Wormholes in AdS

living on the edges

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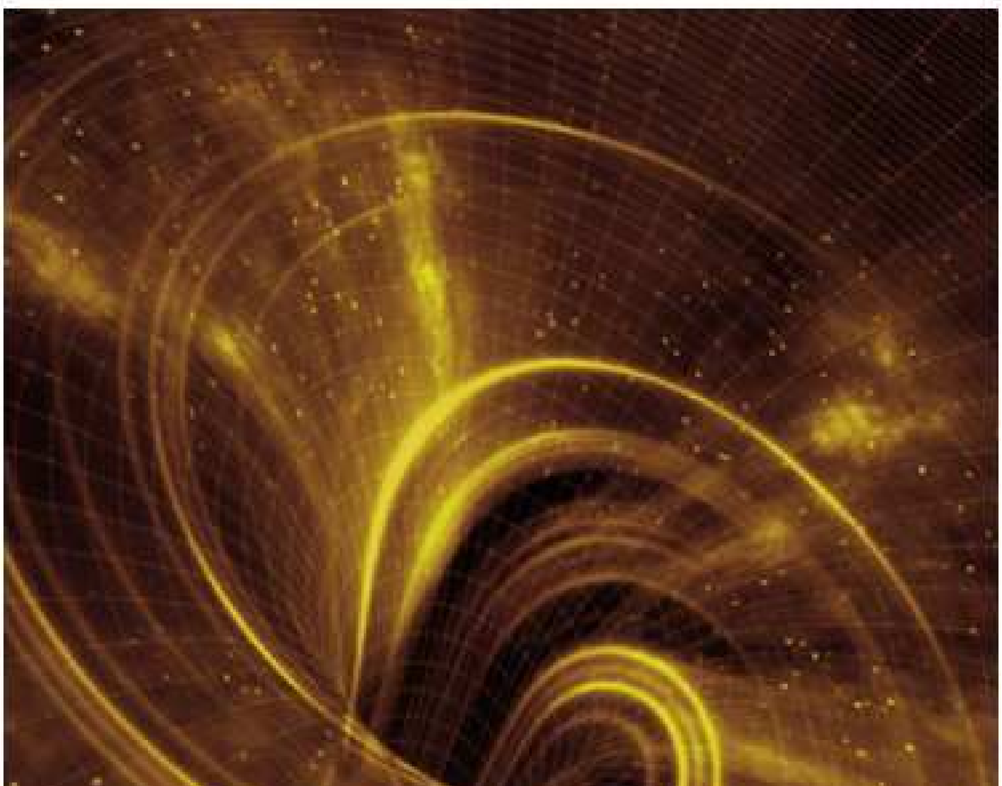
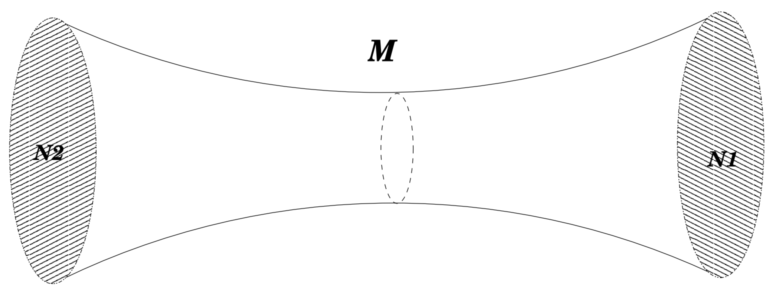
STRINGS 2004

J.Maldacena, L. Maoz, hep-th/0401024
JHEP02(2004)053



Wormholes:



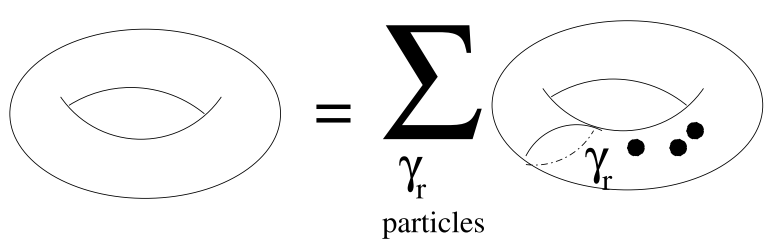
Configurations with two disconnected boundaries $N_{1,2}$, which are connected through the interior M .

Puzzling from the AdS/CFT point of view.

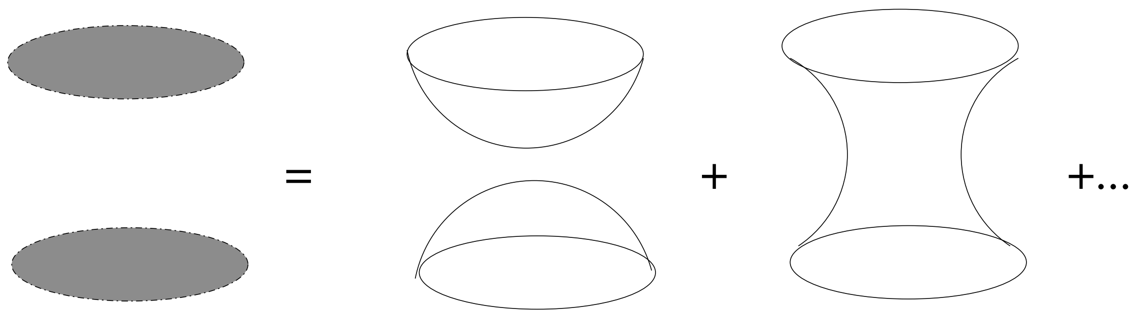
The partition function of a CFT on the boundary is same as the sum over all geometries with fixed boundary conditions

$$< e^{-\int \varphi_j^{(0)} O_j(x) d^d x} >_{CFT} = Z_{string}[\varphi_j] \sim Z_{sugra}[\varphi_j] \sim \sum_{cl.sols} e^{-S[\varphi_j]}$$

e.g. CFT with target space $Hilb^k(K3) \sim$ IIB string theory on $AdS_3 \times S^3 \times K3$ (R. Dijkgraaf, J. Maldacena, G. Moore, E. Verlinde)



HOWEVER:



CFT: the CFT on $\mathcal{N} = \cup_i \mathcal{N}_i$ is the product of the theories on the different $\mathcal{N}_i$'s. Completely independent CFTs $\rightarrow$ Correlations should factorize.

BULK: expect correlations between the two regions.

(*) The puzzle is even more apparent when the wormhole is not only a classical solution, but also a stable solution (perturbatively and non-perturbatively)

OUTLINE

1. The AdS/CFT puzzle

2. The conditions for existence of wormholes

3. General features of the wormholes

4. 3 different types of solutions:
(a) construction as a reduction of 10d or 11d sugra
(b) perturbative instabilities
(c) nonperturbative instabilities
(d) boundary to boundary correlators

5. Summary and open issues

Lorentzian Geometries:

Topological censorship theorems (Galloway, Schleich, Witt, Woolgar)

M: Globally hyperbolic spacetime.

$\mathcal{I}$: timelike boundary satisfying the ANEC.

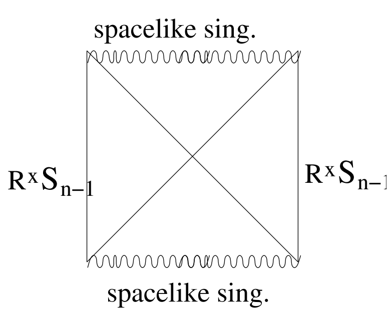
$\mathcal{I}_0$: a connected component of $\mathcal{I}$.

If either $\mathcal{I}_0$ admits a compact spacelike cut, or M satisfies the generic condition

then $\mathcal{I}_0$ CANNOT COMMUNICATE with any other component of $\mathcal{I}$ (i.e. $\mathcal{J}^+(\mathcal{I}_0) \cap (\mathcal{I} \backslash \mathcal{I}_0) = \emptyset$).

$\Rightarrow$ M as above and $\mathcal{I}$ disconnected $\rightarrow$ the spacetime contains black hole horizons.

e.g. Extended AdS Schwarzschild (Maldacena) :



2 copies of the CFT and an initial entangled state.

Euclidean Geometries:

M a complete Einstein manifold in $n+1$ dimensions of negative curvature with boundary $\mathcal{N}$.

(I) If $\mathcal{N}$ has negative curvature, the theory is UNSTABLE. (a codimension 1 BPS brane on M has an action unbounded from below) (Seiberg, Witten)

(II) If $\mathcal{N}$ has positive curvature, then:

- $\mathcal{N}$ is connected.

- $H_n(\overline{M}, Z) = 0$.

- $i_* : \pi_1(\mathcal{N}) \to \pi_1(\overline{M})$ induced by inclusion is onto.

[extended to non-Einstein M's with $Ric \geq -ng$; to $\mathcal{N}$ of non-negative curvature]

(Witten-Yau; Anderson; Cai-Galloway)

Yet: we construct geometries with:

(I) $\mathcal{N}$ of negative curvature: such unstable branes do not exist.

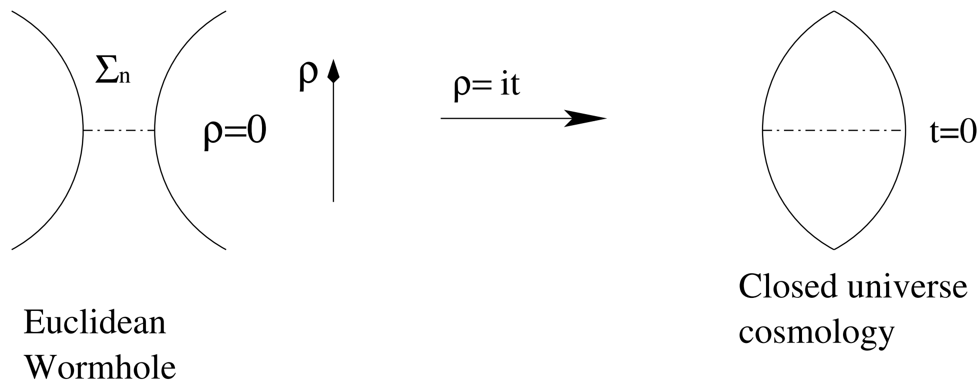
(II) $\mathcal{N}$ of positive curvature: extra SUGRA fields are turned on, and $Ric < -ng$.

General Features

* Euclidean
* Asymptotically AdS
* Solutions of 10d or 11d SUGRA

$$ds^2_{n+1} = d\rho^2 + w^2(\rho) ds^2_{\Sigma_n}$$

where $w(\rho) \to e^{|\rho|}$ as $\rho \to \pm\infty$, and Σ_n is compact.



* Not supersymmetric

Issue of instabilities:

Perturbative : near each boundary, or negative modes in the bulk.

Non-perturbative : there exist other configurations with the same asymptotics, but a lower action.

(A) Quotients of hyperbolic space

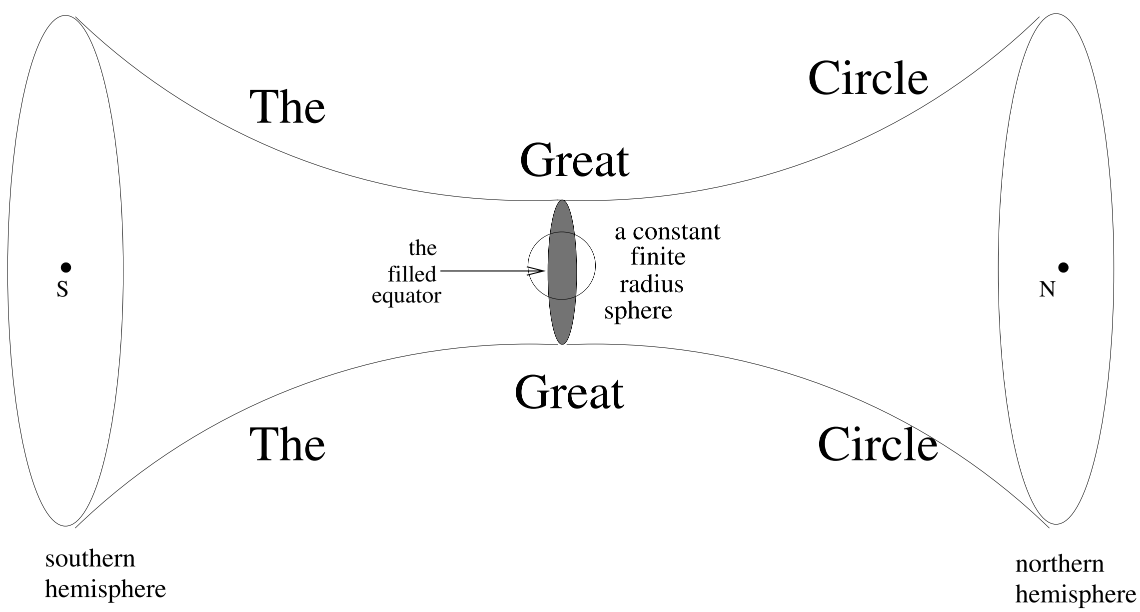
* Spherical slicing:

$$ds^2_{H_{n+1}} = dy^2 + \sinh^2 y \, d\Omega^2_n \quad , \quad y \in [0, \infty)$$

boundary at $y = \infty$: an S^n.

*Hyperbolic slicing:

$$ds^2_{H_{n+1}} = d\rho^2 + \cosh^2 \rho \, ds^2_{H_n} \quad , \quad \rho \in R$$



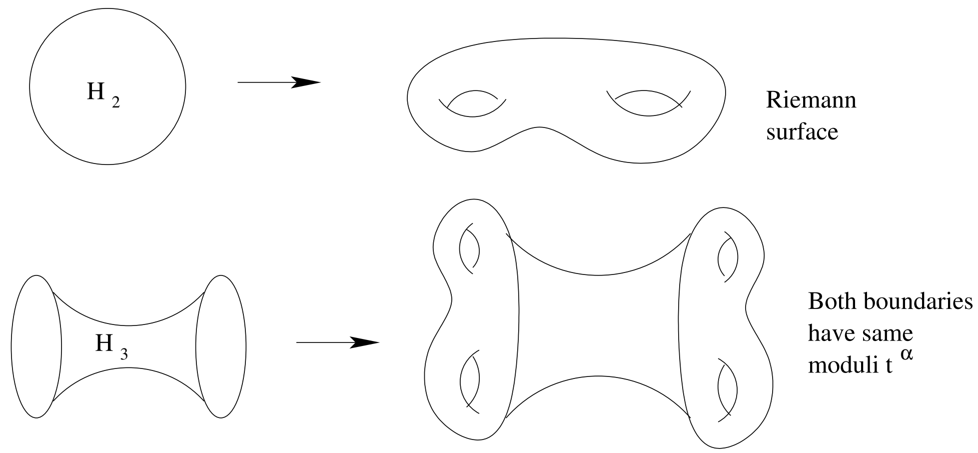
two disks H_n with "transparent" boundary conditions.

Making a quotient by a discrete subgroup $\Gamma \subset SO(1,n)$ such that $\Sigma_n = H_n/\Gamma$ is a compact, smooth, finite volume surface $\Longrightarrow$ two disconnected boundaries.

specific example: AdS_3/CFT_2 (n=2)

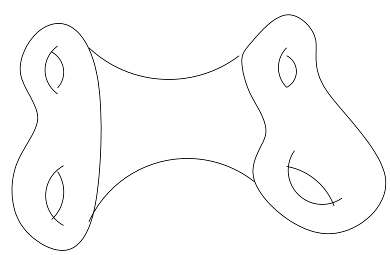
possible quotients $\Gamma \subset SL(2, C)$:

* Fuchsian groups $\Gamma \subset SL(2, R)$



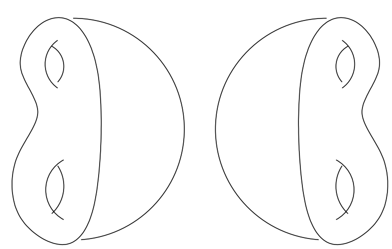
* Quasi-Fuchsian groups:

Bers simultaneous uniformization theorem: $\mathcal{QF}(S_{g,n})$ is homeomorphic to $Teich(S_{g,n}) \times \overline{Teich(S_{g,n})}$.



*Schottky groups:

The retrosection theorem: every compact Riemann surface can be represented as Ω/Σ (where Σ a Schottky group).



Quotients of hyperbolic space
◇ perturbative instabilities - bulk

scalar fields in:

$$ds^2_{H_{n+1}} = d\rho^2 + \cosh^2 \rho \, ds^2_{\Sigma_n}$$

(Σ_n a constant negative curvature compact manifold).

The regular, normalizable solutions of

$$[-\nabla^2 + m^2]\phi = \lambda\phi$$

are: $\lambda_k = \Delta(\Delta - n) + k(n-k)$; $k \in Z\,, 0 < k \leq n/2$,
where $\Delta = \frac{n}{2} + \sqrt{(\frac{n}{2})^2 + m^2}$.

$\Longrightarrow$ $\lambda < 0$ iff $\Delta < n$: relevant operator in the CFT.

Eliminating the $\lambda \leq 0$ modes?

* n=2: IIB $AdS_3 \times S^3 \times K3$:

No negative modes
A zero mode for $\Delta = 1$: (1/2,1/2) under $SU(2)_L \times SU(2)_R$
project it out by a quotient $Z_N \subset U(1)_L \subset SU(2)_L$. $\Rightarrow$ (4,0) susy CFT.
(Larsen, Martinec).

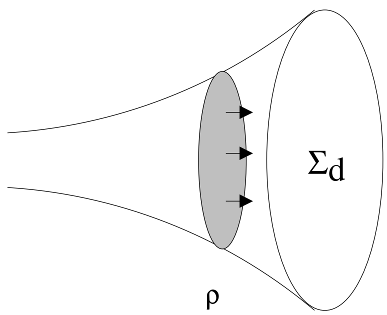
*n=4: IIB $AdS_5 \times S^5$:

Negative modes for $\Delta = 2$.

There is no $\Gamma \subset SO(6)$ such that $AdS_5 \times (S^5/\Gamma)$ has no fixed points and projects out all $\Delta = 2$ operators (in the 20 of $SO(6)$).

Quotients of hyperbolic space

◇ Non-perturbative instabilities - boundary



(Seiberg, Witten)

$$S \sim T\cdot(area) - n\cdot q\cdot(volume) \;\sim\; -\frac{2n}{2^n(n-2)}e^{(n-2)\rho} + \ldots$$

$\Rightarrow$ Instability to creating branes (barrier $\sim \frac{1}{g_s}$) and moving one to $\rho \to \pm\infty$. [In the field theory: conformally coupled scalar with $m^2_{conf} = \frac{n-2}{4(n-1)}R < 0$].

Removing the instabilities?

- Try to add a mass term to the lagrangian. (Giraldello-Petrini-Porrati-Zaffaroni ; Polchinski-Strassler ; Buchel)

- Look at $AdS_3 \times S^3 \times K3$ again: turning on RR fields on $K3$ $\to$ no BPS branes ($T > q$) $\approx$ resolving the small instanton singularities.

$$S \sim \frac{q}{4}[\epsilon e^{2\rho} - 4\rho + 2] \quad ; \quad \epsilon \equiv \frac{T}{q} - 1$$

* large ρ: stable asymptotic boundary conditions.

* For large Q_1, Q_5, there exist branes with $\epsilon \ll 1$

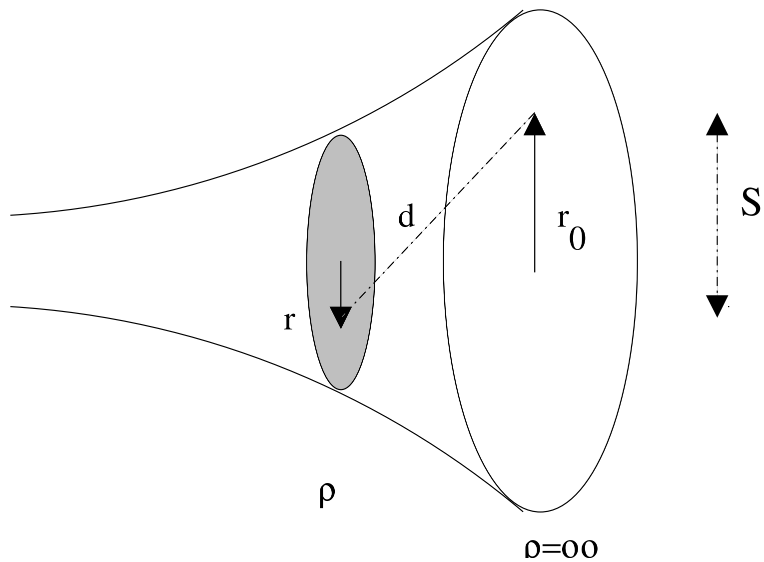
$\to$ non-perturbative instability

$\to$ expect another stable configuration.

Correlation functions of boundary operators

(*) Before the quotient :

the bulk to boundary propagator in hyperbolic coordinates:



$$G(r,\rho;r_0) \sim \frac{1}{[\cosh\rho]^{\Delta}[\cosh s - \tanh\rho]^{\Delta}}$$

Taking $\rho \to \pm\infty$ and renormalizing, we get the 2-point functions:

Same boundary $< O(r,\theta^i)_1 O(r',\theta'^i)_1 > \sim \frac{1}{[\sinh s/2]^{2\Delta}}$

Opposite boundaries $< O(r,\theta^i)_1 O(r',\theta'^i)_2 > \sim \frac{1}{[\cosh s/2]^{2\Delta}}$

(*) Doing the quotient $H_n/\Gamma \Rightarrow$ sum over all images

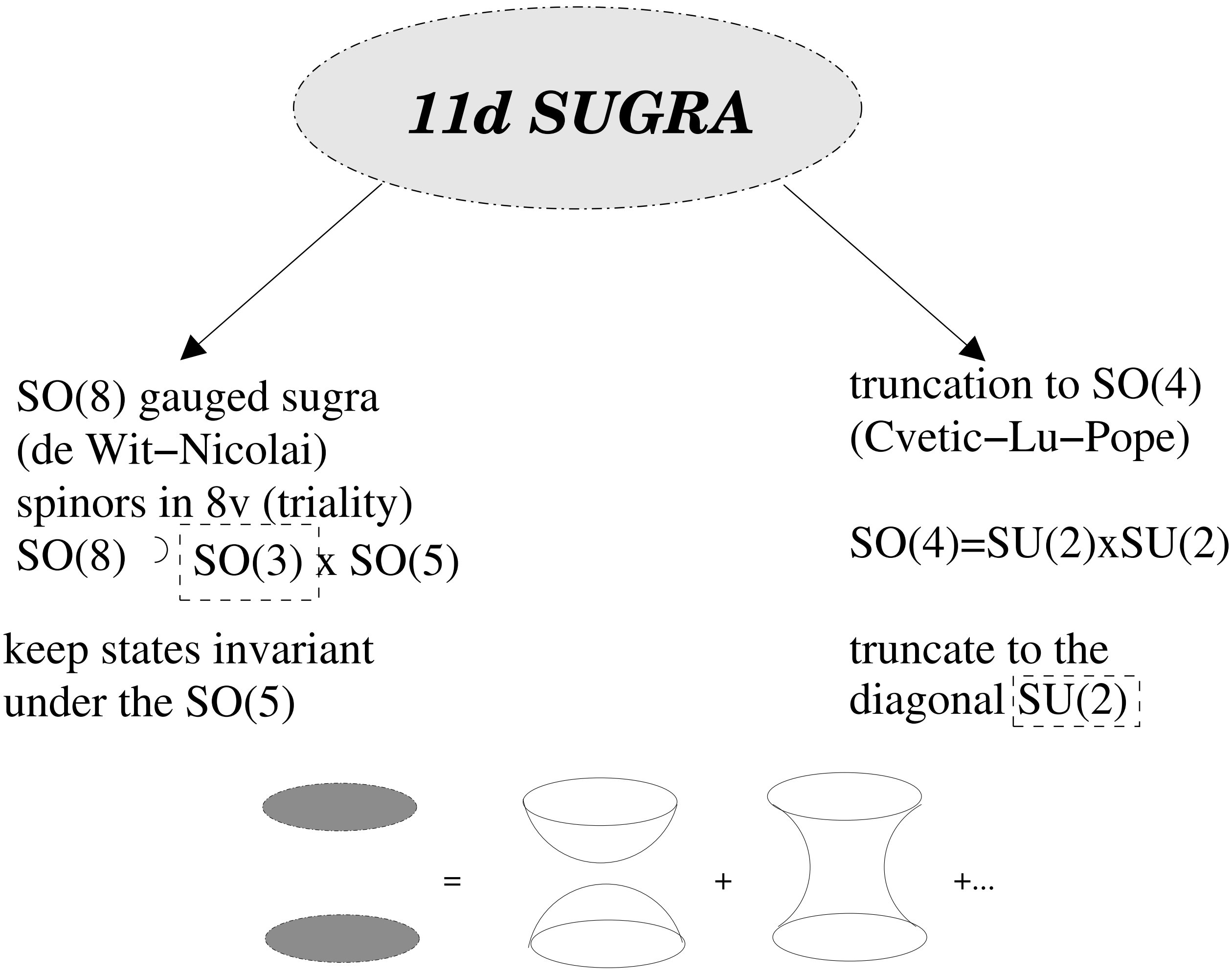
$\Rightarrow$ Generically the correlator across the two boundaries is NONZERO and COORDINATE DEPENDENT

(B) Meron wormholes $\sim AdS_4$

A consistent reduction of 11d SUGRA:

$$S \sim \int d^4x\sqrt{g}[-(R+6) + F^a_{\mu\nu}F^{a\,\mu\nu}]$$

$F^a_{\mu\nu}$: an $SU(2)$ gauge field.



We construct both solutions with boundaries which are S^3 and a constant gauge connection on the boundary.

The Field theory

The theory on coincident M2 branes (the IR limit of 2+1 YM with 16 supercharges) coupled to a fixed background $SU(2)$ gauge connection.

The wormhole solution

$$ds^2 = d\rho^2 + \frac{1}{2}(\sqrt{5}\cosh 2\rho - 1)\, ds^2_{S^3}$$

$A^a = \frac{1}{2}\omega^a$ meron

where ω^a are SU(2) left invariant spin-connections:
$ds^2_{S^3} = \frac{1}{4}\omega^a\omega^a$.   (Hosoya-Ogura ; Rey ; Gupta-Hughes-Preskill-Wise)

* perturbative instabilities?

$$A^a = (\tfrac{1}{2} + \epsilon(\rho)\ )\omega^a \quad ; \quad \lim_{\rho\to\pm\infty}\epsilon(\rho) = 0$$

Find negative eigenvalues $\rightarrow$ perturbatively unstable

* evaluating the action: $S_{2bdy} = (30.296..)N$

The single boundary solution:

Since AdS_4, $F = 0$ is a solution,
also AdS_4 , $F = \pm * F$ is a solution (vanishing E-M tensor).

As AdS_4 is conformal to the unit ball in R^4 we get same instanton solutions but the gauge fields are not necessarily pure gauge on the boundary of AdS:

$ds^2 = dy^2 + \sinh^2 y \, ds^2_{S^3}$ $\quad AdS_4$

$A^a = f(y)\omega^a \quad ; \quad f^{-1}(y) = 1 + \frac{1}{\tanh^2 y/2} \longrightarrow (\frac{1}{2})^{-1}$

* half an instanton in AdS_4

* The instantons in AdS break SUSY

* The action for the 1/2 instanton:
$S_{1bdy} = N[4 + 8]$
so we find $2S_{1bdy} < S_{2bdy} \Rightarrow$ Instability

(C) Instanton - anti instanton wormholes $\sim AdS_5$

A consistent reduction of type IIB on S^5 to a 5-dimensional action with $SO(6)$ gauge fields A^{IJ} and a 6x6 symmetric unimodular matrix of scalars T^{IJ}. The effective action includes a potential term for the scalars and Chern-Simons terms in the Gauge-fields. (Cvetic-Lu-Pope-Sadrzadeh-Tran).

We separate $SO(6) \supset SO(3) \times \widetilde{SO(3)}$ and denote $L^{a,IJ}$ $(I,J = 1,2,3)$ the generators of $SO(3)$; $\tilde{L}^{a,IJ}$ $(I,J = 4,5,6)$ the generators of $\widetilde{SO(3)}$.

Find the solution :

$$ds_5^2 = d\rho^2 + e^{2\omega(\rho)} ds_{S^4}^2 = d\rho^2 + e^{2\omega(\rho)}[d\theta^2 + \sin^2\theta \tfrac{1}{4} w^a w^a]$$
$$e^{2\omega(\rho)} = \tfrac{1}{2}(\sqrt{5}\cosh 2\rho - 1)$$

$$A_\mu^{IJ} = i A_\mu^a L^{aIJ} + i \tilde{A}_\mu^a \tilde{L}^{aIJ}$$
$$A^a = \cos^2(\theta/2) w^a \quad \text{instanton}$$
$$\tilde{A}^a = \sin^2(\theta/2) w^a \quad \text{anti-instanton}$$

$$T^{IJ} = \delta^{IJ}$$

Instabilities?

(*) Note that the gauge field on S^4 is topologically trivial as an $SO(6)$ and can be continuously deformed to a pure gauge configuration. These are indeed negative modes on the S^4.

However, requiring normalizability at $\rho \to \pm\infty$ removes the negative modes $\Rightarrow$ no such negative modes.

(*) Looking at fluctuations of the scalars T^{IJ} gives no negative modes.

(*) other perturbative instabilities?

(*) A single boundary solution with the same asymptotics?

Summary and open issues

We've built a variety of Euclidean geometries, which

-connect two boundaries with AdS asymptotics

-are completely regular

-are consistent reductions of string theory

(1) Quotients of hyperbolic space:
boundaries of negative curvature

In particular $AdS_3/\Gamma \times S^3/Z_N \times K3$ seems rather stable
seem to find correlations between the boundaries

(2) Wormholes with AdS_4 asymptotics (+merons):
boundaries of positive curvature

perturbative instabilities
The action is smaller for two 1-boundary spaces

(3) Wormholes with AdS_5 asymptotics (+instanton, anti-instanton)
boundaries of positive curvature

did not find any instabilities or any lower action solution with the same asymptotics

Possible resolutions

(I) Maybe after summing over all geometries, the gravity correlators do factorize.

inspired by Coleman's analysis of axionic wormholes (in the Lorentzian setting: the sum over all geometries does not induce non-local interactions, or causes loss of Unitarity)

(II) Maybe there is some subtle correlation between the two field theories

A partition function of the form $Z = \sum_i Z_i^1 Z_i^2$ where i is some sector of the field theory. This can arise if the field theory partition functions are not well defined (e.g. a chiral boson in 2d)