Sisyphus effect and paramagnetic relaxation in an oscillating magnetic field

C. Cohen-Tannoudji

Collège de France and Ecole Normale Supérieure, Laboratoire(*) Kastler Brossel, 24 rue Lhomond, 75231 Paris Cedex 05, France

Abstract. — After a brief description of the Sisyphus effect, an atomic cooling process through laser irradiation, we show the complete formal and quantitative analogy between this problem and that of a spin subjected to a periodic magnetic field and to relaxation transitions. This analogy illustrates the usefulness of the fictitious spin concept.

The lectures of Anatole Abragam which I have been fortunate to follow, first at Saclay and then at the Collège de France, have transmitted to me a certain passion for problems of spin dynamics in static or time-dependent magnetic fields. Each time I am confronted with a problem involving a finite number of levels, I try to reformulate it in terms of a fictitious spin. And it is seldom, then, that I am not helped by fruitful analogies and simple physical pictures suggested by such an approach.

In this article, written as a testimony of my affection and of my admiration for Anatole Abragam, I would like to show that one of the most effective mechanisms of laser cooling, the Sisyphus effect, bears close analogies with a physical phenomenon studied at the end of the 30s', before the discovery of magnetic resonance, namely paramagnetic relaxation in an oscillating field [1].

I. The Sisyphus effect.

Let us first recall very briefly, on a very simple system, what the Sisyphus effect consists of [2]. Several laser configurations give rise to light fields whose polarization is spatially modulated. Consider for instance two waves with the same frequency ω_L, the same amplitude, propagating in opposite directions along the axis Oz with orthogonal linear polarizations, parallel to Ox and Oy respectively (Fig. 1a). The dephasing between the two

(*) Laboratory associated with the CNRS and with Université Pierre et Marie Curie.

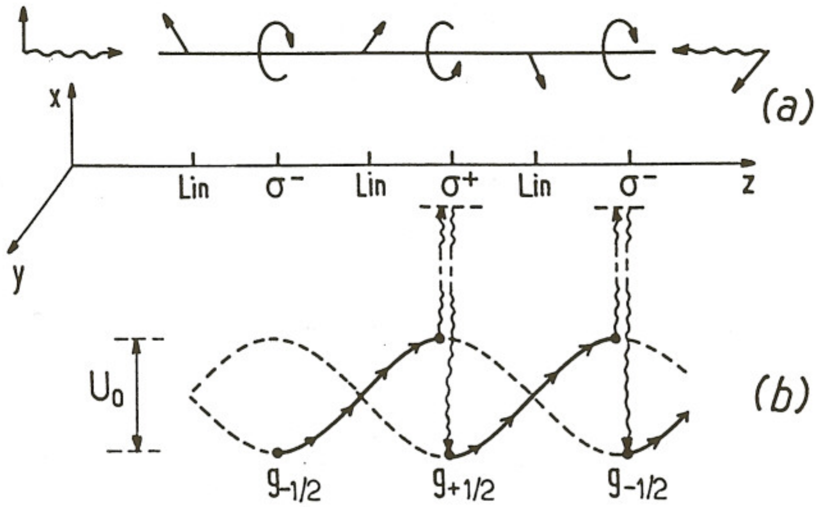

Fig. 1. — a) Polarization of the laser field resulting from the superposition of two laser waves propagating in opposite directions with orthogonal linear polarizations. b) Space variations of the light shifts of the two ground sublevels $g_{\pm 1/2}$. Because of the correlations between the space modulations of the light shifts and of the rates of optical pumping from one sublevel to the other, the potential hills that the moving atom climbs up are more numerous than those it goes down.

waves varies along Oz and the total field has a polarization that changes in a periodic way in space. It goes from σ^+ (right circular) to σ^- (left circular) every quarter wavelength $\lambda/4$, the field having in between an elliptical polarization, indeed even linear, with axes at 45° from Ox and Oy.

Let us suppose that such a laser field excites an atomic transition connecting a ground level g, of angular momentum $J_g = 1/2$, with two Zeeman sublevels $g_{+1/2}$ and $g_{-1/2}$, to an excited level e, of angular momentum $J_e = 3/2$. If the laser frequency $\omega_{\rm L}$ is slightly detuned towards the red of the atomic frequency $\omega_{\rm A}$ (detuning $\delta = \omega_{\rm L} - \omega_{\rm A} < 0$), the two Zeeman sublevels $g_{\pm 1/2}$ undergo negative light shifts, which vary from one sublevel to the other and which depend on the light polarization. Such a phenomenon has been known for several years. It is precisely because the two sublevels are differently shifted that it has been possible, before the advent of laser sources, to detect very small light shifts, in the Hertz range, by the shift of the magnetic resonance line (very narrow) in the ground state [4]. In the case of the laser configuration of Figure 1a, one finds [2] that the light shifts of $g_{+1/2}$ and $g_{-1/2}$ vary sinusoidally as a function of z, and in phase opposition. To within a global constant, the same for $g_{+1/2}$ and $g_{-1/2}$, one has for the energies $E_{\pm 1/2}$ of these two states

$$E_{+1/2}(z) = \frac{U_0}{2}\cos 2kz, \tag{1a}$$

$$E_{-1/2}(z) = -\frac{U_0}{2}\cos 2kz. \tag{1b}$$

The variations with z of $E_{\pm 1/2}(z)$ are represented in Figure 1b. U_0 is the depth of the potential wells. The spatial period of $E_{\pm 1/2}(z)$ is equal to half the laser wavelength

$\lambda = 2\pi/k = 2\pi c/\omega_L$, because the light shifts are proportional to the laser intensity and not to the field. For $\omega_L < \omega_A$, the shift of $g_{+1/2}$ (resp. $g_{-1/2}$) is maximum, downwards, at the points where the polarization is σ^+ (resp. σ^-).

The light shifts are due to virtual photon absorptions and reemissions by the atom. Real photon absorptions, followed by spontaneous emissions, can also take place, if the detuning δ is not too large. They give rise to processes of optical pumping [5] which transfer the atom from one sublevel to the other. At the points where the polarization is σ^+, this transfer takes place from $g_{-1/2}$ to $g_{+1/2}$, whereas it takes place from $g_{+1/2}$ to $g_{-1/2}$ at the points where the polarization is σ^-. More precisely, for the laser configuration of Figure 1a, it can be shown [2] that these transfer rates $\Gamma_{+\to-}$ (from $g_{+1/2}$ to $g_{-1/2}$) and $\Gamma_{-\to+}$ (from $g_{-1/2}$ to $g_{+1/2}$) are given by

$$\Gamma_{+\to-}(z) = A\cos^2 kz = \frac{A}{2}(1+\cos 2kz), \tag{2a}$$

$$\Gamma_{-\to+}(z) = A\sin^2 kz = \frac{A}{2}(1-\cos 2kz), \tag{2b}$$

where A is a positive constant independent of z.

Then it appears that the light shifts and the optical pumping rates are spatially modulated, with the same period $\lambda/2$, and are therefore correlated. More precisely, equations (1) and (2) show that the optical pumping rate is more important from the higher Zeeman sublevel towards the lower one. For instance, when $\cos 2kz = 1$, the Zeeman sublevel $g_{-1/2}$ is according to (1b) the lower (U_0 is positive), and $\Gamma_{+\to-}$ is then maximum whereas $\Gamma_{-\to+}$ vanishes. It is this correlation between the two spatial modulations showing up in equations (1) and (2), due to the same cause, namely the spatial modulation of the laser field, which is at the origin of the Sisyphus effect. Indeed let us consider an atom moving to the right, with velocity v, and starting from the bottom of a valley of the potential curve $E_{-1/2}(z)$ of the state $g_{-1/2}$, the value of $\Gamma_{-\to+}$ being zero at this point. Then the atom has enough time to climb the potential hill up to the top where the probability for it to be transferred by optical pumping to the second sublevel $g_{+1/2}$ is maximum. It may then end up at the bottom of a valley and has again to climb a potential hill (Fig. 1b). Such a situation is quite analogous to that of the hero of Greek mythology, and this is the origin of the name we have chosen with Jean Dalibard for this effect. When the atom climbs a potential hill, it is slowed down because its kinetic energy is turned into potential energy. The potential energy thus won is next dissipated by spontaneous emission. Indeed, it can be seen in Figure 1b that the optical pumping cycle bringing back the atom to the lower sublevel corresponds to an anti-Stokes Raman process, during which a photon is spontaneously emitted with an energy higher than that of the absorbed laser photon.

In the ground state g the atom with its two sublevels $g_{\pm 1/2}$ can be considered as a fictitious spin 1/2. In the following we will try to understand the dynamics of such a spin. We will begin (Sect. II) by showing that the effect of the light shifts and of the optical pumping is equivalent to that of a fictitious magnetic field and to a relaxation process acting on the spin. Then we will relate the power necessary to move the atom along the axis Oz to that supplied by the fictitious field acting on the spin in the rest frame of the atom (Sect. III). When the atom moves with velocity v along Oz, the fictitious field "seen" by the spin is oscillating. The analogy thus established with a phenomenon of paramagnetic relaxation and an oscillating field will then allow us (Sect. IV) to relate the friction force which damps the atom velocity to the dephasing, due to relaxation, between the fictitious oscillating field and the spin response to such an excitation.

II. Fictitious magnetic field and relaxation associated with light shifts and optical pumping.

The light shifts produced by the laser field lift the degeneracy between the two sublevels $g_{\pm 1/2}$ of the ground state. The same effect would be produced by a magnetic field applied along Oz. The effect of the light shifts is then equivalent to that resulting from a fictitious magnetic field $B_0(z)$, parallel to Oz and depending on z. Let us calculate such a field $B_0(z)$.

Let μ be the magnetic moment associated to the state $g_{+1/2}$. For the state $g_{-1/2}$, the magnetic moment is $-\mu$. In the fictitious field $B_0(z)$, the state $g_{+1/2}$ acquires an energy $-\mu B_0(z)$ which must coincide with the energy $E_{+1/2}(z)$ given in (1a). We deduce from this that

$$B_0(z) = -\frac{U_0}{2\mu}\cos 2kz. \tag{3}$$

The magnetic energy of $g_{-1/2}$ in $B_0(z)$ then coincides with (1b).

Now, let us consider the redistribution of populations among the sublevels $g_{+1/2}$ and $g_{-1/2}$ induced by the optical pumping, and show that it can be associated with a relaxation process. More precisely, let us call $\Pi_+(z)$ and $\Pi_-(z)$ the populations of the states $g_{+1/2}$ and $g_{-1/2}$ for an atom at rest at point z and let

$$M_z = \mu\left[\Pi_+(z) - \Pi_-(z)\right] \tag{4}$$

be the magnetization, parallel to Oz, of this atom. We will show that under the effect of the optical pumping $M(z)$ tends towards an equilibrium value $M_0(z)$ with a well-defined time constant. From the very definition of the optical pumping rates $\Gamma_{+\to-}(z)$ and $\Gamma_{-\to+}(z)$, one can write

$$\frac{\partial}{\partial t}\Pi_+(z,t) = -\Gamma_{+\to-}(z)\Pi_+(z,t) + \Gamma_{-\to+}(z)\Pi_-(z,t), \tag{5a}$$

$$\frac{\partial}{\partial t}\Pi_-(z,t) = +\Gamma_{+\to-}(z)\Pi_+(z,t) - \Gamma_{-\to+}(z)\Pi_-(z,t). \tag{5b}$$

Let us multiply both equations (5) by μ and substract the second equation from the first one. Then one obtains, owing to the definition (4), to equations (2), and from the fact that the populations $\Pi_\pm$ are normalized ($\Pi_+ + \Pi_- = 1$),

$$\frac{\partial}{\partial t}M(z,t) = -A\left[M(z,t) - M_0(z)\right], \tag{6}$$

with

$$M_0(z) = -\mu\cos 2kz. \tag{7}$$

Equation (6) expresses indeed that the magnetization at point z tends towards the equilibrium value $M_0(z)$ with a time constant

$$T_1 = 1/A, \tag{8}$$

which is a longitudinal relaxation time.

Note that the equilibrium magnetization $M_0(z)$ has the same z-dependence as the fictitious field (3). It would also be the case if we had a paramagnetic substance at thermal equilibrium at high temperature ($\mu B_0 \ll k_B T$), $M_0(z)$ being then proportional to $B_0(z)$. However, one must emphasize that here $M_0(z)$ reaches its saturation value $+\mu$ or $-\mu$ at the points z where the modulus of $B_0(z)$ is maximum ($\cos 2kz = \pm 1$). If we defined a susceptibility χ through

$$M_0(z) = \chi B_0(z), \tag{9}$$

we would then find that χ is inversely proportional to the maximum value of $B_0(z)$.

III. Energy exchange upon displacement of the atom.

The analysis of energy exchange presented in this section is very analogous to that of reference [3].

III.1 Energy required to move the atom and reinterpretation of this energy in terms of fictitious field.

In the laser configuration of Figure 1a, the atom is subjected to a radiative force calculated in reference [2], and whose average value is

$$F(z) = -\Pi_+(z)\mathrm{d}E_{+1/2}(z)/\mathrm{d}z - \Pi_-(z)\mathrm{d}E_{-1/2}(z)/\mathrm{d}z. \tag{10}$$

The interpretation of expression (10) is very clear. When the atom is in the state $g_{+1/2}$, the energy $E_{+1/2}(z)$ of this state looks like a potential energy, giving rise to a force $-\mathrm{d}E_{+1/2}(z)/\mathrm{d}z$. Then the force $F(z)$ is the average of the forces associated with each of the two states $g_{\pm 1/2}$, weighted by the probabilities $\Pi_{\pm 1/2}(z)$ of occupancy of these two states.

In order to move the atom, one must exert on it an external force $F_{\text{ext}}(z) = -F(z)$ which compensates for $F(z)$. The work $\mathrm{d}W$ that one must supply to the atom to move it by $\mathrm{d}z$ is equal to the work of this external force. It is then equal to

$$\begin{aligned} \mathrm{d}W &= F_{\text{ext}}(z)\mathrm{d}z = -F(z)\mathrm{d}z \\ &= \Pi_+(z)\mathrm{d}E_{+1/2}(z) + \Pi_-(z)\mathrm{d}E_{-1/2}(z). \end{aligned} \tag{11}$$

In order to interpret this work in terms of spin and fictitious fields, let us come back to the equations

$$E_{\pm 1/2}(z) = \mp \mu B_0(z) \tag{12}$$

relating $E_{\pm 1/2}(z)$ to $B_0(z)$. By differentiating equations (12) and transferring the result to equations (11), one then obtains, taking (4) into account

$$\mathrm{d}W = -M(z)\mathrm{d}B_0(z), \tag{13}$$

which coincides as it should with the magnetic work of an external magnetic field $B_0(z)$ acting on a magnetization $M(z)$ (see for instance reference [6] p.441. Note however that here we calculate the work supplied to the system and not the work produced by the system, which is its opposite. Furthermore we use MKSA units). The power $P = \mathrm{d}W/\mathrm{d}t$ supplied to the system, that is absorbed by the system is then

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = -M(z)\frac{dB_0(z)}{\mathrm{d}t}. \tag{14}$$

III.2 Energy dissipated in the reservoirs.

We have already mentioned above that the optical pumping transitions went together with an energy dissipation in the reservoir consisting of the radiation. For instance, when the atom undergoes an optical pumping transitions from level $g_{-1/2}$ to level $g_{+1/2}$, a laser photon of energy $\hbar\omega_{\mathrm{L}}$ vanishes and a fluorescence photon of energy $\hbar\omega_{\mathrm{L}} + E_{-1/2} - E_{+1/2}$ is produced, so that the reservoir energy changes by $E_{-1/2} - E_{+1/2}$. Likewise, in an optical pumping transitions from $g_{+1/2}$ to $g_{-1/2}$, the reservoir energy changes by $E_{+1/2} - E_{-1/2}$. Now, in the time interval $\mathrm{d}t$ during which the position change $\mathrm{d}z$ takes place, the number of optical pumping transitions from $g_{-1/2}$ to $g_{+1/2}$ is $\Gamma_{-\to+}(z)\Pi_-(z)\mathrm{d}t$ and the number of optical pumping transitions from $g_{+1/2}$ to $g_{-1/2}$ is $\Gamma_{+\to-}(z)\Pi_+(z)\mathrm{d}t$. Then the energy dissipated in the reservoir is

$$\mathrm{d}W_{\mathrm{diss}} = \left[E_{-1/2}(z) - E_{+1/2}(z)\right]\left[\Gamma_{-\to+}(z)\Pi_-(z) - \Gamma_{+\to-}(z)\Pi_+(z)\right]\mathrm{d}t. \tag{15}$$

Let us multiply both sides of equations (5) by $\mathrm{d}t$ and substract the second equation from the first one. We get

$$\mathrm{d}\Pi_+(z) - \mathrm{d}\Pi_-(z) = 2\left[\Gamma_{-\to+}(z)\Pi_-(z) - \Gamma_{+\to-}(z)\Pi_+(z)\right]\mathrm{d}t. \tag{16}$$

Moreover, according to (12)

$$E_{-1/2}(z) - E_{+1/2}(z) = 2\mu B_0(z). \tag{17}$$

By carrying (16) and (17) over (15), and taking (4) into account, one obtains finally

$$\mathrm{d}W_{\mathrm{diss}} = B_0(z)\mathrm{d}M(z) \tag{18}$$

hence, for the dissipated power

$$P_{\mathrm{diss}} = \frac{\mathrm{d}W_{\mathrm{diss}}}{\mathrm{d}t} = B_0(z)\frac{\mathrm{d}M(z)}{\mathrm{d}t}. \tag{19}$$

In the magnetic field $B_0(z)$ the magnetization $M(z)$ has a potential energy

$$U = -M(z)B_0(z). \tag{20}$$

It follows from equation (20) that

$$\begin{aligned} dU &= -M_z dB_0(z) - B_0(z) dM(z) \\ &= dW - dW_{\text{diss}}. \end{aligned} \tag{21}$$

The first term on the right-hand side, dW, is the work supplied to the system to move it by dz during the time dt. The second term, $-dW_{\text{diss}}$, is the energy supplied by the reservoir to the system. Therefore, this second term has the same status as heat supplied to the system.

IV. Atom moving with an imposed velocity v.

We assume now that the atom moves along the axis Oz with a constant velocity v, so that the coordinate z becomes a function of time

$$z = vt. \tag{22}$$

The fictitious magnetic field "seen" by the spin in the atom rest frame becomes a sine function of time, obtained by inserting (22) into (3).

$$B_0(t) = B_{\text{m}} \cos \Omega t, \tag{23}$$

with

$$B_{\text{m}} = -\frac{U_0}{2\mu}, \tag{24a}$$

$$\Omega = 2\,kv. \tag{24b}$$

IV.1 Forced motion of the magnetization.

We have already mentioned earlier (in Sect. II) that the equilibrium magnetization at point z had the same z-dependence as the fictitious field $B_0(z)$. If the atom moves very slowly, the field $B_0(t)$ "seen" by the spin oscillates very slowly and the magnetization $M(t)$ has time to reach the equilibrium value corresponding to that field. For larger values of v, $M(t)$ will not have time to adjust instantly to the variations of $B_0(t)$ and there will be a dephasing between the oscillation of $B_0(t)$ and the forced oscillation of $M(t)$.

In order to study more precisely the evolution of $M(t)$, let us go back to the relaxation equation (6) of M and replace z by vt in the source term (7). One obtains, taking (8) into account

$$\left(\frac{d}{dt} + \frac{1}{T_1}\right) M(t) = -\frac{1}{T_1} \mu \cos \Omega t. \tag{25}$$

The forced solution of this equation can be written

$$M(t) = \text{Re}\, \tilde{M} \exp(i\Omega t), \tag{26}$$

where

$$\bar{M} = -\mu \, \frac{1}{1 + i\Omega T_1}, \tag{27}$$

which yields

$$\frac{1}{\mu} M(t) = -\frac{1}{1 + (\Omega T_1)^2} \cos \Omega t - \frac{\Omega T_1}{1 + (\Omega T_1)^2} \sin \Omega t. \tag{28}$$

One distinguishes on the right-hand side of (28) the responses in phase and in quadrature of $M(t)$ to the variations of $B_0(t)$. The amplitudes of these responses vary with Ω as absorption and dispersion curves. The critical value Ω_c of Ω corresponding to the width of these curves is given by

$$\Omega_c = \frac{1}{T_1}. \tag{29}$$

IV.2 Absorbed power. Dissipated power.

The power absorbed by the spin is obtained by using (23) and (28) in (14). As required, only the component of $M(t)$ in quadrature with $B_0(t)$ yields a contribution which does not vanish on the average over time. A simple calculation then yields the mean absorbed power

$$\bar{P} = \frac{U_0}{4T_1} \frac{(\Omega T_1)^2}{1 + (\Omega T_1)^2}. \tag{30}$$

The mean dissipated power is calculated in the same way from (19) and one finds

$$\bar{P}_{\text{diss}} = \bar{P}. \tag{31}$$

Such a result is *a priori* evident. In the forced regime the potential energy (20) of the spin recovers the same value for all integer times the period $2\pi/\Omega$. The energy absorbed by the spin during this integer number of periods is therefore necessarily dissipated into the reservoir.

To come back to the problem of Sisyphus cooling, it is sufficient to replace in (30) Ω by $2\,kv$ and T_1 by $1/A$. Since the mean power absorbed by the atom is equal to $-\bar{F}v$, where $\bar{F}$ is the mean radiative force acting on it, one obtains for $\bar{F}$ an analytical expression, function of U_0, kv and A, expression which of course coincides with that of reference [2]. The variations of $\bar{F}$ with v are those of a dispersion curve, whose linear part, in the vicinity of $v = 0$ makes it possible to define a friction coefficient. The force $\bar{F}$ reaches its maximum value for $v = v_c$, where v_c is a critical velocity given by $2\,kv_c = A$. Then, $\bar{F}$ tends towards zero when $v \gg v_c$.

V. Conclusion.

The results obtained above make it then possible to reformulate the problem of the Sisyphus cooling in terms of a fictitious spin. To the atom moving in the potential hills of Figure 1b and undergoing optical pumping transitions, we have associated a spin 1/2 subjected to an oscillating magnetic field and to a relaxation process which, at any moment, causes its magnetization to tend to an equilibrium value proportional to the field value at the same time. Furthermore, we have verified that there existed a perfect equivalence between the energy necessary to move the atom along Oz and the work of the magnetic field acting on the spin magnetization.

The problem of Sisyphus cooling is therefore completely equivalent to that of paramagnetic relaxation in an oscillating field [1]. A paramagnetic sample is placed in an oscillating magnetic field $B_{\mathrm{m}} \cos \Omega t$. When the field oscillates very slowly, the induced magnetization is in phase with the field and there is no energy absorption. If the oscillation period $2\pi/\Omega$ of the field is not very long compared with the relaxation time T_1 of the magnetization $M(t)$, the latter can no longer adjust instantly to the field variations and there appears a dephasing between the oscillation of $M(t)$ and that of the field, responsible for an energy absorption.

Retruning to the Sisyphus cooling problem, one then understands why the friction force against which one must work in order to move the atom has its physical origin in retardation effects. When the atom moves, the laser field it "sees" changes. The internal state can adapt to this change only with a certain time constant, which is the time of optical pumping. If during this pumping time the atom moves by a non-negligible amount compared with the optical wavelength (which yields the scale of space variation of the laser field), the oscillation of the internal atomic state will be dephased with respect to that of the laser field, and it will be necessary to spend energy to move the atom, which characterizes the existence of a friction force.

I thank Maurice Goldman for his help in seeking references relative to paramagnetic relaxation in an oscillating field, and for his translation into English of my original article.

References.

[1] See for instance CASIMIR H.B.G., "Magnetism at very low temperatures"(Cambridge University Press, 1940) Chapter VII and references therein.

[2] DALIBARD J., and COHEN-TANNOUDJI C., *JOSA* **B6** (1989) 2023. The Sisyphus effect considered in this article is about an atom with several ground sublevels interacting with a laser field of low intensity. There also exists a Sisyphus effect for an atom with two levels interacting with an intense stationary laser wave: see reference [3].

[3] DALIBARD J., and COHEN-TANNOUDJI C., *JOSA* **B2** (1985) 1707.

[4] COHEN-TANNOUDJI C., *C.R. Acad. Sci. Paris* **252** (1961) 394.

[5] KASTLER A., *J. Phys. Rad.* **11** (1950) 255.

[6] REIF F., "Fundamentals of statistical and thermal physics"(Mc Graw-Hill, New York, 1965) p. 441.