To appear in Proceedings of TENICOLS'91
(Edited by M. DUCLOY, E. GIACOBINO, G. CAMY, World Scientific)

REVIEW ON FUNDAMENTAL PROCESSES IN LASER COOLING

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ABSTRACT

Laser cooling is based on photon scattering. New physical insights can be obtained on photon scattering by considering it as a quantum measurement process or by associating a series of quantum jumps with a sequence of scattering processes.

1. Introduction

Laser cooling and trapping is an expanding field of research where spectacular developments have occurred during the last few years[1]. Very low kinetic temperatures, in the microkelvin range, have been obtained[2], opening the way to the realization of new schemes, such as atomic fountains, which seem quite promising for the improvement of atomic clocks[3]. Another important feature of such ultracold atoms is their long de Broglie wavelength which makes the wave aspects of atomic motion easier to detect. Several papers of these proceedings are devoted to the new subject of atomic interferometry.

All these developments provide a great stimulation for a deeper understanding of the quantum features of atomic motion in laser light. New types of questions may be asked leading to new physical insights in photon-atom interactions. In this paper, we present and discuss a few examples of such problems which exhibit the connection existing between photon scattering and a quantum measurement process.

We first consider in Section 2 a single photon scattering process and we show how it can be considered as a quantum measurement of the atom's position in the von Neumann's sense. Such an analogy clearly explains why photon scattering destroys spatial coherences and why therefore it should be avoided in atomic interferometers. The theoretical analysis of Section 2 is then applied in Section 3 to

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the discussion of a question which has been asked about the detection of spatial coherences. If an atom has been prepared in a coherent superposition of two non overlapping wave packets, is it possible to detect this spatial coherence by interference fringes on the scattering cross section of an incoming photon ? Finally, we consider in Section 4 a sequence of scattering processes and we give a description of the atomic evolution in terms of a series of coherent evolutions separated by quantum jumps occurring at random times.

2. Photon Scattering and Spatial Coherences

2.1 Spatial Coherence Length

Consider an atom in a translational state described by a density operator σ. Spatial coherence for such an atom is related to the off-diagonal elements of σ, $\langle \mathbf{r}'|\sigma|\mathbf{r}''\rangle$, in the position representation. More precisely, one can introduce the global spatial coherence at a distance $\boldsymbol{\rho}$, defined by

$$F(\boldsymbol{\rho}) = \int d^3r \ \langle \mathbf{r}|\sigma|\mathbf{r}+\boldsymbol{\rho}\rangle \tag{1}$$

and which is the sum of all spatial coherences between two points separated by a fixed distance $\boldsymbol{\rho}$. Changing from the position representation to the momentum representation, one can easily derive the following relation

$$F(\boldsymbol{\rho}) = \int d^3p \ e^{i\mathbf{p}\cdot\boldsymbol{\rho}/\hbar}\langle \mathbf{p}|\sigma|\mathbf{p}\rangle \tag{2}$$

which shows that $F(\boldsymbol{\rho})$ is the Fourier transform of the momentum distribution $\langle \mathbf{p}|\sigma|\mathbf{p}\rangle$. The normalization of σ results in $F(\mathbf{0}) = 1$.

The spatial coherence length ξ is the typical length characterizing the decrease of $F(\boldsymbol{\rho})$ with $|\boldsymbol{\rho}|$. From Eq. 2, it clearly appears that the narrower the momentum distribution, i.e. the colder the atom, the broader is $F(\boldsymbol{\rho})$, i.e. the larger is ξ. For example, for a particle of mass M, in thermal equilibrium at temperature T, ξ is, within a numerical factor, the well known thermal de Broglie wavelength $\lambda_T = \left(2\pi\hbar^2/Mk_BT\right)^{1/2}$. Laser cooling is thus interesting for achieving large spatial coherence lengths. An important question concerns then the limits which can be reached by such methods. If the cooled atoms don't stop absorbing and reemitting photons, it seems impossible to avoid the random recoil due to spontaneous emission, so that the momentum distribution has a width Δp larger than the photon momentum $\hbar k$. It then follows from Eq. 2 that $F(\boldsymbol{\rho})$ has a width smaller than $1/k$. The fundamental limit for the spatial coherence lengths which can be achieved by usual laser cooling methods seems therefore to be the laser wavelength $\lambda = 2\pi/k$[(*)]. We try now to give a new physical insight in this problem in terms of quantum measurement theory.

(*) Such a limitation can be removed in certain cases, for example if the photon absorption probability varies rapidly with the atomic momentum $\mathbf{p}$ around $\mathbf{p} = \mathbf{0}$ (see Section 4).

2.2 Main features of a quantum measurement process

We first briefly recall the main features of a quantum measurement process in the von Neumann's sense[4]. Let $\mathcal{S}$ be the measured system and $\mathcal{A}$ the observable of $\mathcal{S}$ which is measured by the measuring apparatus $\mathcal{M}$. If $\mathcal{S}$ is initially in an eigenstate $|a\rangle$ of $\mathcal{A}$, and if $\mathcal{M}$ is initially in the state $|\mathcal{X}_{\text{in}}\rangle$, the $\mathcal{S}$-$\mathcal{M}$ interaction is supposed to lead the total $\mathcal{S}+\mathcal{M}$ system in the state $|a\rangle \otimes |\mathcal{X}_a\rangle$

$$|a\rangle \otimes |\mathcal{X}_{\text{in}}\rangle \longrightarrow |a\rangle \otimes |\mathcal{X}_a\rangle \tag{3}$$

In other words, $\mathcal{S}$ remains in the eigenstate $|a\rangle$ of $\mathcal{A}$ whereas $\mathcal{M}$ ends in a state $|\mathcal{X}_a\rangle$ which is correlated with $|a\rangle$. This correlation is perfect if, for any pair $|a\rangle$, $|a'\rangle$ of orthogonal eigenstates of $\mathcal{A}$, the two corresponding states of $\mathcal{S}$, $|\mathcal{X}_a\rangle$ and $|\mathcal{X}_{a'}\rangle$, are themselves orthogonal

$$\langle a'|a\rangle = 0 \Longrightarrow \langle \mathcal{X}_{a'}|\,\mathcal{X}_a\rangle = 0 \tag{4}$$

The measurement is then ideal in so far as observing the final state of $\mathcal{M}$ determines unambiguously the initial state of $\mathcal{S}$.

From the linearity of Schrödinger equation, it then follows that, if $\mathcal{S}$ is initially in a linear superposition of the states $|a\rangle$, $\mathcal{S}+\mathcal{M}$ ends in the same linear superposition of the states $|a\rangle \otimes |\mathcal{X}_a\rangle$

$$\left(\sum_a c_a|a\rangle\right) \otimes |\mathcal{X}_{\text{in}}\rangle \longrightarrow \sum_a c_a|a\rangle \otimes |\mathcal{X}_a\rangle \tag{5}$$

Suppose now that, after the measurement process, $\mathcal{S}$ and $\mathcal{M}$ no longer interact. It is then well known that all predictions concerning $\mathcal{S}$ alone can be deduced from the reduced density operator of $\mathcal{S}$ obtained by tracing $|\psi_{\text{fin}}\rangle\langle\psi_{\text{fin}}|$ over $\mathcal{M}$, where $|\psi_{\text{fin}}\rangle$ is the final state of $\mathcal{S}+\mathcal{M}$ appearing on the right hand side of Eq. 5

$$\sigma_{\text{fin}} = Tr_{\mathcal{M}}\,|\psi_{\text{fin}}\rangle\langle\psi_{\text{fin}}| = \sum_a \sum_{a'} c_a c_{a'}^*\,\langle\mathcal{X}_{a'}|\,\mathcal{X}_a\rangle\,|a\rangle\langle a'| \tag{6}$$

If one compares Eq. 6 with the initial density operator of $\mathcal{S}$, $\sigma_{\text{in}} = \sum_a \sum_{a'} c_a c_{a'}^* |a\rangle\langle a'|$ one concludes that the diagonal elements of σ remain unchanged during the measurement process since

$$\langle a\,|\sigma_{\text{fin}}|\,a\rangle = |c_a|^2\,\langle\mathcal{X}_a|\,\mathcal{X}_a\rangle = |c_a|^2 = \langle a\,|\sigma_{\text{in}}|\,a\rangle \tag{7}$$

whereas the off diagonal elements

$$\langle a\,|\sigma_{\text{fin}}|\,a'\rangle = c_a c_{a'}^*\,\langle\mathcal{X}_{a'}|\,\mathcal{X}_a\rangle = \langle a\,|\sigma_{\text{in}}|\,a'\rangle\,\langle\mathcal{X}_{a'}|\,\mathcal{X}_a\rangle \tag{8}$$

become multiplied by $\langle \mathcal{X}_{a'} | \mathcal{X}_a \rangle$ which vanishes if the measurement is perfect. In other words, in the basis of eigenstates of the measured observable $\mathcal{A}$, the measurement process appears as a pure T_2 relaxation process.

2.3 Photon Scattering as a Quantum Measurement Process

We come back to photon scattering. Such a process is entirely characterized by the scattering $\mathcal{S}$-matrix which gives the amplitudes of the elementary processes

$$|\mathbf{K}\rangle \otimes |\mathbf{k}_i\rangle \longrightarrow |\mathbf{K} + \mathbf{k}_i - \mathbf{k}_f\rangle \otimes |\mathbf{k}_f\rangle \tag{9}$$

where an atom with momentum $\hbar\mathbf{K}$ scatters a photon whose momentum changes from $\hbar\mathbf{k}_i$ to $\hbar\mathbf{k}_f$. Conservation of the total momentum explicitly appears in Eq. 9 since the atom momentum changes from $\hbar\mathbf{K}$ to $\hbar\,(\mathbf{K} + \mathbf{k}_i - k_f)$. We will denote $\mathcal{S}\,(\mathbf{k}_i, \mathbf{k}_f; \mathbf{K})$ the amplitude of such a process. The $\mathbf{K}$ dependence of this amplitude is due for example to the Doppler effect which is proportionnal to the atomic velocity $\hbar\mathbf{K}/M$, where M is the atom's mass.

The most general initial state $|\varphi\rangle$ for the atom can be written

$$\begin{aligned} |\varphi\rangle &= \int d^3r\ |\mathbf{r}\rangle\langle\mathbf{r}|\varphi\rangle = \int d^3r\ \varphi(\mathbf{r})|\mathbf{r}\rangle \\ &= \int d^3K\ |\mathbf{K}\rangle\langle\mathbf{K}|\varphi\rangle = \int d^3K\ \tilde{\varphi}(\mathbf{K})|\mathbf{K}\rangle \end{aligned} \tag{10}$$

where $\varphi(\mathbf{r}) = \langle\mathbf{r}|\varphi\rangle$ and $\tilde{\varphi}(\mathbf{K}) = \langle\mathbf{K}|\varphi\rangle$ are the wave functions associated with $|\varphi\rangle$ in the position representation and the momentum representation respectively. From the linearity of the Schrödinger equation, one deduces that, if the initial state of the "atom + photon" system is

$$\begin{aligned} |\psi_{\text{in}}\rangle &= |\varphi\rangle \otimes |\mathbf{k}_i\rangle \\ &= \int d^3r\ \varphi(\mathbf{r})|\mathbf{r}\rangle \otimes |\mathbf{k}_i\rangle = \int d^3K\ \tilde{\varphi}(\mathbf{K})|\mathbf{K}\rangle \otimes |\mathbf{k}_i\rangle \end{aligned} \tag{11}$$

the final state is

$$|\psi_{\text{fin}}\rangle = \int d^3k_f \int d^3K\ \mathcal{S}\,(\mathbf{k}_i, \mathbf{k}_f; \mathbf{K})\,\tilde{\varphi}(\mathbf{K})\,|\mathbf{K} + \mathbf{k}_i - \mathbf{k}_f\rangle \otimes |\mathbf{k}_f\rangle \tag{12}$$

We introduce now an approximation which consists in neglecting the $\mathbf{K}$ dependence of $\mathcal{S}^{(*)}$. In several cases, this is a very good approximation (see however the

(*) $\mathcal{S}$ could eventually depend on the average value of $\mathbf{K}$. What we neglect here is the variation of $\mathcal{S}$ with $\mathbf{K}$ over the width ΔK of $\tilde{\varphi}(\mathbf{K})$.

section 4 of this paper, for an example of situation where the $\mathbf{K}$ – dependence of $\mathcal{S}$ plays an essential role)

$$\mathcal{S}(\mathbf{k}_i, \mathbf{k}_f; \mathbf{K}) \simeq \mathcal{S}(\mathbf{k}_i, \mathbf{k}_f) \tag{13}$$

Introducing Eq. 13 in Eq. 12 and using

$$|\mathbf{K} + \mathbf{k}_i - \mathbf{k}_f\rangle = e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{R}}|\mathbf{K}\rangle \tag{14}$$

where $\mathbf{R}$ is the position operator of the atom, we can transform Eq. 12 into

$$|\psi_{\text{fin}}\rangle \simeq \int d^3k_f \; \mathcal{S}(\mathbf{k}_i, \mathbf{k}_f)\, e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{R}}|\varphi\rangle \otimes |k_f\rangle \tag{15}$$

We replaced $\int d^3\mathbf{K}\; \tilde{\varphi}(\mathbf{K})|\mathbf{K}\rangle$ by $|\varphi\rangle$. Using Eq. 10 to replace $|\varphi\rangle$ by $\int d^3\mathbf{r}\; \varphi(\mathbf{r})|\mathbf{r}\rangle$, and the fact that $|\mathbf{r}\rangle$ is an eigenstate of $\mathbf{R}$ with eigenvalue $\mathbf{r}$, we finally get

$$|\psi_{\text{fin}}\rangle = \int d^3r \; \varphi(\mathbf{r})|\mathbf{r}\rangle \otimes |\mathcal{X}_\mathbf{r}\rangle \tag{16}$$

where

$$|\mathcal{X}_\mathbf{r}\rangle = \int d^3k_f \; e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{r}} \; \mathcal{S}(\mathbf{k}_i, \mathbf{k}_f)\, |\mathbf{k}_f\rangle \tag{17}$$

is a photon state which depends on $\mathbf{r}$. Comparing Eq. 11 and Eq. 16, we can describe the scattering process by the transformation

$$|\psi_{\text{in}}\rangle = \left(\int d^3r \; \varphi(\mathbf{r})|\mathbf{r}\rangle\right) \otimes |k_i\rangle \longrightarrow |\psi_{\text{fin}}\rangle = \int d^3r \; \varphi(\mathbf{r})|\mathbf{r}\rangle \otimes |\mathcal{X}_\mathbf{r}\rangle \tag{18}$$

which is quite similar to Eq. 5.

As in a von Neumann's measurement process, each position state of the atom becomes correlated with a photon state $|\mathcal{X}_\mathbf{r}\rangle$ which depends on $\mathbf{r}$. The probability for the atom to be in $\mathbf{r}$, after the scattering process, is equal to $|\varphi(r)|^2 \langle \mathcal{X}_\mathbf{r} | \mathcal{X}_\mathbf{r}\rangle$, and remains unchanged since $\langle \mathcal{X}_\mathbf{r} | \mathcal{X}_\mathbf{r}\rangle = 1$ as a consequence of the unitarity of the $\mathcal{S}$-matrix. More precisely, one deduces from Eq. 17 that

$$\langle \mathcal{X}_{\mathbf{r}'} | \mathcal{X}_{\mathbf{r}''}\rangle = \int d^3k_f \, |\mathcal{S}(\mathbf{k}_i, \mathbf{k}_f)|^2 \, e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot(\mathbf{r}' - \mathbf{r}'')} \tag{19}$$

Since the variations of $|\mathcal{S}(\mathbf{k}_i, \mathbf{k}_f)|^2$ with $\mathbf{k}_f$ are restricted to an interval on the order of $2\hbar k$, it follows from Eq. 19 that $\langle \mathcal{X}_{\mathbf{r}'} | \mathcal{X}_{\mathbf{r}''}\rangle$ is a function of $\mathbf{r}' - \mathbf{r}''$, which is equal to 1 for $\mathbf{r}' = \mathbf{r}''$ and which tends to 0 if $|\mathbf{r}' - \mathbf{r}''| \gg \lambda$. The fact that $|\mathcal{X}_{\mathbf{r}'}\rangle$ and $|\mathcal{X}_{\mathbf{r}''}\rangle$ become orthogonal only if $|\mathbf{r}' - \mathbf{r}''| \gg \lambda$ means that the measurement of the atom's position by photon scattering is not perfect, but has a finite resolution, on the order of the photon wavelength λ. This is in agreement with the well known

result of wave optics according to which two points cannot be resolved optically if their distance is smaller than λ.

The calculation which leads from Eq. 5 to Eq. 8 can be repeated for Eq. 18 and gives

$$\langle \mathbf{r}' | \sigma_{\text{fin}} | \mathbf{r}'' \rangle = \varphi(\mathbf{r}')\varphi^*(\mathbf{r}'') \langle \mathcal{X}_{\mathbf{r}'} | \mathcal{X}_{\mathbf{r}''} \rangle = \langle \mathbf{r}' | \sigma_{\text{fin}} | \mathbf{r}'' \rangle \langle \mathcal{X}_{\mathbf{r}'} | \mathcal{X}_{\mathbf{r}''} \rangle \qquad (20)$$

This shows that, after a scattering process, the spatial coherence length is necessarily smaller than λ, whatever the initial state may be. We find again the result derived above in Subsection 2.1, but here in the context of quantum measurement theory.

So far, we have considered only a single scattering process. One can show that, if the atom undergoes a sequence of independent scattering processes, its spatial coherences $\langle \mathbf{r}' | \sigma | \mathbf{r}'' \rangle$ are damped, even if $|\mathbf{r}' - \mathbf{r}''|$ is smaller than λ, with a rate proportional to $|\mathbf{r}' - \mathbf{r}''|^2$. This explains why macroscopic systems, with large scattering cross-sections, are rapidly localized by their interaction with the environment and how classical properties emerge as a result of this coupling[5].

To sum up, photon scattering may be considered as a measurement process of the atom's position. This measurement has a finite resolution given by λ and destroys spatial coherences beyond a range which is also given by λ. These results provide also some physical insight in the quantum state of an atom in an optical molasses. Because of the quantum correlations which appear as a result of photon-atom interactions, the state of the atom cannot be described by a wave packet, but rather by a statistical mixture of wave packets. Since the spatial coherence length is necessarily smaller than λ, as a consequence of scattering processes, each of the wave packets forming the statistical mixture has a width which is smaller than λ, but the centers of these wave packets are distributed over a range Δr which may be much larger than λ. In other words, a clear distinction must be made between the width Δr of the position distribution $\langle \mathbf{r} | \sigma | \mathbf{r} \rangle$, which can increase indefinitely by spatial diffusion (if the atom is not trapped), and the spatial coherence length ξ which is reduced to very low values by photon scattering.

3. Are Photon Scattering Cross-sections Sensitive to Atomic Spatial Coherences ?

Consider a single atom whose wave function $\varphi(\mathbf{r})$ is a linear superposition of two wave packets $\varphi_a(\mathbf{r})$ and $\varphi_b(\mathbf{r})$, centered on two points $\mathbf{r}_a$ and $\mathbf{r}_b$, the width of each of these wave packets being small compared to $|\mathbf{r}_a - \mathbf{r}_b|$.

$$\varphi(\mathbf{r}) = c_a \varphi_a(\mathbf{r}) + c_b \varphi_b(\mathbf{r}) \qquad (21)$$

Suppose that a photon with momentum $\hbar \mathbf{k}_i$, is impinging on this atom and that one looks at the scattered photon along a direction $\mathbf{k}_f / k_f$ different from $\mathbf{k}_i / k_i$. Intuitively, one is tempted to consider that the incident light wave is scattered

simultaneously by the two wave packets φ_a and φ_b, so that one expects to have interference effects between the two outgoing waves emerging from $\mathbf{r}_a$ and $\mathbf{r}_b$. Is such a picture correct ? Does the differential scattering cross-section exhibit interference fringes depending on $\mathbf{r}_a - \mathbf{r}_b$ when $\mathbf{k}_f/k_f$ is varied ? (*)

The calculations of the previous section allow one to give a clear answer to this question, since we know the final state of the "photon+atom" system after the scattering process. The probability to find the photon in the state $|\mathbf{k}_f\rangle$ is equal to the norm of the vector multiplying $|\mathbf{k}_f\rangle$ in Eq. 15, i.e. to(**)

$$|\mathcal{S}(\mathbf{k}_i, \mathbf{k}_f)|^2 \left\langle \varphi \left| e^{-i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{R}} e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{R}} \right| \varphi \right\rangle = |\mathcal{S}(\mathbf{k}_i, k_f)|^2 \qquad (22)$$

This is independent of the double peak structure of $\varphi(\mathbf{r})$, which shows that there are no corresponding interference fringes in the scattering cross-section.

Such a result, which remains valid even if the approximation corresponding to Eq. 13 is not made, has actually a simple physical meaning. There are indeed two scattering paths for the incoming photon, one through $|\varphi_a\rangle$ and one through $|\varphi_b\rangle$. But the final states of the atom corresponding to these two paths are not the same : the scattering through $|\varphi_a\rangle$ leaves the atom localized near $\mathbf{r}_a$, whereas the scattering through $|\varphi_b\rangle$ leaves the atom localized near $\mathbf{r}_b$. It follows that the two scattering paths cannot interfere because they correspond to orthogonal final atomic states.

This would be no longer true if, instead of a single atom, we had two atoms, one in the state $|\varphi_a\rangle$, the other in the state $|\varphi_b\rangle$. Now, the two paths could interfere, provided however that the momentum transfer $\hbar(\mathbf{k}_f - \mathbf{k}_i)$ occurring after a scattering through $|\varphi_i\rangle$ ($i = a$ or b) does not transform $|\varphi_i\rangle$ into a state orthogonal to $|\varphi_i\rangle$. For such a condition to be fulfilled, the momentum spread in $|\varphi_i\rangle$ must be large compared to $\hbar k$, which means that the spatial extent of the wave packet $\varphi_i(\mathbf{r})$ must be small compared to λ.

4. Photon Scattering and Quantum Jumps

4.1 Coming Back to the Approximation Made on the $\mathcal{S}$-Matrix

In the calculations of the previous two sections, we have neglected the $\mathbf{K}$ – dependence of the amplitude $\mathcal{S}(\mathbf{k}_i, \mathbf{k}_f; \mathbf{K})$ associated with the process (see Eq. 13). To interpret Doppler cooling, it is necessary to introduce the first order corrections in $\mathbf{K}$, which describe how the scattering cross-section depends on the atomic velocity through first-order Doppler effect. It is then possible to describe

(*) We are grateful to W.D. Phillips and G.P. Lafyatis for bringing this problem to our attention.

(**) Note that we don't specify here the final state of the atom. We measure only the final state of the photon, so that the cross-section calculated here is the total one (elastic plus inelastic).

the competition between the broadening of the momentum distribution due to the terms of $\mathcal{S}$ independent of $\mathbf{K}$ and the narrowing due to the first order corrections. Note however that, since scattering processes never stop, the coherence length ξ remains always smaller than λ.

It may happen that the amplitude $\mathcal{S}(\mathbf{k}_i, \mathbf{k}_f; \mathbf{K})$ vanishes for certain values $\mathbf{K}_0$ of $\mathbf{K}$. For example, one can have two distinct absorption amplitudes whose interference is perfectly destructive for $\mathbf{K} = \mathbf{K}_0$. This is the case for velocity selective coherent population trapping[6] (V.S.C.P.T.). Since scattering processes stop for $\mathbf{K} = \mathbf{K}_0$, one can show that, as a result of optical pumping and filtering in momentum space, it is now possible to get coherent lengths larger than λ. Because of the rapid variations of $\mathcal{S}$ around $\mathbf{K} = \mathbf{K}_0$, the calculations of Section 3 are no longer valid for analyzing such a process. We present now a new approach to photon scattering in terms of quantum jumps which can be applied to V.S.C.P.T., and which provides a new insight in the time evolution of the system.

4.2 A New Method for Describing a Sequence of Scattering Processes

We introduce the principe of this method on the simple case of one dimensional V.S.C.P.T. (Fig. 1)

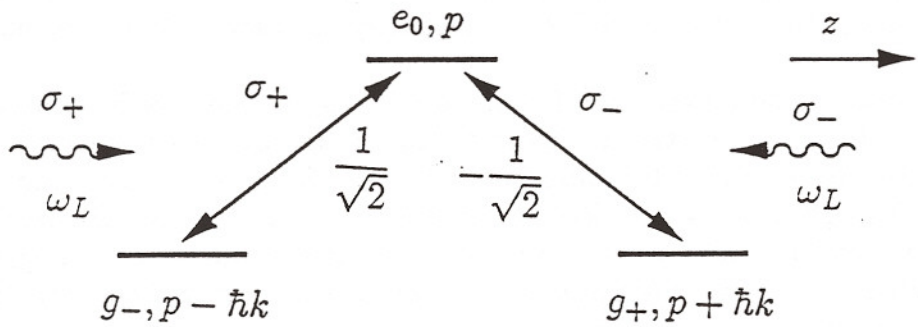

Fig. 1 Laser configuration and atomic level scheme used in one dimensional V.S.C.P.T.

Two σ^+ and σ^- laser beams, propagating along the positive and negative directions of the Oz axis, drive respectively the $g_{-1}, p - \hbar k \longleftrightarrow e_0, p$ and the g_{+1}, $p + \hbar k \longleftrightarrow e_0, p$ transitions between the three atomic levels e_0, g_{-1}, g_{+1} with angular momenta along Oz equal to $0, -\hbar, +\hbar$, respectively (e_0, p represents a state where the atom is in the excited sublevel e_0, with momentum p along Oz). Note the selection rules resulting from the conservation of the total linear and angular momenta along Oz. The Clebsch-Gordan coefficients of the two σ^+ and σ^- transitions are equal to $+1/\sqrt{2}$ and $-1/\sqrt{2}$, respectively.

As long as spontaneous emission is not taken into account, we have a coherent evolution between the three states of Fig. 1, which can be described in terms of Rabi precessions and stimulated Raman processes. Spontaneous emission introduces a random character in the atomic evolution. At random times,

the atom "jumps" into the lower states, while a fluorescence photon appears in one of the initially empty modes of the quantized radiation field. Each individual scattering process can thus be associated with a quantum jump of the atom. To study the statistics of this sequence of quantum jumps, it is very convenient to consider the "delay function", introduced for analyzing intermittent fluorescence[7]. We summarize now the main steps of such an approach, as it can be applied to V.S.C.P.T.

(i) When an atom in e_0, with a well defined momentum p' along Oz, spontaneously emits a photon in a given direction with polar angles θ and Φ, and with a given polarization ε, it jumps into a well defined linear superposition of g_{-1} and g_{+1}. In a 1D problem, we are not interested in the azimuthal angle Φ and in the polarization ε. Averaging over Φ and ε leads, for the state of the atom just after the jump, to a statistical mixture with equal weights $1/2$ of g_{-1} and g_{+1}, the momentum of the atom along Oz being $p' - \hbar k\cos\theta$. So we can decide randomly the value of θ (according to the emission diagram) and the sublevel g_{-1} or g_{+1} into which the atom jumps after a spontaneous emission.

(ii) Suppose that the previous random choice has given an atom jumping at time $\tau = 0$ into $g_{-1}, p' - \hbar k\cos\theta$ and let us put $p' - \hbar k\cos\theta = p - \hbar k$. After such a jump the wave function of the total system evolves according to

$$\begin{aligned} |\psi(\tau)\rangle = & \left[c_0(\tau)\,|e_0, p\rangle + c_1(\tau)\,|g_{+1}, p + \hbar k\rangle + c_{-1}(\tau)\,|g_{-1}, p - \hbar k\rangle\right] \\ & \otimes |0 - \text{fluorescence photon}\rangle \\ & + \text{ States with } 1, 2 \ldots \text{ fluorescence photons} \end{aligned} \tag{23}$$

(iii) The probability to have the *next* spontaneous emission occurring between τ and $\tau + d\tau$ is then given by

$$W(\tau)d\tau = \Gamma\,|c_0(\tau)|^2\,dt \tag{24}$$

where Γ is the spontaneous emission rate (natural width of e_0). According to Eq. 24 and Eq. 23, $W(\tau)$ is the departure rate from the $0 -$ fluorescence photon manifold. $W(\tau)$ can also be considered as the distribution of the time intervals $\tau = t_{n+1} - t_n$ between two *successive* spontaneous jumps, the n^{th} one occurring at $t = t_n$ and the next one occurring at $t = t_{n+1}$.

(iv) The three functions $c_0(\tau), c_1(\tau), c_{-1}(\tau)$ appearing in the first two lines of Eq. 23 and describing the evolution within the 0-fluorescence photon manifold can be obtained by solving a Schrödinger equation governed by an effective non Hermitian hamiltonian H_{eff}, obtained by adding and imaginary term $-i\hbar\Gamma/2$ to the

energy of $|e_0, p\rangle$(*)

$$H_{\text{eff}} = \begin{pmatrix} \frac{p^2}{2M} - i\hbar\frac{\Gamma}{2} & -\frac{\hbar\Omega_1}{2\sqrt{2}} & \frac{\hbar\Omega_1}{2\sqrt{2}} \\ -\frac{\hbar\Omega_1}{2\sqrt{2}} & \frac{(p+\hbar k)^2}{2M} + \hbar\delta & 0 \\ \frac{\hbar\Omega_1}{2\sqrt{2}} & 0 & \frac{(p-\hbar k)^2}{2M} + \hbar\delta \end{pmatrix} \tag{25}$$

In Eq. 25, Ω_1 is the Rabi frequency associated with the two σ^+ and σ^- laser fields, assumed to have the same amplitude, and $\delta = \omega_L - \omega_A$ is the detuning between the laser and atom frequencies. For $\tau = 0$, we have $c_{-1}(0) = 1$, $c_0(0) = c_{+1}(0) = 0$.
(v) Once the next spontaneous emission process has occurred at $t = t_{n+1}$, we know a posteriori that, between $t = t_n$ and $t = t_{n+1}$, the system is certainly in the 0-fluorescence photon manifold. Its state is thus described between t_n and t_{n+1}, by the normalized state vector

$$\frac{c_0(t)\,|e_0, p\rangle + c_1(t)\,|g_{+1}, p+\hbar k\rangle + c_{-1}(t)\,|g_{-1}, p-\hbar k\rangle}{\left[|c_0(t)|^2 + |c_1(t)|^2 + |c_{-1}(t)|^2\right]^{1/2}} \tag{26}$$

4.3 Monte-Carlo Simulation of the Quantum Jumps Occurring in V.S.C.P.T.

The procedure outlined in the previous subsection can be applied to V.S.C.P.T. and provides a Monte-Carlo simulation of such a phenomenon, preserving its quantum features.

The key point for V.S.C.P.T. is the existence of atomic states which are not coupled to the laser field. If one introduces the two orthogonal linear combinations of $g_{-1}, p - \hbar k$ and $g_{+1}, p + \hbar k$ given by

$$\begin{aligned} |\psi_{NC}(p)\rangle &= 2^{-1/2}\left[\,|g_{-1}, p-\hbar k\rangle + |g_{+1}, p+\hbar k\rangle\right] \\ |\psi_{C}(p)\rangle &= 2^{-1/2}\left[\,|g_{-1}, p-\hbar k\rangle - |g_{+1}, p+\hbar k\rangle\right], \end{aligned} \tag{27}$$

one can easily check that $|\psi_{NC}(p)\rangle$ is not coupled to $|e_0, p\rangle$ by the laser-atom interaction hamiltonian (terms proportional to Ω_1 in Eq. 25)). On the other hand, the fact that the two states $|g_{\pm 1}, p \pm \hbar k\rangle$ have not the same kinetic energy results in the appearance a motional coupling between $|\psi_{NC}(p)\rangle$ and $|\psi_C(p)\rangle$ proportional to the difference between these two kinetic energies. Actually, one can easily show from Eq. 25 and Eq. 27 that

$$\langle\psi_{NC}(p)|\,H_{\text{eff}}\,|\psi_C(p)\rangle = \hbar kp/M \tag{28}$$

(*) Such a simplification is due to the fact that, in the 0-fluorescence photon manifold, spontaneous emission can be entirely described by departure rates.

It follows that, if $p = 0$, the state $|\psi_{NC}(p = 0)\rangle$ is a perfect trap where the atoms can remain trapped indefinitely, such a trap becoming less and less perfect when $|p|$ increases as a consequence of the indirect coupling between $|\psi_{NC}(p)\rangle$ and $|e_0, p\rangle$ through $|\psi_C(p)\rangle$.

In the quantum jump description, one can say that one of the three complex eigenvalues of Eq. 25 has a damping rate which becomes smaller and smaller when $p \longrightarrow 0$, so that the time delay between two successive quantum jumps can increase considerably when p gets smaller and smaller. Since p can change in a random way after each jump, and since this change of p is taken into account in the procedure of Subsection 4.2, it follows that the length of the "dark periods" (periods between two successive jumps) can change during the time evolution, becoming longer and longer when p gets smaller and smaller. Figure 2 shows the results of a Monte-Carlo type simulation which confirms such predictions.

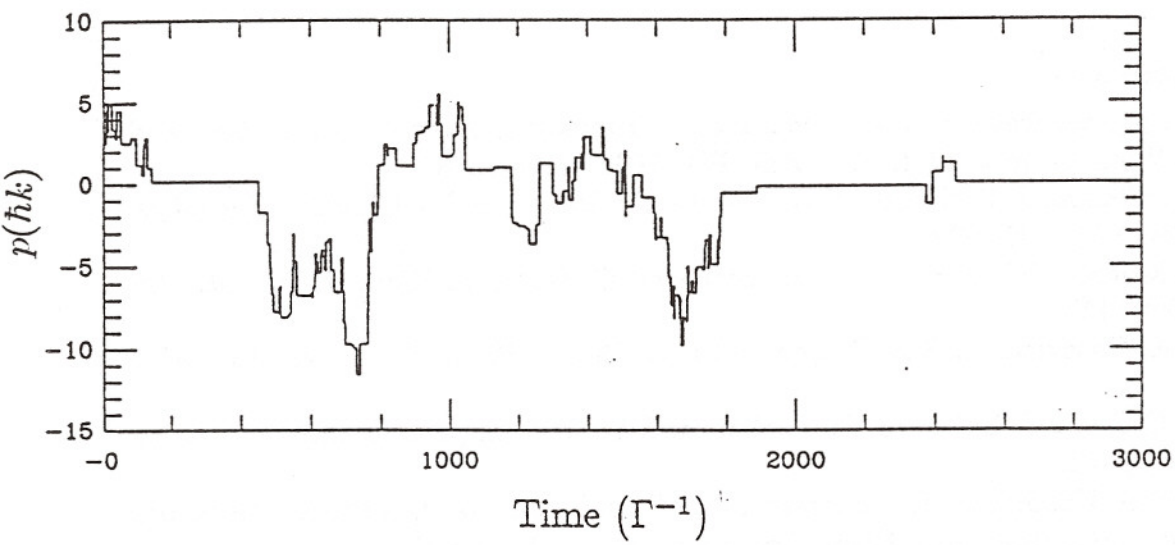

Fig. 2 Monte-Carlo simulation of the quantum jumps occurring in V.S.C.P.T. Curve giving the variations of p with time. Each discontinuity corresponds to a quantum jump. The time interval τ between each jump and the next one (dark period) is a random variable which is chosen according to the distribution $W(\tau)$ which depends on p. Note that the dark periods are longer when p is close to zero.

4.4 Advantages of Such an Approach

Figure 2 shows that the atom spends most of its time in long dark periods. Since its state is then given by Eq. 26 and since only $|\psi_{NC}(p)\rangle$ is associated with a small decay rate, it is clear that the weight of $|\psi_{NC}(p)\rangle$ in Eq. 26 becomes rapidly predominant in these dark periods. We thus clearly understand how an atom which jumps into g_{-1} or g_{+1} after a spontaneous emission process is then rapidly filtered in the dark state $|\psi_{NC}(p)\rangle$ if p is small enough. Averaging over a set of such Monte-Carlo realizations, one can reconstruct the momentum distribution

of the atom. The results obtained in this way are in good agreement with those obtained from a numerical solution of optical Bloch equations.

Similar Monte-Carlo approaches have been recently developed for dealing with dissipative processes in quantum optics[8]. They lead to a picture of the time evolution of the atom which consists of a series of quantum jumps separated by time invervals where the atomic state can be described by a wave function. Such approaches are called for that reason "Monte-Carlo Wave Function". Their complete equivalence with optical Bloch equations has been proven. They not only give a new physical insight in dissipative processes, but they are also numerically simpler since dealing with wave functions requires less computer memory than for density matrices. They look therefore very promising for investigating a whole series of problems.

5. Acknowledgements

We thank C. Westbrook and J. Dalibard for fruitful discussions.

6. Références

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