Cavity QED with Rydberg Atoms

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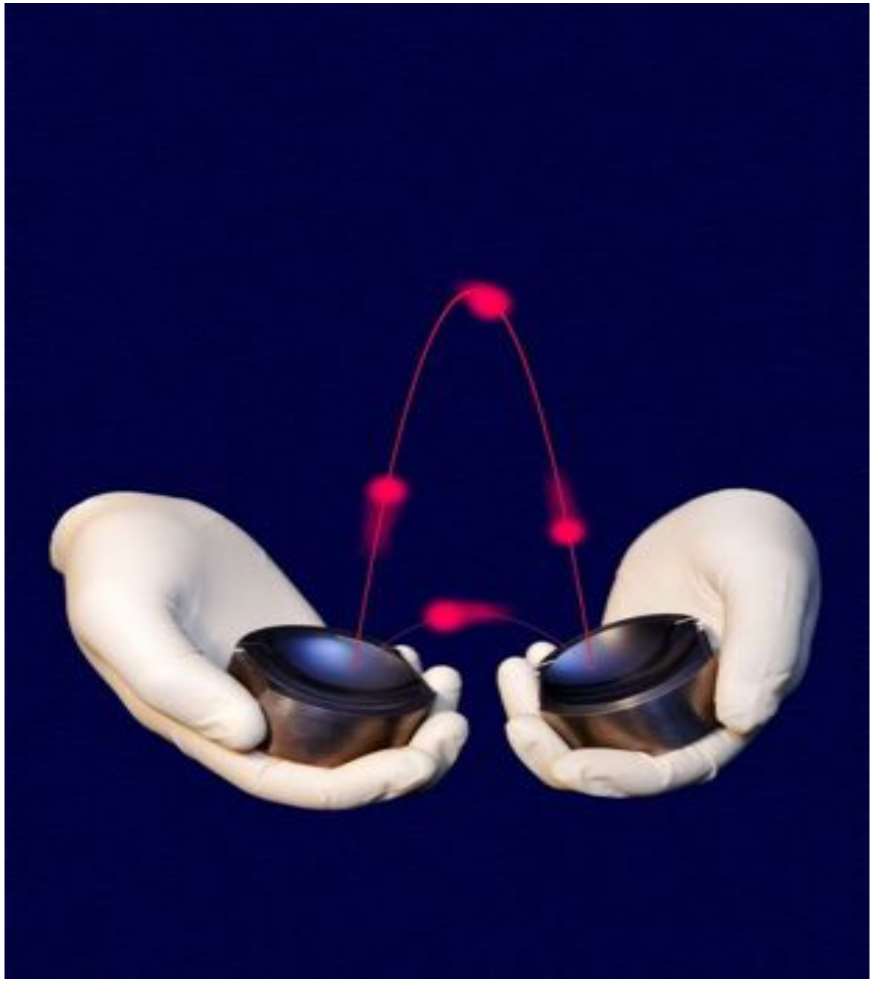
Lecture 3:

Quantum feedback and field state reconstruction in Cavity QED experiments. Introduction to Circuit QED.

III-A
Quantum feedback in Cavity QED experiments

How to combine measurements and actuator actions on a quantum system to drive it towards a target state and protect it against decoherence



A game analogous to « classical » juggling with the added difficulty that observing the photons has an unavoidable back action which must be taken into account…

Back action of single atom detection
(see Lecture 2)

Initial state

$$(j = e, g) \quad \rho_{\text{proj}} = \frac{M_j \, \rho \, M_j^\dagger}{\mathrm{Tr}(M_j \, \rho \, M_j^\dagger)}$$

State after projection

Phase of interferometer

$$M_e = \sin\left(\frac{\phi_r + \phi(N)}{2}\right)$$

$a^\dagger a$

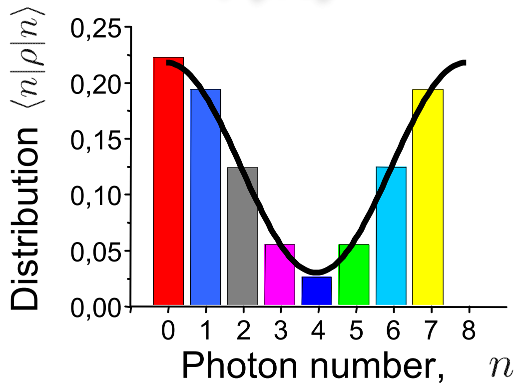
2 operators corresponding to the 2 possible outcomes

phaseshift per photon

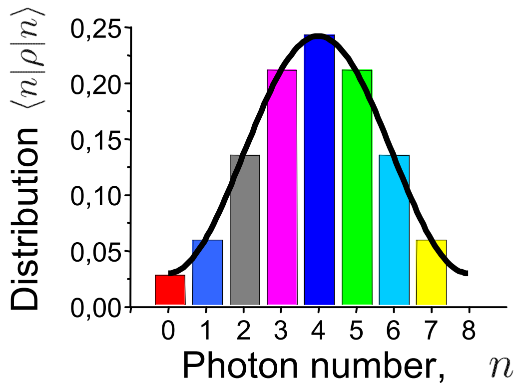
$$M_g = \cos\left(\frac{\phi_r + \phi(N)}{2}\right)$$

Atomic detection changes the photon number distribution

ρ_{proj}



atom in |**e**⟩



atom in |**g**⟩

Applying quantum feedback to the stabilization of Fock states?

Fock states are interesting examples of non-classical states

They are fragile and lose their non-classicality in time scaling as 1/n.

The preparation by projective measurement is random

Is it possible to prepare them in a deterministic way by using quantum feedback procedures?

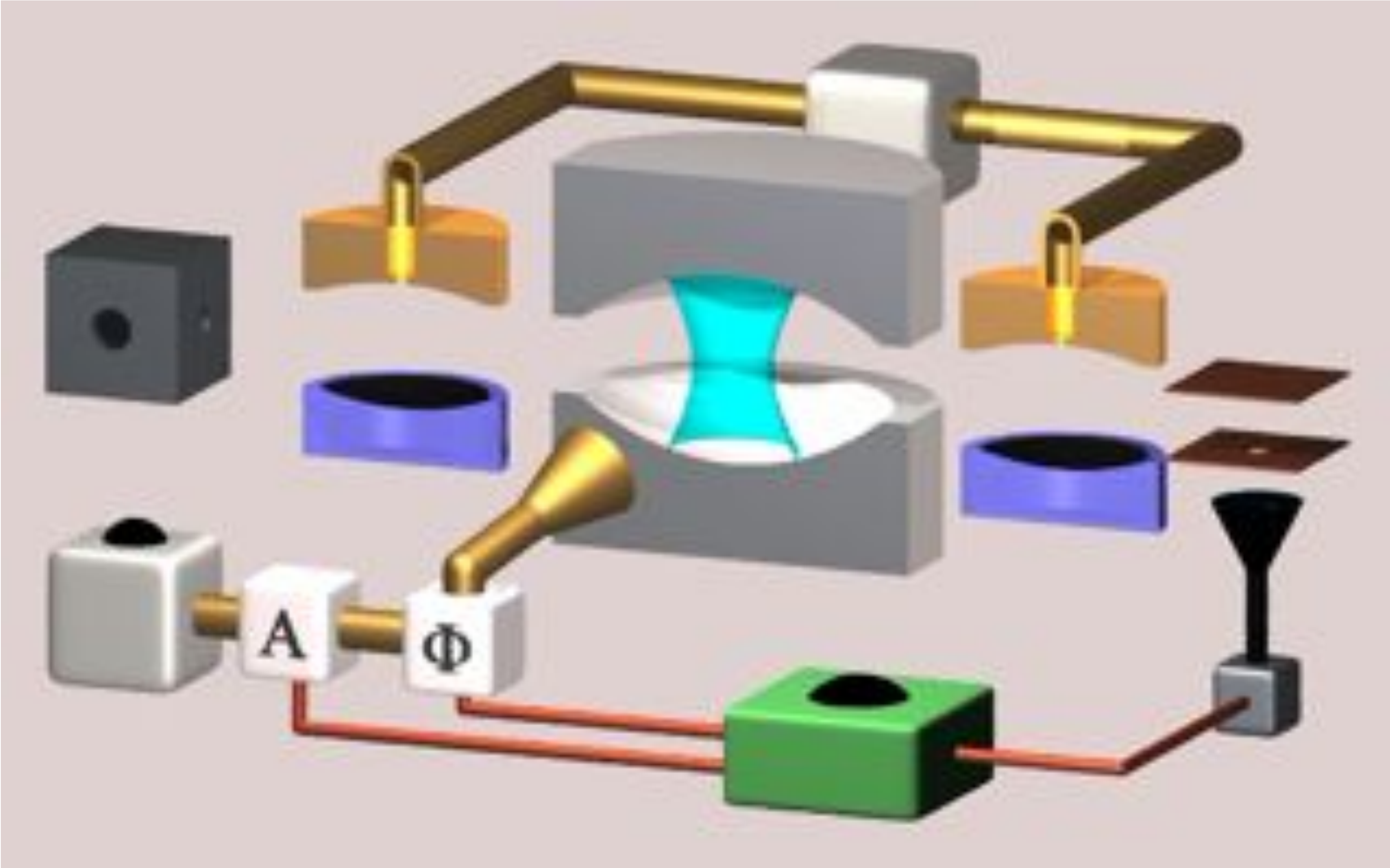
Can these procedures protect them against quantum jumps (loss or gain of photons)?

An ideal sensor for these experiments: QND probe atoms measuring photon number by Ramsey interferometry.
Back action is suppressed when target is reached!

What kind of actuator? Classical or quantum?

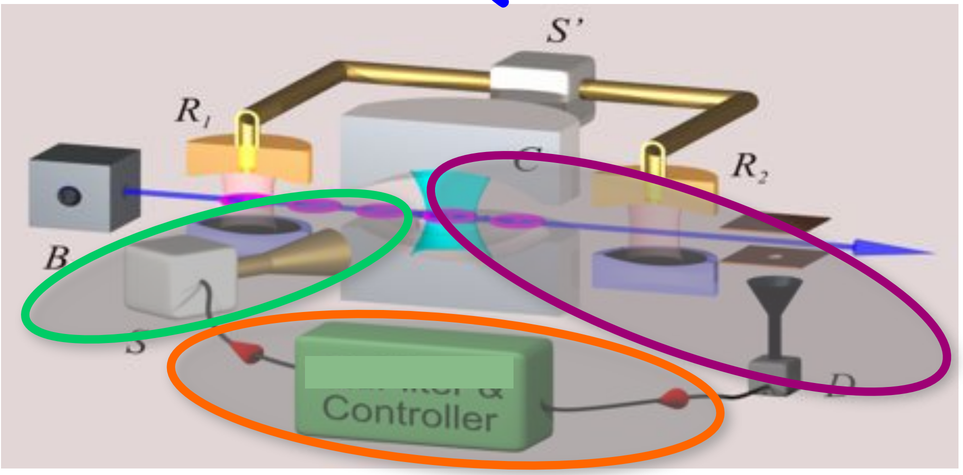
Quantum feedback with classical actuator

C.Sayrin, I.Dotsenko et al, Nature 448, 889 (2011)



Experiment performed with the theoretical coolaboration of
Pierre Rouchon's group at Ecole des Mines

Principle of quantum feedback in Cavity Quantum electrodynamics



Components of feedback loop

- **Sensor** (quantum "eye"):
  atoms and QND measurements
- **Controller** ("brain"):
  computer
- **Actuator** (classical "hand"):
  microwave injection

Feedback protocol:

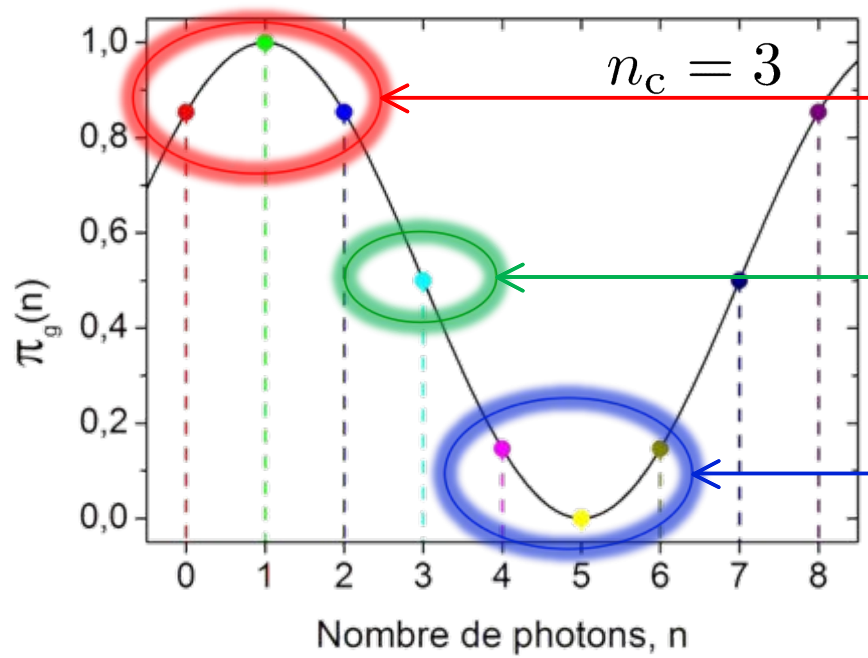
- Send atoms one by one in Ramsey interferometer
- Detect each atom, projecting field density operator ρ in new state estimated by computer
- Compute displacement α which minimises distance D between target and new state
- Close feedback loop by injecting a coherent field with amplitude α in C
- Repeat loop until reaching D ~ 0.

Probe : weak measurement

Fixing the parameters of experiment

$$M_e = \sin\left(\frac{\phi_r + \phi(N)}{2}\right) \qquad M_g = \cos\left(\frac{\phi_r + \phi(N)}{2}\right)$$

- Phaseshift per photon : $\phi_0 = \pi/4 \qquad (\phi(n) = n\phi_0)$
- Ramsey phase : $\phi_r = \pi/2 - \phi(n_c)$



$n < n_c$

$n = n_c$

$n > n_c$

Three well

distinct sets

Probe : weak measurement

Fixing the parameters of experiment

$$M_e = \sin\left(\frac{\phi_r + \phi(N)}{2}\right) \qquad M_g = \cos\left(\frac{\phi_r + \phi(N)}{2}\right)$$

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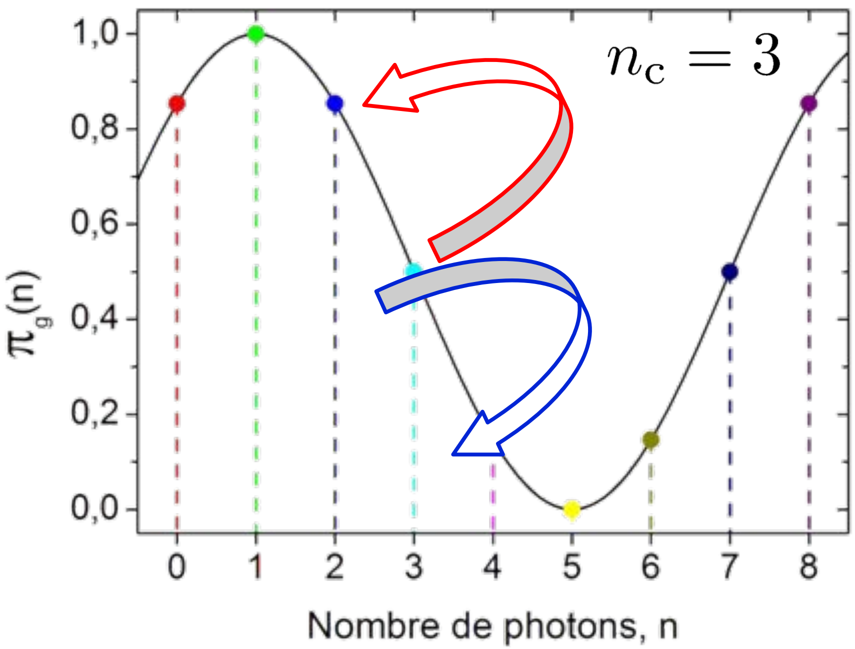


Quantum jumps well detected

- $|n_c\rangle \longleftrightarrow |n_c - 1\rangle$
- $|n_c\rangle \longleftrightarrow |n_c + 1\rangle$

Controler : real time estimation of field state

Before weak measurement, field described by density matrix ρ

- **Weak measurement**

Detected atom : outcome $|j = e, g\rangle$

$$\rho_{\mathrm{proj}} = \frac{M_j \, \rho \, M_j^\dagger}{\mathrm{Tr}(M_j \, \rho \, M_j^\dagger)}$$

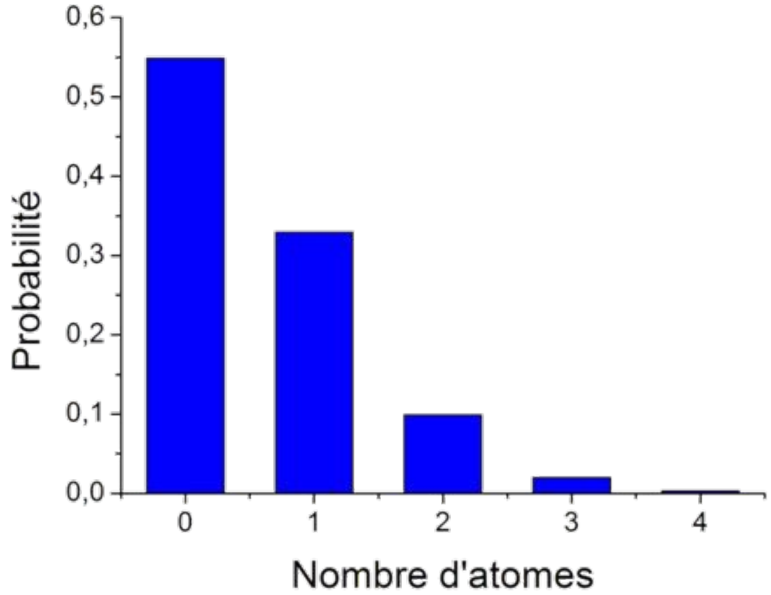
« Ideal » situation: does not take into account the **imperfections** of experimental set-up !

Controler : field state estimation

Difficulty : atomic source is not deterministic

Poisson law for atom number per sample with average : $n_a \approx 0{,}6$ atom

$$P_a(n) = \mathrm{e}^{-n_a} \frac{n^{n_a}}{n!}$$



Most probable : no atom in sample

Two atoms possible

New Kraus operators when 2 atoms detected

$$M_{ee} = M_e \times M_e \qquad M_{gg} = M_g \times M_g$$

$$M_{eg} = \sqrt{2}\, M_e \times M_g$$

Controler : field state estimation

Difficulty : imperfect apparatus

- Detection efficiency : $\epsilon_d \approx$ 35 % of atoms are counted

Unread measurement

$$\rho_{\text{proj}} = M_e\,\rho\,M_e^\dagger + M_g\,\rho\,M_g^\dagger$$

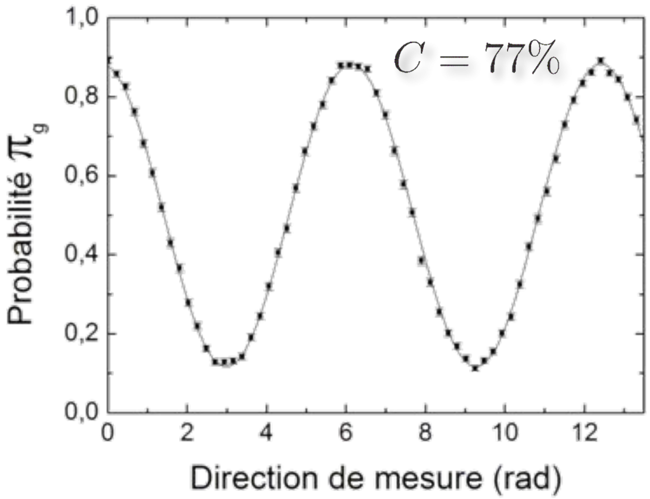
proportion of atoms in |e⟩

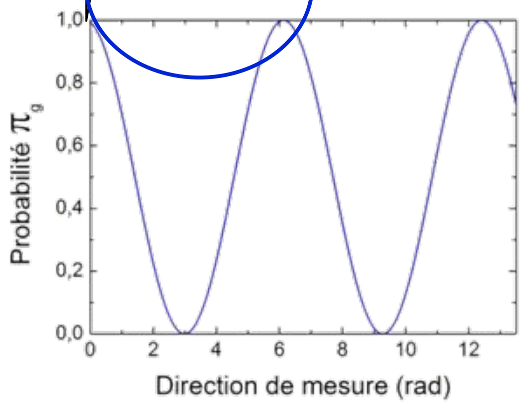
detected in |g⟩

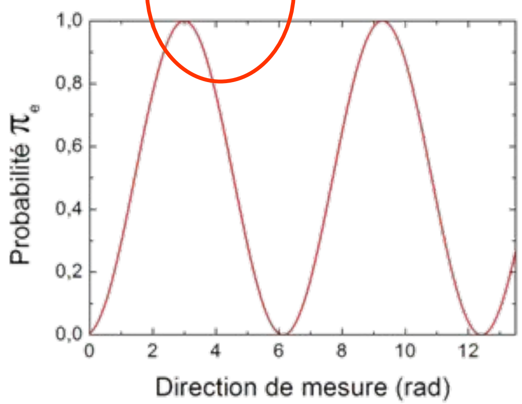
- Limited interferometer contrast

Detection errors

$$1 - \eta_g = 88,5\% \quad + \quad \eta_e = 11,5\%$$







Controler : field state estimation

Difficulty : imperfect apparatus

- Poisson statistics
- Detection efficiency
- Detection errors

Assume 1 atom detected in state $|e\rangle$

- Was really the atom in this state? $|e\rangle$ or $|g\rangle$?
- Was a second atom missed ?
- If so, in which state was it ? $|e\rangle$ or $|g\rangle$?

$$\rho_{\mathrm{proj}} = \frac{M_e \rho M_e^\dagger}{\mathrm{Tr}\left(M_e \rho M_e^\dagger\right)} \equiv \rho_e$$

$$\rho_{\mathrm{proj}} = p(e|e^d)\rho_e$$

All conditional probabilities given by Bayes law, knowing calibrated imperfections

I. Dotsenko *et al.*, Phys. Rev. A **80**, 013805 (2009)

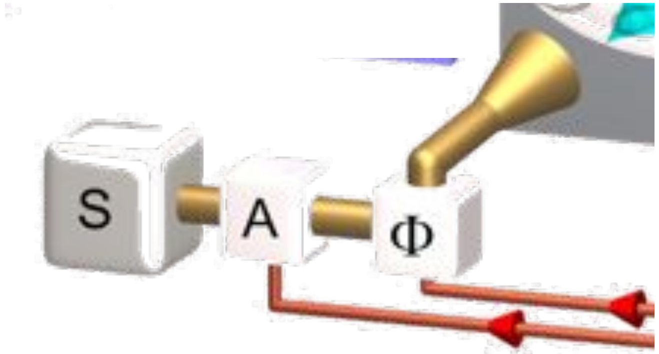
Actuator : field displacement

Change photon number distribution *via* field displacement

Displacement operator $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$: *injection of coherent field in cavity*

$$\rho_{\text{disp}} = D(\alpha)\,\rho\,D(-\alpha) \equiv \mathbf{D}_\alpha \rho$$

amplitude of displacement : *complex amplitude of microwave pulse*



In experiment :

- α real only
- phase is chosen to be 0 or π , with respect to initial field (fixing sign of displacement)
- Modulus $|\alpha|$ is controled *via* duration of microwave pulse

$$\alpha_{\max} = 0,1 \qquad t_{\max} = 60\ \mu\text{s}$$

$$\alpha_{\min} = 0,001 \qquad t_{\min} = 0,6\ \mu\text{s}$$

Controler : computing optimal displacement

Choosing displacement amplitude : moving field closer to target

Minimise proper distance to desired number state

❖ **A straightforward definition :**

$$d_F(\rho, \rho_c) = 1 - \langle n_c|\rho|n_c\rangle$$

Fidelity with respect to target

Drawback : Other Fock states are **undistinguishable**

$$d_F(|n\rangle\langle n|\,, \rho_c) = 1 \quad \textbf{for} \quad n \neq n_c \quad (n = n_c \pm 1, n \gg n_c, \ldots)$$

❖ **A better definition :**

$$d(\rho, \rho_c) = \sum_n \Gamma_n^{(n_c)} \langle n|\rho|n\rangle$$

Controler : computing optimal displacement

Choosing displacement amplitude : moving field closer to target

Minimise proper distance to desired number state

- **A straightforward definition : :**

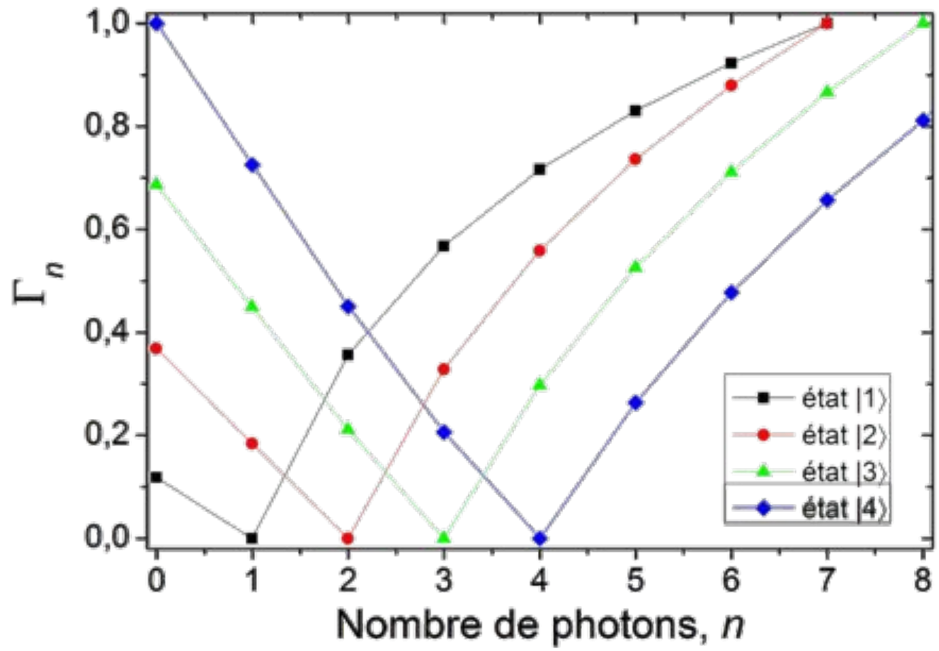
$$d_F(\rho, \rho_c) = 1 - \langle n_c|\rho|n_c\rangle$$

- **A better definition :**

$$d(\rho, \rho_c) = \sum_n \Gamma_n^{(n_c)} \langle n|\rho|n\rangle$$

The further n is from n_c ,
the larger the distance to the target!

$$\Gamma_{n_c}^{(n_c)} = 0$$



Controler : computing optimal displacement

- **Minimisation :** $$\alpha = \arg\min_{\alpha\in\mathbb{R}} d(\mathbf{D}_\alpha \rho, \rho_c)$$

⟹ Very costly in computing time !

- **To speed up the process :** restrict to small displacement amplitudes

⟹ Define a maximum amplitude : α_{max} = 0,1

⟹ Behaviour of $d(\mathbf{D}_\alpha \rho, \rho_c)$ around α = 0 ?

$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

Coefficients $\Gamma_n^{(n_c)}$ chosen so that :

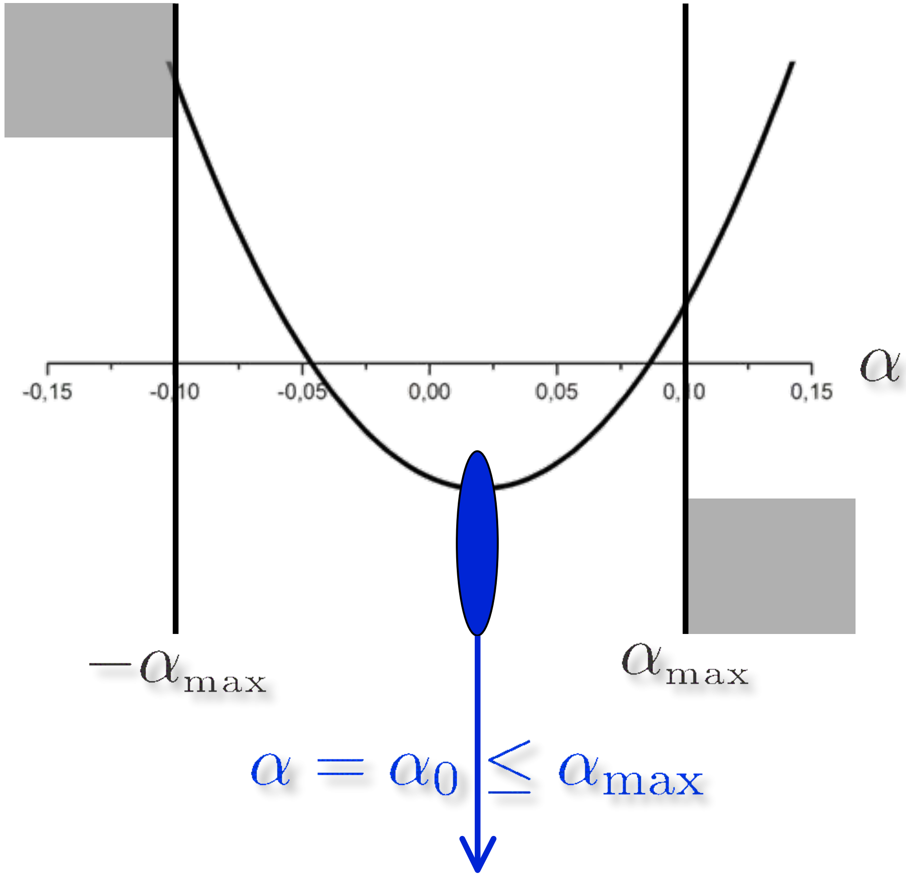
- If $\rho = |n_c\rangle\langle n_c| = \rho_c$ ⟹ $d(\mathbf{D}_\alpha \rho, \rho_c)$ is **minimum** at α=0
  $(a_1(\rho_c) = 0 \quad a_2(\rho_c) < 0)$
- If $\rho = |n \neq n_c\rangle\langle n \neq n_c|$ ⟹ $d(\mathbf{D}_\alpha \rho, \rho_c)$ is **maximum** at α=0
  $(a_1(|n\rangle\langle n|) = 0 \quad a_2(|n\rangle\langle n|) > 0)$

Controler : computing optimal displacement

$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

- **Control law :** studying the function

It has a local minimum on [$-\alpha_{max}$, + α_{max}]

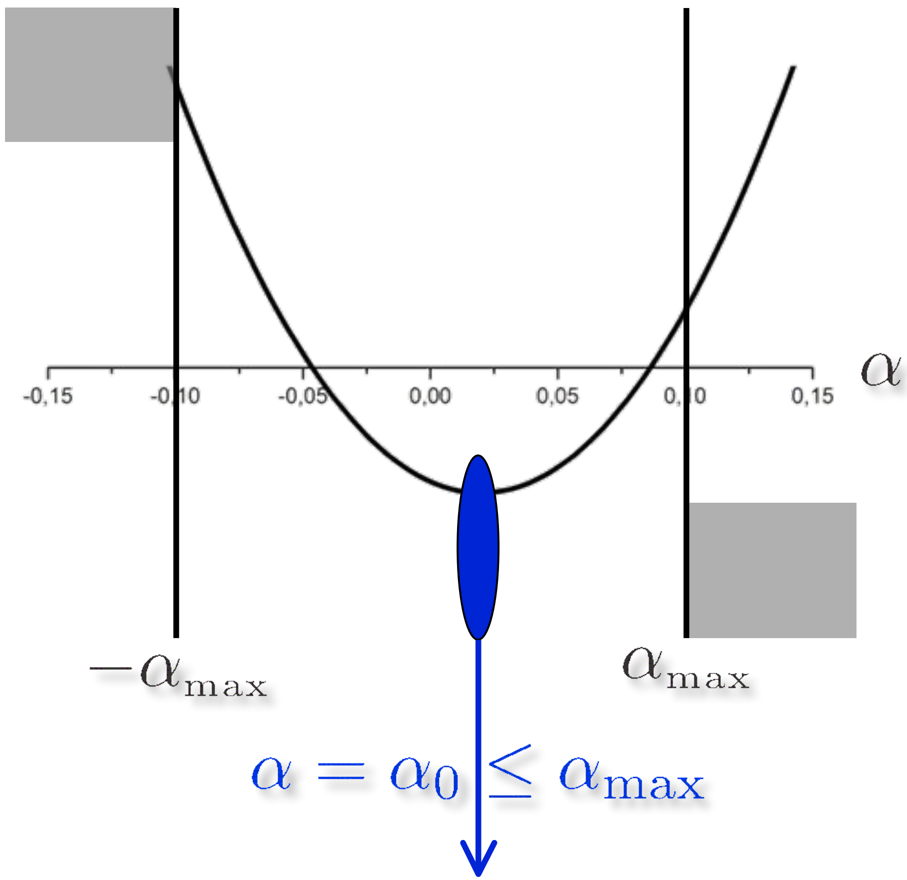


Controler : computing optimal displacement

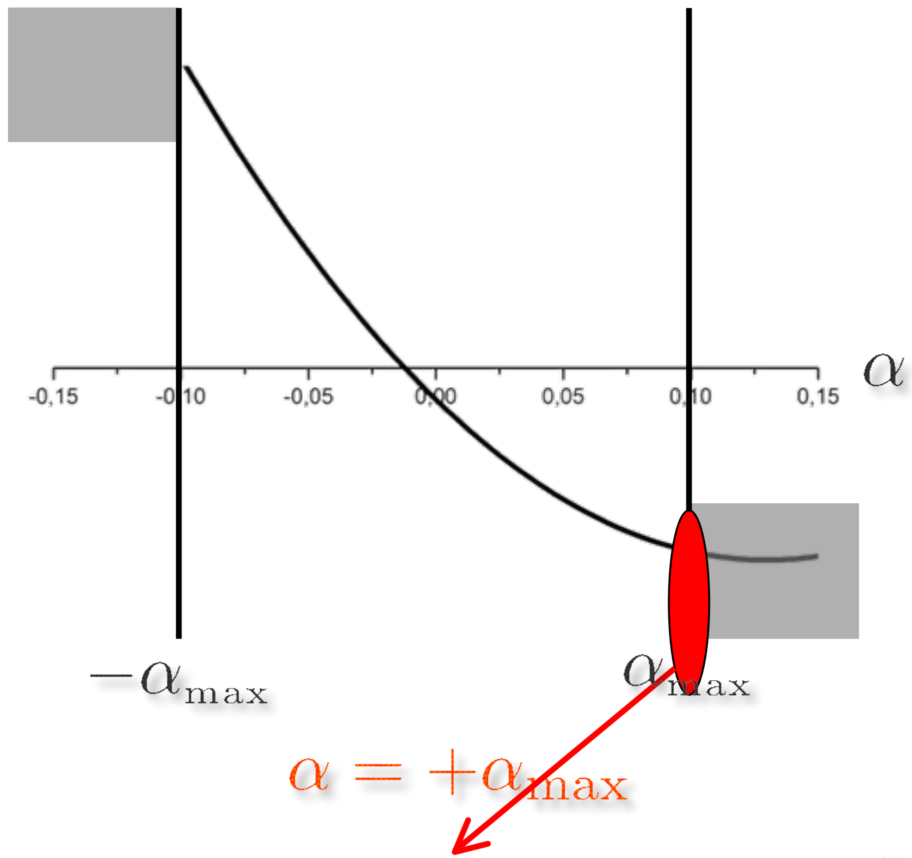
$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

•**Control law :** studying the function $\alpha \mapsto d(\mathbf{D}_\alpha \rho, \rho_c)$

If local minimum on [$-\alpha_{max}$, + α_{max}]



If Local minimum outside [$-\alpha_{max}$, + α_{max}]

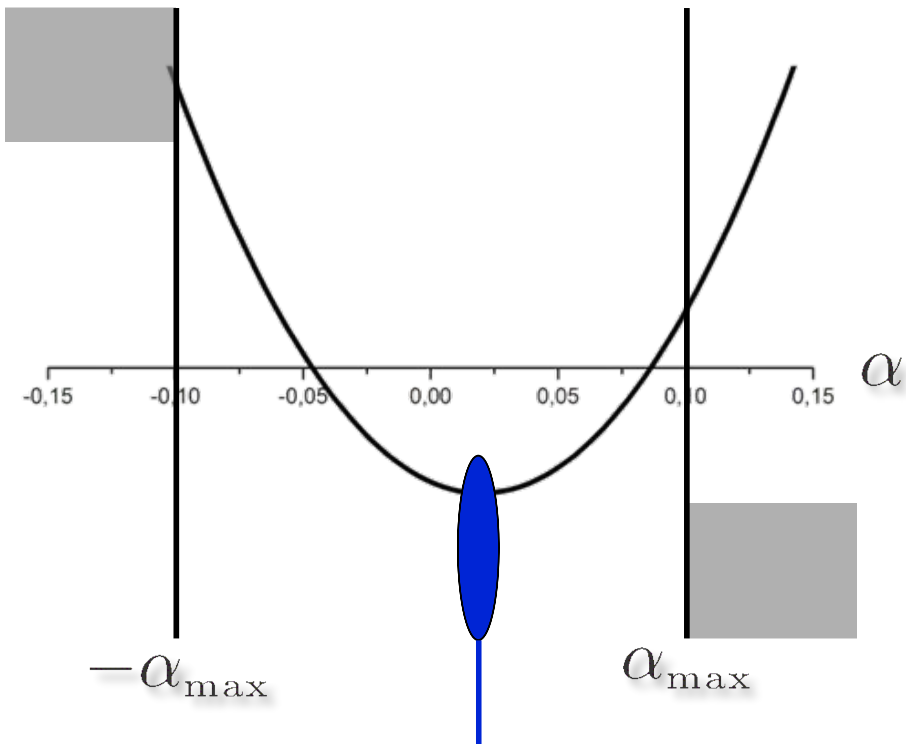


Controler : computing optimal displacement

$$d(\mathbf{D}_\alpha \rho, \rho_c) = d(\rho, \rho_c) - a_1(\rho)\alpha - a_2(\rho)\frac{\alpha^2}{2}$$

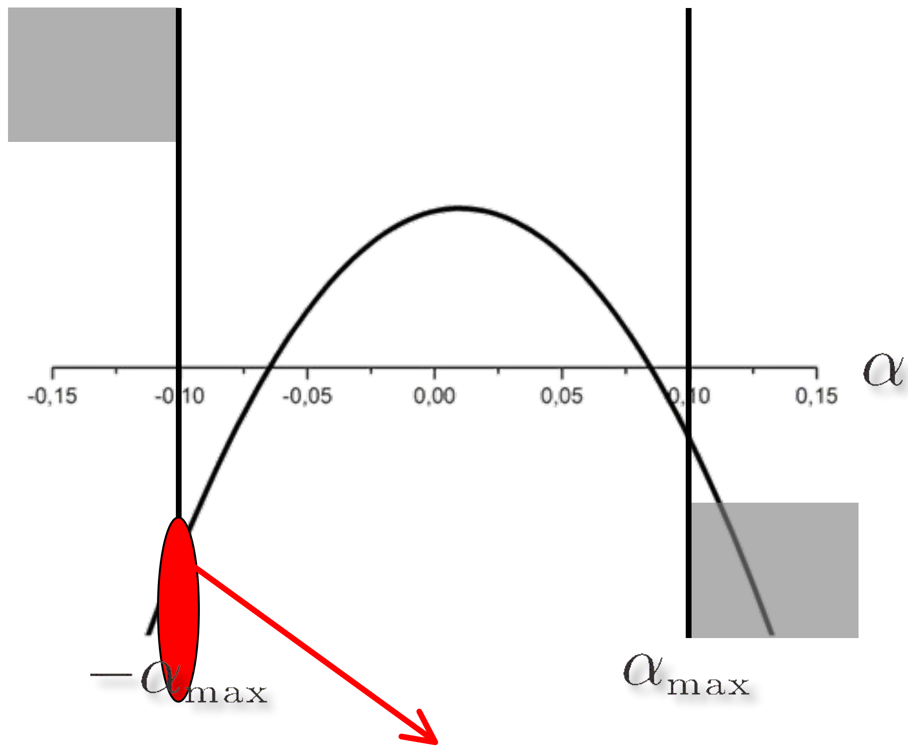
•**Control law :** studying the function $\alpha \mapsto d(\mathbf{D}_\alpha \rho, \rho_c)$ ρ

If local minimum on [-α_{max} , + α_{max}]



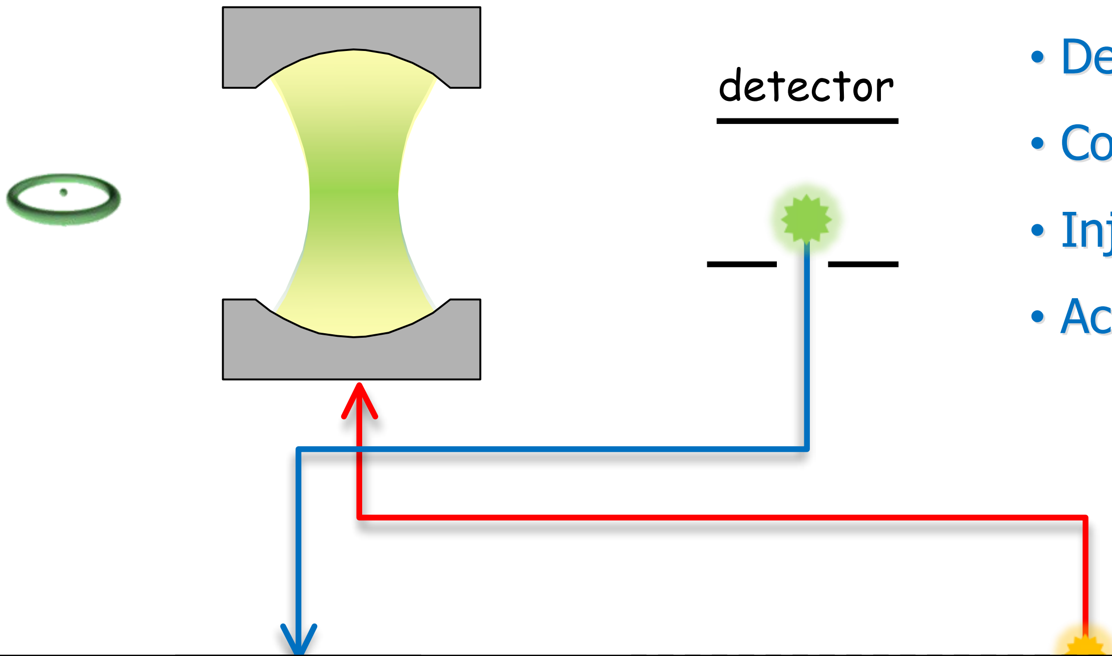
$\alpha = \alpha_0 \leq \alpha_{\max}$

If Local maximum on [-α_{max} , + α_{max}]



$\alpha = -\alpha_{\max}$

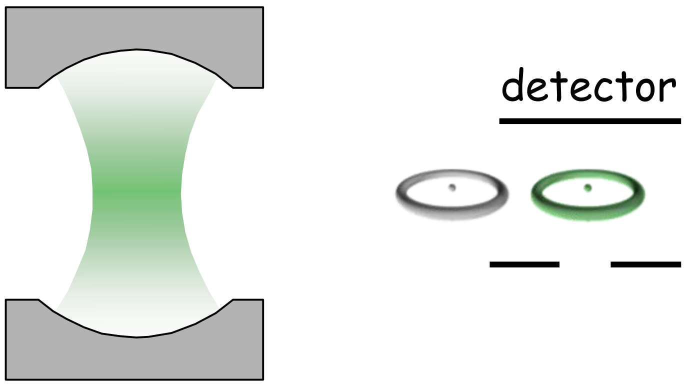
Summing it up: the feedback loop



- Detection of atomic sample
- Computing optimal displacement
- Injecting control field
- Accounting for relaxation

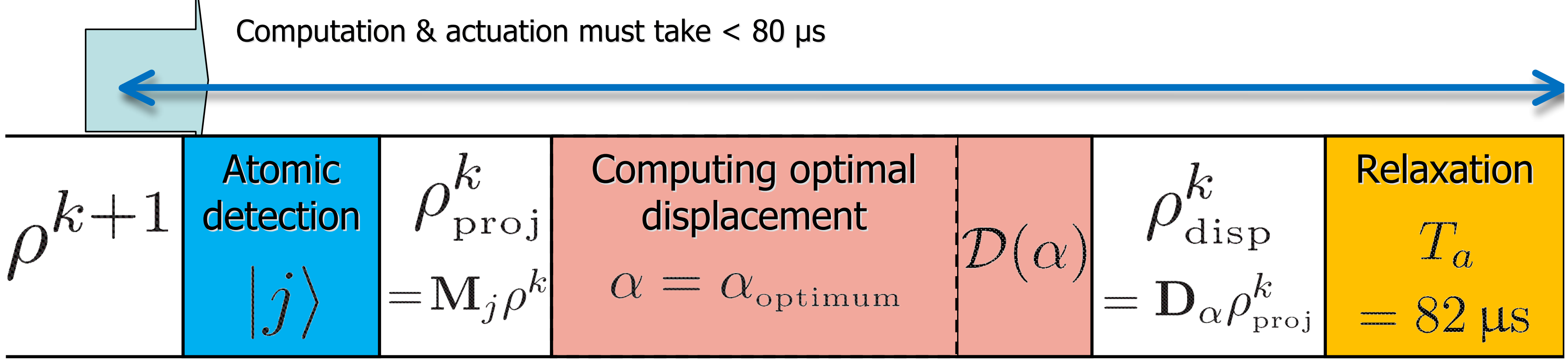
ρ^k ρ^{k+1}	Atomic detection $\vert j\rangle$	$\rho^k_{\mathrm{proj}} = \mathbf{M}_j \rho^k$	Computing optimal displacement $\alpha = \alpha_{\mathrm{optimum}}$	$\mathcal{D}(\alpha)$	$\rho^k_{\mathrm{disp}} = \mathbf{D}_\alpha \rho^k_{\mathrm{proj}}$	Relaxation $T_a = 82\ \mu\mathrm{s}$

Summing it up: the feedback loop



- Detection of atomic sample
- Computing optimal displacement
- Injecting control field
- Accounting for relaxation

Speed requirement : next atom follows after 82 µs !



n_t=3 target

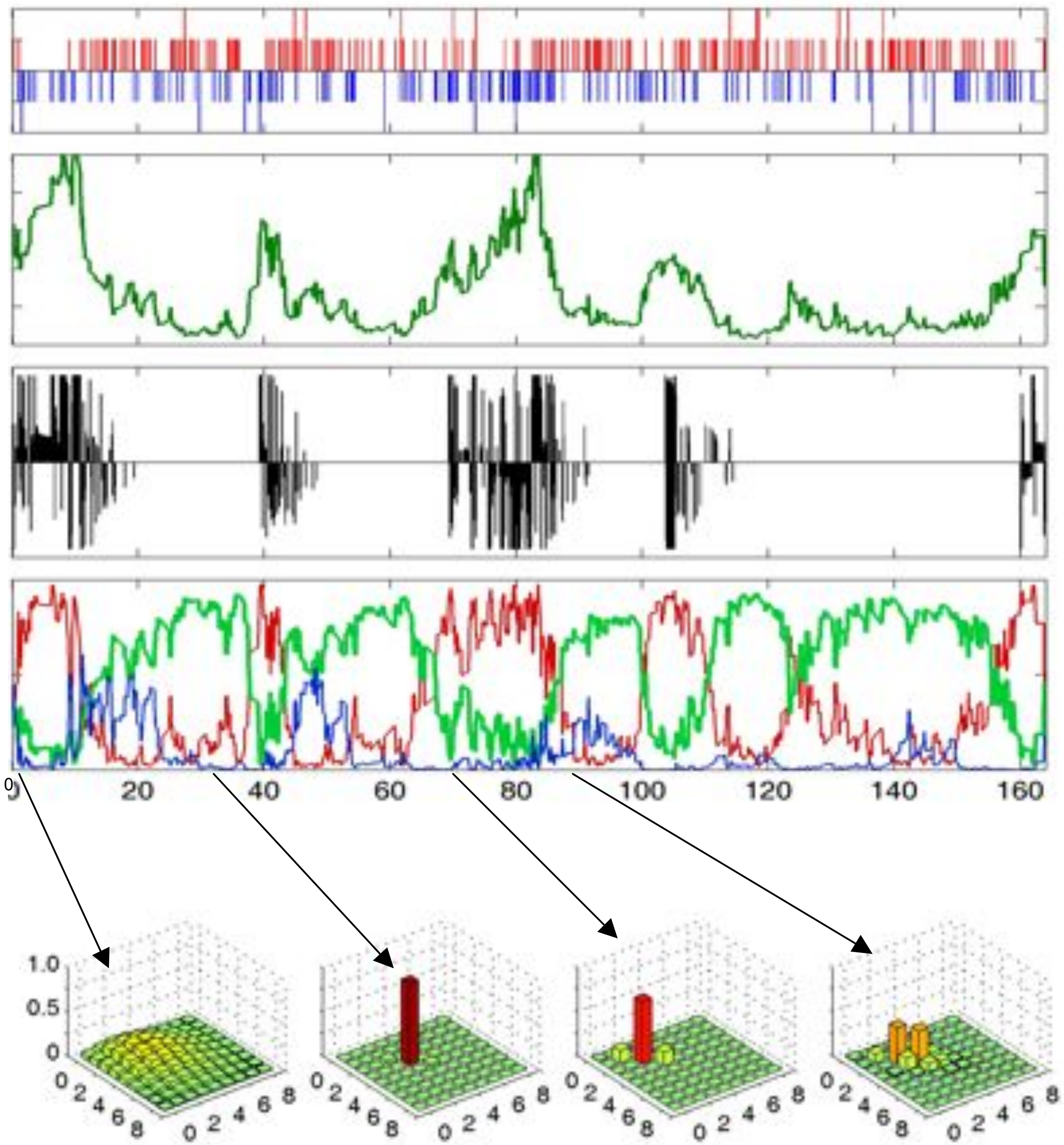
Raw detection

Distance to target

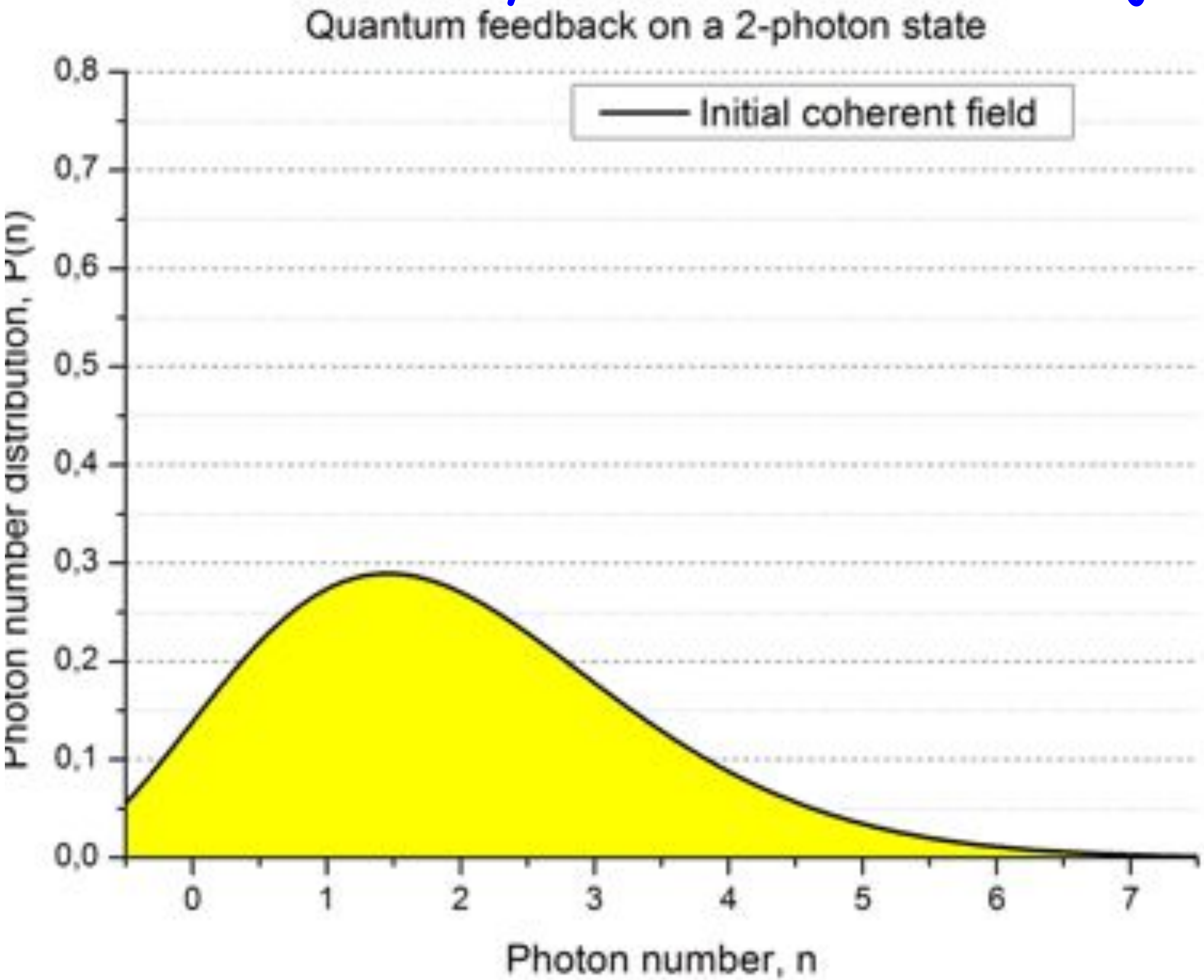
Actuator injection amplitudes

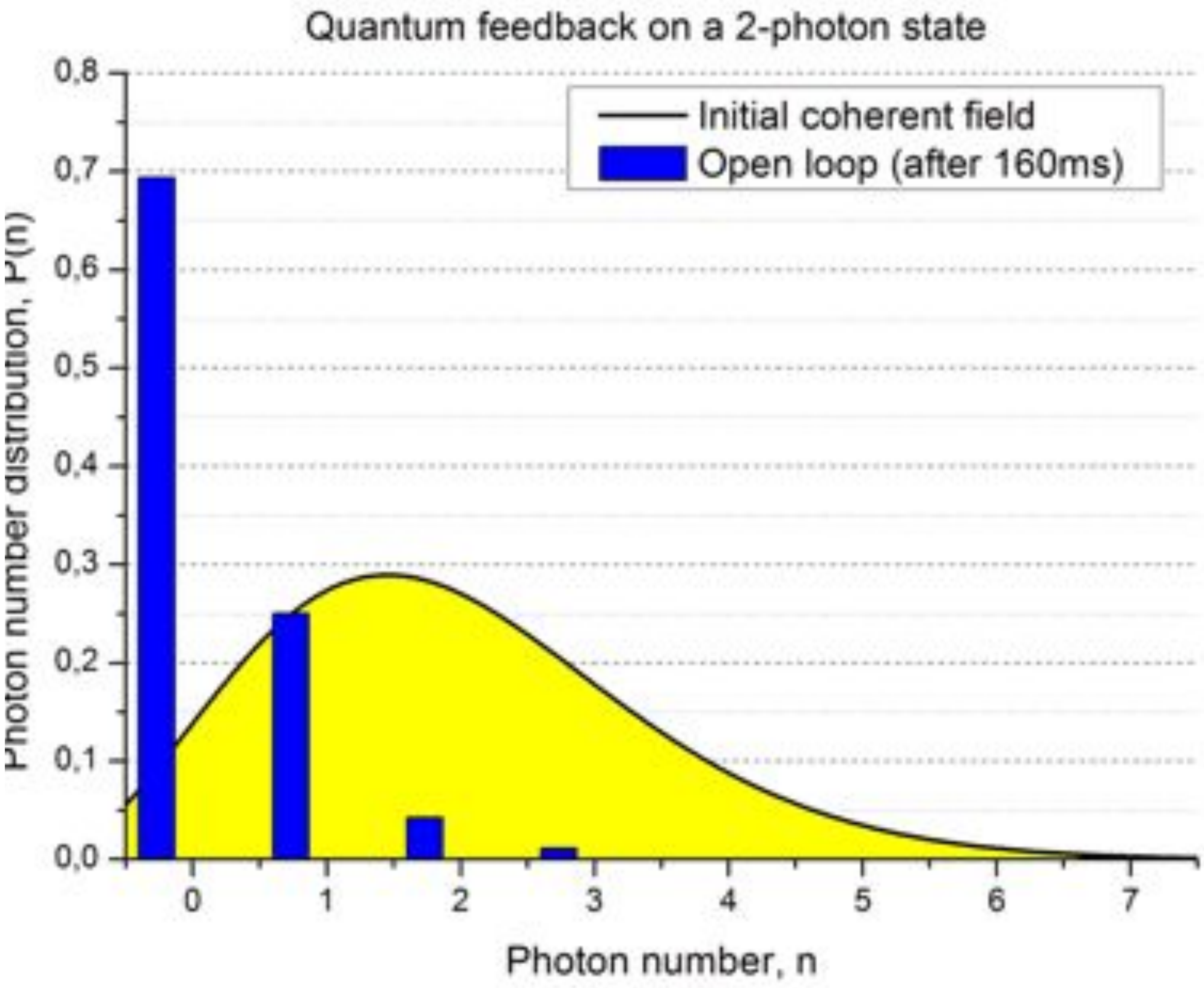
Estimated photon number probabilities: $P(n=n_t)$, $P(n<n_t)$, $P(n>n_t)$

Estimated density operator

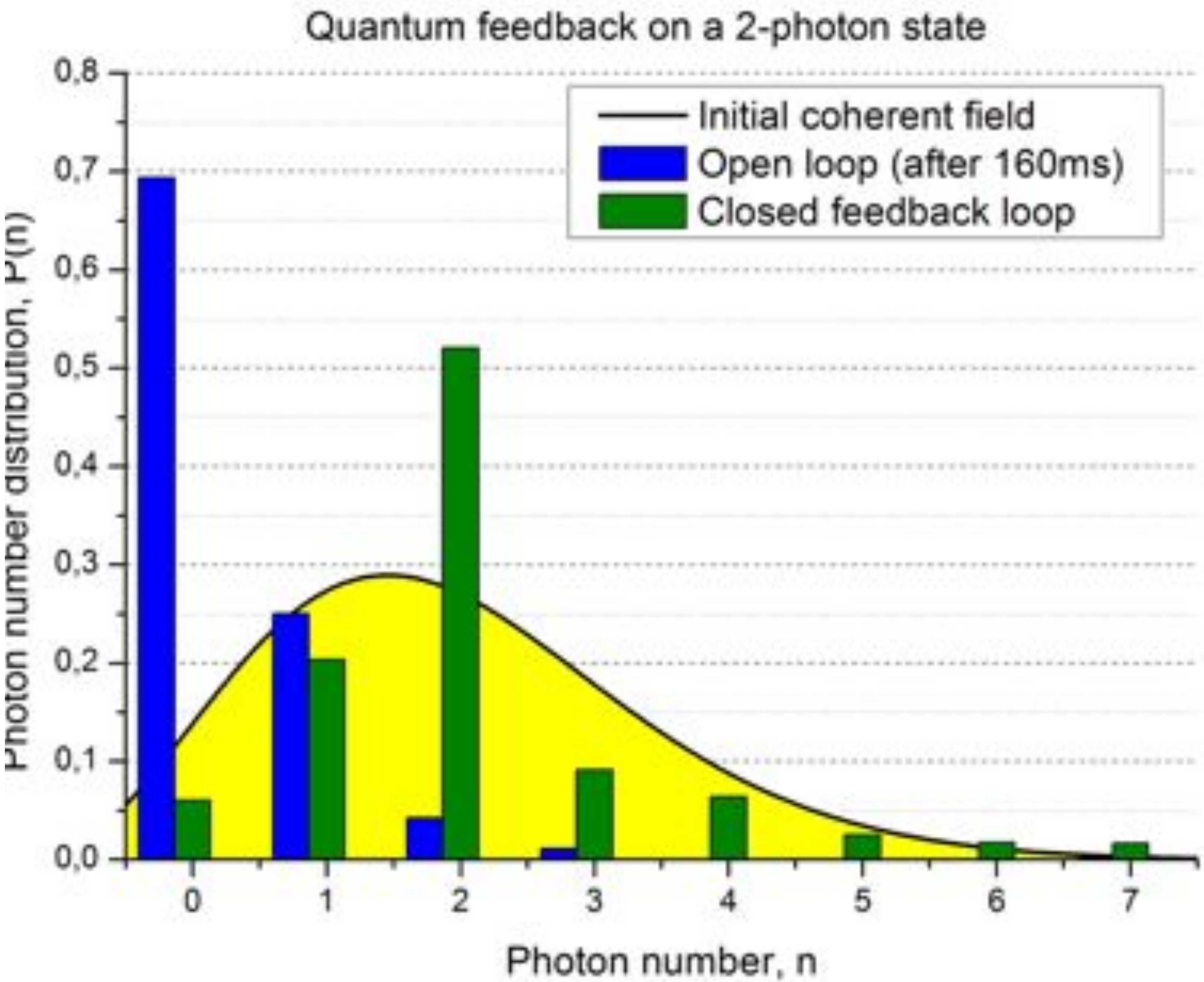


Statistical analyzis of an ensemble of trajectories

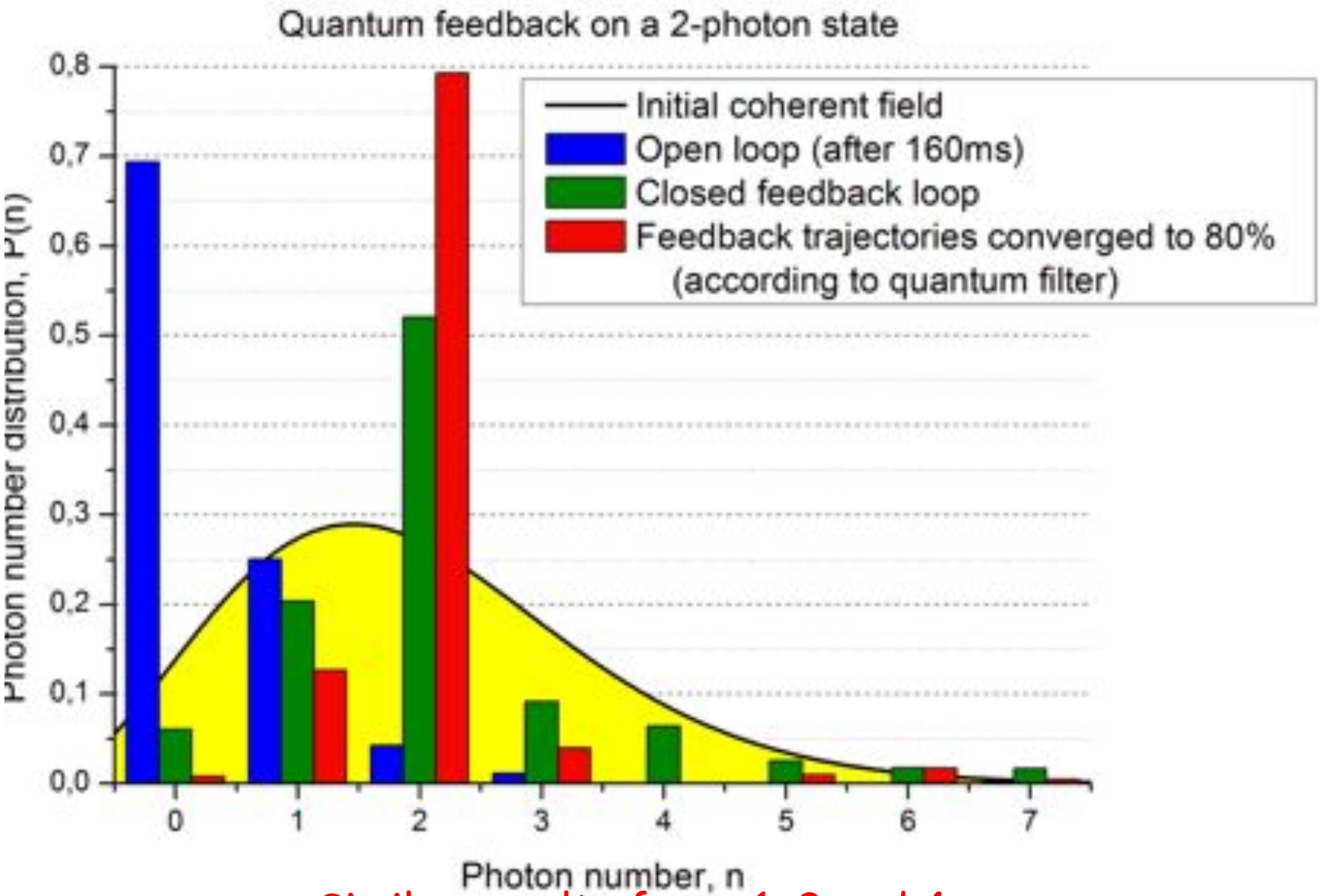




Quantum feedback on a 2-photon state
Initial coherent field
Open loop (after 160ms)
Photon number distribution, P(n)
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
0
1
2
3
4
5
6
7
Photon number, n

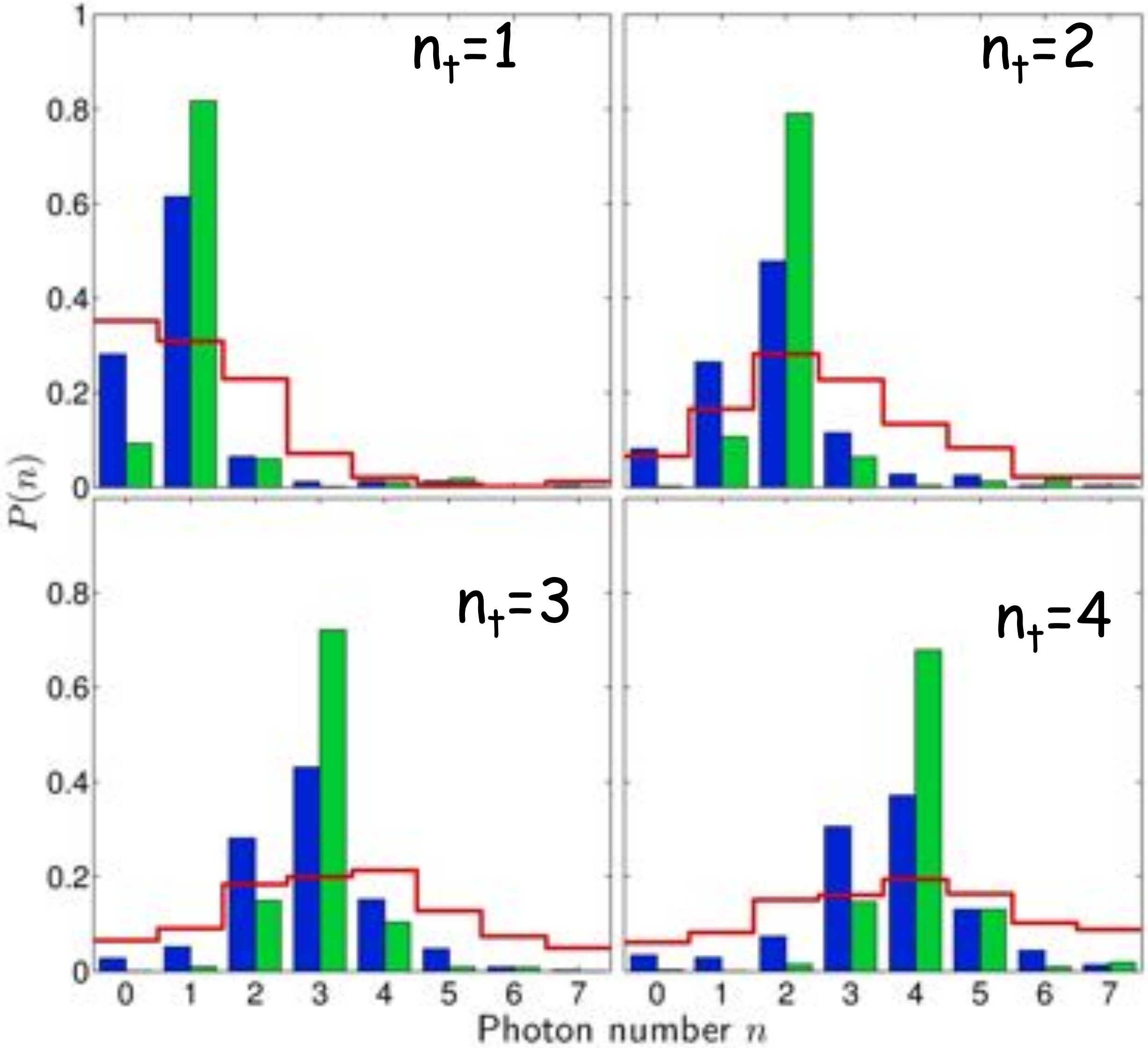


Quantum feedback on a 2-photon state
Initial coherent field
Open loop (after 160ms)
Closed feedback loop
Photon number distribution, P(n)
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0
1
2
3
4
5
6
7
Photon number, n



Similar results for n=1, 3 and 4…

Photon number probability distributions
(statistical average over large number of trajectories)



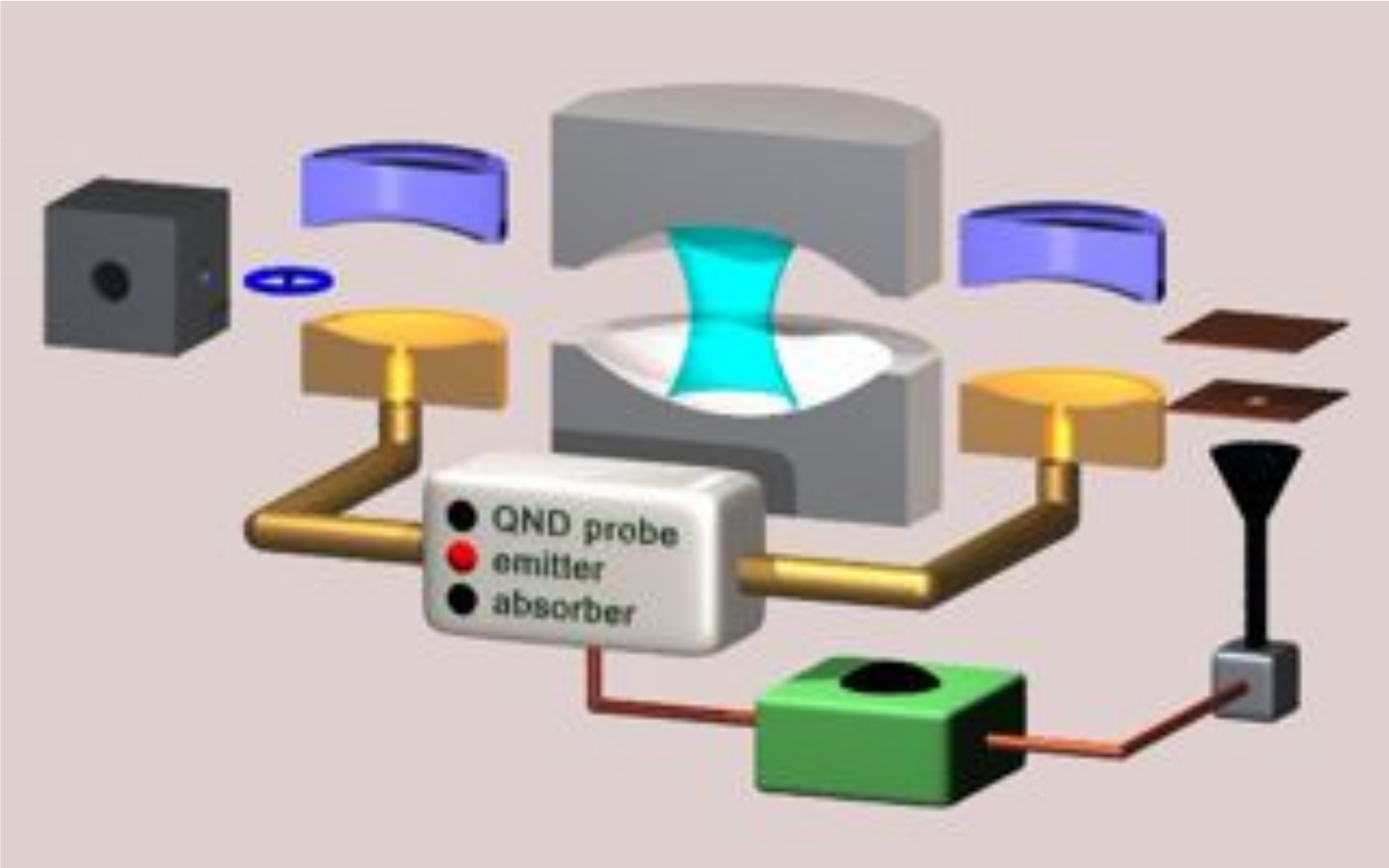
Initial field in red

Field after controller announces convergence in green

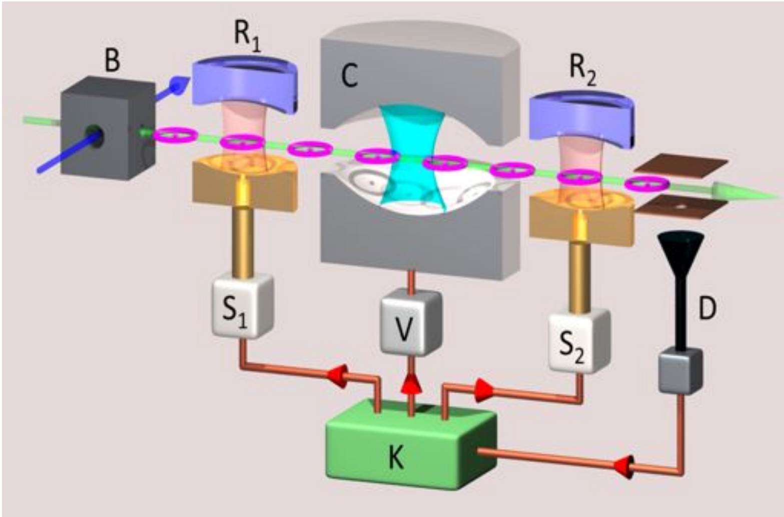
Steady state field in blue

Feedback with quantum actuator; atoms probe the field (dispersively), and also emit or absorb photons (resonantly)

X.Zhou, I Dotsenko et al, PRL, June 2012



The three atom "modes"

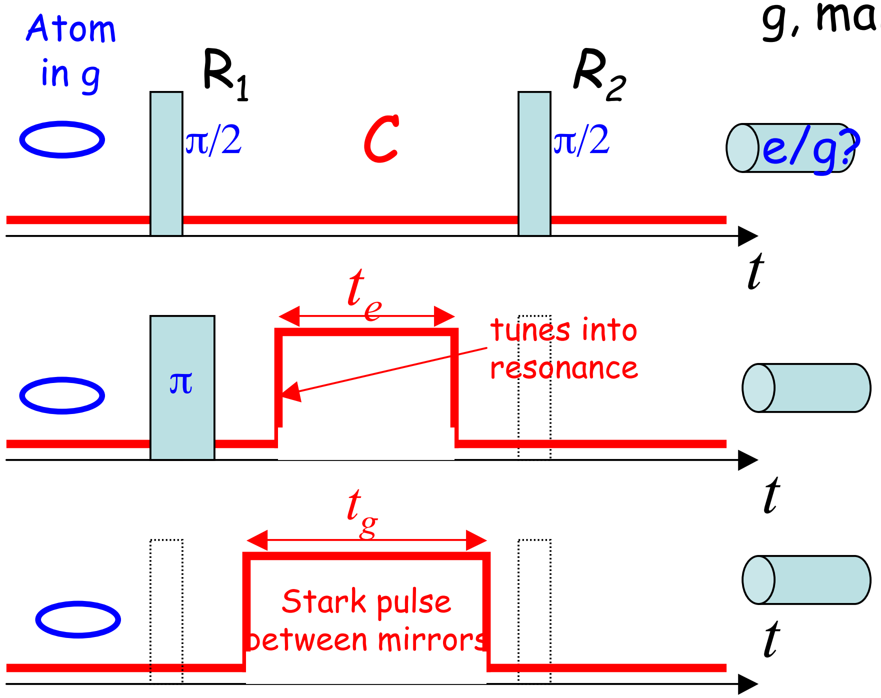


The algorithm relies on three kind of actions:

Non-resonant sensor atoms, prepared in state superposition in R_1, perform QND measurements in Ramsey interferometer

Resonant emitter atoms, prepared in state e in R_1, make the field jump up in Fock state ladder.

Resonant absorber atoms, prepared in state g, make field jump down in Fock state ladder.



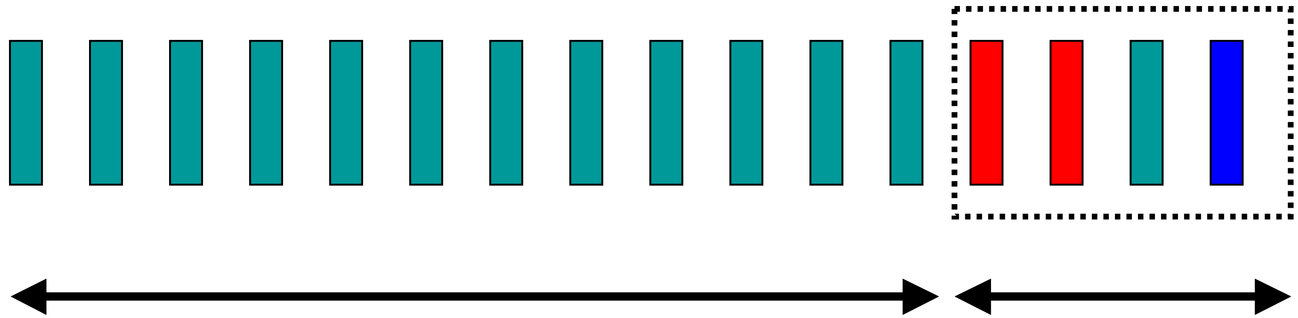
sensor

Emitter actuator

Absorber actuator

Switching between these three modes is controlled by K via microwave pulses applied in R_1,R_2 by S_1 and S_2 and dc voltage V across C mirrors (Stark tuning of atomic transition in and out of resonance)

The quantum feedback loop with atomic sensors and actuators



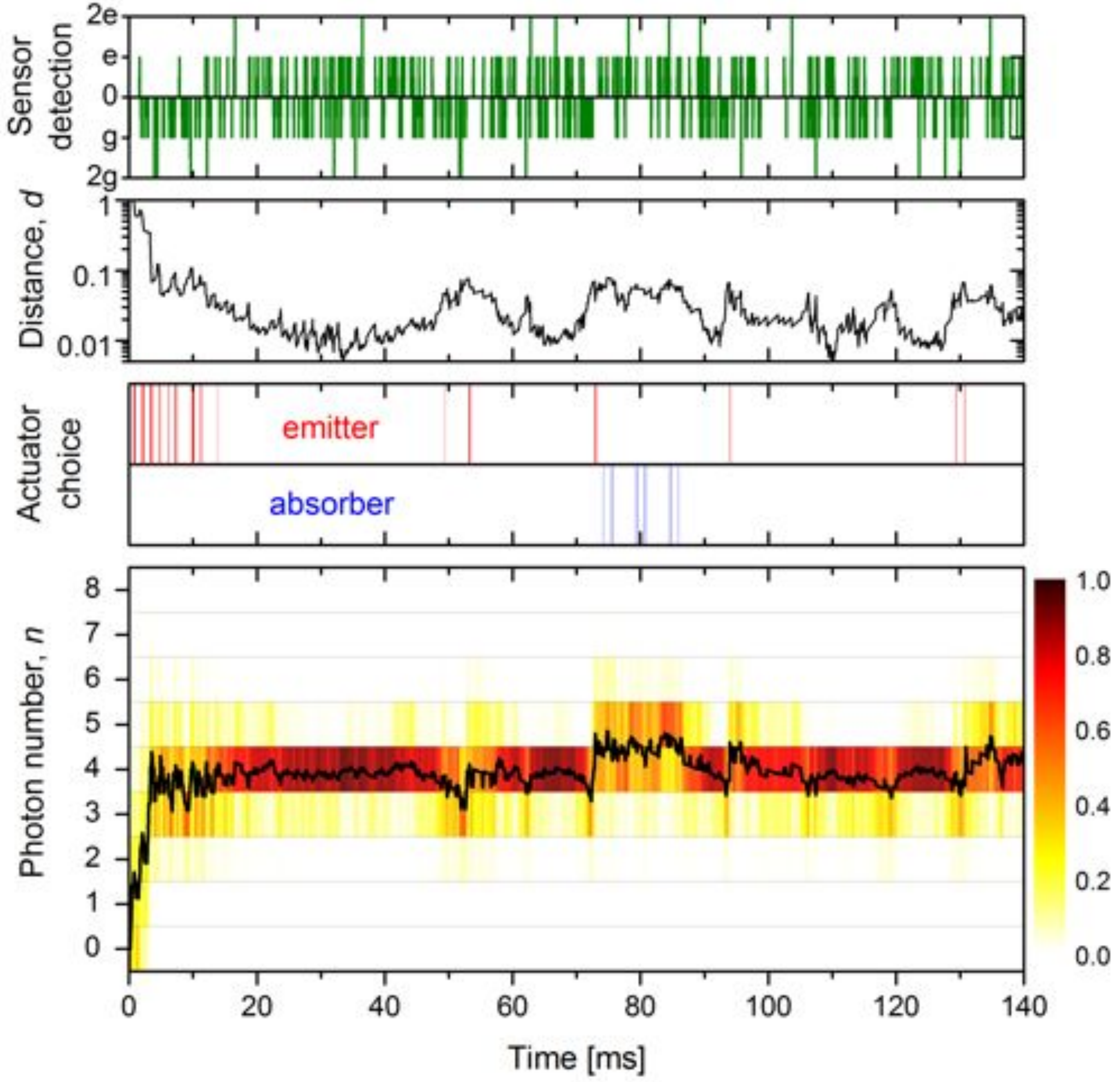
12 QND sensor samples
(0,1 or 2 atoms in each)

4 control samples
(K decides which
mode is best)

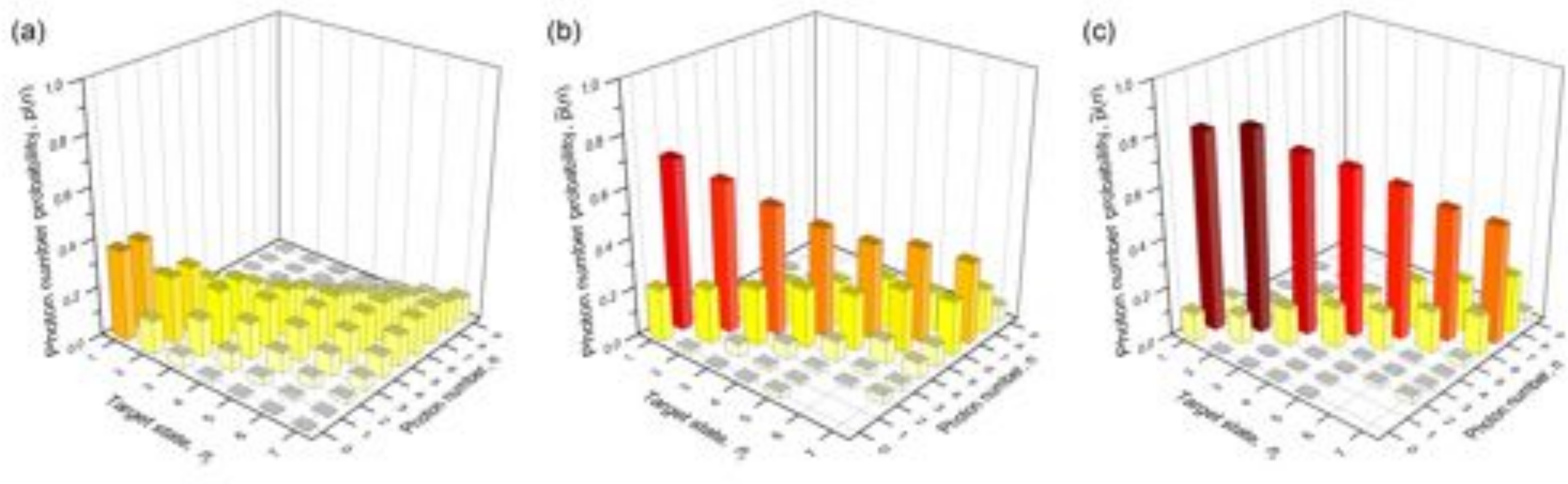
It requires several atoms to acquire info about photon number, but in principle only one atom to correct by ±1 photon: hence, many more sensors than emitter/absorbers

K estimates the field state by Baysian rules, computes the distance to target and decides what to do with the four control samples in each loop: emit, absorb or probe…

Locking the field to the n=4 Fock state



Statistical analysis of 4000 trajectories for each target state

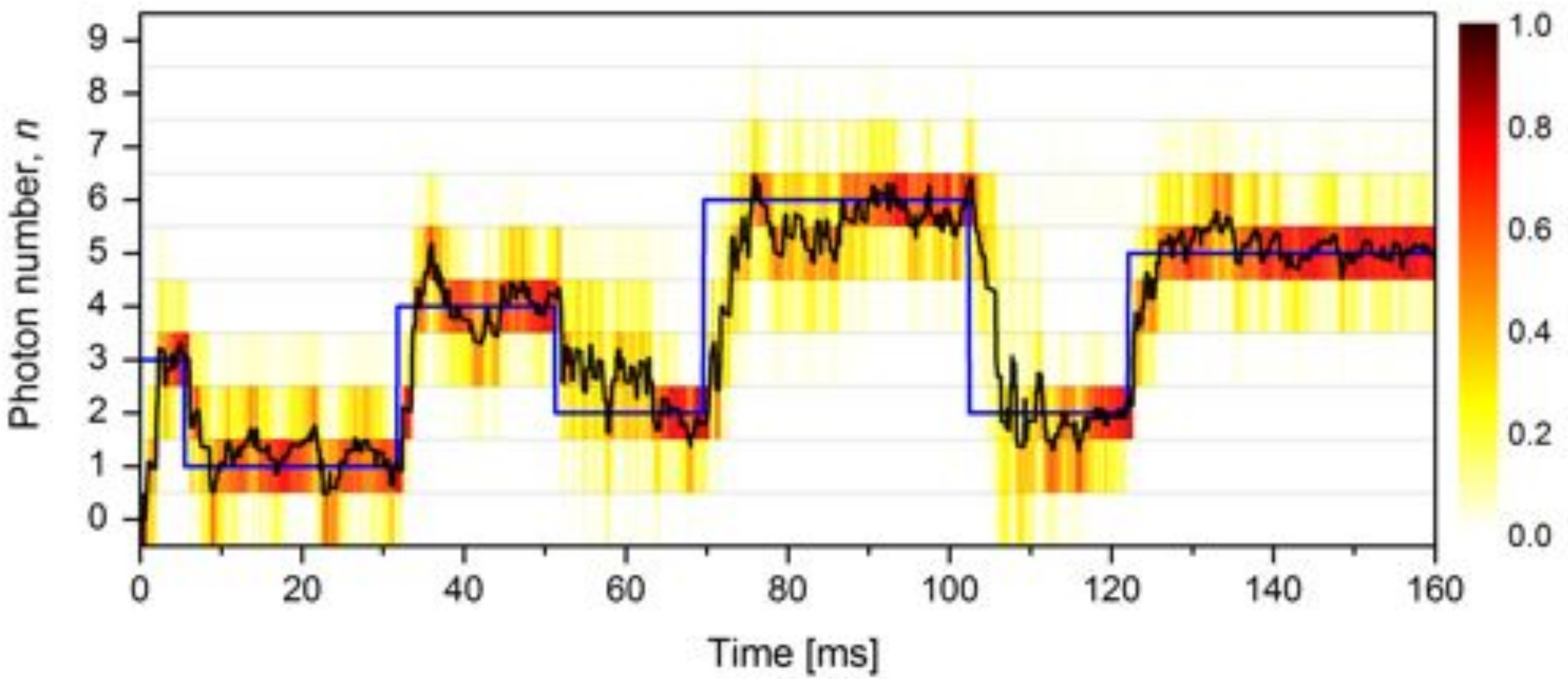


For comparison, Poisson distributions with mean photon numbers 1 to 7

Photon number distributions for the targets n_t=1,2,3,4,5,6,7 when quantum feedback is stopped at fixed time

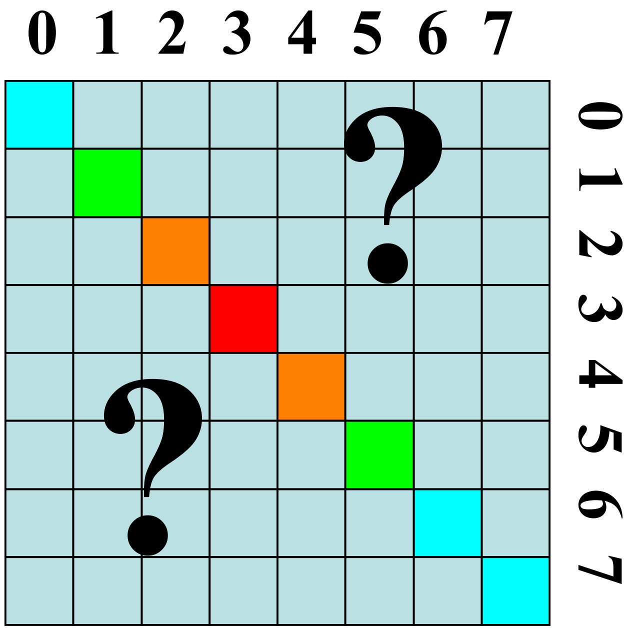
Photon number distributions for same targets when quantum feedback is interrupted after K announces successful locking (with fidelity >0.8)

Programming a walk between Fock state by changing the target state (here the sequence n= 3,1,4,2,6,2,5)



III-B
Field state reconstruction in CQED

QND photon counting and field state reconstruction



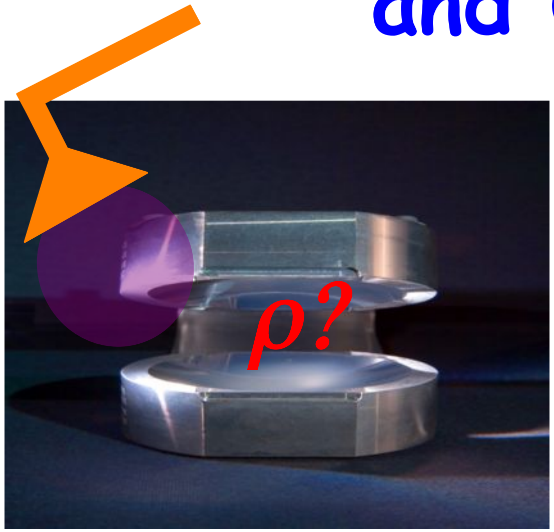
Repeated QND photon counting on copies of field determines the diagonal ρ_{nn} elements of the field density operator in Fock state basis, but leaves the off-diagonal coherences $\rho_{nn'}$ unknown

Recipe to determine the off-diagonal elements and completely reconstruct ρ:

translate the field in phase space by homodyning it with coherent fields of different complex amplitudes and count (on many copies) the photon number in the translated fields

Tomography of trapped light

Reconstructing field state by homodyning and QND photon counting



$$\rho \rightarrow \rho^{(\alpha)} = D(\alpha)\,\rho\,D(-\alpha)$$

Field translation operator (Glauber):

$$D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^{*} a)$$

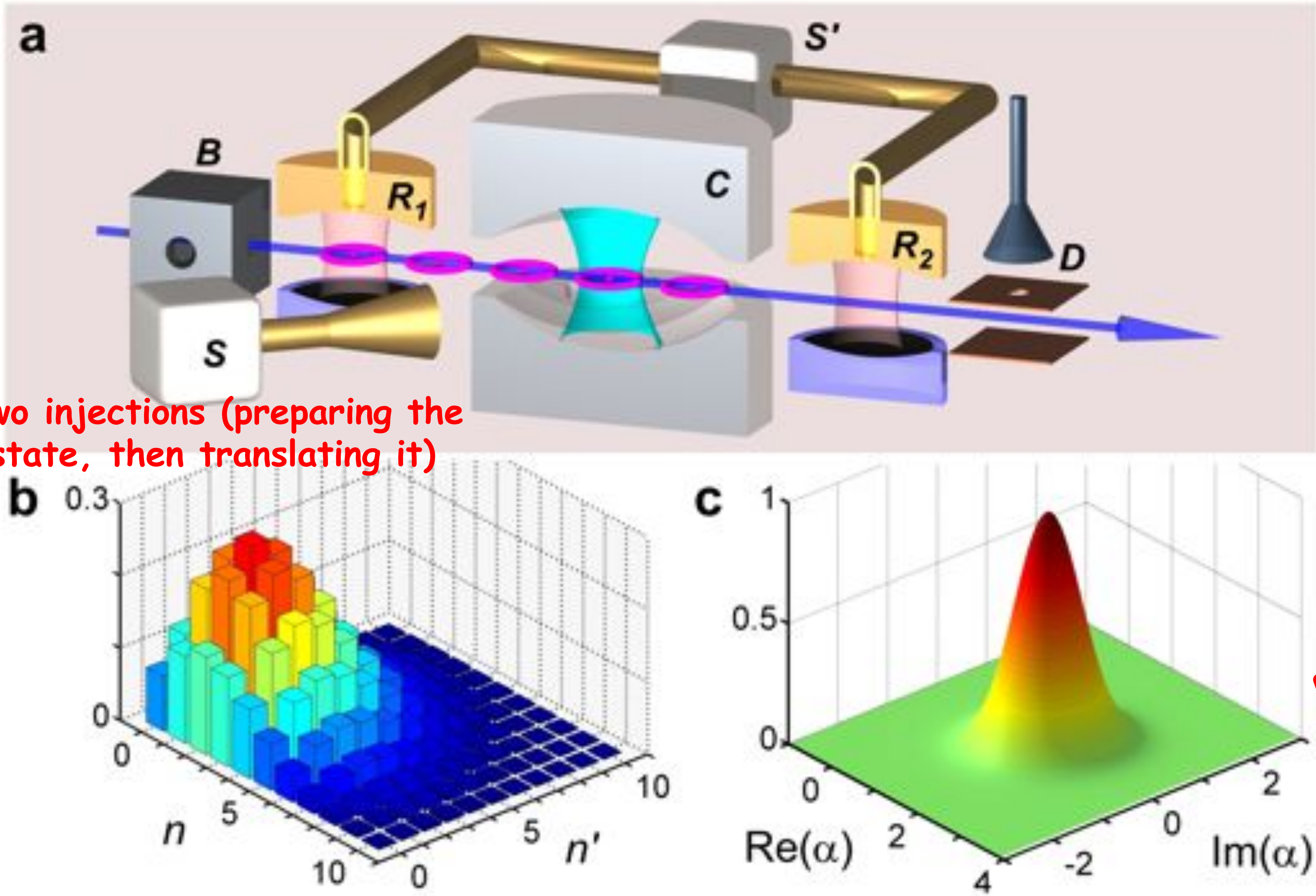
The homodyning translation in phase space admixes field coherences $\rho_{n'n''}$ into the diagonal matrix elements $\rho^{(\alpha)}_{nn}$ of the translated field:

$$\text{measured } \rho^{(\alpha)}_{nn} = \Sigma_{n'n''}\, D_{nn'}(\alpha)\;\rho_{n'n''}\, D_{n''n}(-\alpha)$$

We determine $\rho^{(\alpha)}_{nn}$ by QND photon counting on translated fields, for many α's, and get a set of linear equations constraining all the $\rho_{n'n''}$ s. By inverting these equations, we get the full density operator of the field. This direct reconstruction method has its problems.

Requires many copies: quantum state is a **statistical** concept

Reconstructing a coherent state

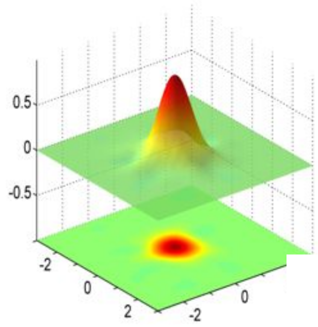


Two injections (preparing the state, then translating it)

ρ

W

A zoo of Fock states



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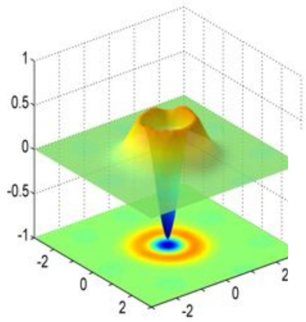
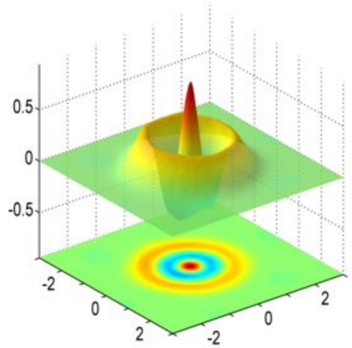


n=1



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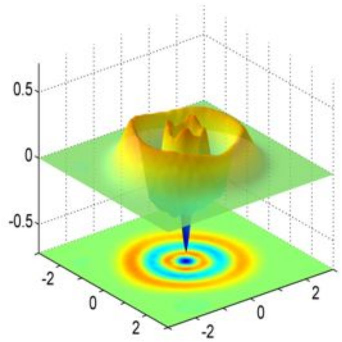
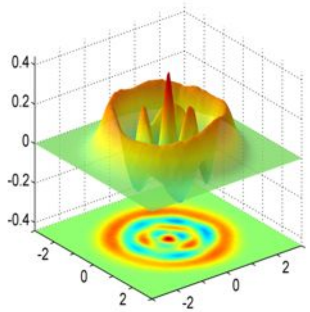
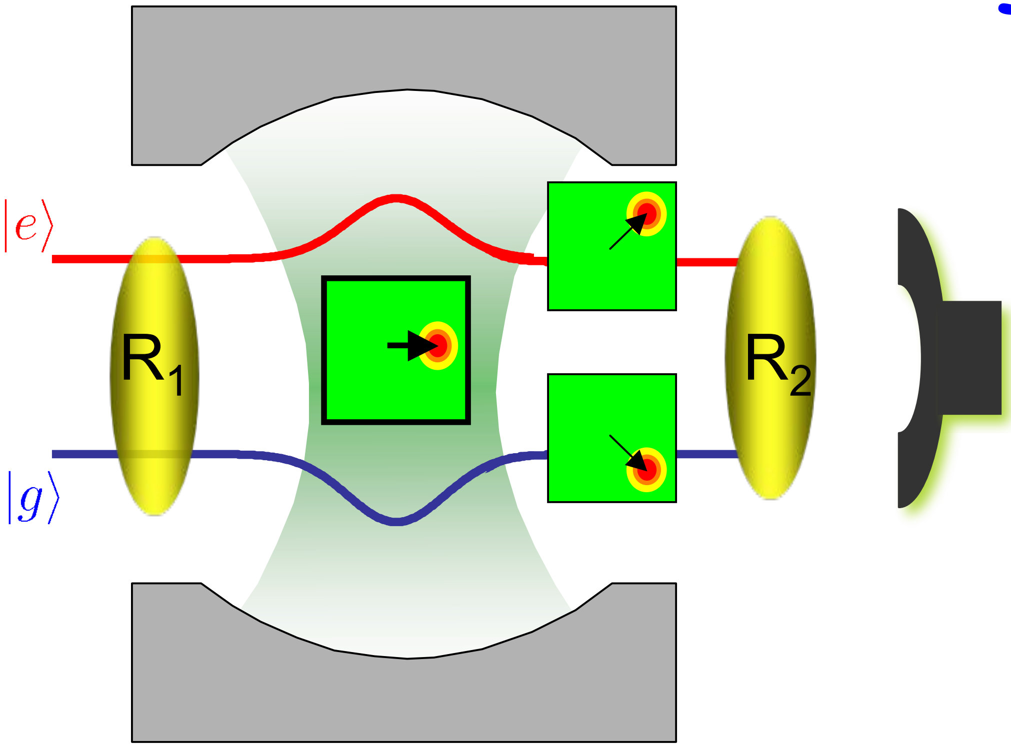


n=3

n=4



How single atom prepares Schrödinger cat state of light

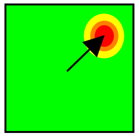
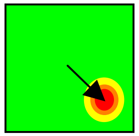
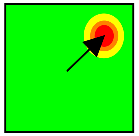
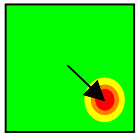
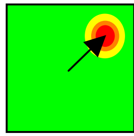
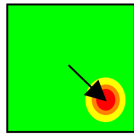


1.Coherent field is prepared in C

2. Single atom is prepared in R_1 in a superposition of e and g

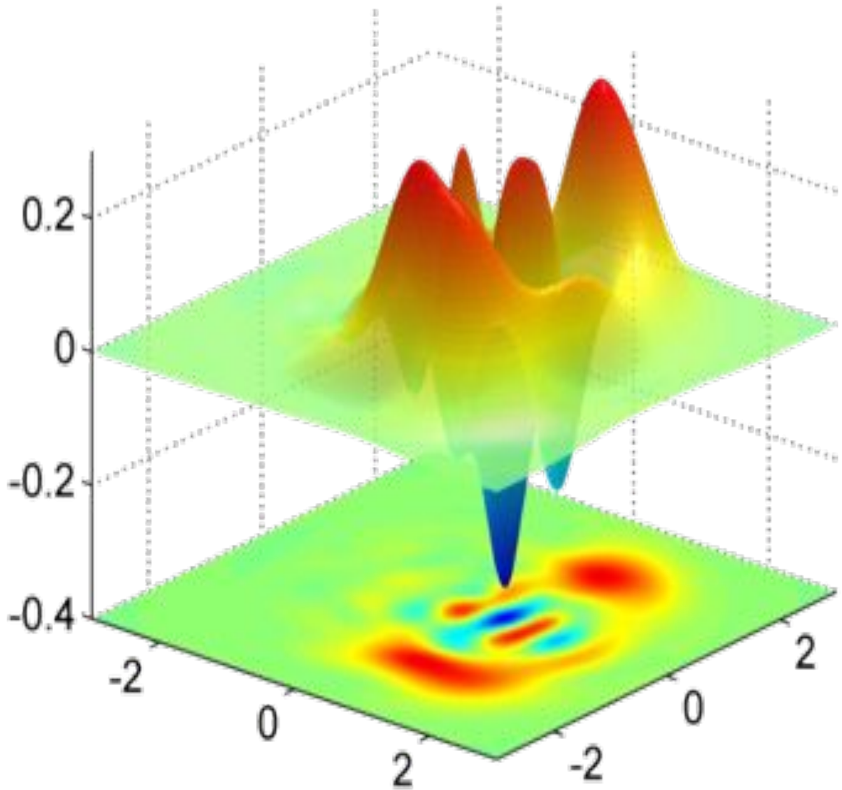
3. Atom shifts the field phase in two opposite directions as it crosses C: superposition leads to entanglement in typical Schrödinger cat situation

4. Atomic states mixed again in R_2 maintains cat's ambiguity:

|  ,e > + |  ,g > →(|  > + |  >)|e>+(|  >- |  >)|g>

Detecting atom in e or g projects field into + or - cat state superposition!

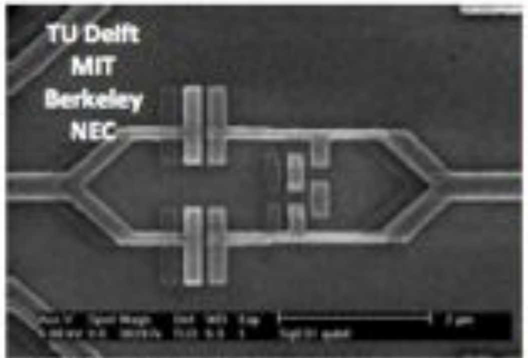
Schrödinger cat state

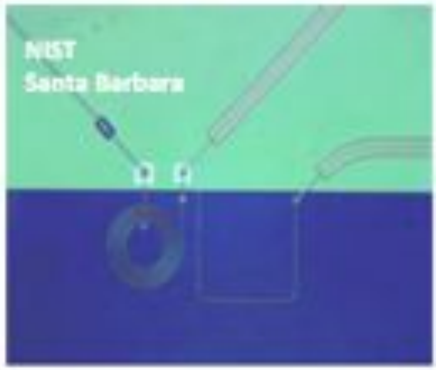


Schrödinger cat
$|\alpha e^{i\varphi}\rangle + |\alpha e^{-i\varphi}\rangle$
generated in C by
single atom index
effect

III-C

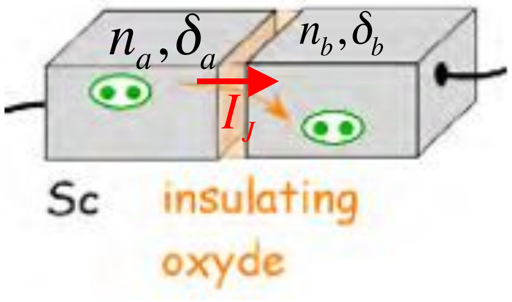
Cavity QED with artificial atoms
(an introduction to my talk at ICAP)





Simple description of an isolated junction

$$\delta_a - \delta_b = \delta$$
$$n_a - n_b = 2p$$



$$V = \frac{Q}{C} = \frac{2ep}{C}$$ C=Capacitance of junction

$$I_J = -2e\frac{dp}{dt} = I_0 \sin\delta$$ dc Josephson effect

$$\frac{d\delta}{dt} = \frac{2eV}{\hbar} = \frac{4e^2 p}{\hbar C}$$ ac Josephson effect

The 2 Josephson relations derive from an Hamiltonian H:

$$\frac{dp}{dt} = -\frac{I_0}{2e}\sin\delta = -\frac{1}{\hbar}\frac{\partial H}{\partial \delta} \quad ; \quad \frac{d\delta}{dt} = \frac{4e^2 p}{\hbar C} = \frac{1}{\hbar}\frac{\partial H}{\partial p} \quad \rightarrow \quad H = \frac{2e^2}{C}p^2 - \frac{\hbar I_0}{2e}\cos\delta$$

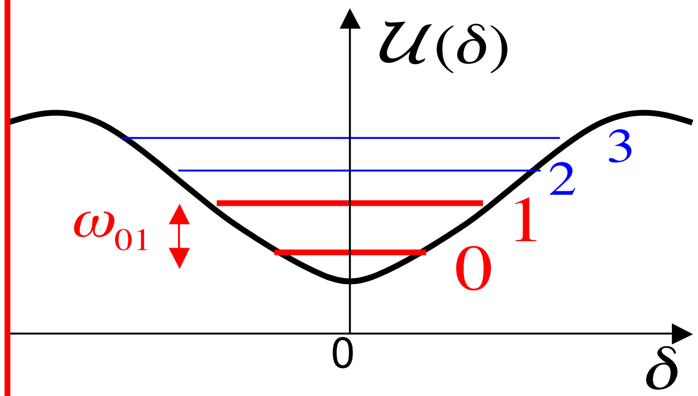
$$\boxed{H = E_C p^2 - E_J \cos\delta \quad ; \quad E_C = \frac{2e^2}{C} \quad , \quad E_J = \frac{\hbar I_0}{2e}}$$

Hamiltonian of a non-linear oscillator

Quantizing the isolated junction

The dimensionless conjugate quantities p and δ become operators (equivalent to momentum and position of a particle) satisfying:

$$[p,\delta] = iI$$



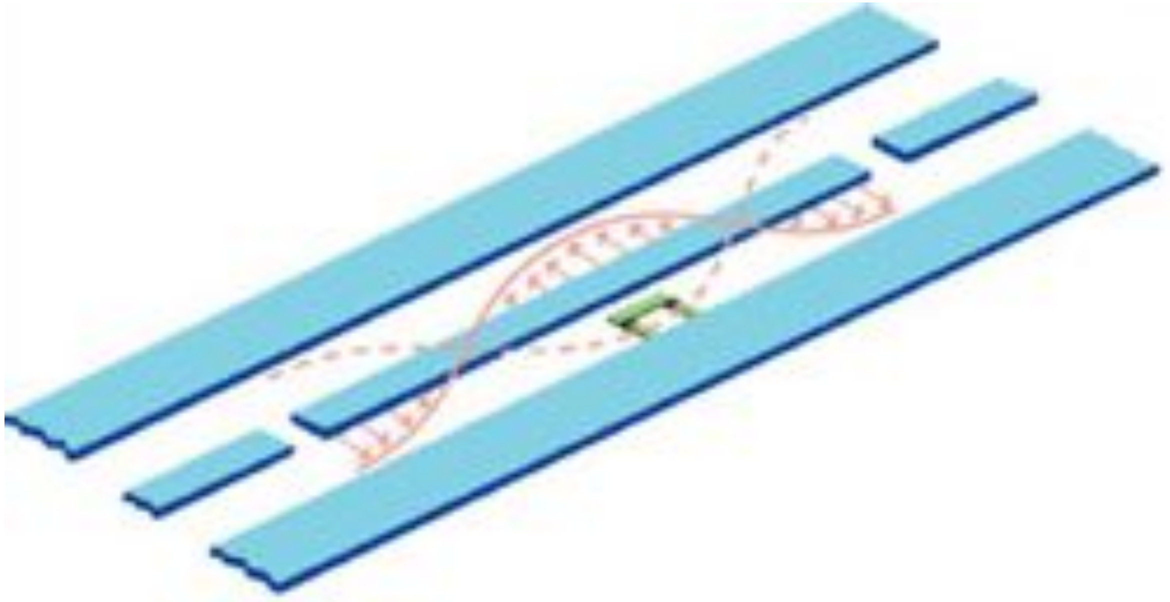
Potential of the "particle" representing the JJ in open circuit, with its eigenstates. The two lowest states 0 and 1 define a qubit.

Non-linearity because $\cos\delta \neq 1-\delta^2/2$

Departure from parabolic potential lifts degeneracy of transitions and makes it possible to isolate a two-level system (qubit)

Shape of potential can be tailored by inserting junction in various circuits: a zoo of different qubits (quantronium, transmon, flux qubit, phase qubit etc..). Control qubit frequency and potential shape by magnetic flux.

Circuit QED



Josephson junctions coupled to coaxial resonator

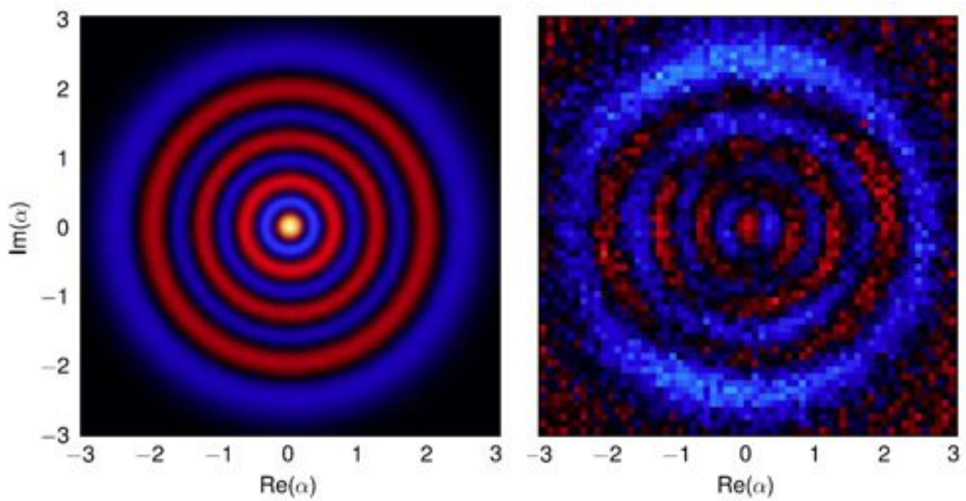
Analogous to Cavity QED with larger coupling and faster dynamics: promising for quantum information.

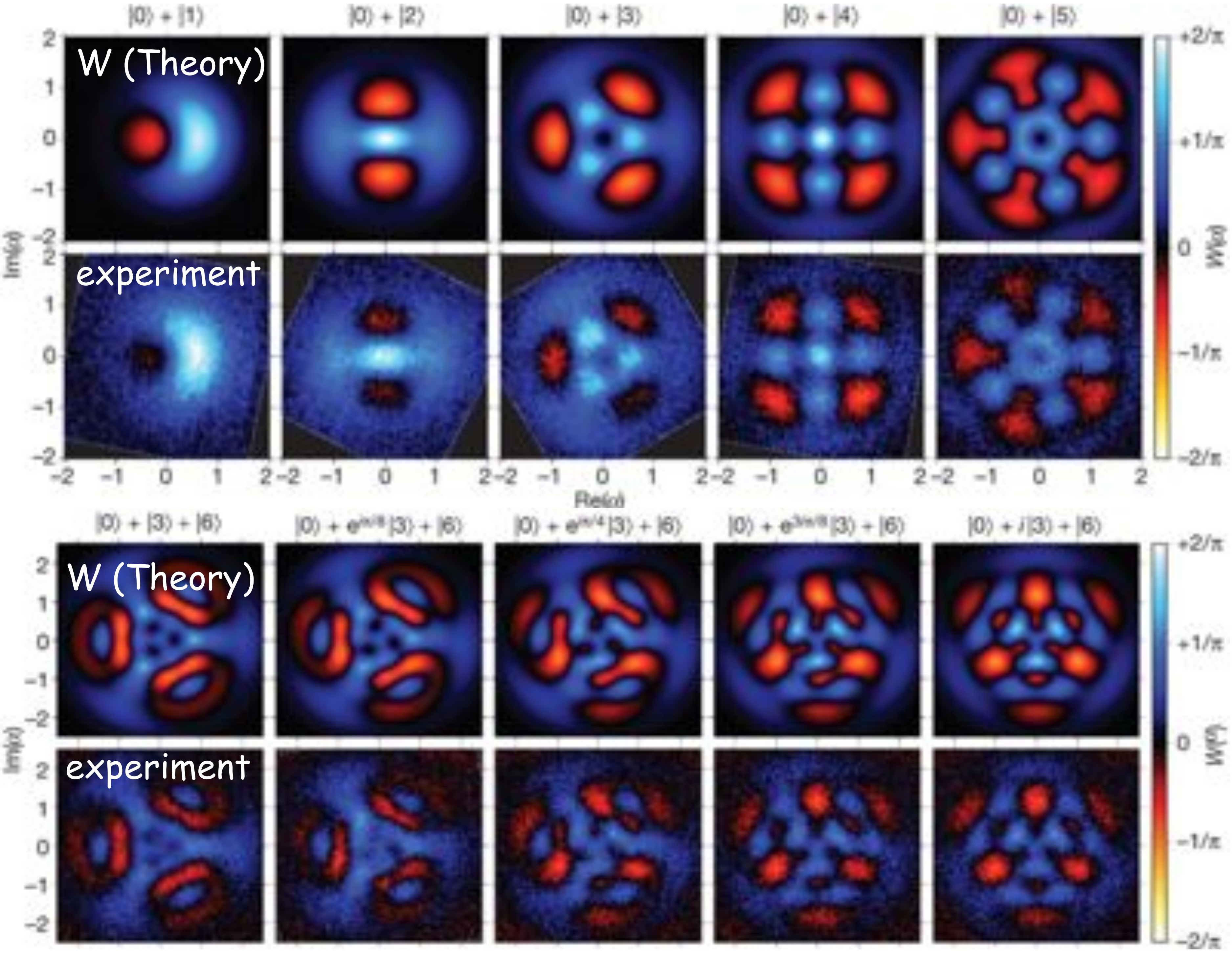
A preview of ICAP talk

A Fock state Wigner function in Circuit QED
(J.Martinis Group,USBC)

n=7

Theory Experiment





|0⟩ + |1⟩
|0⟩ + |2⟩
|0⟩ + |3⟩
|0⟩ + |4⟩
|0⟩ + |5⟩
W (Theory)
experiment
Im(α)
Re(α)
W(α)
+2/π
+1/π
0
−1/π
−2/π
|0⟩ + |3⟩ + |6⟩
|0⟩ + i|3⟩ + |6⟩
W (Theory)
experiment
r

Suggested readings

Cavity QED experiments:

J.M.Raimond, M.Brune and S.Haroche, "Colloquium: Manipulating quantum entanglement with atoms and photons in a cavity", Rev.Mod.Phys. 73, 565 (2001).

S. Gleyzes, S.Kuhr, C. Guerlin, J. Bernu, S. Deléglise, U. Busk Hoff, M. Brune, J-M.Raimond and S. Haroche, «Quantum jumps of light recording the birth and death of a photon in a cavity », Nature, 446, 297 (2007).

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S.Deléglise, I.Dotsenko, C.Sayrin, J.Bernu, M.Brune, J-M.Raimond and S.Haroche, "Reconstruction of non-classical cavity field states with snapshots of their decoherence", Nature, 455, 510 (2008).

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Circuit QED experiments:

•R.Schoelkopf & S.Girvin, Wiring-up quantum systems, Nature, **451** , 664 (2008)

•J.Clarke & F.Wilhelm, Superconducting quantum bits, Nature, **453**, 1031 (2008)

. Deterministic entanglement of photons in two superconducting microwave resonators H. Wang, Matteo Mariantoni, Radoslaw C. Bialczak, M. Lenander, Erik Lucero, M. Neeley, A. O'Connell, D. Sank, M. Weides, J. Wenner, T. Yamamoto, Y. Yin, J. Zhao, John M. Martinis, A. N. Cleland PRL 106, 060401 (2011).

. Synthesizing arbitrary quantum states in a superconducting resonator Max Hofheinz, H. Wang, M. Ansmann, Radoslaw C. Bialczak, Erik Lucero, M. Neeley, A. D. O'Connell, D. Sank, J. Wenner, John M. Martinis, A. N. Cleland Nature 459, 546-549 (2009)

. Generation of Fock states in a superconducting quantum circuit Max Hofheinz, E. M. Weig, M. Ansmann, Radoslaw C. Bialczak, Erik Lucero, M. Neeley, A. D. O'Connell, H. Wang, John M. Martinis, A. N. Cleland Nature 454, 310-314 (2008).

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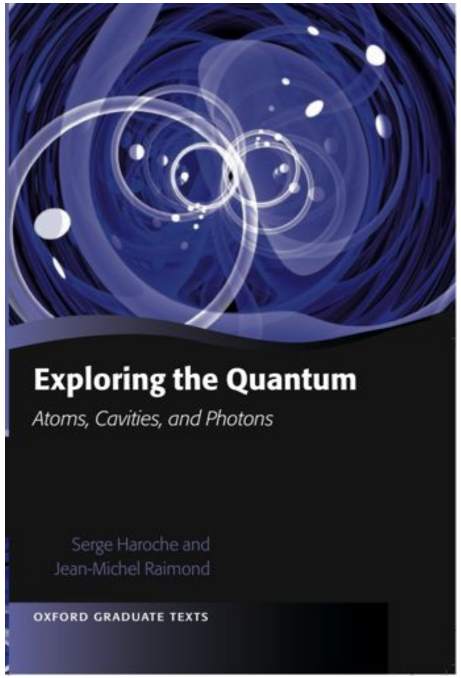
B.Peaudecerf
T.Rybarczyk



Exploring the Quantum
Atoms, cavities and Photons

S.Haroche and J-M.Raimond

Oxford University Press