

GAUGE THEORY WITH

$SU(N)$ GAUGE GROUP

=

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STRING THEORY WITH

$1/N$ AS THE STRING

COUPLING CONSTANT?

AN OLD GOAL, AND

APPARENTLY TRUE...

②

FROM THE AdS/CFT CORRESPONDENCE
WE KNOW THAT GAUGE/STRING
DUALITY HOLDS FOR

$\mathcal{N}=4$ SUPER YANG-MILLS

ALL KINDS OF ORBIFOLDS THEREOF

AND MULTITUDES OF THEORIES

OBTAINED BY RELEVANT

PERTURBATIONS AND R.G. FLOWS

MORALLY SPEAKING, FOR ALL

THEORIES OF MATRIX-VALUED FIELDS

IN $D \leq 4$

③
BUT AT PRESENT, IT IS ONLY A

USEFUL COMPUTATIONAL TOOL

FOR LARGE $g^2 N$, WHERE WE

CAN USE SUPERGRAVITY.

THAT ISN'T ENOUGH IF WE

WISH TO UNDERSTAND THE

MYSTERIES OF QCD, SUCH

AS QUARK CONFINEMENT.

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QCD IS ASYMPTOTICALLY FREE

SO $g^2 N \ll 1$ AT SHORT DISTANCES.

A STRING THEORY DUAL TO QCD

MUST LOOK AT SHORT

DISTANCES LIKE THE STRING

DUAL OF A WEAKLY COUPLED

GAUGE THEORY.

BUT WHAT IS THAT?

... I'LL DESCRIBE A WEAK COUPLING

GAUGE/STRING DUAL, BUT I DON'T
KNOW IF IT IS THE ONE WE NEED.

FOR A STRING DUAL OF $\mathcal{N}=4$ ⁽⁵⁾
SUPER YANG-MILLS, WE WANT A
TARGET SPACE WITH $PSU(4|4)$
SYMMETRY- THE SYMMETRY OF THE
 $\mathcal{N}=4$ THEORY.

THE AdS/CFT STORY IS BASED
ON

$$PSU(4|4)/SO(5) \times SO(5)$$

WHOSE BOSONIC PART IS

$$AdS_5 \times S^5$$

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I'LL CONSIDER INSTEAD A STRING
THEORY WITH TARGET $CP^{3/4}$,
A HOMOGENEOUS SPACE WITH
FOUR BOSE AND FOUR FERMI
COORDINATES

$$Z^I = (X^\alpha, \psi^A)$$

$$\alpha, A = 1 \dots 4$$

X^α NOT ALL ZERO AND

$$Z^I \simeq \lambda Z^I, \quad \lambda \neq 0$$

$$\lambda \in \mathbb{C}^*$$

PRECISELY BECAUSE THE NUMBER ⁽⁷⁾
 OF FERMIONS EQUALS THE NUMBER
 OF BOSONS, $\mathbb{CP}^{3/4}$ IS A
 CALABI-YAU SUPERMANIFOLD

VOLUME FORM

$$\Omega_0 = \epsilon_{\alpha_1 \dots \alpha_4} dx^{\alpha_1} \dots dx^{\alpha_4} \\
\epsilon_{A_1 \dots A_4} d\psi^{A_1} \dots d\psi^{A_4}$$

ON $\mathbb{CP}^{4/4}$ ~~THIS FORM~~ IS C^* -INVARIANT

AND SO DESCENDS TO CALABI-YAU
 VOLUME FORM ON $\mathbb{CP}^{3/4}$

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WE KNOW VARIOUS REASONS

~~AND~~ THAT $\mathcal{N}=4$ SUPER-YANG-MILLS
IS SPECIAL.

THE β FUNCTION VANISHES
PRECISELY FOR $\mathcal{N}=4 \dots$

AND $\mathbb{CP}^{3/\mathcal{N}}$ IS CALABI-YAU
PRECISELY FOR $\mathcal{N}=4 \dots$

⑨

BEING A CALABI-YAU

MANIFOLD, $\mathbb{CP}^{3/4}$ HAS A "TOPOLOGICAL
B-MODEL."

ONE STARTS WITH THE 2-DIM²

$N=(2,2)$ SUPERSYMMETRIC σ

MODEL WITH TARGET $\mathbb{CP}^{3/4}$

$$\mathcal{L} = \int d^2x \left[\partial_\alpha Z^I \partial^\alpha \bar{Z}_I + \text{fermions} \right]$$

AND MAKE A "TOPOLOGICAL
TWIST"

THE TWIST MEANS WE
REINTERPRET ONE OF THE
SUPERCHARGES, Q , WHICH
HAS $Q^2 = 0$, AS A
"BRST OPERATOR"....

SPACE OF PHYSICAL STATES
= COHOMOLOGY OF Q

I WANT TO DO THE

B-MODEL FOR OPEN

STRINGS ... ASSOCIATED WITH

$\mathbb{CP}^{3|4}$ -FILLING D-BRANES.

LET US FIRST REMEMBER

THE TOPOLOGICAL B-MODEL

OF AN ORDINARY (BOSONIC)

CALABI-YAU³-MANIFOLD ~~THE~~

W

(12)

THE BASIC OPEN

STRING EXCITATION IS A

"GAUGE FIELD" $A_{\bar{t}}(x, \bar{x})$

{WITH VALUES IN $U(N)$ FOR

N D-BRANES... AND

ACTION

$$\int dx^{\bar{t}} dx^{\bar{j}} dx^{\bar{k}} \text{Tr} \left(A_{\bar{t}} \partial_{\bar{j}} A_{\bar{k}} + \frac{2}{3} A_{\bar{t}} A_{\bar{j}} A_{\bar{k}} \right)$$

$\wedge \Omega$

FOR $\mathbb{CP}^{3/4}$, IT IS MUCH
THE SAME, EXCEPT THE

GAUGE FIELD $A_{\bar{c}}(X, \bar{X}, \psi)$

DEPENDS ON FERMIONIC

COORDINATES ψ^A AS WELL AS

BOSONIC COORDINATES $X, \bar{X}$

$$\begin{aligned} A_{\bar{c}}(X, \bar{X}, \psi) = & A_{\bar{c}}(X, \bar{X}) \\ & + \psi^A \chi_{\bar{c}A}(X, \bar{X}) \\ & + \psi^A \psi^B \phi_{\bar{c}AB}(X, \bar{X}) \\ & + \dots \end{aligned}$$

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THE ACTION IS STILL

A CHERN-SIMONS (0,3)-FORM

$$I = \int d\bar{x}^{\bar{t}} d\bar{x}^{\bar{j}} d\bar{x}^{\bar{k}}$$

$$\text{Tr} (a_{\bar{t}} \partial_{\bar{j}} a_{\bar{k}} + \frac{2}{3} a_{\bar{t}} a_{\bar{j}} a_{\bar{k}})$$

$$\wedge \Omega$$

WHERE NOW, ROUGHLY,

$$\Omega = d^3x d^4\psi$$

THIS ACTION HAS THE
AMAZING PROPERTY THAT

- ALTHOUGH DEFINED ON

THE SPACE $\mathbb{CP}^{3/4}$ OF THE

WRONG DIMENSION - IT HAS

THE SAME SPECTRUM AS

$n=4$ SUPER YANG-MILLS

ON MINKOWSKI SPACE

... AND SOME OF THE SAME
INTERACTIONS

HELICITIES :

$$Q = A + \psi\chi + \psi\psi\phi + \psi^3\tilde{\chi} + \psi^4\epsilon$$

!	!	!	!	!
1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1

THIS STATEMENT IS EQUIVALENT
TO THE "PENROSE TRANSFORM"

FROM WAVE EQUATIONS IN

MINKOWSKI SPACE TO $\bar{\partial}$ OR

SHEAF COHOMOLOGY IN $\mathbb{CP}^{3/4}$

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HOWEVER, WE DON'T
GET THE INTERACTIONS OF
 $\mathcal{N}=4$ SYM ...

ROUGHLY SPEAKING, WE
CAN'T, SINCE WE HAVE AN
EXTRA $U(1)_R$ SYMMETRY...

$\mathcal{N}=4$ SYM HAS $SU(4)_R$
BUT $\mathbb{CP}^{3|4}$ HAS $U(4)_R$
ACTING ON ψ^A , $A=1\dots 4$

THE EXTRA $U(1)$

$$Z^I = \left(\underset{S=0}{X^\alpha}, \underset{S=1}{\psi^A} \right)$$

DOES NOT LEAVE INVARIANT

THE VOLUME-FORM

$$\Omega = d^3x d^4\psi$$

SO IT IS ANOMALOUS

IN THE B-MODEL

THE $\mathcal{N}=4$ SYM ACTION ⁽¹⁹⁾
IS A SUM OF TERMS WITH
DIFFERENT VALUES OF THE
S-CHARGE.

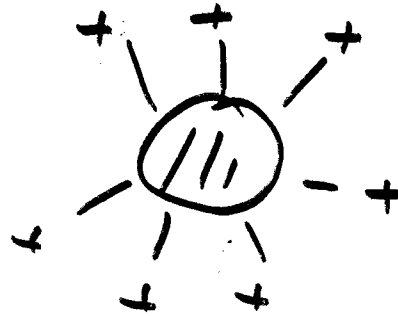
THE TERMS OF $S = -4$
(IN SOME CONVENTION I WON'T
EXPLAIN) COME FROM THE
CHERN-SIMONS $(0,3)$ -FORM,
AND THE REST FROM A
D-WINSTANTON CORRECTION

TREE-LEVEL YANG-MILLS

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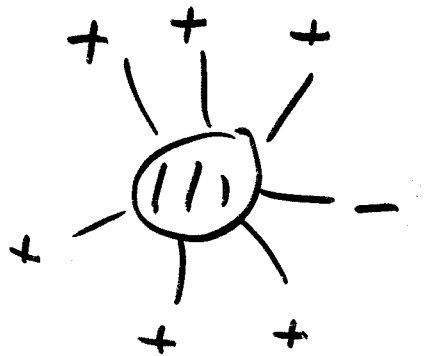
SCATTERING AMPLITUDES ARE

REPRODUCED AS FOLLOWS:

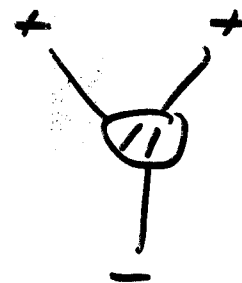


$$= 0, \quad A$$

STANDARD RESULT
de Witt, ...

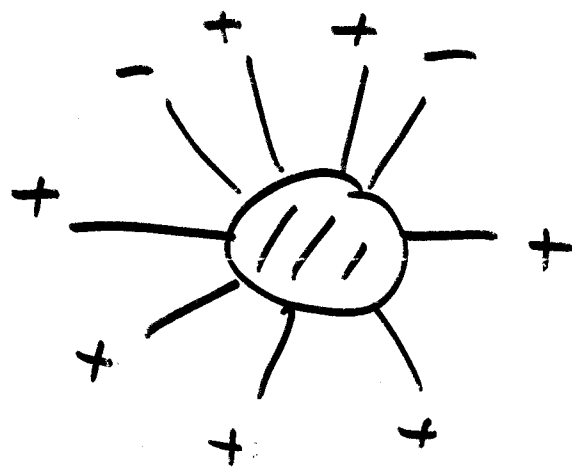


$$= 0 \quad \text{EXCEPT}$$



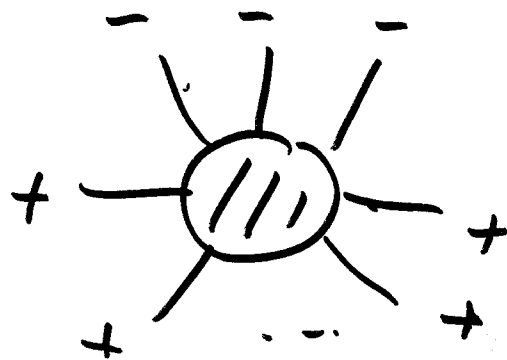
FROM THE CHERN-SIMONS
(0,3)-FORM

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= Parke-Taylor
MHV
amplitude

arises as a
one-instanton effect



two instantons

$$\left(\underbrace{++\dots+}_P \underbrace{--\dots-}_8 \right)$$

FROM 8-1 INSTANTONS

THE APPROACH VIA
INSTANTONS IN $\mathbb{CP}^{3/4}$ HAS
REPRODUCED AND ILLUMINATED
STANDARD YM TREE
AMPLITUDES

Roiban, Spradlin, Volovich

Cachazo, Svrcek, EW

HOWEVER NO CALCULATIONS

YET FOR LOOPS

BEAUTIFUL RESULTS TO AIM FOR

-DIXON'S TALK- BERN, DIXON, KOSOWER
....

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THE LOOPS, IN ANY
EVENT, WILL NOT
DESCRIBE PURE SYM
BECAUSE CLOSED STRING
MODES WILL COME IN...
THE CLOSED STRING
MODES DESCRIBE $\mathcal{N}=4$
CONFORMAL SUPERGRAVITY
(BERKOVITS & EW)

I SEE NUMEROUS PROBLEMS,
AMONG THEM

* LEARN TO CALCULATE
THE LOOPS

** FWD A VERSION
OF TWISTOR STRING
THEORY THAT DESCRIBES
PURE SUPER YANG-MILLS
WITHOUT THE CONFORMAL
SUPERGRAVITY

WE NEED TO SOLVE THE
SECOND PROBLEM IF WE
REALLY WANT TO ACHIEVE
THE ORIGINAL GOAL:

FIND A STRING THEORY
DUAL TO WEAKLY COUPLED
STRING THEORY
GAUGE