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6.1 Introduction

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Electromagnetic interactions play a central role in low-energy physics, chemistry, and biology. They are responsible for the cohesion of atoms and molecules and are at the origin of the emission and absorption of light by such systems. They can be described in terms of absorptions and emissions of *photons* by charged particles or by systems of charged particles like atoms and molecules. Photons are the energy quanta associated with a light beam. Since the discoveries of Planck and Einstein at the beginning of the last century, we know that a plane light wave with frequency v, propagating along a direction defined by the unit vector u, can also be considered as a beam of photons with energy E = hv and linear momentum p = (hv/c)u. We shall see later on that these photons also have an angular momentum along u depending on the polarization of the associated light wave.

Conservation laws are very useful for understanding the consequences of atomphoton interactions. They express that the total energy, the total linear momentum, and the total angular momentum are conserved when the atom emits or absorbs a photon. Consider for example the conservation of the total energy. Quantum mechanics tells us that the energy of an atom cannot take any value. It is quantized, the possible values of the energy forming a discrete set E_a , E_b , E_c , ... In an emission process, the atom goes from an upper energy level E_b to a lower one E_a and emits a photon with energy $h\nu$. Conservation of the total energy requires

$$E_{\rm b} - E_{\rm a} = h\nu. \tag{6.1}$$

The energy lost by the atom on going from E_b to E_a is carried away by the photon.

According to Equation (6.1), the only possible frequencies emitted by an atom are those corresponding to the energy differences between pairs of energy levels of this atom. This important result means that light is an essential source of information on the atomic world. By measuring the frequencies emitted or absorbed by an atom, it is possible to determine the differences $E_{\rm b} - E_{\rm a}$ and thus to obtain the energy diagram of this atom. This is what is called *spectroscopy*. High-resolution spectroscopy provides very useful information on the internal dynamics of the atom. Furthermore, each atom

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has its own spectrum. The frequencies emitted by a hydrogen atom are different from those emitted by a sodium or a rubidium atom. The spectrum of frequencies emitted by an atom is in some way its fingerprint. It is thus possible to collect information on the constituents of different types of media by observing the light originating from these media.

During the past few decades, it has been realized that light is not only a source of information on atoms, but also a tool that can be used to act on them, to manipulate them, to control their various degrees of freedom. These methods are also based on conservation laws and use the transfer of angular and linear momentum from photons to atoms. With the development of laser sources, this research field has expanded considerably during the past few years. Methods for polarizing atoms, trapping them, and cooling them to very low temperatures have been developed. New perspectives have been opened in various domains such as atomic clocks, atomic interferometry, and Bose–Einstein condensation. The purpose of this chapter is to review the physical processes which are the basis of this research field, and to present some of its most remarkable applications. Other applications are also described in Chapters 7, 9, and 11.

Two types of degrees of freedom have to be considered for an atom: (i) the internal degrees of freedom, such as the electronic configuration or the spin polarization, in the center-of-mass reference frame; and (ii) the external degrees of freedom, i.e. the position and the momentum of the center of mass of the atom. In Section 6.2 we present the basic concepts used in the control of the internal degrees of freedom. We then turn to the control of the external motion of an atom using an electromagnetic field. We show how one can trap atoms (Section 6.3) and cool them (Section 6.4). Finally, we review in Section 6.5 a few important applications of cold atoms.

6.2 Manipulation of the internal state of an atom

6.2.1 Angular momentum of atoms and photons

Atoms are like spinning tops. They have an internal angular momentum *J*. Like most physical quantities, the projection J_z of *J* along the *z* axis is quantized: $J_z = M\hbar$, where $\hbar = h/(2\pi)$ and *M* is an integer or half-integer, positive or negative.

Consider the simple case of a spin- $\frac{1}{2}$ atom. The *quantum number J* characterizing the angular momentum is $J = \frac{1}{2}$ and there are two possible values of the magnetic quantum number *M*:

$$M = +\frac{1}{2}$$
: spin up \Uparrow ; $M = -\frac{1}{2}$: spin down \Downarrow

At room temperature and in a low magnetic field, the Boltzmann factor $\exp[-\Delta E / (k_{\rm B}T)]$ corresponding to the energy splitting ΔE between the two states is very close to 1 ($k_{\rm B}$ is the Boltzmann constant). The populations of the two spin states are nearly equal and the spin polarization is negligible.

Photons have also an angular momentum J_z , which depends on their polarization (see Figure 6.1). For a right-circular polarization with respect to the *z* axis (called σ_+ polarization), $J_z = +\hbar$. For a left-circular polarization with respect to the *z* axis (called σ_- polarization), $J_z = -\hbar$. Finally, for a linear polarization parallel to the *z* axis (called π polarization), $J_z = 0$.



Figure 6.1. Three different states of polarization of a photon and corresponding values of J_z .

Polarization selection rules

When an atom absorbs a photon, it gains the angular momentum of the absorbed photon and its magnetic quantum number changes from $M_{\rm g}$ to $M_{\rm e}$. The change $(M_{\rm e} - M_{\rm g})\hbar$ of the atomic angular momentum must be equal to the angular momentum of the absorbed photon. It follows that $M_{\rm e} - M_{\rm g}$ must be equal to +1 after the absorption of a σ_+ -polarized photon, to -1 after the absorption of a σ_- -polarized photon, and to 0 after the absorption of a π -polarized photon. These *polarization selection rules* result from the conservation of the total angular momentum and express the clear connection which exists between the variation $M_{\rm e} - M_{\rm g}$ of the magnetic quantum number of an atom absorbing a photon and the polarization of this photon (see Figure 6.2).

6.2.2 Optical pumping

Optical pumping, which was developed in the early 1950s by Alfred Kastler and Jean Brossel, is our first example of manipulation of atoms with light. To explain optical pumping, let us consider the simple case of a transition connecting a ground state g with an angular momentum $J_g = \frac{1}{2}$ to an excited state e with an angular momentum $J_e = \frac{1}{2}$, so that there are two ground-state Zeeman sublevels $g_{+1/2}$ and $g_{-1/2}$ and two excited Zeeman sublevels $e_{+1/2}$ and $e_{-1/2}$ (see Figure 6.3). If one excites such an atom







Figure 6.3. The principle of optical pumping for a $1/2 \rightarrow 1/2$ transition. Atoms are transferred from $g_{-1/2}$ to $g_{+1/2}$ by an *optical pumping cycle*, which consists of an absorption of a σ_+ -polarized photon followed by the spontaneous emission of a π -polarized photon.



Figure 6.4. Magnetic-resonance imaging of the chest of a patient. Left: image obtained with ordinary proton-based resonance. Right: image obtained with gaseous-helium-based resonance. The patient has inhaled a mixture of air and helium-3, and the latter has been polarized using optical pumping. (Photographs courtesy of *Physics World*. Figure taken from the article of G. Allan Johnson, Laurence Hedlund, and James MacFall, November 1998.)

with σ_+ -polarized light, one drives only the transition $g_{-1/2} \rightarrow e_{+1/2}$ because this is the only transition corresponding to the selection rule $M_e - M_g = 1$ associated with a σ_+ excitation (see Section 6.2.1). Once the atom has been excited to $e_{+1/2}$, it can fall back by spontaneous emission either into $g_{-1/2}$, in which case it can repeat the same cycle, or into $g_{+1/2}$ by emission of a π -polarized photon. In the last case, the atom remains trapped in $g_{+1/2}$ because there is no σ_+ transition starting from $g_{+1/2}$ (see Figure 6.3). This gives rise to an optical-pumping cycle transferring atoms from $g_{-1/2}$ to $g_{+1/2}$ through $e_{+1/2}$. During such an absorption–spontaneous-emission cycle, the angular momentum of the impinging photons has been transferred to the atoms, which thus become polarized.

It clearly appears in Figure 6.3 that atoms absorb resonant light only if they are in $g_{-1/2}$. If they are in $g_{+1/2}$, they cannot absorb light because there is no σ_+ transition starting from $g_{+1/2}$. This means that any transfer of atoms from $g_{+1/2}$ to $g_{-1/2}$ that would be induced by a resonant radio-frequency (RF) field or by a relaxation process can be detected by monitoring the amount of light absorbed with a σ_+ polarization.

A first interesting feature of these optical methods is that they provide a very efficient scheme for polarizing atoms at room temperature and in a low magnetic field. Secondly, they have very high sensitivity. A single RF transition between the two ground-state sublevels is detected by the subsequent absorption or emission of an optical photon, and it is much easier to detect an optical photon than a RF photon because it has a much higher energy. Finally, these optical methods allow one to study and to investigate non-equilibrium situations. Atoms are removed from their thermodynamic equilibrium by optical pumping. By observing the temporal variations of the absorbed or emitted light, one can study how the system returns to equilibrium.

Magnetic-resonance imaging with optical pumping

Optical pumping has recently found an interesting application for imaging the human body. One prepares a sample of polarized gaseous helium-3, using optical pumping





Figure 6.5. Light shift of the ground state g of an atom produced by a non-resonant light excitation detuned to the red side of the atomic transition (right) or to the blue side (left).

with a laser. This gas is inhaled by patients (this is harmless!) and it is used to perform magnetic-resonance imaging (MRI) of the cavities in their lungs (Figure 6.4). Current proton-based MRI provides information only on solid or liquid parts of the human body such as the muscles, the brain, and the blood (the left-hand part of Figure 6.4). The use of gaseous polarized helium with high degrees of spin polarization provides MRI signals strong enough to be detected even with a system as dilute as a gas. These signals allow internal spaces of the body, such as the cavities of the lung, to be visualized at unprecedented resolutions (the right-hand part of Figure 6.4). The use of this polarized gas is a promising tool for improving our understanding of lung physiology and function.

6.2.3 Light broadening and light shifts

The interaction of an atom with an electromagnetic field perturbs atomic energy levels. This perturbation exists even in the absence of any incident light beam, in the "vacuum" of photons. Atomic energy levels are shifted. The interpretation of this effect, which was discovered by Willis Lamb in 1947, has stimulated the development of quantum electrodynamics, which is the prototype of modern quantum field theories. It can be interpreted as due to "virtual" emissions and reabsorptions of photons by the atom. Atomic excited states are also broadened, the corresponding width Γ being called the natural width of the atomic excited state e. In fact, Γ is the rate at which an excited atom spontaneously emits a photon and falls into the ground state g. It can be also written $\Gamma = 1/\tau_R$, where τ_R is called the radiative lifetime of the excited state, i.e. the mean time after which an atom leaves the excited state by spontaneous emission.

Irradiation with light introduces another perturbation for the ground state of an atom, which can be also described, at low enough intensities, as a broadening Γ' and a shift δE_g of the atomic ground state g. Both quantities depend on the intensity of light I_L and on the detuning $\Delta = \omega_L - \omega_A$ between the frequency of light ω_L and the atomic frequency ω_A . The broadening Γ' is the rate at which photons are scattered from the incident beam by the atom. The shift δE_g is the energy displacement of the ground level, as a result of virtual absorptions and stimulated emissions of photons by the atom within the light-beam mode; it is called the light shift or AC Stark shift. It was predicted and observed by Jean-Pierre Barrat and one of the authors of this article (C.C.-T.) in the early 1960s. At resonance ($\Delta = 0$), $\delta E_g = 0$ and Γ' takes its maximum value. For large enough detunings, δE_g is much larger than $\hbar\Gamma'$. One can show that δE_g is then proportional to I_L/Δ . The light shift produced by irradiation with nonresonant light is thus proportional to the intensity of light and inversely proportional to the detuning. In particular, it has the same sign as the detuning (see Figure 6.5).

6.3 Electromagnetic forces and trapping

The most well-known action of an electromagnetic field on a particle is the Lorentz force. Consider a particle with charge q moving with velocity v in an electric field E and a magnetic field B. The resulting force $F = q(E + v \times B)$ is used in all electronic devices. For neutral particles, such as neutrons, atoms, and molecules, the Lorentz force is zero. However, one can still act on these particles by using the interaction of their electric dipole moment D or magnetic dipole moment μ with a gradient of electric or magnetic field. The interaction energy of the dipole with the field is $-D \cdot E$ or $-\mu \cdot B$, and it constitutes a potential energy for the motion of the center of mass of the particle. Depending on the relative orientation between the dipole and the field, the resulting force is directed toward regions of large or small field.

In the following we shall first address the possibility of trapping charged particles with electromagnetic fields (Section 6.3.1). We shall then turn to neutral particles and we shall discuss separately the case of magnetic forces (Section 6.3.2) and electric (Section 6.3.3) forces. The physical concepts involved are quite different since one deals with a permanent dipole moment in the first case, and an induced moment in the latter. We shall finally turn to the radiation-pressure force (Section 6.3.4), and present two spectacular applications of this force to atom manipulation: the deceleration of an atomic beam and the magneto-optical trap.

6.3.1 Trapping of charged particles

At first sight, the simplest trap for charged particles should consist in a pure electrostatic field E(r) such that the electrostatic force F = qE in the vicinity of a given point 0 would be a restoring force. This is unfortunately forbidden by the Gauss equation for electrostatics $\nabla \cdot E = 0$, which entails $\nabla \cdot F = 0$. This result, which makes it impossible to have a force pointing inwards for a sphere centered at any point 0, is known as the Earnshaw theorem. We present hereafter two possibilities for how to circumvent this theorem. The first one takes advantage of the time dependence of the applied electric field. The second possibility uses a combination of electric and magnetic fields.

The Paul trap

The principle of this trap, which was invented by Wolfgang Paul in Bonn in the mid 1950s, places the particle to be trapped in an electric field that is rapidly oscillating at frequency Ω : $E(\mathbf{r}, t) = \mathcal{E}(\mathbf{r}) \cos(\Omega t)$, the amplitude $\mathcal{E}(\mathbf{r})$ vanishing at the center of the trap 0. The motion of the particle can then be decomposed into a fast oscillation at frequency Ω (the micro-motion) superimposed upon a motion with a slower characteristic frequency. After averaging over a time period $2\pi/\Omega$, the kinetic energy associated with the micro-motion is $U(\mathbf{r}) = q^2 \mathcal{E}^2(\mathbf{r})/(4m\Omega^2)$. This kinetic energy plays the role of a potential energy for the slow motion. It is null at 0, since \mathcal{E} vanishes at this point, and positive everywhere else. One thus achieves in this way a potential well centered on 0, which confines the particle. The consistency of the above treatment is ensured by choosing the field amplitude \mathcal{E} such that the oscillation frequency of the particle in the potential $U(\mathbf{r})$ is indeed much smaller than the fast frequency Ω .





Figure 6.6. Trapping charged particles. Left: the scheme of electrodes used in a Penning trap, with a vertical magnetic field. Right: a chain of four calcium ions confined in a linear Paul trap; the distance between two ions is 15 μm. (Photograph by courtesy of Rainer Blatt, Innsbruck.)

The Penning trap

The Penning trap is formed by the superposition of a quadrupole electrostatic potential V(r) and a uniform magnetic field *B* parallel to the *z* axis (Figure 6.6, left-hand side). The electrostatic potential ensures that trapping along the *z* direction occurs, and it expels the particle in the perpendicular *x*-*y* plane; it can be written $V(r) = \kappa (2z^2 - x^2 - y^2)$, where κ is a constant. The Newtonian equations of motion for the particle are linear in *r* and *v*; hence they can be solved exactly. For $qB > (8\kappa m)^{1/2}$, one finds that the motion in the *x*-*y* plane is stabilized by the magnetic field, while the trapping along the *z* axis by the quadrupole electric field is not affected. One thus achieves a stable three-dimensional confinement of the particles.

Applications

Both Paul and Penning traps are extensively used in modern physics. They are often associated with efficient cooling of the particles using either quasiresonant laser light (see Section 6.4.3) or resonant coupling with a damped electrical circuit. They allow a precise measurement of the cyclotron frequency qB/m of the trapped particle. By facilitating comparison of this ratio for a single electron and a single positron, or for a single proton and a single antiproton, Penning traps have allowed tests of the symmetry between matter and antimatter with unprecedented precision. One can also use this trap to measure accurately the gyromagnetic ratio of a particle, i.e. the ratio between the cyclotron frequency and the Larmor frequency, characterizing the evolution of the magnetic moment. Using a single trapped electron, Hans Dehmelt and his group have employed this system to test quantum electrodynamics at the 10^{-12} level of accuracy. Single ions trapped in Penning or Paul traps are used for ultra-high-resolution spectroscopy and metrology. It is also possible to trap large assemblies of ions in these traps and to study in great detail the macroscopic behavior of these plasmas. Last but not least, a string of a few ions trapped in a Paul trap is considered a very promising system with which to implement the basic concepts of the new field of quantum information (Figure 6.6, right-hand side - see also Chapter 11).

6.3.2 The magnetic dipole force

One of the most celebrated experiments performed in the early days of quantum mechanics was the Stern–Gerlach experiment. In that experiment, performed in 1921, a beam of silver atoms was sent into a region with a large magnetic gradient. Otto



Figure 6.7. Magnetic trapping of neutral atoms. Left: a loffe–Pritchard trap, consisting of four linear conductors and two circular coils. The arrows indicate the current direction in each conductor. The modulus of the magnetic field has a nonzero local minimum at the center of symmetry 0 of the system. Atoms with a magnetic moment anti-parallel to the local magnetic field are confined around 0. Right: a photograph of 10⁷ cesium atoms confined in a loffe–Pritchard trap. The image of the cigar-shaped atom cloud has been obtained by recording the absorption of a short resonant laser pulse and making the image of the shadow of the atom cloud onto a CCD camera. The temperature is of the order of 10 microkelvins. (Photograph: ENS.)

Stern and Walter Gerlach observed that the beam was split into two components, one being deflected toward the region of large magnetic field, the other being deflected away from this region.

The modern interpretation of this experiment is straightforward within the framework that we outlined in Section 6.2. The silver atoms have a spin $J = \frac{1}{2}$, and they possess a magnetic moment μ proportional to their angular momentum $\mu = \gamma J$, where γ is a constant. Therefore the projection of the magnetic moment along the axis of the magnetic field can take only two values, $\pm \hbar \gamma/2$, corresponding to an orientation of μ either parallel or anti-parallel to *B*. The magnetic-interaction energy is then $-\mu \cdot B = \mp \mu B$. Atoms with μ parallel to *B* (energy $-\mu B$) are attracted by the highfield region, whereas atoms with μ anti-parallel to *B* are deflected toward the low-field region. The magnetic force can be quite large in practice. Consider a hydrogen atom in its ground state; μ is the Bohr magneton, of the order of 10^{-23} J T⁻¹. In the vicinity of a strong permanent magnet, the gradient is ~10 T m⁻¹, hence a force 6000 times larger than gravity.

Magnetic trapping of neutral atoms

The magnetic force is now widely used to trap neutral particles such as atoms, neutrons, and molecules. Static magnetic traps are centered around a point 0 where the amplitude *B* of the magnetic field is minimum. Atoms prepared with a magnetic moment μ anti-parallel with *B* (*low-field seekers*) have a magnetic energy $\mu B(r)$ and they feel a restoring force toward 0. It is not possible to achieve a local maximum of *B* (except on the surface of a conductor). Therefore one cannot form a stable magnetic trap for *high-field seekers*, i.e. atoms with μ parallel to *B*.

The first magnetic trap for neutrons was demonstrated by Wolfgang Paul and his group in 1975. William Phillips and his team at the NIST, Gaithersburg, observed the first magnetically trapped atomic gas in 1985. Nowadays the most commonly used magnetic trap is the Ioffe–Pritchard trap, which ensures confinement around a location 0 where the field B_0 is nonzero, typically from 0.1 to 1 mT (Figure 6.7). This ensures

that the Larmor frequency characterizing the evolution of μ is large, on the order of 1–10 MHz. Since the oscillation frequency of the atom in the trap is usually much smaller (a few hundred hertz only), this allows the magnetic moment μ to adjust adiabatically to the direction of *B* during the displacement of the atom in the trap: an atom initially prepared in a low-field-seeking state will remain in this state during the course of its evolution.

Magnetic traps are simple to design and to build, and atoms can be stored for some time (several minutes) at very low temperatures (microkelvins), without any appreciable heating. In fact the lifetime of an atom in a magnetic trap is mostly determined by the quality of the vacuum in the chamber containing the trap. Indeed, the trap depth is relatively low, so a collision with a molecule from the background gas ejects the atom out of the trap. Magnetic traps have played a key role in the achievement of Bose– Einstein condensation with atomic gases. They are also used to trap atomic fermionic species and molecules.

6.3.3 The electric dipole force

Permanent dipole moments: molecules

For a system with a permanent electric dipole D such as a hetero-molecule (CO, NH₃, H₂O), one can transpose the reasoning given above for a magnetic dipole. When the molecule is placed in an electric field, the projection D_z of its dipole moment in the electric-field direction is quantized. It can take 2J + 1 values, where J is the angular momentum of the molecular state under consideration; the electrostatic energy $-D \cdot E = -D_z E$ gives rise to 2J + 1 potential-energy surfaces. If the molecular beam propagates in a region where the electric field is inhomogeneous, a different force corresponds to each surface and the beam is split into 2J + 1 components.

This electric dipole force is used in many devices, such as the ammonia maser, for which it is at the basis of the preparation of the population inversion. A recent spectacular application of this force has been developed by the group of Gerard Meijer in Nijmegen. It consists of decelerating a pulsed beam of molecules using electric-field gradients. The beam is sent into a region of increasing field, so that molecules with D anti-parallel to E are slowed down. With a maximum field of $10^7 \,\mathrm{V \,m^{-1}}$ the decrease in kinetic energy is on the order of $k_{\rm B} \times 1 \,\mathrm{K}$. The electric field is then switched off as soon as the pulsed beam of molecules has reached the location where this field is large. Using a carefully designed stack of electrodes, one repeats this operation a large number of times over the total length of the beam (typically 1 m) and the pulsed beam of molecules can be brought nearly to rest.

Induced dipole moments: atoms

For atoms, the permanent dipole moment is null, as a consequence of the symmetry of the physical interactions at the origin of the atom's stability. However, it is still possible to act on them using an electric-field gradient, through an *induced* electric dipole moment. When an atom is placed in a static electric field *E*, it acquires a dipole moment $D = \alpha_0 E$, where α_0 is the static polarizability. For simplicity we shall assume here that α_0 is a scalar, although it may also have a tensorial part.

The potential energy of an atom in an electric field is $W = -\alpha_0 E^2/2$ and the corresponding force is $F = -\nabla W = \alpha_0 \nabla (E^2)/2$. For an atom in its ground state g, α_0 is positive. Therefore the atom is always attracted to the regions where the electric field is the largest. The potential energy W is nothing but the shift of the relevant atomic internal state induced by the electric field. It is calculated here at the second order of perturbation theory, assuming a linear response of the atom with respect to the field. We neglect saturation effects, which is valid as long as the applied electric field is small compared with the inner field of the atom created by the nucleus.

This analysis can be generalized to the case of a time-dependent electric field. Consider a field oscillating with the angular frequency ω_L . The static polarizability must then be replaced by the dynamic polarizability $\alpha(\omega_L)$. When ω_L is much smaller than the relevant atomic Bohr frequencies ω_A of the atom, then $\alpha(\omega_L) \simeq \alpha_0$. This is the case in many experiments in which one manipulates ground-state alkali atoms (Bohr frequencies $\omega_A \sim 3 \times 10^{15} \,\mathrm{s}^{-1}$) with very-far-detuned laser light, such as the radiation from a CO₂ laser ($\omega_L \sim 2 \times 10^{14} \,\mathrm{s}^{-1}$).

The resonant dipole force

A very important practical case concerns an atom in its ground internal state, which is irradiated with a laser wave whose frequency ω_L is comparable to the atomic Bohr frequency ω_A corresponding to the resonance transition $g \leftrightarrow e$. In this case the dipole potential $W(\mathbf{r}) = -\alpha(\omega_L)E^2(\mathbf{r})/2$ is nothing but the light shift δE_g of the ground state that we derived in Section 6.2.3. We recall that the sign of the light shift, and hence the direction of the dipole force, depends on the sign of the detuning to resonance $\Delta = \omega_L - \omega_A$. When Δ is negative, the result is qualitatively the same as for a static field; the atom is attracted to the region of large laser intensities. On the contrary, when Δ is positive, the force on the atom tends to push it away from high-intensity regions. The dipole force is nonzero only if the light intensity is spatially inhomogeneous. One can show that it can be interpreted as resulting from a redistribution of photons between the various plane waves forming the laser wave in absorption-stimulatedemission cycles.

When the laser intensity is increased to a large value, the atom spends a significant time in the excited internal state e. In this case the preceding expression for the dipole potential must be modified. A convenient point of view on the system is obtained through the *dressed-atom* formalism, in which one deals with the energy levels of the combined system "atom + laser photons." Two types of dressed states are found, connecting respectively to the ground and to the excited atomic states when the laser intensity tends to zero. The forces associated with the two dressed states are opposite. Since spontaneous-emission processes cause random jumps between the two types of dressed states, the atomic motion is stochastic, with an instantaneous force oscillating back and forth between two opposite values in a random way. Such a dressed-atom picture provides a simple interpretation of the mean value and of the fluctuations of dipole forces.

Dipole traps for neutral atoms

One of the most spectacular uses of the resonant dipole force is the possibility of trapping atoms around local maxima or minima of the laser intensity. The first laser trap was demonstrated in 1985 at Bell Labs, by the group of Steven Chu and Arthur Ashkin,





Figure 6.8. Manipulation of atoms using the resonant dipole force. Left: a photograph of cesium atoms captured in a hexagonal optical lattice with a period of 29 μ m. Each spot contains 10⁴ atoms. (Picture courtesy of C. Salomon, ENS Paris.) Right: an atom mirror formed by an evanescent wave propagating at the surface of a glass prism. For a positive detuning Δ of the laser beam with respect to the atomic resonance, the atoms are repelled from the high-intensity region. If their incident kinetic energy is low enough, they bounce elastically on the light sheet without touching the glass surface.

using a single focused traveling laser wave. Atoms were accumulated at the vicinity of the focal point of the light wave. Later on, several other traps were investigated, such as hollow tubes used as atom guides. The research on dipole traps has led to another spectacular development, *optical tweezers*. The object being trapped is no longer a single atom, but a micrometer-sized dielectric sphere. It can be attached to objects of biological interest, such as a DNA molecule, and it allows the microscopic manipulation of these objects.

Optical lattices

Optical lattices are formed by the periodic modulation of the light intensity in a laser standing wave. Depending on the sign of the detuning Δ , the atoms accumulate at the nodes or the antinodes of the standing wave (Figure 6.8, left-hand side). Optical lattices, which were initially studied by the groups of Gilbert Grynberg at the Ecole Normale Supérieure (Paris) and William Phillips at the NIST, have led to several spectacular developments, from both theoretical and experimental points of view. The tunneling between adjacent wells plays a significant role in the dynamics of the atoms, and these lattices constitute model systems for studying quantum transport in a periodic potential. As an example, the group of Christophe Salomon has shown that atoms submitted to a constant force in addition to the lattice force undergo periodic Bloch oscillations, instead of being uniformly accelerated as in the absence of a lattice. Also, the team of Theodor Hänsch and Immanuel Bloch at Munich has observed the superfluid-insulator Mott transition for an ultra-cold gas placed in an optical lattice. The superfluid phase corresponds to a Bose-Einstein condensate, in which each atom is delocalized over the whole lattice. The isolating phase is obtained by increasing the lattice depth so that the tunneling between adjacent wells is reduced. The repulsion between atoms then favors a situation in which the number of atoms at each lattice node is fixed.

Atomic mirrors

For a positive detuning of the laser wave with respect to the atomic frequency, the dipole force repels the atoms from the high-intensity region. It is thus possible to create a potential barrier on which the atoms can be elastically reflected. Following a suggestion by Richard Cook and Richard Hill, several groups have used an evanescent wave propagating at the surface of a glass prism to form an atomic mirror (Figure 6.8, right-hand side). The incident atoms arrive on the vacuum side and feel the repulsive dipole force as they enter the evanescent wave. If their incident kinetic energy is smaller than the potential barrier created by the light, atoms turn back before touching the glass.

In practice, with a laser intensity of 1 W focused on a surface of the order of 1 mm², an atom can be reflected if the component of its velocity normal to the mirror is lower than a few meters per second. Such atomic mirrors are therefore well suited for manipulating laser-cooled atoms (see Section 6.4). They constitute very useful components for the development of atomic optics. Using a curved dielectric surface, one can focus or defocus an atomic beam. Using an evanescent wave whose intensity is modulated in time, one can make a vibrating mirror. The corresponding modulated Doppler shift introduces a frequency modulation of the reflected de Broglie wave (Section 6.5).

6.3.4 The radiation-pressure force

Recoil of an atom emitting or absorbing a photon

Consider an atom in an excited electronic state e, with its center of mass initially at rest. At a certain time, the atom emits a photon and drops to its electronic ground state g. The total momentum of the system is initially zero and it is conserved throughout the whole process. Therefore, in the final state, since the emitted photon carries away the momentum $\hbar k$, the atom recoils with momentum $-\hbar k$. This recoil phenomenon also occurs when an atom absorbs a photon. Consider the atom in its ground state g and with its center of mass initially at rest; suppose that a photon with wavevector k is sent toward this atom. If the atom absorbs the photon, it jumps to the excited state and recoils with the momentum $\hbar k$.

To the change $\hbar k$ of the atom's momentum there corresponds a change $v_{\rm rec} = \hbar k/m$ of the atom's velocity, where *m* is the atom's mass. For a hydrogen atom absorbing or emitting a photon on the Lyman- α line (2p \rightarrow 1s transition), this recoil velocity is 3 m s⁻¹. For a sodium atom, a photon emitted or absorbed on its resonance line (wavelength 590 nm) corresponds to a change in velocity of $v_{\rm rec} = 3 \text{ cm s}^{-1}$. These are very low velocities compared with those of atoms or molecules at room temperature, which are on the order of several hundreds of meters per second. This explains why the changes in velocity due to recoil effects have most of the time been neglected in the past. However, as we see below, the repetition of these changes in velocity can lead to large forces.

The radiation pressure in a resonant light wave

Consider an atom placed in a traveling laser wave with wavevector *k*. We assume that the laser frequency ω_L is resonant with the atomic transition $g \Leftrightarrow e$ at frequency ω_A . The atom then undergoes a succession of *fluorescence cycles*. The atom initially



Figure 6.9. Manipulation of atoms using the radiation-pressure force. Left: a photograph of a beam of sodium atoms stopped by the radiation pressure of a counter-propagating laser beam. (Photograph courtesy of W. D. Phillips, NIST Gaithersburg.) Right: the principle of the magneto-optical trap. The two counter-propagating laser waves have the same intensity and the same frequency, and they are respectively σ_+ - and σ_- -polarized. In the presence of a magnetic field, the balance between the two radiation-pressure forces is broken; using a gradient of magnetic field, one achieves a situation in which atoms feel a restoring force toward the center O.

in its ground state absorbs a photon from the laser beam and gains momentum $\hbar k$. After a time on the order of the radiative lifetime τ_R of the electronic excited state e, the atom decays back to the ground state by emitting spontaneously a photon. The direction of emission of this fluorescence photon is random; the symmetry properties of spontaneous emission are such that the probabilities of the photon being emitted in two opposite directions are equal. Therefore the change of momentum in a spontaneous emission averages out to zero. It follows that, in a fluorescence cycle, the average variation of the atomic velocity is related only to the absorption process and is equal to $\hbar k/m$.

The repetition rate of these cycles is limited only by the lifetime τ_R of the excited state e. Since τ_R is on the order of 10^{-8} s, about 10^8 (one hundred million!) fluorescence cycles can take place per second. During each cycle, the velocity of the atom changes on average by an amount $v_{rec} \sim 1 \text{ cm s}^{-1}$. Being repeated 100 million times per second, this produces a change in velocity per second 100 million times larger than the recoil velocity, corresponding to an acceleration or deceleration on the order of 10^6 m s^{-2} . Radiation-pressure forces are therefore 10^5 times larger than the force due to gravity!

Stopping an atomic beam

This considerable radiation-pressure force makes it possible to stop an atomic beam. Consider a sodium atomic beam coming out of an oven at a temperature of 500 K, corresponding to an average speed of 1 km s^{-1} . We irradiate this atomic beam with a counter-propagating resonant laser beam, so that the radiation-pressure force slows the atoms down. If the available distance is large enough, the atoms may even stop and return in the opposite direction: an atom with an initial velocity of 1 km s^{-1} subjected to a deceleration of 10^6 m s^{-2} is brought to rest in one millisecond. During this deceleration time it travels 50 cm only, which makes such an atomic decelerator very practicable (Figure 6.9, left-hand side).

A complication in the deceleration process originates from the Doppler effect. As the atom's velocity v changes, the laser frequency in the atomic frame $\tilde{\omega}_{L} = \omega_{L} - kv$ also changes and the resonance condition $\tilde{\omega}_L = \omega_A$ is no longer satisfied. In order to circumvent this problem, several solutions have been proposed and demonstrated: use of an inhomogeneous magnetic field so that the Zeeman effect also changes the atomic resonance frequency ω_A as the atom progresses in the decelerator, chirping of the laser frequency during the deceleration process, use of a broadband laser, etc. The first atoms stopped by radiation pressure were observed in 1984 by the groups of William Phillips (NIST) and John Hall (JILA, Boulder).

The magneto-optical trap

The radiation-pressure force can be used to trap neutral atoms quite efficiently. The trap is based upon the imbalance between the opposite radiation-pressure forces created by two counter-propagating laser waves. The imbalance is made position-dependent through a spatially dependent Zeeman shift produced by a magnetic-field gradient. The principle of the trap takes advantage of both the linear and the angular momenta carried by the photons. For simplicity we present its principle for a one-dimensional configuration, as was first suggested by one of the authors (J. D.) in 1986; we assume that the angular momenta of the ground g and excited e internal levels involved in the trapping are respectively $J_g = 0$ and $J_e = 1$ (Figure 6.9, right-hand side). The two counter-propagating waves have the same negative detuning Δ ($\omega_L < \omega_A$) and they have opposite circular polarizations; they are thus in resonance with the atom at different places. At the center O of the trap, the magnetic field is zero. By symmetry the two radiation-pressure forces have the same magnitude and opposite directions. They balance each other and an atom at 0 feels no net force. Consider now an atom to the left of 0. The laser wave coming from the left, which is σ_+ -polarized, is closer to resonance with the allowed transition $g \leftrightarrow e, m = +1$ than it is for an atom at 0. The radiation pressure created by this wave is therefore increased with respect to its value at O. Conversely, the radiation-pressure force created by the wave coming from the right is decreased with respect to its value at 0. Indeed, the wave is σ_{-} -polarized and it is further from resonance with the transition $g \leftrightarrow e, m = -1$ than it is at 0. Therefore the net force for an atom to the left of 0 is pointing toward 0. For an atom located to the right of 0, the reverse phenomenon occurs: the radiation-pressure force created by the wave coming from the right now dominates, so the resulting force also points toward 0. One therefore achieves a stable trapping around 0.

Such a scheme can be extended to three dimensions, as was first demonstrated in a collaboration between the groups at MIT and Bell Labs, and it leads to a robust, large, and deep trap called a *magneto-optical trap*. It has a large velocity-capture range and it can be used for trapping atoms in a cell filled with a low-pressure vapor, as was shown by the JILA Boulder group. Furthermore, the nonzero value of the detuning provides cooling of the trapped atoms, along the lines that will be discussed in the next section.

6.4 Cooling of atoms

The velocity distribution of an ensemble of atoms is characterized by the mean velocity and the velocity dispersion around the mean value. In physics, temperature is associated with this velocity spread, i.e. with the disordered motion of the atoms. The hotter the temperature of the medium, the higher the velocity dispersion of its constituents. To cool a system, this velocity spread has to be reduced.



Figure 6.10. Doppler cooling in one dimension, resulting from the Doppler-induced imbalance between the radiation-pressure forces of two counter-propagating laser waves. The laser detuning is negative ($\omega_L < \omega_A$).

6.4.1 Doppler cooling

The simplest cooling scheme uses the Doppler effect and was first suggested in 1975 by Theodor Hänsch and Arthur Schawlow for free atoms and by David Wineland and Hans Dehmelt for trapped ions. The concept is basically simple; we explain it for free atoms, for which case it is very reminiscent of the principle of the magneto-optical trap discussed previously. Consider an atom being irradiated by two counter-propagating laser waves (Figure 6.10). These two laser waves have the same intensity and the same frequency $\omega_{\rm L}$ slightly detuned below the atomic frequency $\omega_{\rm A}$. For an atom at rest with zero velocity, there is no Doppler effect. The two laser waves have then the same apparent frequency. The forces being exerted have the same value with opposite signs; they balance each other and no net force is exerted on the atom. For an atom moving to the right with a velocity v, the frequency of the counter-propagating beam seems higher because of the Doppler effect. The wave becomes closer to resonance, more photons are absorbed, and the force created by this beam increases. Conversely, the apparent frequency of the co-propagating wave is reduced because of the Doppler effect and becomes farther from resonance. Fewer photons are absorbed and the force decreases. For a moving atom, the two radiation-pressure forces no longer balance each other. The force opposite to the atomic velocity finally prevails and the atom is thus subjected to a nonzero net force opposing its velocity. For a small velocity v, this net force can be written as $F = -\alpha v$, where α is a friction coefficient. The atomic velocity is damped out by this force and tends to zero, as if the atom were moving in a sticky medium. This laser configuration is called optical molasses.

The limit of Doppler cooling

The Doppler friction responsible for the cooling is necessarily accompanied by fluctuations due to the fluorescence photons which are spontaneously emitted in random directions and at random times. Each emission process communicates to the atom a random recoil momentum $\hbar k$, which is responsible for a momentum diffusion described by a diffusion coefficient D. As in usual Brownian motion, competition between friction and diffusion leads to a steady state, with an equilibrium temperature proportional to D/α . A detailed analysis shows that the equilibrium temperature obtained with such a scheme is always larger than a certain limit T_D , called the Doppler limit. This limit is given by $k_{\rm B}T_{\rm D} = \hbar\Gamma/2$, where Γ is the natural width of the excited state. It is reached for a detuning $\Delta = \omega_{\rm L} - \omega_{\rm A} = -\Gamma/2$, and its value is on the order of 100 μ K for alkali atoms. In fact, when the measurements became precise enough, the group of William Phillips showed in 1988 that the temperature in optical molasses was much lower than had been expected. This indicated that other laser-cooling mechanisms, more powerful than Doppler cooling, are operating. They were identified in 1998 by the Paris and Stanford groups. We describe one of them in the next subsection, namely the Sisyphus cooling mechanism, which was proposed by the authors of this chapter.



Figure 6.11. One-dimensional Sisyphus cooling. The laser configuration is formed by two counter-propagating waves along the *z* axis with orthogonal linear polarizations. The polarization of the resulting field is spatially modulated with a period $\lambda/2$. For an atom with two ground Zeeman sublevels $m_g = \pm \frac{1}{2}$, the spatial modulation of the laser polarization results in correlated spatial modulations of the light shifts of these two sublevels and of the optical-pumping rates between them. Because of these correlations, a moving atom runs up potential hills more frequently than down.

6.4.2 Sisyphus cooling

The ground level g of most atoms, in particular alkali atoms, has a nonzero angular momentum J_g . This level is thus composed of several Zeeman sublevels. Since the detuning used in laser-cooling experiments is not large compared with Γ , both differential light shifts and optical-pumping transitions exist for the various Zeeman sublevels of the ground state. Furthermore, the laser polarization and the laser intensity vary in general in space, so the light shifts and optical-pumping rates are position-dependent. We show now, with a simple one-dimensional example, how the combination of these various effects can lead to a very efficient cooling mechanism.

Consider the laser configuration of Figure 6.11, consisting of two counterpropagating plane waves along the *z* axis, with orthogonal linear polarizations and with the same frequency and the same intensity. Because the phase shift between the two waves varies linearly with *z*, the polarization of the total field changes from σ_+ to σ_- and vice versa every $\lambda/4$. In between, it is elliptical or linear. We address here the simple case in which the atomic ground state has angular momentum $J_g = \frac{1}{2}$. The two Zeeman sublevels $m_g = \pm \frac{1}{2}$ undergo different light shifts, depending on the laser polarization, so the Zeeman degeneracy in zero magnetic field is removed. This gives the energy diagram of Figure 6.11, showing spatial modulations of the Zeeman splitting between the two sublevels with a period $\lambda/2$.

If the detuning Δ is not very large compared with Γ , there are also real absorptions of photons by the atom, followed by spontaneous emission, which give rise to optical-pumping transfers between the two sublevels, whose direction depends on the polarization: $m_{\rm g} = -\frac{1}{2} \longrightarrow m_{\rm g} = +\frac{1}{2}$ for a σ_+ polarization, $m_{\rm g} = +\frac{1}{2} \longrightarrow m_{\rm g} = -\frac{1}{2}$ for a σ_- polarization. Here also, the spatial modulation of the laser polarization results in a spatial modulation of the optical-pumping rates with a period of $\lambda/2$.

The two spatial modulations of light shifts and optical-pumping rates are of course correlated because they are due to the same cause, the spatial modulation of the light polarization. These correlations clearly appear in Figure 6.11. With the proper sign of the detuning, optical pumping always transfers atoms from the higher Zeeman sublevel to the lower one. Suppose now that the atom is moving to the right, starting from the bottom of a valley, for example in the state $m_g = +\frac{1}{2}$ at a place where the polarization is σ_+ . Because of the finite value of the optical-pumping time, there is a time lag between the dynamics of internal and external variables. The atom can climb up the potential hill before absorbing a photon. It then reaches the top of the hill, where it has the maximum probability of being optically pumped in the other sublevel, i.e. at the bottom of a valley, and so on.

Like Sisyphus in Greek mythology, who was always rolling a stone up the slope, the atom is running up potential hills more frequently than down. When it climbs a potential hill, its kinetic energy is transformed into potential energy. Dissipation then occurs by emission of light, since the spontaneously emitted photon has an energy higher than that of the absorbed laser photon. After each Sisyphus cycle, the total energy *E* of the atom decreases by an amount on the order of U_0 , where U_0 is the depth of the optical potential wells of Figure 6.11. When *E* becomes smaller than U_0 , the atom remains trapped in the potential wells.

Limits of Sisyphus cooling

The previous discussion shows that Sisyphus cooling leads to temperatures T_{Sis} such that $k_{\text{B}}T_{\text{Sis}} \simeq U_0$. We have seen in Section 6.2.3 that the light shift U_0 is proportional to I_{L}/Δ . Such a dependence of T_{Sis} on the laser intensity I_{L} and on the detuning Δ has been checked experimentally.

At low intensity, the light shift is much smaller than $\hbar\Gamma$. This explains why Sisyphus cooling leads to temperatures much lower than those achievable with Doppler cooling. One cannot, however, decrease the laser intensity to an arbitrarily low value. The previous discussion ignores the recoils due to the spontaneously emitted photons. Each recoil increases the kinetic energy of the atom by an amount on the order of $E_{\rm R}$, where

$$E_{\rm R} = \hbar^2 k^2 / (2M) \tag{6.2}$$

is the recoil energy of an atom absorbing or emitting a single photon. When U_0 becomes on the order of or smaller than E_R , the cooling due to Sisyphus cooling becomes weaker than the heating due to the recoil, and Sisyphus cooling no longer works. This shows that the lowest temperatures which can be achieved with such a scheme are on the order of a few E_R/k_B . This is on the order of a few microkelvins for heavy atoms such as rubidium and cesium. This result is confirmed by a full quantum theory of Sisyphus cooling and is in good agreement with experimental results.

Under the optimal conditions of Sisyphus cooling, atoms become so cold that they are trapped in the few lowest quantum vibrational levels of each potential well, more precisely the lowest allowed energy bands of this periodic potential. This is an example of the optical lattices discussed in Section 6.3.3. The steady state corresponds to an antiferromagnetic ordering, since two adjacent potential wells correspond to opposite spin polarizations.

6.4.3 Sub-recoil cooling

In Doppler cooling and Sisyphus cooling, fluorescence cycles never cease. Since the random recoil hk communicated to the atom by the spontaneously emitted photons



Figure 6.12. Sub-recoil cooling. The random walk in velocity space is characterized by a jump rate R vanishing at v = 0. As a result, atoms that fall within a small interval around v = 0 remain trapped there for a long time and accumulate.

cannot be controlled, it seems impossible to reduce the atomic momentum spread δp below a value corresponding to the photon momentum $\hbar k$. The condition $\delta p = \hbar k$ defines the *single-photon recoil limit*, the effective recoil temperature being set as $k_{\rm B}T_{\rm R}/2 = E_{\rm R}$. The value of $T_{\rm R}$ ranges from a few hundred nanokelvins for heavy alkalis to a few microkelvins for a light atom such as metastable helium, irradiated on its resonance line 2 ³S \leftrightarrow 2 ³P.

Sub-recoil cooling of free particles

It is possible to circumvent the recoil limit and to reach temperatures T lower than T_R . The basic idea is to create a situation in which the photon-absorption rate Γ' , which is also the jump rate R of the atomic random walk in velocity space, depends on the atomic velocity v = p/M and vanishes for v = 0 (Figure 6.12). For an atom with zero velocity, the absorption of light is quenched. Consequently, there is no spontaneous re-emission and no associated random recoil. In this way one protects ultra-slow atoms (with $v \simeq 0$) from the "bad" effects of the light. On the contrary, atoms with $v \neq 0$ can absorb and re-emit light. In such absorption–spontaneous-emission cycles, their velocities change in a random way and the corresponding random walk in v-space can transfer atoms from the $v \neq 0$ absorbing states into the $v \simeq 0$ dark states, where they remain trapped and accumulate.

Up to now, two sub-recoil cooling schemes have been proposed and demonstrated. In the first one, called *velocity-selective coherent population trapping* (VSCPT), which was investigated by the Paris group in 1988, the vanishing of R(v) for v = 0 is achieved by using quantum interference between different absorption amplitudes, which becomes fully destructive when the velocity of the atom vanishes. The second one, called Raman cooling, which was investigated by the Stanford group in 1992, uses appropriate sequences of stimulated Raman and optical-pumping pulses for tailoring the desired shape of R(v). Using these two schemes, it has been possible to cool atoms down to a few nanokelvins.

Sideband cooling of trapped ions

The states $v \simeq 0$ of Figure 6.12 are sometimes called "dark states" because an atom in these states does not absorb light. Dark-state cooling also exists for ions and is called



Figure 6.13. Sideband cooling. A laser excitation at frequency $\omega_{\rm A} - \omega_{\rm V}$ excites selectively transitions g, $v \longrightarrow e, v - 1$, where v is the vibrational quantum number, if the natural width Γ of the excited state is small relative to the vibration frequency ω_V . The most intense spontaneous transitions bringing the ion back to the ground state obey the selection rule $\Delta v = 0$, so v decreases after such a cycle. When the ion reaches the ground state g, 0, it remains trapped there because there are no transitions at frequency $\omega_{\rm A} - \omega_{\rm V}$ that can be excited from this state.

sideband cooling (see Figure 6.13). Consider an ion trapped in a parabolic potential well. The vibrational motion of the center of mass of this ion is quantized. The corresponding levels are labeled by a vibrational quantum number v = 0, 1, 2, ... and the splitting between two adjacent levels is equal to $\hbar\omega_V$, where ω_V is the vibrational frequency of the ion in the parabolic potential well. The motion of the center of mass is due to the external electric and magnetic forces acting on the charge of the ion and is, to a very good approximation, fully decoupled from the internal motion of the electrons. It follows that the parabolic potential well and the vibrational levels in the ground electronic state g and in the electronic excited state e are the same (Figure 6.13). The absorption spectrum of the ion therefore consists of a set of discrete frequencies $\omega_A \pm n\omega_V$, where ω_A is the frequency of the electronic transition, and where $n = 0, \pm 1, \pm 2, \ldots$. We have a central component at frequency ω_A and a series of "sidebands" at frequencies $\omega_A \pm \omega_V, \omega_A \pm 2\omega_V, \ldots$. We suppose here that these lines are well resolved, i.e. that the natural width Γ of the excited state, which is also the width of the absorption lines, is small compared with their frequency spacing ω_V : $\Gamma \ll \omega_V$.

The principle of sideband cooling is to excite the ion with laser light tuned at the lower sideband frequency $\omega_A - \omega_V$. One excites in this way the transitions g, $v \rightarrow e, v - 1$. For example, Figure 6.13 shows the excitation of the transition g, $2 \rightarrow e$, 1. After such an excitation in e, v - 1 the ion falls to the ground state by spontaneous emission of a photon. One can show that the most probable transition obeys the selection rule $\Delta v = 0$. For the example of Figure 6.13, the ion falls preferentially into g, 1. There are also much weaker transitions corresponding to $\Delta v = \pm 1$, the ion falling into g, 0 and g, 2. It is thus clear that the cycle consisting of the excitation by a lower sideband followed by spontaneous emission decreases the vibrational quantum number v. After a few such cycles the ion reaches the vibrational ground state g, v = 0. It is then trapped in this state because there is no possible resonant excitation from g, 0 with laser light of frequency $\omega_A - \omega_V$. The transition with the lowest frequency from g, 0 is the transition g, $0 \rightarrow e$, 0 with frequency ω_A (see Figure 6.13). The state

Figure 6.14. A few characteristic velocities associated with the various cooling schemes.



g, 0 is therefore a dark state and, after a few cycles, the ion is put into this state. Sideband cooling is a very convenient way for preparing a single ion in the vibrational ground state. Strictly speaking, the population of the first vibrational excited state is not exactly equal to zero, because of the nonresonant excitation of the transitions $\Delta v = 0$ by the laser light at frequency $\omega_A - \omega_V$, but, if Γ is small enough relative to ω_V , this population is negligible.

Velocity scales for laser cooling

To conclude this section, we show in Figure 6.14 a few characteristic velocities given by the previous analysis and appearing in the velocity scale. The first, $v_R = \hbar k/M$, is the recoil velocity. The second, $v_D = \sqrt{\hbar\Gamma/M}$, is such that $Mv_D^2 = \hbar\Gamma$ and thus gives the velocity dispersion which can be achieved by Doppler cooling. The last, $v_N = \Gamma/k$, satisfies $kv_N = \Gamma$, which means that the Doppler effect associated with v_N is equal to the natural width Γ . It gives therefore the velocity spread of the atoms which can be efficiently excited by a monochromatic light. It is easy to check that v_N/v_D and v_D/v_R are both equal to $\sqrt{\hbar\Gamma/E_R}$, where $E_R = \hbar^2 k^2/(2M)$ is the recoil energy. For most allowed transitions, we have $\hbar\Gamma \gg E_R$, so that $v_R \ll v_D \ll v_N$ (the *v* scale in Figure 6.14 is not linear). This shows the advantage of laser cooling. Laser spectroscopy with a monochromatic laser beam produces lines whose widths expressed in velocity units cannot be smaller than v_N . Doppler cooling reduces the velocity dispersion of the atoms to a much lower value, which, however, cannot be smaller than v_D . Sisyphus cooling reduces this lower limit to a few times v_R . Finally, sub-recoil cooling allows one to go below v_R .

6.5 Applications of ultra-cold atoms

Ultra-cold atoms move with very small velocities. This opens new possibilities for basic research and applications. Firstly, ultra-cold atoms can be kept a much longer time in the observation zone than can thermal atoms, which leave this zone very rapidly because of their high speed. This lengthening of the observation time considerably increases the precision of the measurements. Fundamental theories can be tested with a higher accuracy. Better atomic clocks can be built. Secondly, we know since the work of Louis de Broglie that a wave is associated with each material particle, the so-called de Broglie wave. The wave-particle duality, which was established initially for light, applies also to matter. The wavelength λ_{dB} of this wave, called the *de Broglie wavelength*, is given, in the nonrelativistic limit, by the formula $\lambda_{dB} = h/(Mv)$, where *h* is the Planck constant, *M* the mass of the particle, and *v* its velocity. Very small values of *v* thus correspond to large values of λ_{dB} , which means that the wave aspects of atoms will be easier to observe with ultra-cold atoms than with thermal atoms. We now illustrate these general considerations by a few examples.

6.5.1 Atomic clocks

The principle of an atomic clock is sketched in Figure 6.15 (on the left-hand side). An oscillator (usually a quartz oscillator) drives a microwave source and its frequency



Figure 6.15. Atomic clocks. Left: the principle of an atomic clock. Right: an atomic fountain

is scanned through an atomic resonance line. This resonance line is centered at the frequency v_0 of a transition connecting two sublevels a and b of the ground state of an atom and has a width Δv . A servo loop maintains the frequency of the oscillator at the center of the atomic line. In this way, the frequency of the oscillator is locked at a value determined by an atomic frequency and is the same for all observers. Most atomic clocks use cesium atoms. The transition connecting the two hyperfine sublevels a and b of the ground state of this atom is used to define the unit of time: the second. By convention, the second corresponds to 9 192 631 770 periods of oscillations v_0^{-1} . In standard atomic clocks, atoms from a thermal cesium beam pass through two microwave cavities fed by the same oscillator. The average velocity of the atoms is several hundred m s⁻¹; the distance between the two cavities is on the order of 1 m. The microwave resonance line exhibits *Ramsey interference fringes*. The width Δv of the central component of the signal varies as 1/T, where *T* is the time of flight of the atoms from one cavity to another: the larger *T*, the narrower the central line. For the longest devices, *T* can reach 10 ms, leading to values of Δv on the order of 1 00 Hz.

Much narrower Ramsey fringes, with sub-hertz linewidths can be obtained in the so-called Zacharias atomic fountain (see Figure 6.15, right-hand side). Atoms are captured in a magneto-optical trap and laser-cooled before being launched upward by a laser pulse through a microwave cavity. Because of gravity they are decelerated, so they return and fall back, passing a second time through the cavity. Atoms therefore experience two coherent microwave pulses when they pass through the cavity, the first time on their way up, the second time on their way down. The time interval between the two pulses can now be on the order of 1 s, i.e. about two orders of magnitude longer than with usual clocks. Atomic fountains have been realized for sodium by Steven Chu's group in Stanford and for cesium by the group of André Clairon and Christophe Salomon in Paris. A short-term relative frequency stability of $4 \times 10^{-14} \tau^{-1/2}$, where τ is the integration time, has recently been measured for a 1-m-high cesium fountain. This stability reaches now the fundamental quantum noise induced by the measurement process: it varies as $N^{-1/2}$, where N is the number of atoms detected. The long-term stability of 6×10^{-16} is most likely limited by the hydrogen maser which is used as a reference source. The real fountain stability, which will be more precisely determined by beating the signals of two fountain clocks, is expected to reach $\Delta \nu / \nu \sim 10^{-16}$ for a one-day integration time. In addition to the stability, another very important property of a frequency standard is its accuracy. Because of the very low velocities in a fountain device, many systematic shifts are strongly reduced and can be evaluated with great precision. With an accuracy of 2×10^{-15} , the Paris fountain is at present the most accurate primary standard. Improvement of this accuracy by a factor of ten is expected

Figure 6.16. Atomic-forcemicroscopy image of lines of chromium atoms channeled by a laser standing wave and deposited onto a silicon substrate. The width of the lines is 63 nm and the period is 213 nm. (Photograph courtesy of T. Pfau and J. Mlynek.)



in the near future. In addition, cold atomic clocks designed for a reduced-gravity environment are currently being built and tested, in order to increase the observation time beyond 1 s. These clocks should operate in space in the relatively near future.

Atomic clocks working with ultra-cold atoms can of course provide an improvement of the Global Positioning System. They could also be used for basic studies. A first line of research consists in building two fountain clocks, one with cesium and one with rubidium atoms, in order to measure with high accuracy the ratio of the hyperfine frequencies of these two atoms. Because of relativistic corrections, the hyperfine frequency is a function of $Z\alpha$, where α is the fine-structure constant and Z is the atomic number. Since Z is not the same for cesium and rubidium, the ratio of the two hyperfine frequencies depends on α . By making several measurements of this ratio over long periods of time, one could check cosmological models predicting a variation of α with time. The present upper limit for $\dot{\alpha}/\alpha$ in laboratory tests could be improved by two orders of magnitude. Another interesting test would be to measure with a higher accuracy the gravitational redshift and the gravitational delay of an electromagnetic wave passing near a large mass (the Shapiro effect).

6.5.2 Atomic optics and interferometry

Atomic lithography

The possibility of controlling the transverse degrees of freedom of an atomic beam with laser light opens interesting perspectives in the domain of lithography. One uses the resonant dipole force created by a laser to guide the atoms of a beam and deposit them onto a substrate, where they form the desired pattern. Using, for example, a standing laser wave orthogonal to the beam axis to channel the atoms at the nodes or antinodes of the wave (depending on the sign of the detuning Δ , see Section 6.3.3), several groups have succeeded in depositing regular atomic patterns (see Figure 6.16). The typical length scale of these patterns is a few tens of nanometers, which makes this technique competitive with other processes of nanolithography. Efforts are currently being made to adapt this technique to atoms of technological interest (indium and gallium).



Figure 6.17. Young fringes observed with metastable neon atoms. (Photograph courtesy of F. Shimizu.)

Young-slit interferometers

Because of the large value which can be achieved for atomic de Broglie wavelengths, a new field of research, atomic interferometry, has developed considerably during the past few years. It consists of extending to atomic de Broglie waves the various experiments which were previously performed with electromagnetic waves. For example, Young fringes have been observed in the laboratory of Fujio Shimizu in Tokyo by releasing a cloud of cold atoms in a metastable state above a screen pierced with two slits. The impact of the atoms on a detection plate is then observed, giving clear evidence of the wave-particle duality. Each atom gives rise to a localized impact on the detection plate. This is the particle aspect. But, at the same time, the spatial distribution of the impacts is not uniform (see Figure 6.17). It exhibits dark and bright fringes, which are nothing but the Young fringes of the de Broglie waves associated with the atoms. Each atom is therefore at the same time a particle and a wave, the wave aspect giving the probability of observing the particle at a given place.

Ramsey-Bordé interferometers

The existence of internal atomic levels brings an important degree of freedom for the design and application of atomic interferometers. The general scheme of an interferometer using this degree of freedom is represented in Figure 6.18. The atoms are modeled by a two-level system, g being the ground state and e an excited state. They interact with two pairs of laser beams, perpendicular to the atomic beam and separated in space. We suppose that spontaneous-emission processes are negligible during the whole interaction time. An atom initially in g with a momentum $p_z = 0$ in the direction z of the laser beams emerges from the first interaction zone in a coherent linear superposition of g, $p_z = 0$ and e, $p_z = \hbar k$ because of the transfer of linear momentum in the absorption process. This explains the appearance of two distinct trajectories differing in both internal and external quantum numbers in the region between the two lasers of the first pair. After the second interaction zone, there is a certain amplitude that the state e, $p_z = \hbar k$ is transformed into g, $p_z = 0$ by a stimulated-emission process while the state g, $p_z = 0$ remains unaffected (see the left-hand side of Figure 6.18). Another possibility is that the state e, $p_z = \hbar k$ remains unaffected while the state g, $p_z = 0$ is transformed into e, $p_z = \hbar k$ (see the right-hand side of Figure 6.18). Finally, the interaction with the second pair of laser beams propagating in the opposite direction can close the two paths of each interferometer. Two relevant interference diagrams therefore occur in such a scheme. In the first one (on the left-hand side of Figure 6.18), the atoms are in the ground state in the central zone between the two pairs of beams.



Figure 6.18. The two interferometers arising in the Ramsey–Bordé type geometry. Two-level atoms interact with two pairs of laser waves, which induce a coherent coupling between the two internal levels g and e. The horizontal axis may represent time as well as space. The vertical axis is used to represent the recoil of an atom after the absorption or the stimulated emission of a photon. Depending on the experimental conditions, this interferometer can be used to measure rotations perpendicular to the plane of the figure or accelerations. If one adjusts the frequency of the lasers to maximize the interference signal, it constitutes the prototype of an optical clock.

In the second interferometer (on the right), the atoms are in the excited state in the central zone. Note that the four interactions are separated here in space. The scheme can be easily transposed to a situation in which these four interactions are separated in time, the atom being initially in g at rest.

Depending on the geometry of the experiment, the two interferometers of Figure 6.18 are sensitive to the frequency of the exciting lasers (thus forming an atomic clock in the optical domain), to the acceleration of the system, or to its rotation (a gyroscope). In all cases, cold atoms have brought an important increase in sensitivity, thanks to their small velocity. Consider for example the case of the detection of a rotation around an axis perpendicular to the plane of the interferometer. The measurement is based on the Sagnac effect: the rotation modifies the difference in length between the two interfering paths of Figure 6.18. The best demonstrated sensitivity of atom interferometers to rotation is 6×10^{-10} rad s⁻¹ for an integration time T = 1 s, and the sensitivity improves as \sqrt{T} . This result, obtained by the group of Mark Kasevich, is comparable to the best optical gyroscopes. One can show that the sensitivity of these atomic gyroscopes is higher than the sensitivity of laser gyroscopes with the same area between the two arms of the interferometer by a factor that can be as large as $mc^2/(h\nu)$, where *m* is the mass of the atom and ν the frequency of the photon, a factor that can reach values on the order of 10^{11} .

These atomic interferometers can also be used to measure fundamental constants accurately. For example, the measurement of the interference pattern as a function of the frequency of the lasers reveals two different resonances for the two interferometers shown in Figure 6.18. The difference in frequency between the two resonances is related to the recoil shift $\hbar k^2/m$, where *k* is the wavevector of the light and *m* the mass of the atom. The measurement of this recoil shift, which has been performed by Steven Chu's group at Stanford, can be combined with the measurement of the Rydberg constant and the ratio between the proton and the electron masses m_p/m_e . It then yields a value of the fine-structure constant α with a relative accuracy of 10^{-8} . The precision of this method, whose advantage is that it does not depend on quantum-electrodynamics calculations, is comparable to those of the other most accurate methods.

6.6 Concluding remarks

The manipulation of atomic particles by electromagnetic fields has led to spectacular new results during the past two decades. The combination of the trapping and cooling methods described in this chapter allows the temperature of atomic gases to be decreased by several orders of magnitude and to reach the sub-microkelvin region. Conversely, the thermal wavelength λ_T of the particles is increased by four orders of magnitude with respect to its room-temperature value. It reaches values on the order of an optical wavelength when the atoms are cooled at the recoil temperature.

For these low temperatures and large wavelengths, the quantum features of the motion of the atomic center of mass become essential. For example, an assembly of cold atoms placed in an optical lattice is a model system for the study of quantum transport in a periodic potential. This allows one to draw profound and useful analogies with condensed-matter physics, where one deals with the motion of electrons or holes in the periodic potential existing in a crystal. Another field of application of these large wavelengths is atomic interferometry, which is now becoming a common tool for realizing ultra-sensitive sensors of acceleration or rotation.

Systems of cold atoms have played an important role in the development of theoretical approaches to quantum dissipation based either on a master equation for the density operator of the system, or on a stochastic evolution of its state vector. The study of sub-recoil laser cooling has also brought some interesting connections with modern statistical-physics problems, by pointing out the possible emergence of Lévy flights in the dynamics of these atoms.

When the spatial density ρ of the laser-cooled atoms increases, collective processes occur. The formation of molecules assisted by light, or *photoassociation*, has been a very fruitful theme of research whereby new information could be collected from these ultra-cold molecular systems. Conversely this formation of molecules in laser-cooled atomic gases limits the achievable spatial density. This has so far prevented one from reaching the threshold for quantum degeneracy ($\rho\lambda^3 \ge 1$) with purely optical cooling: Bose–Einstein condensation of a bosonic-atom gas is observed only when a final step of evaporative cooling is used, after an initial pre-cooling provided by the optical methods described above (see Chapter 7).

To summarize, the manipulation of atoms with light is nowadays a tool that is encountered in most atomic physics laboratories. It is a key step in the production of quantum degenerate gases, both for bosonic and for fermionic species – see Chapter 7. It plays a central role in modern metrology devices, and is at the heart of the new generation of atomic clocks. It is also an essential element for the practical implementation of quantum information concepts (see Chapter 10). Thanks to the development of miniaturized systems, cold atoms can be used in very diverse environments, on Earth or even in space: the time provided by a cold atomic clock will soon be available from the International Space Station!

FURTHER READING

- A more detailed presentation can be found in the texts of the three Nobel lectures of 1997: S. Chu, *Rev. Mod. Phys.* 70 (1998) 685, C. Cohen-Tannoudji, *Rev. Mod. Phys.* 70 (1998), 707, and W. D. Phillips, *Rev. Mod. Phys.* 70 (1998) 721.
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- E. Arimondo, W. D. Phillips, and F. Strumia (eds), *Laser Manipulation of Atoms and Ions*, Proceedings of the 1991 Varenna Summer School, Amsterdam, North Holland, 1992.
- 4. H. Metcalf and P. van der Straten, Laser Cooling and Trapping Springer, 1999.
- [5] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Atom–Photon Interactions Basic Processes and Applications, New York, Wiley, 1992.
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Claude Cohen-Tannoudji

Born in Constantine, Algeria (then part of France), Claude Cohen-Tannoudji studied physics at the Ecole Normale Supérieure (ENS) in Paris, where he went on to do diploma work under Alfred Kastler. After military service, he returned to the ENS for further research under Kastler and Jean Brossel on atom–photon interactions. He was awarded his Ph.D. in 1962 and obtained a teaching position at the University of Paris. In 1973 he was appointed Professor of Atomic and Molecular Physics at the Collège de France, where he formed an influential group investigating new methods of laser trapping and cooling. These went on to open up the fruitful study of physics in the microkelvin and nanokelvin range. He has played an important part in the teaching of modern quantum physics in France, and with his collaborators has written standard books on quantum mechanics, quantum electrodynamics, quantum optics, and Lévy statistics, together with about 200 theoretical and experimental papers on various problems of atomic physics and quantum optics. In 1997, he shared the Nobel Prize for Physics with Steven Chu and William D. Phillips for his work on the development of methods to cool and trap atoms with laser light.

Jean Dalibard

Following his studies at the Ecole Normale Supérieure (ENS) in Paris, Jean Dalibard began his research career at the ENS Kastler–Brossel laboratory, where his thesis work on the laser cooling of atoms was under the supervision of Claude Cohen–Tannoudji. As well as making important contributions to the physics of atoms and photons and to quantum optics, he has also worked with Alain Aspect on experiments to test fundamental quantum mechanics. More recently he has been investigating the detailed properties of Bose–Einstein condensates.

He is research director at the French Centre National de la Recherche Scientifique and Professor at the Ecole Polytechnique, Palaiseau. He was invited to the US National Institute of Standards and Technology in 1991 and has also taught at Innsbruck, the Technion in Haifa, Tokyo, and Connecticut. He was awarded the Gustave Ribaud Prize of the French Academy of Science in 1987, its Mergier Bourdeix Prize in 1992, and the Jean Ricard Prize of the French Physical Society in 2000.