Higher Spin Gauge Theories in Any Dimension

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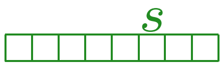
Symmetric Massless Free Fields



Fronsdal (1978)

$$\varphi_{n_1 \ldots n_s}(x) \qquad \varphi^{m}{}_{m}{}^{k}{}_{k\, n_5 \ldots n_s}(x) = 0$$

Abelian HS gauge symmetry

$$\delta \varphi_{n_1 \ldots n_s}(x) = \partial_{\{n_1} \varepsilon_{n_2 \ldots n_s\}}(x) \qquad \varepsilon^{r}{}_{r n_3 \ldots n_{s-1}} = 0$$

Invariant Free Action

$$S^s = \frac{1}{2}(-1)^s \int d^d x \, \{ \partial_n \varphi_{m_1 \ldots m_s} \partial^n \varphi^{m_1 \ldots m_s}$$

$$- \frac{s(s-1)}{2} \partial_n \varphi^{r}{}_{r m_1 \ldots m_{s-2}} \partial^n \varphi^{k}{}_{k}{}^{m_1 \ldots m_{s-2}} + s(s-1) \partial_n \varphi^{r}{}_{r m_1 \ldots m_{s-2}} \partial_k \varphi^{n k m_1 \ldots m_{s-2}}$$

$$- s \partial_n \varphi^{n}{}_{m_1 \ldots m_{s-1}} \partial_r \varphi^{r m_1 \ldots m_{s-1}} - \frac{s(s-1)(s-2)}{4} \partial_n \varphi^{r}{}_{r}{}^{n}{}_{m_1 \ldots m_{s-3}} \partial_k \varphi^{t}{}_{t}{}^{k m_1 \ldots m_{s-3}} \}$$

$s = 1$ **Maxwell**

$s = 2$ **Einstein**

$s > 2$ **Unifying gauge principle ?**

Geometric formulations

Metric-like

de Wit, Freedman (1980)
Siegel, Zwiebach (1987)
Francia, Sagnotti (2002)
Bekaert, Boulanger (2003)
de Medeiros, Hull (2003)

Frame-like

M.V.(1980)
Alkalaev, Shaynkman, M.V.(2003)

Higher Spin Problem

Find a nonlinear HS gauge theory

- Correct free field limit
- Unbroken HS gauge symmetries
- Non-Abelian global HS symmetry of a vacuum solution

Motivations

- Massive HS modes in Superstring from breaking of HS gauge symmetries?!

Gross, Mende (1988)

- AdS/CFT: HS symmetries are unbroken in the Sunborg–Witten limit

$$\lambda = g^2 N \to 0 \qquad\qquad l_{str}^2 \Lambda_{AdS} \to \infty$$

Sundborg hep-th/0103247
Witten (2001)
Sezgin, Sundell hep-th/0105001
Klebanov, Polyakov hep-th/0204051

Difficulties

- $S-$ matrix argument

Coleman, Mandula (1967)

- HS-gravity interaction problem

Aragone, Deser (1979)

$$\partial_n \to D_n = \partial_n - \Gamma_n \qquad [D_n\, D_m] = \mathcal{R}_{nm}\ldots$$

$$\delta\varphi_{nm\ldots} \to D_n \varepsilon_{m\ldots}$$

$$\delta S_s^{cov} = \int \underset{\uparrow}{\mathcal{R}_{\ldots}} (\varepsilon_{\ldots} D\varphi_{\ldots}) \neq 0 \qquad ?!$$

Weyl tensor for $s > 2$

Higher Derivatives in HS Interactions

A.Bengtsson, I.Bengtsson, L.Brink (1983)

Berends, Burgers, van Dam (1984)

$$S = S^2 + S^3 + \dots$$

$$S^3 = \sum_{p,q,r} (D^p \varphi)(D^q \varphi)(D^r \varphi) \ell^{p+q+r+\frac{1}{2}d-3}$$

Role of AdS Background

HS theories:

Fradkin, M.V. (1987)

$$\ell = \Lambda^{-\frac{1}{2}} = R_{AdS} \qquad [D_n, D_m] \sim \Lambda \sim O(1)$$

String:

$$\ell \sim \sqrt{\alpha'}$$

SUGRA by Gauging SUSY Algebra

$o(d-1,2)$ $\qquad$ $o(N)$

$T^{A\,B}$ $\qquad$ Q^p_α $\qquad$ $t^{p\,q}$ $\qquad$ $A, B, \cdots = 0, \ldots d$

${\omega_n}^{A\,B}$ $\qquad$ ${\Psi_n}^p_\alpha$ $\qquad$ ${A_n}^{p\,q}$ $\qquad$ $p, q, \cdots = 1, \ldots N$

$a, b, \cdots = 0, \ldots d-1$

$s = 2$ $\qquad$ $g_{nm} \longrightarrow e^a_n \longrightarrow \{e^a_n, \omega^{ab}_n\} \longrightarrow \omega^{AB}_n$ $\qquad$ $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$

MacDowell, Mansouri (1978)

Stelle, West (1980)

Higher Spin Gauge Fields

$s \geq 2$

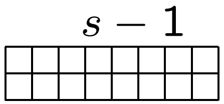
$$\varphi_{n_1\dots n_s} \longrightarrow e_n{}^{a_1\dots a_{s-1}} \longrightarrow \{e_n{}^{a_1\dots a_{s-1}}, \omega_n{}^{a_1\dots a_{s-1},b_1\dots b_{s-1}}\} \longrightarrow$$

$$\longrightarrow \omega_n{}^{A_1\dots A_{s-1},B_1\dots B_{s-1}}$$

Lopatin, M.V. (1988)

M.V. hep-th/0106200

$$\left.\begin{array}{l} \omega_n{}^{\{A_1\dots A_{s-1},A_s\}\,B_2\dots B_{s-1}} = 0 \\ \omega_n{}^{A_1\dots A_{s-3}\,C}{}_{C,}{}^{B_1\dots B_{s-1}} = 0 \end{array}\right\}$$



$o(d-1,2)$

Dynamical frame-like field

$$e_n{}^{a_1\ldots a_{s-1}} = \omega_n^{a_1\ldots a_{s-1}}{}_{,}{}^{B_1\ldots B_{s-1}} V_{B_1}\ldots V_{B_{s-1}}$$

$V_B = \delta_B^{\hat{d}}$ $\hat{d} : (d+1)^{th}-$ **direction**

Fronsdal field $\varphi_{n_1 n_2 \ldots n_s} = e_{\{n_1\,,\,n_2\ldots n_s\}}$

Auxiliary Lorentz–connection type fields

$$\omega_n{}^{a_1\ldots a_{s-1}}{}_{,}{}^{b_1\ldots b_t} = \omega_n^{a_1\ldots a_{s-1}}{}_{,}{}^{b_1\ldots b_t\, B_{t+1}\ldots B_{s-1}}\, V_{B_{t+1}}\ldots V_{B_{s-1}} : \qquad \left(\frac{1}{\sqrt{\Lambda}}\frac{\partial}{\partial x}\right)^t (e)$$

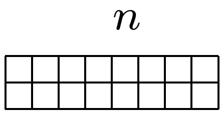
$$t > 0$$

HS Algebra $hu(1|2{:}[d-1,2])$

As CFT_{d-1} HS algebra: Eastwood hep-th/0206233

Generators

$$T_{A_1 \ldots A_n, B_1 \ldots B_n}$$



$o(d-1,2)$

$$T_{\{A_1 \ldots A_n, A_{n+1}\} B_2 \ldots B_n} = 0 \qquad T^C{}_{C\, A_3 \ldots A_n, B_1 \ldots B_n} = 0$$

Oscillator Realization

M.V. hep-th/0304049

$[Y^A_i\,,\,Y^B_j]_* \;=\; \epsilon_{ij}\eta^{AB}$ **canonical pair** $Y^A_1 = Y^A\,,\,Y^A_2 = P^A$

$\eta_{A\,B} = \eta_{B\,A}$ $A, B = 0, \dots d$ $\epsilon_{i\,j} = -\epsilon_{j\,i}$ $i, j = 1, 2$

$T^{A,B} = -T^{B,A} \;=\; \frac{1}{2}\{Y^A_i\,,\,Y^B_j\}_*\epsilon^{ji}$ $o(d-1, 2)$

$t_{i\,j} = t_{j\,i} \;=\; \frac{1}{2}\{Y^A_i\,,\,Y_{j\,A}\}_*$ $sp(2)$

$[t_{i\,j}\,,\,T^{A\,B}]_* \;=\; 0$ **Howe duality**

$$hu(1|2{:}[d-1, 2])$$

Lie bracket: $[f(Y)\,,\,g(Y)]_*$

- $[f(Y)\,,\,t_{ij}]_* \;=\; 0$
- $f(Y) \;=\; t^{i\,j} * f_{l\,i\,j} \;=\; f_{r\,i\,j} * t^{i\,j} \;\sim\; 0$

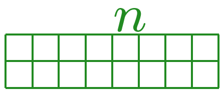
Gauging HS Algebra

$$\omega(Y|x) = \sum_{n=0}^{\infty} dx^m \omega_{m\,A_1 \ldots A_n , B_1 \ldots B_n}(x) Y_1^{A_1} \ldots Y_1^{A_n} Y_2^{B_1} \ldots Y_2^{B_n}$$

$$0 \;=\; Dt_{i\,j} \;=\; \overset{\mathbf{0}}{\overset{\|}{dt_{i\,j}}} + [\omega\,,\,t_{i\,j}]_* = 0$$

imposes Young properties

$$\omega_{m\,\{A_1 \ldots A_n , A_{n+1}\} B_2 \ldots B_n} = 0$$



Field strength

$$R = d\omega(Y|x) + \omega(Y|x) \wedge * \, \omega(Y|x)$$

Different Spins: Homogeneous Polynomials

$$\omega(\mu Y|x) = \mu^{2(s-1)}\omega(Y|x)$$

maximal finite dimensional subalgebra

$o(d-1,2)$ $\oplus$ $u(1)$

bilinears constants

$s{=}2$ $s{=}0$

Matrix valued fields:

$$\omega \rightarrow \omega_q{}^p(Y|x) \qquad p,q = 1,\dots,n \quad u(1) \rightarrow u(n)$$

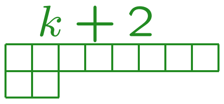
Lower Spin Examples

$s = 2:$ $R^a = 0$ zero torsion

↓ Weyl tensor

$R^{ab} = e_c \wedge e_d \, C^{ac,\,bd}$ Ricci is zero

Bianchi identities imply $\partial_{n_1} \ldots \partial_{n_k} C_{a_1 a_2, b_1 b_2} \sim C_{c_1 \ldots c_{k+2}, d_1 d_2}$

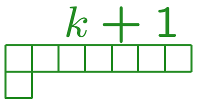


Einstein equations: $C_{a_1 \ldots a_{k+2}, b_1 b_2}$ traceless

Linearized $s = 2$ **equations in flat space**

$$dC_{a_1 \ldots a_l, b_1 b_2} = e^c \left(k C_{a_1 \ldots a_l c, b_1 b_2} + 2 C_{a_1 \ldots a_l \{b_1, b_2\} c} \right)$$

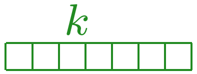
$s = 1$: Maxwell equations



traceless

$$F = e_c \wedge e_d \, C^c{,}^d \,, \qquad dC_{a_1\ldots a_l,b} = e^c \Big((l+1) C_{a_1\ldots a_l c,b} + C_{a_1\ldots a_l b,c} \Big)$$

$s = 0$: Klein-Gordon equations

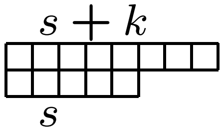


traceless

$$dC_{a_1\ldots a_k} = e^c (k+2) C_{a_1\ldots a_k c}$$

$$C_{a_1\ldots a_k b}{}^b = 0$$

Any s :



traceless

$$C_{a_1\ldots a_{s+k}\,,\,b_1\ldots b_s}$$

AdS_d **Vacuum**

$\omega_0 \in o(d-1,2)$:

$$\omega_0^{A\,B} \neq 0\,, \qquad \omega_0^{A_1\ldots A_{s-1},B_1\ldots B_{s-1}} = 0 \qquad s > 2$$

AdS_d **vacuum:**

$$R^{A\,B}(\omega_0) = 0$$

Linearization:

$$\omega(Y|X) = \omega_0(Y|X) + \omega_1(Y|X)$$

Central On-Mass-Shell Theorem

All spins: free equations + constraints on auxiliary fields

M.V. hep-th/0106200

$$\boxed{\begin{aligned} R_1(Y|x) &= \tfrac{1}{2}\, e_0^a \wedge e_0^b\, \frac{\partial^2}{\partial Y_i^a \partial Y_j^b}\, \varepsilon_{ij}\, C(Y|x)\Big|_{V_A Y_i^A = 0} \\ \widetilde{D}_0(C) &= 0 \end{aligned}}$$

$$R_1 = d\omega + \omega_0 * \omega_1 + \omega_1 * \omega_0\,, \qquad \widetilde{D}_0(C) = dC + \omega_0 * C - C * \widetilde{\omega}_0$$

$$\widetilde{f}(Y) = f(\widetilde{Y})\,, \qquad \widetilde{Y}_i^A = Y_i^A - \frac{2}{V^2}\, V^A V_B\, Y_i^B$$

$$t_{ij} * C = C * t_{ij}$$

Unfolded Dynamics

$\omega^{\Phi}(x)$: **a set of differential forms**

$$d\omega^{\Phi} = F^{\Phi}(\omega) \qquad\qquad d = dx^{n}\frac{\partial}{\partial x^{n}}$$

Consistency condition

$$F^{\Phi} \wedge \frac{\delta F^{\Omega}}{\delta \omega^{\Phi}} = 0$$

FDA's

- (HS) gauge invariance

$$\delta\omega^{\Phi} = d\varepsilon^{\Phi} - \varepsilon^{\Omega}\frac{\delta F^{\Phi}}{\delta\omega^{\Omega}} = 0 \qquad \deg\varepsilon^{\Phi} = \deg\omega^{\Phi} - 1$$

- Invariance under diffeomorphisms
- Interactions: nonlinear deformation of F^{Φ}
- Degrees of freedom: 0-forms

  Infinite dimensional module dual to

  the space of one-particle states ($\forall x$)
- Universality

Nonlinear Construction

- Find a larger algebra g' such that

$$\star \qquad \omega \to W = \omega + \omega C + \omega C^2 + \dots$$

in

$$dW + W \wedge W = 0$$

reconstructs nonlinear equations

- Find restrictions on W that reconstruct $\star$ in all orders

Guiding principle: unbroken $sp(2)$

Result: interaction is unique up to field redefinitions.

YM constant $g^2 = |\Lambda|^{\frac{d-2}{2}}\kappa^2$ can be rescaled away in the classical HS model

Doubling of Oscillators

$g \longrightarrow g'$: $\qquad Y_i^A \rightarrow (Z_i^A, Y_i^A)$ $\qquad$ M.V. (1992)

$$\omega(Y|x) \rightarrow W(Z,Y|x)\,, \qquad C(Y|x) \rightarrow B(Z,Y|x)$$

$S(Z,Y|x) = dZ_i^A \, S_A^i$: connection along Z_i^A

Noncommutative connection

$$\mathcal{W} = d + W + S$$

Star Product

$$(f * g)(Z,Y) = \int dS dT f(Z+S, Y+S) g(Z-T, Y+T) \exp 2 S^i_A T_i^A$$

$$[Y_i^A, Y_j^B]_* = -[Z_i^A, Z_j^B]_* = \varepsilon_{ij}\eta^{AB}$$

$Z - Y : Z + Y$ **normal ordering**

Klein operator

$$\mathcal{K} = \exp \frac{2}{V^2} V_A Z^{Ai} V_B Y^B{}_i \,, \qquad \mathcal{K} * f = \tilde{f} * \mathcal{K} \,, \qquad \mathcal{K} * \mathcal{K} = 1$$

Nonlinear HS Equations

M.V. hep-th/0304049

$$\boxed{\mathcal{W}*\mathcal{W}=\frac{1}{2}(dZ^i_A dZ^A_i+4\Lambda^{-1}dz^i dz_i B*\mathcal{K})\qquad \mathcal{W}*B=B*\widetilde{\mathcal{W}}}$$

$$\widetilde{\mathcal{W}}(dZ,Z,Y)=\mathcal{W}(\widetilde{dZ},\tilde{Z},\tilde{Y})\qquad dz_i=\frac{1}{\sqrt{V^2}}V_B dZ^B_i$$

Manifest gauge invariance

$$\delta\mathcal{W}=[\varepsilon,\mathcal{W}]_*\,,\qquad \delta B=\varepsilon*B-B*\tilde{\varepsilon}\,,\qquad \varepsilon=\varepsilon(Z,Y|x)$$

Nonlinear $sp(2)$: two dimensional fuzzy hyperboloid in noncommutative space of Y^A_i and Z^A_j.

Radius depends on HS curvature $B(x)$.

Perturbative Analysis:

$$W = W_0 + W_1\,, \qquad S = dZ_i^A Z^i_A + S_1\,, \qquad B = B_1$$

$$W_0 = \frac{1}{2}\omega_0^{AB}(x) Y^i_A Y_{iB} \qquad \omega_0^{AB}(x):\qquad AdS_d$$

Central On-Mass-Shell Theorem

reproduced to lowest order

Singletons in any Dimension

M.V. hep-th/0404124

$hu(1|(1,2){:}[d-1,2])$: **fermionic extension**

$$sp(2) \to osp(1,2) \qquad Y_i^A \to (Y_i^A, \phi^A)\,, \qquad \{\phi^A, \phi^B\} = -2\eta^{AB}$$

Fock modules of $hu(1|2{:}[M,2])$ and $hu(1|(1,2){:}\,[M,2])$: massless scalar S_M and spinor F_M in M dimensions

Bars, Deliduman, Andreev hep-th/9803188

Generalized Flato-Fronsdal theorem

$$S_{d-1} \otimes S_{d-1} = \sum_{s=0}^{\infty} \oplus \overbrace{\square\square\square\square\square\square\square\square}^{s} \qquad m = 0 \quad \textbf{bosons in } AdS_d$$

Mikhailov (2002); Das, Jevicki (2003); Gopakumar (2003)

$$F_{d-1} \otimes S_{d-1} = \sum_{s=\frac{1}{2}}^{\infty} \oplus \overbrace{\square\square\square\square\square\square\square\square}^{s-1/2} \qquad m = 0 \quad \textbf{fermions in } AdS_d$$

$$F_{d-1} \otimes F_{d-1} = \sum_{p,q} \oplus \;\; q \begin{array}{l} \overbrace{\square\square\square\square\square\square\square\square}^{p+1} \\ \square \\ \square \\ \square \\ \square \end{array} \qquad m = 0 \quad \textbf{bosons in } AdS_d$$

$$\oplus m > 0 \qquad \textbf{antisymmetric tensors}$$

Matching: gauge fields –UIRs

HS superalgebras in any d

Finite dimensional subsuperalgebras: $d = 3, 4, 5.$

Conclusions

- **Nonlinear HS theories exist**

any d

- **HS geometry**

fuzzy hyperboloid

Other Topics

- **Twistor realization:**

$Y_i^A \rightarrow$ **spinor oscillators** y_α M.V. (1988)

AdS_4, AdS_5, $N=4$ SYM

Sezgin, Sundell

Alkalaev, M.V.

- **HS fields: superfields in space-time with matrix (central charge) coordinates**

Fronsdal (1985)

Bandos, Lukierski, Sorokin hep-th/9904109

M.V. hep-th/0106149

Nonlinear nonlocal HS currents

Gelfond, M.V. hep-th/0304020

- **Applications of Unfolded Dynamics**

Classification of all (free) conformal equations

Tipunin, Shaynkman, M.V. hep-th/0401086

To Do

- Free and interacting mixed symmetry HS gauge fields

  in AdS_d

- Nonlinear HS dynamics at the action level

- $sp(2M)$ covariant formulation of nonlinear HS theories

- **Solutions of nonlinear HS field equations:**

  to break down HS gauge symmetries =

  to introduce a massive parameter

  $$m^2 \neq \Lambda \qquad\qquad \Lambda^{-\frac{1}{2}} D \sim 1$$

  Low energy expansion in $\left(\frac{1}{m}\frac{\partial}{\partial x}\right)^p$

- **To understand HS symmetries in String Theory?!**