Particles and strings in six-dimensional (2, 0) theory

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The six-dimensional $(2,0)$ theories

In 1995, we learned of the existence of

quantum theories in six space-time dimensions with $(2,0)$ *superconformal symmetry.*

Here are three reasons to study these theories:

Finding the right conceptual framework to define them is a challenging problem.
This is likely to involve new interesting mathematical structures.

They give rise to certain Yang-Mills theories with maximally extended supersymmetry upon compactification on a two-torus.
This may be a way to find an S*-dual formulation of these lower dimensional theories.*

They arise within string/M*-theory as decoupled subsectors localized on certain space-time impurities (branes or singularities).*
This may provide an opportunity to study aspects of these higher dimensional theories that are not related to quantum gravity.

Tensor multiplet particles

The best known representation of the $(2,0)$ supersymmetry algebra is the tensor multiplet of massless space-time fields:

field	$SO(5,1)$	$SO(5)_R$
ϕ	scalar	vector
ψ	anti-chiral spinor	spinor
$h = db$	self-dual three-form	scalar

Second quantization of these fields yields a Fock space of massless particle states.
These particles transform as scalars, spinors, and self-dual tensors under the little group $SO(4) \subset SO(5,1)$ of transverse rotations.

Dimensional reduction of these fields gives rise to a massless vector multiplet of maximally extended supersymmetry.

But the two-form b does not naturally couple to a particle.
So it appears difficult to construct a 'non-abelian' tensor multiplet theory in six dimensions.

Self-dual strings

Instead, b naturally couples to a self-dual string.

This is also indicated by the $(2,0)$ supersymmetry algebra extended with 'central charges':

$$\{Q, Q\} = P + Z.$$

generator	$SO(5,1)$	$SO(5)_R$
Q (supercharge)	chiral spinor	spinor
P (six-momentum)	vector	scalar
Z (string charge)	vector	vector

A straight string element in the spatial direction V coupled to a tensor multiplet ϕ, ψ, h gives rise to the charge $Z = \langle\phi\rangle \otimes V$. Note that $|P| \geq |Z|$.

A multiplet of straight string states transforms as scalars, spinors, and vectors under the little group $SO(4) \subset SO(5,1)$ of transverse rotations.

Dimensional reduction along a tensile string gives rise to the particles of a massive vector multiplet of maximally extended supersymmetry.

So (spontaneously broken) non-abelian gauge symmetry may appear after compactification!

The Lagrangian formulation

Continuous symmetries spontaneously broken by a string gives rise to Goldstone fields on the world-sheet $\Sigma \subset \mathbb{R}^{1,5}$:

field	$SO(4) \subset SO(5,1)$
$X^\perp$ (non-chiral)	vector
Θ^+ (left-moving)	chiral spinor
Θ^- (right-moving)	anti-chiral spinor

The dynamics is governed by the action

$$\begin{aligned} S &= \frac{1}{4\pi\lambda^2}\int_{\mathbb{R}^{1,5}} d^6x\,(\partial\phi\partial\phi + \bar{\psi}\partial\!\!\!/\psi + h^2) \\ &\quad + \int_\Sigma d^2\sigma\sqrt{\phi\phi}\,(DX^\perp DX^\perp + \bar{\Theta}^+ D\!\!\!\!/\,\Theta^+ + \bar{\Theta}^- D\!\!\!\!/\,\Theta^-) \\ &\quad + e\int_\Sigma b \quad + \ldots, \end{aligned}$$

where h fulfills the modified Bianchi identity

$$dh = 2\pi q\,\delta_\Sigma$$

and

$$\begin{aligned} \lambda &= \text{coupling constant} \\ e &= \text{string electric charge} \\ q &= \text{string magnetic charge.} \end{aligned}$$

Decoupling of the anti self-dual part of h (not part of the tensor multiplet) requires $e = \frac{1}{\lambda^2}q$.

The coupling constant and the charges

The quantum theory of a chiral two-form b (with self-dual field strength h) can only be defined for a rational value of the coupling constant λ^2. The correct value appears to be $\lambda^2 = 2$. (Consistent embedding into M-theory, commutation relations of Wilson-'t Hooft surface observables for two linked surfaces,...)

The topological class of the 'wave function' of a system of two strings with electric-magnetic charges (e, q) and (e', q') is determined by the quantity

$$e \cdot q' + q \cdot e' \in \mathbb{Z}.$$

The plus sign means that even two identical strings give a non-zero result...
(Cf. dyonic particles in four dimensions.)

With $e = \frac{1}{2}q$ and $e' = \frac{1}{2}q'$ (decoupling of anti self-dual part of h), we get the condition

$$q \cdot q' \in \mathbb{Z}$$

for all magnetic string charges q and q'.

The normal bundle anomaly

Diffeomorphisms of $\mathbb{R}^{1,5}$ that leave Σ invariant appear as $SO(4)$ gauge transformations on the normal bundle $N = (T\Sigma)^{\perp}$. The theory must respect this symmetry.

But because of the modified Bianchi identity $dh = 2\pi q\, \delta_\Sigma$, the electric coupling

$$e \int_\Sigma b = e \int_{\mathbb{R}^{1,5}} b \wedge \delta_\Sigma$$

classically suffers from an anomaly inflow described by descent on the four-form

$$I^{\text{class}} = q \cdot e\, \delta_\Sigma|_\Sigma = q \cdot e\, \chi(N).$$

The chiral and anti-chiral fermions Θ^+ and Θ^- give a further one-loop contribution:

$$I^{\text{quant}} = -\chi(N).$$

Anomaly cancellation and the relation $e = \frac{1}{2} q$ thus imply that

$$q \cdot q = 2$$

for all magnetic string charges q.

The ADE-classification

The general solution to the conditions $q \cdot q' \in \mathbb{Z}$ and $q \cdot q = 2$ are labeled by the series $A_{1,2,\ldots}$, $D_{4,5,\ldots}$, or $E_{6,7,8}$.

This is in agreement with the classification of simple singularities of hyper-Kähler four-folds $K \simeq \mathbb{C}^2/\Gamma$ for $\Gamma =$ discrete subgroup of $SU(2)$. (Type IIB string theory on $\mathbb{R}^{1,5} \times K$ gives a realization of $(2,0)$ theory.)

More mysteriously, it also indicates that a compactified $(2,0)$ theory may only give rise to Yang-Mills theories with maximally extended supersymmetry and a simply laced gauge group (*i.e.* $SU(r+1)$, $SO(2r)$, or $E_{6,7,8}$).

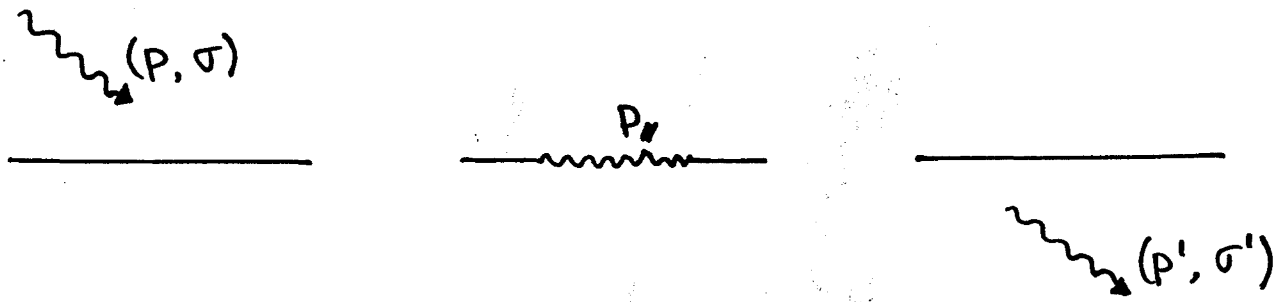
In the Coulomb phase of the Yang-Mills theory:

Massless vector multiplets associated with Cartan generators originate from massless tensor multiplets.
Massive vector multiplets associated with root generators originate from tensile self-dual strings.

Scattering processes

Consider an infinitely extended, approximately straight and static 'bare' string. It is surrounded by a non-vanishing configuration of the space-time tensor multiplet fields ϕ, ψ, and h.
We would like to change variables and work with a 'dressed' string, which includes this self-field.

Because of the $\sqrt{\phi\phi}$ string tension, tensor multiplet quanta of energy E then couple to string wave quanta with strength $E/(\sqrt{\phi\phi})^{1/2}$. Tree-level scattering of tensor multiplets off a string:



For $E^2 << \sqrt{\phi\phi}$, the tensor multiplets are described by a free space-time field theory, and the string waves are described by some decoupled world-sheet conformal field theory.

But which two-dimensional CFT?

The decoupled world-sheet theory

The degenerate ground states of the string are a representation of the Clifford algebra of the Θ^+ and Θ^- zero modes.
Above these ground states, there are four bosonic ($X^\perp$) and four fermionic (Θ^+ and Θ^-) polarizations of string wave excitations.

But because of the electro-magnetic interaction, the bosonic string waves are not free.

Replace $X^\perp$ with 'spherical' variables $r \in \mathbb{R}$ and $g \in S^3 \simeq SU(2)$, modulo $(r,g) \sim (-r,-g)$. The normal bundle structure group acts as $g \mapsto ugv^{-1}$ for $(u,v) \in SU(2) \times SU(2) \simeq Spin(4)$.

The 'radial' coordinate r is a free boson, but g is governed by a 'level one' $SU(2)$ Wess-Zumino-Witten model. This reproduces the classical bosonic normal bundle anomaly.

The WZW kinetic term of g implies that the effective space-time geometry probed by the string wave fluctuations is blown up at the locus of the string.

Conclusion...

The six-dimensional $(2, 0)$ theories exhibit a tightly constrained structure, apparent already in the effective low-energy description in terms of particles and strings:

The classical theory is inconsistent.
Supersymmetry is essential.
There is an ADE-classification.
The string world-sheet theory is non-trivial.

...and outlook

Investigations of scattering processes, particularly in a compactified situation, may give insights into the six-dimensional origins of (spontaneously broken) non-abelian gauge symmetry in certain lower-dimensional Yang-Mills theories.

Particles and strings are useful at a generic point in the moduli space with non-vanishing $\langle \phi \rangle$.
A major challenge is to instead use the full superconformal symmetry of the theory. But we do not yet know the correct framework for such a construction.