Strings 2004, Paris

Minimal Topological Strings

Open/closed duality from non-critical strings to AdS

Leonardo Rastelli

Basic Motivation

Large N gauge theory $\longrightarrow$ string theory?

D-branes have given a clear spacetime picture,
but time to go back seriously to the worldsheet:
how to really "close the holes" ?

Ideal goal: understanding AdS/CFT
from old-fashioned channel duality

We can learn from solvable models

Outline

1. Introduction: open $\rightarrow$ closed worldsheets

2. Liouville D-branes and the Kontsevich model

D. Gaiotto, L.R., hep-th/0312196

D.Gaiotto, T. Takayanagi, L.R, work in progress

3. Strings in $AdS_3 \times S^3$ and matrix models

D. Gaiotto, L.R., work in progress

Open/closed duality and moduli spaces

$\mathcal{M}^{open}_{g,h} \equiv$ moduli space of open Riemann surfaces of genus g, with h holes
$\dim(\mathcal{M}^{open}_{g,h}) = 6g - 6 + 3h$

$\mathcal{M}^{closed}_{g,p} \equiv$ moduli space of closed Riemann surfaces of genus g, with p punctures
$\dim(\mathcal{M}^{closed}_{g,p}) = 6g - 6 + 2p$

Natural isomorphism Penner, Kontsevich

$$\mathcal{M}^{open}_{g,h} \cong \mathbf{R}^h_+ \times \mathcal{M}^{closed}_{g,p=h}$$

At least formally

$$\int [d\,\mathcal{M}^{open}_{g,h}] \int [\mathcal{D}X]\,[\mathcal{D}\,bc]\,e^{-S[X,bc]} \longrightarrow \int [d\,\mathcal{M}^{closed}_{g,p=h}]\,\langle \mathcal{W}_1 \ldots \mathcal{W}_h \rangle\,,$$

we can replace each hole with a puncture.

In favorable cases, sum over holes $\sim \exp\left(\int d^2z\,\mathcal{W}(z)\right)$, a smooth deformation of the closed string background.

Open/closed duality

Open string side

Open string field theory (OSFT) on N D-branes.

Vacuum amplitude as a function of open string moduli $\{z_i\}$, $i = 1, \dots N$

$$\log \mathcal{Z}^{open}(g_o, N) = \sum_{g=0}^{\infty} \sum_{h=1}^{\infty} {g_o}^{-2+2g} ({g_o}^2 N)^h F_{g,h}^{open}(\{z_i\}) \,.$$

Closed string side

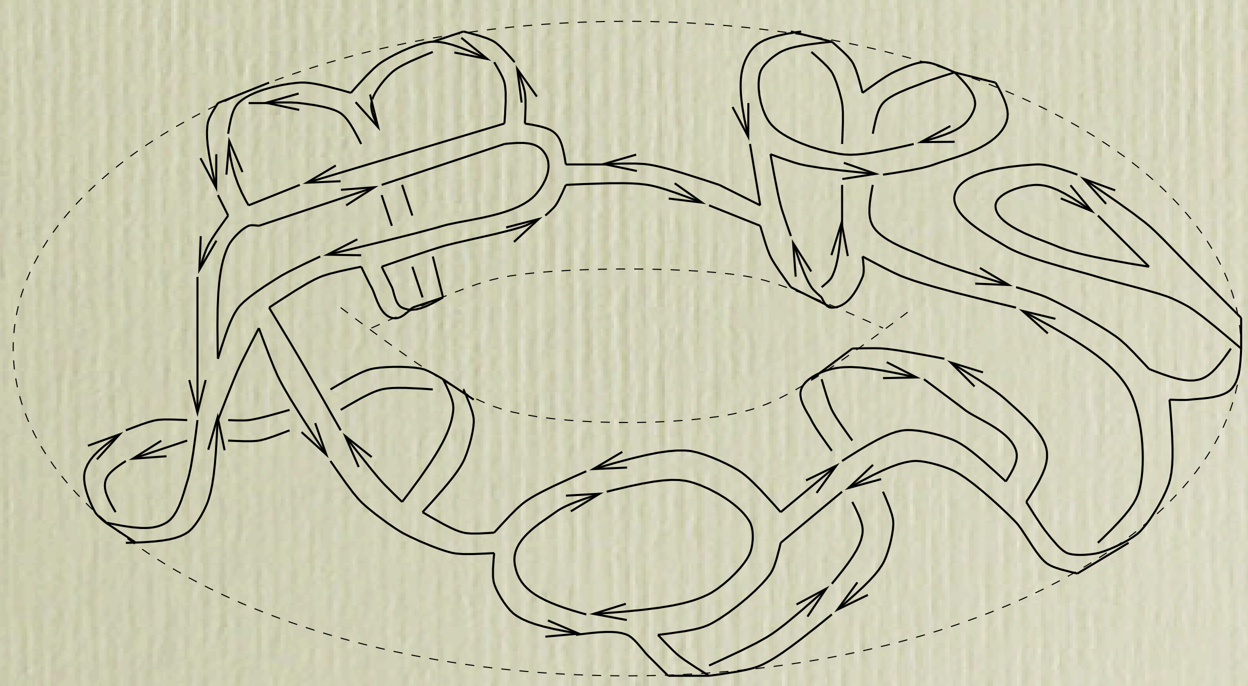
Correlators of closed strings physical states $\{\mathcal{O}_k\}$, encoded in

$$\log \mathcal{Z}^{closed}(g_s, \{t_k\}) = \sum_{g=0}^{\infty} {g_s}^{2g-2} \langle \exp(\sum_k t_k \mathcal{O}_k) \rangle_g \,.$$

Open/closed dictionary

$$b_0 \int d\rho \rho^{L_0} |\mathcal{B}_z\rangle \leftrightarrow \sum_k c_k(z)\mathcal{O}_k$$

$$\mathcal{Z}^{open}(g_o, \{z_i\}) = \mathcal{Z}^{closed}\left(g_s = g_o^2, t_k = \sum_i c_k(z_i)\right)$$



Examples

Large N transitions for topological strings,

e.g. topological A model on the conifold Gopakumar Vafa, Ooguri Vafa

D-branes in imaginary time Maloney Strominger Yin, Gaiotto Itzhaki L.R., Lambert Liu Maldacena, Bergman Razamat

$$t\, b_0 \int d\rho\, \rho^{L_0}\, |\mathcal{B}\rangle_P \leftrightarrow t\, \mathcal{W}(P)$$

Does AdS/CFT work this way?

Same pattern: N can be kept finite, $t = g_{YM}^2 N$ is a geometric modulus.

SYM theory = full OSFT on N appropriate branes? Polyakov

't Hooft fatgraphs $\equiv$ open string field Feynman diagrams.

CFT $\rightarrow$ AdS by gluing worldsheets Gopakumar

Non-critical strings ideal laboratory for these ideas.

Liouville CFT and open/closed duality

$$S_{Liouville} = \int d^2z\, \partial\phi\bar{\partial}\phi + Q\, R\, \phi + \mu\, e^{2b\phi}\,, \qquad Q \equiv b + \frac{1}{b}$$

• $\{e^{2\alpha\phi}\}$ with $\alpha \in \mathbf{R}$, $\alpha < Q/2$: local closed string vertex operators, localized at $\phi \to -\infty$.

• ZZ branes: localized at $\phi = +\infty$, unstable

Intepretation of "old-matrix model" for $c = 1$:
exact OSFT on N ZZ branes $\equiv$ quantum mechanics of N free fermions in inverted harmonic potential McGreevy Verlinde, Klebanov Maldacena Seiberg, Sen

• FZZT branes: "Neumann" branes dissolving at $\phi \sim \log(\mu_B)$

$$\partial\phi - \bar{\partial}\phi = \mu_B\, e^{b\phi}$$

Massless open "tachyon" on FZZT worldvolume $\mu_B \to \infty$
as a smooth "tachyon condensation"

Of course, all of this is very reminiscent of AdS/CFT:

$\phi = -\infty \leftrightarrow$ AdS boundary where local CFT operators are inserted

Two dressings $e^{2\alpha\phi}$, $e^{2(Q-\alpha)\phi} \leftrightarrow$ Two roots $\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2}$

Klebanov Witten

In this analogy, FZZT are perhaps more natural,
since they sit in the right place.

Moreover, FZZT branes can be replaced by a sum of
local closed string operators.

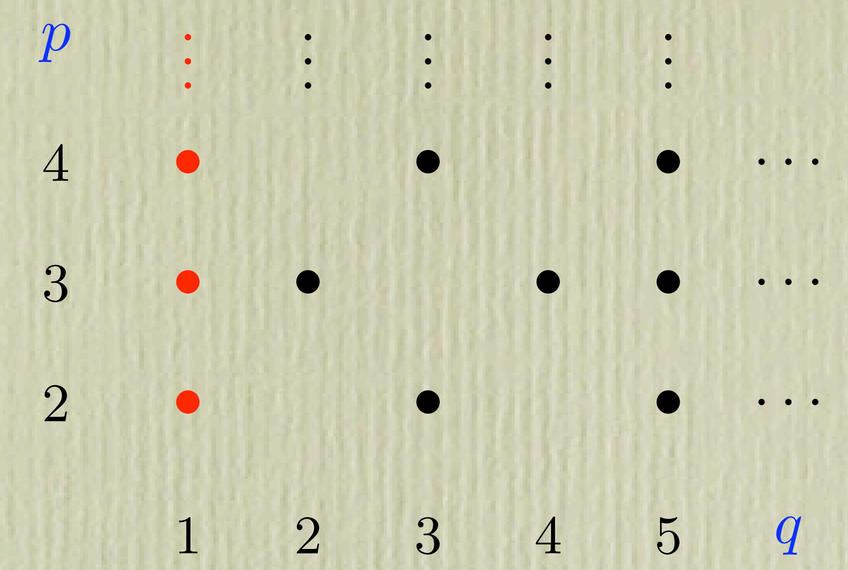
Quantum gravity interpretation

$$|\mathcal{B}\rangle_{\mu_B} \leftrightarrow \int_0^\infty e^{-\mu_B\, l} W(l) \sim \sum_{k=0}^{\infty} \frac{\mathcal{O}_k}{\mu_B^{\alpha_k}}$$

$W(l)$ macroscopic loop operator Banks Douglas Seiberg Shenker, Ambjorn Makeenko ...

- What is then the worldvolume theory living on N FZZT branes?

Minimal Topological Strings



Noncritical bosonic string theories from (p, q) BPZ minimal models $\oplus$ Liouville.
Exactly solvable. Douglas Shenker, Brezin Kasakov, Gross Migdal

Models in the same row related by turning on deformations,
$S = S_0 + t_n \mathcal{O}_n$, "times" t_n of the p-KP hierarchy Douglas

$(p, 1)$ column: topological ancestors. Witten, Dijkgraaf Verlinde Verlinde, . . .
The matter CFT is non-minimal.

Alternative formulations:
twisted $\mathcal{N} = 2$ MM coupled to topological gravity,
B-model on CY $zw + y^p + x = 0$ Aganagic Dijkgraaf Klemm Marino Vafa

(2,1) model: strings in d=-2

$$\mathcal{S} = \frac{1}{2\pi}\int d^2z\,\epsilon_{\alpha\beta}\partial\Theta^\alpha\bar{\partial}\Theta^\beta + S_\phi^{c=28} + S_{bc} \qquad \alpha,\beta = 1,2\,.$$

Θ^1 and Θ^2 real and Grassmann odd.

Closed string local operators

$$\mathcal{O}_{2k+1} = e^{\sqrt{2}(1-k)\phi}\,\mathcal{P}_k(\partial\Theta^\alpha)\,c\bar{c}$$

Canonical choice of $(\frac{k(k+1)}{2}, \frac{k(k+1)}{2})$ primaries $\mathcal{P}_k$ from SL(2) invariance.
Already in the correct "picture".

Look at FZZT branes

Consider

$$|\mathcal{B}\rangle_z = (\text{FZZT brane for Liouville with } \mu = 0,\ \mu_B = z) \otimes |\mathcal{B}^{c=-2}_{Dirichlet}\rangle$$

1. Expansion in local operators
$$|\mathcal{B}\rangle_z \longrightarrow \sum_{k=0}^{\infty} \frac{\mathcal{O}_{2k+1}}{(2k+1)z^{2k+1}}$$

2. Boundary CFT is topological. Bosonization Distler

$$\beta = \partial\Theta^1 e^{b\phi} \qquad \gamma = \partial\Theta^2 e^{-b\phi}$$

BCFT = (2,-1) $\beta\gamma$ system $\oplus$ (2,-1) bc system

Scalar Supercharge

$$Q_S = \oint b(z)\gamma(z) = \oint b(z)e^{-\phi(z)}\partial\Theta^2(z)\,,$$

$$Q_S^2 = \{Q_B, Q_S\} = 0\,.$$

Full OSFT on N such branes

$$S[\Psi] = -\frac{1}{g_o^2}\left(\frac{1}{2}\langle\Psi_{ij}, Q_B\Psi_{ji}\rangle + \frac{1}{3}\langle\Psi_{ij}, \Psi_{jk}, \Psi_{ki}\rangle\right) .$$

$$\Psi_{ij} = X_{ij}T_{ij} + \cdots = X_{ij}e^{b\phi}c_1|0\rangle_{ij} + \cdots .$$

OSFT localizes to a cubic matrix integral, the Kontsevich model,

$$S[X, Z] = -\frac{1}{g_o^2}\mathrm{Tr}\left[\frac{1}{2}ZX^2 + \frac{1}{6}X^3\right]$$

Eigenvalues $\{z_i\}$ of Z are the N boundary cosmological constants

Using open/closed duality, extract correlators of close strings $\{\mathcal{O}_{2k+1}\}$ from $\mathcal{Z}^{open}(g_o, z_i) \equiv$ partition function of Kontsevich integral.

$$\mathcal{Z}^{closed}\left(g_s = g_o^2, t_{2k+1} = g_s \sum \frac{1}{(2k+1){z_i}^k}\right) = \mathcal{Z}^{open}(g_o, z_i)$$

Some lessons

- As expected,
  Genus g Kontsevich diagrams with h holes $\leftrightarrow$ genus g closed correlators with h punctures

- Duality makes sense for finite N:
  OSFT on N branes $\cong$ subsector of the full theory.
  To span full closed string Hilbert space, need $N \to \infty$

- OSFT holographic despite FZZT branes are "extended"!

- Open/closed duality can also be understood using "spacetime" Ward identities
  DVV, C. Johnson, Aganagic et al., Gaiotto and L.R.

A few generalizations

• All $(p, 1)$ models follow similar pattern D. Gaiotto, L.R,, Aganagic et al.

• Some OB models related to orbifolds of the bosonic models, simplest example: $\mathbf{Z}_2$ orbifold of $c = -2$ bosonic string $\{\Theta_\alpha \to -\Theta_\alpha\} \leftrightarrow$ pure supergravity.
D. Gaiotto, T. Takayanagi, L.R.

•Extension to $c = 1$ at the self-dual radius Ghoshal Mukhi Murthy:
OSFT on N FZZT $= SU(2)$ symmetric model, still holographic.

How general are these ideas?

Strings in $AdS_3 \times S^3$

IIB on $AdS_3 \times S^3 \times \mathcal{M}_4$ with NSNS flux, near horizon of Q_1 F1, $Q_5 = k$ NS5.
(Here Euclidean AdS_3)

Exact matter CFT for $AdS_3 \times S^3$ Giveon Kutasov Seiberg

$$\widehat{SU}(2)_{k-2} \oplus \{\psi^+, \psi^-, \psi^3\} \oplus \widehat{SL}(2,R)_{k+2} \oplus \{\chi^+, \chi^-, \chi^3\}$$

$\{\psi^a\}$, $\{\chi^a\}$ free fermions.

Usual choice of complex structure: $\psi^+\psi^-$, $\chi^+\chi^-$, $\psi^3\chi^3$.

Instead, pair symmetrically $AdS_3 \leftrightarrow S^3$ ($SL(2,C)$ complex structure)

$$\eta_1 \equiv \chi_1 + i\psi_1 \quad \eta_2 \equiv \chi_2 + i\psi_2 \quad \eta_3 \equiv \chi_3 + \psi_3$$

$$\xi_1 \equiv \chi_1 - i\psi_1 \quad \xi_2 \equiv \chi_2 - i\psi_2 \quad \xi_3 \equiv \chi_3 - \psi_3$$

Topological twist $T \rightarrow T - \frac{1}{2}\partial J\,,\quad J \equiv \eta^a \xi_a$

makes (η^a, ξ^a) of spins (1,0) and $c_{tot} = 6 + 3(-2) = 0.$

B-twist gives precisely G/G topological model,
with $G = SU(2)_{k-2}$!

Under the $U(1)$, supercharge $G = G_+ + G_- + G_{---}$

Generators close in a Kazama algebra.

$$J^a_{tot} = j^a + k^a + i\epsilon a_{bc}\chi^b\eta^c = \{G_+, \eta^a\}\,.$$

(Different choice of twist explored by Y. Sugawara)

Physical states of G/G model = (subset of) chiral primaries in spacetime.

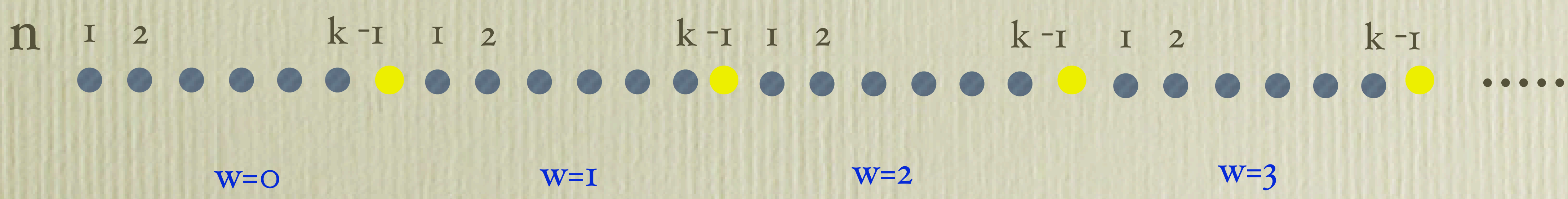
"Short string" chiral primaries are of the form Kutasov Larsen Leigh

$$\mathcal{T}_j = \mathcal{V}_j^{SU(2)}\,\mathcal{V}_j^{AdS}\,(\text{fermions}) \quad j = \frac{n}{2} - 1\,,\ n = 1, 2, \ldots k-1\,,$$

Each short string has an infinite tower of spectral flowed descendants,

Maldacena Ooguri, Argurio Giveon Shomer

$$\mathcal{T}_j^{(w)} = t_+^w\,\mathcal{T}_j\,,\quad w = 1, 2, 3\ldots \qquad \Delta_j^{(w)} = j + w\,\frac{k}{2}\,.$$



Missing every k-th state. Related to singular behavior of boundary CFT?

Seiberg Witten

Further twist $T \to T + \partial J^3_{tot}$ re-organizes the theory into $(k,1)$ bosonic string

Aharony Ganor Sonneschein Yankielowicz

Wakimoto representation of AdS_3 CFT in terms of (ϕ, β, γ),
and of $SU(2)$ in terms of $(\tilde{\phi}, \tilde{\beta}, \tilde{\gamma})$

Hamiltonian reduction $\jmath^+ = \beta = \sqrt{\mu}$ reduces
AdS_3 CFT $\to$ Liouville with $b = 1/\sqrt{k}$,
similarly $SU(2)$ CFT $\to$ matter CFT with $b = 1/\sqrt{k}$.

$(1,0)$	$(1,0)$	$(2,-1)$
$\eta^3\ \xi^3$	$\xi^-\ \eta^+$	$\eta^-\ \xi^+$
$\gamma\ \beta$	$\tilde{\gamma}\ \tilde{\beta}$	

All in all, field content and screening charges of $(k,1)$ bosonic string.

Interaction $\beta\bar{\beta}\exp(\frac{2\phi}{\sqrt{k}}) \to \mu \exp(2b\phi)$.

Liouville $\equiv$ radial AdS direction!

Mapping local operators

$\mathcal{T}_j \rightarrow$ small phase space

$\mathcal{T}_j^w \rightarrow$ gravitational descendants

Missing states well-known to be absent in the matrix model.
"Boundary operators" (holes with extra open puncture)? Martinec Moore Seiberg

Mapping branes

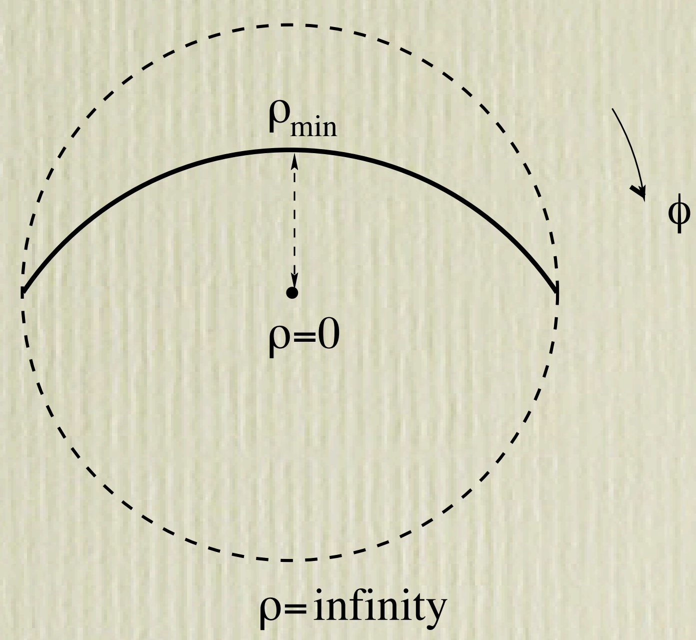
Tentative identification

$$|\mathrm{FZZT}(\mu_B)\rangle \otimes |\mathrm{matter}\rangle \leftrightarrow \mathrm{AdS}_2 \times S^2 \text{ brane} \qquad C \sim \mu_B$$

AdS_2 branes: static (1,q) strings at Bachas Petropoulos, $\cdots$

$$\sinh\rho \sin\phi = C = q\,\frac{T_F}{T_D}\,,$$

in global AdS_3.



"Permeable domain walls in boundary CFT " Bachas de Boer Dijkgraaf Ooguri.

In our picture, expect again $|B\rangle \to \sum \mathcal{O}_k$.

Conclusions

- Exactly solvable models confirm general picture of open/closed duality described in the introduction
- A role for OSFT on infinitely many branes.
- A new class of topological string theories?
- Does AdS/CFT work similarly?