Cavity QED with Rydberg Atoms

Serge Haroche,

Collège de France & Ecole Normale Supérieure, Paris

A three lecture course

Goal of lectures

Manipulating states of simple quantum systems has become an important field in quantum optics and in mesoscopic physics, in the context of quantum information science. Various methods for state preparation, reconstruction and control have been recently demonstrated or proposed.

Two-level systems (qubits) and quantum harmonic oscillators play an important role in this physics. The qubits are information carriers and the oscillators act as memories or quantum bus linking the qubits together. Coupling qubits to oscillators is the domain of Cavity Quantum Electrodynamics (CQED) and Circuit Quantum Electrodynamics (Circuit-QED). In microwave CQED, the qubits are Rydberg atoms and the oscillator is a mode of a high Q cavity while in Circuit QED, Josephson junctions act as artificial atoms playing the role of qubits and the oscillator is a mode of an LC radiofrequency resonator.

The goal of this course is to present the field of CQED, whose concepts are readily extended to circuit QED, and to analyze various ways to measure, manipulate and reconstruct quantum fields in cavities. These lectures will give us an opportunity to review basic concepts of measurement theory in quantum physics.

Outline of lectures (tentative)

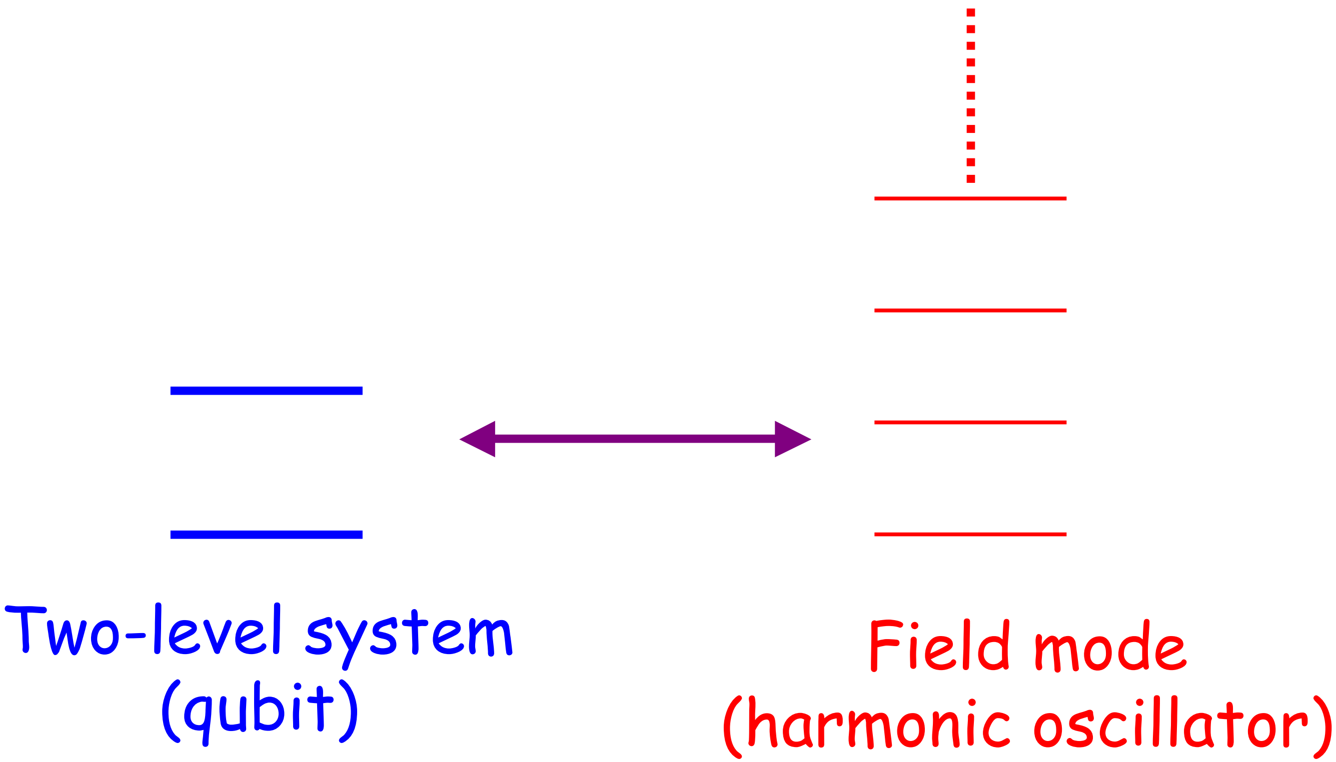
•**Lecture 1 (July 17th):**
Introduction to Cavity QED with Rydberg atoms interacting with microwave fields in a high Q superconducting resonator.

• **Lecture 2 (July 19th):**
Review of measurement theory illustrated by the description of quantum non-demolition photon counting in Cavity QED.

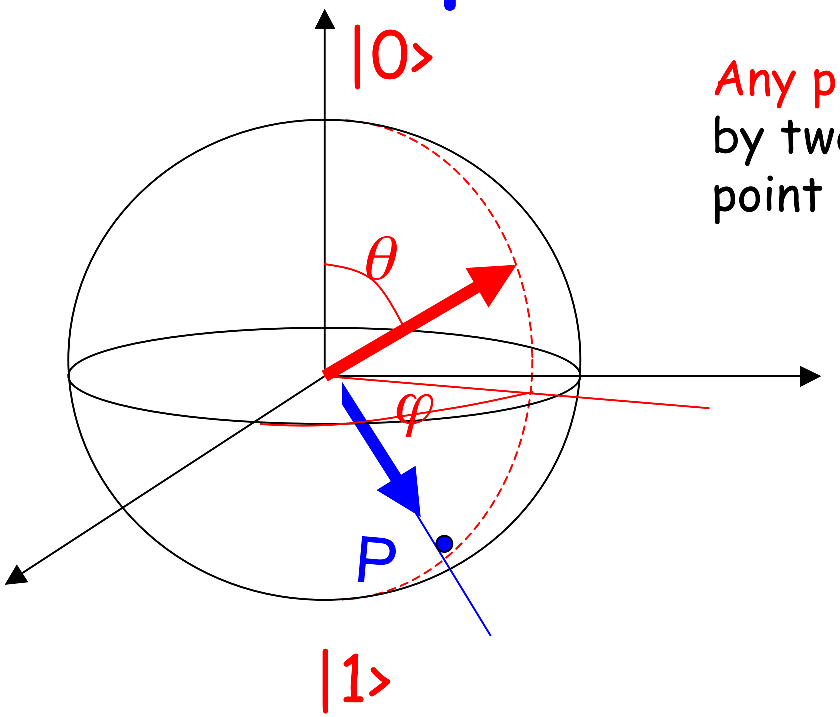
• **Lecture 3 (July 20th):**
State reconstruction and quantum feedback experiments in Cavity QED. Link with Circuit QED.

I-A

The basic ingredients of Cavity QED: qubits and oscillators



Description of a qubit (or spin 1/2)



Any pure state of a qubit (0/1) is parametrized by two polar angles θ,φ and is represented by a point on the Bloch sphere :

$$|\theta,\varphi\rangle = \cos\frac{\theta}{2} e^{-i\varphi/2}|0\rangle + \sin\frac{\theta}{2} e^{i\varphi/2}|1\rangle$$

A statistical mixture is represented by a density operator:

$$\rho = |\psi_{qubit}\rangle\langle\psi_{qubit}| \quad (pure\ state)$$

$$\rho = \sum_i p_i |\psi_{qubit}^{(i)}\rangle\langle\psi_{qubit}^{(i)}| \quad (\sum_i p_i = 1) \quad (mixture)$$

which can be expanded on Pauli matrices with coeffcients defining a Bloch vector:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_i^2 = I \quad (i = x, y, z)$$

$$\rho = \frac{1}{2}\left(I + \vec{P}.\vec{\sigma}\right) \quad ; \quad |\vec{P}| \leq 1$$

ρ hermitian, with unit trace and ≥ 0 eigenvalues

$$|\vec{P}| = 1 : pure\ state \quad ; |\vec{P}| < 1 : mixture\, ; |\vec{P}| > 1 : non\ physical\ (negative\ eigenvalue)$$

A qubit quantum state (pure or mixture) is fully determined by its Bloch vector

Description of a qubit (cont'd)

The Bloch vector components are the expectation values of the Pauli operators:

$$P_i = Tr\rho\sigma_i = \langle\sigma_i\rangle \quad (i = x, y, z) \quad ; \quad \rho = \frac{1}{2}\left[I + \sum_i \langle\sigma_i\rangle\sigma_i\right] \qquad \left(Tr\sigma_i\sigma_j = 2\delta_{ij}\right)$$

The qubit state is determined by performing averages on an ensemble of realizations: **the concept of quantum state is statistical.**

Calling p_{i+} et p_{i-} the probabilities to find the qubit in the eigenstates of σ_i with eigenvalues ±1, we have also:

$$P_i = p_{i+} - p_{i-} \quad ; \quad \rho = \frac{1}{2}\left[I + \sum_i \left(p_{i+} - p_{i-}\right)\sigma_i\right]$$

Manipulation and measurement of qubits: Qubit rotations are realized by applying resonant pulses whose frequency, phase and durations are controlled. In general, it is easy to measure the qubit in its energy basis (σ_z component). To measure an arbitrary component, one starts by performing a rotation which maps its Bloch vector along 0z, and then one measures the energy.

Qubit rotation induced by microwave pulse

Coupling of atomic qubit with a classical field:

$$H = H_{at} + H_{\text{int}}(t) \quad ; \quad H_{at} = \hbar \frac{\omega_{eg}}{2}\sigma_z \quad ; \quad H_{\text{int}}(t) = -\vec{D}.\vec{E}_{mw}(t)$$

Microwave electric field linearly polarized with controlled phase φ_0:

$$E_{mw}(t) = E_0 \cos(\omega_{mw} t - \varphi_0) = \frac{E_0}{2}\left[\exp i(\omega_{mw} t - \varphi_O) + \exp - i(\omega_{mw} t - \varphi_O)\right]$$

Qubit electric-dipole operator component along field direction is off-diagonal and real in the qubit basis (without loss of generality):

$$D_{along\ field} = d\sigma_x \quad (d \quad real)$$

Hence the Hamiltonian:

$$H = \hbar \frac{\omega_{eg}}{2}\sigma_z - \hbar \frac{\Omega_{mw}}{2}\sigma_x\left[\exp i(\omega_{mw} t - \varphi_O) + \exp - i(\omega_{mw} t - \varphi_O)\right] \quad ; \quad \Omega_{mw} = d.E_0 / \hbar$$

and in frame rotating at frequency ω_{mw} around Oz:

$$i\hbar \frac{d\left|\Psi\right\rangle}{dt} = H\left|\Psi\right\rangle \quad ; \quad \left|\tilde{\Psi}\right\rangle = \exp\left(i\frac{\sigma_z \omega_{mw}}{2}t\right)\left|\Psi\right\rangle \quad \rightarrow \quad i\hbar \frac{d\left|\tilde{\Psi}\right\rangle}{dt} = \tilde{H}\left|\tilde{\Psi}\right\rangle \quad ;$$

$$\tilde{H} = \hbar \frac{(\omega_{eg} - \omega_{mw})}{2}\sigma_z - \hbar \frac{\Omega_{mw}}{2} e^{i\frac{\sigma_z \omega_{mw} t}{2}} \sigma_x e^{-i\frac{\sigma_z \omega_{mw} t}{2}}\left[\mathrm{e}^{i(\omega_{mw} t - \varphi_O)} + \mathrm{e}^{-i(\omega_{mw} t - \varphi_O)}\right]$$

Bloch vector rotation (ctn'd)

$$\tilde{H} = \hbar \frac{(\omega_{eg} - \omega_{mw})}{2}\sigma_z - \hbar \frac{\Omega_{mw}}{2} e^{i\frac{\sigma_z \omega_{mw} t}{2}} \sigma_x e^{-i\frac{\sigma_z \omega_{mw} t}{2}} \left[\mathrm{e}^{i(\omega_{mw}t - \varphi_O)} + \mathrm{e}^{-i(\omega_{mw}t - \varphi_O)} \right]$$

$$e^{i\frac{\sigma_z \omega_{mw} t}{2}} \sigma_x e^{-i\frac{\sigma_z \omega_{mw} t}{2}} = \sigma_x \cos\omega_{mw} t \; - \; \sigma_y \sin\omega_{mw} t = \left(\frac{\sigma_x + i\sigma_y}{2}\right) e^{i\omega_{mw} t} + \left(\frac{\sigma_x - i\sigma_y}{2}\right) e^{-i\omega_{mw} t}$$

The rotating wave approximation (rwa) neglects terms evolving at frequency $\pm 2\omega_{mw}$:

$$\tilde{H}_{rwa} = \hbar \frac{(\omega_{eg} - \omega_{mw})}{2}\sigma_z - \hbar \frac{\Omega_{mw}}{2}\left(\sigma_+ e^{i\varphi_0} + \sigma_- e^{-i\varphi_0}\right) \quad ; \quad \sigma_\pm = (\sigma_x \pm i\sigma_y)/2$$

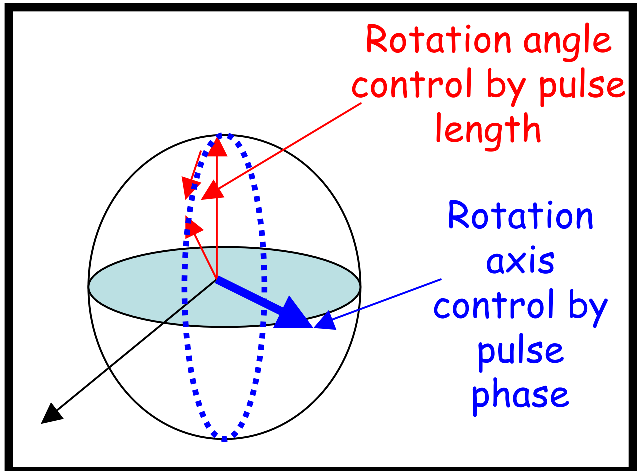
The rwa hamiltonian is t-independant. At resonance (ω_{eg}=ω_{mw}), it simplifies as:

$$\tilde{H}_{rwa} = -\hbar \frac{\Omega_{mw}}{2}\left(\sigma_+ e^{i\varphi_0} + \sigma_- e^{-i\varphi_0}\right) = -\hbar \frac{\Omega_{mw}}{2}\left(\sigma_x \cos\varphi_0 - \sigma_y \sin\varphi_0\right)$$

A resonant mw pulse of length τ and phase φ_0 rotates Bloch vector by angle $\Omega_{mw}\tau$ around direction Ou in Bloch sphere equatorial plane making angle -φ_0 with Ox:

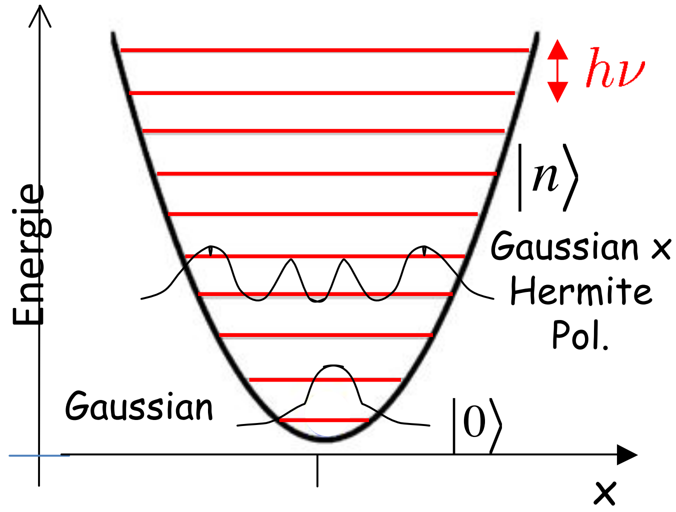
$$\tilde{U}(\tau) = \exp(-i\tilde{H}_{rwa}\tau / \hbar) = \exp(-i\frac{\Omega_{mw}\tau}{2}\sigma_u)$$

$$\sigma_u = \sigma_x \cos\varphi_0 - \sigma_y \sin\varphi_0$$



A method to prepare arbitrary pure qubit state from state |e> or |g>. By applying a convenient pulse prior to detection in qubit basis, one can also detect qubit state along arbitrary direction on Bloch sphere.

Description of harmonic oscillator (phonons or photons)

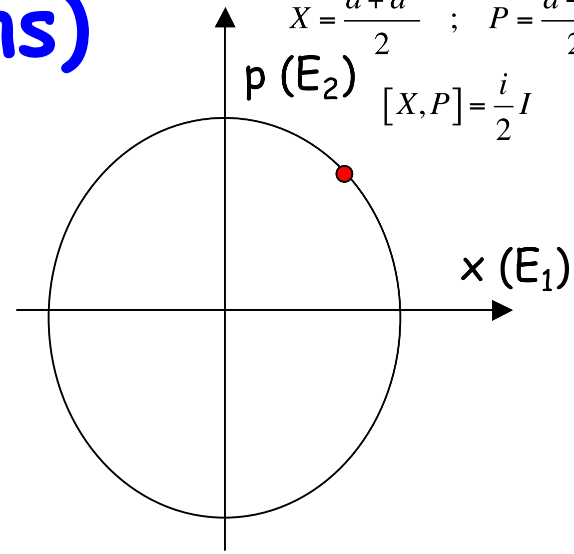


Particle in a parabolic potential or field mode in a cavity

Mechanical or electromagnetic oscillation. Coupling qubits to oscillators is an important ingredient in quantum information.

$$X = \frac{a + a^\dagger}{2} \quad ; \quad P = \frac{a - a^\dagger}{2i}$$

$$[X, P] = \frac{i}{2} I$$



Phase space (with conjugate coordinates x,p or E_1,E_2)

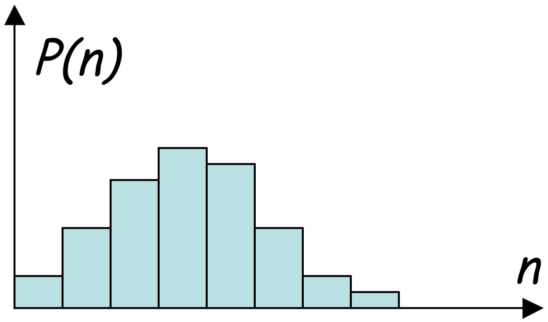
Basic formulae with photon annihilation and creation operators

$$\left[a, a^\dagger\right] = I \quad ; \quad a|n\rangle = \sqrt{n}\,|n-1\rangle \,;\, a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle \quad ; \quad |n\rangle = \frac{a^{\dagger n}}{\sqrt{n!}}|0\rangle$$

$$N = a^\dagger a \quad ; \quad H_{field} = \hbar\omega N \quad ; \quad e^{iN\omega t} a e^{-iN\omega t} = e^{-i\omega t} a$$

$$Displacement\ operator : D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

$$Coherent\ state : |\alpha\rangle = D(\alpha)|0\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$



Photon (or phonon) number distribution in a coherent state: Poisson law

Coherent state

Coupling of cavity mode with a small resonant classical antenna located at r=0:

$$V = -\vec{J}(t).\vec{A}(0) \quad ; \quad J(t) \sim \cos\omega t \quad ; \quad A(0) \sim a + a^\dagger$$

Hence, the hamiltonian for the quantum field mode fed by the classical source:

$$H_Q = \hbar\omega a^\dagger a + V = \hbar\omega a^\dagger a + \Lambda\left(e^{i\omega t} + e^{-i\omega t}\right)\left(a + a^\dagger\right)$$

$$(\Lambda : constant\ \ proportional\ to\ current\ amplitude\ in\ antenna)$$

Interaction representation:

$$\left|\tilde{\psi}_{field}\right\rangle = \exp(i\omega a^\dagger a t)\left|\psi_{field}\right\rangle \rightarrow i\hbar\frac{d\left|\tilde{\psi}_{field}\right\rangle}{dt} = \tilde{H}_Q\left|\tilde{\psi}_{field}\right\rangle \quad with \quad \tilde{H}_Q = \Lambda\left(e^{i\omega t} + e^{-i\omega t}\right)e^{i\omega a^\dagger at}(a + a^\dagger)e^{-i\omega a^\dagger at}$$

Rotating wave approximation (keep only time independent terms):

$$\tilde{H}_Q(rwa) = \Lambda\left(a + a^\dagger\right)$$

Field evolution in cavity starting from vacuum at t=0:

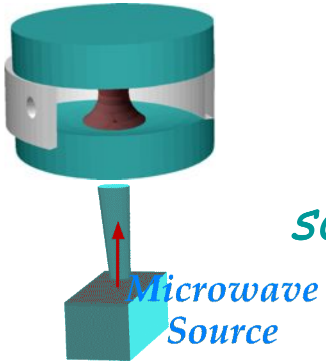
$$\left|\tilde{\psi}_{field}(t)\right\rangle = \exp\left(-i\frac{\Lambda}{\hbar}\left[a + a^\dagger\right]t\right)\left|0\right\rangle = \exp\left(\alpha a^\dagger - \alpha^* a\right)\left|0\right\rangle = e^{-\alpha\alpha^*/2}e^{\alpha a^\dagger}e^{-\alpha^* a}\left|0\right\rangle \quad (\alpha = -i\Lambda t/\hbar)$$

We have used Glauber formula to split the exponential of the sum in last expression. Expanding exp($\alpha a^\dagger$) in power series, we get the field in Fock state basis:

$$\left|\tilde{\psi}_{field}(t)\right\rangle = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}\left|n\right\rangle$$

Coupling field mode to classical source generates coherent state whose amplitude increases linearly with time.

Coherent state (ctn'd)

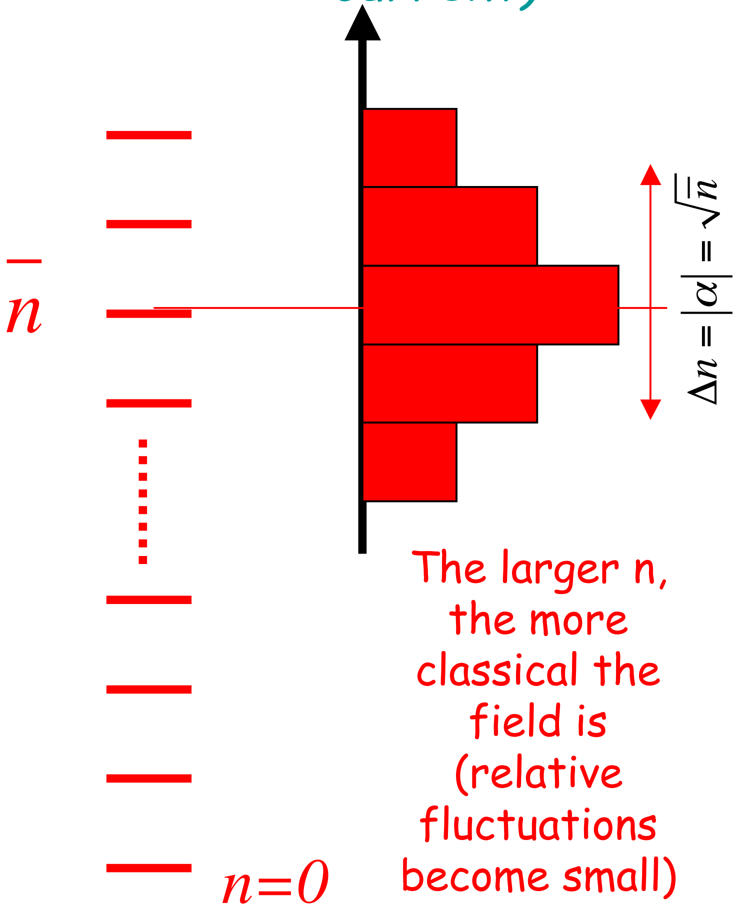


Produced by coupling the cavity to a source (classical oscillating current)

A Poissonian superposition of photon number states:

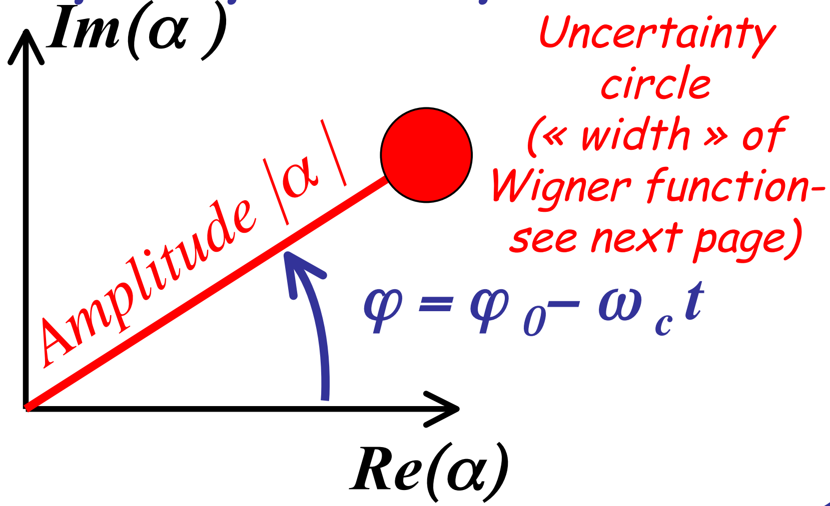
$$|\alpha\rangle = \sum_n C_n |n\rangle \quad , \quad C_n = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \quad ,$$

$$\bar{n} = |\alpha|^2 \quad , \quad P(n) = |C_n|^2 = e^{-\bar{n}} \frac{\bar{n}^n}{n!}$$



The larger n, the more classical the field is (relative fluctuations become small)

Complex plane representation



Representations of a field state

Density operator for a field mode:

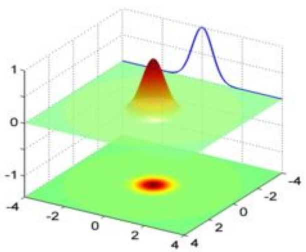
$$\rho = \left|\psi_{field}\right\rangle\left\langle\psi_{field}\right| \quad (pure\ state) \quad ; \quad \rho = \sum_i p_i \left|\psi_{field}^{(i)}\right\rangle\left\langle\psi_{field}^{(i)}\right| \quad (\sum_i p_i = 1) \quad (mixture)$$

Matrix elements of ρ can be discrete ($\rho_{nn'}$ in Fock state basis) or continuous ($\rho_{xx'}$ in quadrature basis where $|x\rangle$ are the eigenstates of $a+a^\dagger$). Going from one representation to the other is easy knowing the amplitudes $\langle x|n\rangle$ expressing the oscillator energy eigenstates in the x basis (Hermite polynomial multiplied by gaussian functions).

Representation in phase space: the Wigner function:

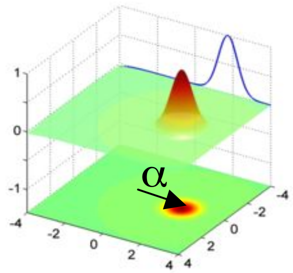
$$W(x,p) = \frac{1}{\pi}\int du \exp(-2ipu)\left\langle x+u/2\right|\rho\left|x-u/2\right\rangle$$

W is a real distribution in (x,p) space (equivalently, in complex plane), whose knowledge is equivalent to that of ρ. The «shadow» of W on p plane yields the x- distribution in the state.



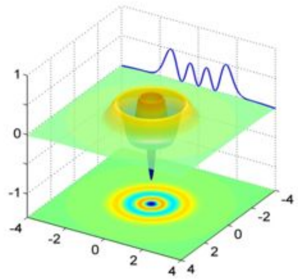
Gaussian function

Ground state



Gaussian function translatec

Coherent state



Positive and negative rings

Fock state

W<0 : non-classical

I-B

Coupling a qubit to a quantized field mode: the Jaynes-Cummings Hamiltonian

	Matter	Radiation		
Semi-classical	Classical current	Quantum field mode	→	Translation in phase space and Coherent states
	Quantum (qubit)	Classical field	→	Rotation in Bloch sphere and arbitrary qubit states
Fully quantum	Quantum (qubit)	Quantum field mode	→	Cavity QED

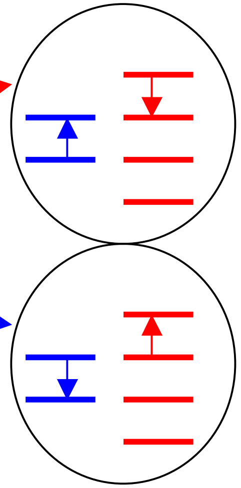
Qubit-quantum field mode coupling

Coupling qubit dipole to quantum electric field operator:

$$V = -\vec{D}.\vec{E}_Q(0) \quad ; \quad E_Q(0) = iE_0\left(a - a^\dagger\right)$$

$$V = -idE_0\sigma_x(a - a^\dagger) = -idE_0\left(\sigma_+ + \sigma_-\right)(a - a^\dagger)$$

Absorption and emission of photons while qubit jumps between its energy states.



Rotating wave approximation:

$$V_{rwa} = -i\frac{\hbar\Omega_0}{2}\left(\sigma_+ a - \sigma_- a^\dagger\right) \quad ; \quad \Omega_0 = 2d.E_0/\hbar$$

with:

$$\sigma_+ = \frac{\sigma_x + i\sigma_y}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = |e\rangle\langle g| \quad ; \quad \sigma_- = \frac{\sigma_x - i\sigma_y}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = |g\rangle\langle e|$$

Vacuum Rabi frequency in cavity mode of volume V:

$$\langle 0|E_Q^2(0)|0\rangle = E_0^2\langle 0|aa^\dagger + a^\dagger a|0\rangle = E_0^2$$

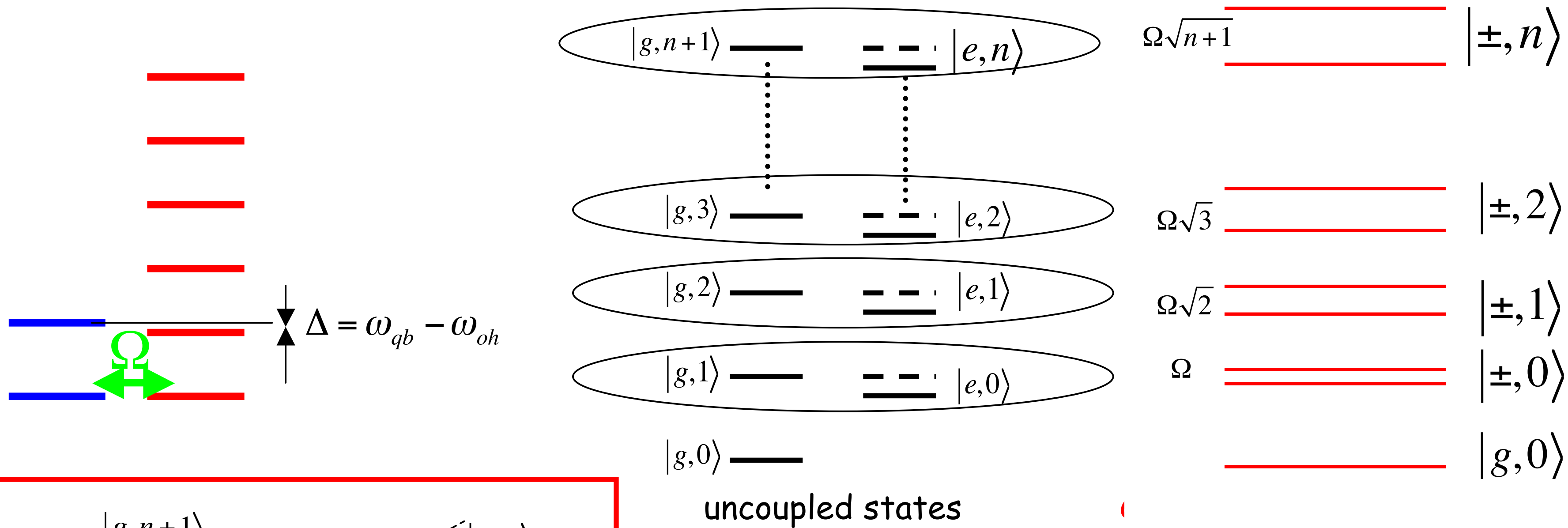
$$\varepsilon_0 V E_0^2 = \frac{\hbar\omega}{2} \quad \rightarrow \quad E_0 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}$$

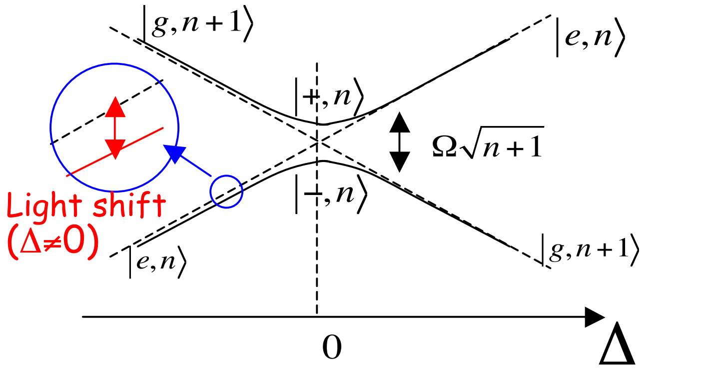
$$\Omega_0 = \sqrt{\frac{2d^2\omega}{\varepsilon_0\hbar V}}$$

Requires large dipole d and small cavity volume V

Qubit-oscillateur coupling: the Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_{qb}\frac{\sigma_z}{2} + \hbar\omega_{oh}a^\dagger a - i\frac{\hbar\Omega}{2}\left[\sigma_+ a - \sigma_- a^\dagger\right]$$





$$E_{\pm,n} = (n+1/2)\hbar\omega_{oh} \pm \frac{\hbar}{2}\sqrt{\Delta^2 + \Omega^2(n+1)}$$

Anticrossing of dressed qubit

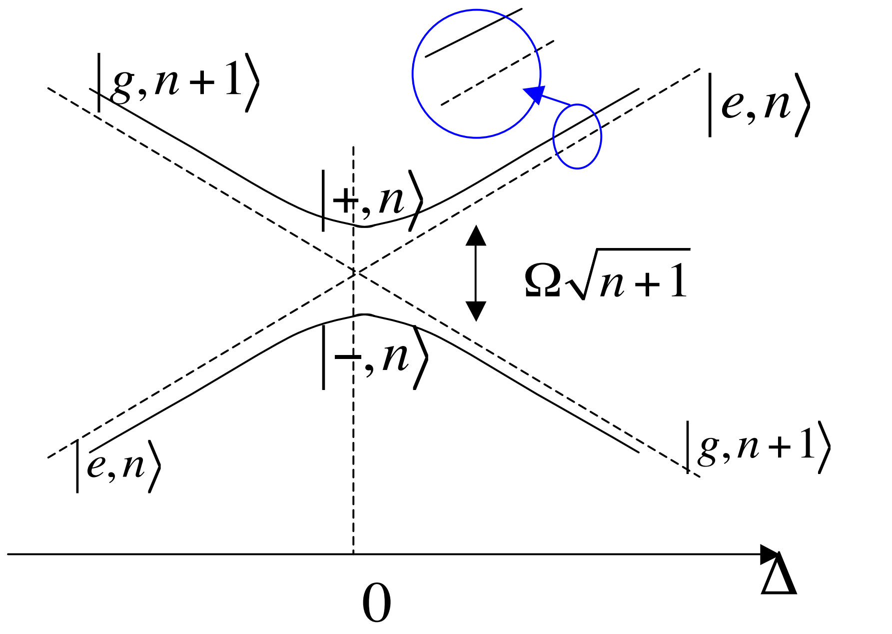
at resonance (Δ=0)

$$|\pm,n\rangle = \frac{1}{\sqrt{2}}\left(|e,n\rangle \pm i\,|g,n+1\rangle\right)$$

$$|e,n\rangle \longrightarrow \cos\frac{\Omega\sqrt{n+1}\,t}{2}|e,n\rangle + \sin\frac{\Omega\sqrt{n+1}\,t}{2}|g,n+1\rangle$$

$$|g,n+1\rangle \longrightarrow -\sin\frac{\Omega\sqrt{n+1}\,t}{2}|e,n\rangle + \cos\frac{\Omega\sqrt{n+1}\,t}{2}|g,n+1\rangle$$

Non-Resonant coupling: light shifts in CQED



$$E_{\pm,n} = (n+1/2)\hbar\omega_{oh} \pm \frac{\hbar}{2}\sqrt{\Delta^2+\Omega^2(n+1)}$$

Anticrossing of dressed qubit

Second order perturbation theory:

$$E_{\pm,n} \approx (n+1/2)\hbar\omega_C \pm \hbar\left(\frac{\Delta}{2}+\frac{\Omega^2(n+1)}{4\Delta}\right)$$

Energy shift due to off-resonant coupling (distance between energy level and asymptot):

$$E_{+,n} - E_{e,n} \approx \hbar\frac{\Omega^2(n+1)}{4\Delta} + \dots \quad ;$$

$$E_{-,n} - E_{g,n+1} \approx -\hbar\frac{\Omega^2(n+1)}{4\Delta} + \dots$$

Vacuum shift (Lamb-shift):

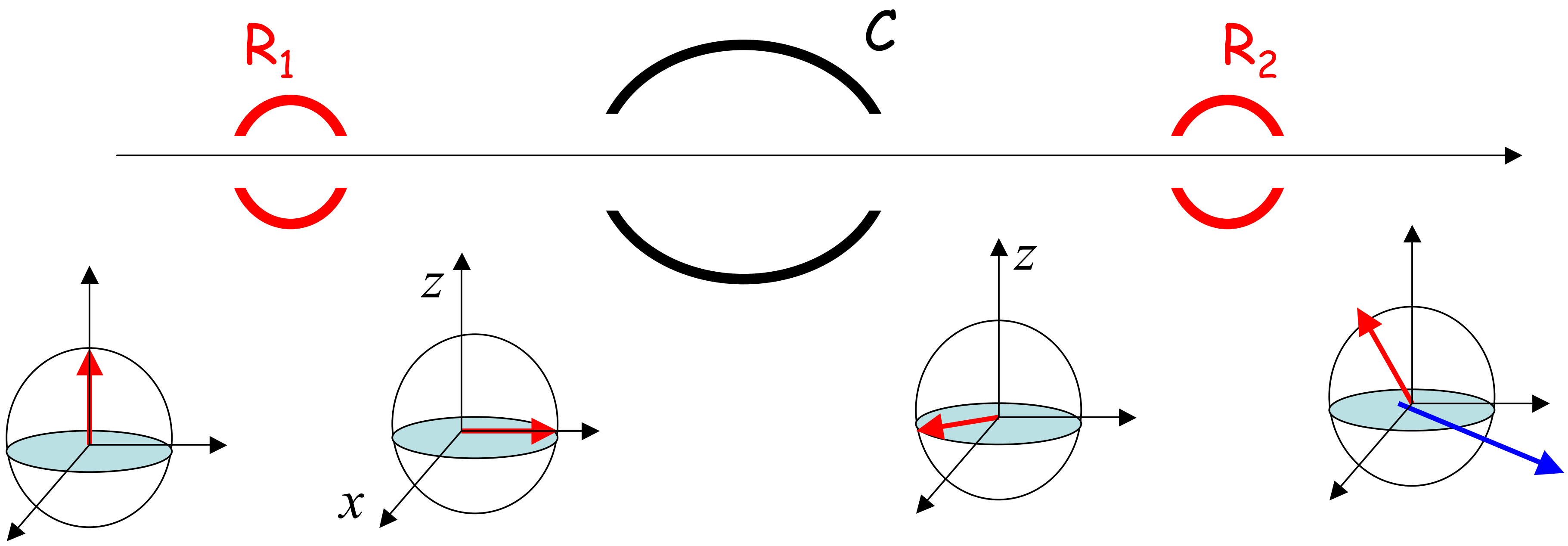
$$\delta_{g,0} = 0 \quad ; \quad \delta_{e,0} = E_{+,0} - E_{e,0} \approx \hbar\frac{\Omega^2}{4\Delta}$$

Light shift induced on eg transition by n-photons:

$$E_{+,n} - E_{-,n-1} = \hbar\omega_{eg} + \hbar\frac{\Omega^2}{2\Delta}n + \dots \rightarrow \delta(\omega_{eg}) = \frac{\Omega^2}{2\Delta}n \quad ; \quad \varphi_0 = \frac{\Omega^2 t}{2\Delta}$$

φ_0=Phase shift per photon accumulated during time t

Measuring qubit phase shift in CQED: the Ramsey interferometer



Qubit initially in e

R_1 pulse rotates Bloch vector by $\pi/2$ around Ox

$$\exp\left(-i\frac{\pi}{4}\sigma_x\right)$$

Qubit phase shift φ_c during C crossing amounts to Bloch vector rotation around Oz

$$\exp\left(-i\frac{\varphi_C}{2}\sigma_z\right)$$

R_2 pulse realizes $\pi/2$ rotation around Ou whose direction depends on R_2-R_1 relative phase φ_r

$$\exp\left(-i\frac{\pi}{4}\sigma_u\right) =$$

$$\exp\left(-i\frac{\pi}{4}\left[\sigma_x\cos\varphi_r - \sigma_y\sin\varphi_r\right]\right)$$

Ramsey interferometer (ctn'd)

A useful formula:

$$\exp(-i\varphi\sigma_u) = \cos\varphi\; I - i\sin\varphi\;\sigma_u \qquad (\textit{for any Pauli operator}\;)$$

Hence the rotation induced by Ramsey interferometer:

$$R = \exp\left(-i\frac{\pi}{4}\sigma_u\right)\exp\left(-i\frac{\varphi_C}{2}\sigma_z\right)\exp\left(-i\frac{\pi}{4}\sigma_x\right)$$

Rotation induced by R_2

Cavity phase-shift

Rotation induced by R_1

$$= \frac{1}{2}\begin{pmatrix} 1 & -ie^{i\varphi_r} \\ -ie^{-i\varphi_r} & 1 \end{pmatrix}\begin{pmatrix} e^{-i\varphi_C/2} & 0 \\ 0 & e^{i\varphi_C/2} \end{pmatrix}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$= -i\begin{pmatrix} e^{i\varphi_r/2}\sin\left(\dfrac{\varphi_c+\varphi_r}{2}\right) & e^{i\varphi_r/2}\cos\left(\dfrac{\varphi_c+\varphi_r}{2}\right) \\ e^{-i\varphi_r/2}\cos\left(\dfrac{\varphi_c+\varphi_r}{2}\right) & e^{-i\varphi_r/2}\sin\left(\dfrac{\varphi_c+\varphi_r}{2}\right) \end{pmatrix}$$

and the evolution of |e> and |g> states:

$$R|e\rangle = -ie^{i\varphi_r/2}\sin\left(\frac{\varphi_c+\varphi_r}{2}\right)|e\rangle - ie^{-i\varphi_r/2}\cos\left(\frac{\varphi_c+\varphi_r}{2}\right)|g\rangle$$

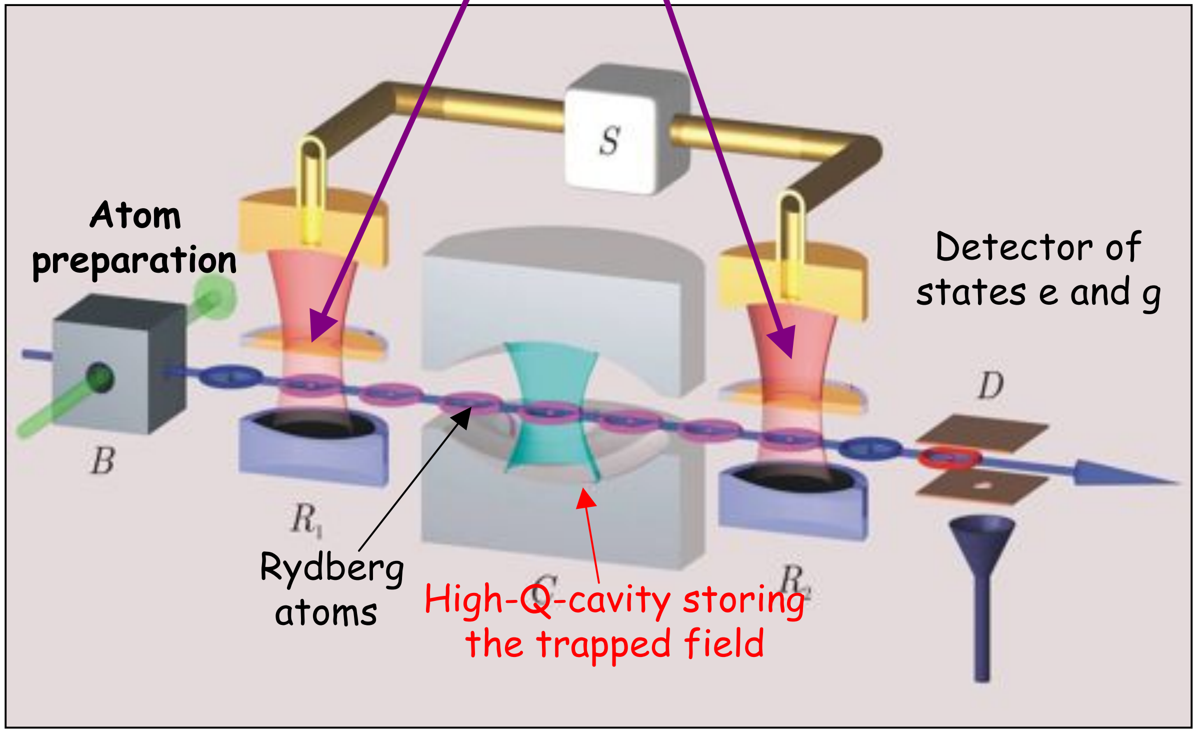
$$R|g\rangle = -ie^{i\varphi_r/2}\cos\left(\frac{\varphi_c+\varphi_r}{2}\right)|e\rangle - ie^{-i\varphi_r/2}\sin\left(\frac{\varphi_c+\varphi_r}{2}\right)|g\rangle$$

$$P_{e\to g} = \cos^2\left(\frac{\varphi_c+\varphi_r}{2}\right)$$

Ramsey fringes detected by sweeping φ_r. Fringe-phase used to measure cavity shift φ_C.

General scheme of cavity QED experiments

Microwave pulses preparing (left) the atomic qubit and rotating it (right) in the detection direction



ENS experiments: Rydberg atoms in states e and g behave as qubits. They are prepared in B in state e and cross one at a time the high-Q cavity C where they are coupled to a field mode. The atom-field system evolution is ruled by the Jaynes-Cummings hamiltonian. A microwave pulse applied in R_1 prepares each atom in a superposition of e and g. After C, a second pulse, applied in R_2, maps the measurement direction of the qubit along the Oz axis of the Bloch sphere, before detection of the qubit by selective field ionization in an electric field (in D). The R_1-R_2 combination constitutes a Ramsey interferometer. This set-up has been used to entangle atoms, realize quantum gates, count photons non-destructively, reconstruct non classical states of the field and demonstrate quantum feedback procedure (lectures 1 to 4).

I-C
A special system: circular Rydberg atoms coupled to a superconducting Fabry-Perot cavity

A qubit extremely sensitive to microwaves: the circular Rydberg atom

Rydberg formulae

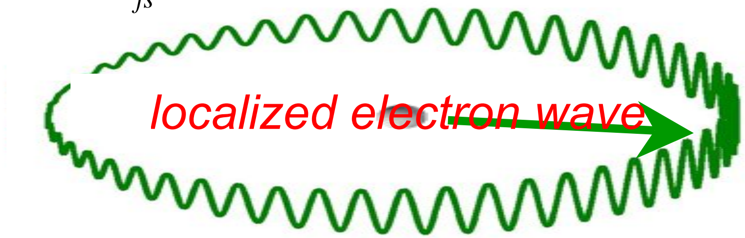
$$n\lambda_{dB} = \frac{nh}{mv} = 2\pi r_R \to r_R = \frac{n\hbar}{mv};$$

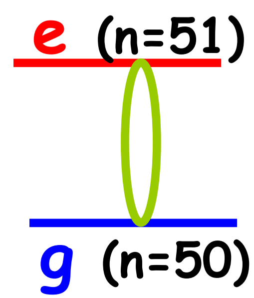
$$|F| = \frac{mv^2}{r_R} = \frac{q^2}{4\pi\varepsilon_0 r_R^2} \to r_R = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{mv^2}$$

$$\to v = \frac{q^2}{4\pi\varepsilon_0\hbar c}\frac{c}{n} = \alpha_{fs}\frac{c}{n} \approx \frac{1}{137}\frac{c}{n} \to |E| = \frac{mv^2}{2} = \frac{1}{2n^2}mc^2\alpha_{fs}^2$$

$$r_R = n^2\frac{\hbar}{\alpha_{fs}mc} = n^2 a_0$$

Atom in circular Rydberg state:
electron on giant orbit
(tenth of a micron diameter)





Atom in ground state:
electron on 10^{-10} m diametre orbit

Very long radiative lifetime T_R

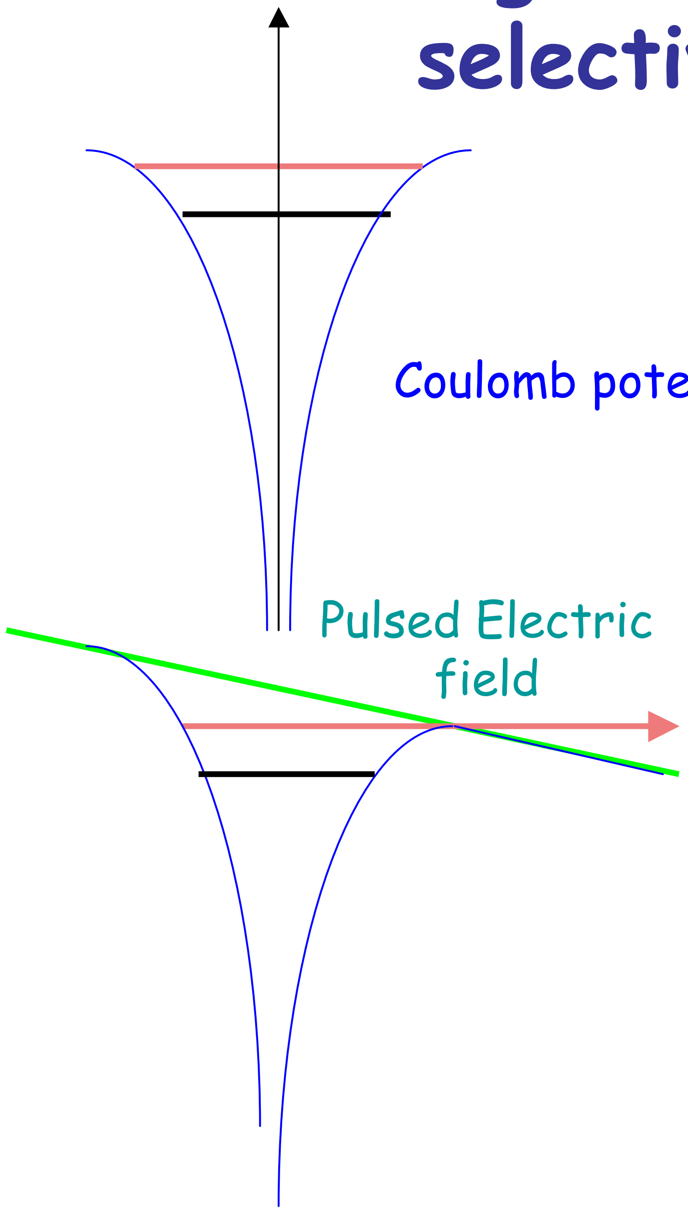
$$\omega_{n,n-1} \sim n^{-3}\ ;\ P_{rad} = \frac{\hbar\omega_{n,n-1}}{T_R} \sim \left(\omega_{n,n-1}^2 r_R\right)^2 \sim n^{-8} \to T_R \sim n^5$$

Electron is localised on orbit by a microwave pulse preparing superposition of two adjacent Rydberg states: |e> → |e> + |g>

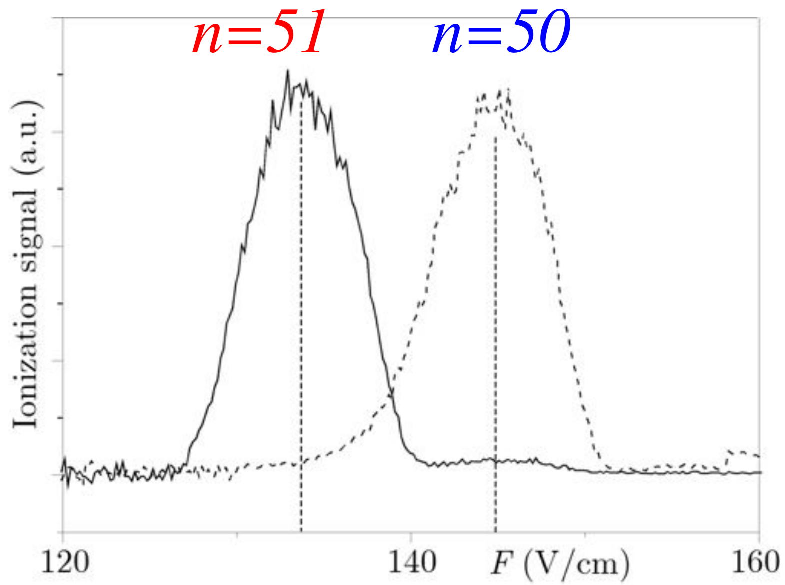
The localized wave packet packet revolves around nucleus at the transition frequency (51 GHz) between the two states like a clock's hand on a dial.

The electric dipole is proportional to the qubit Bloch vector in the equatorial plane of the Bloch sphere.

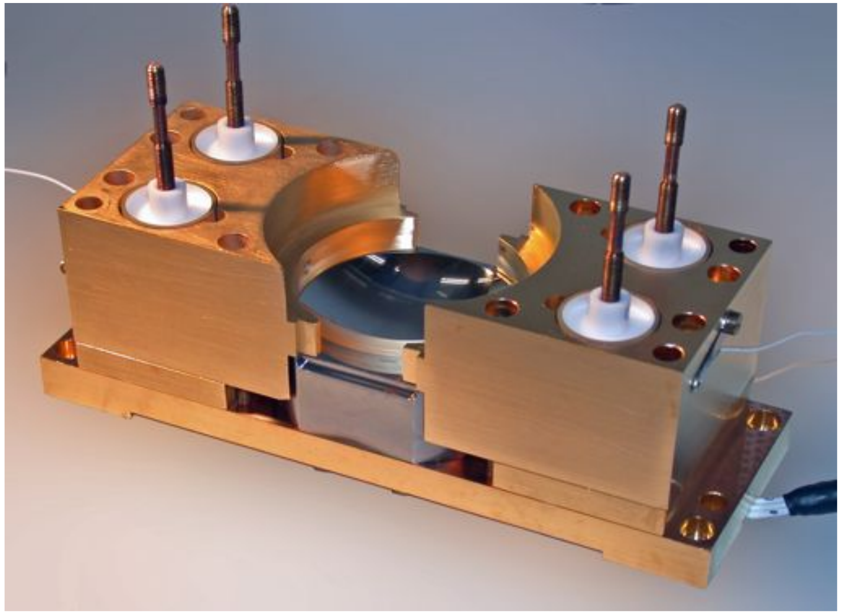
Detecting circular Rydberg states by selective field ionization



Adjusting the ionizing field allows us to discriminate two adjacent circular states. The global detection efficiency can be > 80%.

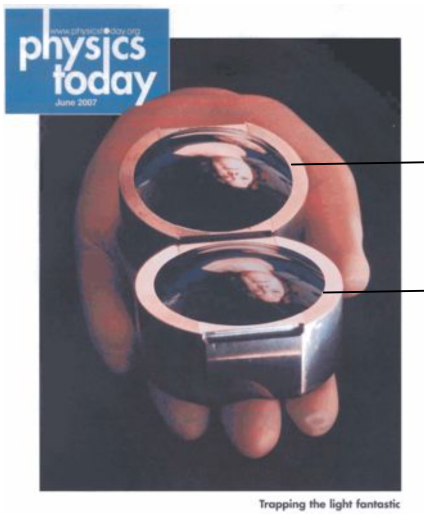


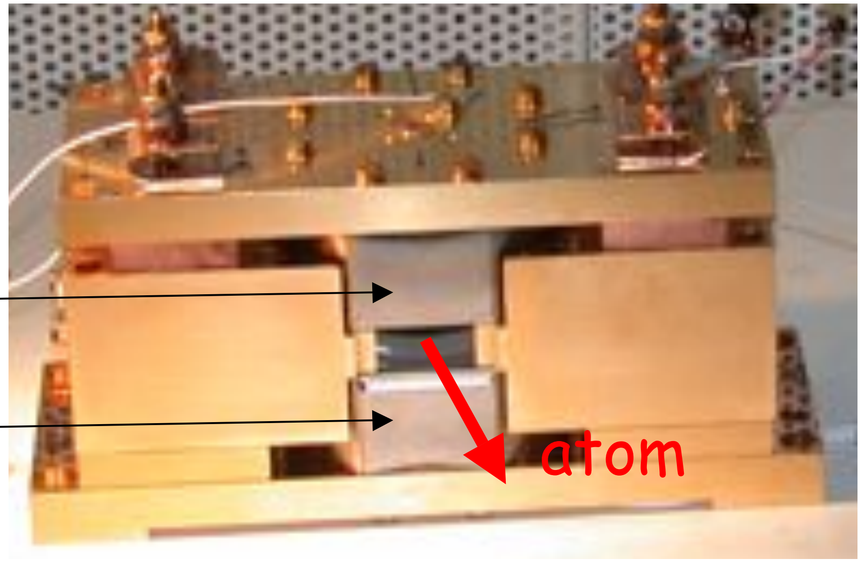
The ENS photon box (latest version)



Cavity half-mounted…

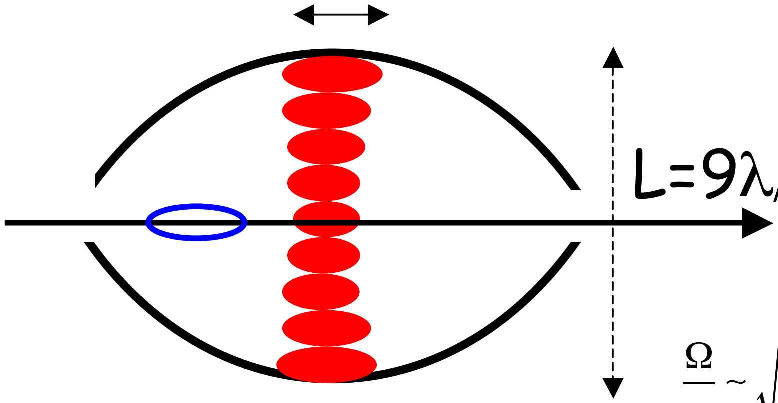
In its latest version, the cavity has a damping time in the 100 millisecond range. Atoms cross it one at a time.





…and fully-mounted

Orders of magnitude



$V \sim \pi w^2 L/4 \sim 4\lambda^3$
$\sim 700\ mm^3$

$$\frac{\Omega}{\omega} \sim \sqrt{\frac{2d^2}{\varepsilon_0 \hbar V \omega}} \sim \sqrt{\frac{q^2}{\pi \varepsilon_0 \hbar c} \times r_a^2 \times \frac{\lambda}{V}} \sim \sqrt{\frac{q^2}{4\pi \varepsilon_0 \hbar c} \times \frac{r_a^2}{\lambda^2}} = \left(\frac{1}{137}\right)^{1/2} \frac{r_a}{\lambda} \sim 10^{-6}$$

$$\Omega \sim 10^{-6} \omega = 2\pi \times 50 kHz$$

Parameter determined by geometric arguments

Number of vacuum Rabi flops during cavity damping time (best cavity):

$$N_{RF} = \frac{\Omega T_C}{2\pi} \sim 5000$$

Atom-cavity interaction time (depends on atom velocity): $t_{\text{int}} \sim \frac{w}{v_{at}} \sim 10 \text{ to } 50 \mu s$

Number of vacuum Rabi flops during atom-cavity interaction time: $\Omega t_{\text{int}} / 2\pi \sim 1 \text{ to } 3$

Strong coupling in CQED	$\frac{1}{\Omega}$	$<$	t_{int}	$\ll$	T_c	Many atoms cross one by one during T_c
	$3.10^{-6} s$		$3.10^{-5} s$		$10^{-1} s$	

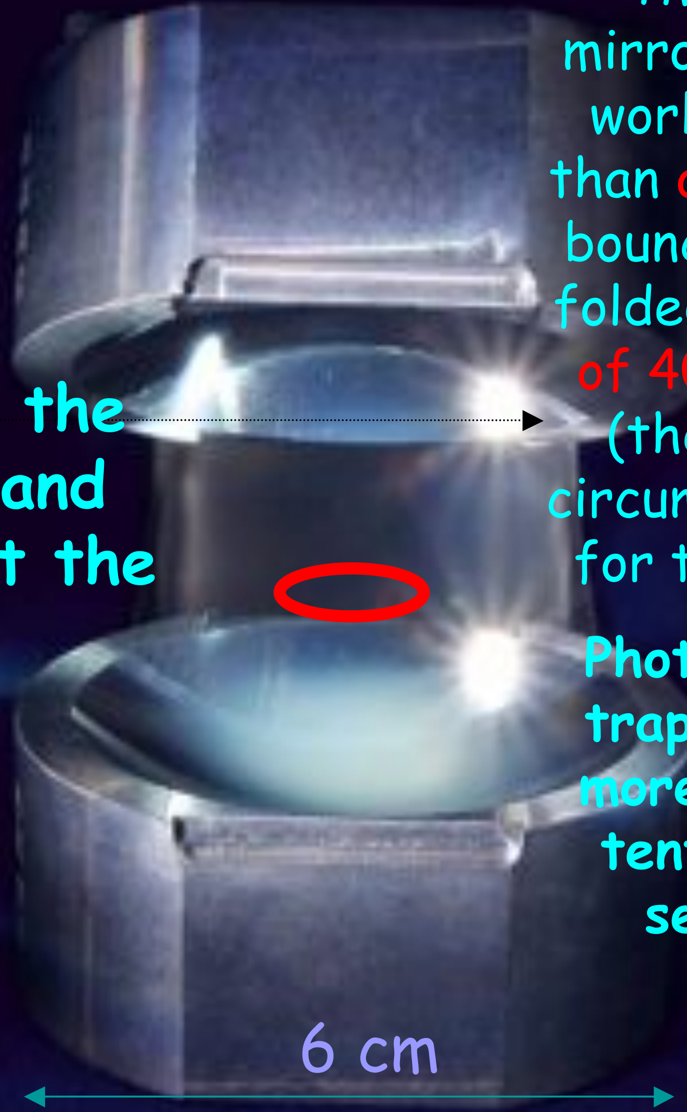
Cavity Quantum Electrodynamics:

a stage to witness the interaction between light and matter at the most fundamental level

One atom interacts with one (or a few) photon(s) in a box

A sequence of atoms crosses the cavity, couples with its field and carries away information about the trapped light

Photons bouncing on mirrors pass many many times on the atom: the cavity enhances tremendously the light-matter coupling



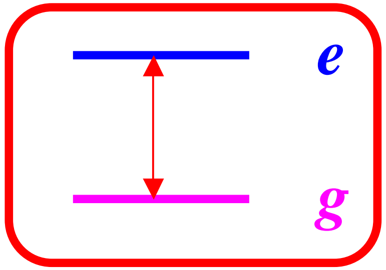
The best mirrors in the world: more than one billion bounces and a folded journey of 40.000km (the earth circumference) for the light!

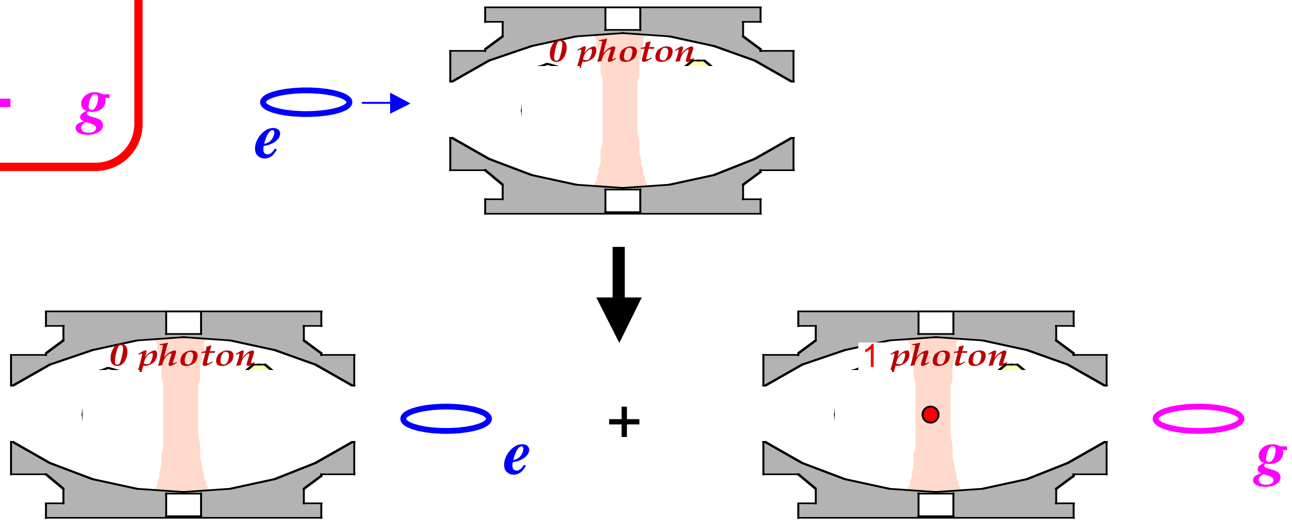
Photons are trapped for more than a tenth of a second!

I-D

Entanglement and quantum gate experiments in Cavity QED

Resonant Rabi flopping





$$\cos\left(\Omega t/2\right)\left|e,0\right\rangle \quad + \quad \sin\left(\Omega t/2\right)\left|g,1\right\rangle$$

Spontaneous emission and absorption involving ***atom-field entanglement***

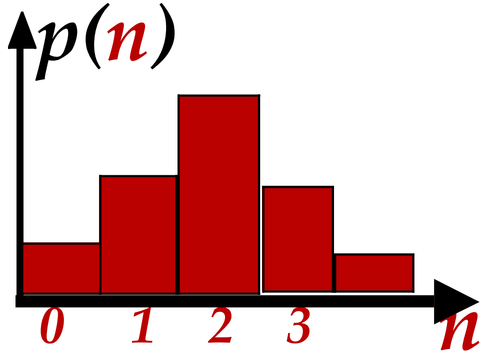
When *n* photons are present, the oscillation occurs faster (stimulated emission):

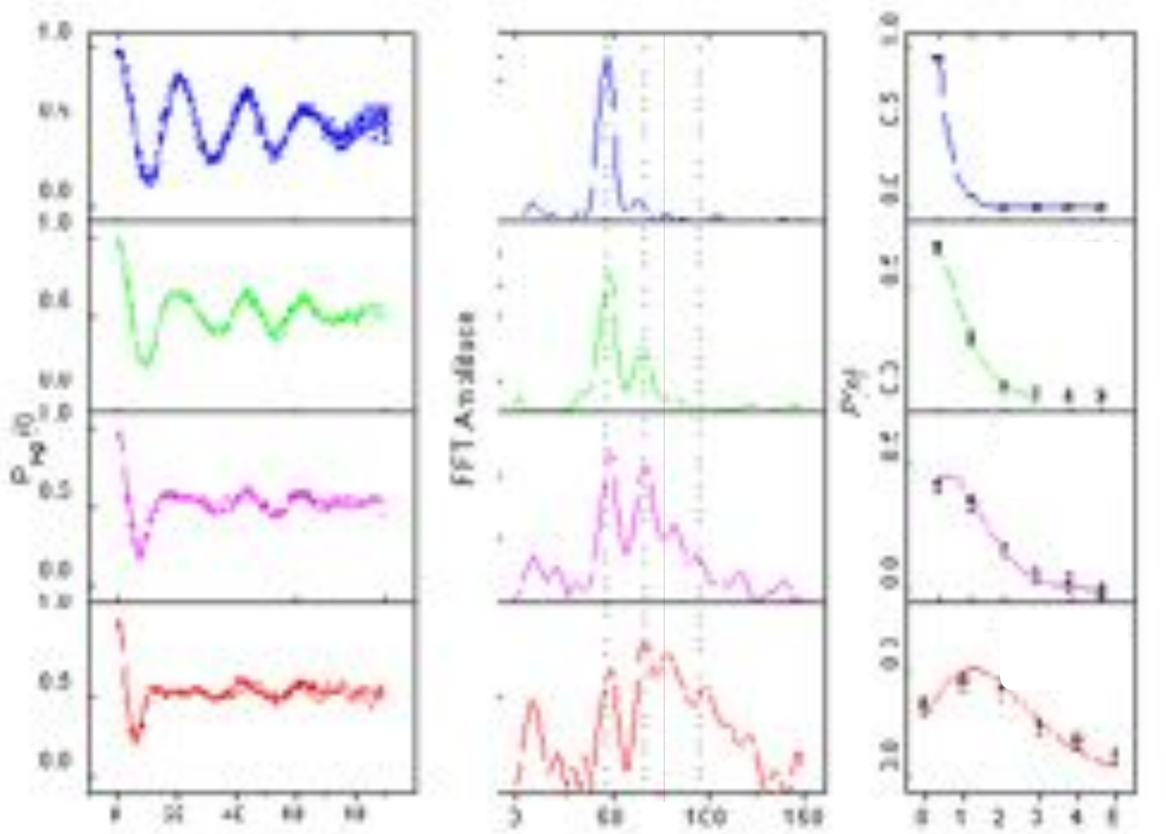
$$\cos\left(\Omega\sqrt{n+1}\,t/2\right)\left|e,n\right\rangle \quad + \quad \sin\left(\Omega\sqrt{n+1}\,t/2\right)\left|g,n+1\right\rangle$$

Simple dynamics of a two-level system ($|e,n\rangle$, $|g,n+1\rangle$)

Rabi flopping in vacuum or in small coherent field: a direct test of field quantization

$$P_e(t) = \sum_n p(n)\cos^2\left(\frac{\Omega\sqrt{n+1}t}{2}\right) \quad ; \quad p(n) = e^{-\bar{n}}\frac{\bar{n}^n}{n!}$$





$\bar{n} = 0 \quad (\bar{n}_{th} = 0.06)$

$\bar{n} = 0.40\ (\pm 0.02)$

$\bar{n} = 0.85\ (\pm 0.04)$

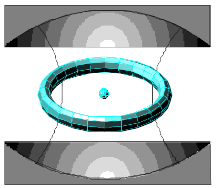
$\bar{n} = 1.77\ (\pm 0.15)$

Brune et al,

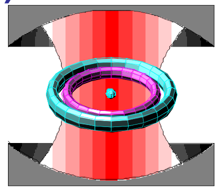
$P_e(t)$ signal　　Fourier transform　　Inferred p(n)

Useful Rabi pulses
(quantum « knitting »)

Initial state

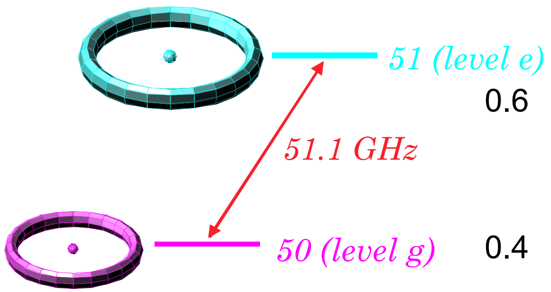


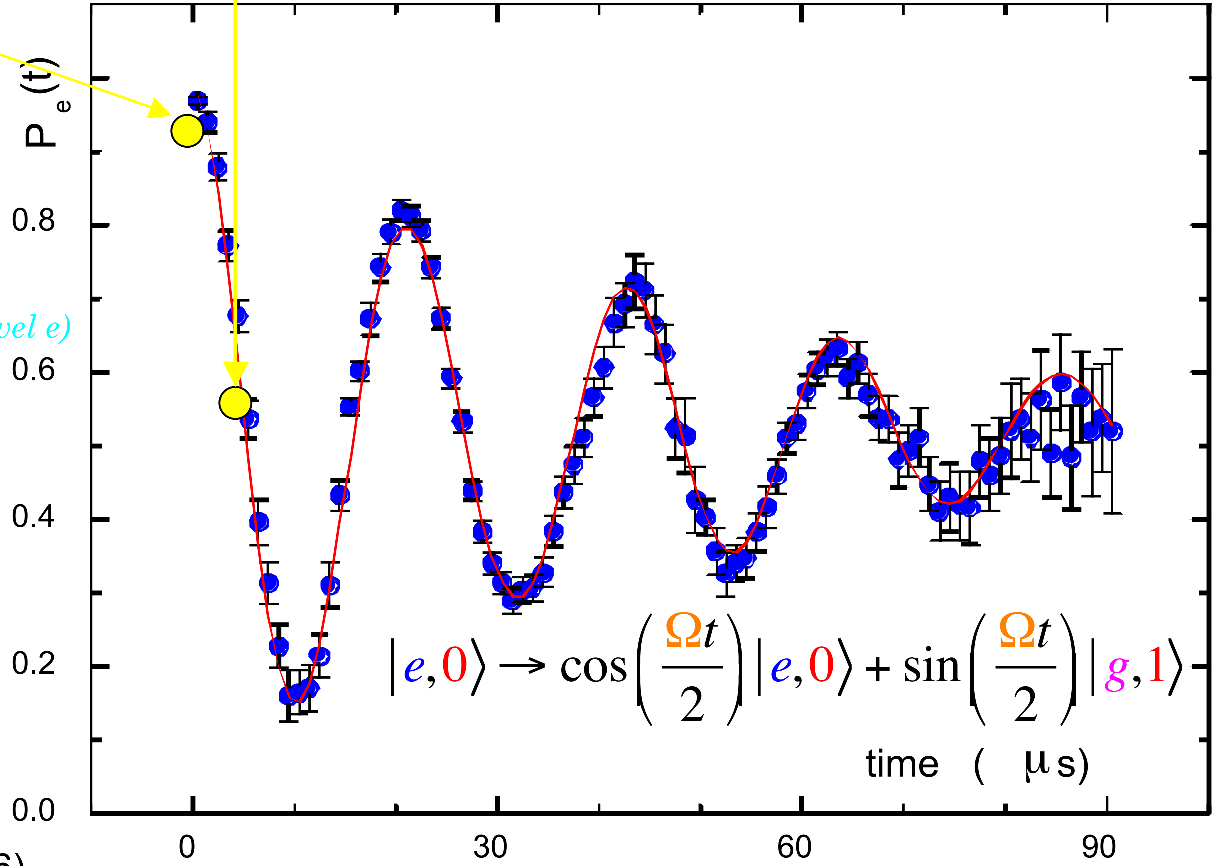
$|e,0\rangle$



$|e,0\rangle \rightarrow \; |e,0\rangle + |g,1\rangle$

π / 2 pulse
creates atom-cavity
entanglement





Brune et al, PRL **76,** 1800 (96)

Microscopic entanglement

Useful Rabi pulses (quantum knitting)

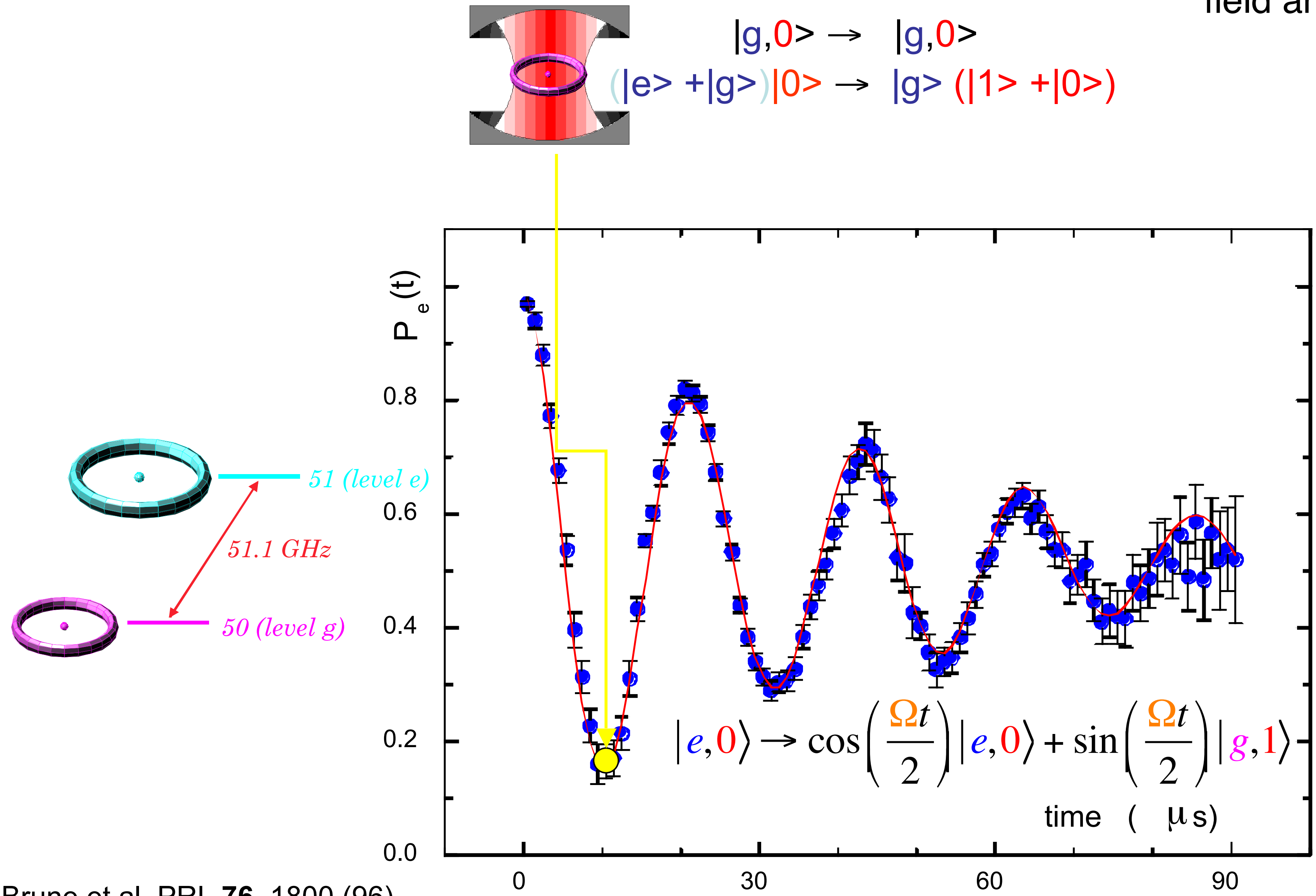
|e,0> → |g,1>

|g,1> → |e,0>

|g,0> → |g,0>

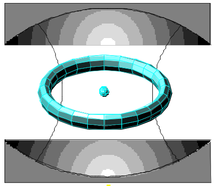
(|e> +|g>)|0> → |g> (|1> +|0>)

π pulse copying atom's state on field and back



Brune et al, PRL **76,** 1800 (96)

Useful Rabi pulses
(quantum knitting)

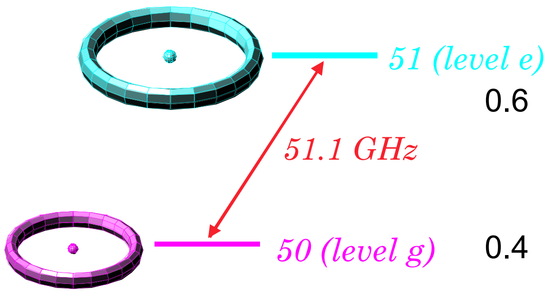


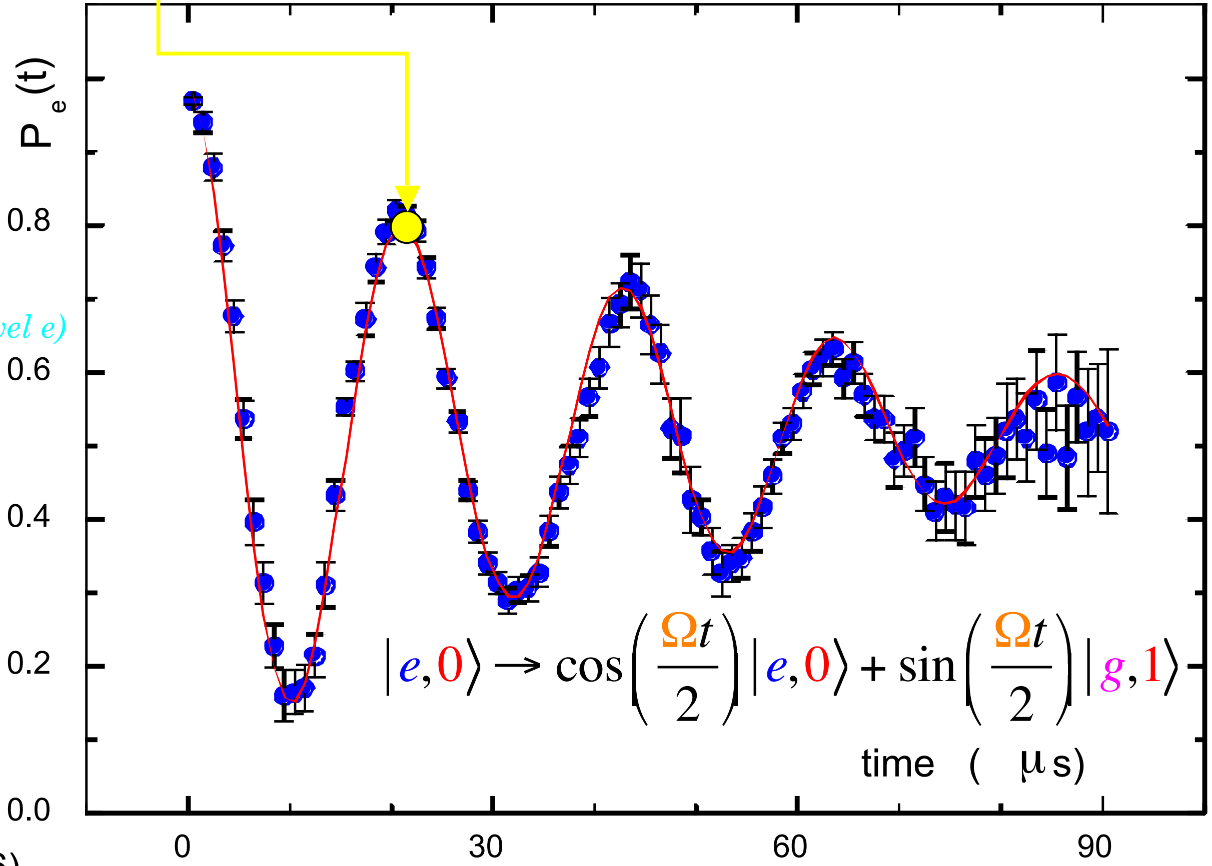
|e,0> → - |e,0>

|g,1> → - |g,1>

|g,0> → |g,0>

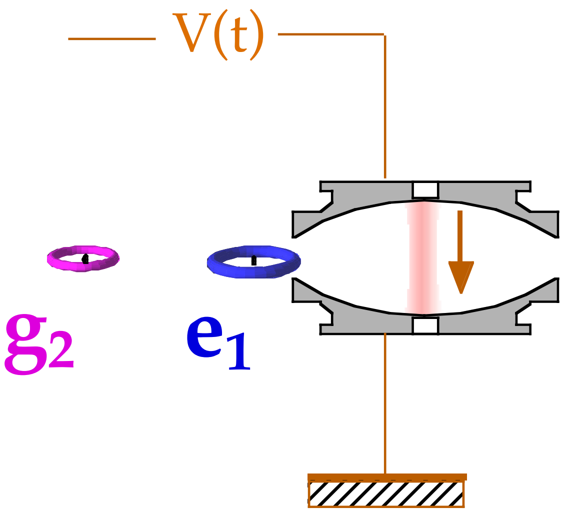
2π pulse :
atomic state changes sign conditioned to photon number: quantum gate





Brune et al, PRL **76,** 1800 (96)

Atom pair entangled by photon exchange



Atom #1 : $\Omega t = \pi/2$

$$|e_1, 0>.|g_2> \longrightarrow \frac{1}{\sqrt{2}}\{|e_1,0> + |g_1,1>\}.|g_2>$$

Atom #1 - Photon entanglement

Atom #2 : $\Omega t = \pi$

Atom - Atom entanglement (a massive E.P.R.pair)

$$\longrightarrow \frac{1}{\sqrt{2}}\{|e_1, g_2> + |g_1, e_2>\}.|0>$$

Atoms entangled in deterministic way without directly interacting

Cavity acts as a catalyst for entanglement

Field generated in a transient stage

Electric field F(t) used to tune atoms 1 and 2 in resonance with C for times t corresponding to $\pi / 2$ or π Rabi pulses

Hagley et al, P.R.L. 79,1 (1997)

Conclusion of first lecture

We have learned how to describe a qubit and a field mode by their Bloch vector and Wigner function respectively, and how one can manipulate these systems by qubit rotations and oscillator state translations in phase space.

The coupling of these two quantum systems is described by the Jaynes-Cummings Hamiltonian. At resonance, the combined qubit-oscillator undergoes a Rabi oscillation corresponding to the reversible emission and absorption of a photon by the qubit.

Out of resonance, the qubit energy states undergo a light shift proportional to the photon number, resulting in a phase shift of the qubit coherence, which can be measured by Ramsey interferometry.

Circular Rydberg atoms interacting with a microwave superconducting cavity realize ideally the Jaynes-Cummings model. The atomic and field relaxation rates are very slow, making it possible to observe coherent atom field coupling effects which occur faster than decoherence. We have analyzed how this system can be manipulated to produce atom-field and atom-atom entanglement, to realize elementary steps of quantum information processing.