau_c = 19.0(8) \mu\text{s}$ which corresponds to $\delta p = \hbar k/25(1)$ and to an effective temperature $T = T_R/625(50)$ [7]. The temperature directly inferred from the width of the peaks of the momentum distribution in the reference VSCPT experiment is of the order of $T_R/70$ in this case. This indicates that the usual TOF method is strongly limited by instrumental effects. From Monte Carlo simulations, we found that the major limitation is due to the initial size of the atomic cloud (radius ~ 1 mm).

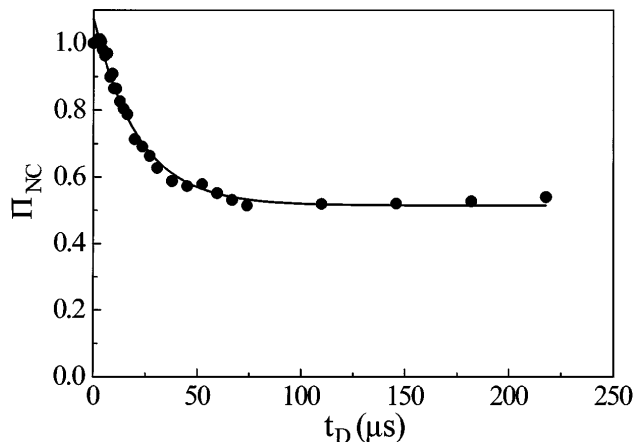


FIG. 2. Fraction of atoms left in noncoupled states after a dark period of duration t_D for $\Omega \approx 0.9\Gamma$ (Γ is the natural width of the excited state) and $\delta \approx 0$. The exponential fit (solid line) gives $\tau_c \approx 19 \mu\text{s}$, which corresponds to $\delta p \approx \hbar k/25$, $T \approx T_R/625$, and $\xi \approx 3.5 \mu\text{m}$.

The possibility to determine the temperature with a very high resolution opens the way to a quantitative comparison with theory based on Levy statistics and quantum Monte Carlo simulations [13], which predicts that $T_R/T = A\theta\Omega^{-2}(M^2/4\hbar^2k^4\Gamma)$, where A is a numerical prefactor of the order of unity. Note that T_R/T does not depend on δ [17]. The experimental dependence of the effective temperature versus θ and Ω is shown in Fig. 3. The linear fit in Fig. 3(a) gives a prefactor $A \sim 3$ in good agreement with theory, in view of the uncertainty in the absolute value of Ω which can only be determined within a factor of 2. The variation of T_R/T versus Ω is shown in Fig. 3(b) and follows a power law of exponent $-1.9(2)$, in very good agreement with the expected value -2 .

The above described measurement of the spatial correlation function is direct evidence of the coherence of the two wave packets involved in a VSCPT state. A perhaps even more convincing demonstration is provided by the following experiment. Starting as before with a dark period of fixed duration $t_D \gg \tau_c$ we now add a second dark period of variable duration t'_D ended by a second probe pulse identical to the first one. If, after this last pulse, we measure the total fraction Π'_{NC} of atoms projected onto noncoupled states in the same way as we did before, we observe an echo signal centered around $t'_D = t_D$ [Fig. 4]. In fact, we have realized a temporal interferometer (see also [18]) that can be described as follows (inset of Fig. 4). After the first dark period, the two wave packets that we now simply denote φ_+ and φ_- are separated by a distance $a \gg \xi$. The first pulse therefore projects half of the atoms in coupled states, leaving the remaining half in superpositions of noncoupled states $|\Psi_{\text{NC}}(p)\rangle$. Hence, during the second dark period, φ_+ splits into φ_{++} and φ_{+-} , while φ_- splits into φ_{-+} and φ_{--} . The overlap between these wave packets is probed by the second pulse. When $|t'_D - t_D|, t'_D \gg \tau_c$, this pulse projects out half of each wave packet which results in $\Pi'_{\text{NC}} = 1/4$. However, when $t'_D \approx t_D$, the two wave packets φ_{-+}

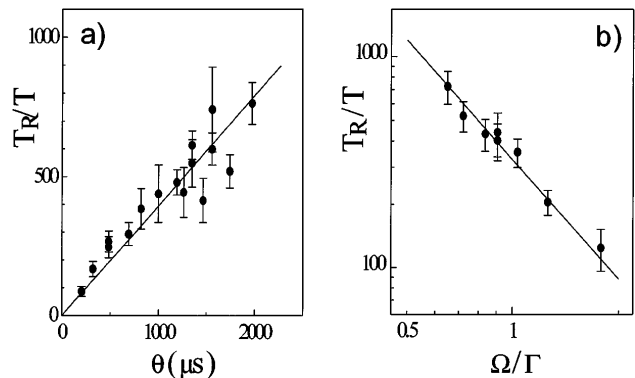


FIG. 3. (a) Dependence of T_R/T on the VSCPT interaction time θ for $\Omega \approx 0.9\Gamma$ and $\delta \approx 0$. The fit (solid line) confirms the linear dependence on θ . (b) Dependence of T_R/T on the Rabi frequency Ω for $\delta \approx 0$ and $\theta \approx 1000 \mu\text{s}$ in log-log scale. The linear fit (solid line) gives a slope of $-1.9(2)$.

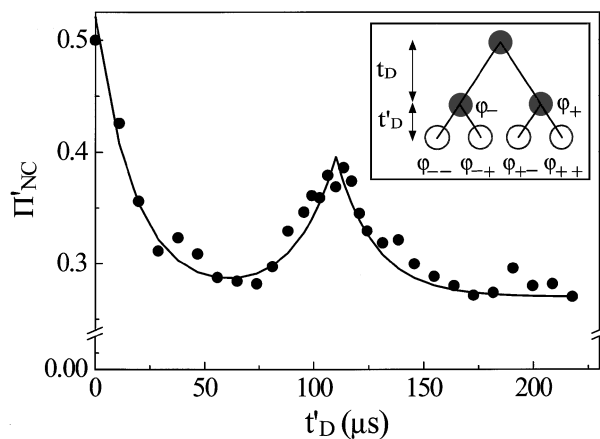


FIG. 4. Experiment with two dark periods for $t_D \approx 110 \mu\text{s}$. The temporal evolution of the wave packets is depicted in the inset. Π'_{NC} is the fraction of atoms left in noncoupled states after the second dark period of duration t'_D and the second pulse. The solid line represents an analytical expression, generalizing Eq. (5) for two dark periods.

and φ_{+-} completely overlap and the destructive interference partially reappears. A simple calculation leads to $\Pi'_{NC}(t'_D = t_D) = 3/8$, which is very close to the observed echo signal (≈ 0.38).

In conclusion, we have used a method analogous to Fourier spectroscopy in optics to measure the spatial correlation function of ultracold atomic wave packets. This method allowed us to determine the temperature of atoms cooled by VSCPT with unprecedented accuracy. We have thus been able to quantitatively verify the predictions of a VSCPT model based on Levy statistics and found a good agreement. Moreover, we extended this method to realize an atomic interferometer with subrecoil cooled atoms. Our approach opens the way to new quantitative studies of ultracold gases. In VSCPT, this includes a generalization to more than one dimension and a detailed analysis of the shape of the momentum distribution. Finally, although the scheme is particularly well adapted to VSCPT, which intrinsically leads to multiple coherent wave packets, the use of coherent beam splitters [9,19] allows the present method to be generalized to a wider range of problems.

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