#### **Collisions of Cosmic F- and D- Strings**

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with Jackson & Polchinski [th/0405229]

(NJ, Stoica, Tye [th/0203163], [th/0303269])

## Field Theory Cosmic String Interactions

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- Inter-vortex reconnection probability is essentially 1 (numerically; Matzner).

- Naively, cosmic string network density scales like  $a^{-2}$   $ds^2 = -dt^2 + a(t)^2 dx_i dx^i$ . Radiation:  $\rho \sim a^{-4}$ ; Dust:  $\rho \sim a^{-3}$ ...

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- *P* < 1, enhancement of cosmic string effects & observables.

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- Irrespective of details (warping, compactification ...), the brane collision ending inflation is hybrid inflation.
- Inflation ends when the complex tachyon rolls.
- Post inflation, string-like defects (only) are formed; no monopoles or domain walls (NJ, Stoica, Tye; Sarangi, Tye).

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- Possibility of having F-, D- and (p,q)-strings.
- (p,q)-strings stable in KKLMMT when no branes are in the inflationary throat (p,q) relatively prime, |p| < M/2).
- General brane inflation (non-stabilised) models can have long-lived F-, D- and (p,q)-strings.

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- Calculate P & (p,q) interactions for scaling estimates and network simulations; detectable differences.
- Results:  $P_{FF} \sim 10^{-2} \leftrightarrow 1$ ;  $P_{DD} \sim 0.1 \leftrightarrow 1$ .

#### String Interaction Types



#### F+F or D+D reconnection.

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- Account for warping by effective confining potential.
  - Quantum fluctuations of string position give an effective volume of compactification.
  - Valid if geometry varying slowly on string scale.

#### **F-F** Interaction



Compactify 2D of 4D spacetime directions on torus and use wound closed strings (Polchinski, 1988).

#### F-F Interaction



Use unitarity to sum over all final states and macroscopic string limit (Polchinksi 1988).

#### F-F Interaction

$$\begin{split} A &= \left\langle \mathcal{V}_{1}^{(0,0)} \mathcal{V}_{2}^{(0,0)} \mathcal{V}_{3}^{(-1,-1)} \mathcal{V}_{4}^{(-1,-1)} \right\rangle \\ &= (2\pi)^{2} \delta^{(2)} (\sum_{i} p_{i}) \frac{N_{\mathbb{S}^{2}} \kappa^{4}}{(2\pi)^{4} V^{2}} \left( \frac{\alpha'}{2} p_{L1} \cdot p_{L2} \right)^{2} \\ &\int d^{2} z_{4} |z_{4}|^{\alpha' p_{L1} \cdot p_{L4}} |1 - z_{4}|^{\alpha' p_{L2} \cdot p_{L4}} \\ &\to - \frac{4\kappa^{2}}{V \alpha'} \frac{\Gamma\left(-\frac{\alpha'}{4}s\right) \Gamma\left(-\frac{\alpha'}{4}t\right) \Gamma\left(-\frac{\alpha'}{4}u\right)}{\Gamma\left(1 + \frac{\alpha'}{4}s\right) \Gamma\left(1 + \frac{\alpha'}{4}t\right) \Gamma\left(1 + \frac{\alpha'}{4}u\right)}, \\ P &= \frac{g_{s}^{2}}{V_{\perp}} (2\pi^{2} \alpha')^{3} \frac{\left(1 - \cos \theta \sqrt{1 - v^{2}}\right)^{2}}{\sin \theta v \sqrt{1 - v^{2}}}. \end{split}$$

## F-(p,q) Interaction



Interaction of a wound closed string with a boundary; add Dirichlet boundary conditions, Chan-Paton factors & constant electric flux.

### F-(p,q) Interaction

(Callan, Lovelace, Nappi, Yost; Seiberg, Witten; ...)

## **F**-(p,q) Interaction

$$\begin{split} A &= \left\langle \mathcal{V}_{2}^{(0,0)} \mathcal{V}_{4}^{(-1,-1)} \right\rangle \\ &= N_{\mathbb{D}^{2}} \frac{\kappa^{2} \sigma 2^{-2\sigma}}{(2\pi)^{2} V} \int_{0}^{1} dx \, (1-x)^{-1-\alpha' t/2} (1+x)^{1+2\sigma+\alpha' t/2} x^{-1-\sigma} \\ &= -N_{\mathbb{D}^{2}} \frac{\kappa^{2}}{(2\pi)^{2} V} \frac{\Gamma(-\frac{\alpha'}{4}t) \Gamma(1-\sigma)}{\Gamma(-\frac{\alpha'}{4}t-\sigma)}, \\ P &= \frac{g_{s}}{V_{\perp}} (2\pi^{2} \alpha')^{3} \frac{q^{2} v^{2} + \left(g_{s} p - \cos \theta \sqrt{1-v^{2}} \sqrt{g_{s}^{2} p^{2} + q^{2}}\right)^{2}}{\sin \theta \, v \sqrt{1-v^{2}} \sqrt{g_{s}^{2} p^{2} + q^{2}}}. \end{split}$$

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- Similar for (p,q)-(p',q') interaction.
## D-D Interaction: Pair Production

$$\begin{split} & \underbrace{\sum}_{0}^{\infty} = -\frac{i}{2} \int_{0}^{\infty} \frac{dt}{t} e^{-ty^{2}/2\pi\alpha'} \left[ \eta^{6}(it) \Theta_{1}\left(i\frac{\theta t}{\pi}\big|it\right)\Theta_{1}\left(\epsilon t\big|it\right) \right]^{-1} \\ & \times \left\{ \sum_{k=2}^{4} (-1)^{k-1} \Theta_{k}\left(0\big|it\right)^{2} \Theta_{k}\left(i\frac{\theta t}{\pi}\big|it\right)\Theta_{k}\left(\epsilon t\big|it\right) \right\}. \end{split}$$

# D-D Interaction: Pair Production

- Total inelastic probability (Schwinger; Bachas), or the probability of producing *at least one* pair of F-strings is

$$1 - \exp\left(-2 \operatorname{Im} \overbrace{\sum}^{i}\right)$$
$$= 1 - \frac{\prod_{\text{fermions},i}(1 - x_i)}{\prod_{\text{bosons},j}(1 + x_j)}, \qquad x = e^{-2\pi\alpha' m^2/\epsilon}.$$

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- Since  $\tau_F/\tau_D = g_s$ , a single pair of F-strings will not cause connection.



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- If many F pairs are produced, their total tension is sufficient to stick the D-strings together; tachyon rolls and D's reconnect (Hashimoto, Taylor; Hashimoto, Nagaoka).

- Precise condition for reconnection  $N > \frac{1}{g_s} \sinh(\pi \epsilon/2)$  $(\epsilon = \frac{2}{\pi} \tanh^{-1} u).$ 

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- Then, 4 fermionic strings always are produced, so need at least  $\left[\frac{1}{g_s}\sinh(\pi\epsilon/2) 4\right]$  tachyonic strings.
- Probability is

$$P \sim \exp\left(\left[4 - \frac{1}{g_s}\sinh(\pi\epsilon/2)\right]e^{-\theta/\epsilon}\right)$$

#### **Compactification Effects**

- Metric and dilaton depend on compact coords,  $Y^i$ :  $ds^2 = H^{-\frac{1}{2}}(Y)\eta_{\mu\nu}dX^{\mu}dX^{\nu} + H^{\frac{1}{2}}(Y)g_{ij}(Y)dY^idY^j.$ 

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- Fluctuations of position about a minimum are

$$\langle Y^{i}Y^{i}\rangle = \frac{\alpha'}{2\tau_{p,q}(0)} \ln \left[1 + \frac{\tau_{p,q}(0)}{2\pi {\alpha'}^{2} H^{\frac{1}{2}}(0) V_{,ii}(0)}\right]$$

#### Fluctuations: F-Strings

- F-F wavefunction overlap:

$$\left(\frac{1}{V_{\perp}}\right)_{FF} \sim \left(\prod_{i=1}^{6} \ln\left[1 + \frac{\tau_{p,q}(0)}{2\pi\alpha'^2 H^{\frac{1}{2}}(0)V_{,ii}(0)}\right]\right)^{-\frac{1}{2}}$$

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- F-D can have separated minima if dilaton depends on  $Y^i$ .
- $\Rightarrow$  supression of wavefunction overlap volume:

$$\left(\frac{1}{V_{\perp}}\right)_{FD} \propto \left(\frac{1}{V_{\perp}}\right)_{FF} \times \exp\left(-\sum_{i} \frac{Y_D^i Y_D^i}{\alpha' \ln[\dots]_i}\right).$$

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- Reconnection probability supressed by

$$\exp\left(-\frac{1}{2\pi\tau_{p,q}(0)\epsilon}\sum_{i}\ln[\ldots]_{i}\right).$$





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- Fluctuations are localised on  $\mathbb{S}^3$ :

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- Fluctuations fill the  $\mathbb{S}^3$ :

$$\langle Y^i Y^i \rangle \sim g_s M \alpha'.$$

#### - FF reconnection:

$$P_{FF} = \mathsf{Max}\left[\frac{1.5g_s^2}{\ln^{3/2}(1+g_sM)}, \frac{100g_s^{1/2}}{M^{3/2}\ln^{3/2}(1+g_sM)}\right]$$

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- $M \simeq 20, P_{FF} \sim 0.25,$  $M \simeq 100, P_{FF} \sim 0.01.$
- F-(p,q) connection probability increased by  $1/g_s$  (if localised at same  $Y^i$ ).

- DD reconnection:

$$P_{DD} \simeq \exp\left(-\frac{1}{g_s}\sinh(\pi\epsilon/2)e^{-\theta/\epsilon+Y^2/2\pi\alpha'\epsilon}\right),$$
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- Lensing: sensitive to  $\tau$ .

### Summary

- $P_{FF} \sim 10^{-2} \leftrightarrow 1.$
- $P_{DD} \sim 0.1 \leftrightarrow 1.$
- In most cases distinguishable from gauge theory strings; enhanced densities at a given tension.
- (p,q)-string networks can be very different from scaling solutions; interesting or disasterous?
## **Observation: Gravitational Waves**



(Damour, Vilenkin [gr-qc/0104026])

- GW burst sensitive to  $\tau$  only, to first order.
- Burst frequency sensitive to network density & therefore *P*.

## **Observation: CMB**



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