

Laser cooling and trapping of neutral atoms: theory

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1. Introduction

The purpose of this report is to present a survey of recent theoretical advances in the field of Laser Cooling and Trapping of Neutral Atoms and to point out a few general trends. The subsequent report, by Aspect, will cover the experimental developments.

It is of course impossible, in a short time, to present an exhaustive review. Several important works will not even be mentioned. I apologize in advance for the incompleteness of this presentation. I have chosen here to put the emphasis on general questions such as:

- What are the physical mechanisms allowing one to manipulate the velocity and the position of an atom?
- What are the fundamental limits for the lowest temperatures which can be achieved by these methods?
- What can we learn from this field? Are there any new concepts, new points of view, new approaches which could be useful for other branches of Physics?

We begin this report by recalling the basic physical processes, which consist of resonant exchanges of energy and momentum between photons and atoms in elementary emission and absorption processes. The atom falls from the excited state e to the ground state g by emitting a photon k , the atomic momentum changing from p to $p - \hbar k$. Or, it can absorb a photon k and jump from g to e , gaining a momentum $\hbar k$. The important point is that the momentum change is always connected with a change of internal state. There is therefore a strong interplay between these two types of atomic degrees of freedom. The description of their coupled evolution involves *a lot of characteristic times*, such as the radiative lifetime τ_R of the excited state e , which is also the inverse of the spontaneous emission rate Γ ; the optical pumping time τ_p between the ground state Zeeman sublevels; the velocity damping time T_{ext} , ... And this raises the important question of finding the *slow variables* of the problem, which "slave" the other ones.

Another very important point is that spontaneous emission introduces *fluctuations and dissipation* in the atomic evolution. This is not surprising, since spontaneous emission results from the coupling of the atom with a large reservoir, which is the quantum radiation field in its vacuum state. The interest of spontaneous emission is that it provides a simple basic model of a *quantum dissipative process*. In a certain sense, laser cooling and trapping can be considered as a continuation of Einstein's pioneering work of 1917, showing for the first time how atoms could reach thermal equilibrium by exchanging

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energy and momentum with a reservoir of photons [1]. A review of the history of the field may be found in ref. [2].

2. Following the onset of dissipation

The amount of dissipation introduced by spontaneous emission can be controlled by varying the interaction time T , which can be chosen small or large compared to the radiative lifetime τ_R of the excited state, or, for a fixed interaction time, by varying the detuning:

$$\delta = \omega_L - \omega_A,$$

between the laser frequency ω_L and the atomic frequency ω_A .

2.1. Dissipation free regime

For very short interaction times, or for very large detunings, spontaneous emission processes can be neglected. Atomic motion can then be described in terms of a pure Hamiltonian evolution of a de Broglie wave in an internal state dependent potential.

Several physical effects, corresponding to such a regime, have been investigated: splitting of an atomic wave packet into two components by a spatially inhomogeneous laser wave (optical Stern and Gerlach effect) [3–5]; reflection of atoms by an evanescent laser wave [6, 7]; diffraction of a de Broglie wave by a laser standing wave (resonant Kapitza–Dirac effect, Bragg diffraction) [8, 9], . . .

There is presently a great renewal of interest in these effects, due to the development of new research fields such as “atom optics” or “atomic interferometry” [10–14]. It turns out that dissipation free laser–atom interactions can be used to achieve mirrors, beam splitters and gratings for atomic de Broglie waves.

2.2. From the diffractive regime to the diffusive regime

By increasing progressively the number of spontaneous emission processes occurring during the interaction time T , one can study how atomic motion changes from a diffractive type (Hamiltonian evolution) to a diffusive one (as in Brownian motion).

Simple models have been proposed, which allow one to get an analytical solution exhibiting the continuous transition from the diffractive regime to the diffusive regime [15]. More recently, more powerful numerical methods have provided exact results concerning the momentum distribution of atoms crossing a laser standing wave [16], which agree quite well with experimental observations.

It may be also interesting in this report to mention a new approach to this problem, where photon scattering is considered as a quantum measurement process, in the von Neumann sense, of the atomic position [17]. Quantum non separable correlations are established between the atom and the photon, which destroy atomic spatial coherences beyond a range determined by the laser wavelength λ_L . Such an approach is similar to those trying to explain why macroscopic objects, with large scattering cross-sections, are rapidly localized by their interaction with the environment, and how classical properties emerge as a result of this coupling [18].

2.3. The diffusive regime – analogy with Brownian motion

For long interaction times, the competition between the destruction of spatial coherences due to photon scattering and the effect of laser cooling which tends to increase the atomic coherence length ξ , leads to a situation where ξ is generally much smaller than λ_L . The atomic state is then a statistical mixture of very well localized wave packets [19]. Most theoretical treatments of laser cooling consider such a “semi-classical limit”, and assume in addition that the external times are much longer than the internal times. Two types of treatments have been developed, which are quite similar to the Langevin equation and Fokker–Planck equation approaches to Brownian motion.

In the first one, which is based on Heisenberg equations, atomic motion appears as being governed by a radiative force operator fluctuating around a mean value [20]. The value of the mean force and of the momentum diffusion coefficient due to the fluctuations of the radiative force are calculated from optical Bloch equations, using the quantum regression theorem (see, for example, ref. [21], ch. V and complement A_v).

The second type of treatment is based on master equations. The quantum equations of motion for the atomic density matrix are expanded in powers of $\hbar k/\Delta p$ (where Δp is the width of the atomic momentum distribution), and the fast internal variables are adiabatically eliminated, which leads to a Fokker–Planck equation for the Wigner function $W(x, p, t)$ describing the state of the atomic center of mass [22–26].

3. Doppler cooling – the Doppler limit

As an illustration of these methods, we discuss now Doppler cooling [27, 28], which is based on an imbalance, due to the Doppler effect, between the radiation pressure forces exerted on a moving atom by two counterpropagating waves, with equal weak intensities and a frequency ω_L smaller than ω_A .

The theoretical approaches mentioned in subsection 2.3 allow one to calculate, for a two-level atom, the mean force $\langle F \rangle$ experienced by the atom and the momentum diffusion coefficient D . For low atomic velocity v , $\langle F \rangle \approx -\alpha v$, where α is a friction coefficient. The equilibrium temperature $T \sim D/\alpha$, is found to be larger than a limit, called the Doppler limit [2, 20, 29],

$$k_B T_D = \hbar \Gamma / 2, \quad (1)$$

which is reached for $\delta = \omega_L - \omega_A = -\Gamma/2$. The velocity damping time $T_{\text{ext}} = M/\alpha$, scales as \hbar/E_R , where

$$E_R = \hbar^2 k^2 / 2M \quad (2)$$

is the recoil energy associated with the absorption or the emission of a single photon. More precisely, one finds

$$T_{\text{ext}} \approx (1/s) \hbar / E_R, \quad (3)$$

where s is the saturation parameter, which is assumed here to be small compared to 1. The condition

$T_{\text{ext}} \gg T_{\text{int}}$, where $T_{\text{int}} = \tau_R = 1/\Gamma$, is satisfied as soon as

$$\hbar\Gamma \gg E_R, \quad (4)$$

a condition which is fulfilled by most optical transitions. Three-dimensional laser cooling provides not only a strong velocity damping, but also a viscous confinement of atoms, leading to the realization of optical molasses [30].

When the measurement of temperatures in optical molasses became precise enough, it became apparent that the Doppler limit (1) could be overcome by orders of magnitude [31] and that other cooling mechanisms, much more efficient than Doppler cooling, were operating. Such a result was confirmed by other groups [32, 33], and the physical origin of these new mechanisms was rapidly identified and related to the multilevel structure of alkali atoms and to the existence of laser polarization gradients [33, 34]. More quantitative theories were then worked out [35, 36]. The key point is that friction is due to the fact that the internal state of a moving atom does not adjust instantaneously to the variations of the laser field "seen" by the moving atom. There is a time lag, which is on the order of the internal time, and which gives rise to a dissipation of energy and to an hysteresis. The longer the time lag, the larger the friction. For multilevel atoms, new internal times appear which are the optical pumping [37] times τ_p between ground state Zeeman sublevels and which become very long at low intensity since:

$$\tau_p \approx 1/\Gamma_s. \quad (5)$$

The appearance of such long internal times, combined with other physical effects such as light shifts of the Zeeman sublevels produced by the light irradiation [38], is at the origin of new cooling mechanisms. We discuss in the next subsection one of them which has been studied in great detail.

4. A simple example of sub-Doppler cooling: Sisyphus cooling

4.1. Semiclassical description [35, 39]

Such a cooling mechanism is due to strong correlations which can appear between the spatial modulation of optical pumping rates between two ground state Zeeman sublevels and the spatial modulation of the light shifts of these sublevels. These correlations result in the fact that the moving atom moves up in potential hills more than down, as did Sisyphus in the Greek mythology.

Consider the specific example of a $J_g = 1/2 \leftrightarrow J_e = 3/2$ atomic transition in a laser configuration consisting of two counterpropagating laser waves with orthogonal linear polarizations. Figure 1a shows the resulting polarization of the total field which changes, every $\lambda/8$, from linearly polarized to circularly polarized. The light shifts of the two ground state Zeeman sublevels $g_{\pm 1/2}$ oscillate in space, as shown in fig. 1b, since the light shift of $g_{-1/2}$ is three times larger (in absolute value) than the light shift of $g_{+1/2}$ in the places where the polarization is σ^+ , the reverse being true in the places where the polarization is σ^- . The inset shows the various Zeeman components of the optical line, and the corresponding relative transition probabilities. It is clear also that optical pumping tends to transfer atoms from the highest Zeeman sublevel to the lowest one since σ^+ (respectively σ^-) optical pumping tends to accumulate atoms in $g_{-1/2}$ (respectively $g_{+1/2}$). Consider now an atom moving to the right and

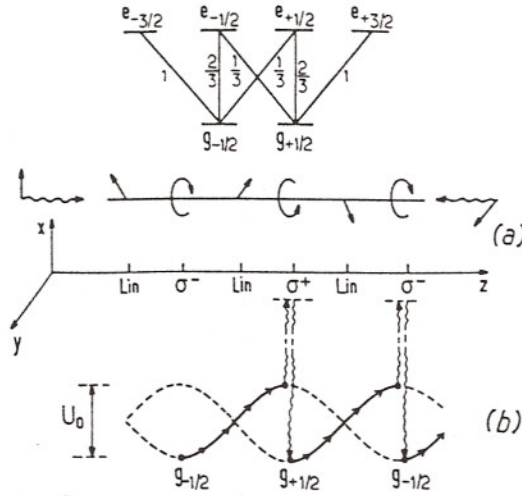


Fig. 1. Principle of Sisyphus cooling. (a) Polarization of the laser field obtained by adding the electric fields of two counterpropagating laser beams with orthogonal linear polarizations. (b) Spatial modulation of the light shifts of the two ground state sublevels. Because of the spatial modulation of optical pumping rates, the moving atom moves up in potential hills more than down. *Inset*: Relative transition probabilities for a $J_g = 1/2 \rightarrow J_e = 3/2$ transition.

starting from the bottom of a valley (fig. 1b). If its velocity v is such that $v\tau_p \sim \lambda/4$, the atom has enough time to climb the potential hill and to reach the top of this hill where it has the highest probability of being optically pumped to the bottom of a potential valley. Part of its kinetic energy has been transformed into potential energy, which is then dissipated by the fluorescence photon which has an energy higher than the energy of the absorbed laser photon. From there, the same sequence can be repeated.

A semiclassical analysis of this cooling scheme [35], valid when $T_{\text{ext}} \gg T_{\text{int}}$, shows that the velocity damping time $T_{\text{ext}} = M/\alpha$ is quite short and independent of the laser intensity,

$$T_{\text{ext}} = (\hbar/6E_R)\Gamma/|\delta|. \quad (6)$$

As for the equilibrium temperature T_s , one finds that $k_B T_s$ is on the order of the depth U_0 of the potential wells of fig. 1b, i.e. of the light shifts, which are proportional to the laser intensity I_L and inversely proportional to the detuning $|\delta|$ (when $|\delta| \gg \Gamma$):

$$k_B T_s \approx I_L/|\delta|, \quad (7)$$

a theoretical prediction which is in quite good agreement with experimental observations [40].

4.2. The limits of Sisyphus cooling [41]

The proportionality of T_s to I_L cannot remain valid for arbitrarily low intensity. The condition of validity of (7), which is $T_{\text{ext}} \gg T_{\text{int}}$, is no longer fulfilled for very low I_L , since $T_{\text{int}} \approx \tau_p$ becomes very long (see eq. (5)), whereas T_{ext} remains constant (see eq. (6)). Physically, the decrease of potential energy after each optical pumping cycle, which is on the order of U_0 , must be larger than the increase of kinetic energy, due to recoil of the absorbed or emitted photon, which is on the order of E_R . This shows

that there is a threshold for U_0 , below which Sisyphus cooling no longer works, and the smallest temperatures which can be achieved by such a cooling scheme scale as E_R and not as $\hbar\Gamma$, as is the case for Doppler cooling.

In order to determine more precisely the optimum of Sisyphus cooling, a full quantum treatment has been worked out, where the quantum equations of motion are solved numerically. For cesium, one finds that the smallest achievable root mean square momentum is on the order of $p_{rms} \approx 5.5\hbar k$, corresponding to a velocity of 1.5 cm/s, these values being reached for $U_0 \approx 95E_R$ and for $|\delta| \gg \Gamma$. For these values, the oscillation period of the atom in the bottom of the optical potential wells, $2\pi/\Omega_{osc}$, becomes shorter than the optical pumping time τ_p . This is an interesting unusual situation where external times, $2\pi/\Omega_{osc}$, associated with the motion of the center of mass, become shorter than internal times.

4.3. Quantization of atomic motion [42]

New theoretical methods have been developed for describing laser cooling in the regime $\Omega_{osc}\tau_p \gg 1$, where, as a result of condition $|\delta| \gg \Gamma$, light shifts are much larger than optical pumping rates. In a first step, one considers only light shifts, i.e. the reactive effects of the atom-laser coupling, which can be described as a pure Hamiltonian evolution of the atom in the bipotential $E_{\pm 1/2}(z)$ of fig. 1. The diagonalisation of the corresponding Hamiltonian gives a series of energy levels, which are actually energy bands because of the periodicity of $E_{\pm 1/2}(z)$. The lowest bands are very narrow because of the smallness of the tunnel effect between two adjacent potential wells. Then, in a second step, one takes into account the dissipative effects of the atom-laser coupling, which corresponds to optical pumping transitions between different energy bands or inside a given band. Condition $\Omega_{osc}\tau_p \gg 1$ allows one to neglect any "non secular" coupling between diagonal and off diagonal elements of the density matrix and to describe the whole process in terms just of transition rates between different energy levels. The momentum and position distributions obtained in this way are in very good agreement with those given by the numerical integration of the quantum equations.

The regime $\Omega_{osc}\tau_p \gg 1$ is an interesting one, not only because it corresponds to the lowest temperatures which can be achieved by Sisyphus cooling, but also because it leads to a quantization of atomic motion for a neutral atom in a light field. Some experimental evidence for such effects seems to have been obtained. Note that the description of laser cooling in terms of optical pumping transitions between bound energy bands is quite analogous to the one given for laser cooling of trapped ions [29]. Reciprocally, it has been recently suggested to extend Sisyphus cooling to trapped ions and temperatures much lower than those obtained by Doppler cooling have been predicted [43].

5. Other theoretical works dealing with sub-Doppler cooling

We first mention other sub-Doppler cooling mechanisms. Other types of polarization gradients have been investigated, for example those associated with a laser configuration consisting of two counter-propagating waves with orthogonal circular polarizations σ^+ and σ^- [35, 36]. The light shifts are then no longer spatially modulated and the cooling mechanism cannot be Sisyphus cooling. Atomic motion produces in this case an ultrasensitive population difference between ground-state sublevels, which gives rise to an imbalance between the radiation pressure forces exerted by the two laser waves. Note that the $\sigma^+ - \sigma^-$ configuration is the one used in the magneto-optical trap and that sub-Doppler

temperatures have been measured in such traps [44]. Very low temperatures have been also obtained in a vapor cell and with a magnetic trap [45]. It has been also mentioned that interesting new effects could be obtained with two counterpropagating waves with linear polarizations making an angle θ different from 0 and $\pi/2$ [46].

Actually, polarization gradients are not always essential for obtaining sub-Doppler temperatures. The combination of a laser standing wave (without polarization gradients) with a dc magnetic field applied transversally to the laser wave can give rise to a "magnetically assisted Sisyphus effect", through Landau-Zener transitions induced by the transverse magnetic field near the nodes of the standing wave [47-49].

One can also mention other theoretical works closely related to sub-Doppler cooling, such as those interpreting the new cooling mechanisms in terms of non linear spectroscopy and wave mixing [50-52], and those classifying the different types of radiative forces [19, 53].

6. The recoil limit - sub-recoil cooling

In usual cooling mechanisms, fluorescence cycles never stop. It seems then impossible to avoid the random recoil due to spontaneously emitted photons and the corresponding single photon recoil limit $k_B T_R = E_R = \hbar^2 k^2 / 2M$. During the last few years, it has been demonstrated theoretically and experimentally [54, 55], that such a limit could be overcome by a combination of two physical effects. First, velocity selective coherent population trapping which selects atoms with very small velocity and which prevents them from absorbing light through a destructive interference between two absorption amplitudes. Secondly, optical pumping in velocity space which uses the random changes of velocity after a fluorescence cycle for transferring atoms from the absorbing $v \neq 0$ classes into the non absorbing $v = 0$ class where they are protected from the "bad" effects of spontaneous emission and where they pile up.

We explain now more precisely the principle of such a cooling in a one-dimensional case (fig. 2). Two counterpropagating laser beams along the 0z axis, with σ^+ and σ^- polarizations, drive respectively the $g_{-1}, p - \hbar k \leftrightarrow e_0, p$ and the $g_{+1}, p + \hbar k \leftrightarrow e_0, p$ transitions between the three atomic sublevels e_0, g_{-1}, g_{+1} of a $J_g = 1 \leftrightarrow J_e = 1$ transition, with angular momenta along 0z equal to 0, $-\hbar, +\hbar$, respectively (e_0, p represents a state where the atom is in the excited sublevel e_0 , with momentum p along 0z). Note the selection rules resulting from the conservation of the total linear and angular momentum along 0z. The Clebsch-Gordan coefficients of the two σ^+ and σ^- transitions are equal to $+1/\sqrt{2}$ and $-1/\sqrt{2}$, respectively. If we introduce the two orthogonal linear combinations of $|g_{-1}, p - \hbar k\rangle$ and $|g_{+1}, p + \hbar k\rangle$,

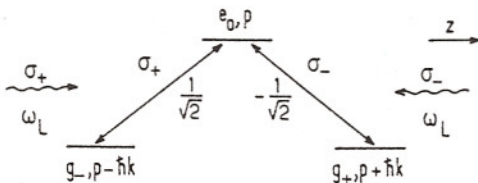


Fig. 2. Laser configuration and atomic level scheme used in one-dimensional velocity selective coherent population trapping.

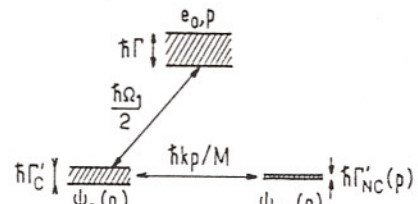


Fig. 3. Various couplings between the three states $|e_0, p\rangle$, $|\psi_C(p)\rangle$, $|\psi_{NC}(p)\rangle$ and radiative widths resulting from these couplings.

$$|\psi_{\text{NC}}(p)\rangle = (|g_{-1}, p - \hbar k\rangle + |g_{+1}, p + \hbar k\rangle)/\sqrt{2}, \quad (8a)$$

$$|\psi_{\text{C}}(p)\rangle = (|g_{-1}, p - \hbar k\rangle - |g_{+1}, p + \hbar k\rangle)/\sqrt{2}, \quad (8b)$$

we get

$$V_{\text{AL}}|\psi_{\text{NC}}(p)\rangle = 0, \quad (9)$$

$$V_{\text{AL}}|\psi_{\text{C}}(p)\rangle = \frac{1}{2}\hbar\Omega_1|e_0, p\rangle, \quad (10)$$

where V_{AL} is the laser-atom interaction Hamiltonian, and where Ω_1 is the Rabi frequency associated with each wave. The two absorption amplitudes from $g_{-1}, p - \hbar k$ to e_0, p and from $g_{+1}, p + \hbar k$ to e_0, p interfere destructively for $|\psi_{\text{NC}}(p)\rangle$, which is thus a non coupled state. The total Hamiltonian of the system contains also the kinetic energy term $P^2/2M$ (we assume that g_{-1} and g_{+1} have the same internal energy). A simple calculation gives:

$$\langle\psi_{\text{C}}(p)|P^2/2M|\psi_{\text{NC}}(p)\rangle = \hbar kp/M, \quad (11)$$

which shows that $|\psi_{\text{C}}(p)\rangle$ and $|\psi_{\text{NC}}(p)\rangle$ are coupled by $P^2/2M$ (motional coupling).

Figure 3 represents the various couplings which exist between the three states $|e_0, p\rangle$, $|\psi_{\text{C}}(p)\rangle$, $|\psi_{\text{NC}}(p)\rangle$, and the radiative widths which result from these couplings. In the absence of laser, e_0 is the only radiatively unstable state with a width Γ (natural width). Because of the contamination induced by V_{AL} between $|\psi_{\text{C}}(p)\rangle$ and $|e_0, p\rangle$ (see eq. (10)), $|\psi_{\text{C}}(p)\rangle$ gets a small width Γ'_c , which can be interpreted as the photon absorption rate from the coupled state $|\psi_{\text{C}}(p)\rangle$. When $p = 0$, the non coupled state is completely isolated from the other states. An atom prepared in $|\psi_{\text{NC}}(p=0)\rangle$ remains there indefinitely, so that $|\psi_{\text{NC}}(p=0)\rangle$ is a perfectly trapping state. As soon as p is non zero, the contamination of $|\psi_{\text{NC}}(p)\rangle$ by $|\psi_{\text{C}}(p)\rangle$, due to the motional coupling (11), transfers to $|\psi_{\text{NC}}(p)\rangle$ a small part, $\Gamma'_{\text{NC}}(p)$, of the radiative width Γ'_c of $|\psi_{\text{C}}(p)\rangle$. The smaller the p , the smaller the $\Gamma'_{\text{NC}}(p)$. The states $|\psi_{\text{NC}}(p)\rangle$, with $p \neq 0$, are therefore imperfect traps, which become more and more perfect when $p \rightarrow 0$. Such an analysis shows that there is no lower limit for the temperature T which can be achieved by such a method. For a given interaction time Θ , one can define a momentum width δp by:

$$\Gamma'_{\text{NC}}(\delta p)\Theta \approx 1. \quad (12)$$

Atoms prepared in $|\psi_{\text{NC}}(p)\rangle$ with $|p| \ll \delta p$ will remain trapped in these states during the whole interaction time, while other atoms will have time to undergo fluorescence cycles bringing them in these non absorbing states. Equation (12) gives $\delta p \sim 1/\sqrt{\Theta}$, which shows that the temperature should vary as $1/\Theta$ and is limited only by the interaction time.

Such a cooling scheme is quite different from the other ones. It is not based on a friction force but on a velocity selection by quantum interference effects combined with momentum diffusion. It works for any detuning. Note also that it cannot be described by a semiclassical approach. The de Broglie wavelength of the atoms, $\lambda_{\text{dB}} = \hbar/\delta p$, becomes larger than the laser wavelength, when δp becomes smaller than $\hbar k$, as is the case for sub-recoil cooling, and atoms can no longer be considered as being localized in the laser wave. A full quantum theory is needed. It predicts [55] that atoms are trapped in double momentum states such as (8a), and that the width δp of the momentum peaks around $\pm \hbar k$ varies as $1/\sqrt{\Theta}$.

Extensions of these subrecoil cooling schemes to two or three dimensions have been proposed [55–57] (see also ref. [19]). Transitions other than $J_g = 1 \leftrightarrow J_e = 1$ have been also considered [58], as well as schemes combining velocity selective coherent population trapping and friction forces [59]. Experiments are in progress for two-dimensional sub-recoil cooling of He atoms, with an interaction time long enough, so that one can hope to reach the nanokelvin range.

7. A new approach to laser cooling in terms of quantum jumps

We present now a new theoretical approach to describe the sequence of photon scattering processes occurring in a laser cooling experiment. We take a specific example, which is the one-dimensional velocity selective coherent population trapping scheme, introduced in the previous section.

Between two successive spontaneous emission events, the system evolves in a three-dimensional manifold $\mathcal{F}(p)$ formed by the three states $\{|e_0, p\rangle, |g_{\pm 1}, p \pm k\rangle\}$, or equivalently $\{|e_0, p\rangle, |\psi_C(p)\rangle, |\psi_{NC}(p)\rangle\}$. Such an evolution can be described in terms of absorption, stimulated emission and stimulated Raman processes. Since the amplitude for the system to be in e_0 decays with a rate $\Gamma/2$, one can show that the evolution within $\mathcal{F}(p)$ is governed by an effective non-Hermitian Hamiltonian H_{eff} obtained by adding, in the projection of the total Hamiltonian H onto $\mathcal{F}(p)$, an imaginary part $-i\hbar\Gamma/2$ to the energy of e_0 (see, for example, ref. [21], ch. III).

Spontaneous emission is the dissipative process which introduces a random character in the atomic evolution. At random times, the atom “jumps” from the excited state e_0 into the lower states $g_{\pm 1}$, while a fluorescence photon appears in one of the initially empty modes of the quantized radiation field. Since the fluorescence photon can be emitted in a random direction, the atomic momentum changes in a random way after each jump. The atomic evolution thus appears as a sequence of coherent evolutions within manifolds $\mathcal{F}(p)$ separated by quantum jumps occurring at random times and during which the atom moves from one manifold $\mathcal{F}(p)$ to another one $\mathcal{F}(p')$ determined by the momentum carried away by the fluorescence photon.

It is possible to make a statistical analysis of the sequence of quantum jumps by solving the Schrödinger equation associated with the effective Hamiltonian H_{eff} and by calculating from this solution the “delay function” giving the distribution of the time intervals between two successive spontaneous emission processes. Such a method has been already introduced [60, 61] for analyzing the “intermittent fluorescence” which can be observed on a single three-level trapped ion. Figure 4 gives the result of a Monte Carlo simulation of the sequence of quantum jumps occurring in velocity selective coherent population trapping [17]. Each vertical discontinuity corresponds to a quantum jump during

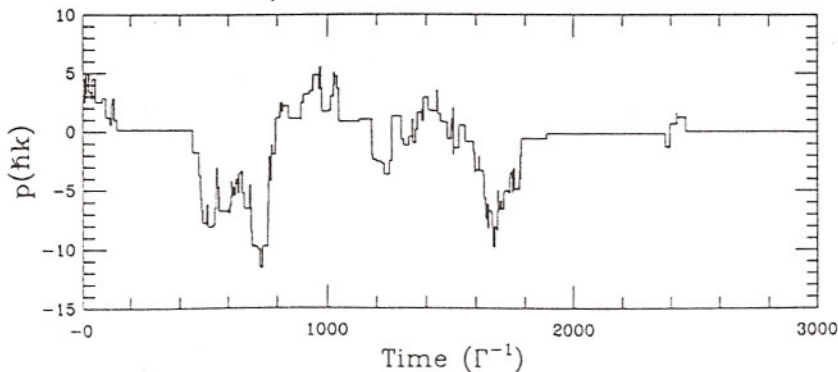


Fig. 4. Monte Carlo simulation of the sequence of quantum jumps occurring in velocity selective coherent population trapping.

which p changes abruptly. Between two successive spontaneous emissions, we have a coherent evolution within $\mathcal{F}(p)$ and p remains constant. It appears clearly in fig. 4 that the length of the “dark periods” (corresponding to coherent evolutions without any fluorescence photon) increases when p gets closer to zero. Such a result is easy to understand. It means that, when p is close to zero, there is in $\mathcal{F}(p)$ a state in which the atom can remain trapped for a long time. One of the three eigenvalues of H_{eff} has an imaginary part which tends to zero when p tends to zero, and which is actually the width $\Gamma'_{\text{NC}}(p)$ introduced in fig. 3. This gives rise to a long tail in the delay function and results in the fact that one can then wait a long time before observing a fluorescence photon.

Such a Monte Carlo simulation of the time evolution of the system preserves the quantum features of the problem. Averaging over a set of simulations, one can reconstruct the atomic momentum distribution obtained from the full quantum optical Bloch equations. We get in this way a new insight into the solution of optical Bloch equations from an analysis of what happens to a single atom. Note also that such an approach provides a simple interpretation of the filtering process by which the trapping state $|\psi_{\text{NC}}(p)\rangle$ becomes populated. Between two successive spontaneous emission processes, the state of the system is described by a normalized state vector $|\psi(t)\rangle$ given by the solution of the Schrödinger equation corresponding to the Hamiltonian H_{eff} . The three eigenstates of H_{eff} have different lifetimes, and if no spontaneous emission has occurred after a time on the order of Γ^{-1} or $(\Gamma'_C)^{-1}$, the weight of $|\psi_{\text{NC}}(p)\rangle$, which has a much longer lifetime $(\Gamma'_{\text{NC}})^{-1}$, becomes predominant in $|\psi(t)\rangle$. Extensions of the method to include the effect of an incoherent excitation by a broadband radiation field are possible. They provide for example simple interpretations for the mechanisms of amplification without inversion.

Similar Monte Carlo approaches have been recently proposed for dealing with dissipative processes in quantum optics [62]. Such approaches are more general than the one described above, insofar as they are not based on the delay function. One follows the evolution of the wave function during a set of time intervals dt , where dt is long compared to the correlation time of the dissipative process, but short compared to the relaxation time. At the end of each interval, a quantum jump may or may not occur, with well defined probabilities. One gets in this way a picture of the time evolution which consists of a series of quantum jumps separated by time intervals where the atomic state is described by a wave function. Such approaches are called for that reason “Monte Carlo wave function”. Their complete equivalence with optical Bloch equations has been proven. They allow one to calculate not only one-time averages but also two-time averages. Their main interest is that they are much simpler numerically than optical Bloch equations since dealing with wave functions requires less computer memory than for density matrices (N complex numbers instead of N^2). They look therefore very promising for investigating a whole series of problems.

8. Conclusion

We will conclude this report by summarizing a few important points which emerge from the previous discussion.

The first one concerns reactive effects versus dissipative effects, i.e. Hamiltonian terms such as light shifts versus relaxation terms such as absorption rates. They both play an important role in the new cooling mechanisms. Their respective contributions can be controlled by varying the detuning or the interaction time. The correlations between them play a crucial role. For example, Sisyphus cooling results from strong correlations between the spatial modulation of light shifts and optical pumping rates.

The second point concerns classical versus quantum effects. Semiclassical treatments work quite well

because photon scattering destroys atomic spatial coherences and produces a very strong localization of atoms within the laser wave. It is clear however that a full quantum treatment is needed to determine the ultimate limits of laser cooling.

Another important point concerns the interplay between internal and external degrees of freedom. An interesting idea is that it is possible to control the exchanges of linear momentum between atoms and photons via the internal angular momentum. One can explain in this way the anomalous momentum diffusion in a $\sigma^+ - \sigma^-$ laser configuration, which is due to the fact that an atom which has been optically pumped after the absorption of a σ^+ photon has a higher probability to absorb a second σ^+ photon than a σ^- one [35, 36, 63]. The atom then performs a random walk in momentum space with large steps, of several $\hbar k$. Other new types of random walk have been mentioned in this report, in a bipotential, or with variable delays between steps, which can become very long. Note finally that internal variables are not necessarily the fastest ones. They can become the slowest ones, as, for example, in Sisyphus cooling, when the oscillation period in the optical potential wells becomes shorter than the optical pumping time.

An important new trend of the field concerns the development of connections with quantum measurement theory. For example, photon scattering can be considered as a quantum measurement process. The quantum evolution of the system can also be described in terms of quantum jumps and with Monte Carlo simulations. More generally, it appears that a description in terms of wave functions, supplemented by appropriate quantum jumps, could be more efficient than the traditional description in terms of density matrices.

Finally, the new field of atomic waves and atomic optics begins to be explored. We have seen that atomic motion can be quantized in a light wave, with a band structure, even in the long time limit. Interference between atomic waves has been observed [14]. Here also, the flexibility provided by internal degrees of freedom can be very useful. Multiwave interferometry can be also envisioned with atomic modes in a cavity [14]. Finally, a fascinating possibility would be to try to trap several ultra-cold atoms [64] and to observe quantum statistical effects, such as Bose-Einstein condensation or macroscopic wave functions.

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