ICAP Summer School, Paris, 2012

Three lectures on quantum gases

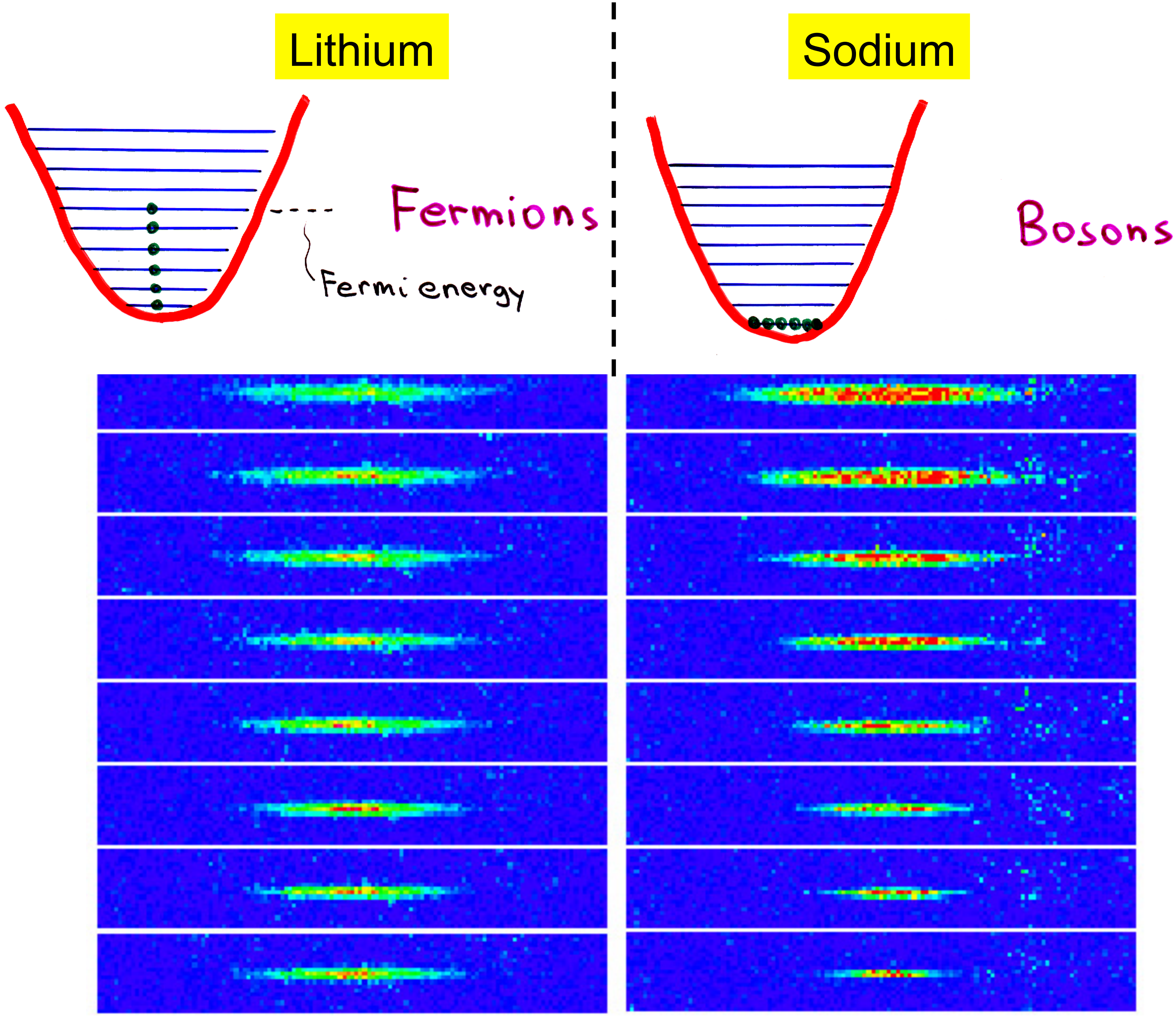
Wolfgang Ketterle, MIT

Cold fermions

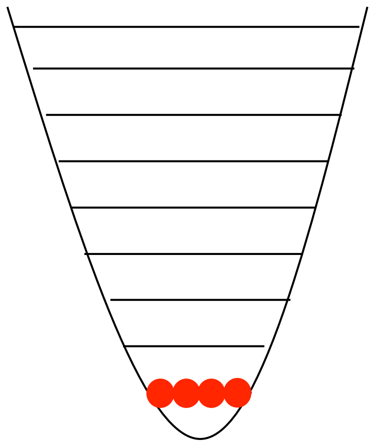
Reference for most of this talk:

W. Ketterle and M. W. Zwierlein:
Making, probing and understanding ultracold Fermi gases.
in *Ultracold Fermi Gases* , Proceedings of the International School of Physics "Enrico Fermi", Course CLXIV, Varenna, 20 - 30 June 2006, edited by M. Inguscio, W. Ketterle, and C. Salomon (IOS Press, Amsterdam) 2008, pp. 95-287; e-print, arXiv: 0801.2500; Rivista del Nuovo Cimento **31** , 247-422 (2008).

Li Na cooling movie

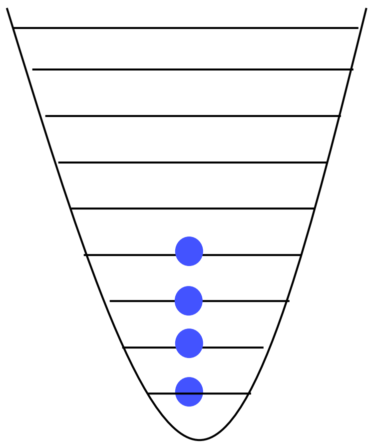


At absolute zero temperature …



Bosons
Particles with an even number of protons, neutrons and electrons

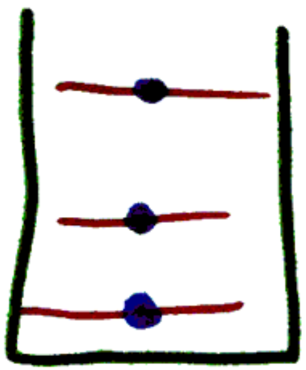
Bose-Einstein condensation
$\Rightarrow$ atoms as waves
$\Rightarrow$ superfluidity



Fermions
Particles with an odd number of protons, neutrons and electrons

Fermi sea:
$\Rightarrow$ Atoms are not coherent
$\Rightarrow$ No superfluidity

Fermions in a box

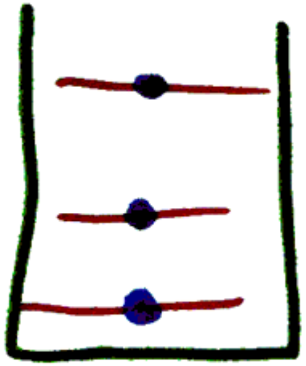


$$p_F = \hbar\left(6\pi^2 n\right)^{1/3}$$

$$E_F = p_F^2/2m$$

$$n = \left(\frac{E_F}{2m}\right)^{3/2}\frac{1}{6\pi^2\hbar^3}$$

Fermions in a box



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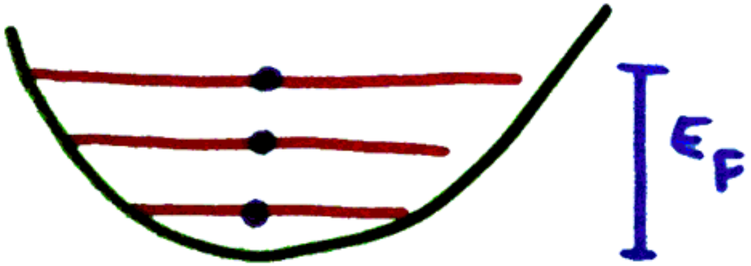
Fermions in an HO

$$E_F = (6N)^{1/3}\hbar\omega$$

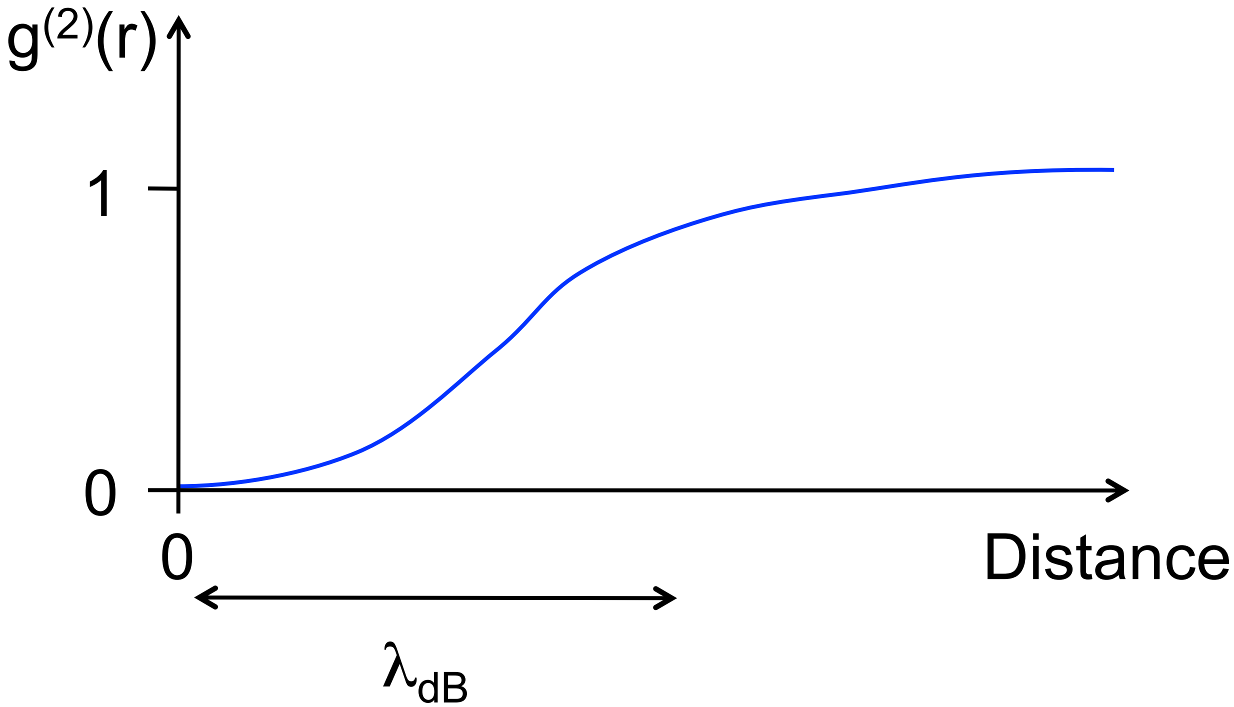


$$n(r) = \left(\frac{E_F - V(r)}{2m}\right)^{3/2}\frac{1}{6\pi^2\hbar^3}$$

local density approximation

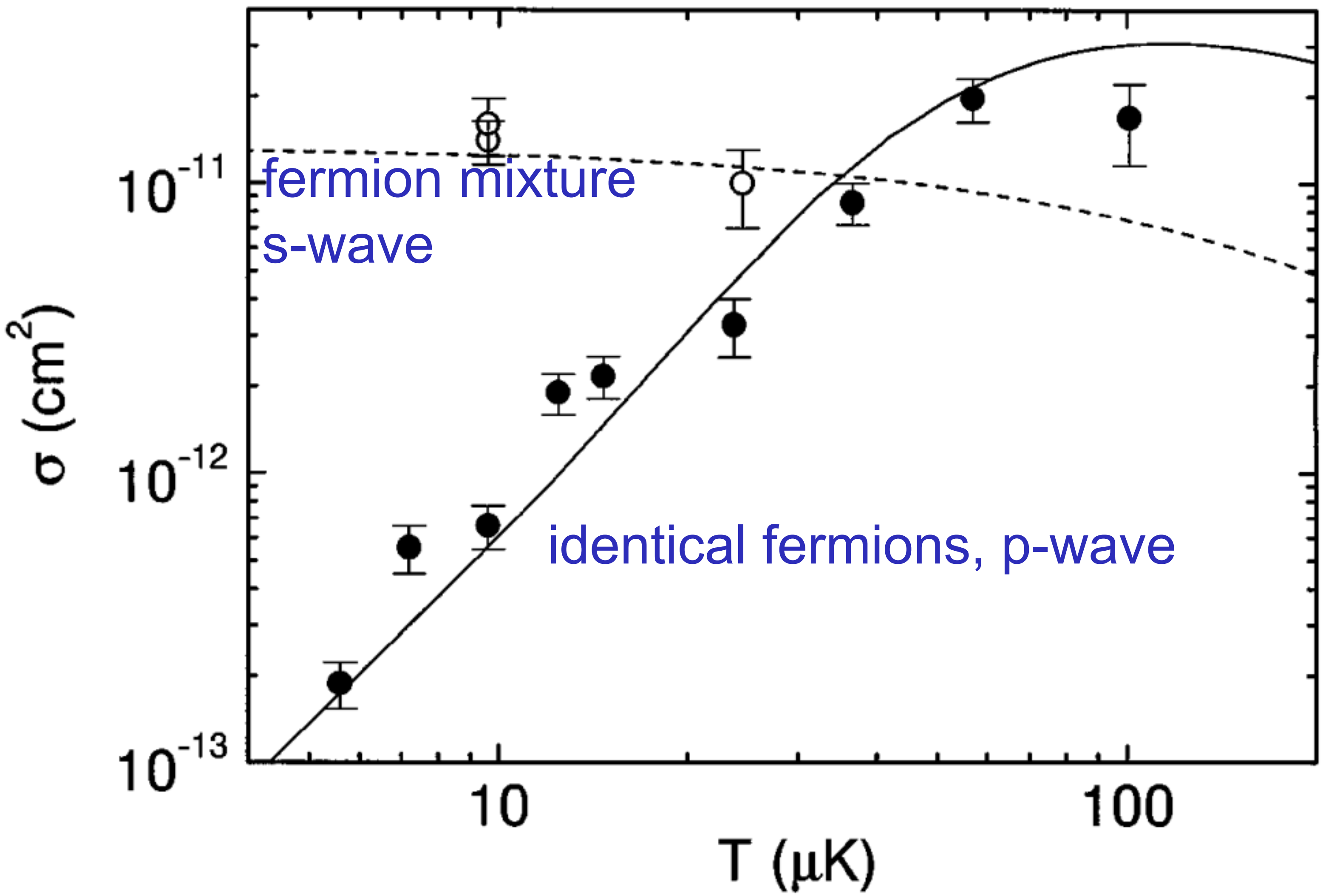
Freezing out of collisions

Pair correlations in a Fermi gas:

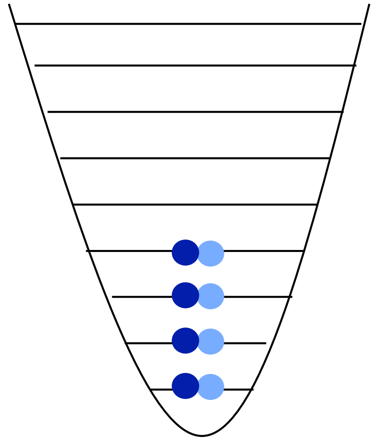


No interactions if range of potential is < λ_{dB}

Elastic collisions suppressed below T_{pwave}

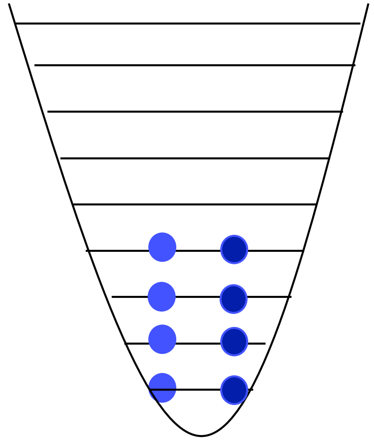


Elastic cross section for K-40 (Jin group, PRL 1999)



Pairs of fermions

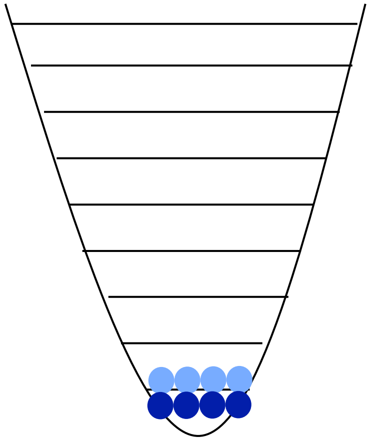
Particles with an even number of protons, neutrons and electrons



Two kinds of fermions

Fermi sea:
⇒ Atoms are not coherent
⇒ No superfluidity

At absolute zero temperature …



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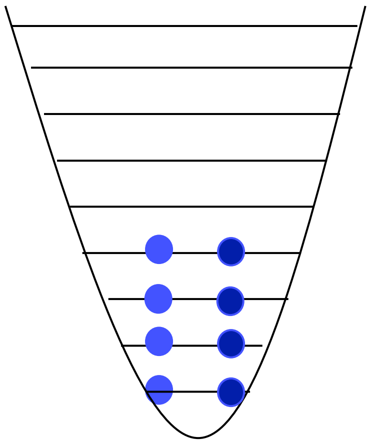
⇒ atoms as waves
⇒ superfluidity



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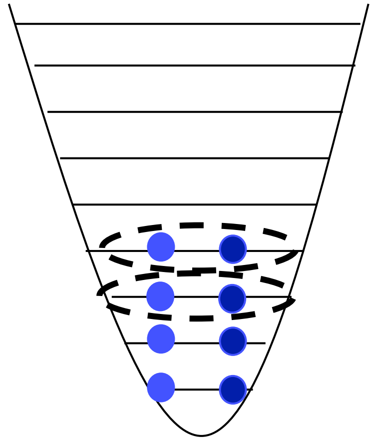
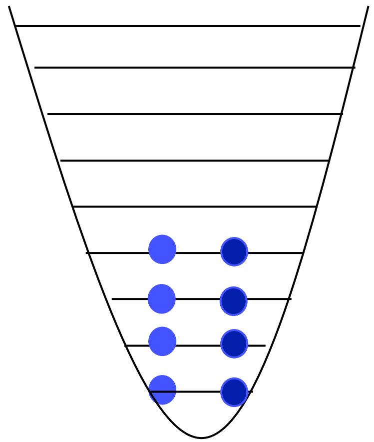


Weak attractive interactions

Cooper pairs
larger than interatomic distance
momentum correlations
$\Rightarrow$ BCS superfluidity



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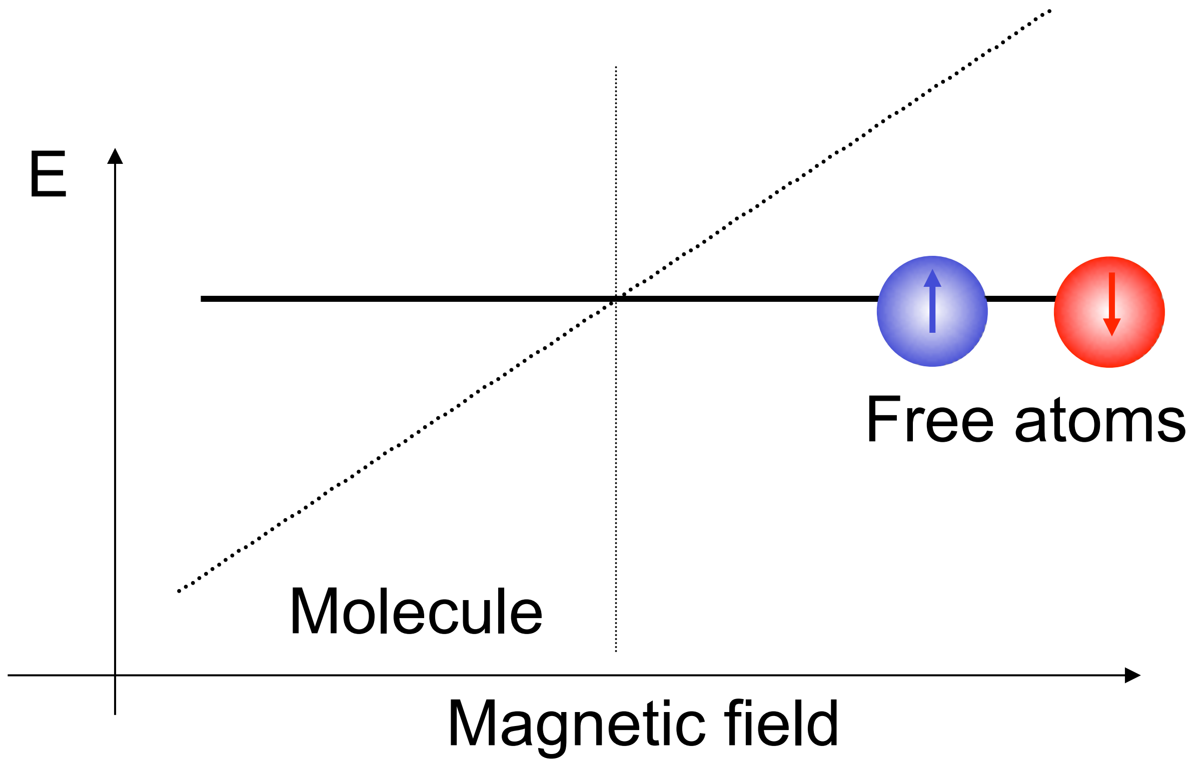
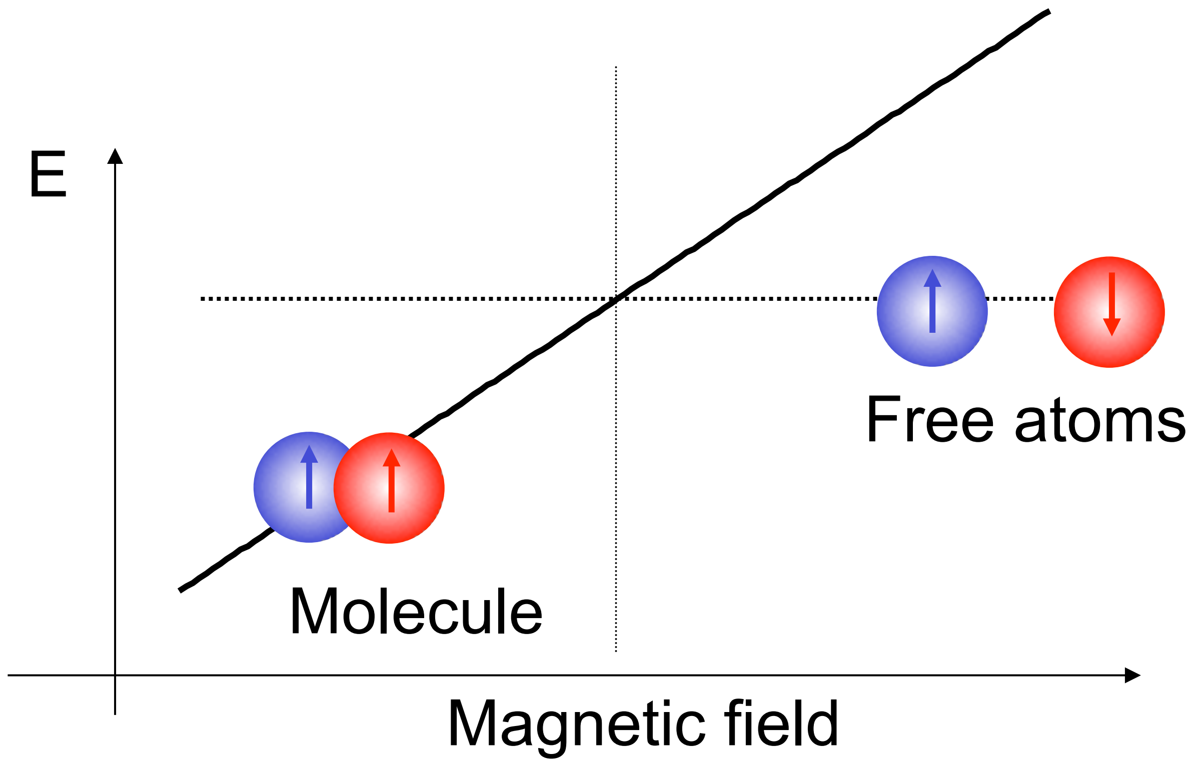


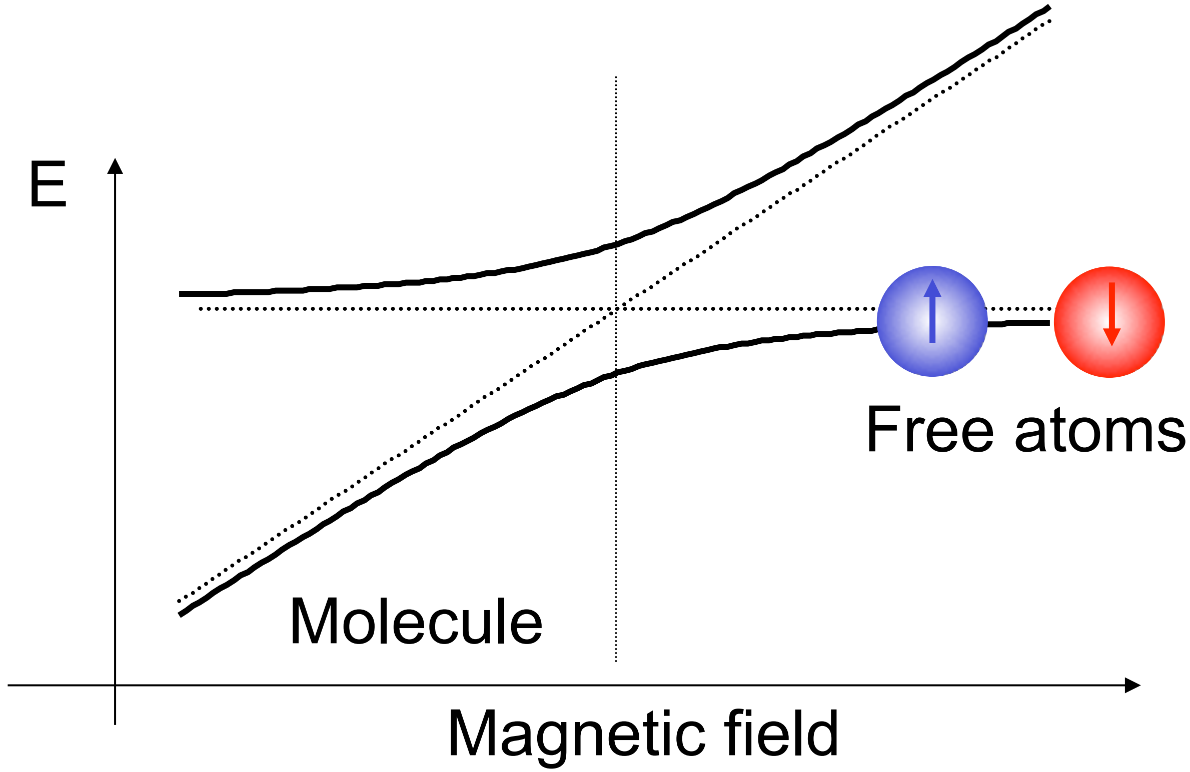
Feshbach resonance

Disclaimer: Drawing is schematic and does not distinguish nuclear and electron spin.



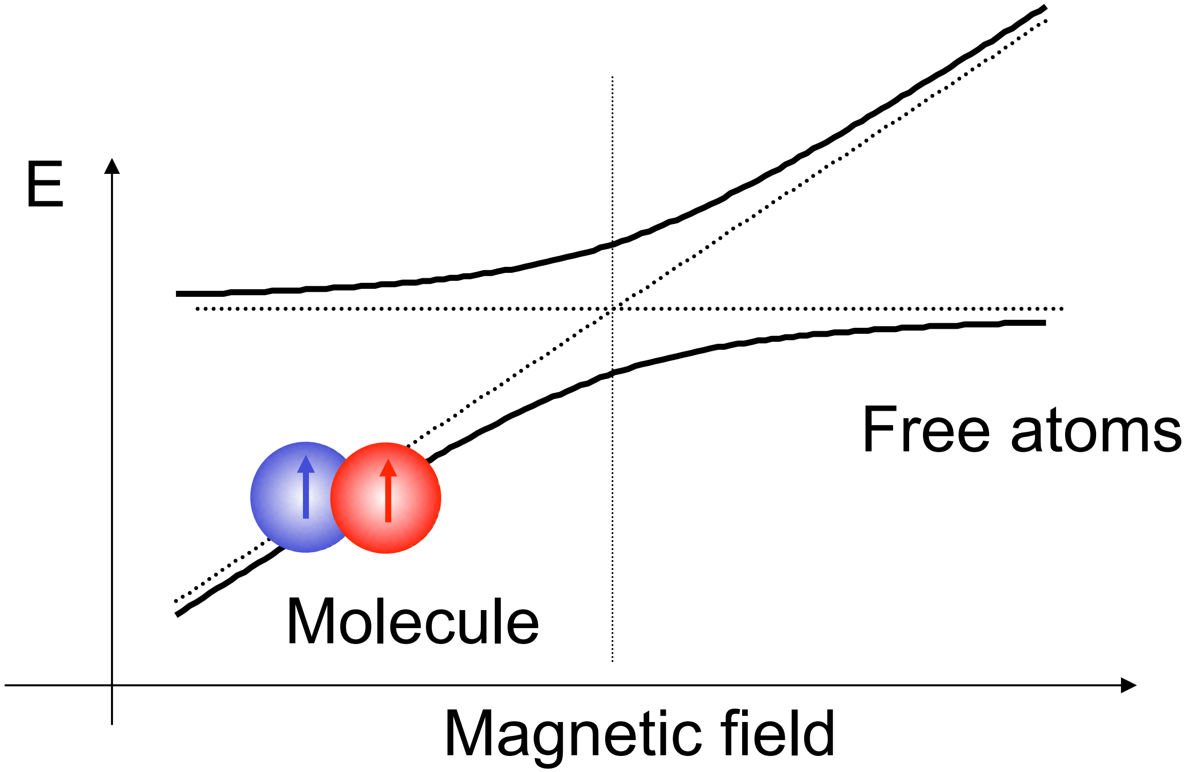
Feshbach resonance

Two atoms ….



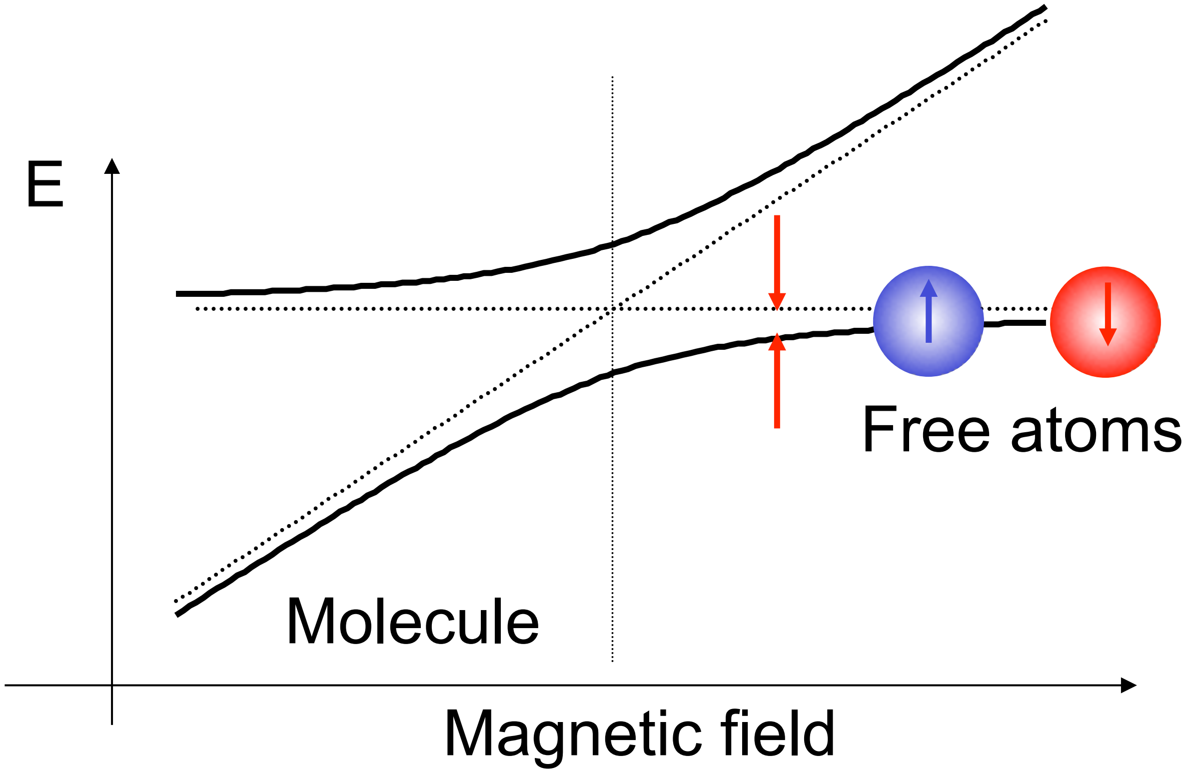
Feshbach resonance

… form a stable molecule



Feshbach resonance

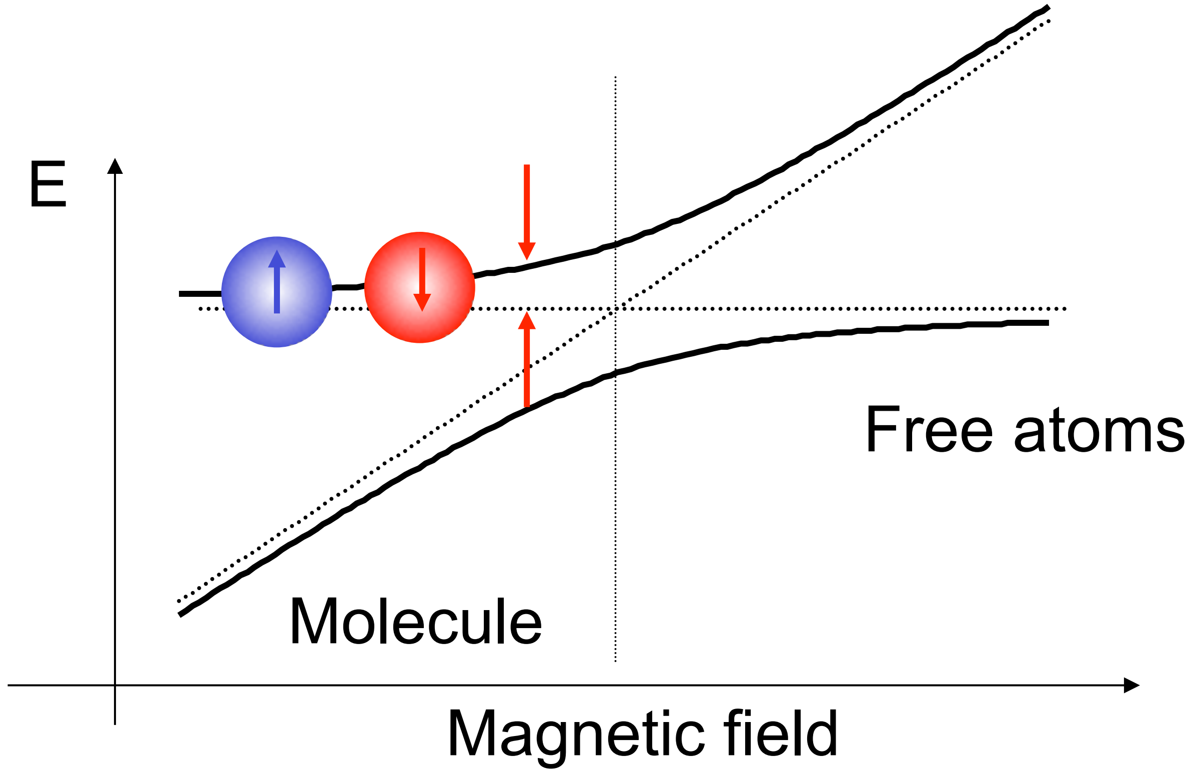
Atoms attract each other



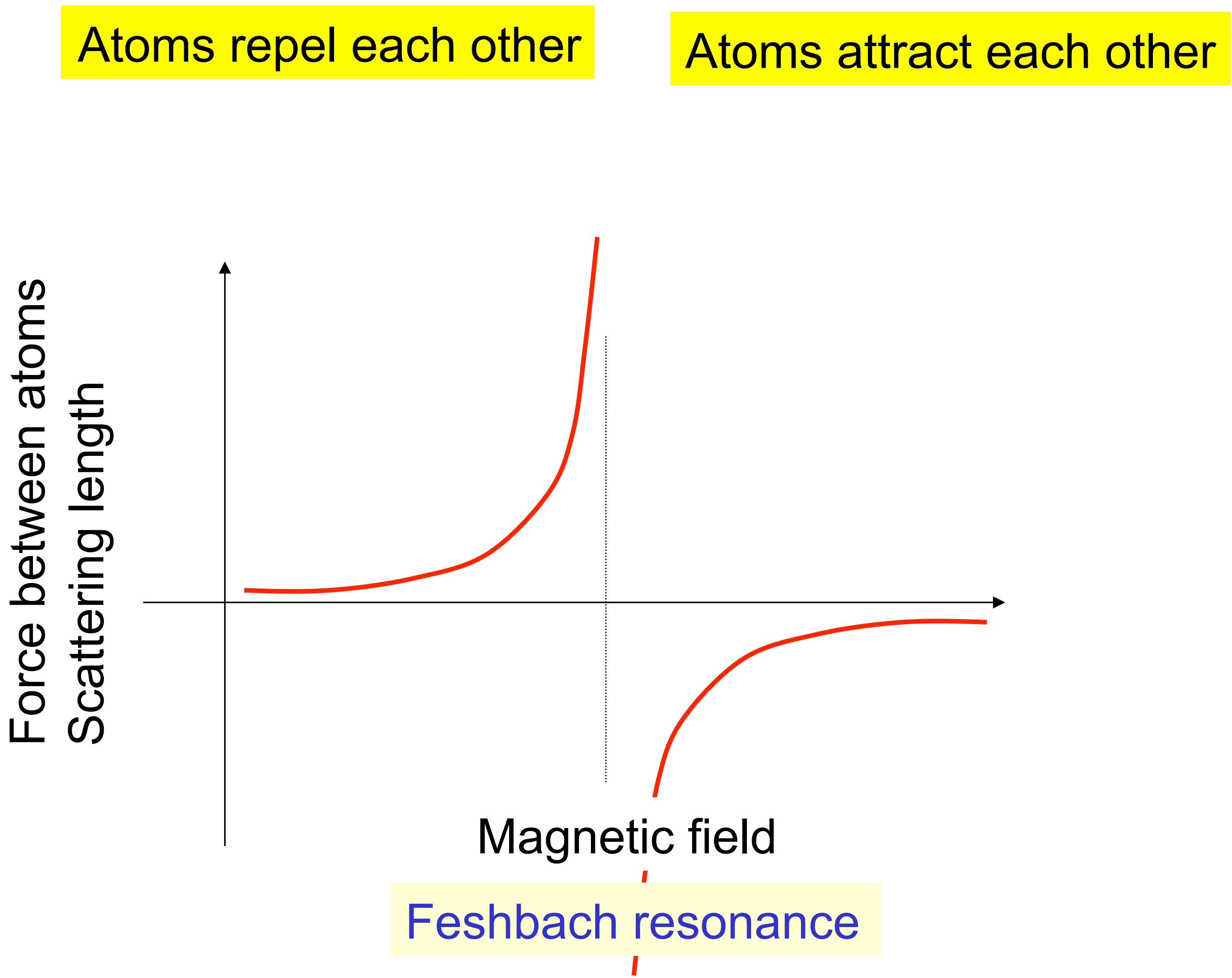
Feshbach resonance

Atoms repel each other

Atoms attract each other

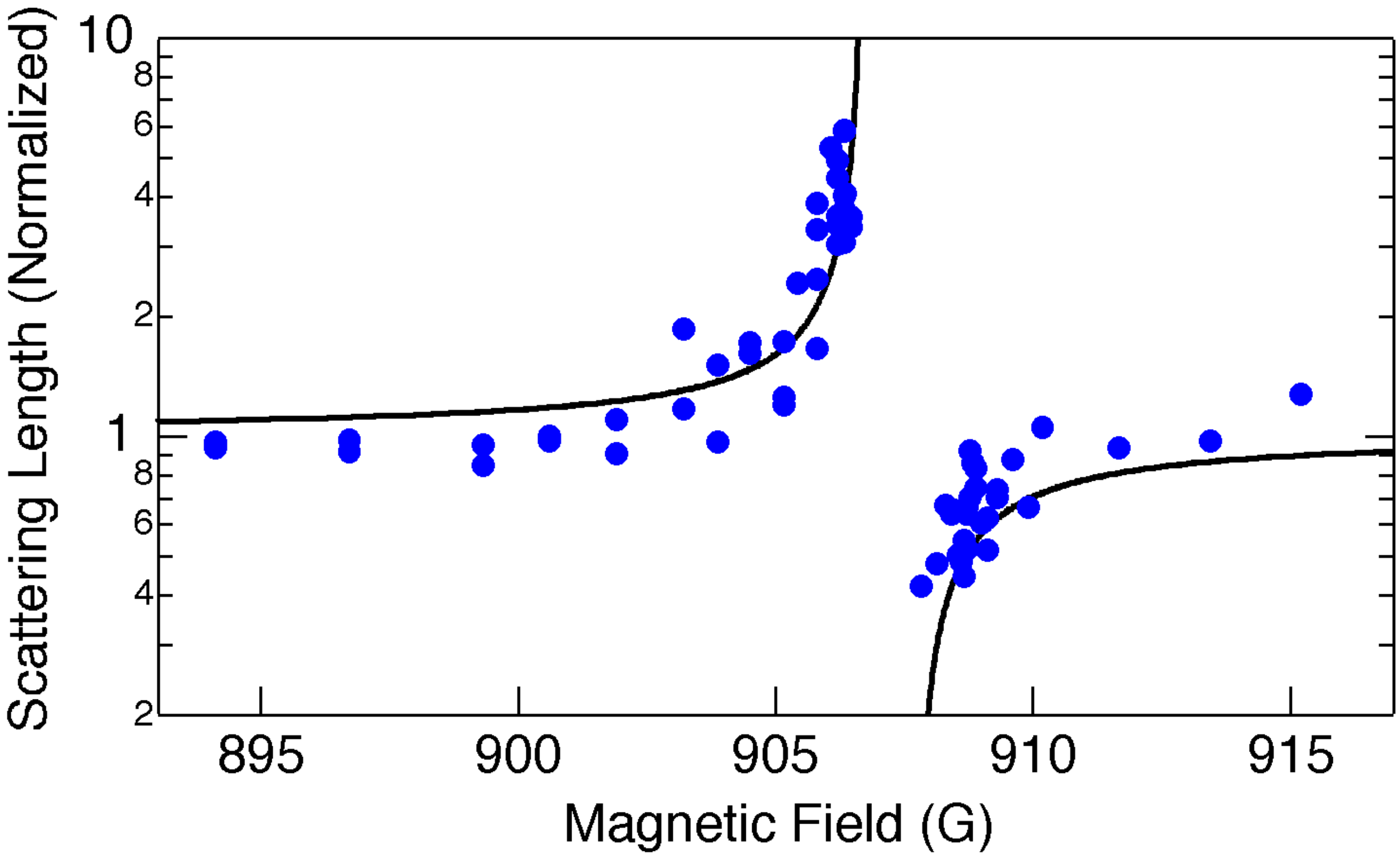


Feshbach resonance

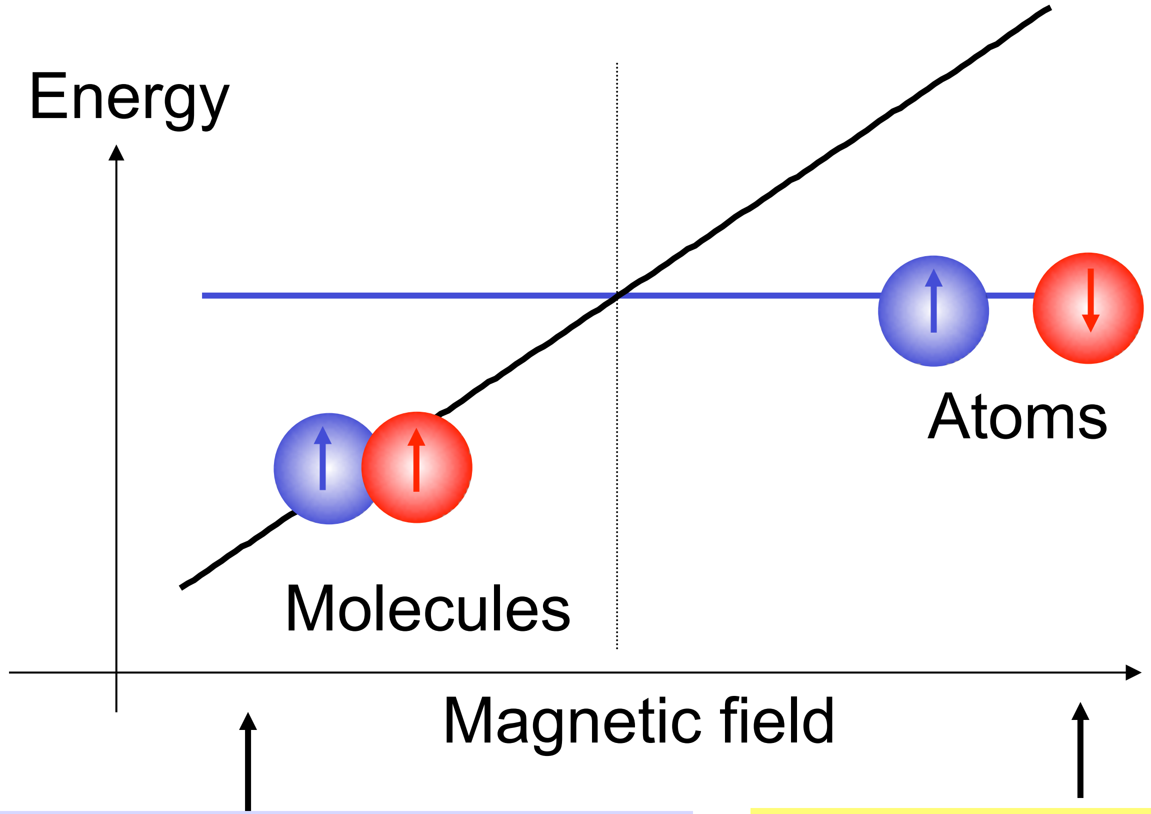

Atoms repel each other
Atoms attract each other
Force between atoms
Scattering length
Magnetic field
Feshbach resonance

Feshbach loss and a (JPEG)

Observation of a Feshbach resonance



S. Inouye, M.R. Andrews, J. Stenger, H.-J. Miesner, D.M. Stamper-Kurn, WK, Nature **392** (1998).



Atoms form stable molecules

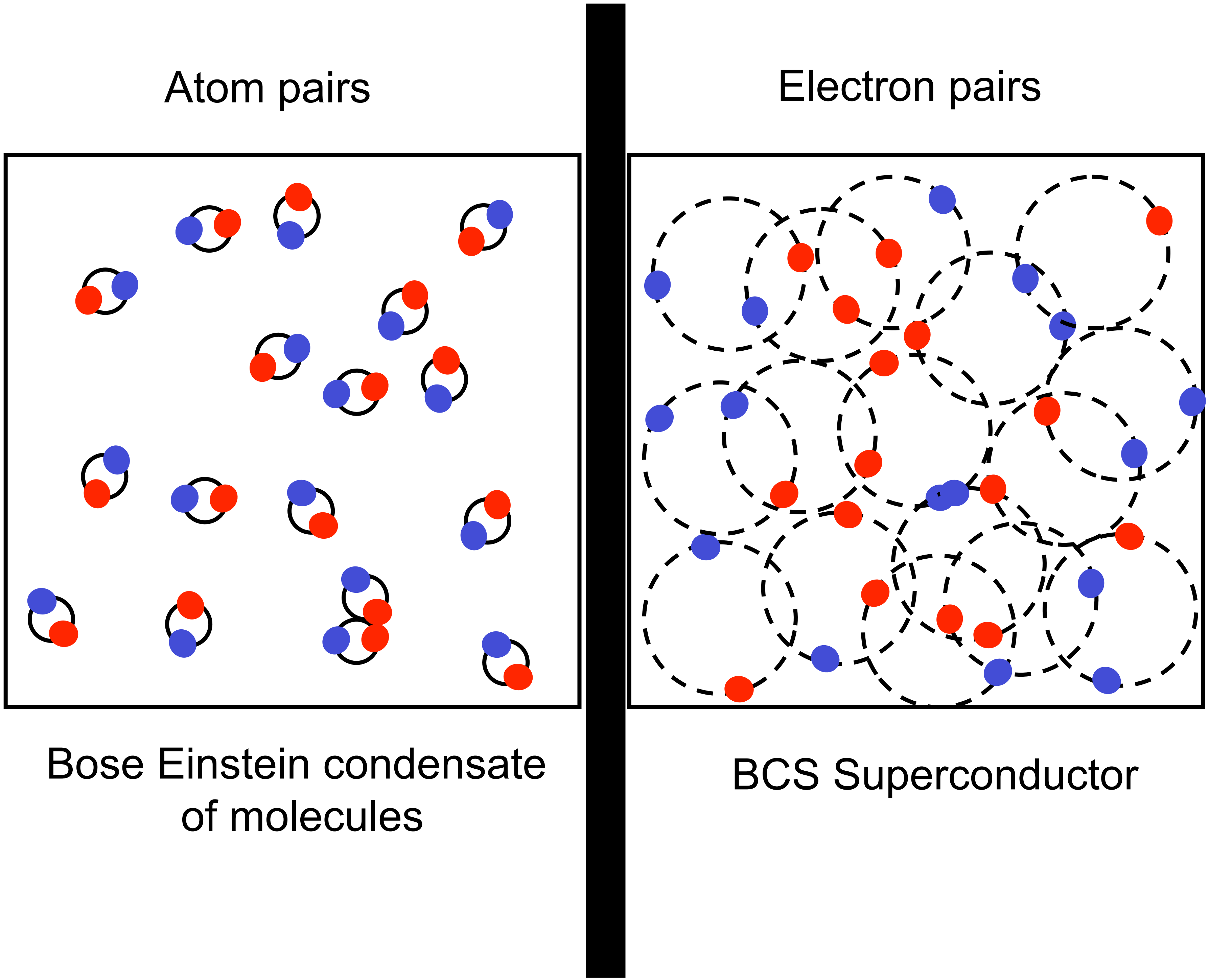
Atoms repel each other
a>0

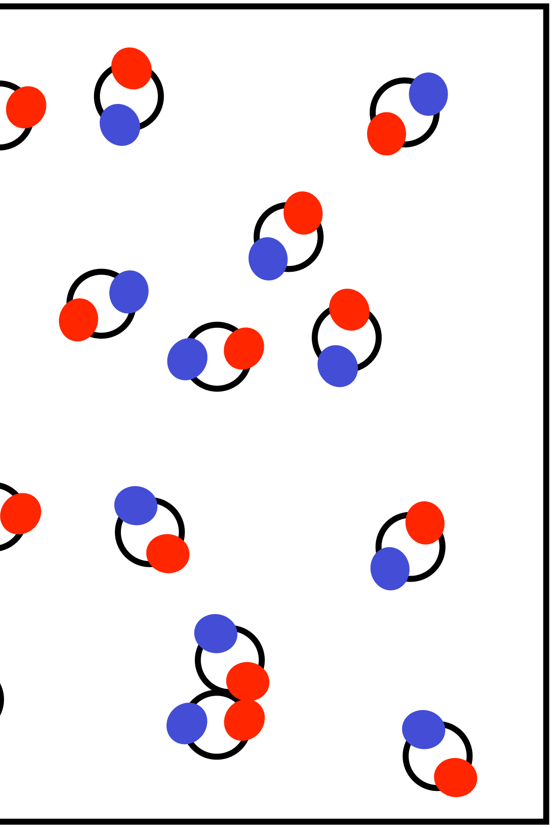
BEC of Molecules:
Condensation of
tightly bound fermion pairs

Molecules are unstable

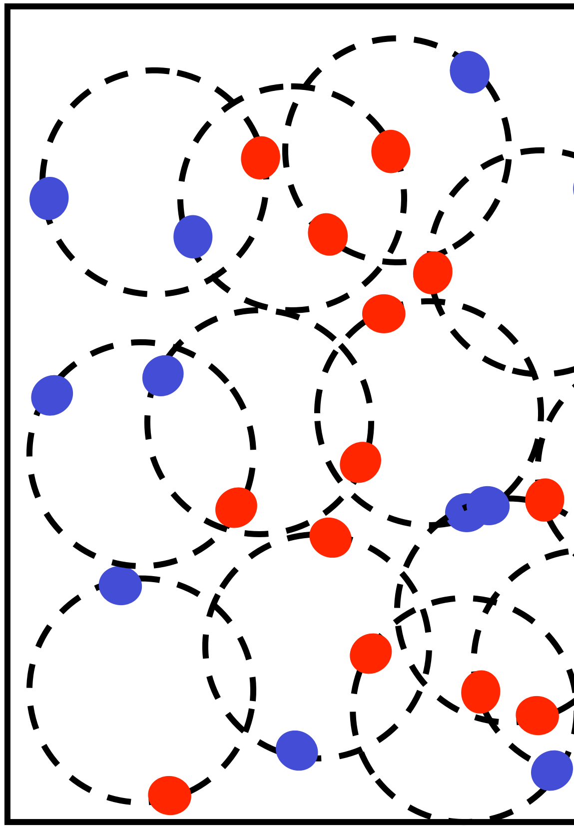
Atoms attract each other
a<0

BCS-limit:
Condensation of
long-range Cooper pairs

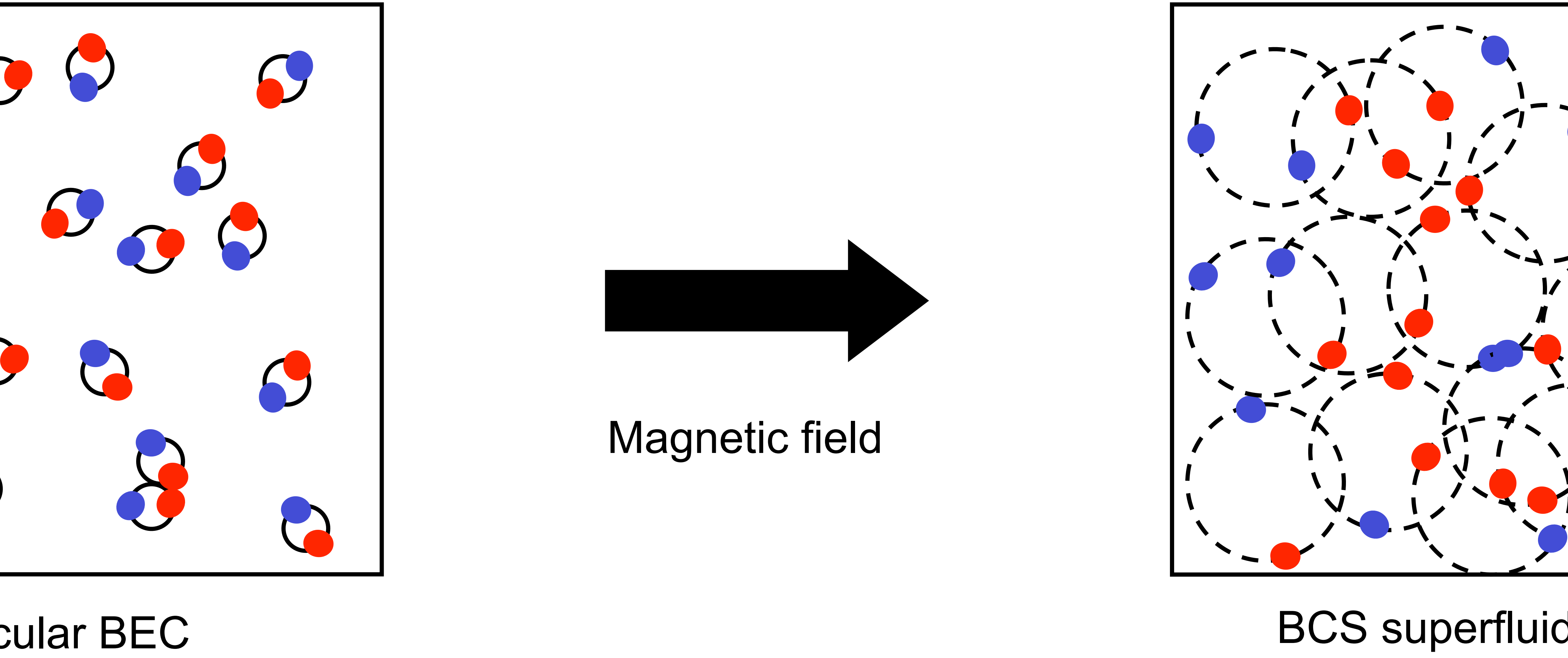

Atom pairs
Electron pairs
Bose Einstein condensate of molecules
BCS Superconductor

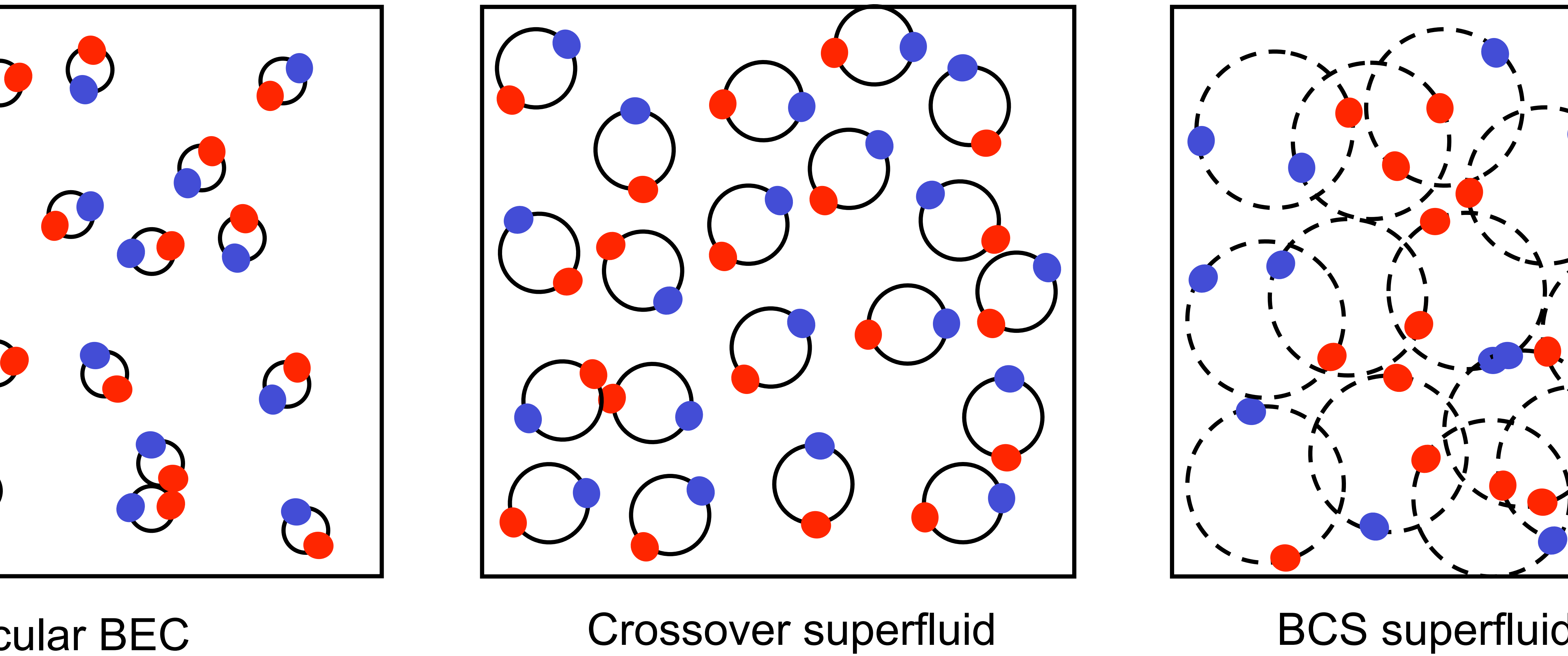


cular BEC



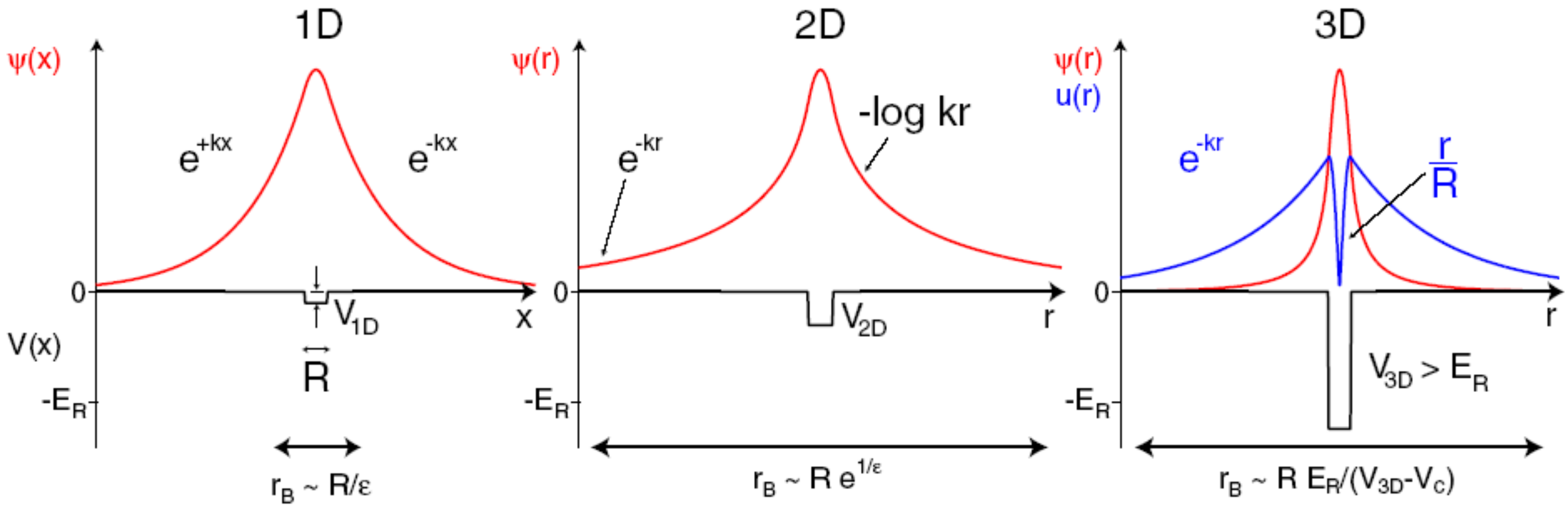
BCS superfluid


cular BEC
Magnetic field
BCS superflui


cular BEC
Crossover superfluid
BCS superfluid

How do atoms pair?

Two-body bound states in 1D, 2D, and 3D



1D, 2D: bound state for arbitrarily small attractive well

3D: Well depth has be larger than threshold

Connection to the density of states

$$\frac{\hbar^2}{m}(\nabla^2 - k^2)\psi = V\psi$$

In momentum space

$$\psi_{\mathbf{k}}(\mathbf{q}) = -\frac{m}{\hbar^2}\frac{1}{q^2+k^2}\int \frac{d^n q'}{(2\pi)^n} V(\mathbf{q}-\mathbf{q}')\psi_{\mathbf{k}}(\mathbf{q}')$$

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Short range potential: $V(q)=V_0$ for $q<1/R$

$$\psi_{\mathbf{k}}(\mathbf{q}) = -\frac{mV_0}{\hbar^2}\frac{1}{q^2+k^2}\int_{q'<\frac{1}{R}} \frac{d^n q'}{(2\pi)^n}\psi_{\mathbf{k}}(\mathbf{q}')$$

Integrate over q, divide by common factor $\int_{q<\frac{1}{R}} \frac{d^n q}{(2\pi)^n}\psi_{\mathbf{k}}(\mathbf{q})$.

$$-\frac{1}{V_0} = \frac{m}{\hbar^2}\int_{q<\frac{1}{R}} \frac{d^n q}{(2\pi)^n}\frac{1}{q^2+k^2} = \frac{1}{\Omega}\int_{\epsilon<E_R} d\epsilon \frac{\rho_n(\epsilon)}{2\epsilon+|E|}$$

Bound state for arbitrarily small V_0 only if integral diverges for $E \rightarrow 0$

$$-\frac{1}{V_0} = \frac{m}{\hbar^2}\int_{q<\frac{1}{R}} \frac{d^n q}{(2\pi)^n}\frac{1}{q^2+k^2} = \frac{1}{\Omega}\int_{\epsilon<E_R} d\epsilon \frac{\rho_n(\epsilon)}{2\epsilon+|E|}$$

Bound state for arbitrarily small V_0 only if integral diverges for $E \rightarrow 0$

In 2D (constant density of states): logarithmic divergence

$$E_{2\mathrm{D}} = -2E_R\, e^{-\frac{2\Omega}{\rho_{2\mathrm{D}}|V_0|}}$$

The Cooper problem:

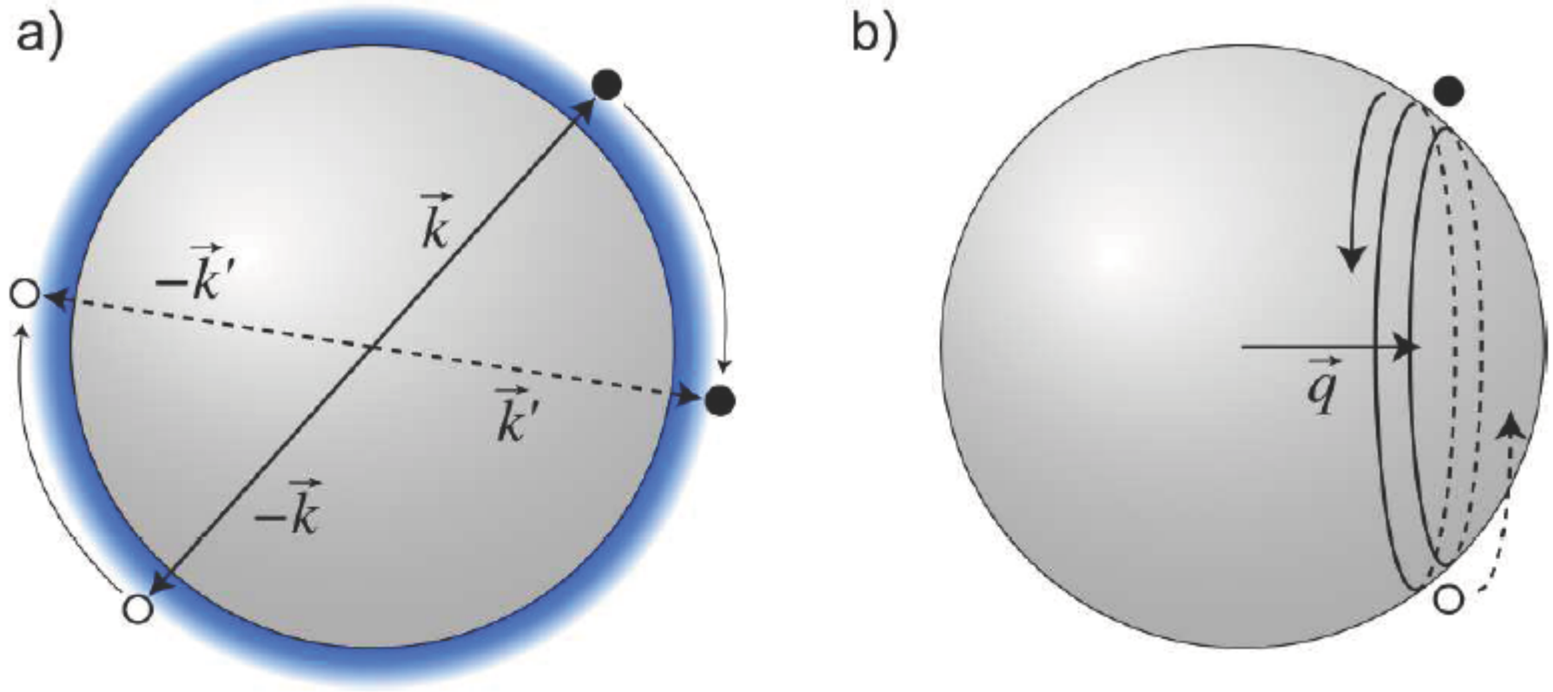
Bound Electron Pairs in a Degenerate Fermi Gas*

LEON N. COOPER

Physics Department, University of Illinois, Urbana, Illinois

(Received September 21, 1956)

Two fermions with weak interactions on top of a filled Fermi sea



Total momentum zero

Total momentum non-zero 2q

search for a small binding energy $E_B = E - 2E_F < 0$

$$-\frac{1}{V_0} = \frac{1}{\Omega}\int_{E_F<\epsilon<E_F+E_R} d\epsilon \frac{\rho_{3\mathrm{D}}(\epsilon)}{2(\epsilon - E_F) + |E_B|}$$

Pauli blocking

$$E_B = -2\, E_R\; e^{-2\Omega/\rho_{3\mathrm{D}}(E_F)|V_0|}$$

Compare with previous result for single particle bound state

$$-\frac{1}{V_0} = \frac{m}{\hbar^2}\int_{q<\frac{1}{R}} \frac{d^n q}{(2\pi)^n}\frac{1}{q^2+k^2} = \frac{1}{\Omega}\int_{\epsilon<E_R} d\epsilon \frac{\rho_n(\epsilon)}{2\epsilon + |E|}$$

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$$-\frac{1}{V_0} = \frac{1}{\Omega} \int_{E_F<\epsilon<E_F+E_R} d\epsilon \frac{\rho_{3\mathrm{D}}(\epsilon)}{2(\epsilon - E_F) + |E_B|}$$

Pauli blocking

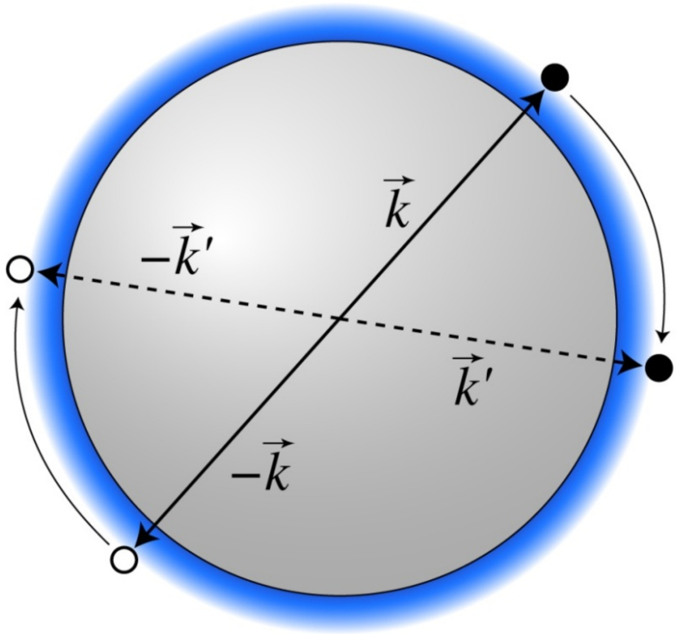
$$E_B = -2\, E_R \, e^{-2\Omega/\rho_{3\mathrm{D}}(E_F)|V_0|}$$

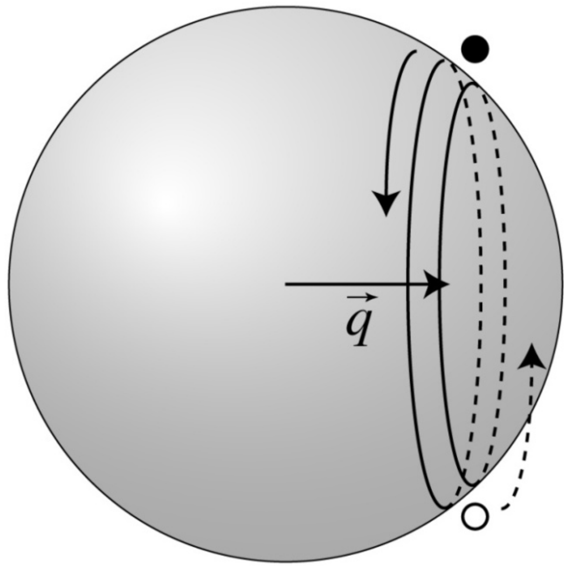
After replacing the bare interaction V_0 by the scattering length a

$$E_B = -\frac{8}{e^2} E_F \, e^{-\pi/k_F|a|}$$

Cooper Pairing

Consider two particles ↑, ↓, on top of a filled, “inert” Fermi sea





Total momentum zero

Total momentum non-zero
- Reduced density of states
- Much smaller binding energy

The important pairs are those with zero momentum

BCS Wavefunction

How can we find a state in which all fermions are paired in a self-consistent way?

John Bardeen

Leon N. Cooper

John R. Schrieffer

BCS Wavefunction

- Many-body wavefunction for a condensate of Fermion Pairs:

$$\Psi\left(\mathbf{r}_1,\ldots,\mathbf{r}_N\right) = \varphi(|\mathbf{r}_1-\mathbf{r}_2|)\chi_{12}\ldots\varphi(|\mathbf{r}_{N-1}-\mathbf{r}_N|)\chi_{N-1,N}$$

Spatial pair wavefunction

Spin wavefunction

$$\chi_{ij} = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_i\left|\downarrow\right\rangle_j - \left|\downarrow\right\rangle_i\left|\uparrow\right\rangle_j\right)$$

- Second quantization:

$$\left|\Psi\right\rangle_N = \int\prod_i d^3r_i\,\varphi(\mathbf{r}_1-\mathbf{r}_2)\Psi_\uparrow^\dagger(\mathbf{r}_1)\Psi_\downarrow^\dagger(\mathbf{r}_2)\ldots\varphi(\mathbf{r}_{N-1}-\mathbf{r}_N)\Psi_\uparrow^\dagger(\mathbf{r}_{N-1})\Psi_\downarrow^\dagger(\mathbf{r}_N)\left|0\right\rangle$$

- Fourier transform: Pair wavefunction: $\varphi(\mathbf{r}) = \sum_k \varphi_k \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$

  Operators: $\Psi_\sigma^\dagger(\mathbf{r}) = \sum_k c_{k\sigma}^\dagger \frac{e^{-i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$

- Pair creation operator: $b^\dagger = \sum \varphi_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger$

- Many-body wavefunction: $\left|\Psi\right\rangle_N = {b^\dagger}^{N/2}\left|0\right\rangle$

  a fermion pair condensate

$|\Psi\rangle_N$ is not a Bose condensate

$$|\Psi\rangle_N = {b^\dagger}^{N/2} |0\rangle$$

- Commutation relations for pair creation/annihilation operators

$$[b^\dagger, b^\dagger]_- = \sum_{kk'} \varphi_k \varphi_{k'} \left[c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}, c^\dagger_{k'\uparrow} c^\dagger_{-k'\downarrow}\right]_- = 0 \quad ✓$$

$$[b, b]_- = \cdots = 0 \quad ✓$$

$$[b, b^\dagger]_- = \cdots = \sum_k |\varphi_k|^2 (1 - n_{k\uparrow} - n_{k\downarrow}) \neq 1 \quad ✗$$

Occupation of momentum k

- pairs do not obey Bose commutation relations, *unless* $n_k \ll 1$

$$[b, b^\dagger]_- \approx \sum_k |\varphi_k|^2 = 1$$

BEC limit of tightly bound molecules

BCS Wavefunction

- Introduce coherent state / switch to grand-canonical description:

$$\mathcal{N}\,|\Psi\rangle \;=\sum_{J_{\text{even}}}\frac{N_p^{J/4}}{(J/2)!}\,|\Psi\rangle_J \;=\sum_M\frac{1}{M!}N_p^{M/2}\,{b^\dagger}^M\,|0\rangle$$

$$= e^{\sqrt{N_p}\,b^\dagger}\,|0\rangle$$

$$= \prod_k e^{\sqrt{N_p}\,\varphi_k c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}}\,|0\rangle \qquad c^\dagger_k \text{ and } c^\dagger_{k'} \text{ commute}$$

$$= \prod_k\left(1+\sqrt{N_p}\,\varphi_k\,c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\right)|0\rangle \quad \text{because } c_k^{\dagger 2}=0$$

- Normalization: $\mathcal{N}=\prod_k\frac{1}{u_k}=\prod_k\sqrt{1+N_p|\varphi_k|^2}$

- BCS wavefunction: $\boxed{|\Psi_{\text{BCS}}\rangle=\prod_k\left(u_k+v_k c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\right)|0\rangle}$

with $v_k=\sqrt{N_p}\,\varphi_k u_k$ and $|u_k|^2+|v_k|^2=1$

BCS Wavefunction

$$|\Psi\rangle_N = {b^\dagger}^{N/2}\,|0\rangle$$

and

$$|\Psi_{\mathrm{BCS}}\rangle = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})\,|0\rangle$$

are identical!

Many-Body Hamiltonian

- Second quantized Hamiltonian for interacting fermions:

$$\hat{H} = \sum_\sigma \int \mathrm{d}^3 r \hat{\Psi}^\dagger_\sigma(\mathbf{r}) \left(-\frac{\hbar^2\nabla^2}{2m}\right) \hat{\Psi}_\sigma(\mathbf{r}) + \int \mathrm{d}^3 r \int \mathrm{d}^3 r' \hat{\Psi}^\dagger_\uparrow(\mathbf{r}) \hat{\Psi}^\dagger_\downarrow(\mathbf{r}') V(\mathbf{r}-\mathbf{r}') \hat{\Psi}_\downarrow(\mathbf{r}') \hat{\Psi}_\uparrow(\mathbf{r})$$

- Contact interaction: $V(\mathbf{r}) = V_0 \delta(\mathbf{r})$
- Fourier transform via $\Psi^\dagger_\sigma(\mathbf{r}) = \sum_k c^\dagger_{k\sigma} \frac{e^{-i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$

$$\hat{H} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{V_0}{\Omega} \sum_{k,k',q} c^\dagger_{k+\frac{q}{2}\uparrow} c^\dagger_{-k+\frac{q}{2}\downarrow} c_{k'+\frac{q}{2}\downarrow} c_{-k'+\frac{q}{2}\uparrow}$$

- BCS Approximation:

  Only include scattering between zero-momentum pairs

$$\hat{H} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{V_0}{\Omega} \sum_{k,k'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{k'\downarrow} c_{-k'\uparrow}$$

- Solve via 1) Variational Ansatz, 2) via Bogoliubov transformation

Variational Ansatz:

- Insert BCS wavefunction into Many-Body Hamiltonian.
- Minimize Free Energy:

$$\mathcal{F} = \left\langle \hat{H} - \mu \hat{N} \right\rangle = \sum_k 2\xi_k v_k^2 + \frac{V_0}{\Omega} \sum_{k,k'} u_k v_k u_{k'} v_{k'}$$

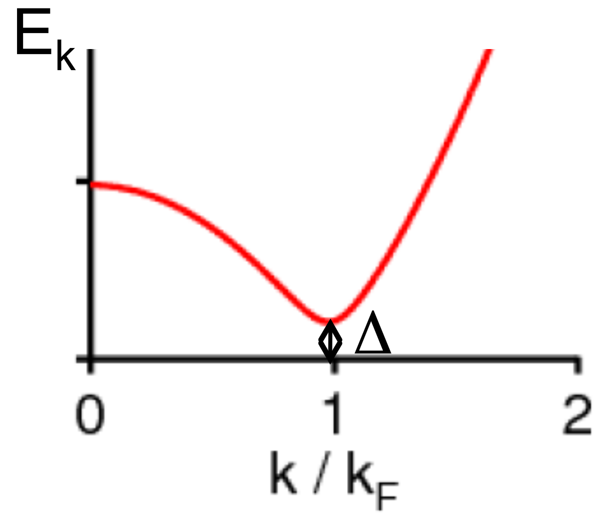
$$\text{with} \quad \xi_k = \epsilon_k - \mu$$

- Result:

$$v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$$

$$u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$$

$$\text{with} \quad E_k = \sqrt{\xi_k^2 + \Delta^2}$$



- Gap equation:

$$\Delta = -\frac{V_0}{\Omega} \sum_k u_k v_k = -\frac{V_0}{\Omega} \sum_k \frac{\Delta}{2E_k}$$

Solution via Bogoliubov Transform

- BCS Hamiltonian is quartic:

$$\hat{H} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{V_0}{\Omega} \sum_{k,k'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{k'\downarrow} c_{-k'\uparrow}$$

- Introduce pairing field (mean field or decoupling approximation):

$$C_k = \langle c_{k\uparrow} c_{-k\downarrow} \rangle$$

$$c_{k\uparrow} c_{-k\downarrow} = C_k + \underbrace{\left(c_{k\uparrow} c_{-k\downarrow} - C_k \right)}_{\text{small fluctuations (assumption)}}$$

- Neglect products (correlations) of those small fluctuations
- Define

$$\Delta = \frac{V_0}{\Omega} \sum_k C_k$$

This plays the role of the condensate wavefunction

Solution via Bogoliubov Transform

- Rewrite Hamiltonian, drop terms quadratic in *C*'s:

$$\hat{H} = \sum_k \epsilon_k (c^\dagger_{k\uparrow} c_{k\uparrow} + c^\dagger_{k\downarrow} c_{k\downarrow}) - \Delta \sum_k \left(c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + c_{k\downarrow} c_{-k\uparrow} + \sum_{k'} C_{k'} \right)$$

Hamiltonian is now bilinear

- Solve via Bogoliubov transformation to quasiparticle operators:

$$\begin{aligned} \gamma_{k\uparrow} &= u_k c_{k\uparrow} - v_k c^\dagger_{-k\downarrow} \\ \gamma^\dagger_{-k\downarrow} &= u_k c^\dagger_{-k\downarrow} + v_k c_{k\uparrow} \end{aligned}$$

- With the choice $v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$ and $u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$ with $E_k = \sqrt{\xi_k^2 + \Delta^2}$ we get

$$\hat{H} - \mu\hat{N} = \underbrace{-\frac{\Delta^2}{V_0/\Omega} + \sum_k (\xi_k - E_k)}_{\text{Ground state energy}} + \underbrace{\sum_k E_k (\gamma^\dagger_{k\uparrow}\gamma_{k\uparrow} + \gamma^\dagger_{k\downarrow}\gamma_{k\downarrow})}_{\text{Non-interacting gas of fermionic quasi-particles}}$$

$$\gamma_{k\uparrow} \left|\Psi\right\rangle = 0$$

Solution of the gap equation

$$\Delta \equiv \frac{V_0}{\Omega}\sum_k \langle c_{k\uparrow}c_{-k\downarrow}\rangle = -\frac{V_0}{\Omega}\sum_k u_k v_k = -\frac{V_0}{\Omega}\sum_k \frac{\Delta}{2E_k}$$

$$\Delta = -\frac{V_0}{\Omega}\sum_k \frac{\Delta}{2E_k}$$

$$1 = -\frac{V_0}{\Omega}\sum_k \frac{1}{2E_k}$$

$$-\frac{\Omega}{V_0} = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2\sqrt{(\epsilon_k-\mu)^2+\Delta^2}}$$

$$\boxed{-\frac{\Omega}{V_0} = \int d\epsilon\, \frac{\rho_3(\epsilon)}{2\sqrt{(\epsilon-\mu)^2+\Delta^2}}}$$

Looks similar to equation for bound state in 2D (and Cooper problem)

$$-\frac{1}{V_0} = \frac{m}{\hbar^2}\int_{q<\frac{1}{R}} \frac{d^nq}{(2\pi)^n}\frac{1}{q^2+k^2} = \frac{1}{\Omega}\int_{\epsilon<E_R} d\epsilon \frac{\rho_n(\epsilon)}{2\epsilon+|E|}$$

Solution of the gap equation

- Gap equation:

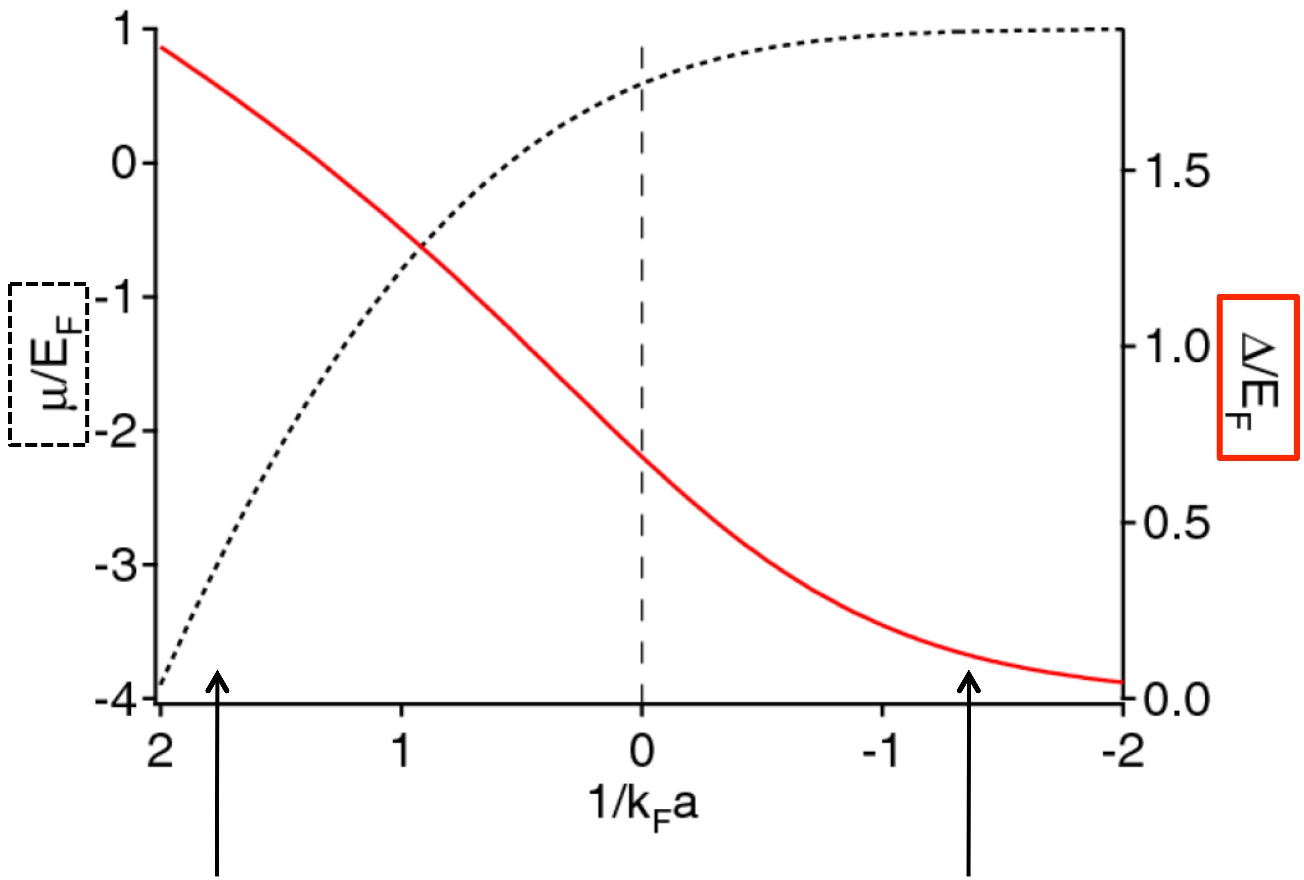
$$-\frac{\Omega}{V_0} = \int d\epsilon \, \frac{\rho_3(\epsilon)}{2\sqrt{(\epsilon - \mu)^2 + \Delta^2}}$$

- Number equation:

$$n = <\hat{n}>= \sum_{k,\sigma} \left\langle c^{\dagger}_{k,\sigma} c_{k,\sigma} \right\rangle$$

- Simultaneously solve for μ and Δ

Solution of the gap equation



BEC-side: Molecules

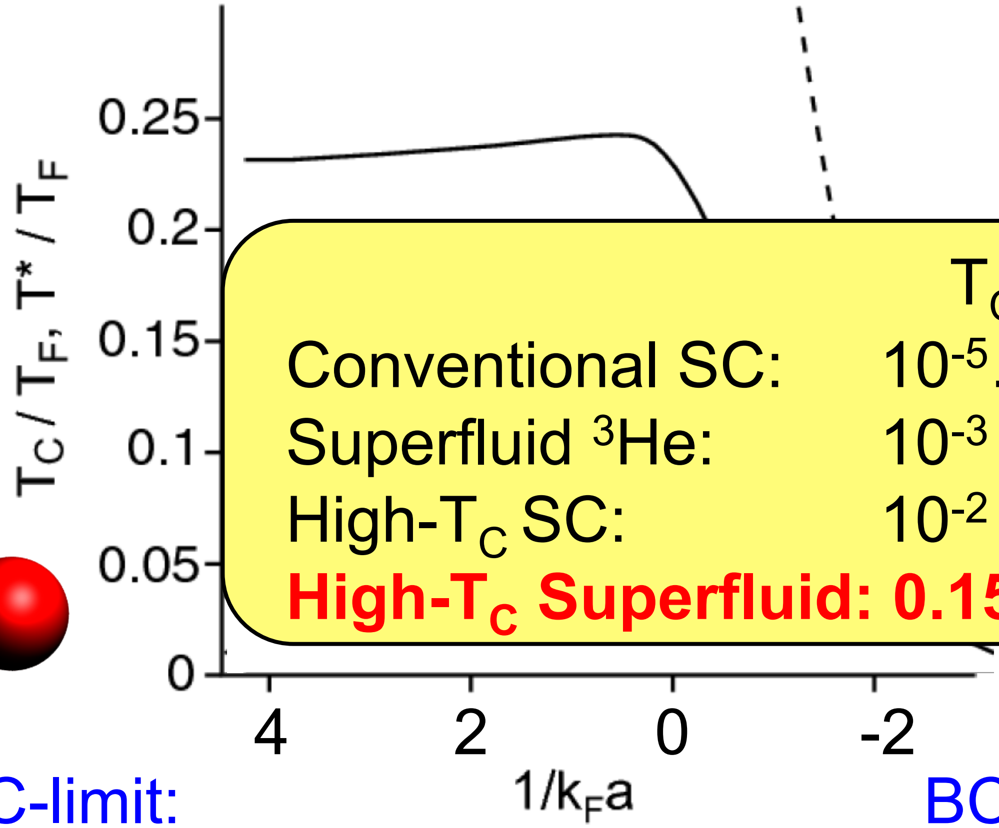
$$\mu \approx -\frac{\hbar^2}{2ma^2} = \frac{1}{2}E_B$$

BCS-side: Gap exponentially small

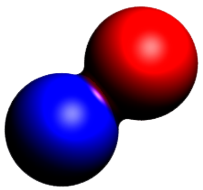
$$\begin{aligned} \mu &\approx E_F \\ \Delta &\approx \frac{8}{e^2} e^{-\pi/2k_F|a|} \end{aligned}$$

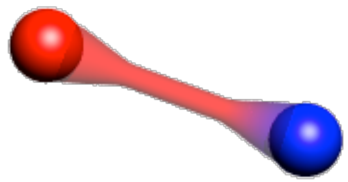
Critical temperature

- Can be derived from Bogoliubov Hamiltonian with fluctuations



	T_C/T_F
Conventional SC:	$10^{-5} \dots 10^{-4}$
Superfluid ^{3}He:	10^{-3}
High-T_C SC:	10^{-2}
High-T_C Superfluid: 0.15	





BEC-limit:

$$n_{\mathrm{Mol}} \lambda^3_{\mathrm{dB,Mol}} \approx 2.612$$

$$k_B T_C = 0.22 E_F$$

BCS-limit:

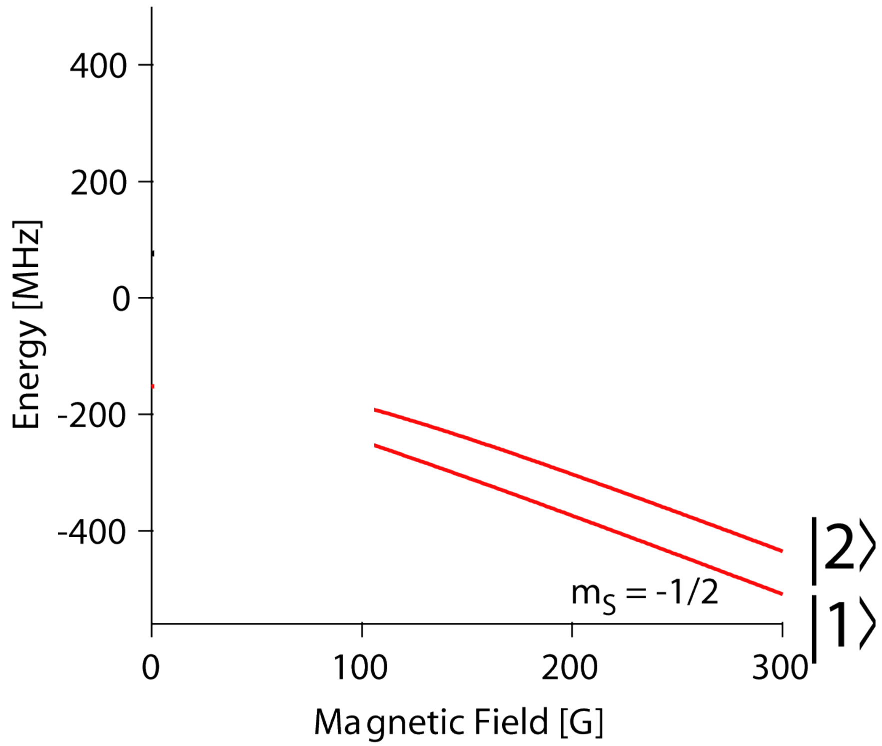
$$\begin{aligned} k_B T_C &= \frac{e^\gamma}{\pi} \frac{8}{e^2} e^{-\pi/2k_F|a|} \\ &= \frac{e^\gamma}{\pi} \Delta_0 = 0.57 \Delta_0 \end{aligned}$$

Experimental realization of the BEC-BCS Crossover

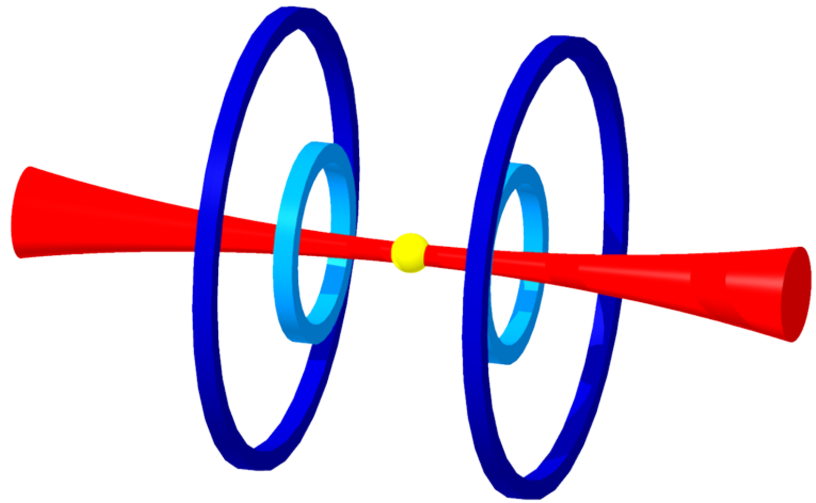
Preparation of an interacting Fermi system in Lithium-6

Electronic spin: S = ½, Nuclear Spin: I = 1

→ (2I+1)(2S+1) = 6 hyperfine states



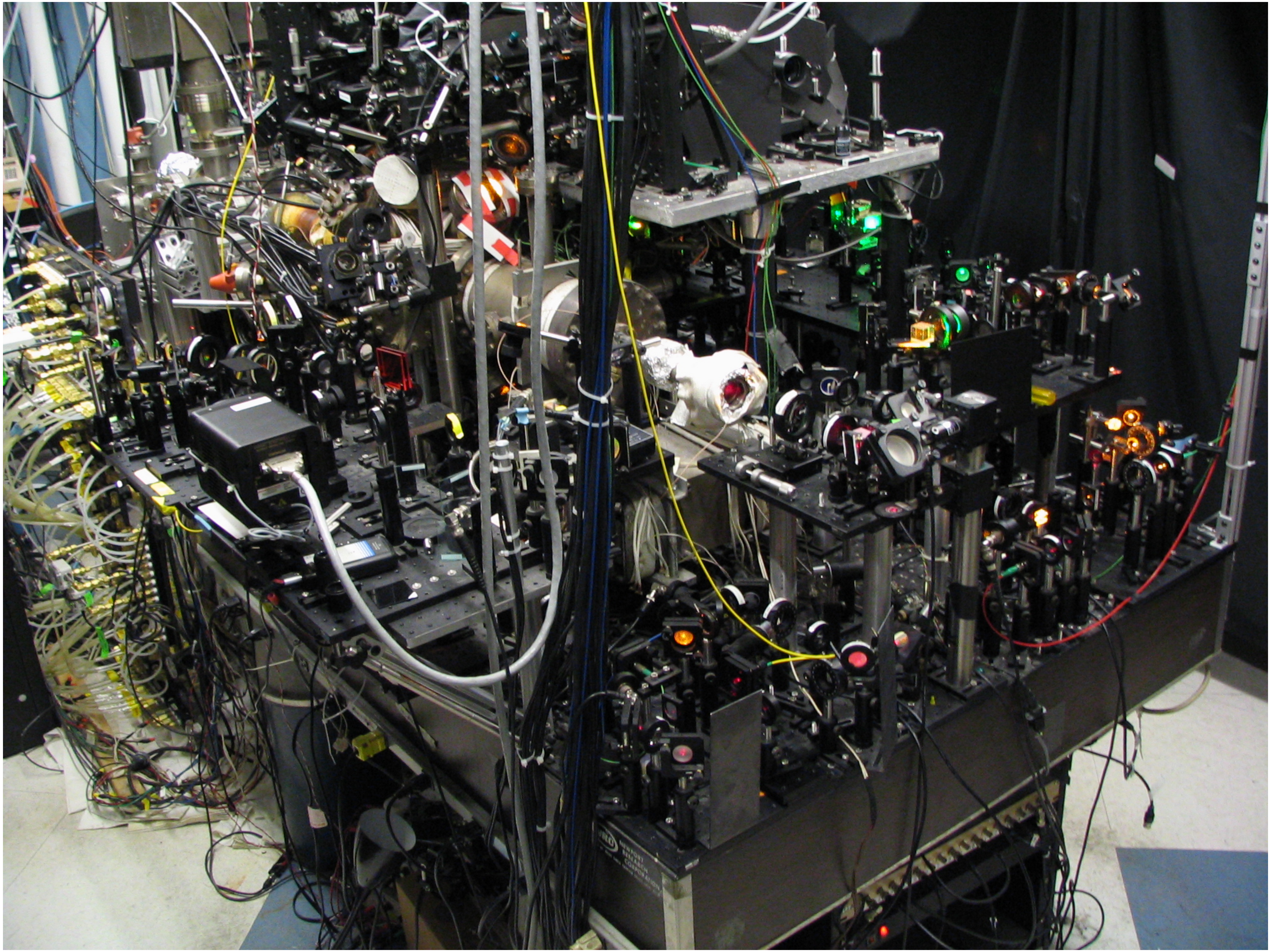
Optical trapping @ 1064 nm

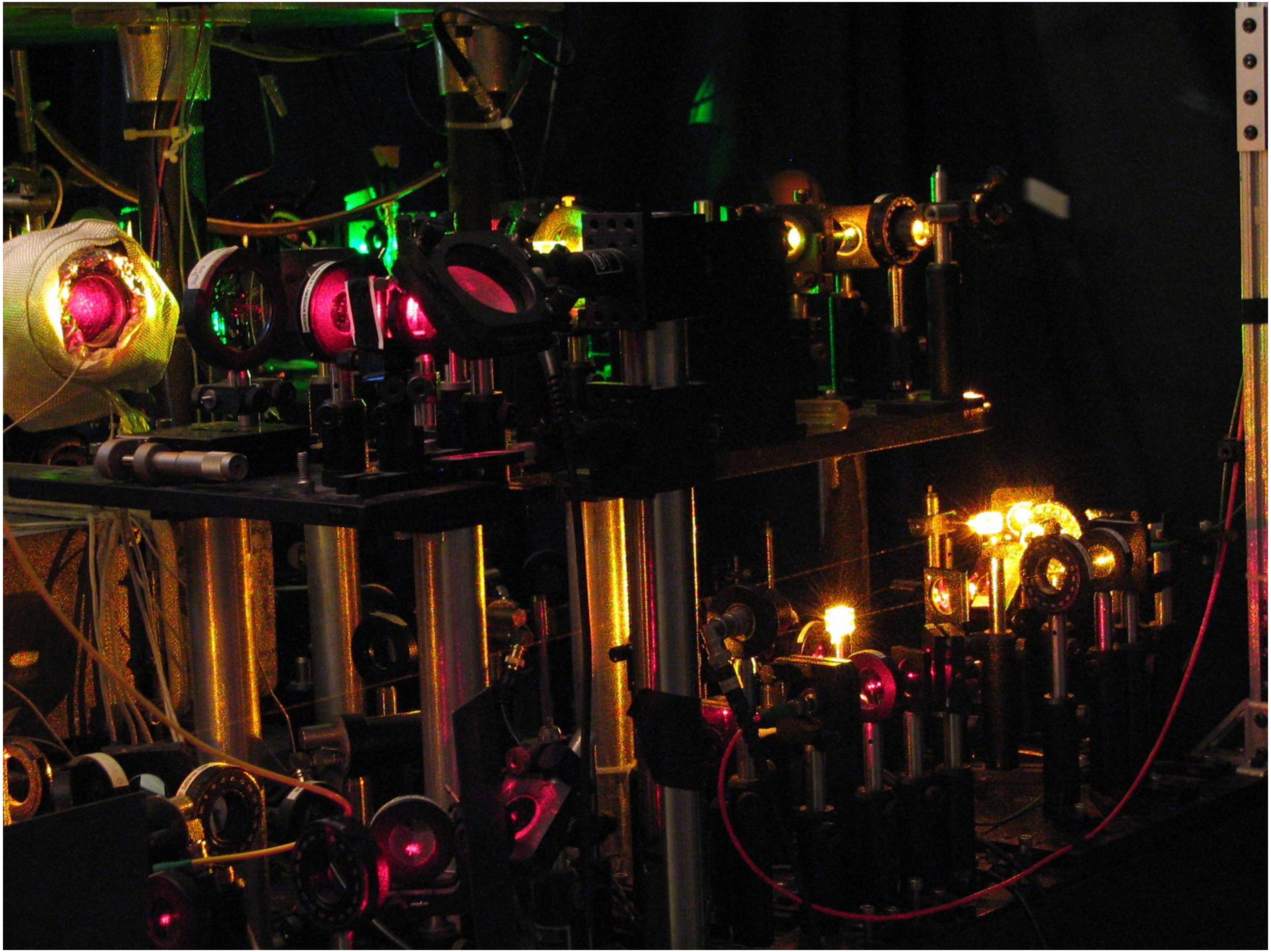


ν_{axial} = 10-20 Hz

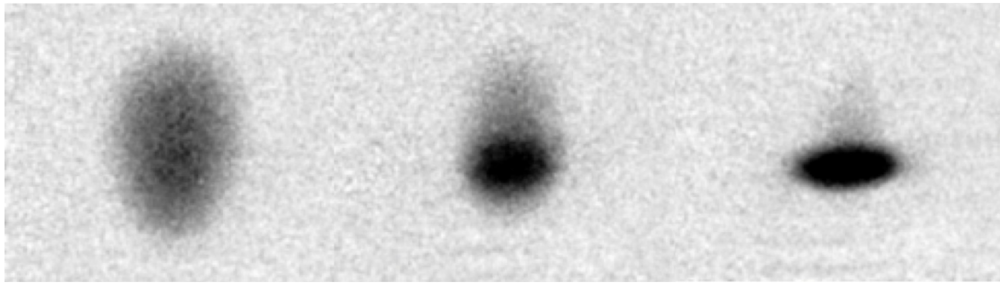
ν_{radial} = 50–200 Hz

E_{trap} = 0.5 - 5 μK





BEC of Fermion Pairs (Molecules)

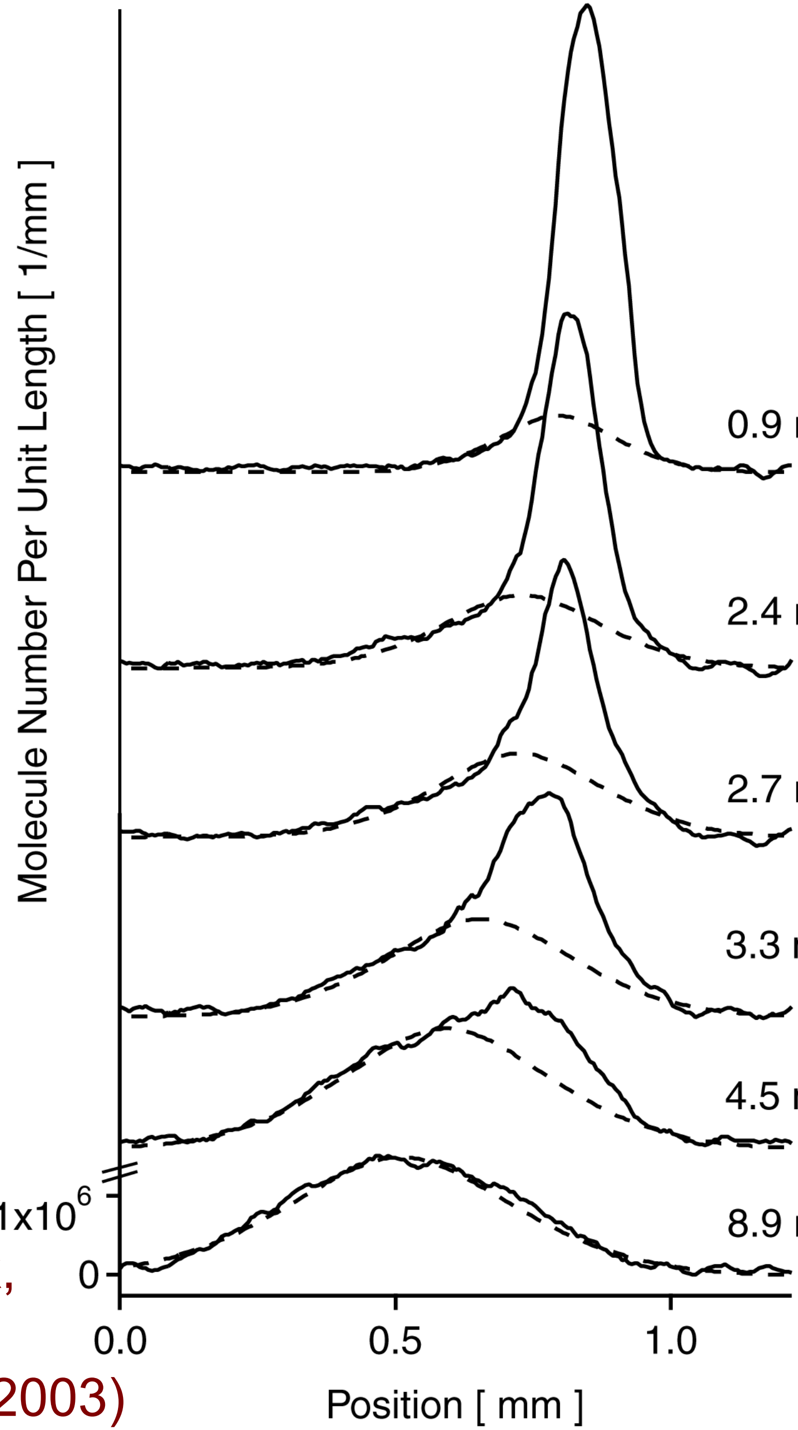


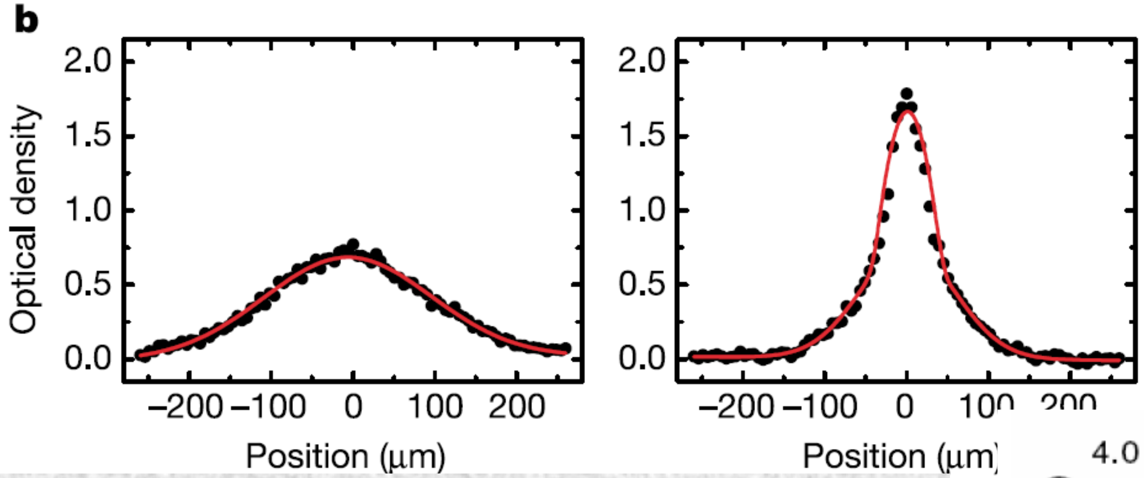
$T > T_C$ $T < T_C$ T T_C

These days: Up to 10 million condensed molecules

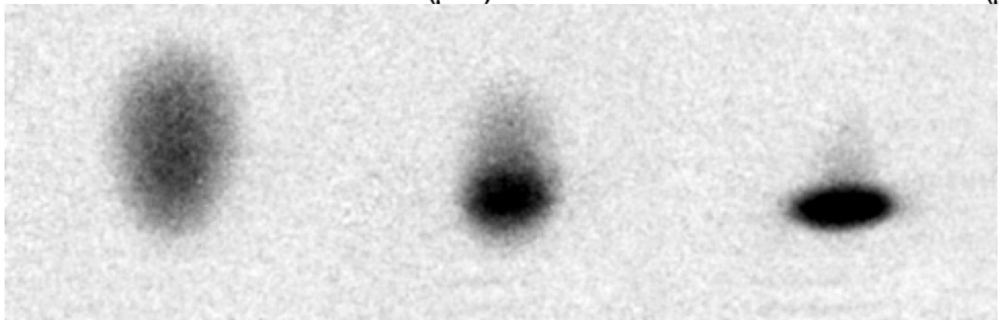
Boulder	Nov ‘03
Innsbruck	Nov ‘03, Jan ’04
MIT	Nov ’03
Paris	March ’04
Rice, Duke	

M.W. Zwierlein, C. A. Stan, C. H. Schunck, S.M.F. Raupach, S. Gupta, Z. Hadzibabic, W. Ketterle, Phys. Rev. Lett. 91, 250401 (2003)

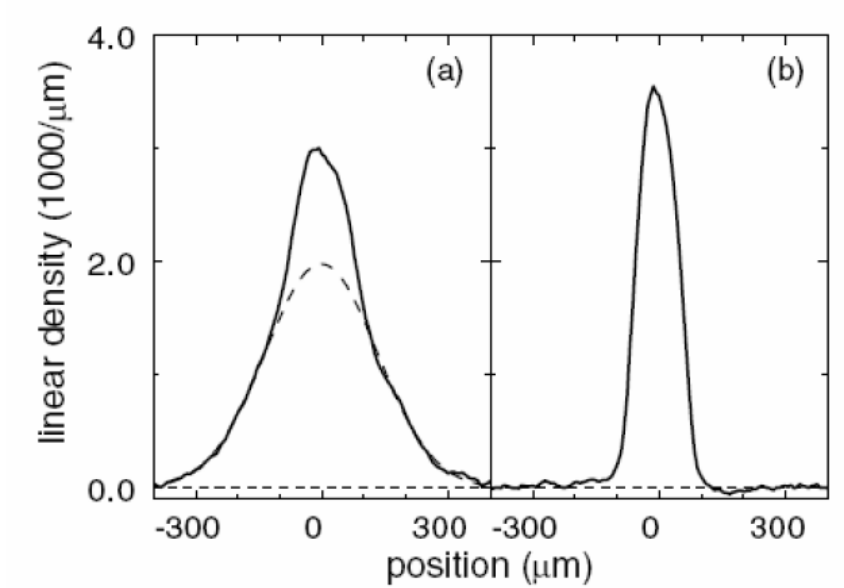




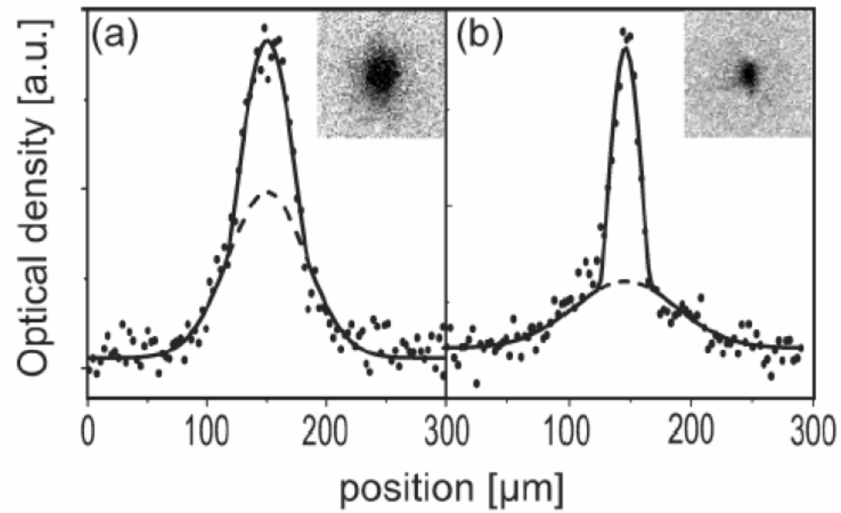
JILA, Nature 426, 537 (2003).



MIT, PRL 91, 250401 (2003)



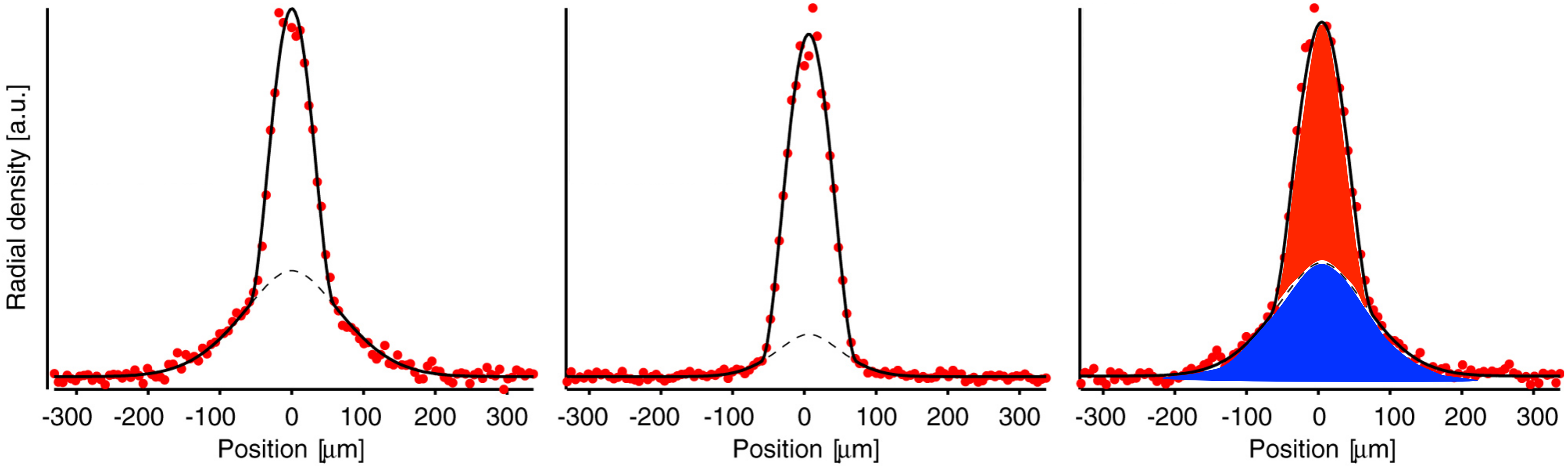
Innsbruck, PRL 92, 120401 (2004).



ENS, PRL 93, 050401 (2004).

Observation of Pair Condensates

BEC-Side | Resonance | BCS-Side (above dissociation limit for molecules)

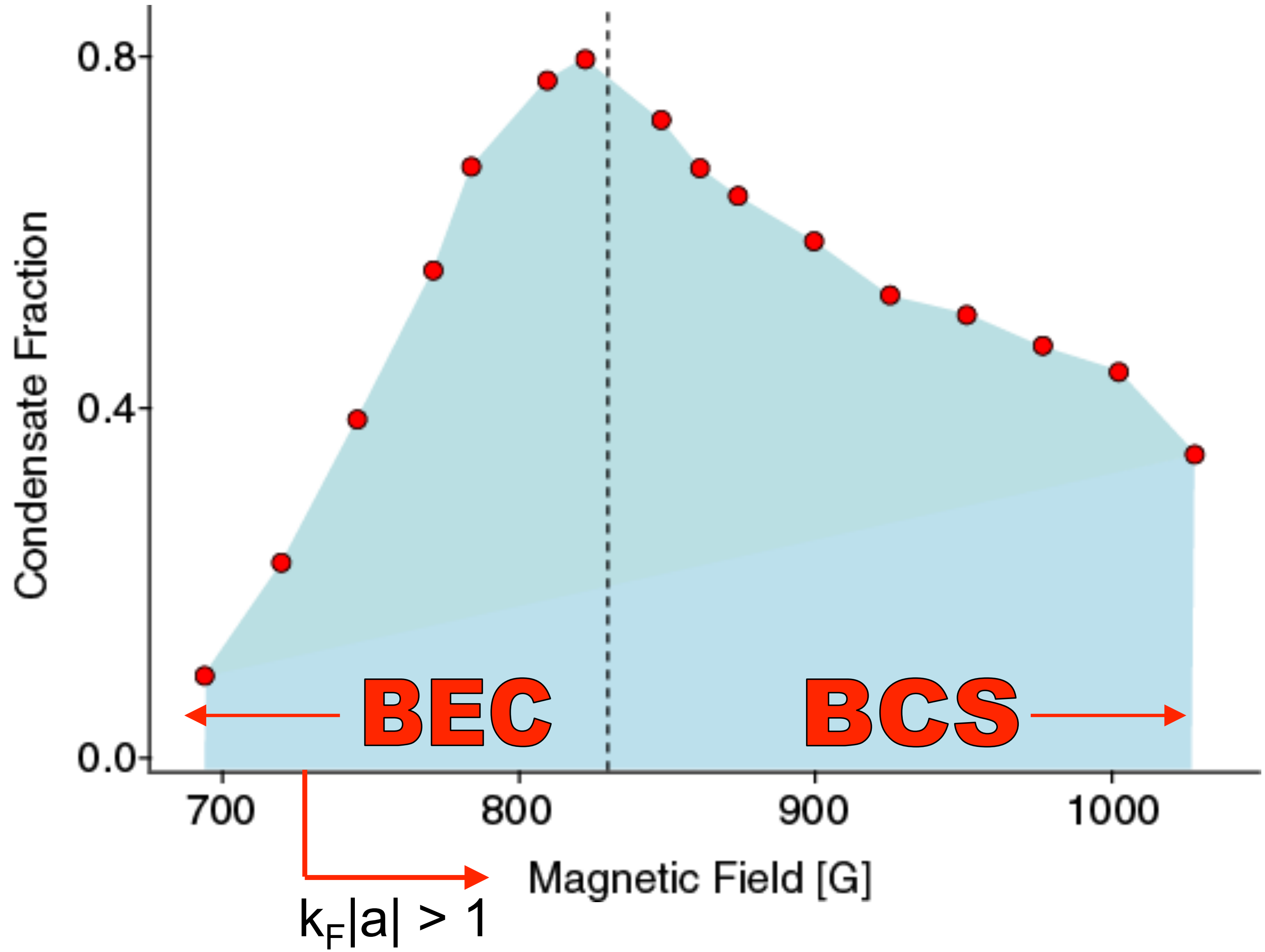


Thermal + condensed pairs

First observation: C.A. Regal et al., Phys. Rev. Lett. **92**, 040403 (2004)

M.W. Zwierlein, C.A. Stan, C.H. Schunck, S.M.F. Raupach, A.J. Kerman, W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

Condensate Fraction vs Magnetic Field

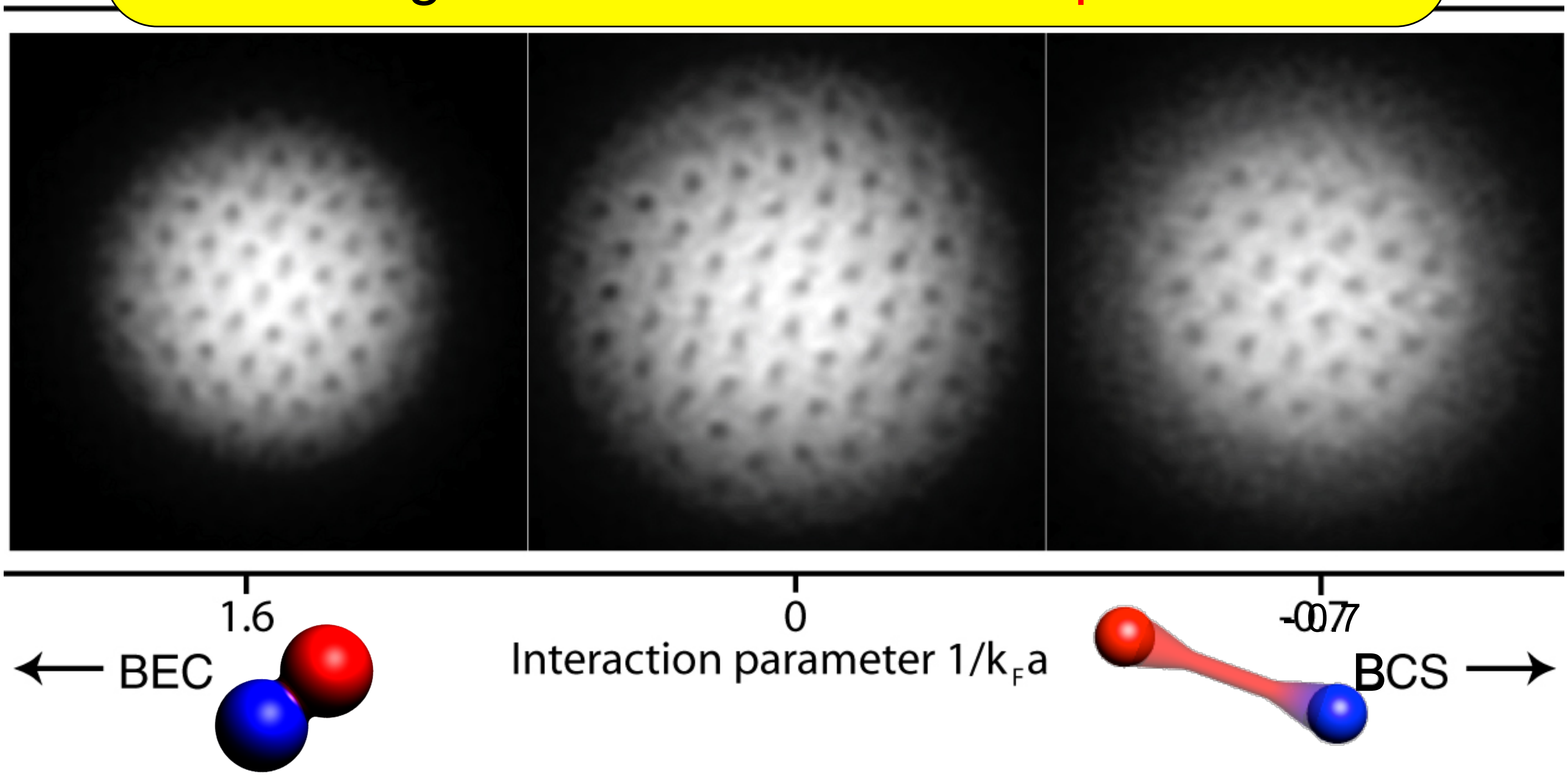


M.W. Zwierlein, C.A. Stan, C.H. Schunck, S.M.F. Raupach, A.J. Kerman, W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

How can we show that these gases are superfluid?

Vortex lattices in the BEC-BCS crossover

Establishes *superfluidity* and *phase coherence* in gases of fermionic atom pairs

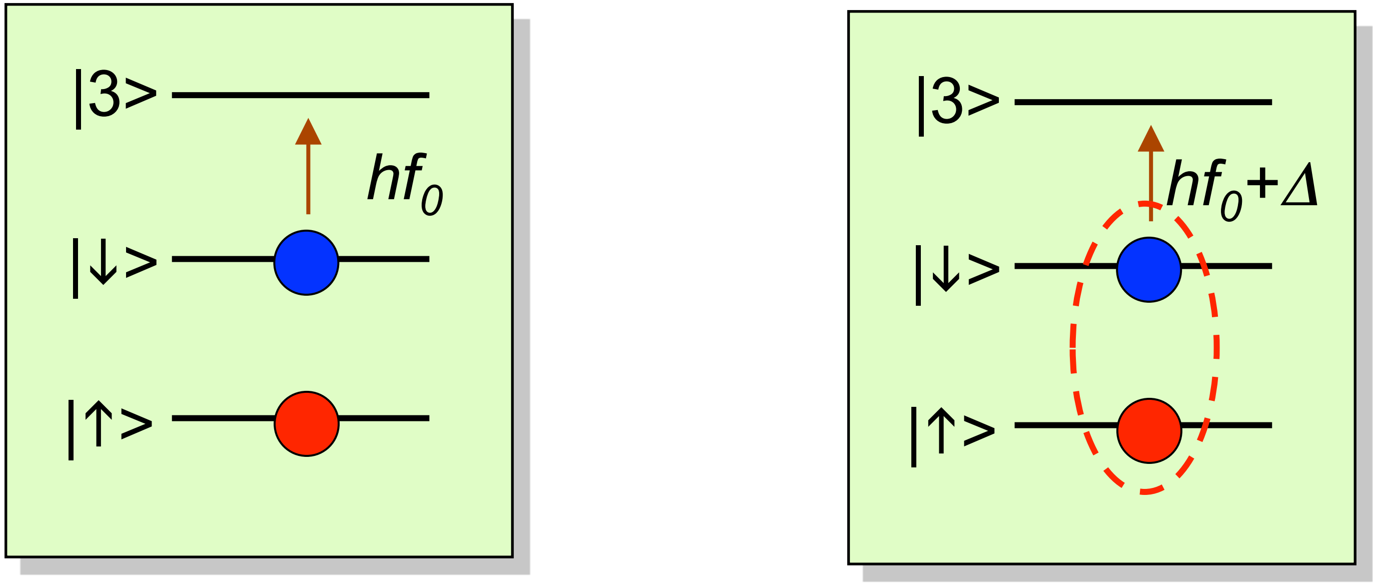


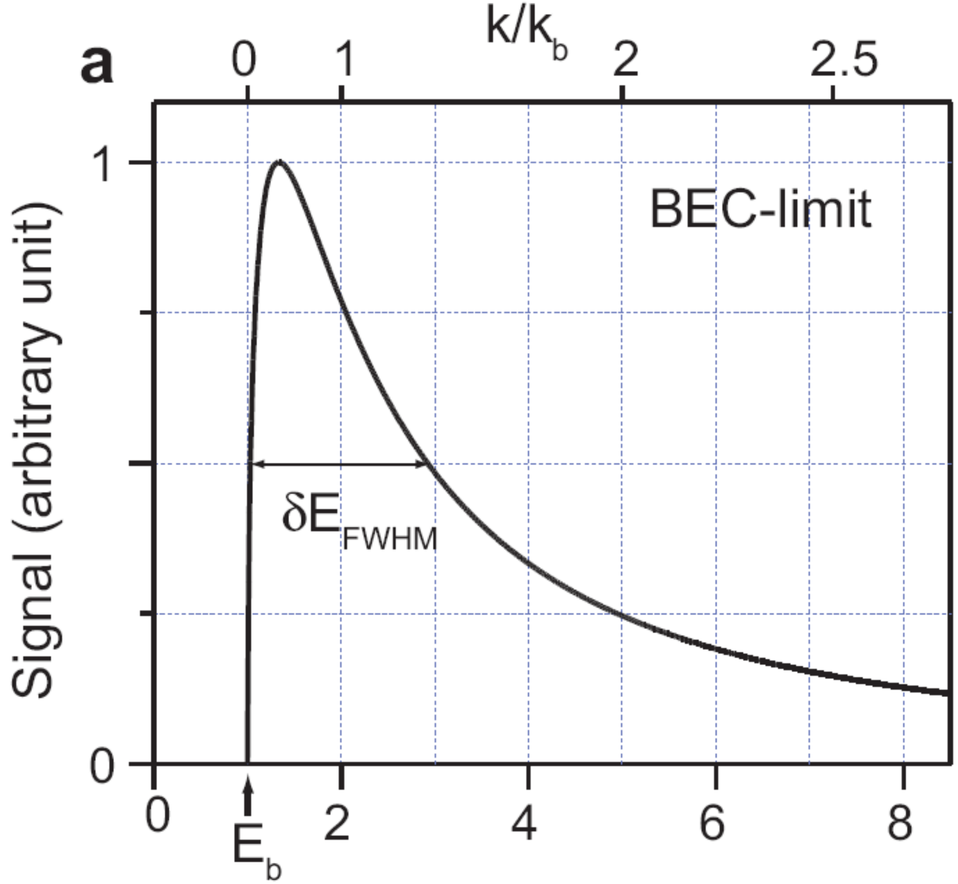
M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle, Nature 435, 1047-1051 (2005)

Superfluidity of fermions requires pairing of fermions

Microscopic study of the pairs by RF spectroscopy

RF spectroscopy

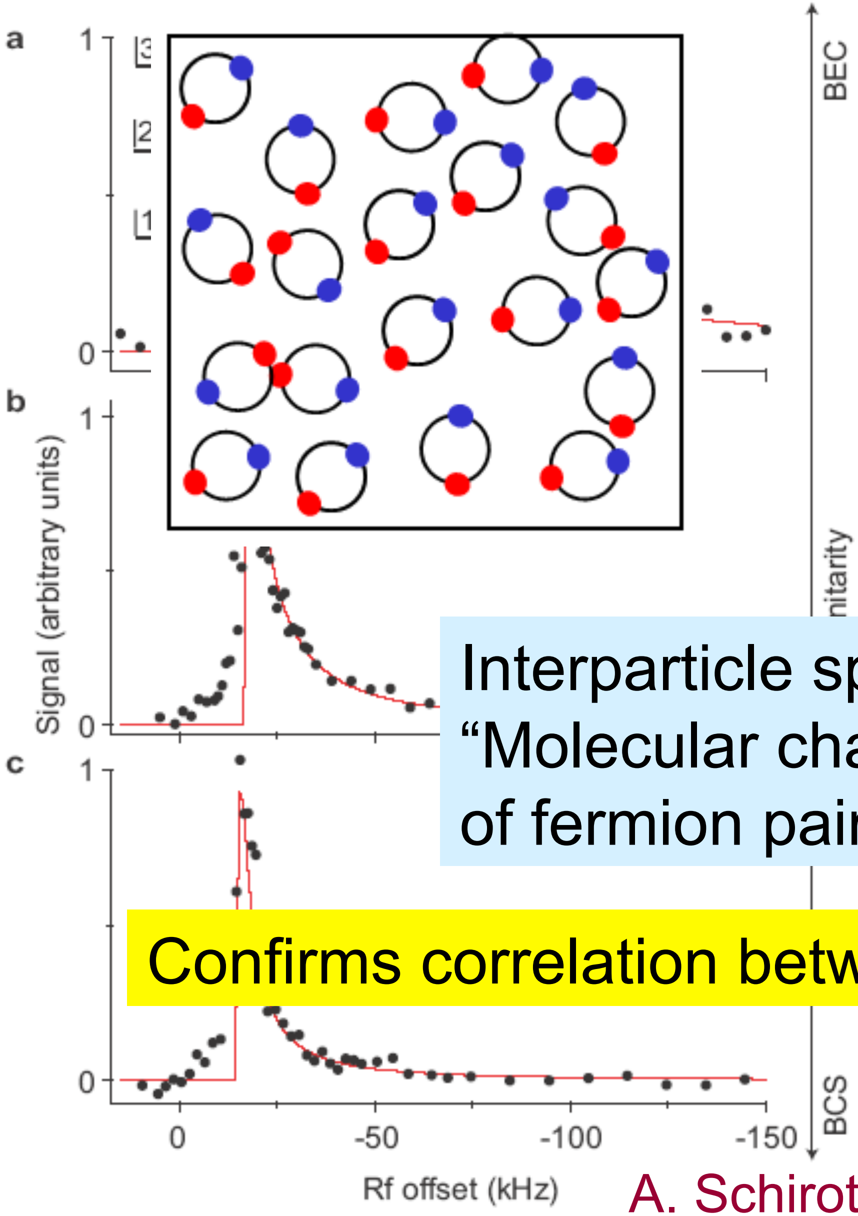




Dissociation spectrum measures the Fourier transform of the pair wavefunction

Width $\propto$ (1/pair size)^2
Threshold $\propto$ (1/pair size)^2

Rf spectra in the crossover



Standard superconductors
$\xi >> 1/k_F$

High T_c superconductors
$\xi \approx 6 \ldots 10 \; (1/k_F)$

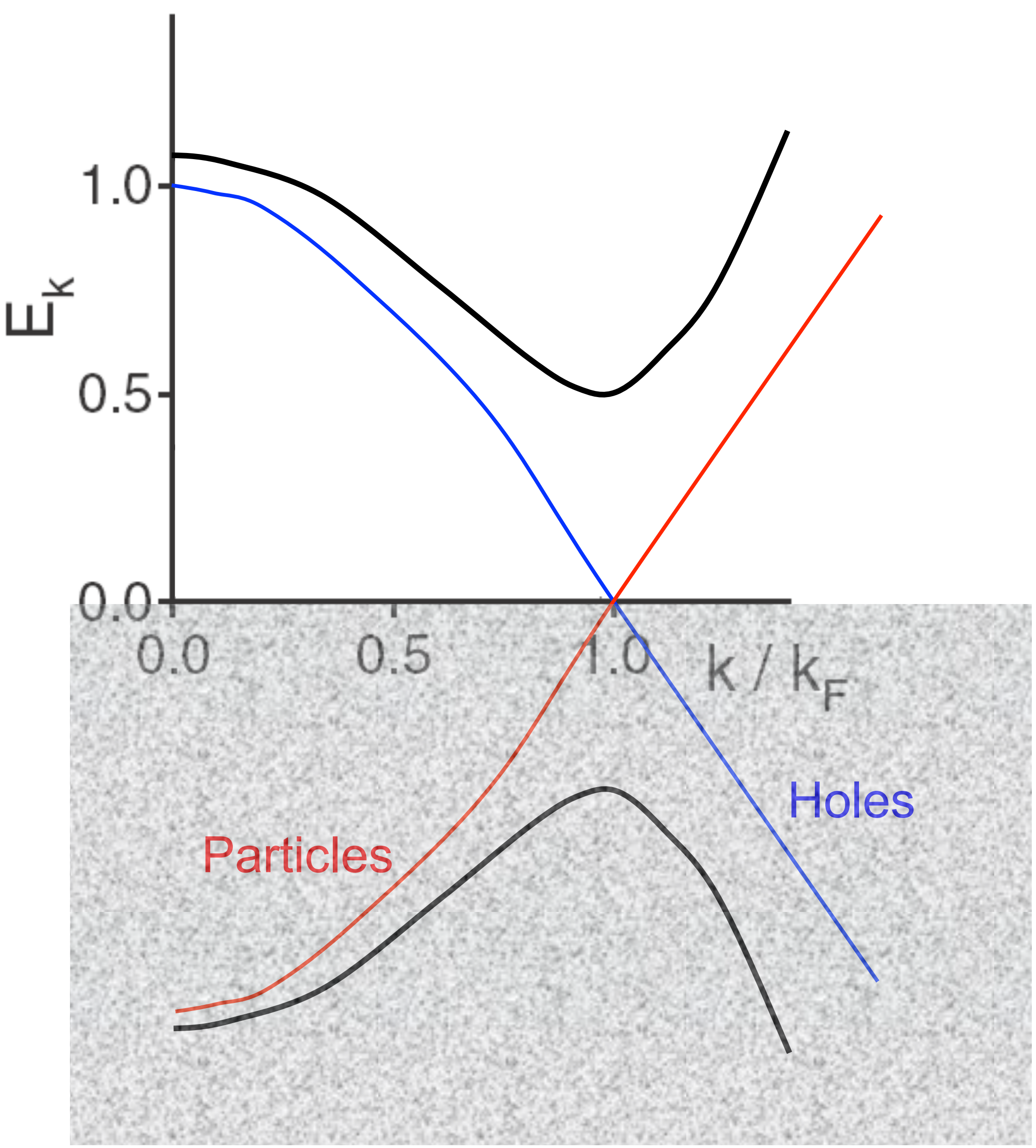
Superfluid at unitarity
$\xi = 2.6 \; (1/k_F)$

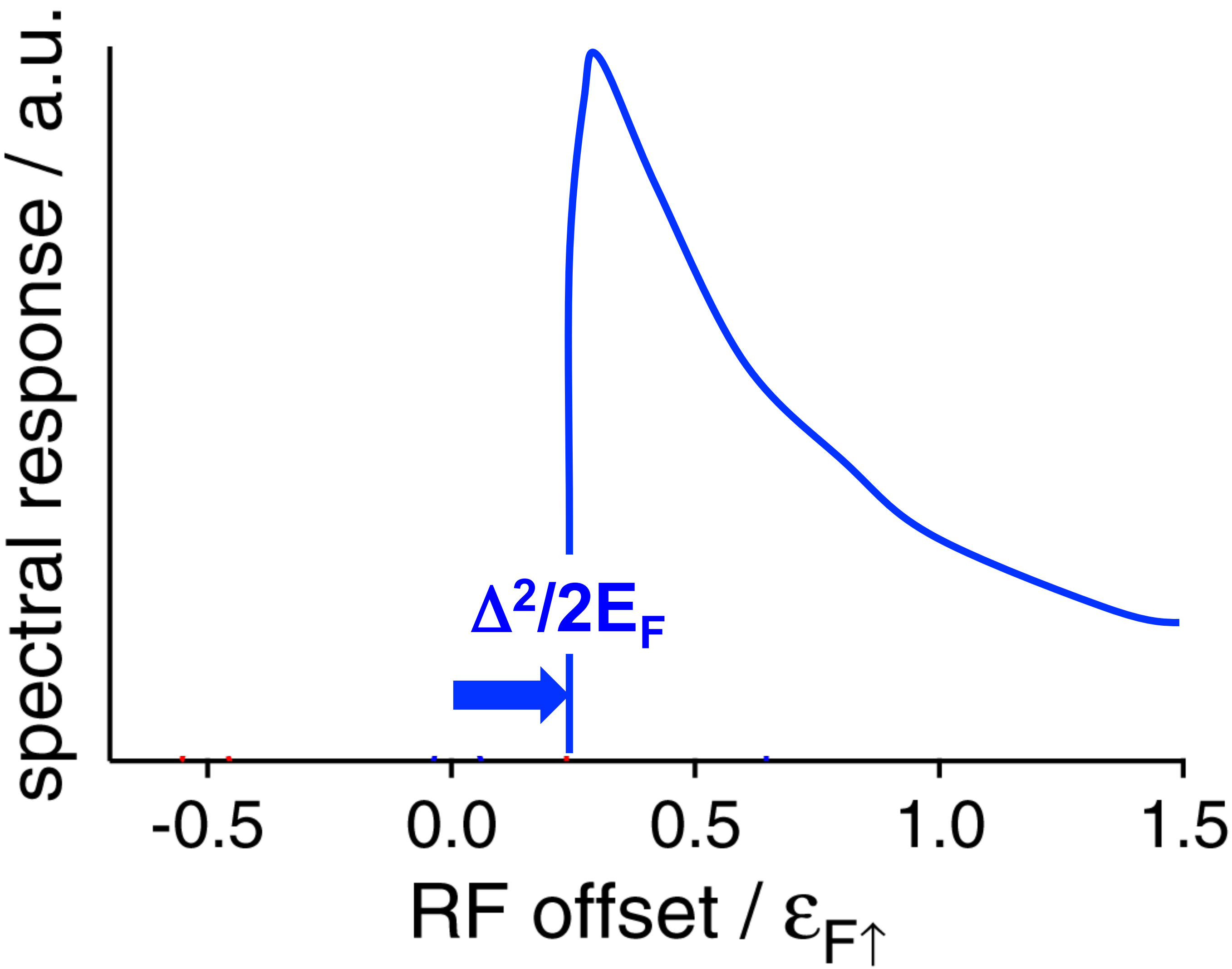
Interparticle spacing ~ 3.1 $(1/k_F)$
“Molecular character”
of fermion pairs

Confirms correlation between high T_c and small pairs

C. H. Schunck, Y. Shin,
A. Schirotzek, W. Ketterle, Nature 454, 739 (2008).

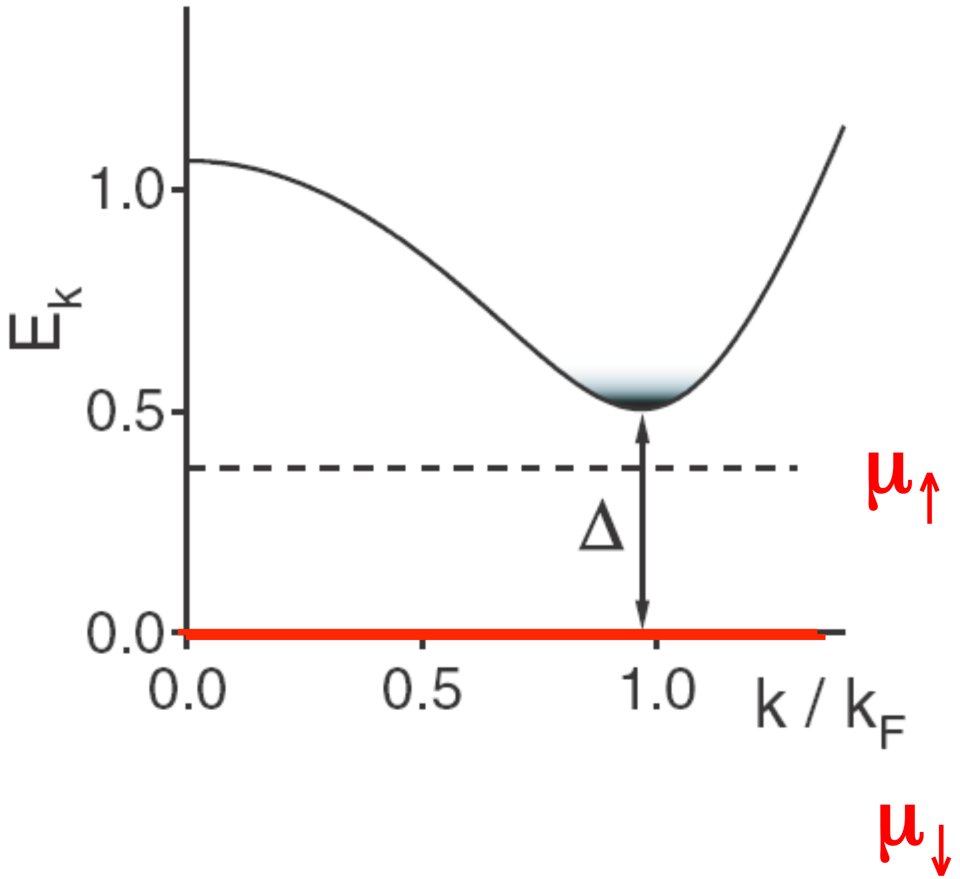
Excitations in a superfluid





spectral response / a.u.
$\Delta^2/2E_F$
-0.5
0.0
0.5
1.0
1.5
RF offset / $\varepsilon_{F\uparrow}$

How to inject quasi-particles near the Fermi surface?

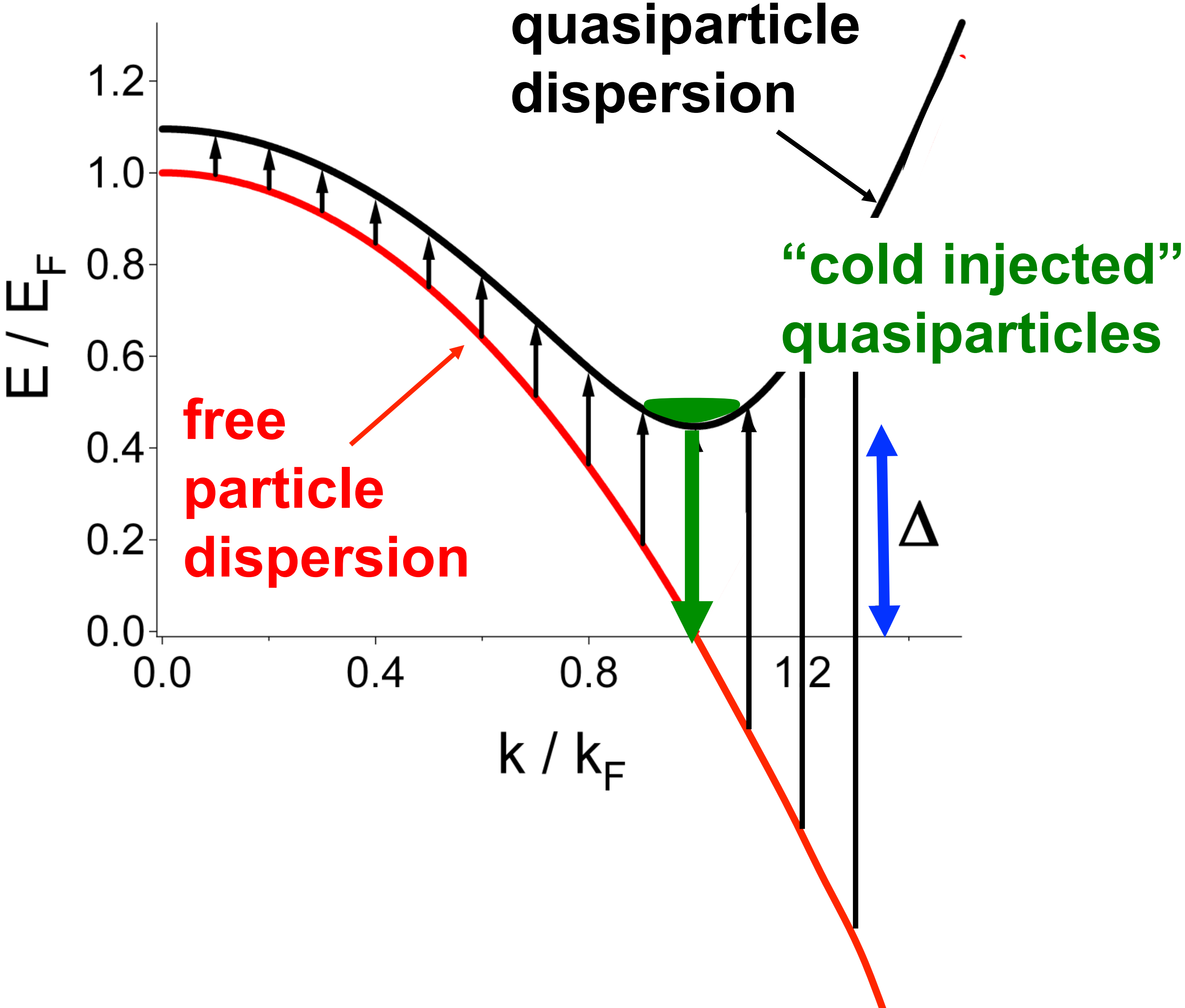


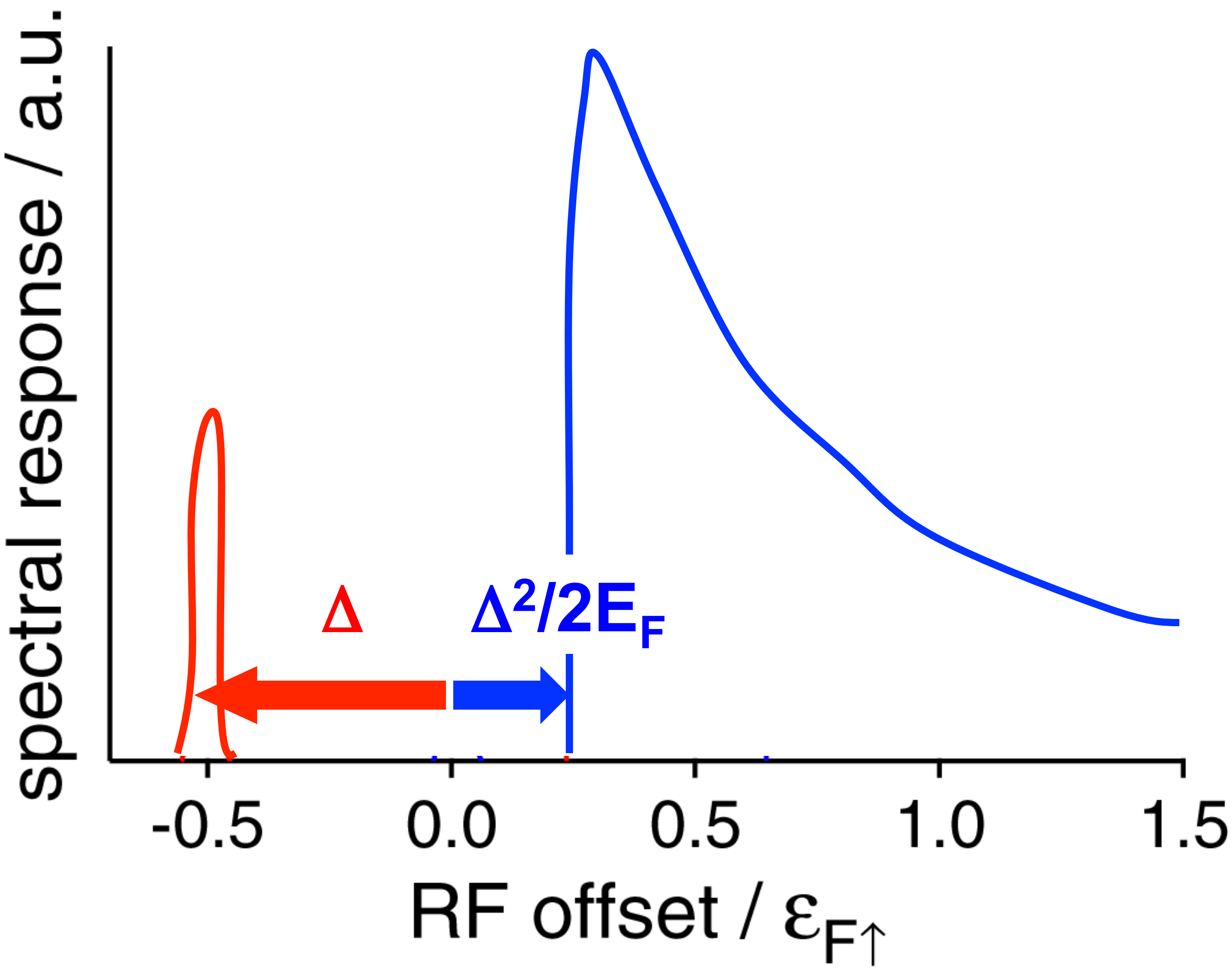
Population imbalance

$$kT \approx \Delta - \mu_\uparrow$$

RF Spectroscopy of a BCS superfluid

- Final state empty, measures *integrated* (over k) spectrum
- RF photon creates quasiparticle and free particle in third state





BCS limit: splitting becomes exactly Δ

Polarized Superfluid

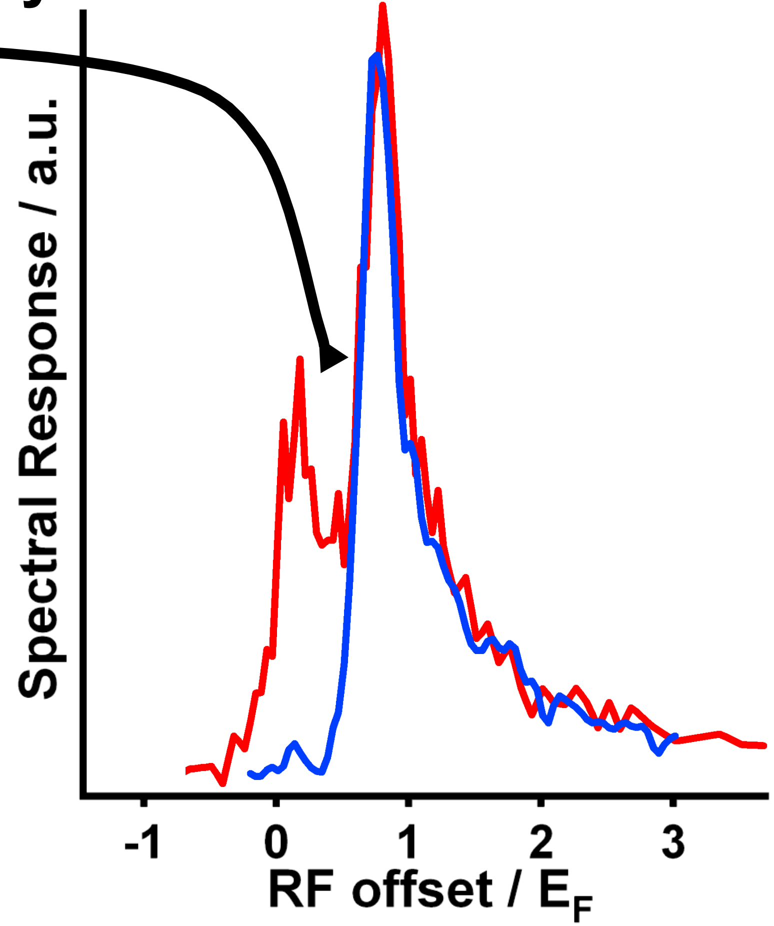
local double peak: Stokes and Anti-Stokes peak

BCS T→0 limit: Splitting is exactly Δ

Splitting allows a direct determination of the superfluid gap Δ

$\Delta = 0.44\ E_F$

QMC (Carlson, Reddy 2008)
$\Delta = 0.45\ E_F$

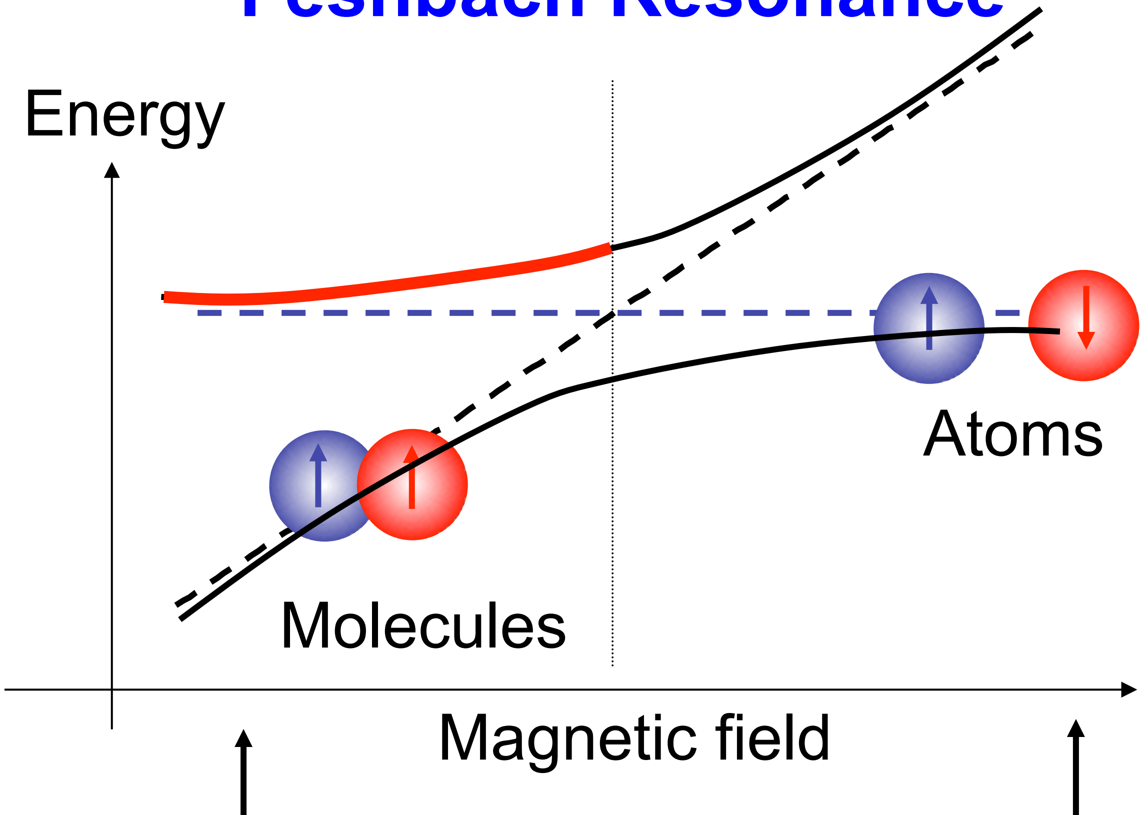


Related experiment
JILA: RF photoemission

A. Schirotzek, Y. Shin, C.H. Schunck, W.K., Phys. Rev. Lett. 101, 140403 (2008).

Now: Fermions with repulsive interactions

Feshbach Resonance



Atoms form stable molecules

Molecules are unstable

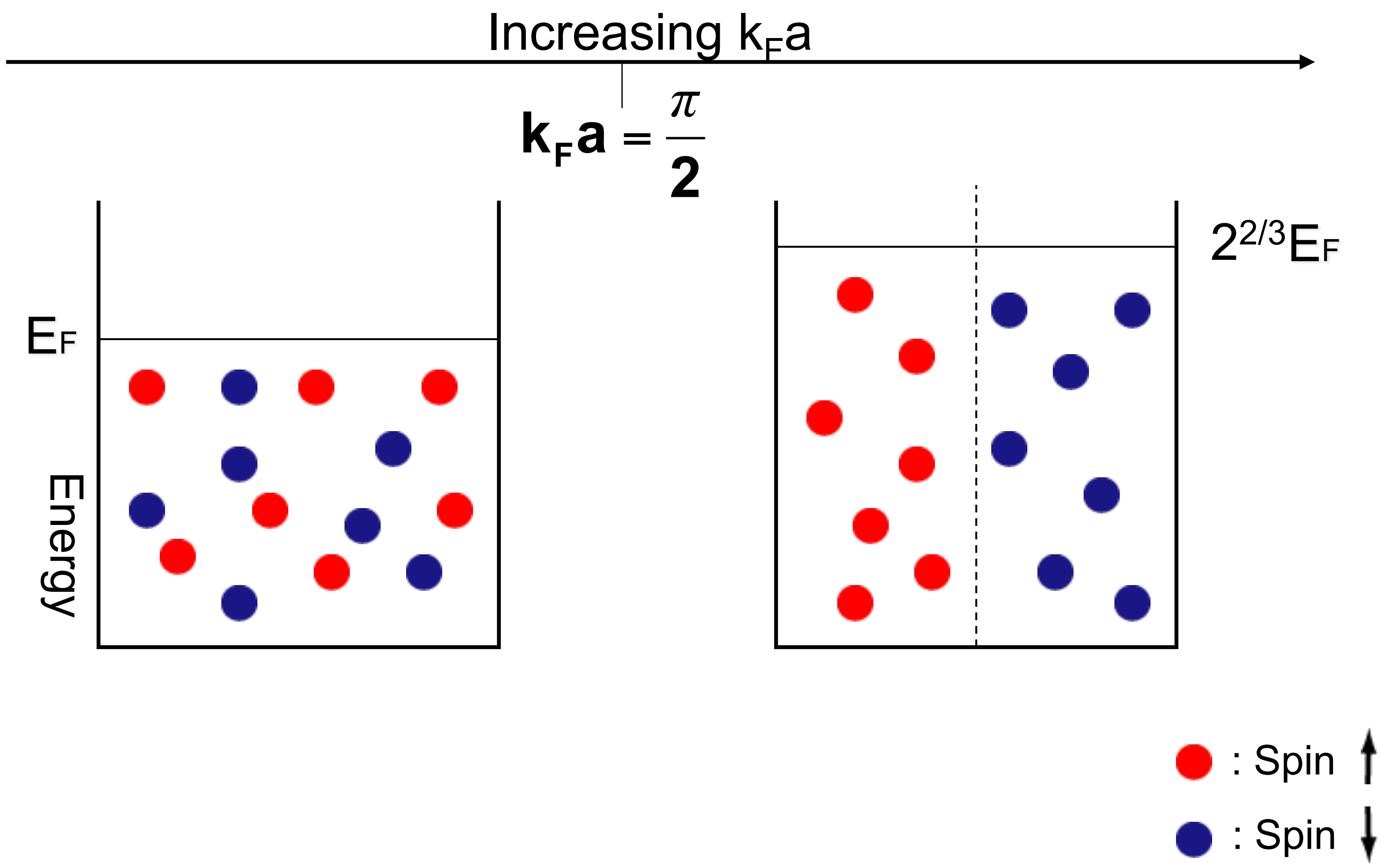
Atoms repel each other
a>0

Atoms attract each other
a<0

Itinerant Ferromagnetism
Stoner instability
in a free gas

BCS-limit:
Condensation of
long-range Cooper pairs

Itinerant Ferromagnetic Phase Transition in Ultracold gases



$$\hat{H} = \int d^3x \left[\sum_{\sigma} a_{\sigma}^{+} \left(\frac{p^2}{2m}\right) a_{\sigma} + g a_{\uparrow}^{+} a_{\downarrow}^{+} a_{\downarrow} a_{\uparrow} \right]$$

Stoner model

$$\hat{H} = \int d^3x \left[\sum_\sigma a_\sigma^+ (\frac{p^2}{2m}) a_\sigma + g a_\uparrow^+ a_\downarrow^+ a_\downarrow a_\uparrow \right]$$

A Fermi gas with short-range repulsive interactions

Kinetic energy

$$E_{kin} = \frac{3}{5} N_1 E_{F,1} + \frac{3}{5} N_2 E_{F,2} = (\ldots)(N_1^{5/3} + N_2^{5/3})$$

$$\Delta N = N_1 - N_2$$

$$\eta = 2\Delta N / N$$

$$N_{1,2} = \frac{N}{2} \pm \frac{\Delta N}{2} = \frac{N}{2}(1 \pm \eta)$$

$$E_{kin} = (\ldots)((1+\eta)^{5/3} + (1-\eta)^{5/3})$$

Mean-field approximation for interaction term:

$$\propto \frac{k_F a}{V^2} N_1 N_2 \quad = (\ldots) k_F a)(1+\eta)(1-\eta)$$

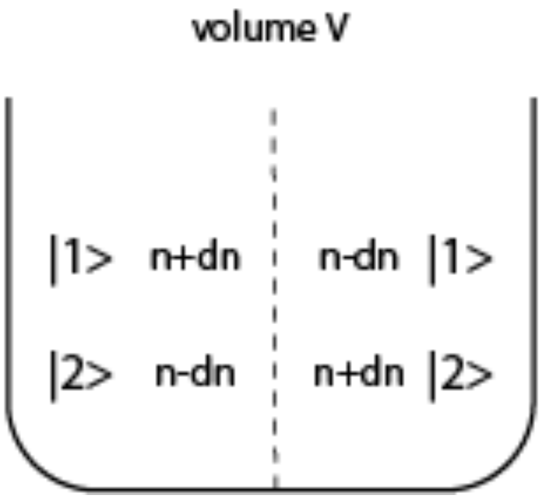
Itinerant Ferromagnetism

T=0 Mean-field Stoner model for the phase transition

local magnetization η

$$E = E_F 2Vn[\underbrace{\frac{3}{10}\{(1+\eta)^{5/3}}_{\text{K.E. spin up}} + \underbrace{(1-\eta)^{5/3}\}}_{\text{K.E. spin down}}$$

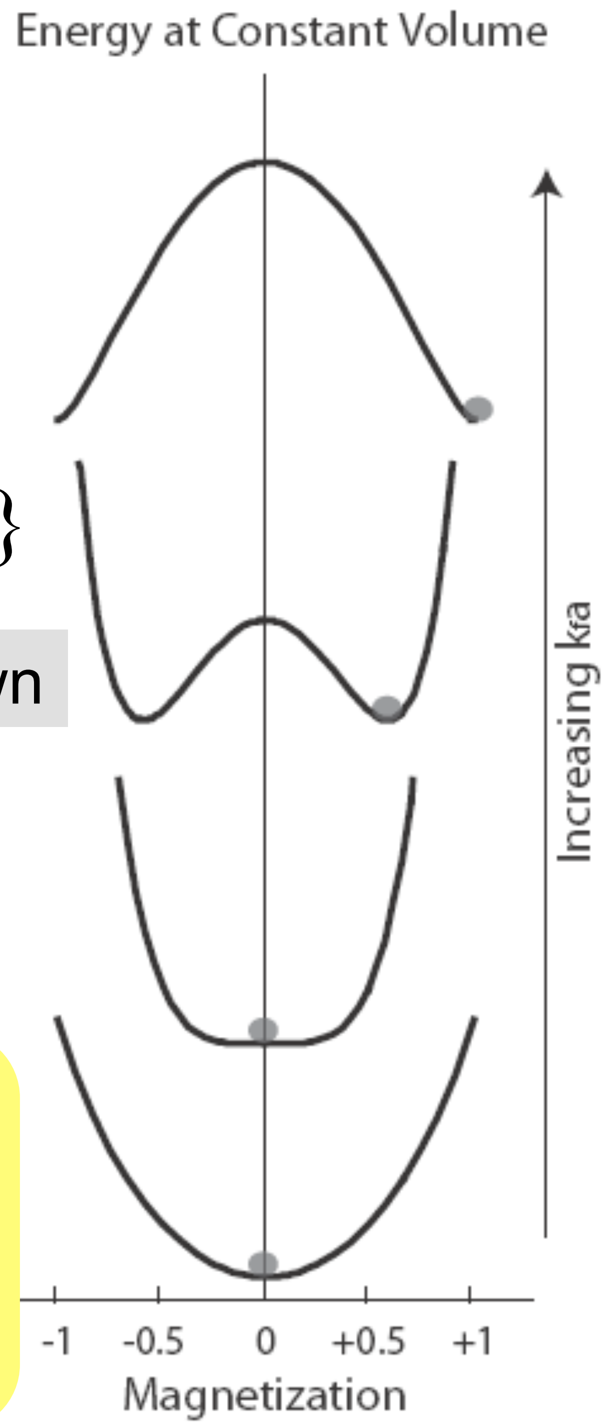
$$+ \underbrace{\frac{2}{3\pi}k_F a(1+\eta)(1-\eta)]}_{\text{repulsive mean field interaction}}$$



phase transition for

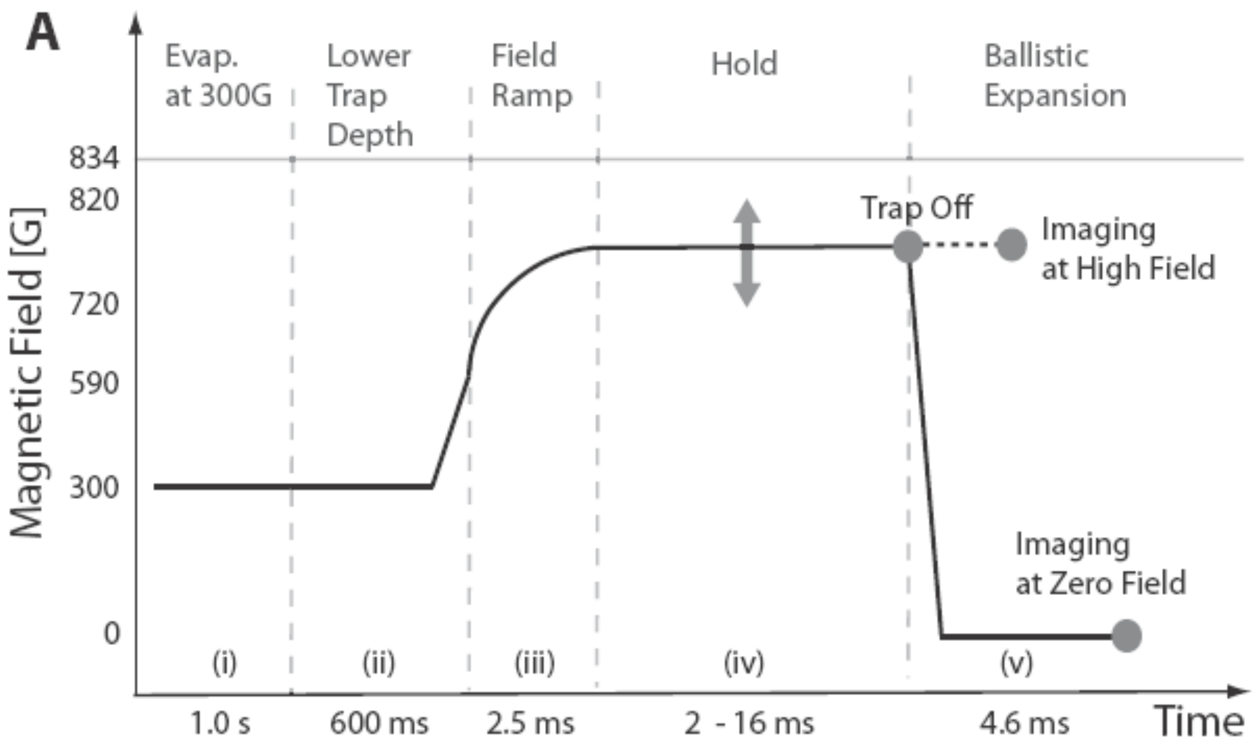
$$\frac{\partial^2 E}{\partial \eta^2} = 0$$

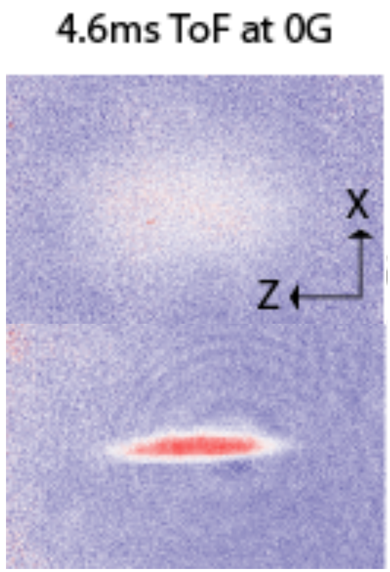
$$k_F a = \frac{\pi}{2}$$

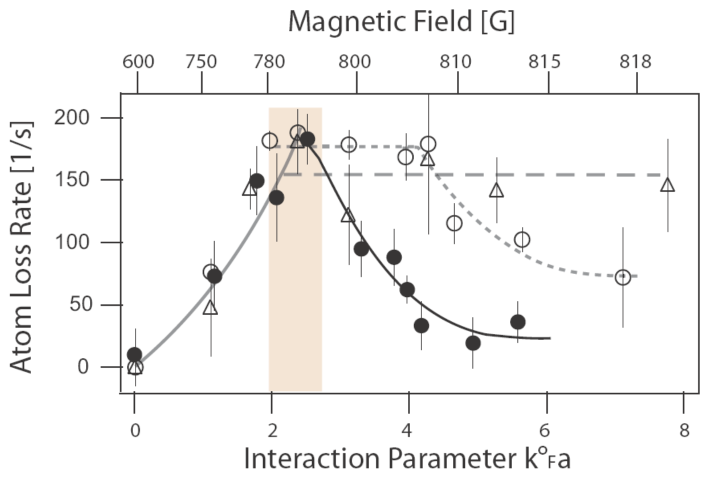


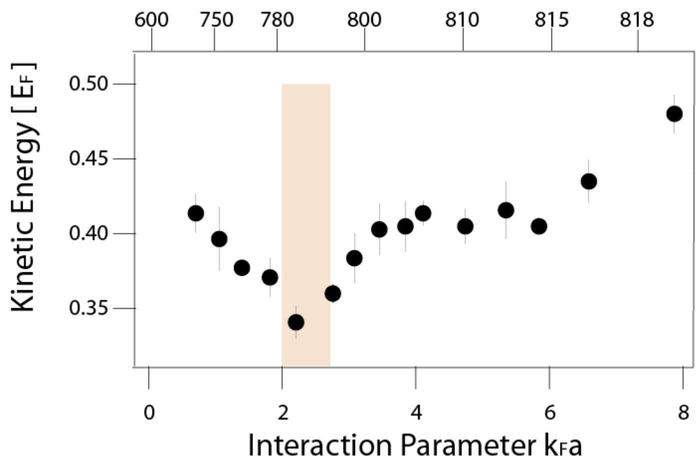
Experiments

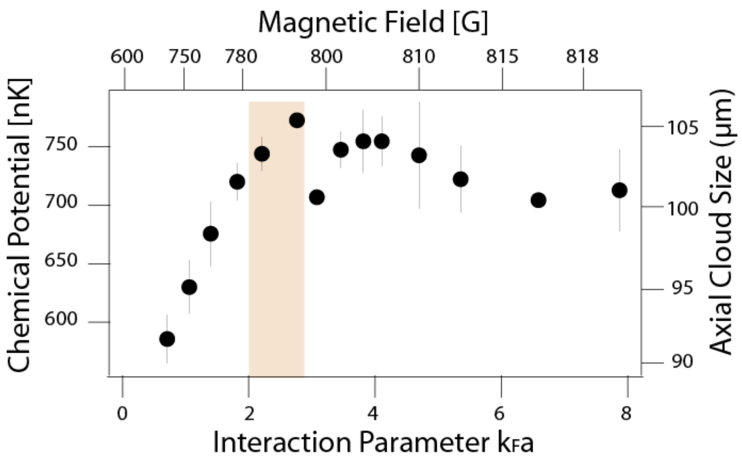
- Prepared a two-component Fermi gas(~ 0.65 million per each spin state)
- Vary repulsive interactions near Feshbach resonance located at 834 G











Three observations of non-monotonic behavior when approaching the Feshbach resonance

- Suggests that itinerant FM can occur for a free gas with short-range interactions
- First study of quantum magnetism in cold fermionic atoms
- Quantum simulation of a Hamiltonian for which even the existence of a phase transition is unknown

BUT:

- Lifetime only 10 ms
- Molecular fraction 25 %
- Magnetic domains not resolved
- Ferromagnetic fluctuations vs. ferromagnetic ground state

G.B. Jo, Y.R. Lee, J.H. Choi, C.A. Christensen, H. Kim, J. Thywissen, D.E. Pritchard, W.K., Science 325, 1521-1524 (2009).

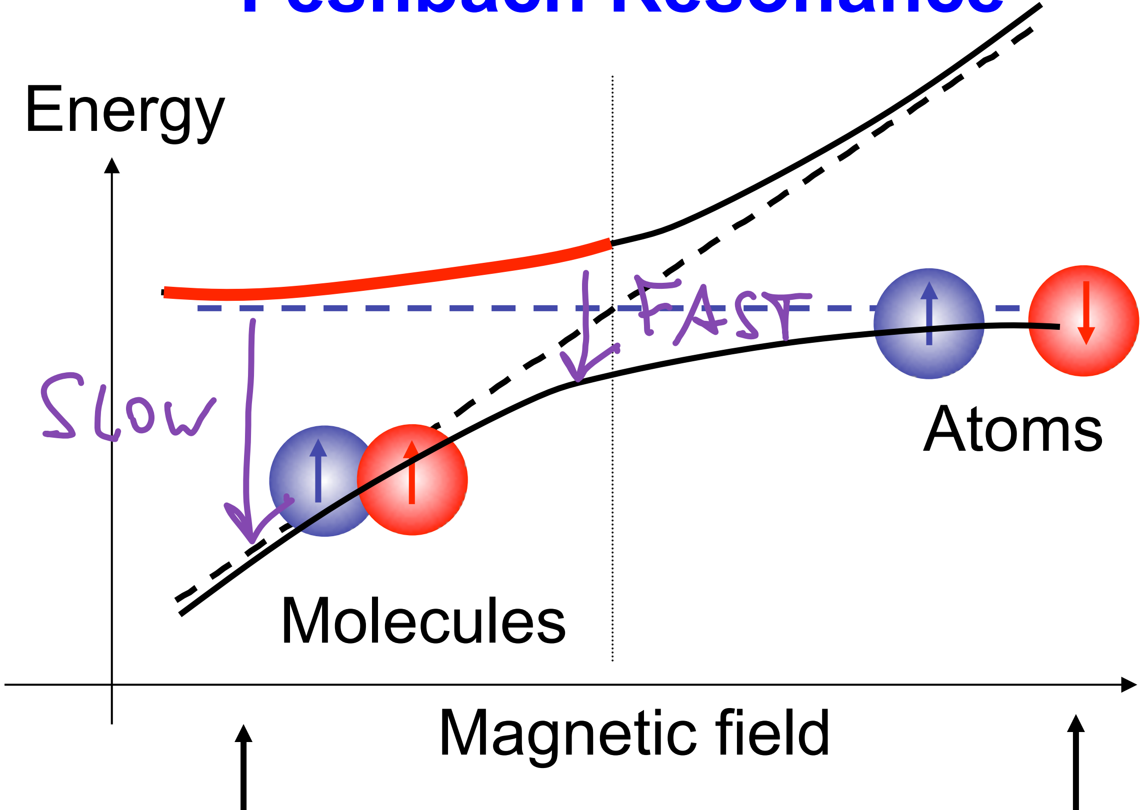
More recent work:

- No ferromagnetic transition
- Rapid decay into pairs

$\Rightarrow$ Highly correlated gas, breakdown of mean field description
$\Rightarrow$ Atoms with strong repulsion cannot be isolated from molecules

C. Sanner, E.J. Su, W. Huang, A. Keshet, J. Gillen, W.K.,
Phys. Rev. Lett. **108**, 240404 (2012)

Feshbach Resonance



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a>0

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Stoner instability
in a free gas

Molecules are unstable

Atoms attract each other
a<0

BCS-limit:
Condensation of
long-range Cooper pairs

Cold atomic gases provide the building blocks of quantum simulators

Quantum “engineering” of interesting Hamiltonians

Ultracold Bose gases: superfluidity (like ^{4}He)

Ultracold Fermi gases (with strong interactions near the unitartiy limit): pairing and superfluidity (BCS, like superconductors)

Optical lattices: crystalline materials

Soon: magnetism in strongly correlated systems