Minimal String Theory

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Strings, 2004
Paris

Klebanov, Maldacena and N. S., hep-th/0309168

N.S. and Shih, hep-th/0312170

Kutasov, Okuyama, Park , N. S., Shih, hep-th/0406030

Motivation

- Simple (minimal) and tractable string theory

- Explore D-branes, nonperturbative phenomena

- Other formulations of the theory – matrix models, holography

Approach

Minimal String Theory $=$

(p, q) Minimal CFT $+$ Liouville $+$ Ghosts

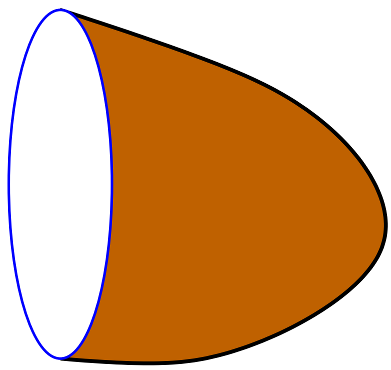
Use worldsheet techniques to derive

- geometric description (similar to topological string theory)

- matrix model

Review of Branes in Liouville

FZZT branes (Fateev, Zamolodchikov and Zamolodchikov, Teschner) – macroscopic loops in the worldsheet



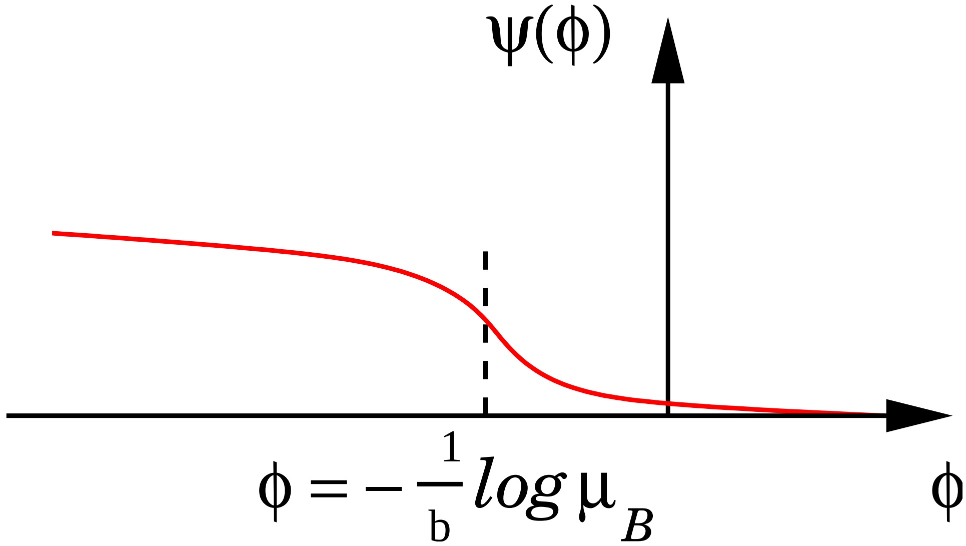
Labelled by the "boundary cosmological constant"

$$\delta S = \mu_B \oint e^{b\phi}$$

Minisuperspace wavefunction

$$\Psi(\phi) = \langle \phi | \mu_B \rangle = e^{-\mu_B e^{b\phi}}$$

The brane comes from infinity and dissolves at $\phi \approx -\frac{1}{b} \log \mu_B$.



In Cardy's formalism a brane is labelled by a representation in the open string channel

$$\mu_B = \cosh \pi\, b\, \sigma \qquad \longleftrightarrow \qquad \Delta = \frac{1}{4}\sigma^2 + \frac{Q^2}{4}$$

$$Q = b + \frac{1}{b}, \qquad c = 1 + 6Q^2$$

For the degenerate representations

$$\sigma = i\Big(\frac{m}{b} + nb\Big)$$

Subtracting the null vectors in the representation leads to the ZZ (Zamolodchikov and Zamolodchikov) branes

$$|m,n\rangle = |\sigma(m,n)\rangle - |\sigma(m,-n)\rangle$$

Same

$$\mu_B = (-1)^m \cos \pi\, n\, b^2$$

at $\sigma(m, \pm n)$ (Martinec).

These branes are localized in the strong coupling region $\phi \to +\infty$.

Branes in Minimal String Theory

FZZT branes – extended branes: Tensor a Liouville brane labelled by σ and a matter brane

ZZ branes – localized branes: Tensor a Liouville brane labelled by (m, n) and a matter brane

Simplification: the independent ZZ branes are

$$1 \leq m < p \ , \quad 1 \leq n < q \ , \quad np < mq$$

Geometric Interpretation

The disk amplitude $Z(\mu_B)$ is not a single valued function of

$$x \equiv \mu_B = \cosh \pi b\, \sigma \ , \qquad b^2 = \frac{p}{q}$$

Instead, x and

$$y \equiv \partial_{\mu_B} Z(\mu_B) = \cosh \frac{\pi\sigma}{b}$$

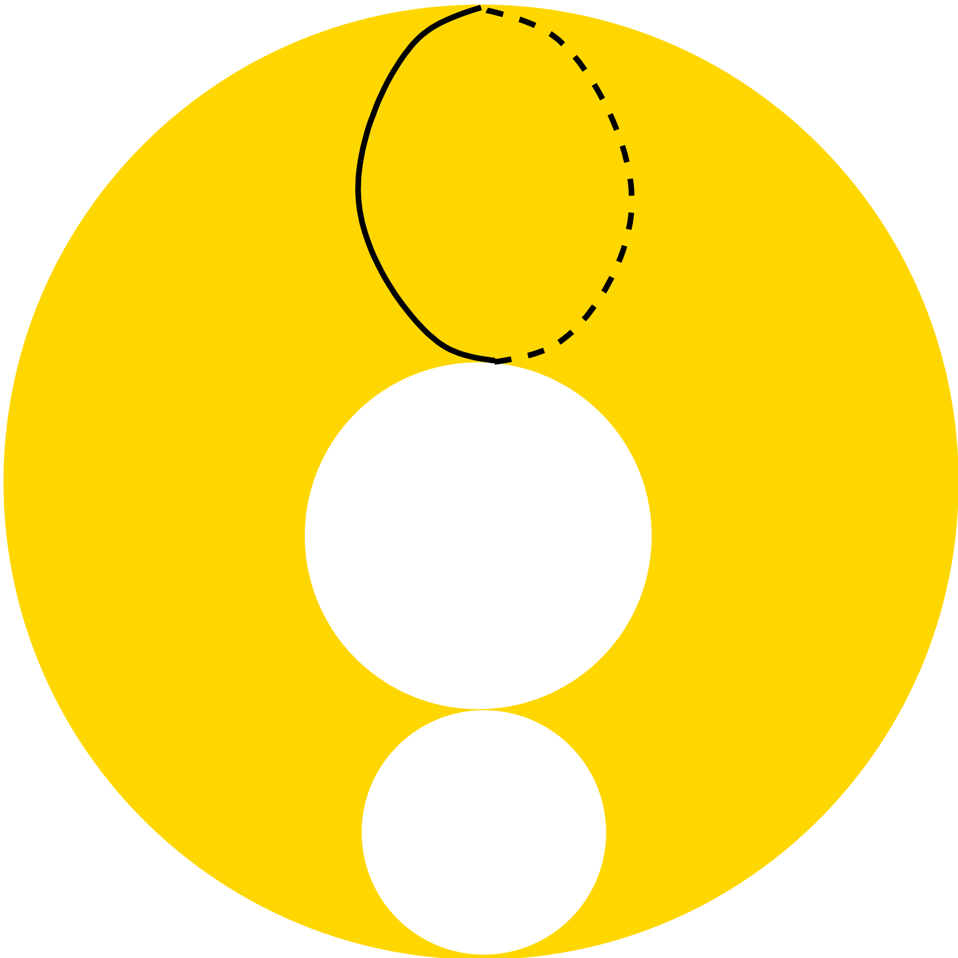
satisfy

$$T_p(y) = T_q(x)$$

($T_p(y = \cos\theta) = \cos p\,\theta$ are Chebyshev)

Related to earlier work of Kazakov, Kostov and collaborators

This is a genus $\frac{(p-1)(q-1)}{2}$ Riemann surface $\mathcal{M}$ with $\frac{(p-1)(q-1)}{2}$ pinched A-cycles



Line integrals of $\omega \equiv y\,dx$ lead to branes:

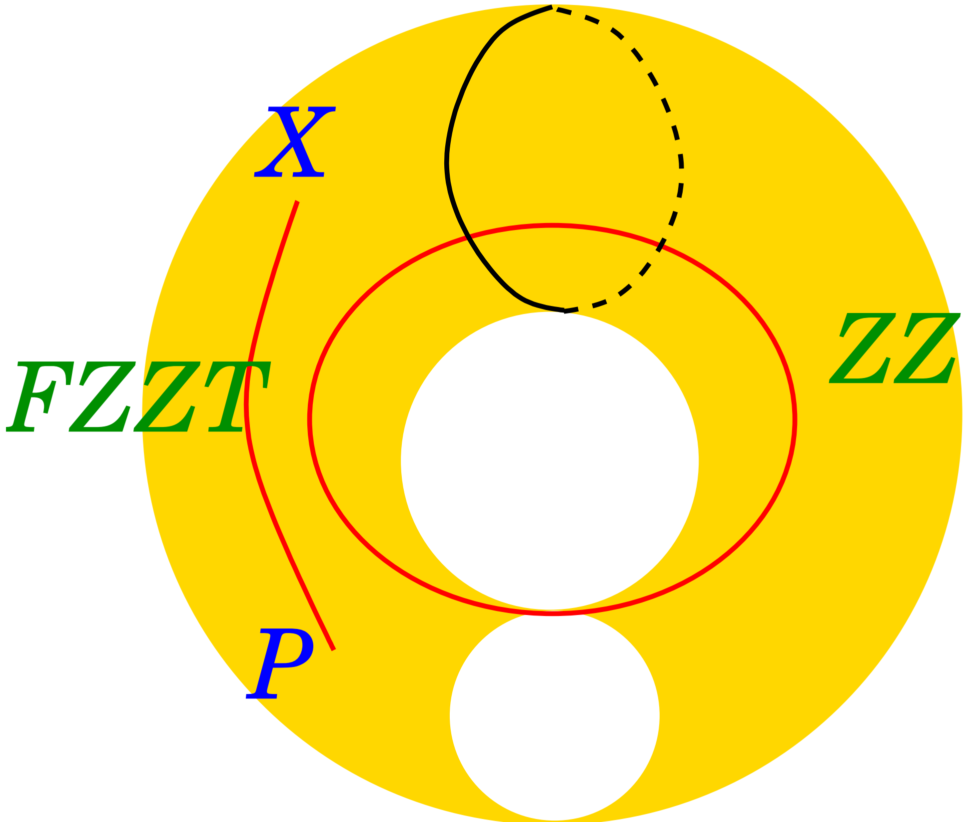
An FZZT brane is an open line integral

$$Z(x) = \int_P^x \omega$$

A ZZ brane is a difference between two FZZT branes. It turns out to pass through a singularity; *i.e.* it is an integral along a B-cycle

$$Z(m,n) = \oint_{B_{m,n}} \omega$$

FZZT and ZZ branes on the Riemann surface $\mathcal{M}$:



$x_{m,n}$ at the singularities are the values of μ_B of the ZZ branes.

Deformations of $\mathcal{M}$

Closed string states $\longleftrightarrow$ singularity preserving deformations; $\oint_A \omega = 0$.

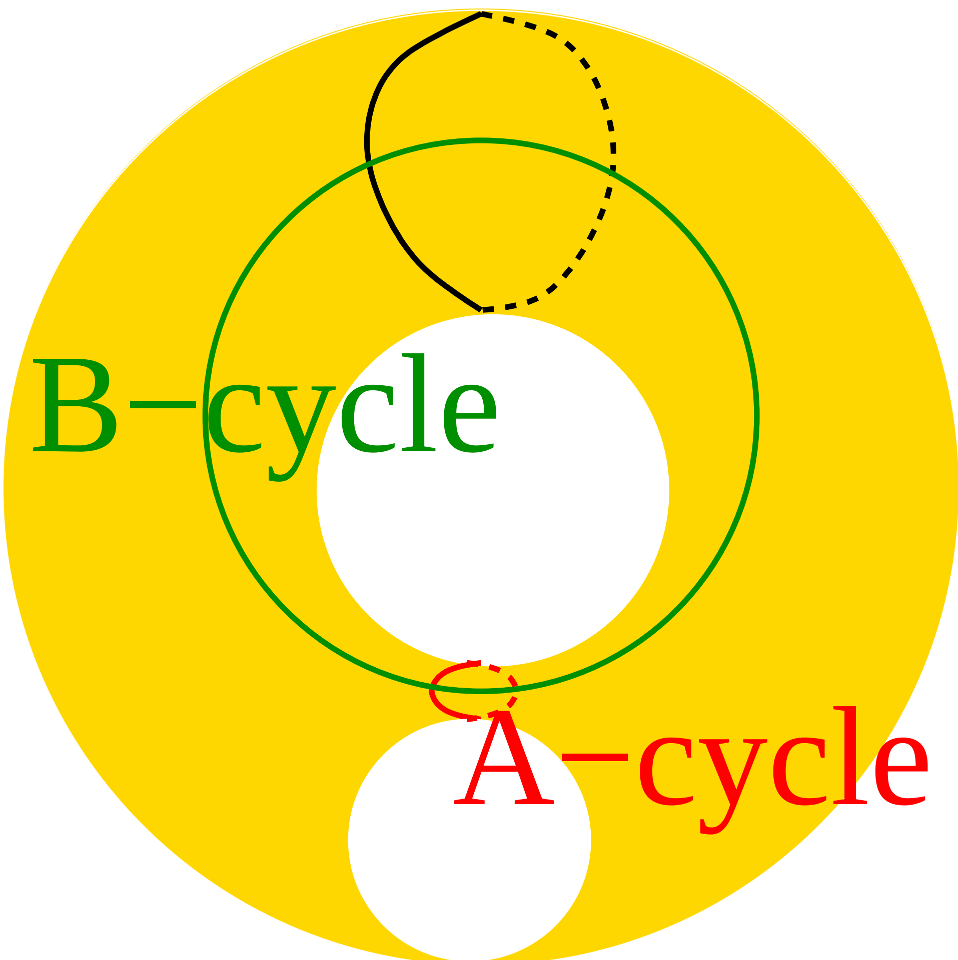
Here we find all the physical closed string states at all ghost numbers.

Adding $\mathcal{O}\left(\frac{1}{g_s}\right)$ ZZ branes $\longleftrightarrow$ open a pinched cycle (smooth out a singularity); $\oint_A \omega \neq 0$.

These lead to background tachyons with the "wrong" Liouville dressing $(\alpha \geq \frac{Q}{2})$; *i.e.* they diverge in the strong coupling region.

$\oint_B \omega$ creates ZZ branes. Their number is measured by the period of the conjugate A-cycle

$$\oint_A \omega = g_s N_{ZZ}$$



Matrix Model

Consider $(p = 2, q = 2l + 1)$, which corresponds to the one matrix model

Our surface is

$$2y^2 - 1 = T_q(x)$$

It has two copies of the complex x plane which are connected along a cut $(-\infty, -1)$ and l singularities (pinched cycles)

$$\left(x_n = \cos\frac{2\pi n}{q}\ ,\ y_n = 0\right)\ , \qquad n = 1, ..., l$$

Interpretation:

Discontinuity along the cut

$$\rho(x) = \mathrm{Im}\sqrt{2 + 2\,T_q(x)}$$

is the eigenvalue density.

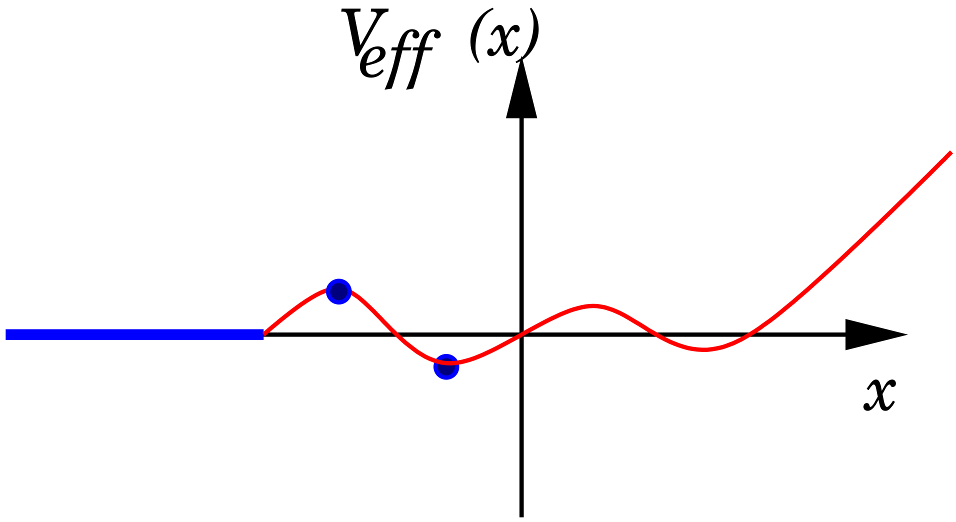
y is the force on an eigenvalue. $y = 0$ at the singularities.

The disk amplitude of FZZT brane

$$Z(x) = \int^x y\,dx' = -\frac{1}{2}V_{eff}(x)$$

is the effective potential of a probe eigenvalue.

ZZ brane: Eigenvalue at a stationary point of $V_{eff}(x)$ (where $y = 0$).



The ZZ branes decay (condense) and fill the Fermi sea

Matrix model M $\longleftrightarrow$ open strings between $N \to \infty$ condensed ZZ branes

FZZT brane in the matrix model

$$\left\langle \mathrm{Tr}\, \frac{1}{x - M} \right\rangle \quad \longleftrightarrow \quad y$$

or after exponentiation

$$\langle \det(x - M) \rangle \quad \longleftrightarrow \quad e^{\int^x y\, dx}$$

Can write the FZZT brane as

$$\det(x - M) = \int d\psi^\dagger d\psi e^{\psi^\dagger (x - M)\psi}$$

$\psi,\ \psi^\dagger \longleftrightarrow$ fermionic open strings between ZZ and FZZT branes.

Conclusions

- A "target space" Riemann surface $\mathcal{M}$ with a one form ω emerges as the moduli space of branes.

- Branes:
  - $\int^x \omega \longleftrightarrow$ creates extended branes
  - $\oint_B \omega \longleftrightarrow$ creates localized branes
  - $\oint_A \omega \longleftrightarrow$ measures # of localized branes

- Deformations of $\mathcal{M} \longleftrightarrow$ closed strings:
  - preserving $\oint_A \omega \longleftrightarrow$ ordinary closed strings
  - changing $\oint_A \omega \longleftrightarrow$ create localized branes, their background fields are "wrong branch" closed strings

This gives a worldsheet derivation of the matrix model, and adds a new perspective to the understanding that

the eigenvalues are associated with D-branes (Polchinski, McGreevy, Verlinde, Klebanov, Maldacena, N.S., Martinec...)