Dressed-Atom Approach to Resonance Fluorescence

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I. INTRODUCTION

Resonance fluorescence, which is the subject of this session, has been studied for a long time. The first quantum treatment of the scattering of resonance radiation by free atoms was given by Weisskopf and Wigner, in the early days of quantum mechanics [1].

The importance of this process in various fields such as spectroscopy, optical pumping, lasers,... is obvious and does not require further discussion. In the last few years, the interest in the problem of resonance fluorescence has been renewed by the development of tunable laser sources which made it possible to irradiate atomic beams with intense monochromatic laser waves and to study the characteristics of the fluorescence light. For example, the fluorescence spectrum, which is monochromatic at very low laser intensities, as predicted by lowest order QED for elastic Rayleigh scattering, exhibits more complex structures at higher intensities when absorption and induced emission predominate over spontaneous emission. Some of these experiments, which have been initiated by the work of Schuda, Stroud and Hercher [2], here in Rochester, will be discussed in subsequent papers [3].

From the theoretical point of view, a lot of papers have been devoted to this problem, and it would be impossible here to review all of them [4]. Let's just mention the publication of Mollow [5], who, in 1969, presented a complete and correct treatment of the problem, starting from the Bloch equations for the atomic density matrix driven by a c-number applied field, and using the quantum regression theorem for evaluating the correlation function of the atomic dipole moment. Perhaps the theoretical activity in this field can be interpreted as an attempt to build some simple physical pictures of resonance fluoresecence at high intensities in terms of photons. Actually this problem is not so simple and, before entering into any calculations, it seems interesting to point out some of these difficulties.

Let us first introduce some important physical parameters. It's well known that an atom, irradiated by a resonant

monochromatic wave, oscillates between the ground state g and the excited state e with a frequency $\omega_1,$ which is the Rabi nutation frequency, and which is equal to the product of the atomic dipole moment d and the electric field amplitude E. ω_1 characterizes the strength of absorption and stimulated emission processes.

 Γ , the natural width of the excited state, is the spontaneous emission rate. In intense fields, when $\omega_1 >> \Gamma$, each atom can oscillate back and forth between e and g several times before spontaneously emitting a fluorescence photon.

T is the transit time of atoms through the laser beam and is usually much longer than the radiative lifetime Γ^{-1} of the excited state e.

From the preceding considerations, it appears first that one cannot analyze the situation in terms of a single fluorescence process. In intense fields, when each atom spends half of its time in e, there is on the average, for each atom, a sequence of several ($\sim \Gamma T/2 >> 1$) fluorescence processes, which cannot be considered as independent, as a consequence of the coherent character of the laser driving field.

Secondly, we have clearly a non equilibrium situation. Any "steady-state" which can be eventually reached by the system is actually a dynamical equilibrium: photons are constantly transferred, through fluorescence processes, from the laser mode to the empty modes of the electromagnetic field.

Finally, and this is perhaps the most difficult point, one must not forget that, in quantum theory, the corpuscular and wave aspects of light are *complementary*. There is not a unique physical description of the sequence of fluoresecence processes which can be applied to all possible experiments.

Suppose for example we are interested in the time aspect of the problem, more precisely in the probability $p(\theta)$ for having 2 successive fluorescence photons emitted by the same atom separated by a time interval θ . This can be achieved by measuring, with a broad-band photomultiplier, the intesity correlations of the fluorescence light emitted by a very dilute atomic beam (for a theoretical analysis of this problem, see references 6 and 7). Note also that, throughout this paper, we restrict ourselves to very dilute atomic systems so that we can ignore any cooperative effects such as those discussed in reference 8. Once we have detected one fluorescence photon, the atom is certainly in the ground state because of the "reduction of the wave packet". In order to be able to emit a second photon, it must be reexcited in the upper state by the laser light. It is therefore not surprising that $p(\theta)$ is given by the Rabi transient describing the excitation of the atom from the ground state. Note in particular that $p(\theta) \to 0$ when $\theta \to 0$, showing an "antibunching" of the fluorescence photons emitted by a single atom.

One can also be interested in the frequency distribution of the fluorescence light, and from now we will only consider this type of problem. In such experiments, the fluorescence photons are sent into an interferometric device, such as a high finesse Fabry-Perot etalon, inside which they are kept for such a long time that we lose all information concerning their order of emission. This clearly shows the complementarity between time and frequency which cannot be simultaneously determined. This means also that, for each ensemble of fluorescence photons with frequencies $\omega_A,\ \omega_B,\dots,\omega_N,$ we have several indistinguishable sequences of fluorescence processes, differing by the order of emission of photons, and that we must take into account possible interferences between the corresponding quantum amplitudes. There is another example of such a difficulty which is well known in atomic physics: the paradox of spontaneous emission from an harmonic oscillator [9]. It is well known that the linewidth of the spontaneously emitted radiation is independent of the initial excitation of the oscillator. Such a result can be derived quantum mechanically only if one takes into account the interferences between the N! possible cascades through which the oscillator decays from its initial excited state N to the ground state 0.

II. THE DRESSED ATOM APPROACH

In this paper, we would like to present a dressed-atom approach to resonance fluorescence, discussed in detail in reference 10, and which, in our opinion, solves the previous difficulties and leads to simple interpretations for the fluorescence and absorption spectra of atoms irradiated by intense resonant laser beams.

Let's emphasize that such an approach does not lead to new results which could not be derived from a c-number description of the laser field. Actually, the c-number description may be shown to be strictly equivalent to the quantum description provided that the initial state of the field is a coherent one [11]. What we would like to show here is that introducing from the beginning the energy levels of the combined system [atom+laser mode] leads, in the limit of high intensities, to simpler mathematical calculations and more transparent physical discussions.

We give now the general idea of this method. In a first step, one neglects spontaneous emission and one considers only the total isolated system "atom + laser mode interacting together". We call such a system the dressed-atom or the atom dressed by laser photons. One easily determines the energy diagram of such a system, which exhibits a quasiperiodicity associated with the quantization of the radiation field.

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Then, we introduce the coupling with the empty modes, responsible for the transfer of photons from the laser mode to the empty modes (Fig. 1). Resonance fluorescence can therefore be considered as spontaneous emission from the dressed atom. Similarly, a sequence of fluorescence processes corresponds to a radiative cascade of the dressed atom downwards its energy diagram.

Due to the very broad frequency spectrum of the empty modes, it is always possible to describe the spontaneous decay of the dressed atom by a master equation. In the limit of high intensities, more precisely in the limit of well resolved spectral lines, we will see that this equation takes a much simpler form.

Finally, it would be in principle necessary to introduce the coupling of the laser mode with the lasing atoms and with the cavity losses, in order to describe the injection of photons into the laser mode and the fluctuations of the laser light.

We will suppose here that the laser fluctuations are negligible and we will forget this coupling. We will describe the laser beam as a free propagating wave corresponding to a single mode of the radiation field, initially excited in a coherent state, with a Poisson distribution $p_0(n)$ for the photon number. The width, Δn , of this distribution is very large in absolute value, but very small compared to the mean number of photons n (quasi classical state):

$$1 \ll \Delta n \simeq \sqrt{\bar{n}} \ll \bar{n} \tag{1}$$

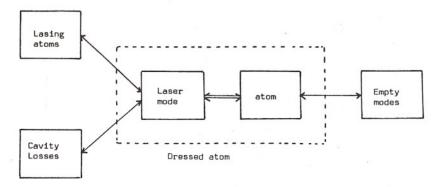


FIGURE 1. Various couplings appearing in the dressed atom approach.

Furthermore, n has no real physical meaning: we can always let n and the quantization volume V tend to ∞, the ratio n/V remaining constant and related to the electromagnetic energy density experienced by the atom. This is why it will be justified to consider the dressed atom energy diagram as periodic over a very large range An, and to neglect the variation with n of any matrix element when n varies within Δn .

III. APPLICATION TO A TWO-LEVEL ATOM

We now show how this method works in the simple case of a two-level atom.

A. Energy Diagram

The unperturbed states of the dressed atom are labelled by two quantum numbers: e, g for the atom; n for the number of photons in the laser mode. They are bunched in two-dimensional multiplicities E_n , E_{n-1} separated by ω_L (laser frequency) (Fig. 2a). The splitting between the two states $|g,n+1\rangle$ and $|e,n\rangle$ of E_n is the detuning δ between the atomic and laser frequencies ω_0 and ω_L :

$$\delta = \omega_{O} - \omega_{L} \tag{2}$$

An atom in g can absorb one laser photon and jump to e. This means that the two states |g,n+1> and |e,n> are coupled by the laser mode-atom interaction Hamiltonian V, the corresponding matrix element being

$$\langle e, n | V | g, n + 1 \rangle = \frac{\omega_1}{2}$$
 (3)

Since δ and ω_1 are small compared to ω_L , one can neglect all other couplings between different multiplicities, which is equivalent to making the rotating wave approximation. Thus, one is led to a series of independent two-level problems. One immediately finds that the two perturbed states associated with \mathcal{E}_n (Fig. 2b) are separated by a splitting

$$\omega_{12} = \sqrt{\omega_1^2 + \delta^2} \tag{4}$$

and are given by

$$|1,n\rangle = \cos\phi |e,n\rangle + \sin\phi |g, n + 1\rangle$$

 $|2,n\rangle = -\sin\phi |e,n\rangle + \cos\phi |g, n + 1\rangle$ (5)

where the angle ϕ is defined by

$$tg 2\phi = \omega_1/\delta \tag{6}$$

For the following discussion, it will be useful to know the matrix elements of the atomic dipole operator D. This operator does not act on the number n of laser photons and therefore the only non-zero matrix elements of D in the unperturbed basis are

$$\langle g, n | D | e, n \rangle = \langle g | D | e \rangle = d$$
 (7)

From the expansion (5) of the dressed atom states, one deduces that D only couples states belonging to two adjacent multiplicities. We will call d_{ii} these matrix elements

$$d_{ji} = \langle j, n-1|D|i,n \rangle \tag{8}$$

One finds immediately the matrix elements corresponding to the various allowed Bohr frequencies (arrows of Fig. 2b):

$$d_{21} = d \cos^{2}\phi \qquad (frequency \omega_{L} + \omega_{12})$$

$$d_{12} = -d \sin^{2}\phi \qquad (frequency \omega_{L} - \omega_{12})$$

$$d_{11} = -d_{22} = d \sin\phi \cos\phi \qquad (frequency \omega_{L})$$
(9)

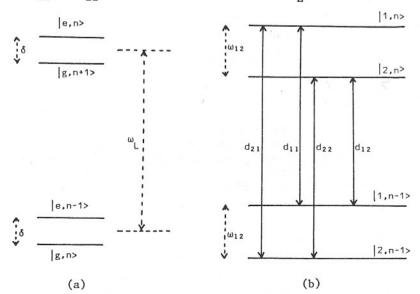


FIGURE 2. (a) Unperturbed states of the total system "atom + laser mode";(b) perturbed states of the same system.

B. Secular Approximation

Let us introduce now spontaneous emission, characterized by Γ . We will not discuss the general case, but restrict ourselves to the limit of well resolved lines:

$$\omega_{12} \gg \Gamma$$
 (10)

From Eq. (4), this condition means either intense fields $(\omega_1 >> \Gamma)$ or large detunings $(|\delta| >> \Gamma)$ or both.

In such a case, the master equation describing spontaneous emission can be considerably simplified. Any coupling, which is of the order of Γ , between two density matrix elements evolving at different frequencies, differing at least by ω_{12} , can be neglected. This is the so called secular approximation which is the starting point of an expansion in powers of Γ/ω_{12} and which leads to independent sets of equations only coupling the elements of the dressed atom density matrix σ evolving at the same Bohr frequency.

C. Evolution of the Populations

As a first illustration of this discussion, let us consider the set of equations coupling the elements of σ evolving at frequency 0, i.e. the diagonal elements of σ which represent the populations \mathbb{I} of the dressed atom energy levels:

$$\Pi_{i,n} = \langle i, n | \sigma | i, n \rangle \tag{11}$$

In these equations, important parameters appear which are the transition rates Γ_{ji} (Γ_{ji} is the transition rate from $|i,n\rangle$ to $|j,n-1\rangle$) and which are simply related to the dipole matrix elements introduced above:

$$\Gamma_{ji} = |\langle j, n-1 | D | i, n \rangle|^2 = d_{ji}^2$$
 (12)

The evolution equations of the populations $\mathbf{II}_{\mathbf{i}\,,\,\mathbf{n}}$ can then be written

$$\dot{\Pi}_{1,n} = -(\Gamma_{11} + \Gamma_{21})\Pi_{1,n} + \Gamma_{11}\Pi_{1,n+1} + \Gamma_{12}\Pi_{2,n+1}
\dot{\Pi}_{2,n} = -(\Gamma_{12} + \Gamma_{22})\Pi_{2,n} + \Gamma_{21}\Pi_{1,n+1} + \Gamma_{22}\Pi_{2,n+1}$$
(13)

These equations have an obvious physical meaning in terms of transition rates: for example, the population $\Pi_{1,n}$ decreases because of transitions from $|1,n\rangle$ to the lower levels with a total rate

$$\Gamma_1 = \Gamma_{11} + \Gamma_{21}$$
 (14)

and increases because of transitions from upper levels $|1,n+1\rangle$ (transition rate Γ_{11}) and $|2,n+1\rangle$ (transition rate Γ_{12}) (Fig. 3):

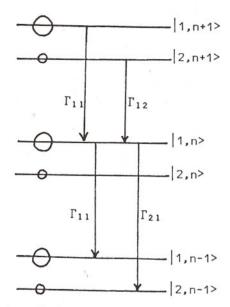


FIGURE 3. Evolution of the population $\Pi_{1,n}$.

Because of the quasi-classical character of the laser mode state (condition 1), all matrix elements can be considered as periodic within the width Δn of the photon distribution $p_0(n)$. Thus $\mathbb{I}_{1,n+1}$ and $\mathbb{I}_{1,n}$ are practically equal and they can be written as

$$\Pi_{i,n+1} = \Pi_{i,n} = P_o(n) \Pi_i$$
(15)

where I_i is a reduced population.

One deduces from Eqs. (13) and (15) that the reduced populations $\mathbb{I}_{\dot{1}}$ obey the simple equations

$$\dot{\Pi}_{1} = -\Gamma_{21} \Pi_{1} + \Gamma_{12} \Pi_{2}
\dot{\Pi}_{2} = -\Gamma_{12} \Pi_{2} + \Gamma_{21} \Pi_{1}$$
(16)

One easily derives the following results for the evolution of the populations $\mathbb{I}_{\mathbf{i}}$. Starting from the initial values $\mathbb{I}_{\mathbf{i}}(o)$ obtained by expanding on the dressed atom states the initial state of the laser mode-atom system, the populations exhibit a transient behavior, on a time scale of the order of Γ^{-1} , and then reach a steady state $\mathbb{I}_{\mathbf{i}}(\infty)$, or more precisely a dynamical equilibrium, where they do not vary any more.

The populations $\Pi_{\dot{\mathbf{1}}}(\infty)$ are determined by the two equations

$$\Pi_1(\infty) + \Pi_2(\infty) = 1 \tag{17}$$

$$\Gamma_{21} \Pi_{1}(\infty) = \Gamma_{12} \Pi_{2}(\infty) \tag{18}$$

The first one is the normalization condition. The second is the detailed balance condition, obtained from Eq. (16), and expressing that the number of transitions from $|2,n+1\rangle$ to $|1,n\rangle$ compensates the number of transitions from $|1,n\rangle$ to $|2,n-1\rangle$.

Solving these two equations leads to

$$\Pi_{1}(\infty) = \frac{\Gamma_{12}}{\Gamma_{12} + \Gamma_{21}} = \frac{\sin^{4}\phi}{\sin^{4}\phi + \cos^{4}\phi}$$

$$\Pi_{2}(\infty) = \frac{\Gamma_{21}}{\Gamma_{12} + \Gamma_{21}} = \frac{\cos^{4}\phi}{\sin^{4}\phi + \cos^{4}\phi}$$
(19)

D. Positions and Weights of the Various Components of the Fluorescence Spectrum

The positions of these components are given by the allowed Bohr frequencies of the atomic dipole moment which, according to the previous discussion, are $\omega_L - \omega_{12}$, $\omega_L, \omega_L + \omega_{12}$. So, we have a triplet of three well resolved lines since $\omega_{12} >> \Gamma$.

It is clear that the total number of photons emitted on the component $\omega_L + \omega_{ij}$ is equal to the total number of transitions |i,n>+|j,n-1>, corresponding to this frequency, and occurring during the transit time T of atoms through the laser beam. Since T is larger than Γ^{-1} , one can consider only the dynamical equilibrium regime. It follows that the weight $G_F(\omega_L + \omega_{ij})$ of the $\omega_L + \omega_{ij}$ component is given by:

$$G_{\mathbf{F}}(\omega_{\mathbf{L}} + \omega_{\mathbf{i}\mathbf{j}}) = \mathbf{T} \Gamma_{\mathbf{j}\mathbf{i}} \Pi_{\mathbf{i}}(\infty)$$
 (20)

From the detailed balance condition (18), one deduces

$$G_{\rm F}(\omega_{\rm L} + \omega_{12}) = G_{\rm F}(\omega_{\rm L} - \omega_{12})$$
 (21)

We have therefore a close connection between the detailed balance and the symmetry of the spectrum.

Since we know the $\Gamma_{j\,i}$ and the Π_{i} , we can easily write analytical expressions for the weights of the two sidebands and for the weight $G_{F}(\omega_{L})$ of the central component:

$$G_{F}(\omega_{L} + \omega_{12}) = G_{F}(\omega_{L} - \omega_{12}) = \Gamma T \frac{\sin^{4}\phi \cos^{4}\phi}{\sin^{4}\phi + \cos^{4}\phi}$$
 (22)

$$G_{\mathbf{F}}(\omega_{\mathbf{L}}) = \mathrm{T}(\Gamma_{11} \ \Pi_{1}(\infty) + \Gamma_{22} \ \Pi_{2}(\infty)) = \mathrm{\Gamma} \mathrm{T} \sin^{2} \phi \cos^{2} \phi$$

They coincide with the now well known results concerning two level atoms at the limit of well resolved spectral lines.

E. Widths of the Components

In order to obtain the width of the lateral components at $\omega_L \pm \omega_{12}$, we consider now the evolution of the off diagonal element of the density matrix connecting |1,n> and |2,n-1> and which we note σ_{12n}^{\dagger}

$$\sigma_{12n}^{+} = \langle 1, n | \sigma | 2, n-1 \rangle$$
 (23)

The evolution equation of σ_{12n}^+ can be written:

$$\dot{\sigma}_{12n}^{+} = -i(\omega_{L} + \omega_{12})\sigma_{12n}^{+} - \frac{1}{2}(\Gamma_{1} + \Gamma_{2})\sigma_{12n}^{+} + d_{11}d_{22}\sigma_{12n}^{+} + d_{11}d_{22}\sigma_{12n}^{+}$$
(24)

Three terms appear in this equation: the first one describes the free evolution of σ_{12n}^+ at the Bohr frequency $\omega_L+\omega_{12};$ the second one describes the damping of σ_{12n}^+ by spontaneous emission with a rate equal to the half sum of the total decay rates Γ_1 and Γ_2 from the two levels $|1,n\rangle$ and $|2,n-1\rangle$. Finally, one must not forget the coupling of σ_{12n}^+ with another off diagonal element of $\sigma, \ \sigma_{12}^+$ n+1, which connects $|1,n+1\rangle$ and $|2,n\rangle$ and evolves at the same Bohr frequency, the coupling coefficient being the product of the two dipole matrix elements d_{11} and d_{22} (Fig. 4).

As above, we can use the periodicity property for replacing σ_{12}^+ σ_{12n}^+ in Eq. (24), which gives the damping rate L_{12} of σ_{12n}^+

$$L_{12} = \frac{1}{2} (\Gamma_1 + \Gamma_2) - d_{11} d_{22}$$
 (25)

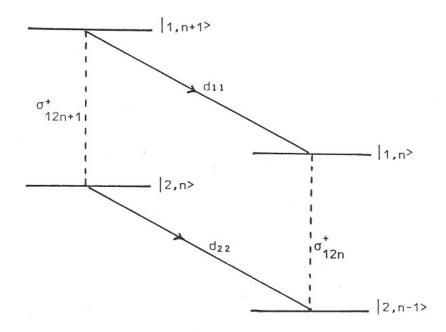


FIGURE 4. Coupling of off-diagonal density matrix elements evolving at the same Bohr frequency.

This rate is also the damping rate of the component of the mean dipole moment oscillating at frequency $\omega_L+\omega_{12}$, so that L_{12} is also the width of the lateral components. One therefore concludes that the width of the line emitted on the transition $|1,n\rangle \rightarrow |2,n-1\rangle$ is not simply given by the half sum of the natural widths Γ_1 and Γ_2 of the two involved levels. Because of the periodicity of the energy diagram, there is a phase transfer in the radiative cascade which is responsible for a correction equal to $-d_{11}d_{22}$. The explicit expression of L_{12} in terms of Γ and φ is:

$$L_{12} = \Gamma(\frac{1}{2} + \cos^2\phi \sin^2\phi)$$
 (26)

The problem of the central component is a little more complicated. There are now two off diagonal elements

$$\sigma_{iin}^{+} = \langle i, n | \sigma | i, n-1 \rangle \quad \text{with } i = 1, 2$$
 (27)

which connect the same multiplicities \mathcal{E}_n and \mathcal{E}_{n-1} and which evolve at the same frequency ω_L . One can show that, except for the free evolution terms, the σ_{iin}^+ and the populations $\Pi_{\text{i,n}}$

obey the same equations. But the time evolution of the populations is given by the superposition of a transient regime and of a steady state one. It follows that the central component is actually the superposition of two lines: a δ -function corresponding to the coherent scattering due to the undamped oscillation of the mean dipole moment driven at ω_L by the laser wave and an inelastic central component. Simple calculations (10) give the weights G_{el} and $G_{\text{inel}}(\omega_L)$ of the elastic and inelastic central components and the width L_{c} of the inelastic one:

$$G_{e1} = T(d_{11} \Pi_{1}(\infty) + d_{22} \Pi_{2}(\infty))^{2}$$

$$= \Gamma T \cos^{2} \phi \sin^{2} \phi \left[\frac{\cos^{4} \phi - \sin^{4} \phi}{\cos^{4} \phi + \sin^{4} \phi} \right]$$
(28)

$$G_{\text{inel}}(\omega_{L}) = G_{F}(\omega_{L}) - G_{\text{el}}$$

$$= \Gamma T \frac{4 \cos^{6} \phi \sin^{6} \phi}{(\cos^{4} \phi + \sin^{4} \phi)^{2}}$$
(29)

$$L_{c} = \Gamma_{12} + \Gamma_{21} = \Gamma(\sin^{4}\phi + \cos^{4}\phi)$$
 (30)

F. Absorption Spectrum

We now show how this dressed atom approach provides a straightforward interpretation of very recent experimental results [12].

Atoms are always irradiated by an intense laser beam at $\omega_L.$ Instead of looking at the fluorescence light emitted by these atoms, one measures the absorption of a second weak probe beam $\omega.$ ω_L is fixed and ω is varied. One can say that, in this experiment, one measures the absorption spectrum of the dressed atom.

Since the perturbation introduced by the weak probe beam can be neglected, the energy levels $|\text{i,n}\rangle$ $|\text{j,n-l}\rangle\dots$ of the dressed atom and their populations $\Pi_{\underline{i}}$ $\Pi_{\underline{j}}\dots$ are the same as before. To the transition $|\text{i,n}\rangle + |\text{j,n-l}\rangle$ of the dressed atom corresponds, in the absorption spectrum, a component centered at $\omega_L + \omega_{\underline{i}\,\underline{j}}$, with a width $L_{\underline{i}\,\underline{j}}$, and a weight $(\Pi_{\underline{j}} - \Pi_{\underline{i}})$ $\Gamma_{\underline{j}\,\underline{i}}T$ determined by the difference between absorption and stimulated emission processes.

This is the main difference between fluorescence and absorption spectra. An absorption signal is proportional to the dif-ference of populations between the two involved levels; a spontaneous emission signal only depends on the population of the upper state.

We therefore arrive at the following conclusions:

- (i) Because of the periodicity property, the two levels $|i,n\rangle$ and $|i,n-1\rangle$ (with i=1,2) have the same populations. So, the central component at ω_L disappears since it corresponds to transitions between two equally populated levels.
- (ii) If Π_2 is larger than Π_1 , the lateral component at $\omega_L + \omega_{12}$ (transition |i,n>+|2,n-1>) is absorbing since the lower level |2,n-1> is the most populated (Fig. 5). But, then, the second lateral component at $\omega_L \omega_{12}$ (transition |2,n>+|1,n-1> is amplifying since it is now the upper level |2,n> which is the most populated.

(iii) Finally, at resonance, one easily finds that $\Pi_1=\Pi_2$ so that all levels are equally populated and all components disappear in the absorption spectrum.

Let's recall that all these results are only valid to 0th order in $\Gamma/\omega_{12}.$ Higher order corrections to the secular approximation, which is used here, would introduce smaller signals, which do not vanish at resonance or near $\omega_{_{\! I}}$.

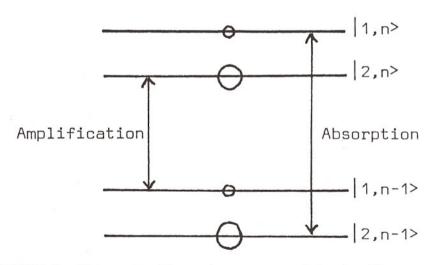


FIGURE 5. If $\pi_2 > \pi_1$, the $\omega_L + \omega_{12}$ component is absorbing, whereas the $\omega_I - \omega_{12}$ component is amplifying.

It appears clearly from the previous discussion that good amplification requires a detuning between the laser and atomic frequencies ω_L and ω_o . One can easily compute the optimum conditions for such an amplification.

The maximum of the amplifying line is obtained by dividing the weight of this line by its width. Let us call G the ratio of this maximum amplification to the maximum absorption of free atoms. One obtains

$$G = \frac{(\Pi_2 - \Pi_1) \Gamma_{12} T}{L_{12}} \frac{\Gamma/2}{\Gamma T}$$

$$= \frac{\sin^4 \phi (\cos^2 \phi - \sin^2 \phi)}{(\cos^4 \phi + \sin^4 \phi) (1 + 2 \sin^2 \phi \cos^2 \phi)}$$
(31)

The value of the detuning which maximizes this amplification is given by

$$\frac{\left|\omega_{L} - \omega_{O}\right|}{\omega_{1}} = 0.334\tag{32}$$

and it corresponds to a maximum amplification

$$G_{\text{max}} = 4.64\%$$
 (33)

IV. CONCLUSION

To conclude, let us mention some further applications of the dressed atom approach.

First, this method can be directly applied to multilevel systems. Similar mathematical expressions having the same physical meaning can be derived for the characteristics of the various components of fluorescence and absorption spectra. We have already used it to study several problems such as the modification of the Raman effect at very high intensities [13]; the simultaneous saturation of two atomic transitions sharing a common level [14], a situation which occurs frequently in stepwise excitation experiments (in that case, we have to consider atoms dressed by two types of photons); polarization effects related to Zeeman degeneracy and which could be observed by exciting atoms with a given polarization and by observing the fluorescence spectrum with a different one [15].

Second, this method is very well suited to the study of the effect of collisions in the presence of resonant fields [16,17, 18]. One has to add in the master equation new terms describing the transition rates induced by collisions. The detailed balance condition, giving the dynamical equilibrium, depends now on both radiative and collisional rates. But. on the other hand, the weights of the lines only depend on radiative rates. This provides very simple interpretations for the asymmetries which appear in the fluorescence spectrum and which are due to collisions.

Finally, one can easily introduce the Doppler effect in the theory by plotting energy diagrams giving the dressed atom energy levels versus the atomic velocity. We have shown that these diagrams can provide very simple interpretations for the various saturation signals observed in laser spectroscopy. We are presently investigating some new effects suggested by such an approach [18].

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