ICAP Summer School, Paris, 2012

Three lectures on quantum gases

Wolfgang Ketterle, MIT

Subtitle

Bose-Einstein condensation

- (Ideal Bose gas)
- Weakly interacting homogenous Bose gas
- Inhomogeneous Bose gas
- (Superfluid hydrodynamics)

Subtitle

Ideal BEC

See thermodynamics textbooks

To remember:

(1) Whether BEC occurs or not depends on density of states:

Power law, depends on dimension and confinement

(2) Even for interacting BEC, normal component is described as ideal gas

T_C

Condensate fraction

BEC B&W

The shadow of a cloud of bosons
as the temperature is decreased

(Ballistic expansion for a fixed time-of-flight)

Temperature is linearly related to the rf frequency
which controls the evaporation

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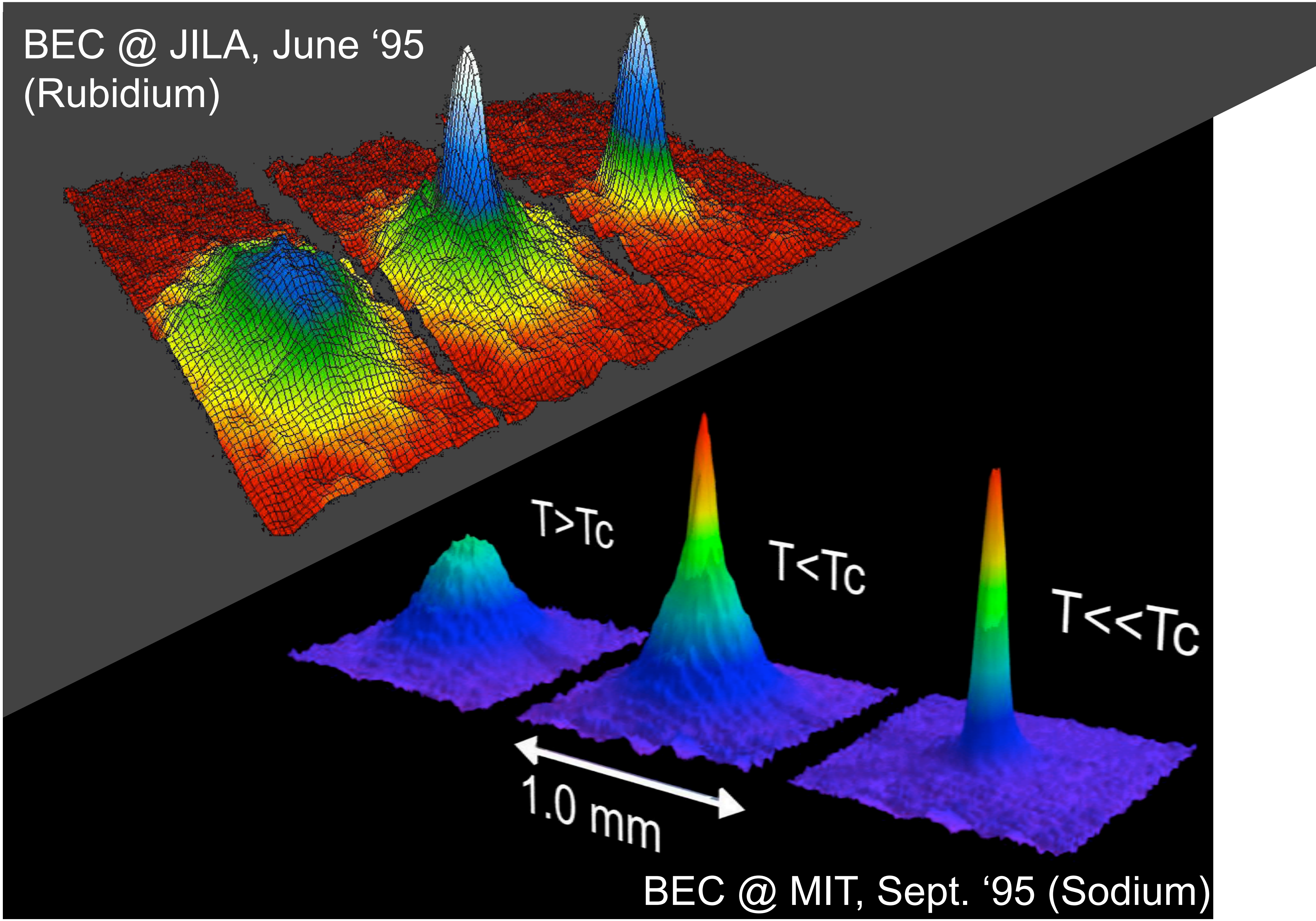
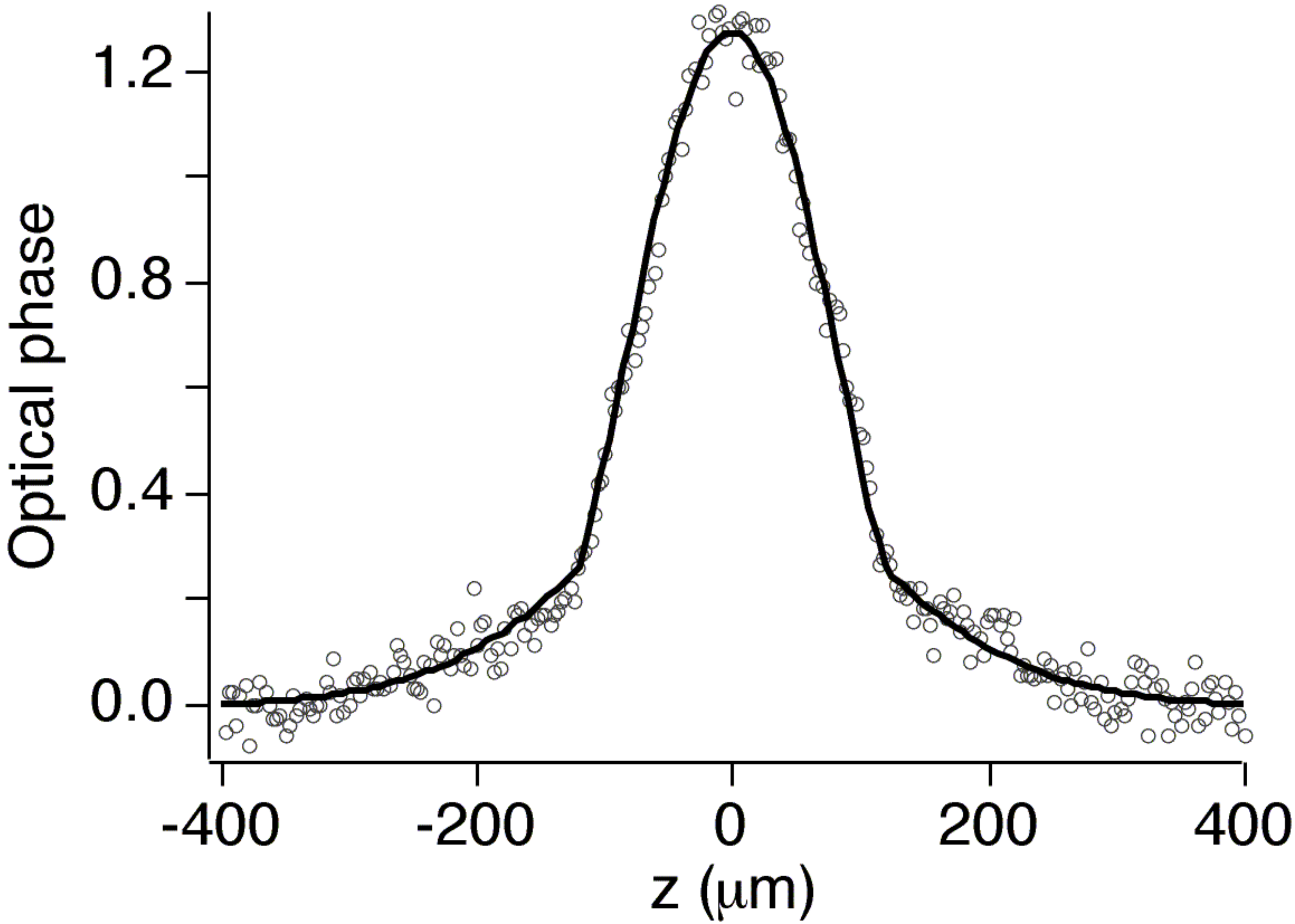
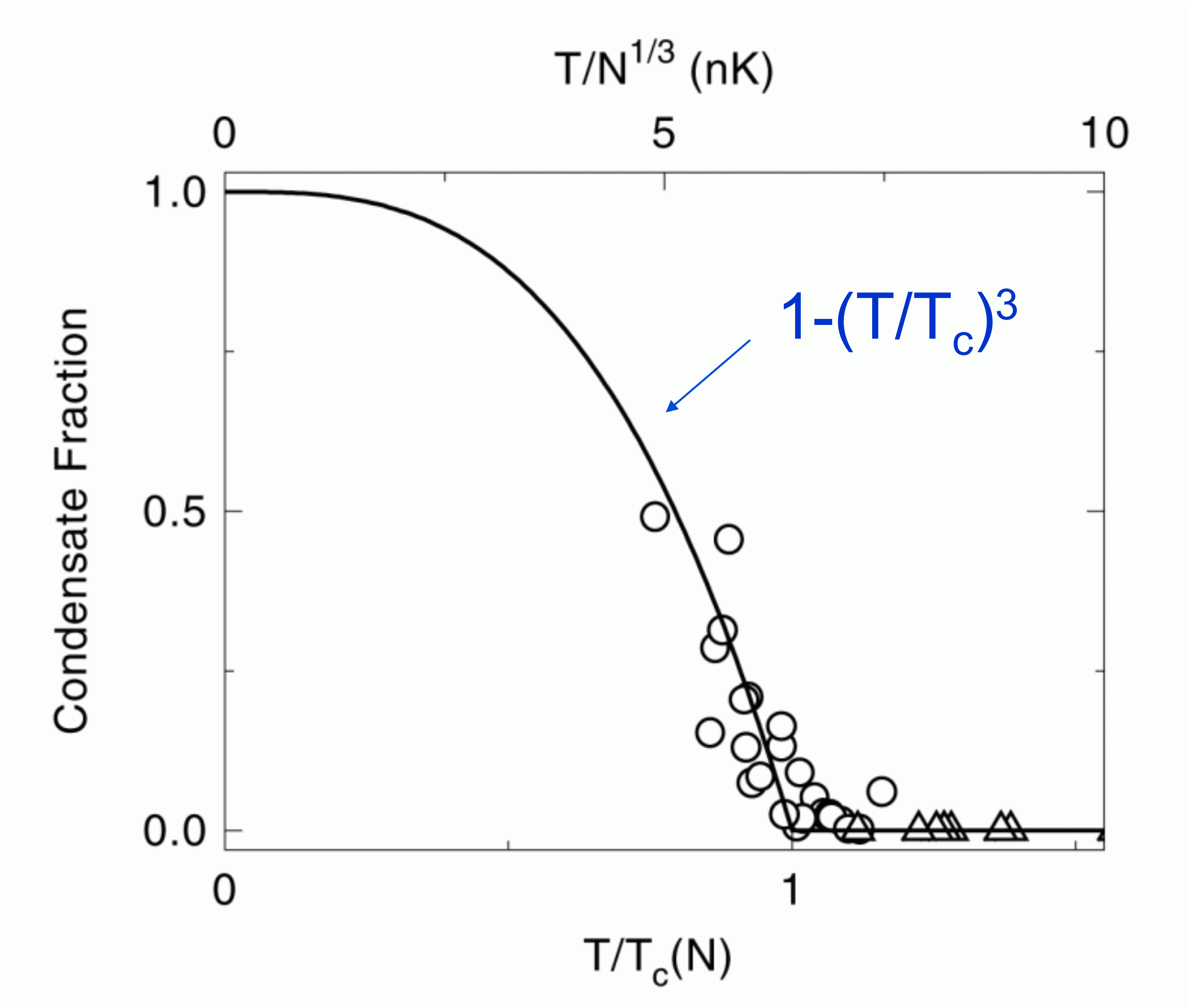
BEC at JILA and MIT



Mixed cloud in phase contrast



Condensate fraction



Subtitle

Homogeneous BEC

Weak interactions

Weakly interacting Bose gas at $T=0$

$$\hat{H}_1 = \frac{1}{2V} \sum U_q \, a^{+}_{p+q} \, a^{+}_{k-q} \, a_k \, a_p$$

$$\llcorner \; U_q = U_0 \Rightarrow U(r) = U_0 \, \delta(r)$$

Weakly interacting Bose gas at $T=0$

$$\hat{H}_1 = \frac{1}{2V}\sum U_q\, a^+_{p+q}\, a^+_{k-q}\, a_k\, a_p$$

$U_q = U_0 \Rightarrow U(r) = U_0\,\delta(r)$

$$U_0 = \frac{4\pi\hbar^2}{m}\,a$$

a scattering length

$$a = \lim_{k\to 0}\left(-\frac{\delta_0}{k}\right) = -f$$

$$\hat{H}_1 = \frac{4\pi a\hbar^2}{m}\sum_{i<j}\delta(r_i - r_j)\cdot\underbrace{\frac{\partial}{\partial r_{ij}}}_{=1 \text{ in first order perturbation theory}} r_{ij}$$

Homogeneous interacting gas

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \frac{U_0}{2V} \sum_{k,k',q} a_{k+q}^\dagger a_{k'-q}^\dagger a_{k'} a_k$$

BEC in $k=0$ state

$$a_0^\dagger |N_0\rangle = \sqrt{N_0+1}\,|N_0+1\rangle$$

$$a_0\,|N_0\rangle = \sqrt{N_0}\,|N_0-1\rangle$$

N_0 large $\quad N_0 \simeq N_0+1, \quad a_0 \simeq a_0^\dagger \simeq \sqrt{N_0}$

(Bogoliubov)

Homogeneous interacting gas

$$H = \sum_{k} \varepsilon_k a_k^{+} a_k + \frac{U_0}{2V} \sum_{k,k',q} a_{k+q}^{+} a_{k'-q}^{+} a_{k'} a_k$$

BEC in $k=0$ state

$$a_0^{+} | N_0 \rangle = \sqrt{N_0+1}\, | N_0+1 \rangle$$

$$a_0 | N_0 \rangle = \sqrt{N_0}\, | N_0-1 \rangle$$

N_0 large $\quad N_0 \simeq N_0+1, \quad a_0 = a_0^{+} = \sqrt{N_0}$

(Bogoliubov)

$$H = \frac{U_0 N_0^2}{2V} + \sum_{k} \varepsilon_k a_k^{+} a_k + \frac{U_0 N_0}{2V} \sum_{k \neq 0} a_k^{+} a_{-k}^{+} + a_k a_{-k} + 2 a_k^{+} a_k + 2 a_{-k}^{+} a_{-k}$$

Homogeneous interacting gas

$$H = \sum_{k} \varepsilon_k a_k^+ a_k + \frac{U_0}{2V} \sum_{k,k',q} a_{k+q}^+ a_{k'-q}^+ a_{k'} a_k$$

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N_0 large $\quad N_0 \simeq N_0+1, \quad a_0 = a_0^+ = \sqrt{N_0}$

(Bogoliubov)

$$H = \frac{U_0 N_0^2}{2V} + \sum_{k} \varepsilon_k a_k^+ a_k + \frac{U_0 N_0}{2V} \sum_{k\neq 0} a_k^+ a_{-k}^+ + a_k a_{-k} + 2 a_k^+ a_k + 2 a_{-k}^+ a_{-k}$$

Introduce $N = N_0 + \frac{1}{2} \sum_{k\neq 0} a_k^+ a_k + a_{-k}^+ a_{-k}$

$$H = \frac{u_0 N^2}{2V} + \frac{1}{2}\sum_{k \neq 0}\left(\varepsilon_k + \frac{N u_0}{V}\right)\left(a_k^+ a_k + a_{-k}^+ a_{-k}\right) +$$

$$\frac{N u_0}{V}\left(a_k^+ a_{-k}^+ + a_k a_{-k}\right)$$

$$H = \frac{u_0 N^2}{2V} + \frac{1}{2}\sum_{k\neq 0}\left(\varepsilon_k + \frac{N u_0}{V}\right)\left(a_k^+ a_k + a_{-k}^+ a_{-k}\right) + \frac{N u_0}{V}\left(a_k^+ a_{-k}^+ + a_k a_{-k}\right)$$

Structure of H:

With $a = a_k$, $b = a_{-k}$

H has only terms of $\mathcal{H} = E_0(a^+a + b^+b) + E_1(a^+b^+ + ba)$

with $[a, a^+] = [b, b^+] = 1$

$$H = \frac{u_0 N^2}{2V} + \frac{1}{2}\sum_{k\neq 0}\left(\varepsilon_k + \frac{N u_0}{V}\right)\left(a_k^+ a_k + a_{-k}^+ a_{-k}\right) + \frac{N u_0}{V}\left(a_k^+ a_{-k}^+ + a_k a_{-k}\right)$$

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bilinear expression solved by Bogoliubov transformation

$a = u\alpha - v\beta^+$ $\qquad$ $b = u\beta - v\alpha^+$

$u^2 - v^2 = 1$ ensures $[\alpha, \alpha^+] = [\beta, \beta^+] = 1$

canonical transformation

$$H = \frac{u_0 N^2}{2V} + \frac{1}{2}\sum_{k \neq 0}\left(\varepsilon_k + \frac{N u_0}{V}\right)\left(a_k^+ a_k + a_{-k}^+ a_{-k}\right) + \frac{N u_0}{V}\left(a_k^+ a_{-k}^+ + a_k a_{-k}\right)$$

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canonical transformation

$\mathcal{H} = (\;) + (\;)(\alpha^+\alpha + \beta^+\beta) + (\;)(\alpha\beta + \beta^+\alpha^+)$

For choice of u, v $\quad = 0$

$$H = \lambda\left(\alpha^{+}\alpha + \beta^{+}\beta\right) + \text{Const}$$

HO with quanta created by α^{+}, β^{+}

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Elementary excitations

$$H = \sum_{k} E_{k}\, a_{k}^{+} a_{k} + \text{const}$$

$$E_{k} = \sqrt{\varepsilon_{k}^{2} + 2\,\varepsilon_{k} N u_{0}/V}$$

$$H = \lambda\left(\alpha^+\alpha + \beta^+\beta\right) + \text{Const}$$

HO with quanta created by α^+, β^+

Elementary excitations

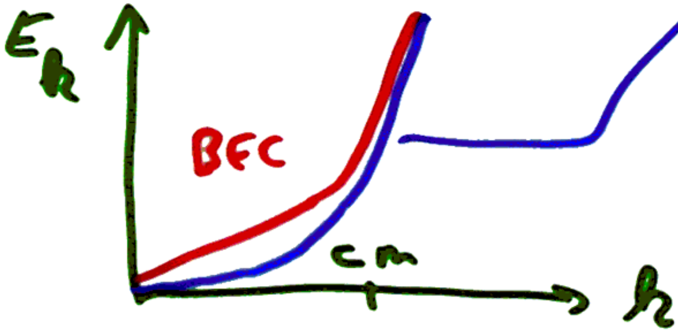
$$H = \sum_k E_k\, a_k^+ a_k + \text{Const}$$

$$E_k = \sqrt{\varepsilon_k^2 + 2\,\varepsilon_k \underbrace{N u_0/V}_{=c^2 m}}$$

$c = \sqrt{u_0 N / V m}$ speed of sound

$$E_k = \sqrt{\left(\frac{\hbar^2 k^2}{2m}\right)^2 + (\hbar c k)^2}$$

$$= \begin{cases} \hbar c k & k \to 0 \quad \text{phonon, sound} \\ \hbar^2 k^2/2m & k \to \infty \quad \text{Free particle} \end{cases}$$

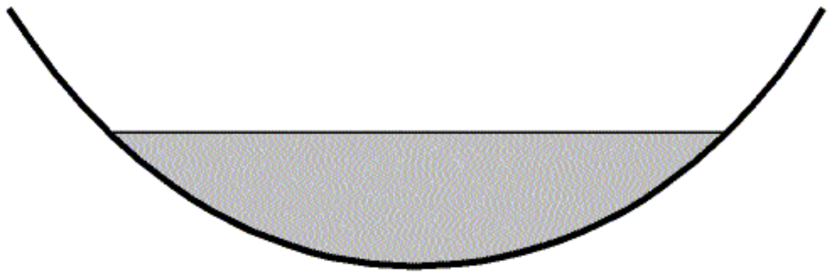


Propagation of sound

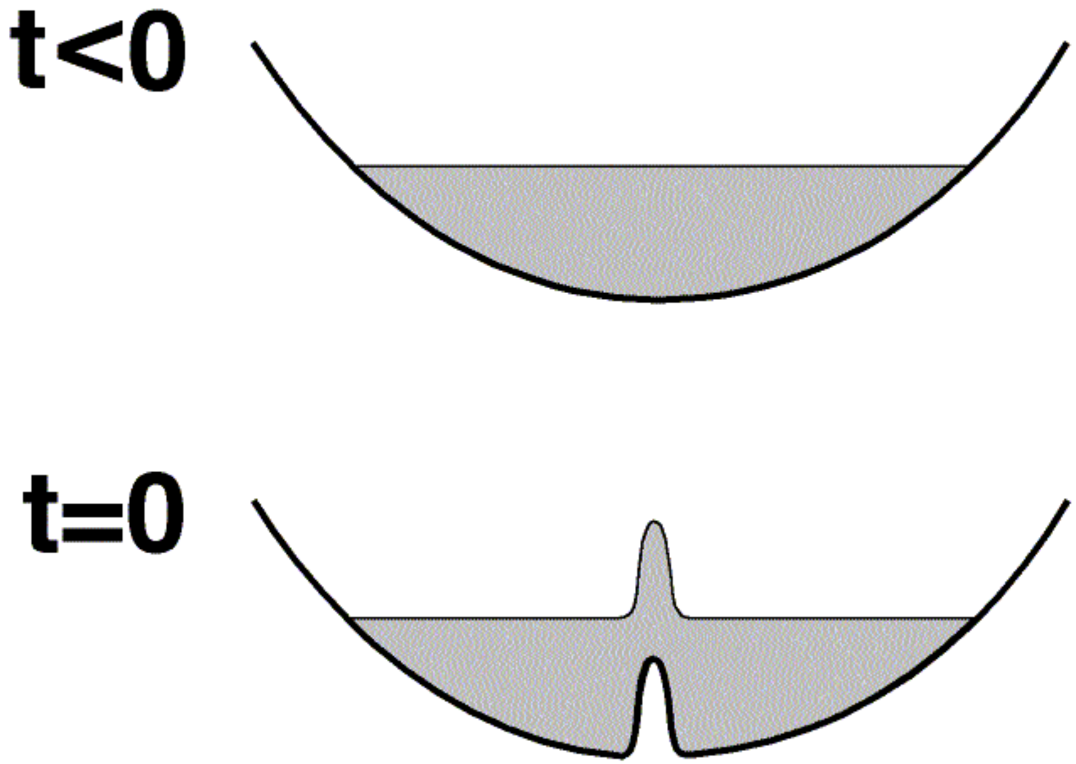
Exciting sound I

Excitation of sound

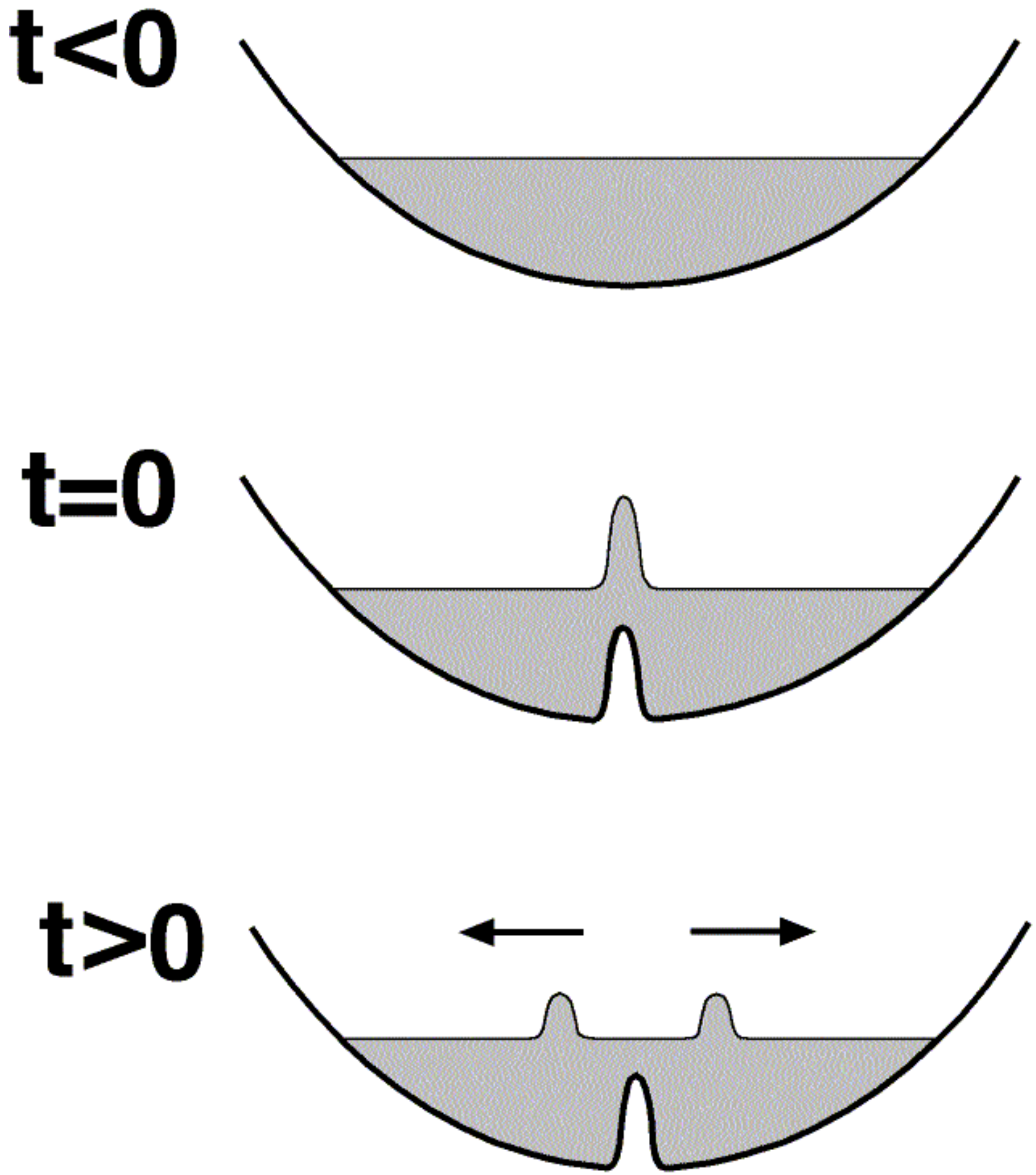
t<0



Excitation of sound

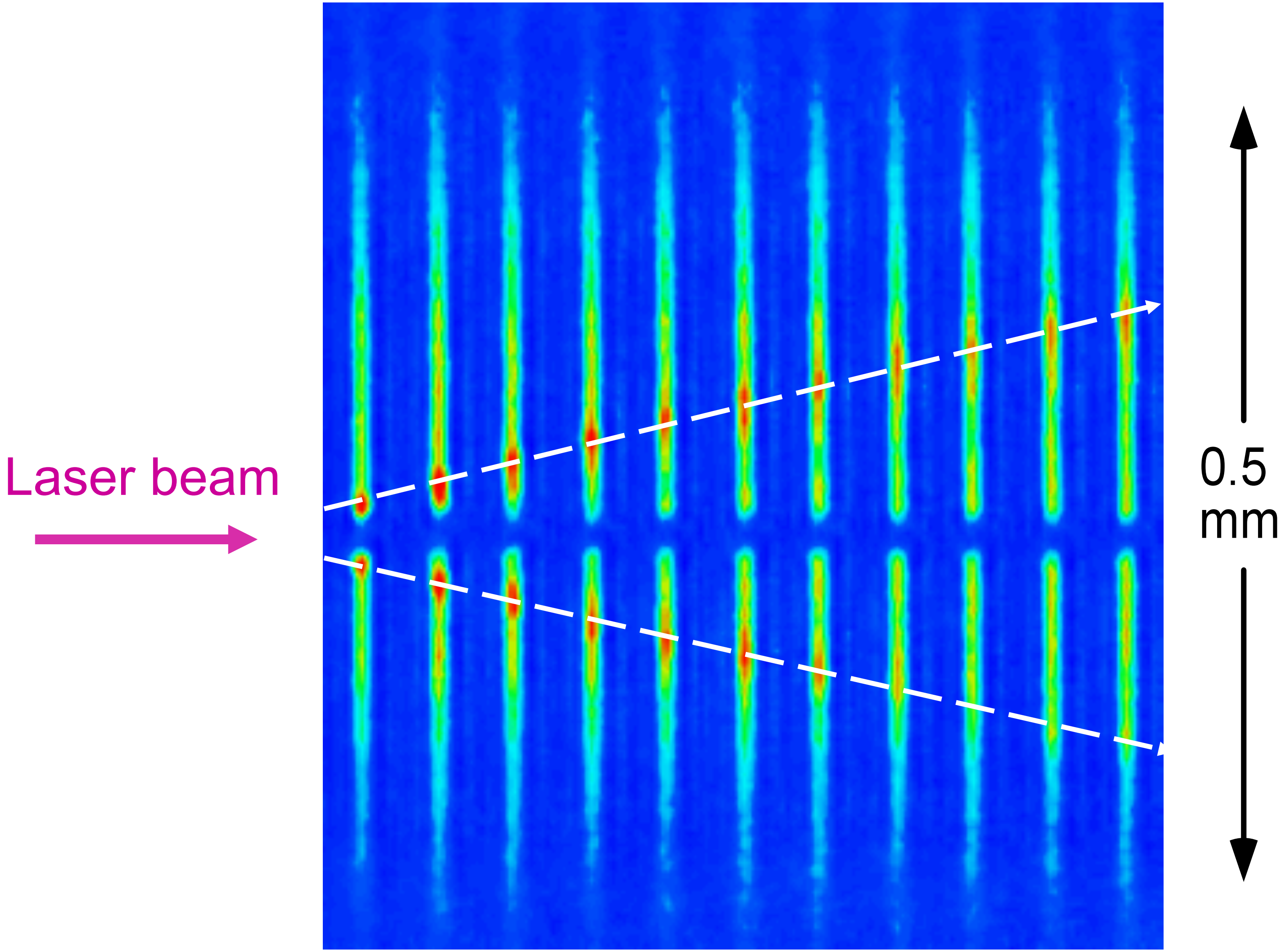


Excitation of sound



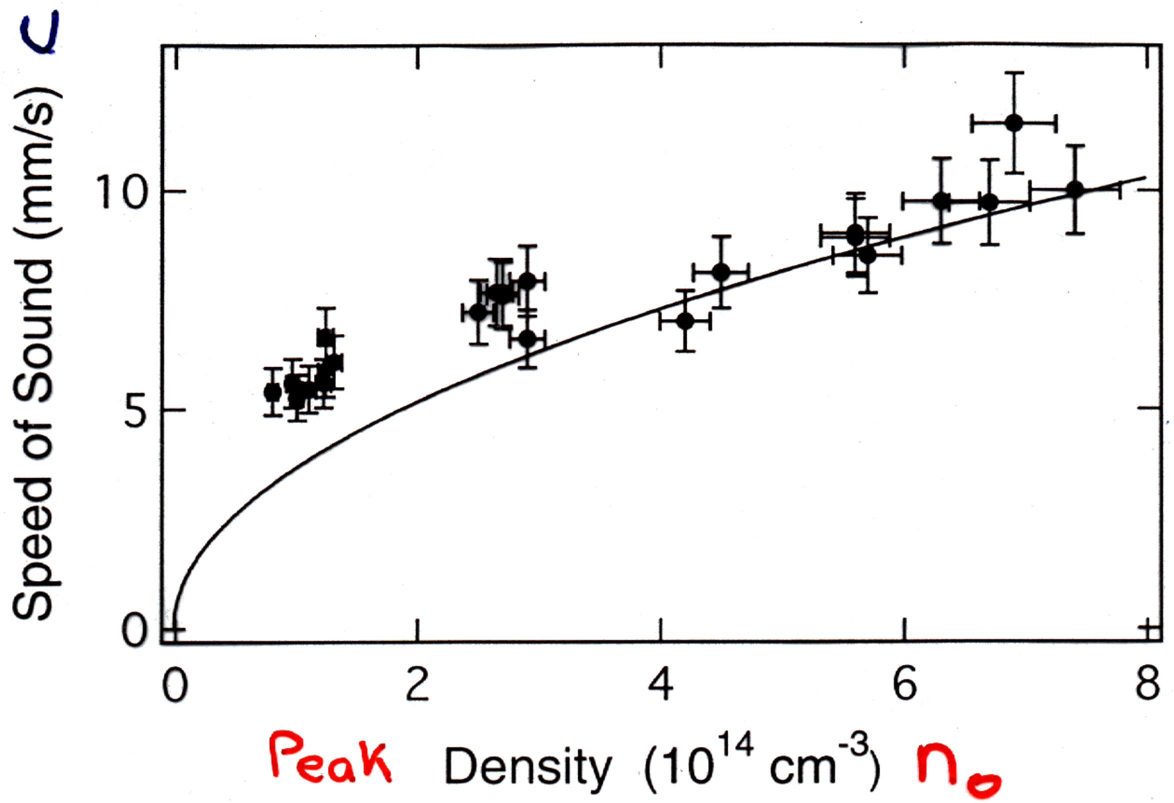
Sound propagation

Sound = propagating density perturbations



1.3 ms per frame

Speed of sound results



$$c = (4\pi\hbar^2 a/m^2)^{1/2}\sqrt{n_0}/\sqrt{2}$$

Bogoliubov 1947

Lee, Huang, Yang 1957

(M. Andrews, D.M. Kurn, H.-J. Miesner, D.S. Durfee, C.G. Townsend, S. Inouye, W.K., PRL 79, 549 (1997))

Bogolinbov solution

$\to E_k$ elementary excitation

Bogoliubov solution

→ E_k elementary excitation

→ ground state energy

$$E_0 = \frac{u_0 n}{2}\left(1 + \frac{128}{15}\sqrt{na^3/\pi}\right)$$

Bogoliubov solution

→ E_k elementary excitation

→ ground state energy

$$E_0 = \frac{U_0 n}{2}\left(1 + \frac{128}{15}\sqrt{na^3/\pi}\right)$$

→ ground state wavefunction

$$\langle n_k \rangle = \frac{v_k^2}{1 - v_k^2}$$

$$\langle n_0 \rangle = N - \sum \langle n_k \rangle = N\Big[1 - \underbrace{\tfrac{8}{3}\sqrt{na^3/\pi}}_{\text{Quantum depletion}}\Big]$$

$$|\psi_0\rangle = \left(|0\rangle\right)^N + \varepsilon$$

90% He
1% alkalis

Dispersion relation

Elementary excitations

$$H = \sum_k E_k \, a_k^+ a_k + \text{const}$$

$$E_k = \sqrt{\varepsilon_k^2 + 2\,\varepsilon_k \underbrace{N u_0 / V}_{= c^2 m}}$$

$c = \sqrt{u_0 N / V m}$ speed of sound

$$E_k = \sqrt{\left(\frac{\hbar^2 k^2}{2m}\right)^2 + (\hbar c k)^2}$$

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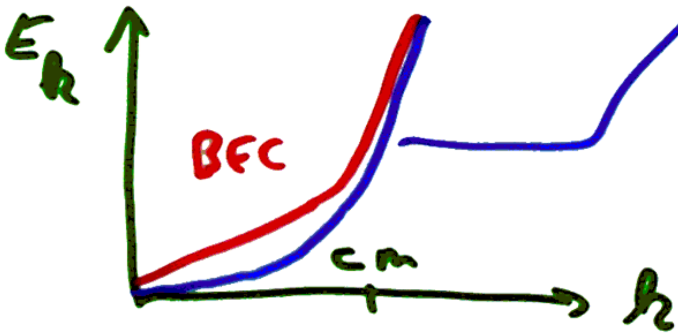
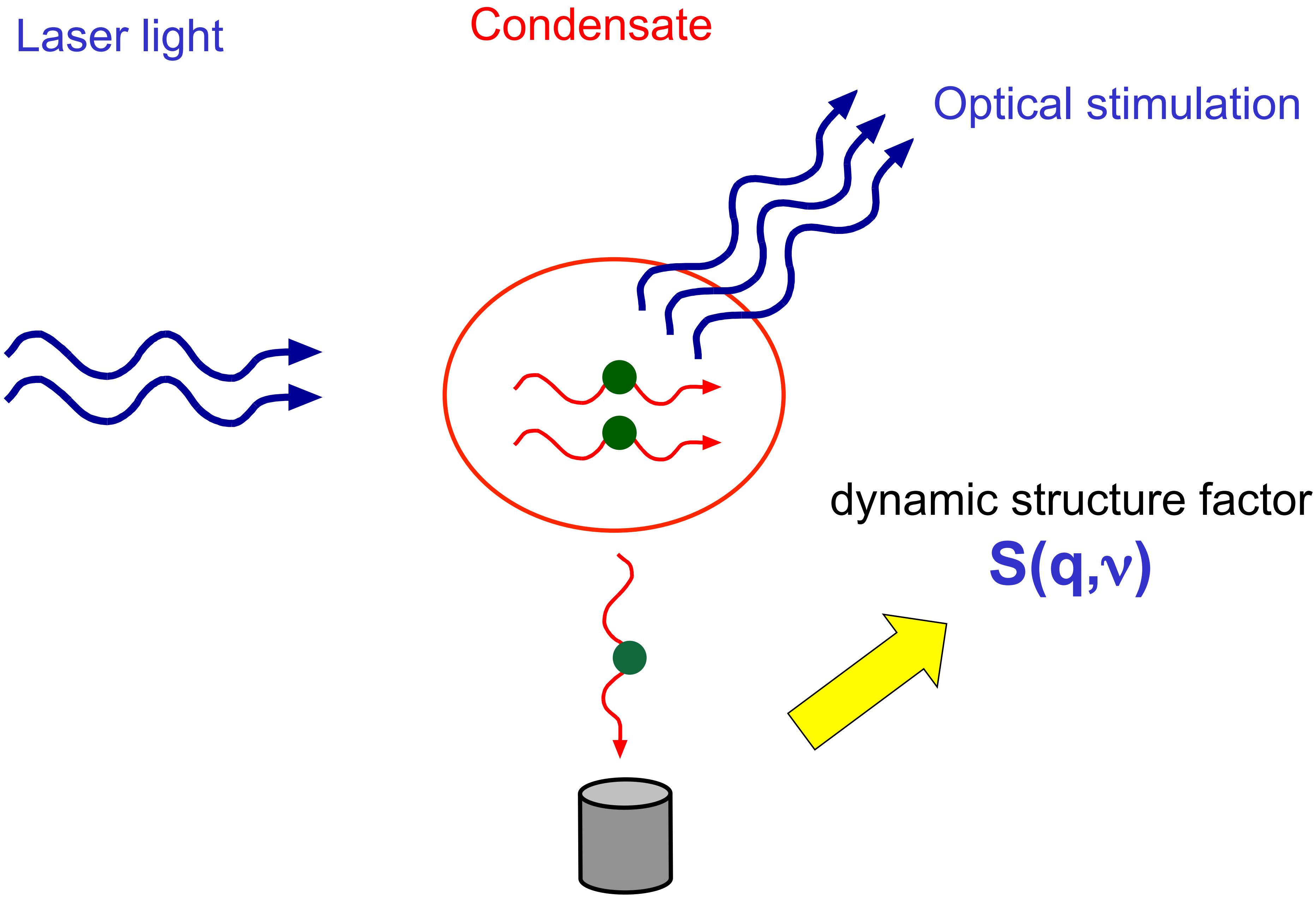


Light scattering VII



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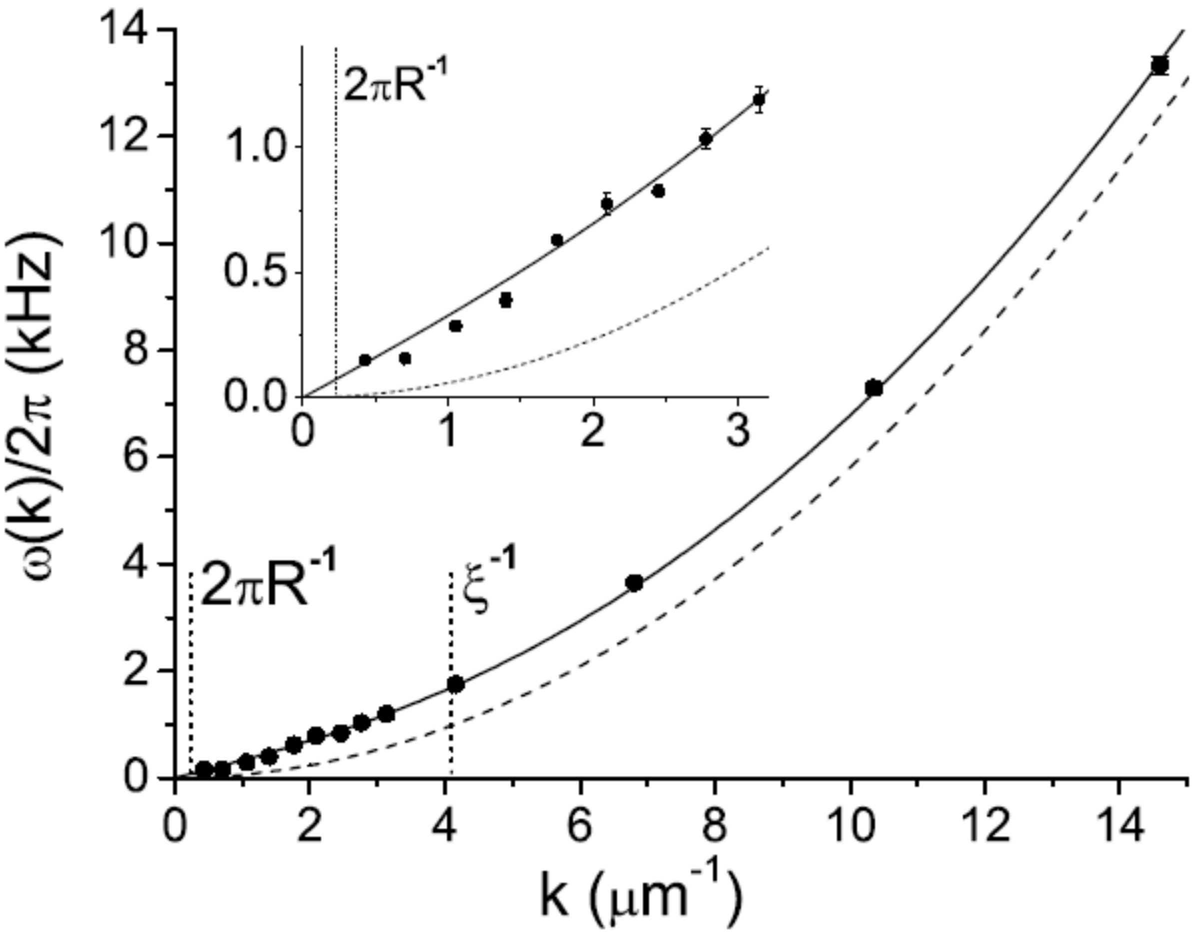
Excitation Spectrum of a Bose-Einstein Condensate

J. Steinhauer, R. Ozeri, N. Katz, and N. Davidson



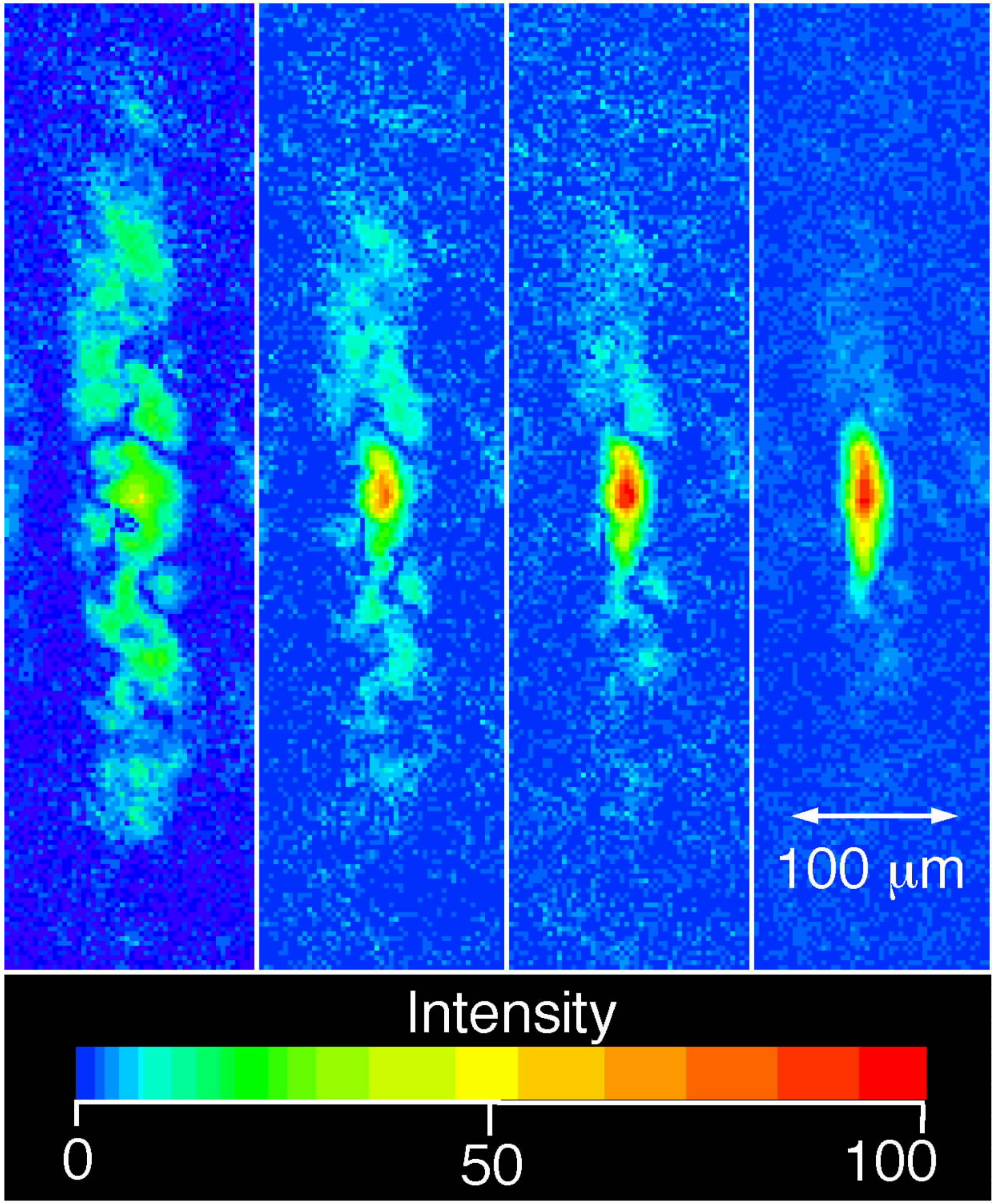
Subtitle

Inhomogeneous BEC

Phase transition, dark ground

A live condensate in the magnetic trap

(seen by dark-ground imaging)



The inhomogeneous Bose gas

New Feature: trapping potential

→ go to $\vec{r}$ space

$$\hat{H} = \int d^3r\, \hat{\psi}^+(r) \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{trap} \right] \hat{\psi}(r)$$
$$+ \frac{1}{2} \int d^3r \int d^3r'\, \hat{\psi}^+(r)\hat{\psi}^+(r')\, U(r-r')\, \hat{\psi}(r')\hat{\psi}(r)$$

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The inhomogeneous Bose gas

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$$+\frac{1}{2}\int d^3r \int d^3r'\, \hat{\Psi}^+(r)\hat{\Psi}^+(r')\underbrace{U(r-r')}_{U_0\,\delta(r-r')}\hat{\Psi}(r')\hat{\Psi}(r)$$

$$\frac{U_0}{2}\int d^3r\, \hat{\Psi}^+(r)\hat{\Psi}^+(r)\hat{\Psi}(r)\hat{\Psi}(r)$$

Heisenberg equation of motion for $\hat{\Psi}$

$$i\hbar\frac{\partial\hat{\Psi}(r,t)}{\partial t} = [\hat{\Psi}, \hat{H}]$$ cannot be solved

Bogoliubov: Condensate operator $\longrightarrow$ c-number

$$\hat{\psi}(r,t) = \psi(r,t) + \tilde{\psi}(r,t)$$

$\langle \hat{\psi}(r,t) \rangle$

quantum (+ thermal) fluctuations

Bogoliubov: condensate operator → c-number

$$\hat{\psi}(r,t) = \psi(r,t) + \tilde{\psi}(r,t)$$

$\psi(r,t)$: $\langle \hat{\psi}(r,t) \rangle$

$\tilde{\psi}(r,t)$: quantum (+ thermal) fluctuations

$T \to 0$ neglect $\tilde{\psi}$

→ NLSE, Gross-Pitaevskii equation for $\psi(r,t)$

$$i\hbar \frac{\partial \psi}{\partial t} = \underbrace{\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{trap} + u_0 \underbrace{N|\psi(r,t)|^2}_{n(r,t)\ \text{density}} \right]}_{\text{mean field potential}} \psi(r,t)$$

$u_0 \sum \delta(r) \to u_0 n(r,t)$

Thomas-Fermi approximation

ξ healing length

$$\frac{\hbar^2}{2m\xi^2} = U_0 n$$

$$\xi = (8\pi a n)^{-1/2}$$

Thomas-Fermi approximation

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neglect kinetic energy term

$$\left[\cancelto{0}{-\frac{\hbar^2}{2m}\nabla^2} + V_{ext} + U_0 |\psi(r)|^2 \right] \psi(r) = \mu \psi(r)$$

$$\left[V_{ext} + U_0 |\psi(r)|^2 - \mu \right] \psi(r) = 0$$

Thomas-Fermi approximation

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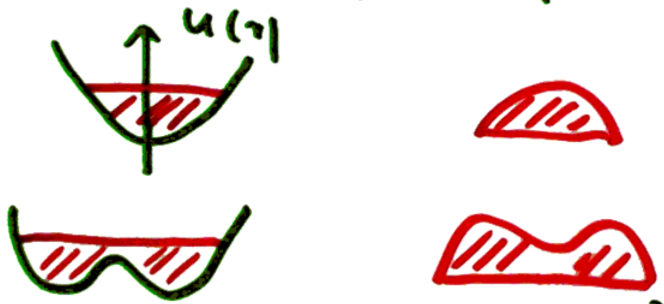
Solution: $|\psi(r)|^2 = \frac{1}{U_0}\left(\mu - V(r)\right)$



Shape is inverted trapping potential

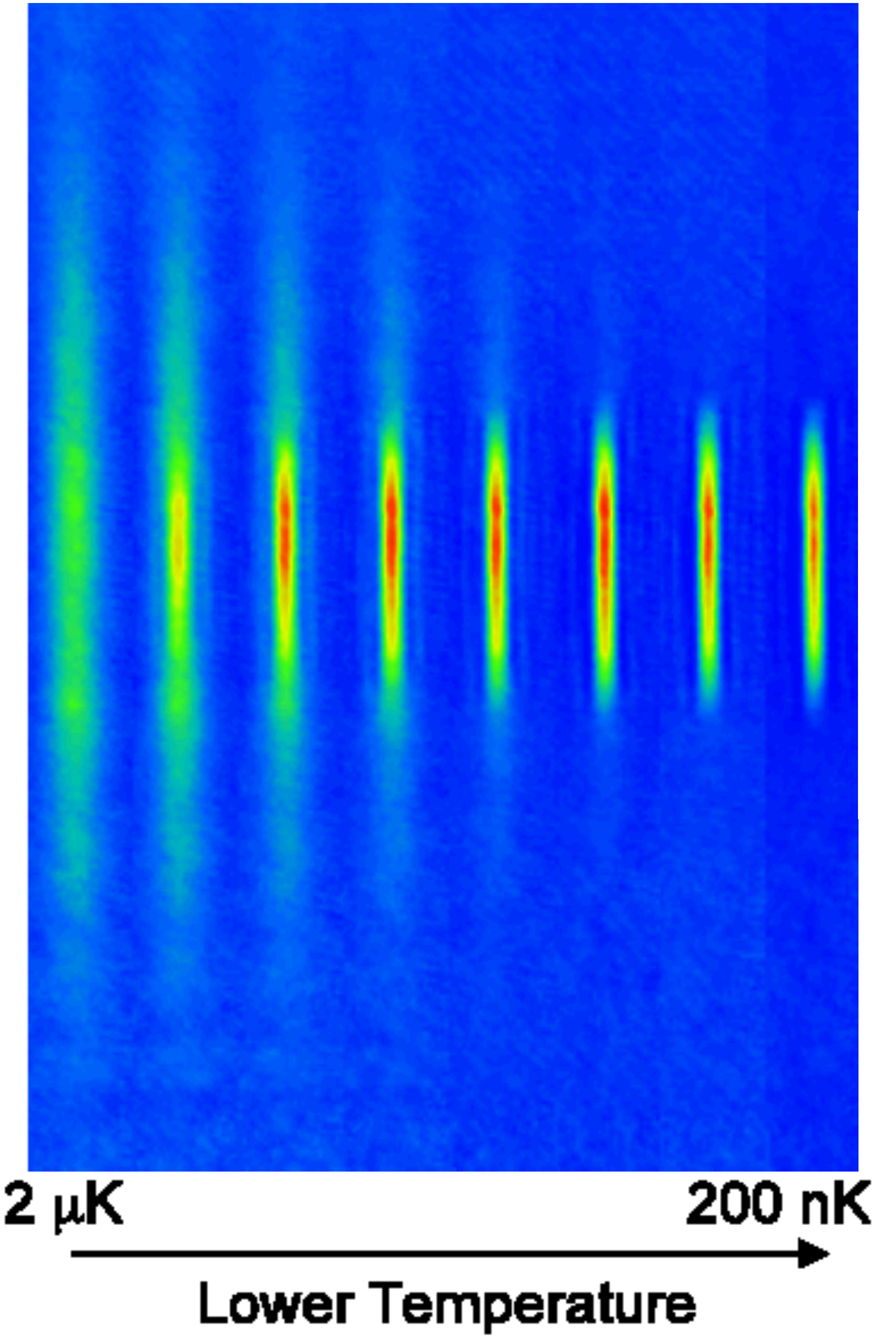
HO: $V(r) = \frac{1}{2} m\omega^2 r^2$

$$\mu = \frac{\hbar\omega}{2}\underbrace{\left(\frac{15 N a}{a_{osc}}\right)^{2/5}}_{\times} \qquad a_{osc} = \sqrt{\frac{\hbar}{m\omega}}$$

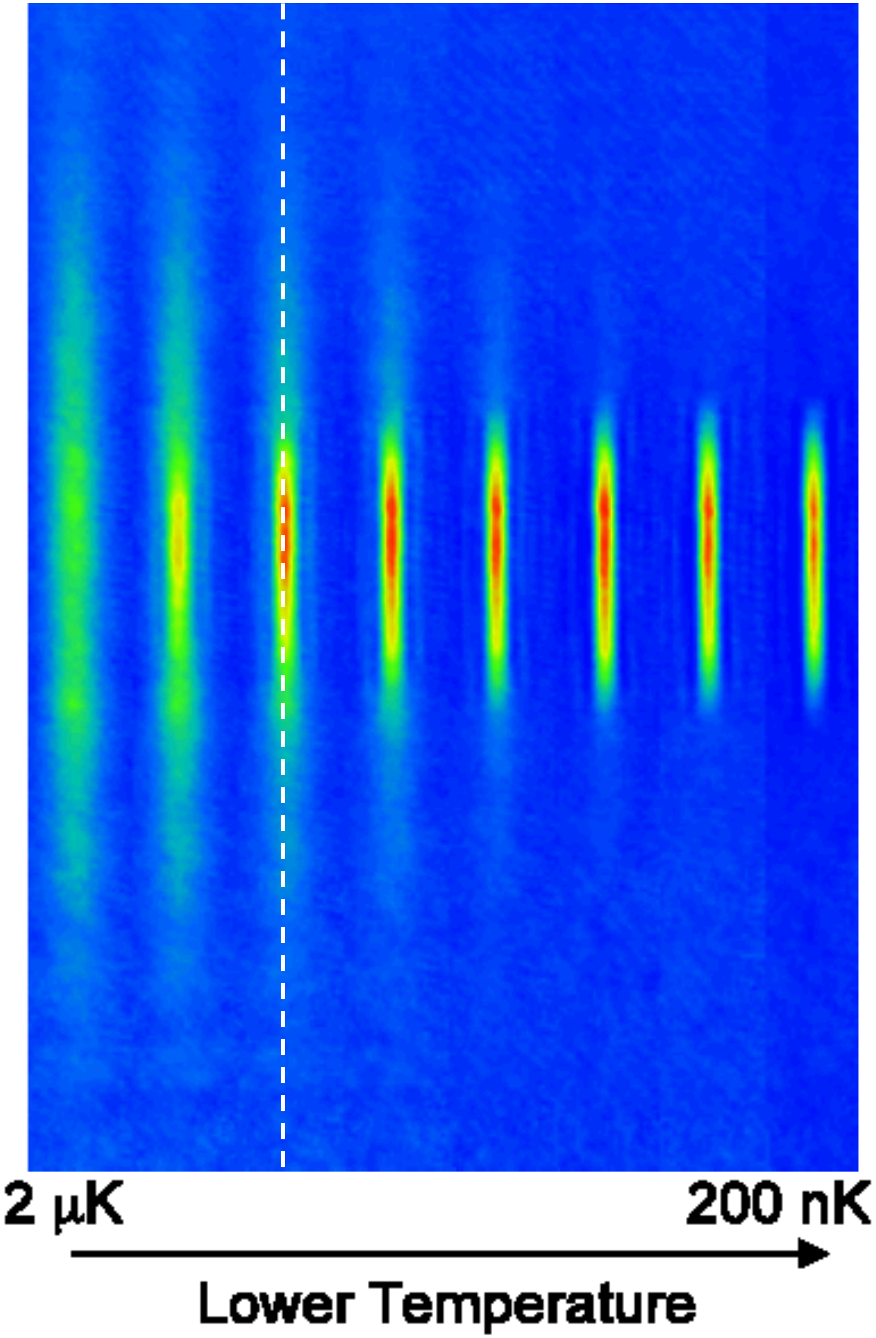
$$\mu = x^{2/5}\,\frac{\hbar\omega}{2} = \frac{1}{2} m \omega^2 R^2$$

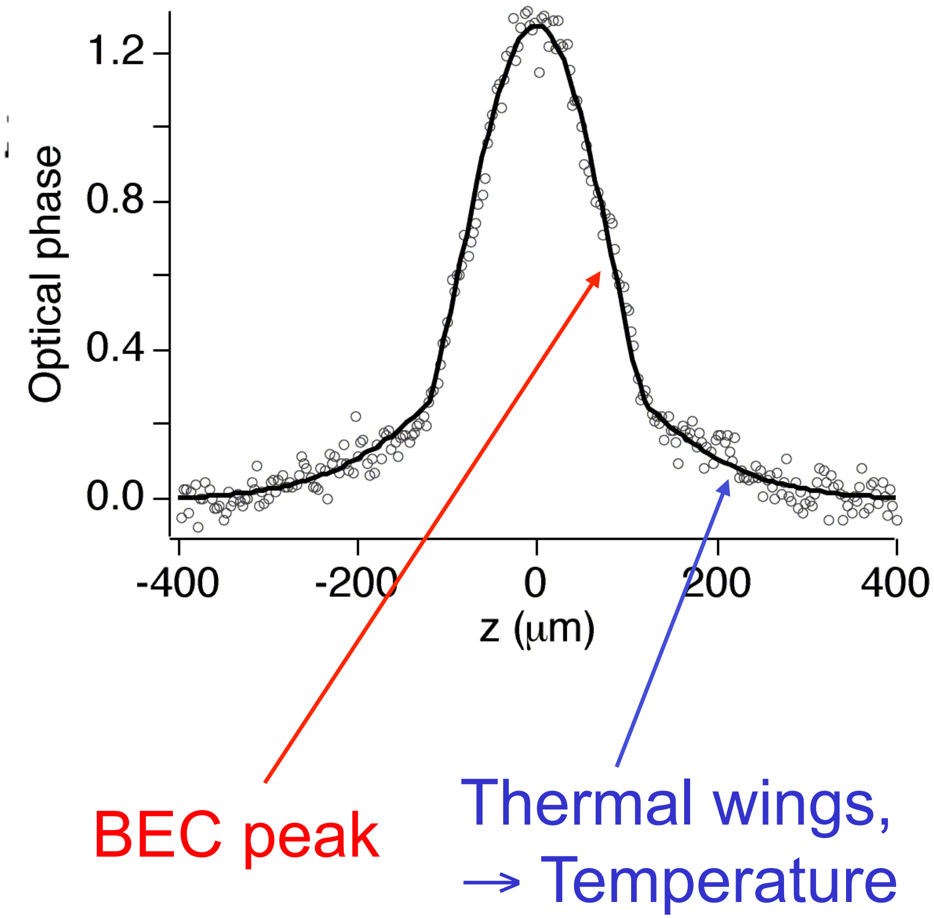
$$R = a_{osc}\, x^{1/5}$$

Phase transition, phase contrast

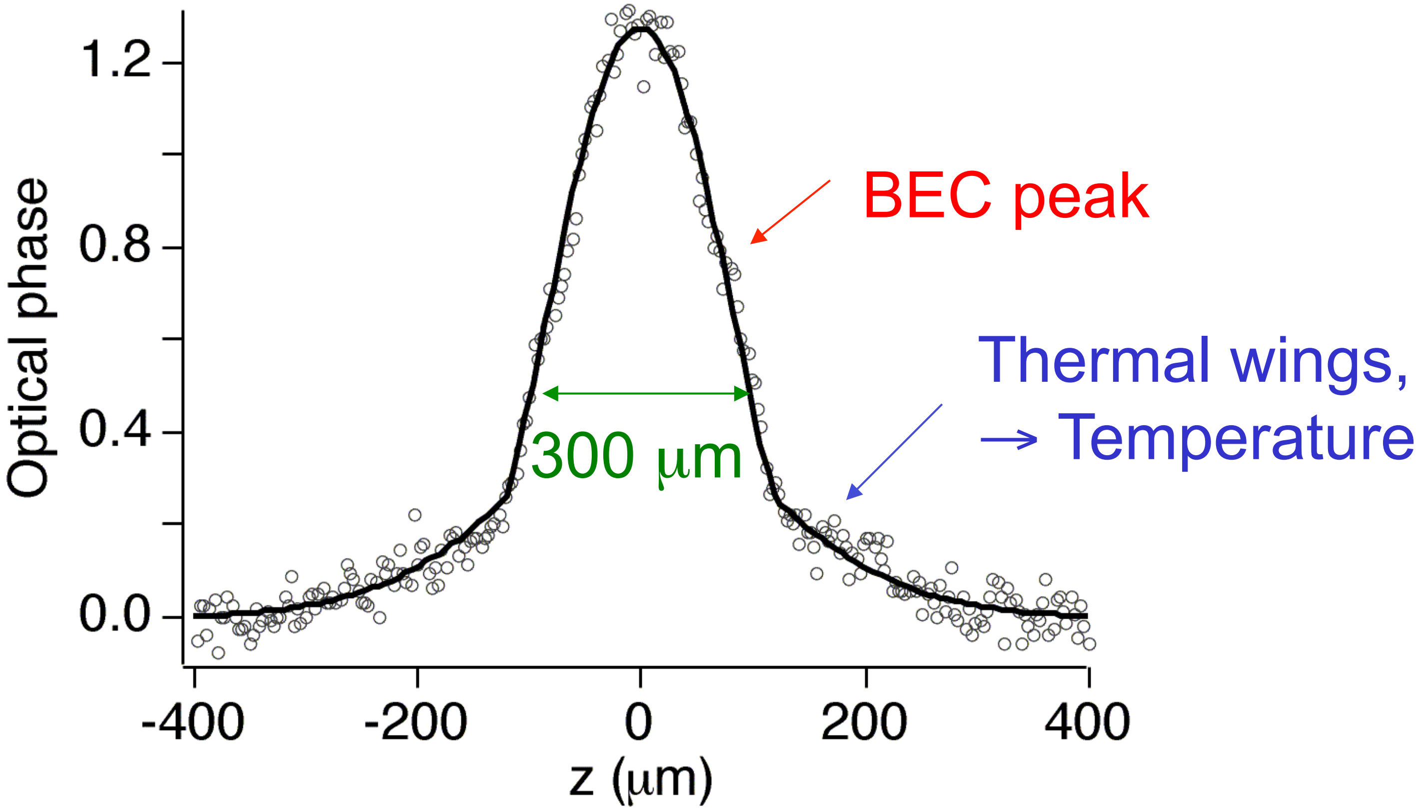


Phase transition, phase contrast





Mixed cloud in phase contrast



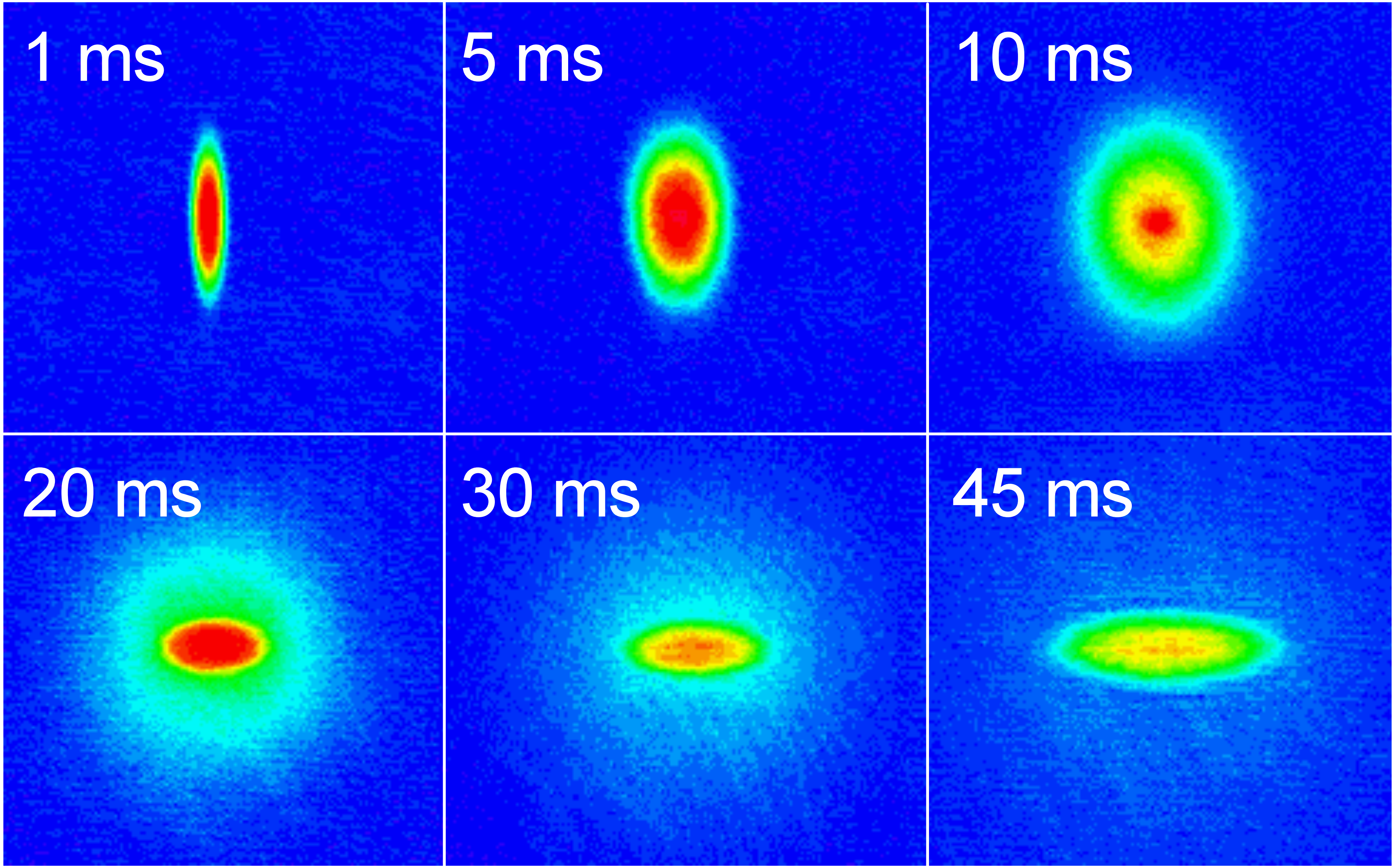
rms width of harmonic oscillator ground state 7 μm

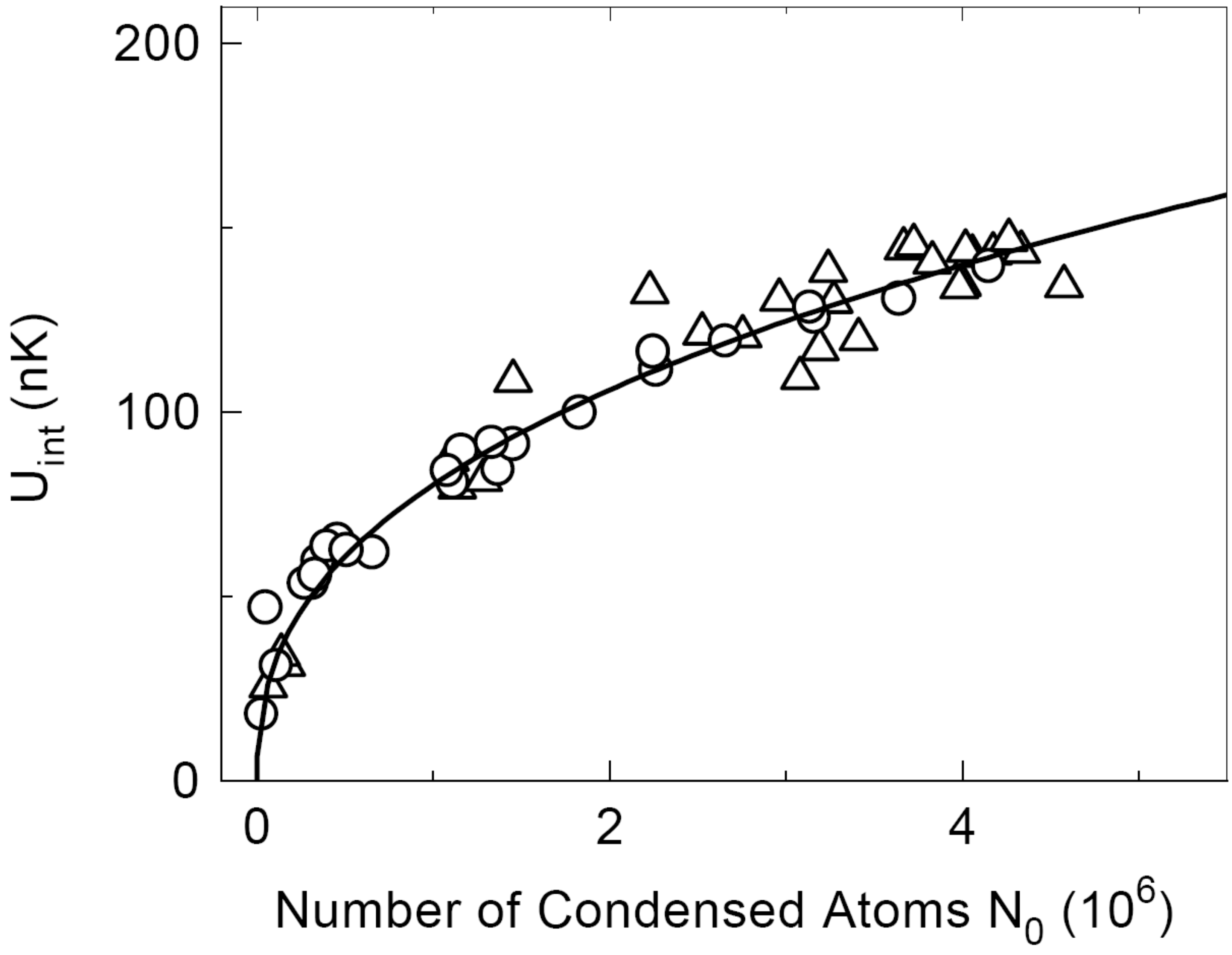
⇒ (repulsive) interactions

⇒ interesting many-body physics

Anisotropic expansion

Signatures of BEC: Anisotropic expansion





200
100
0
U_{int} (nK)
0
2
4
Number of Condensed Atoms N_0 (10^6)

Subtitle

Vortices

→ NLSE, Gross-Pitaevskii equation for $\psi(r,t)$

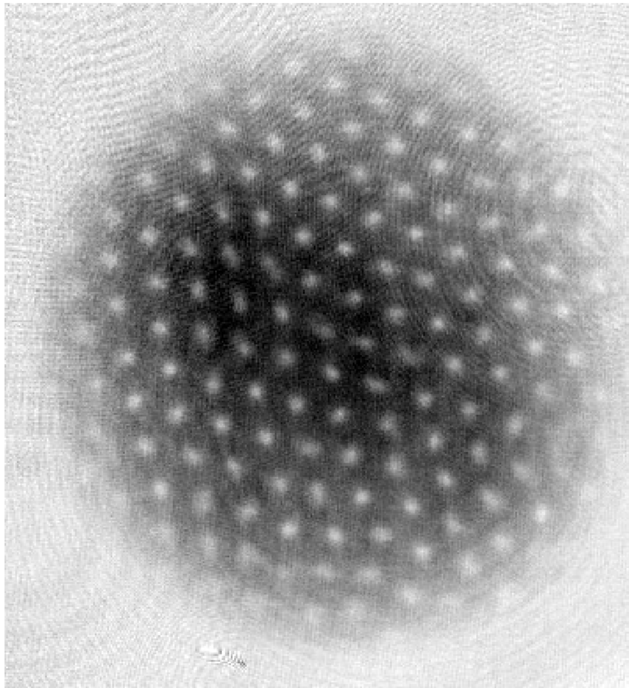
$$i\hbar \frac{\partial \psi}{\partial t} = \Big[-\frac{\hbar^2}{2m}\nabla^2 + V_{trap} + \underbrace{U_0 \underbrace{N|\psi(r,t)|^2}_{n(r,t) \text{ density}}}_{\text{mean field potential}}\Big]\, \psi(r,t)$$

$$U_0 \sum \delta(r) \longrightarrow U_0\, n(r,t)$$

Spinning a Bose-Einstein condensate

The rotating bucket experiment with a superfluid gas
100,000 thinner than air

Rotating
green laser beams



Two-component vortex
- Boulder, 1999

Single-component vortices
- Paris, 1999
- Boulder, 2000
- MIT 2001
- Oxford 2001

J. Abo-Shaeer, C. Raman, J.M. Vogels, W.Ketterle, Science, 4/20/2001

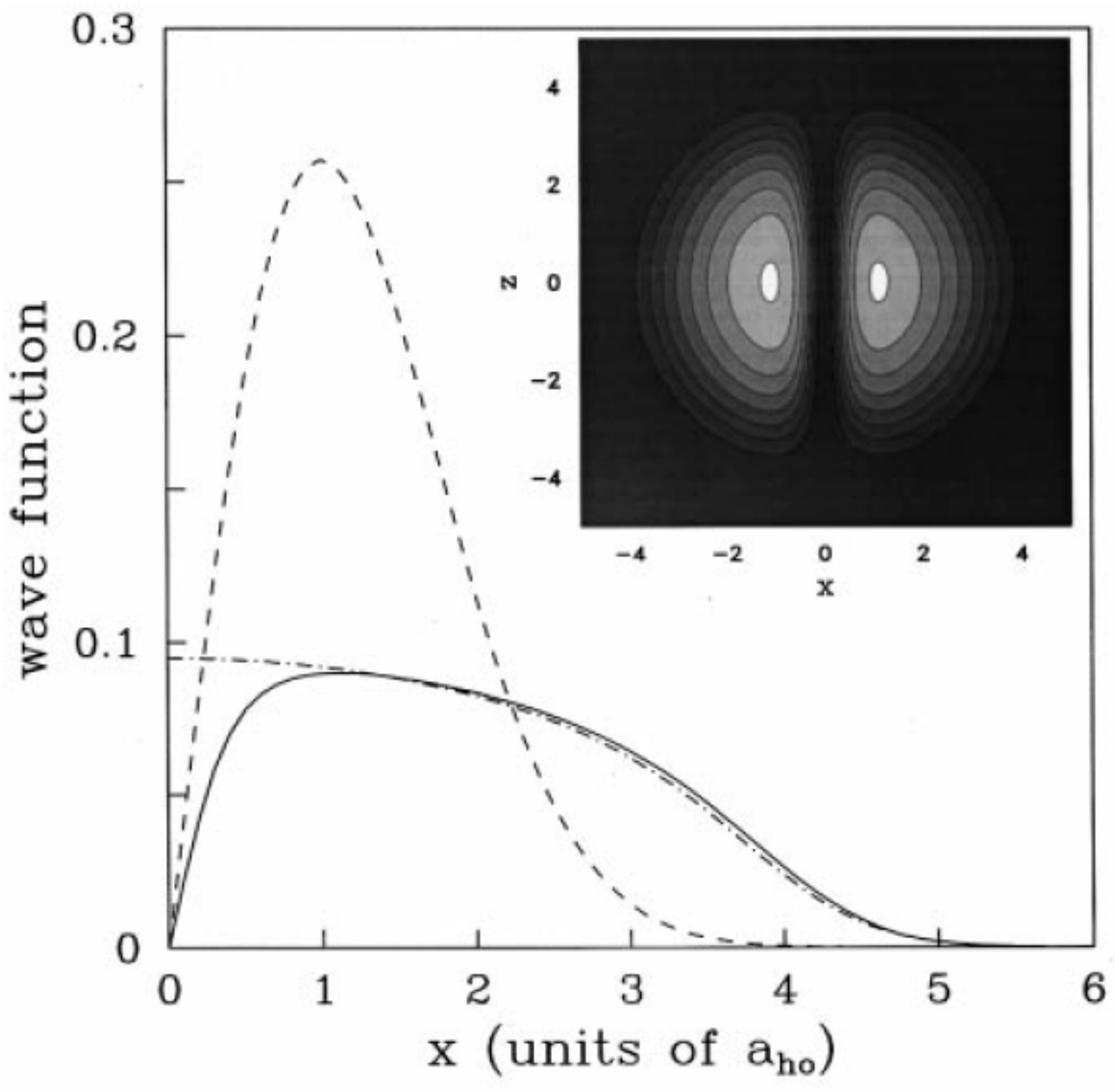
GPE for vortices

Order parameter

$$\phi(\mathbf{r}) = \phi_v(r_\perp, z)\exp[i\kappa\varphi]$$

GPE for modulus

$$\left[-\frac{\hbar^2\nabla^2}{2m} + \frac{\hbar^2\kappa^2}{2mr_\perp^2} + \frac{m}{2}(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2) + g\phi_v^2(r_\perp, z)\right]\phi_v(r_\perp, z) = \mu\phi_v(r_\perp, z)$$



F. Dalfovo, S. Giorgini, L.P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999)

Title

Optical Lattices

One way to achieve strong interactions:
reduce kinetic energy in a lattice
(suppress tunneling, increase effective mass)

Superfluid to Mott Insulator Transition

Optical lattice (cubic)

$$V(x,y,z) = V_0 \left(\sin^2 kx + \sin^2 ky + \sin^2 kz \right)$$

QM in periodic potentials (1D)

$$H = \frac{\hbar^2}{2m} \nabla^2 - V_0 \sin^2(kx)$$

$$\psi_{q,n} = e^{iqx/\hbar} u_{q,n}(x)$$ Bloch theorem

Quasi momentum (q) Band index (n)

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$$\psi_{q,n} = e^{iqx/\hbar} \, u_{q,n}(x) \qquad \text{Bloch theorem}$$

q: Quasi momentum; n: Band index

$$u_{q,n}(x) = \underbrace{\sum_\ell c_\ell^{q,n} e^{i2\ell kx}}_{\text{periodic}} \qquad \text{Fourier expansion}$$

$$-V_0 \sin^2(kx) = \sum_r \tilde{V}_r e^{i2rkx}$$

$$\tilde{V}_{-1} = \tilde{V}_1 = V_0/4$$

$$\tilde{V}_0 = -V_0/2$$

Insert into $H\psi_{q,n} = E_{q,n}\psi_{q,n}$

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$$H = \frac{\hbar^2}{2m} \nabla^2 - V_0 \sin^2(kx)$$

$$\psi_{q,n} = e^{iqx/\hbar} \underbrace{u_{q,n}(x)}_{\text{periodic}} \qquad \text{Bloch theorem}$$

Quasi momentum (q), Band index (n)

Fourier expansion

$$-V_0 \sin^2(kx) = \sum \tilde{V}_r e^{i2rkx} \qquad \Big| \qquad \sum_\ell c_\ell^{q,n} e^{i2\ell kx}$$

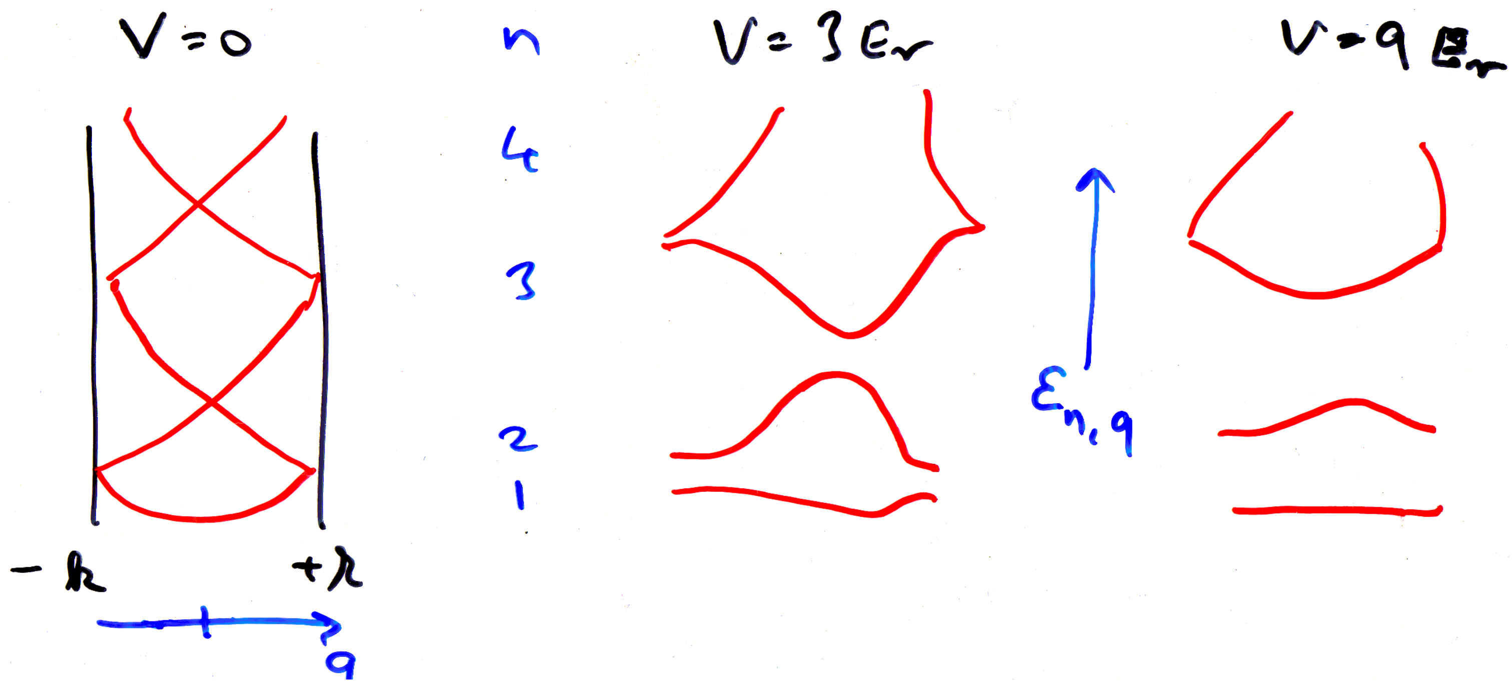
$$\tilde{V}_{-1} = \tilde{V}_1 = V_0/4$$

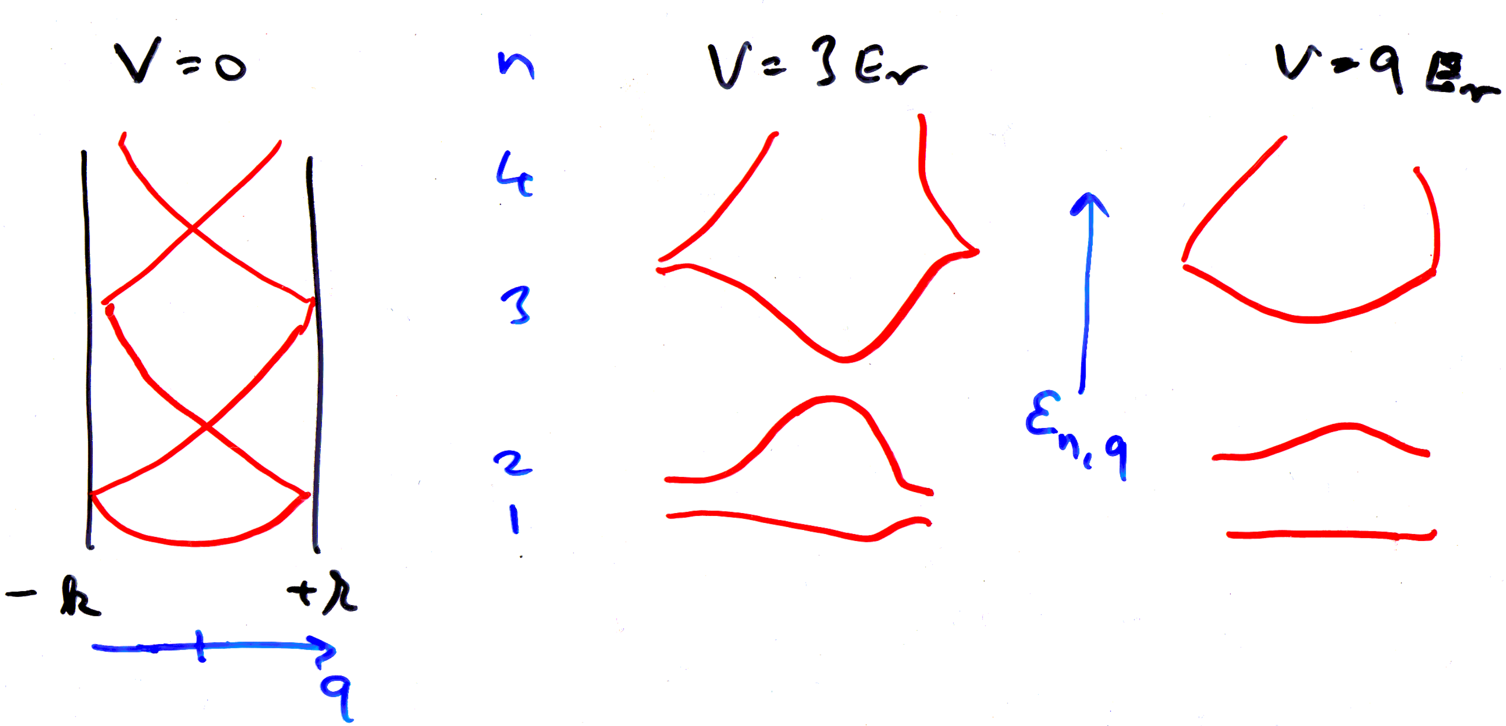
$$\tilde{V}_0 = -V_0/2$$

Insert into $H\psi_{q,n} = E_{q,n}\psi_{q,n}$

$$\sum_{\ell'} e^{i(q+2\ell'\hbar k)x} \underbrace{\left[\left(\frac{(q+2\ell'\hbar k)^2}{2m} - E_{q,n} \right) c_{\ell'}^{q,n} + \sum V_r c_{\ell'-r}^{q,n} \right]}_{=0} = 0$$

Set of linear equations


V=0
n
V = 3 Er
V = 9 Er
4
3
2
1
− k
+ k
q
En,q

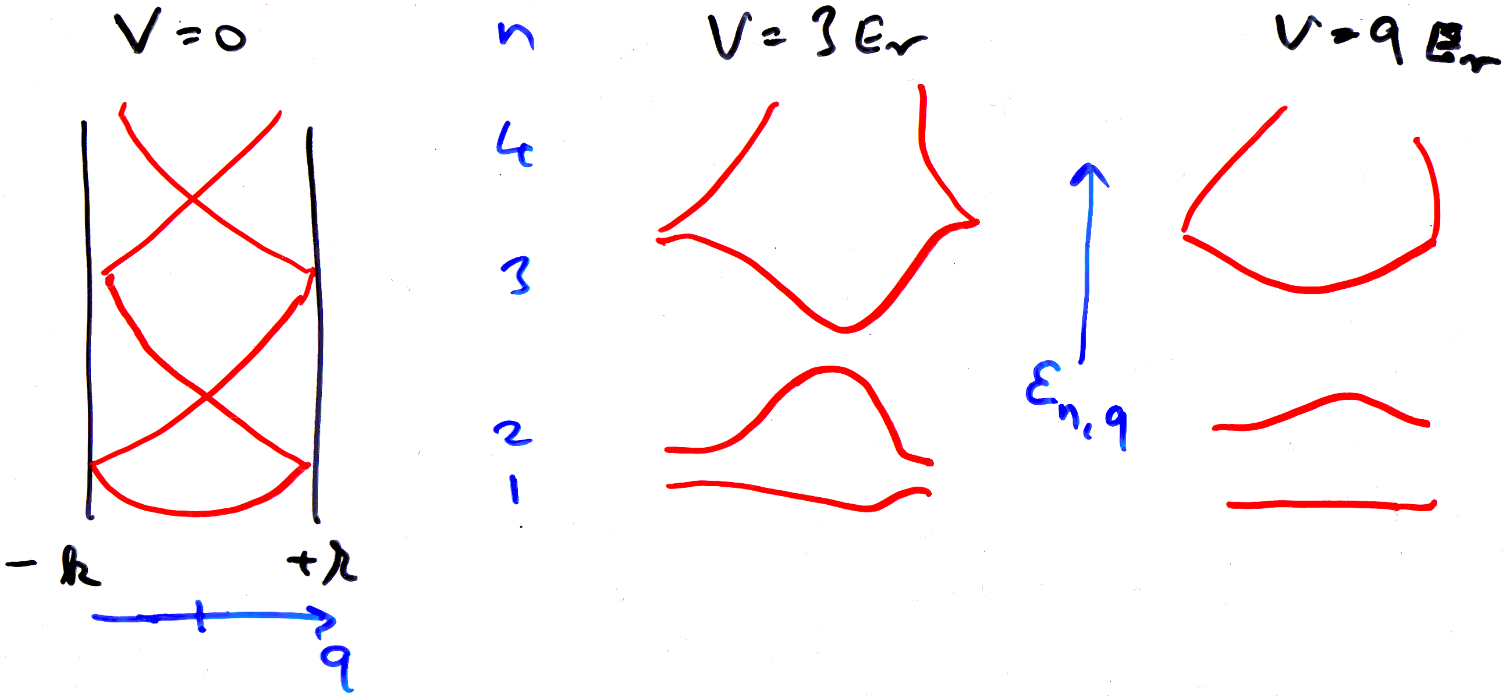


$\frac{V_0}{E_r} \gg 1$ tight binding case

$\Rightarrow$ harm. confinement at each lattice site

$E_r = \frac{\hbar^2 k^2}{2m}$

$\hbar \omega_0 = 2 E_r (V_0/E_r)^{1/2}$



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$$E_r = \frac{\hbar^2 k^2}{2m}$$

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lowest band

$$\mathcal{E}_1(q) = \frac{3}{2}\hbar\omega_0 - 2J(\cos q_x a + \cos q_y a + \cos q_z a)$$

$$a = \frac{\lambda}{2} = \frac{\pi}{k}$$

J : 4 x band width

Wannier Functions (orthogonal basis set)
localized

$$\underbrace{w_n(x - x_j)}_{\text{Site}} = \underbrace{N^{-1/2}}_{\text{normalization}} \sum e^{iqx_j} \psi_{q,n}(x)$$

Wannier Functions (orthogonal localized basis set)

$$w_n(x - \underbrace{x_j}_{\text{site}}) = \underbrace{N^{-1/2}}_{\text{normalization}} \sum e^{iqx_j} \psi_{q,n}(x)$$

$$J = \int w_1(x - x_j) \left[\frac{\hbar^2}{2m} \nabla^2 + V(x) \right] w_1(x - x_l)$$

tunneling from site j to l

J "tunneling energy"

$J/\hbar$ "tunneling rate"

Wannier Functions (orthogonal basis set)
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tunneling from site j to ℓ

J "tunneling energy"

$J/\hbar$ "tunneling rate"

deep lattice $\quad J = \frac{4}{\sqrt{\pi}} E_r \left(\frac{V_0}{E_r}\right)^{3/4} \exp\left[-2\sqrt{V_0/E_r}\right]$

Tight binding: w_1 is Gaussian (solution for HO with freq. ω_0)

Now: Interactions

Mean Field interactions

$$U(x) = \underbrace{\frac{4\pi\hbar^2 a_s}{m}}_{g}\,\delta(x)$$

On-site interactions:

$$U = g \int [w_i(x)]^4 \, d^3x = \sqrt{\frac{8}{\pi}}\, k a_s E_r \left[\frac{V_0}{E_r}\right]^{3/4}$$

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$$H_{Full} = \int d^3x\, \psi^+(x)\left(\frac{p^2}{2m} + V(x)\right)\psi(x)$$
$$+ \frac{g}{2}\int d^3x\, \psi^+(x)\psi^+(x)\psi(x)\psi(x)$$

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Wannier Functions $\quad \hat{\psi}(x) = \sum \hat{b}_i\, w_1(x - x_i)$

Assume: only lowest band occupied

$$\Rightarrow H_{Full} = -\sum_{i,j} J_{ij}\, b_i^+ b_j + \frac{1}{2}\sum_{i,j,k,l} U_{ijkl}\, b_i^+ b_j^+ b_k b_l$$

$$J_{ij} = -\int dx\, w_1(x - x_i) \left(\frac{p^2}{2m} + V(x) \right) w_1(x - x_j)$$

$$U_{ijkl} = g \int dx\, w(x - x_i)\, w(x - x_j)\, w(x - x_k)\, w(x - x_l)$$

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Hubbard model

$J = J_{ij} \neq 0$ nearest neighbors

$U = U_{ijkl} \neq 0$ for $i=j=k=l$ on site

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$$\hat{H} = -J \sum_{\substack{l l' \\ \text{nearest} \\ \text{neighbor}}} b_l^{+} b_{l'} + \frac{U}{2}\sum_l \underbrace{n_l}_{b_l^{+} b_l}(n_l - 1) - \mu \sum_l n_l$$

n_l : # of bosons / site

Refs: Zwerger, J. Opt. B: Quantum Semiclass. Opt. 5, S9 (2003)

Jaksch, Zoller, Annals of Phys. 315, 52 (2005)

Two limiting cases: integer filling $\bar{n}$

$U \gg J$

ground state $|\psi_{MI}\rangle (J=0, \bar{n}) = \prod_{\ell} \left(|\bar{n}\rangle_{\ell} \right)$

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$|\Psi_{SF,N}\rangle (U=0) = \left(\frac{1}{\sqrt{M}} \sum_{\ell=1}^{M} b_{\ell}^{+} \right)^{N} |0\rangle$ M sites

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Note: Bogoliubov approximation $a_0 = a_0^{+} = \sqrt{N_0}$
does not capture the transition to insulating state
Interactions are treated only approximately
Valid only for small depletion $N - N_0$

Quantum depletion
or
How to observe the transition from a quantum gas to a quantum liquid

In 1D: Zürich

K. Xu, Y. Liu, D.E. Miller, J.K. Chin, W. Setiawan, W.K., PRL 96, 180405 (2006).

What is the wavefunction of a condensate?

Ideal gas:

$$\Psi = \left(|q = 0\rangle \right)^N$$

Interacting gas:

$$H' = U_0 \delta(r)$$

$$H' = U_0 \sum a_p^\dagger a_q^\dagger a_r a_s \qquad H' = U_0 a_0 a_0 \sum a_p^\dagger a_{-p}^\dagger$$

$$\Psi = \left(|q = 0\rangle \right)^N + \alpha \left(|q = 0\rangle \right)^{N-2} |q = p\rangle |q = -p\rangle + \ldots$$

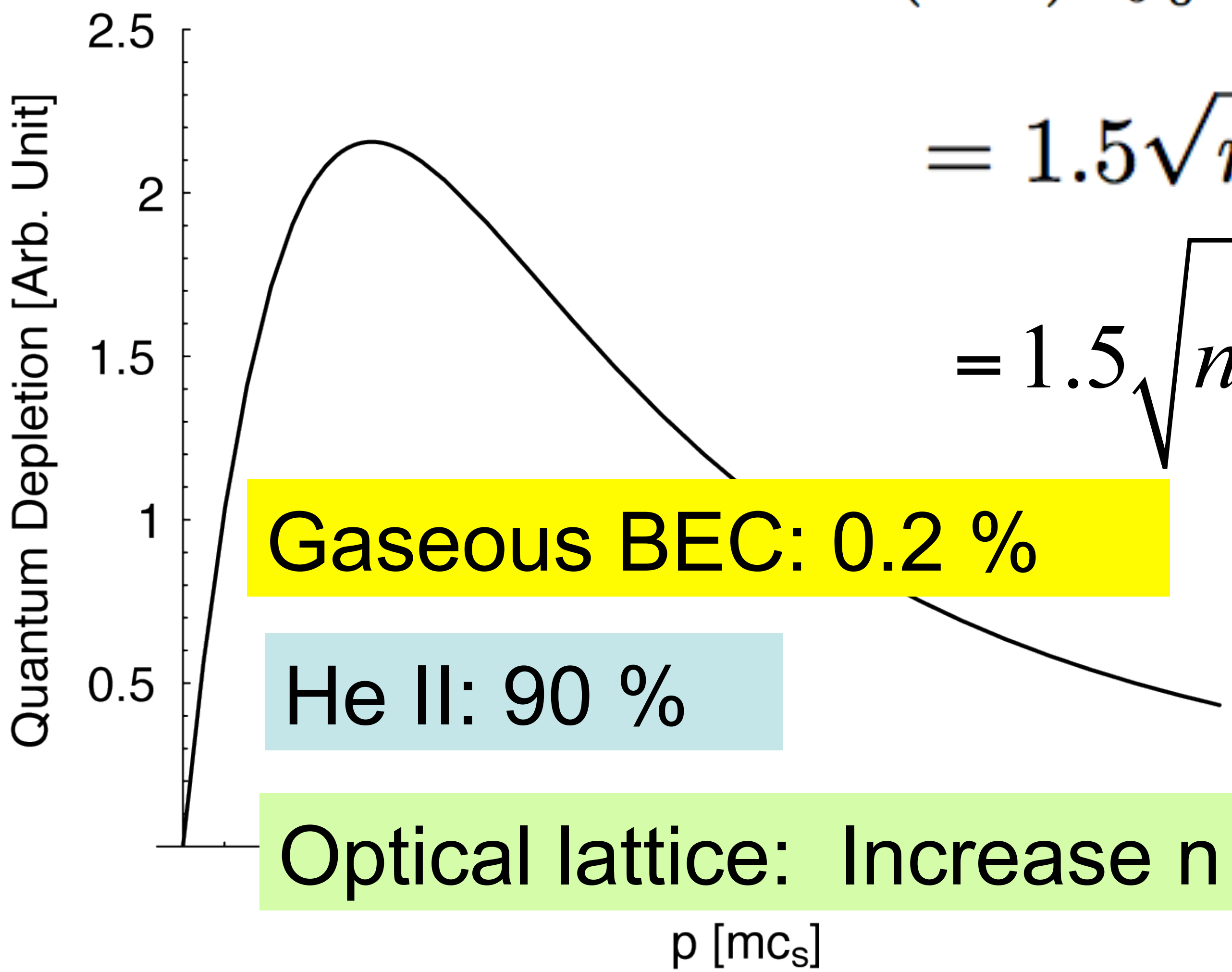
Quantum depletion

Quantum depletion in 3-dimensional free space

$$\frac{V}{(2\pi\hbar)^3}\int_0^\infty v_p^2(4\pi p^2)dp$$

$$= 1.5\sqrt{na^3}$$

$$= 1.5\sqrt{n\frac{M}{4\pi\hbar^2}U_0}$$



Gaseous BEC: 0.2 %

He II: 90 %

Optical lattice: Increase n and M_{eff}

Quantum Depletion

$$v_p^2 = \frac{T(p) + \mu - \sqrt{T^2(p) + 2\mu T(p)}}{2\sqrt{T^2(p) + 2\mu T(p)}}$$

Free space

$$T(p) = \frac{p^2}{2M}$$

$$\mu = \frac{4\pi\hbar^2 a}{M} n = Mc_s^2$$

Lattice

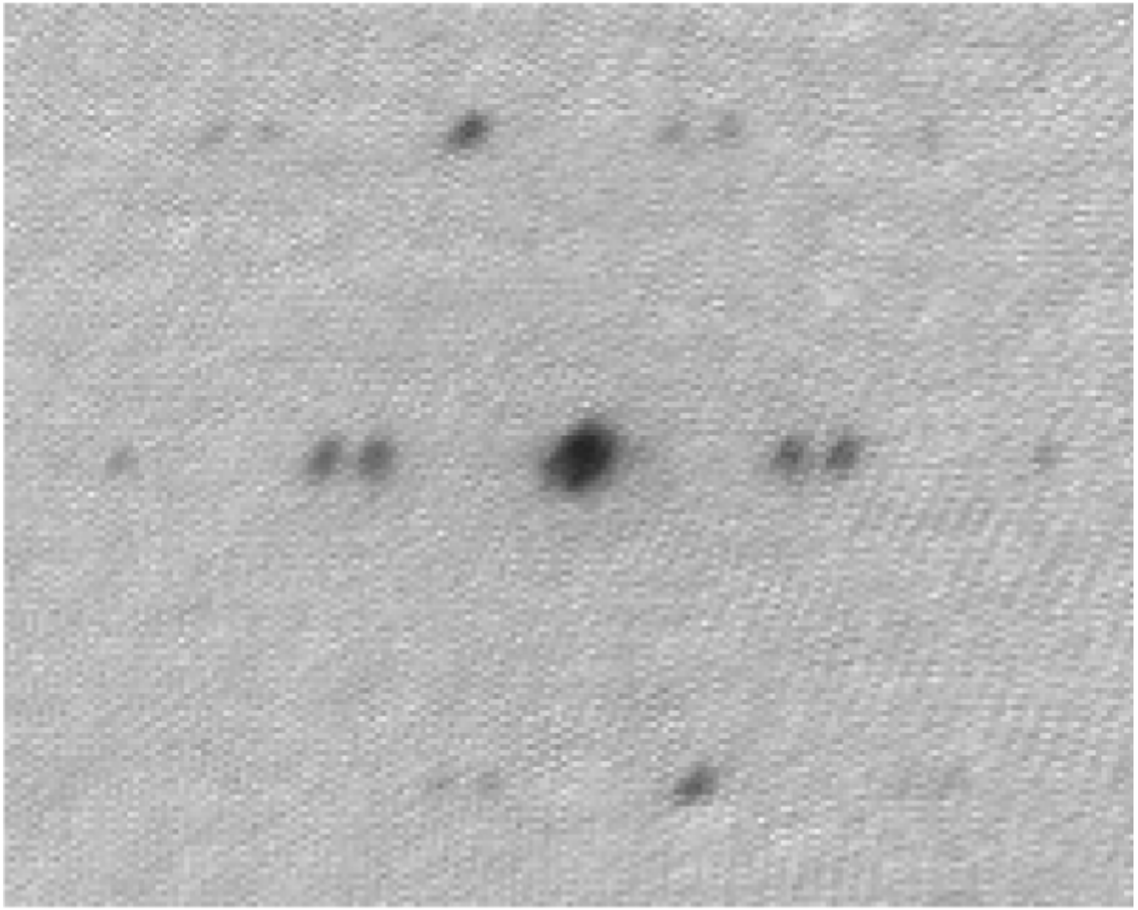
$$4J\sin^2\left(\frac{\lambda_L}{4\hbar}p\right)$$

$$n_0 U$$

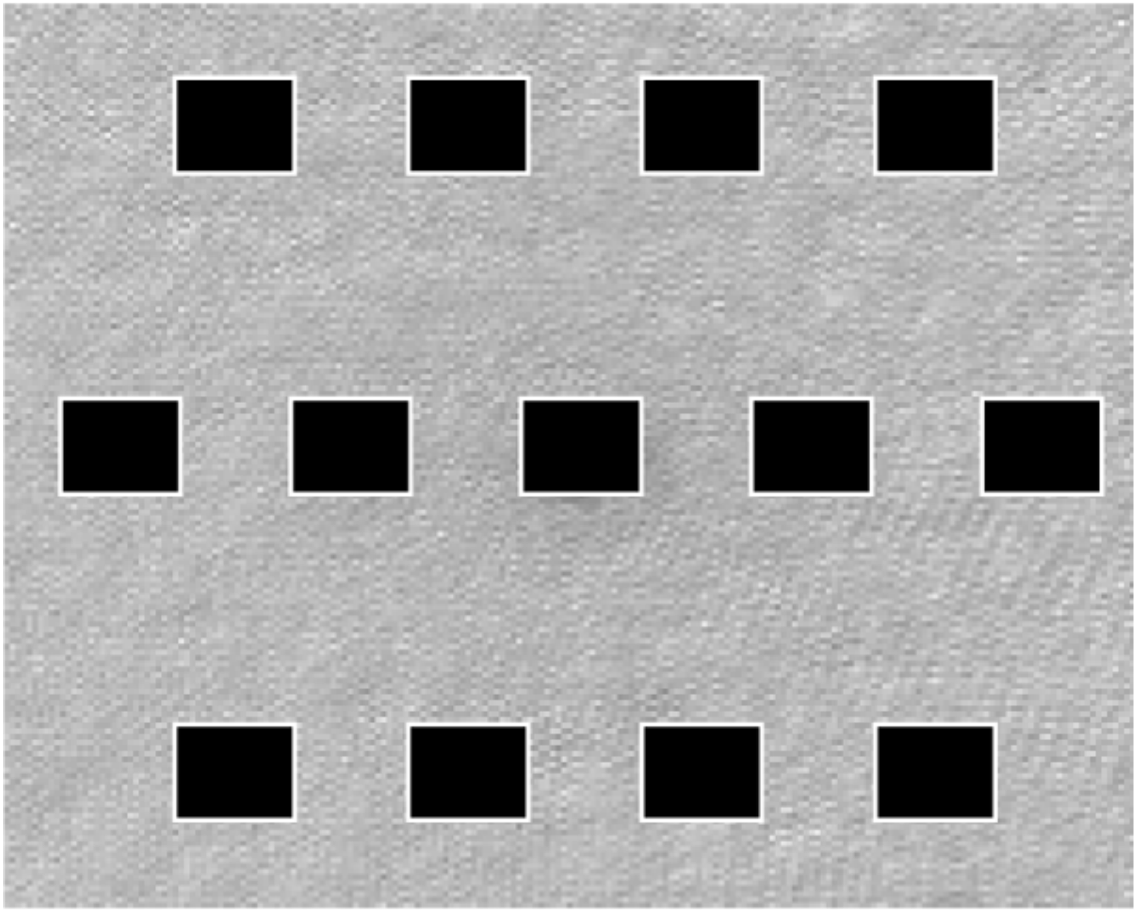
J : tunneling rate

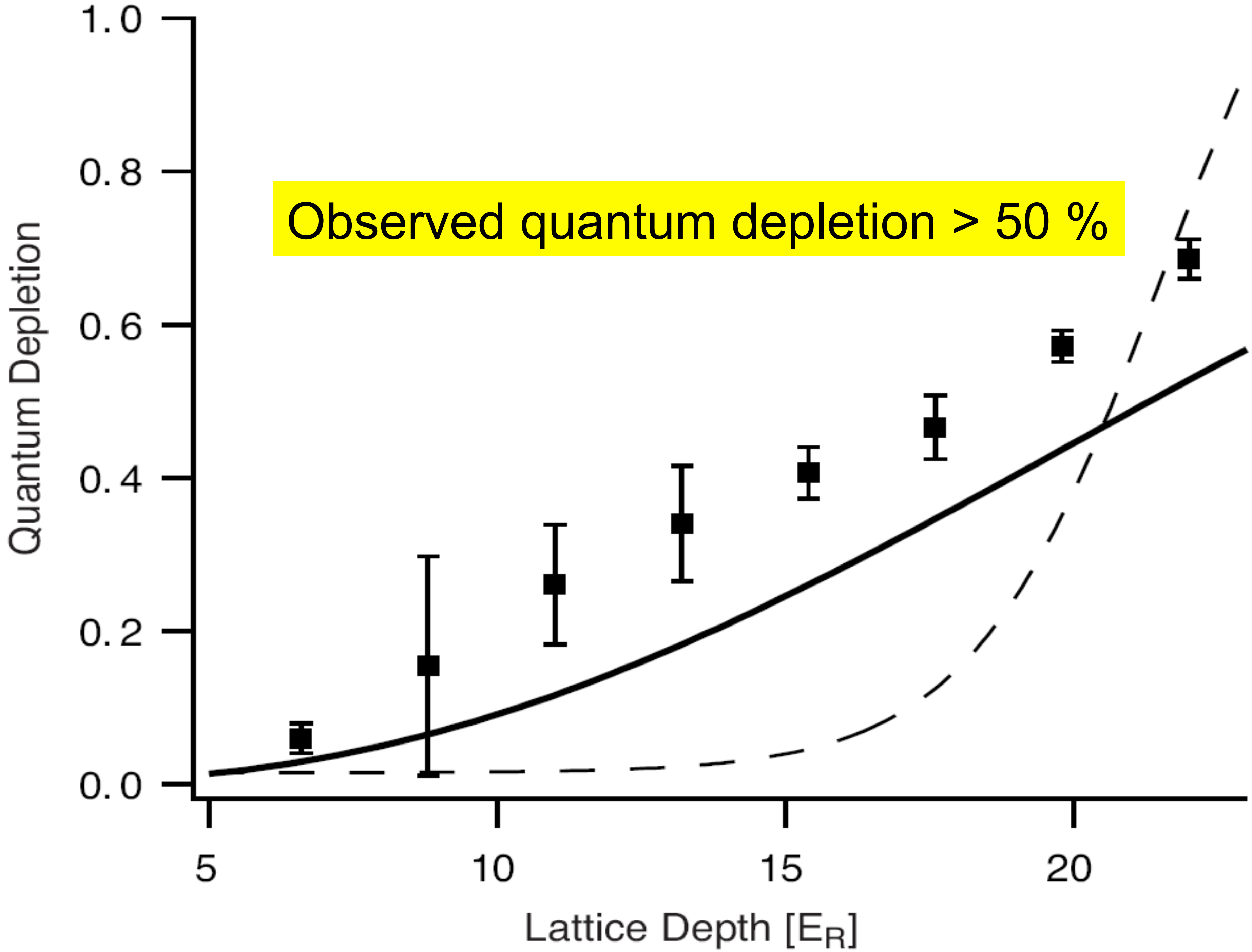
U : on-site interaction

2-D Mask Gaussian Fit



2-D Mask Gaussian Fit




Observed quantum depletion > 50 %
Quantum Depletion
1.0
0.8
0.6
0.4
0.2
0.0
5
10
15
20
Lattice Depth [E_R]

Back to the superfluid to Mott insulator transition …

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$U \gg J$

ground state $|\Psi_{MI}\rangle (J=0,\bar{n}) = \prod_{\ell} \left(|\bar{n}\rangle_{\ell}\right)$

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$|\Psi_{SF,N}\rangle (U=0) = \left(\frac{1}{\sqrt{M}} \sum_{\ell=1}^{M} b_{\ell}^{+}\right)^{N} |0\rangle$ M sites

Note: Bogoliubov approximation $a_0 = a_0^{+} = \sqrt{N_0}$
does not capture the transition to insulating state
Interactions are treated only approximately
Valid only for small depletion $N-N_0$

Goal: Find effective onsite Hamiltonian by mean-field decoupling

van Osten, van der Straten, Stoof, PR A 63 053601 (2001).

$$\hat{A}\,\hat{B} = (\langle A\rangle + \Delta\hat{A})(\langle B\rangle + \Delta\hat{B}) \approx \langle A\rangle \Delta\hat{B} + \Delta\hat{A}\langle B\rangle + \langle A\rangle\langle B\rangle$$
$$= \langle A\rangle \hat{B} + \hat{A}\langle B\rangle - \langle A\rangle\langle B\rangle$$

Coupling between sites: tunneling $J\, b_\ell^+ b_{\ell'}$

$$b_\ell^+ b_{\ell'} \approx \langle b_\ell^+ \rangle b_{\ell'} + b_\ell^+ \langle b_{\ell'} \rangle - \langle b_\ell^+ \rangle \langle b_{\ell'} \rangle$$

Coupling between sites: tunneling $J\, b_\ell^+ b_{\ell'}$

$$b_\ell^+ b_{\ell'} \approx \underbrace{\langle b_\ell^+ \rangle}_{\psi} b_{\ell'} + b_\ell^+ \underbrace{\langle b_{\ell'} \rangle}_{\psi} - \underbrace{\langle b_\ell^+ \rangle \langle b_{\ell'} \rangle}_{\psi^2}$$

SF order parameter $\psi = \sqrt{n_\ell} = \langle b_\ell^+ \rangle = \langle b_\ell \rangle$

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$$\psi \qquad\qquad \psi \qquad\qquad \psi^2$$

SF order parameter $\psi = \sqrt{n_\ell} = \langle b_\ell^+ \rangle = \langle b_\ell \rangle$

z # of nearest neighbors

$$H_{eff} = -zJ\psi \sum_\ell \left(b_\ell^+ + b_\ell \right) + zJ \underset{\text{\# of sites}}{M} \psi^2 + \frac{U}{2} \sum n_\ell (n_\ell - 1) - \mu \sum n_\ell$$

Coupling between sites: tunneling $J\, b_l^+ b_{l'}$

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$$H_{eff} = zJ \sum H_{eff,l} \qquad \bar{U} = U/zJ \qquad \bar{\mu} = \mu/zJ$$

$$H_{eff,l} = \frac{1}{2} \bar{U} n_l (n_l - 1) - \bar{\mu} n_l - \psi \left(b_l^+ + b_l \right) + \psi^2$$

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$$H_{\text{eff},\ell} = \frac{1}{2} \bar{U} n_\ell (n_\ell - 1) - \bar{\mu} n_\ell - \psi \left(b_\ell^+ + b_\ell \right) + \psi^2$$

$$= H^{(0)} + \psi V \qquad \text{with } V = -\left(b_\ell^+ + b_\ell \right)$$

$$H^{(0)} = \frac{1}{2} \bar{U} \hat{n} (\hat{n} - 1) - \bar{\mu} \hat{n} + \psi^2 \quad \text{diagonal in } \hat{n}$$

ground state for $H^{(0)}$

If $\bar{u}(j-1) < \bar{\mu} < \bar{u}\, j$

$\Rightarrow \quad E_j^{(0)} = \frac{1}{2}\bar{u}\, j(j-1) - \bar{\mu}\, j$ $\qquad$ j occupation #

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$\Rightarrow E_j^{(0)} = \frac{1}{2}\bar{u}\, j(j-1) - \bar{\mu}\, j$ $\qquad j$ occupation #

V: couples $\Delta n = \pm 1$

second order perturbation theory

$$E_j^{(2)} = t^2 \sum_{n \neq j} \frac{|\langle j | V | n \rangle|^2}{E_j^{(0)} - E_n^{(0)}}$$

$$= \frac{j}{\bar{u}(j-1) - \bar{\mu}} + \frac{j+1}{\bar{\mu} - \bar{u}\, j}$$

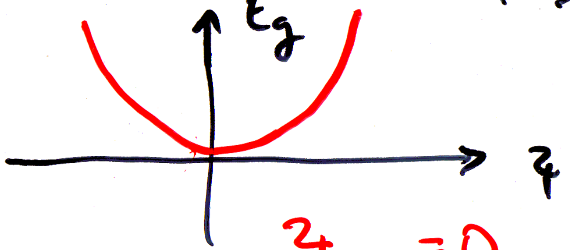
Phase transition

Landau Formalism:

$$E_g(q) = a_0 + a_2 q^2 + o(q^4)$$

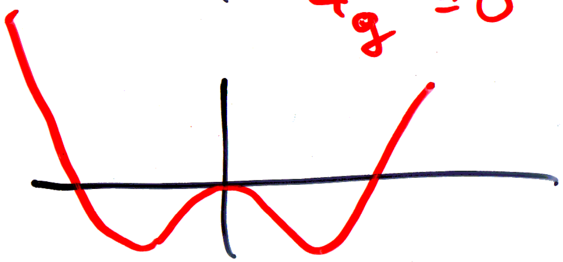
$\hookrightarrow > 0$, see 4th order perturbation theory

$a_2 > 0$



$q_g = 0$

$a_2 < 0$



$q_g \neq 0$

Phase transition for $\boxed{a_2 = 0}$

Phase transition

Landau Formalism:

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↳ >0, see 4th order perturbation theory

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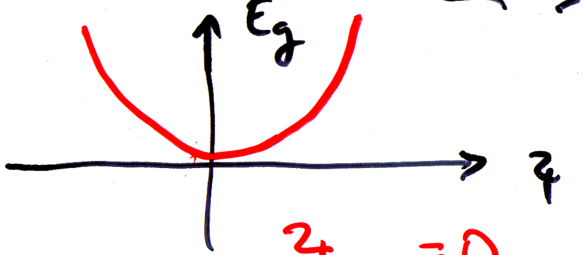
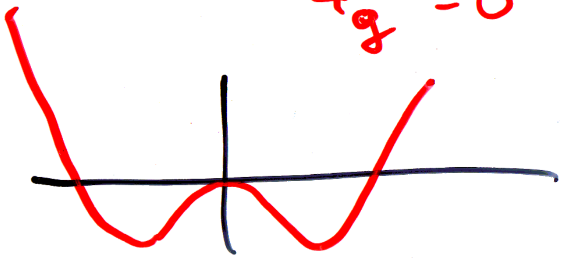


$z_g = 0$

$a_2 < 0$

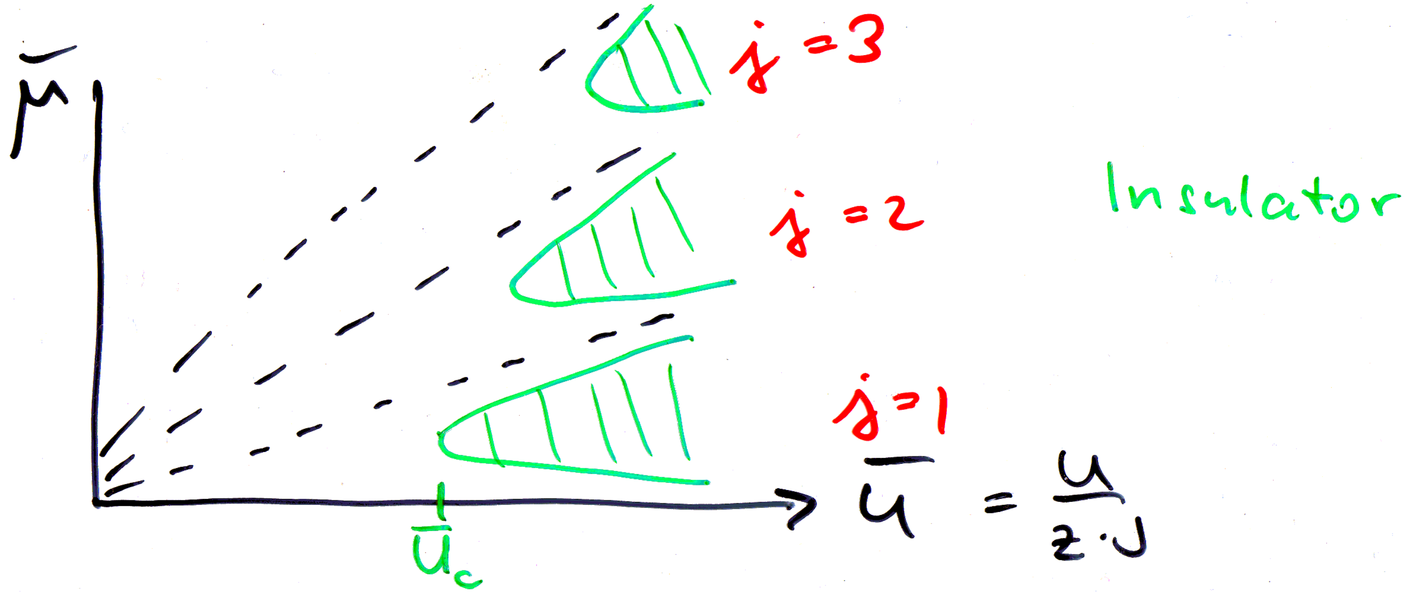


$z_g \neq 0$

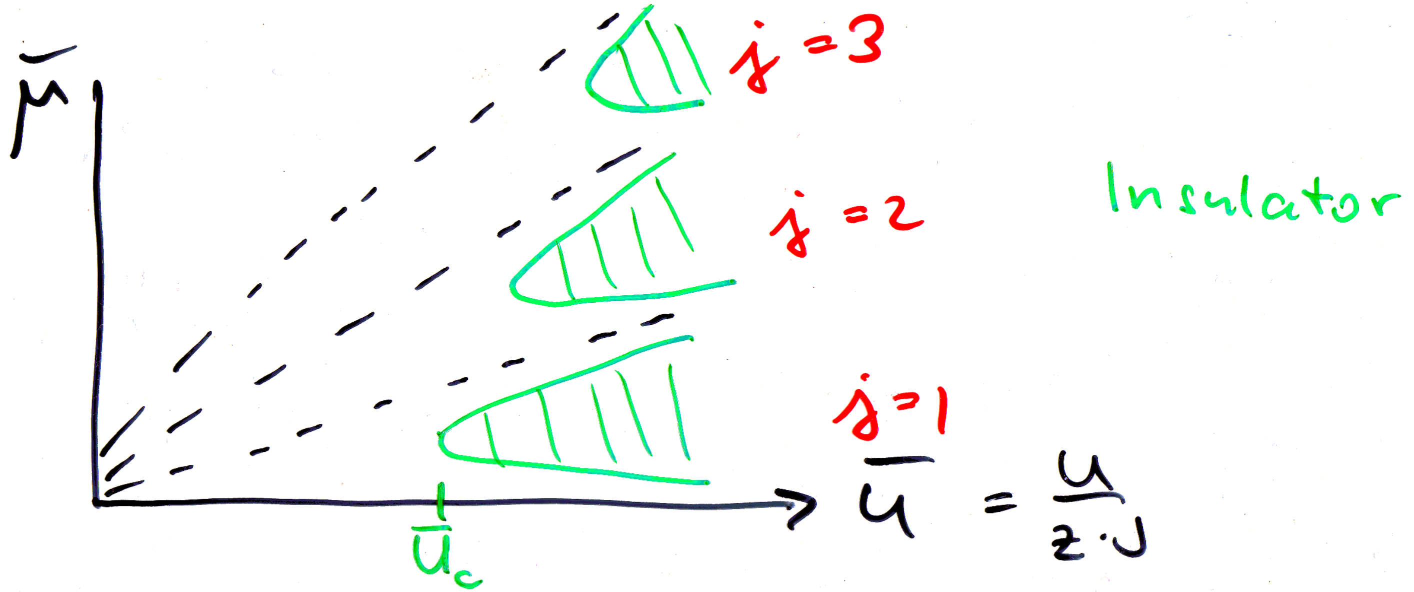
Phase transition for $\boxed{a_2 = 0}$

$$a_2 = \frac{j}{\bar{u}(j-1) - \bar{\mu}} + \frac{j+1}{\bar{\mu} - \bar{u} j} + 1 = 0$$

$$\bar{\mu}_{\pm} = \frac{1}{2}\left[\bar{u}(2j+1) - 1\right] \pm \frac{1}{2}\sqrt{\bar{u}^2 - 2\bar{u}(2j+1) + 1}$$



$\bar{\mu}$
$j=3$
$j=2$
$j=1$
Insulator
$\bar{U} = \frac{U}{z \cdot J}$
$\bar{U}_c$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Insulator
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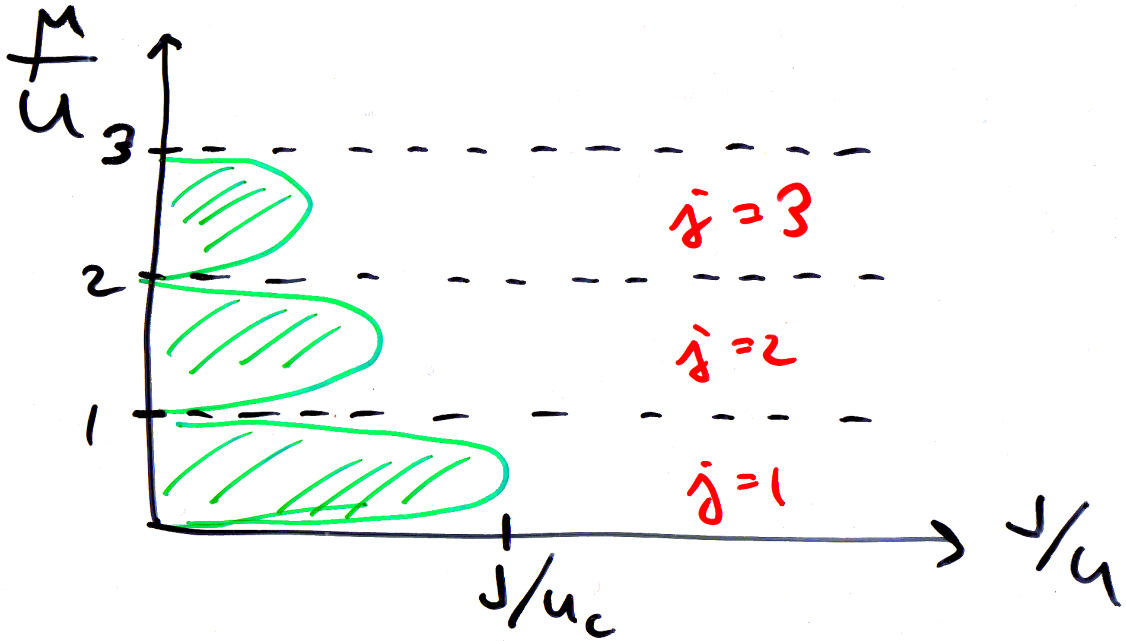
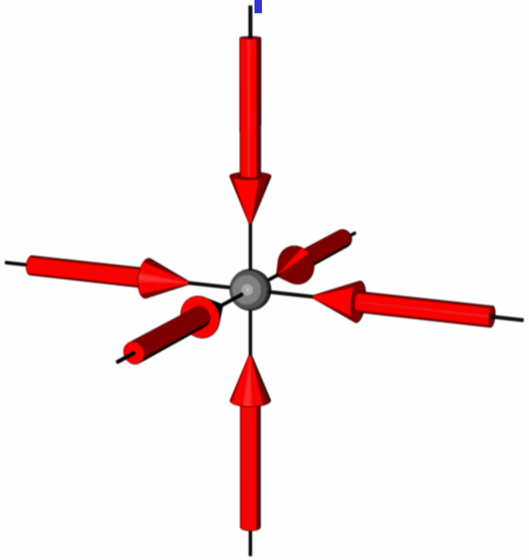
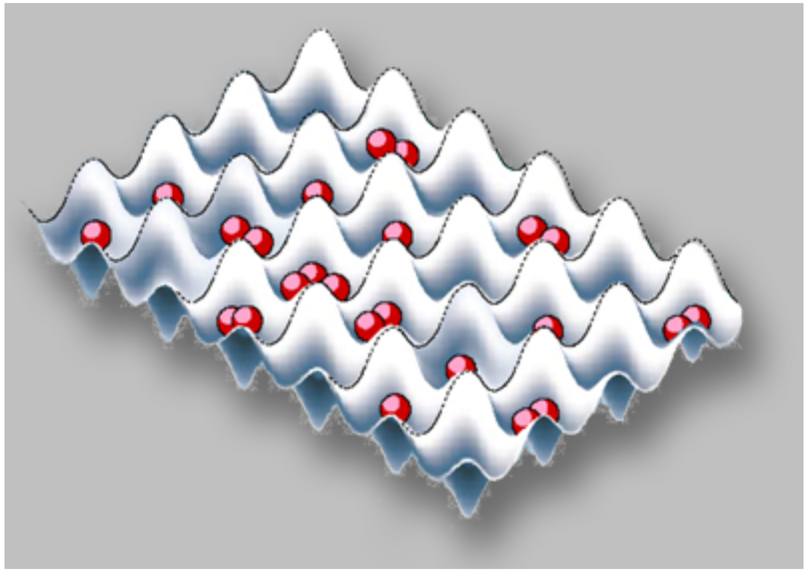
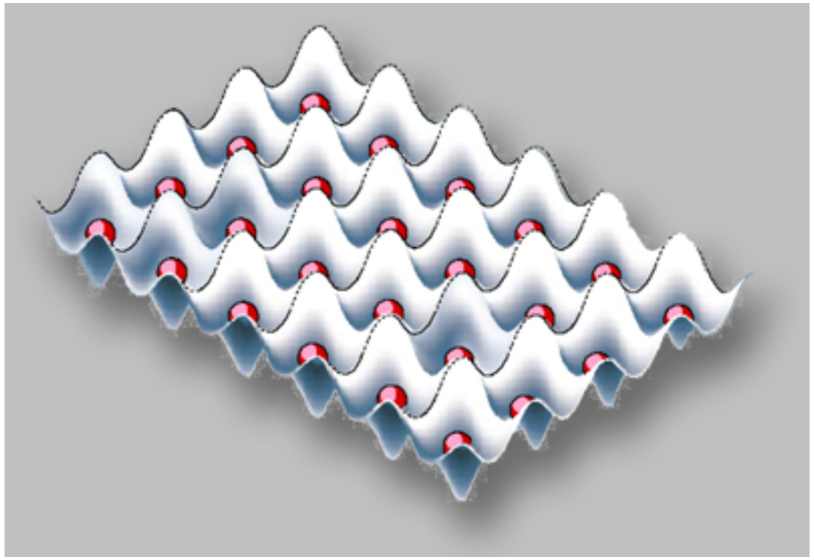

$\frac{\mu}{U}$
3
2
1
$j=3$
$j=2$
$j=1$
J/U_c
J/U

BEC in 3D optical lattice



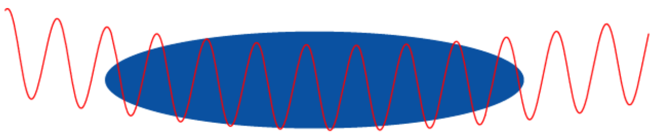




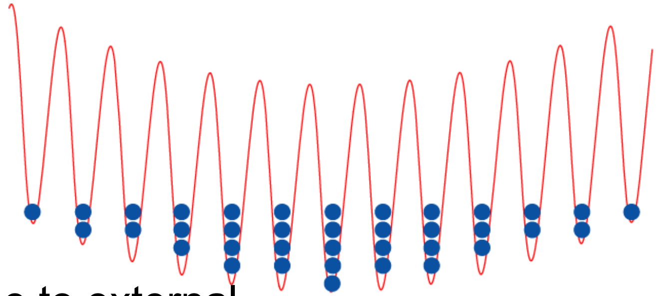
Courtesy Markus Greiner

The Superfluid-Mott Insulator transition

Shallow Lattices - Superfluid



Deep Lattices – Mott Insulator



Energy offset due to external harmonic confinement. Not in condensed matter systems.

$$\hat{H} = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger a_j + \sum_i \frac{1}{2} U \hat{n}_i(\hat{n}_i - 1) + \sum_i (\epsilon_i - \mu)\hat{n}_i$$

tunneling term between neighboring sites

$$J = \frac{4}{\sqrt{\pi}} E_r \left(\frac{V_0}{E_r}\right)^{3/4} \exp -2\left(\frac{V_0}{E_r}\right)^{1/2}$$

on-site interaction

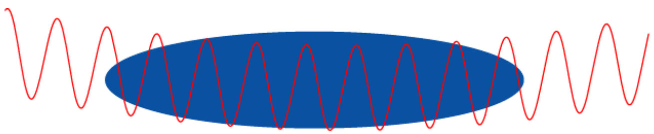
$$U = \left(\frac{4\pi\hbar^2 a}{m}\right)\int |w(x)|^4 d^3x$$

a = s-wave scattering length

Other exp: Mainz, Zurich, NIST Gaithersburg, Innsbruck, MPQ and others

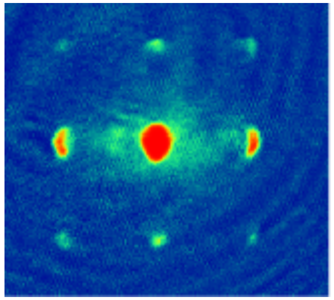
The Superfluid-Mott Insulator transition

Shallow Lattices - Superfluid

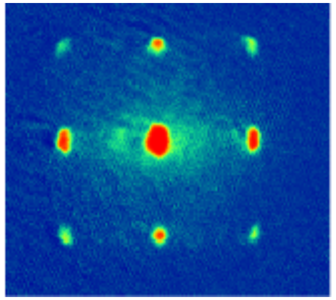


$$|\Psi_{SF}\rangle \propto \left(\sum_{i=1}^{M} \hat{a}_i^\dagger\right)^N |0\rangle$$

$$\hat{H} = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger a_j + \sum_i \frac{1}{2}U\hat{n}_i(\hat{n}_i - 1) + \sum_i(\epsilon_i - \mu)\hat{n}_i$$



5 E_{rec}

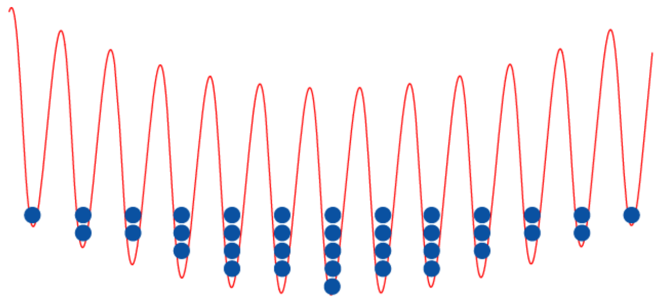


9 E_{rec}

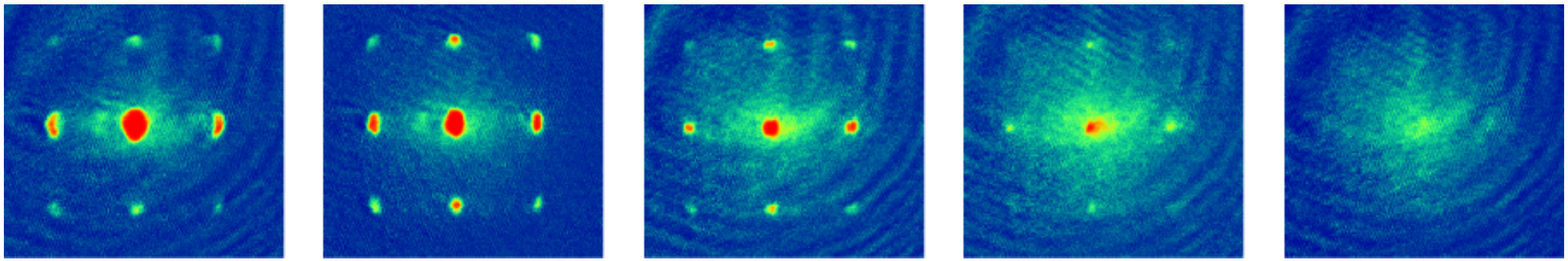
The Superfluid-Mott Insulator transition

$$|\Psi_{MI}\rangle_{J=0} \propto \prod_{i=1}^{M} (\hat{a}_i^\dagger)^n |0\rangle$$

Deep Lattices – Mott Insulator



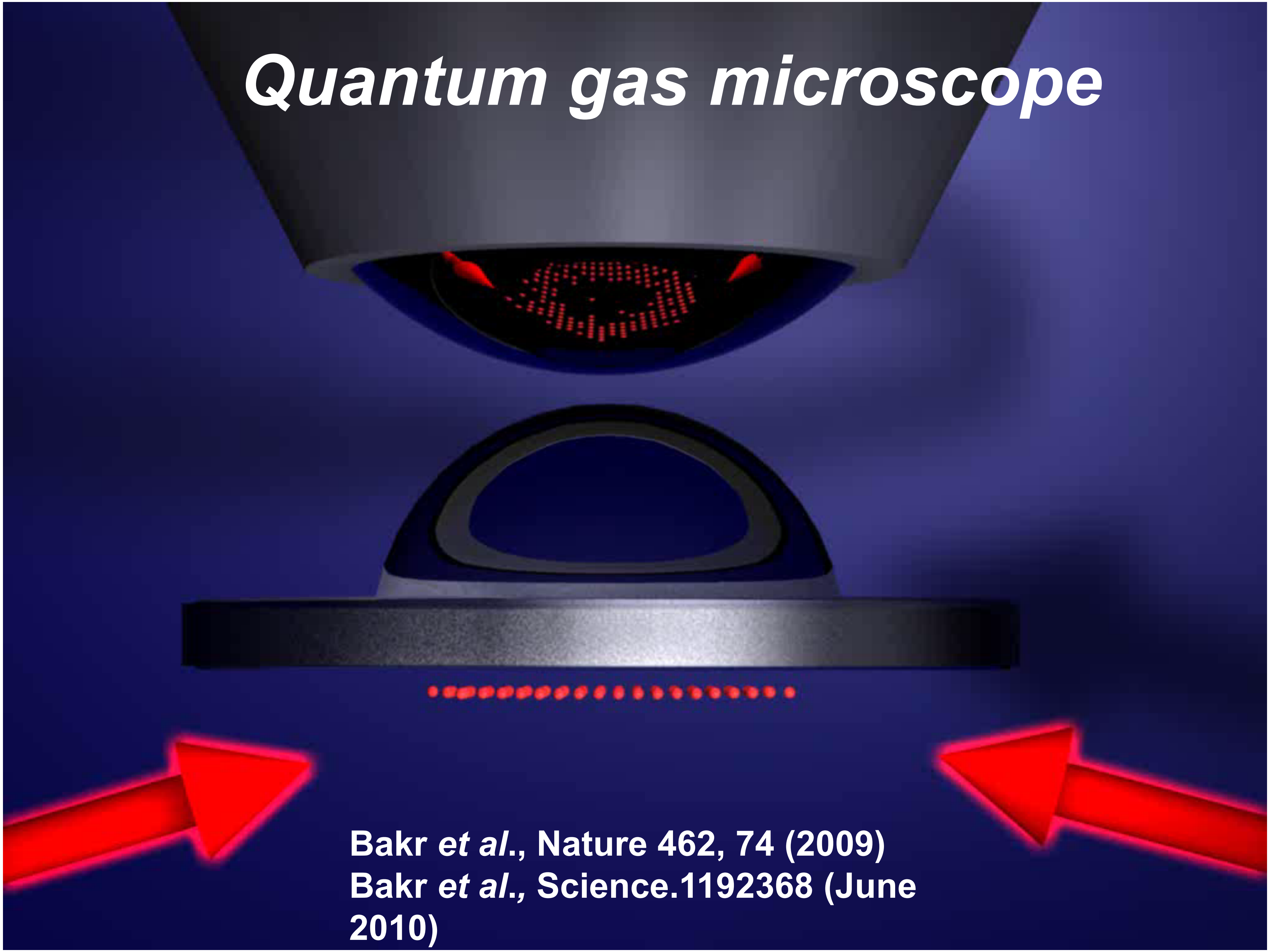
$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger a_j + \sum_i \frac{1}{2} U \hat{n}_i (\hat{n}_i - 1) + \sum_i (\epsilon_i - \mu) \hat{n}_i$$



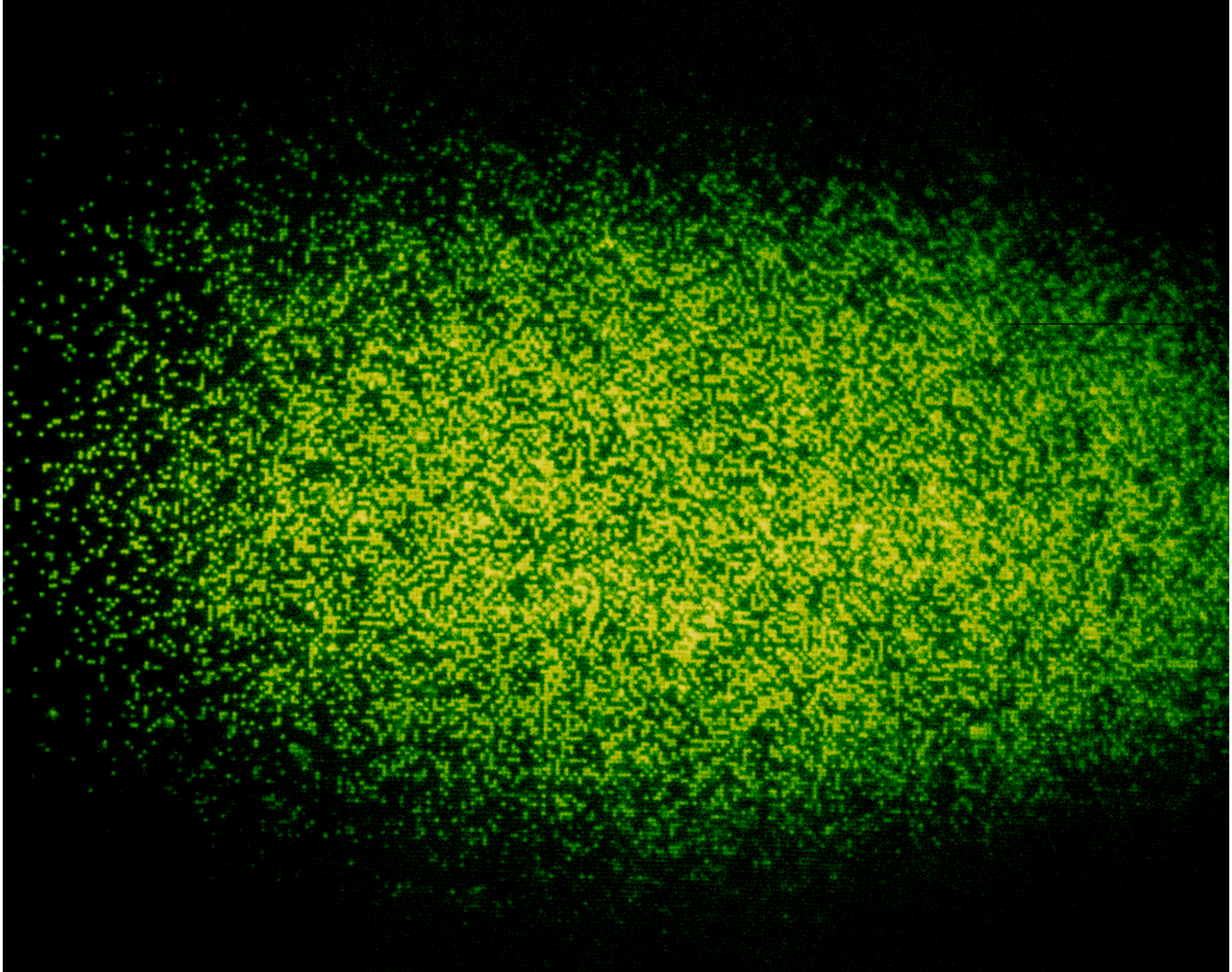
5 E_{rec} | 9 E_{rec} | 12 E_{rec} | 15 E_{rec} | 20 E_{rec}

As the lattice depth is increased, J decreases exponentially, and U increases. For J/U<<1, number fluctuations are suppressed, and the atoms are localized

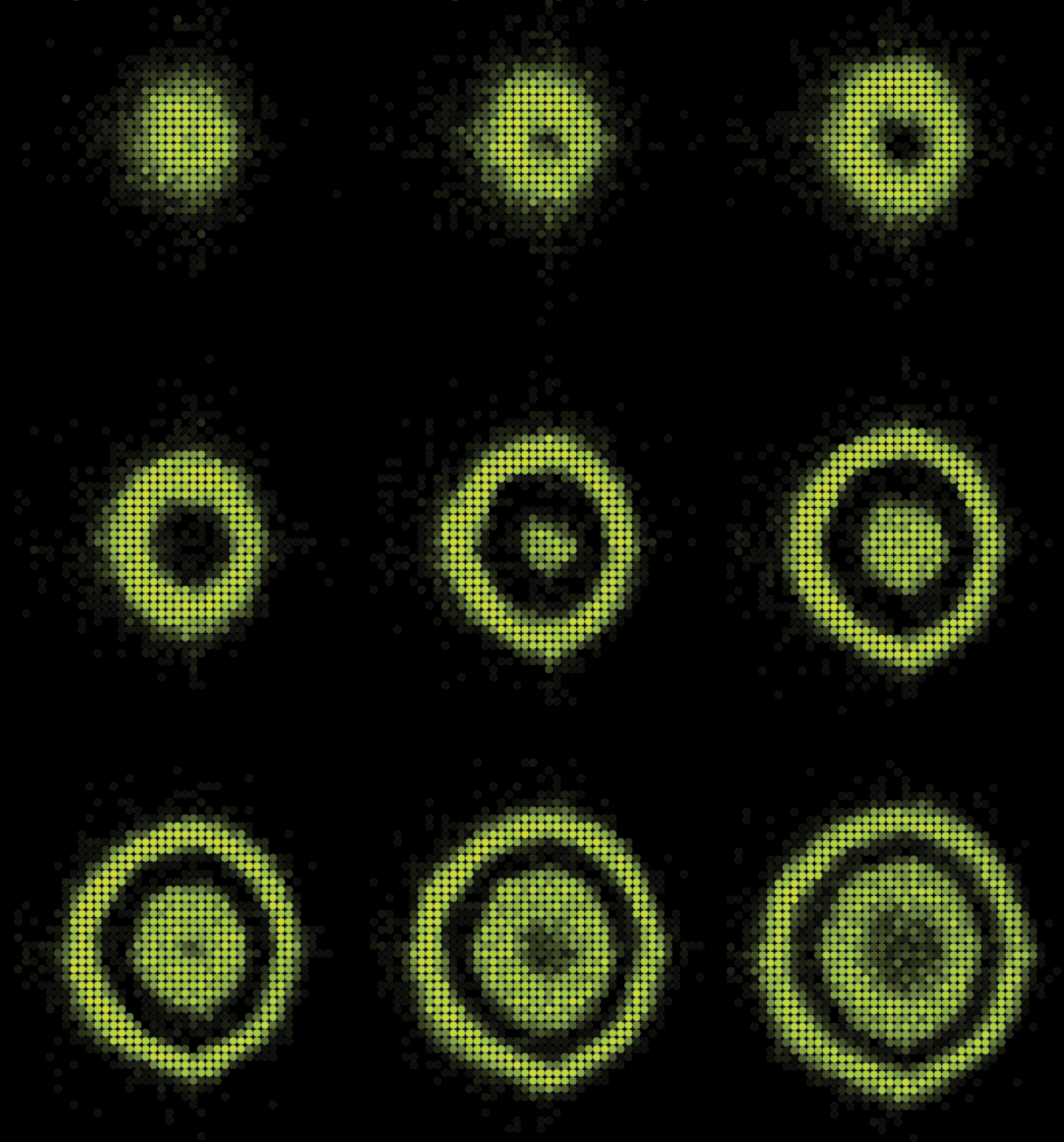
Quantum gas microscope



Bakr *et al*., Nature 462, 74 (2009)
Bakr *et al*., Science.1192368 (June 2010)



Single site imaging of shell structure



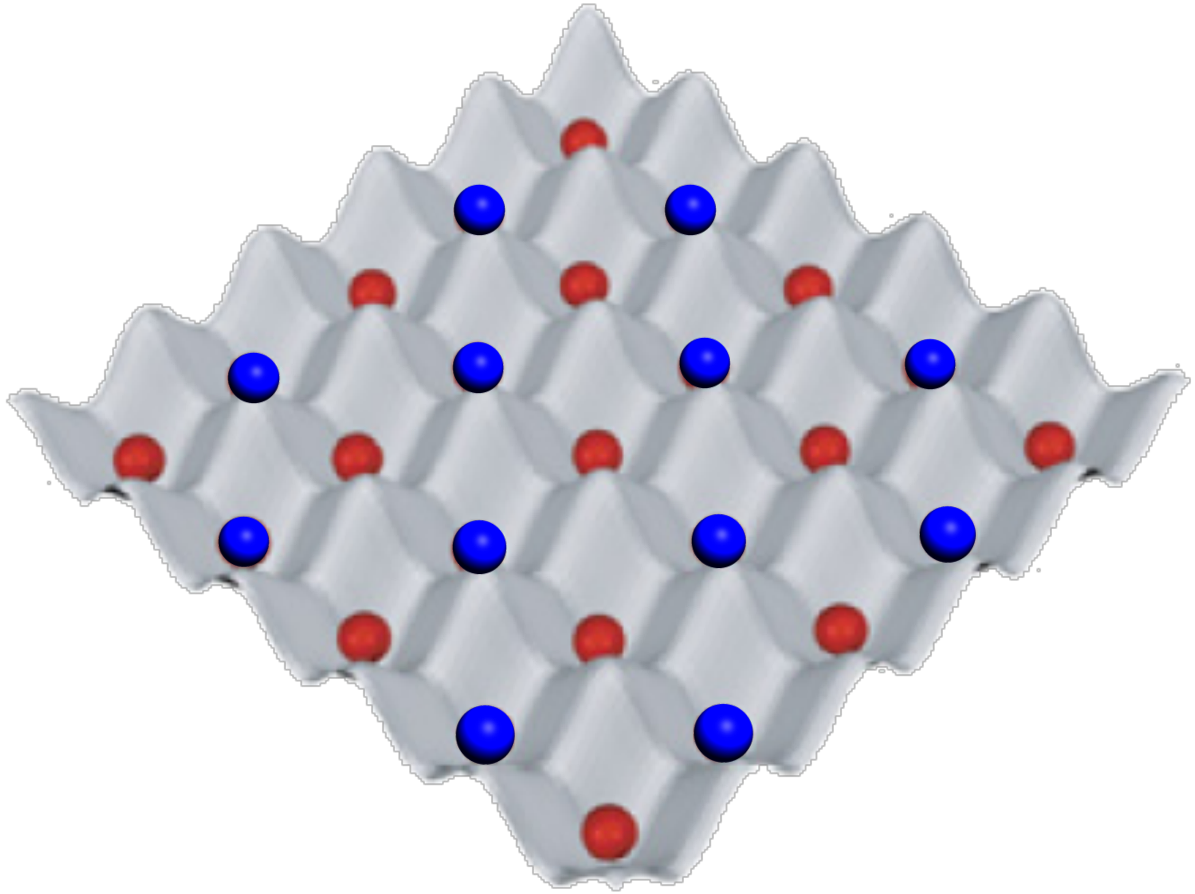
Mott insulator - all motion is frozen out

Degree of freedom left: spin ordering

Goal:

Magnetism of localized spins

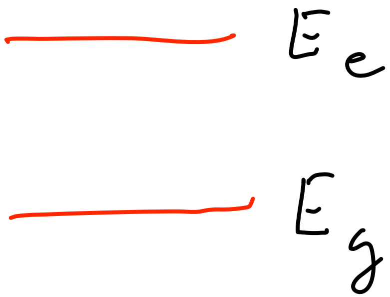
Requires TOTAL entropy to be less than $N\ k_B \log 2$



How to particles (spins) order in a lattice?

How to particles (spins) order in a lattice?

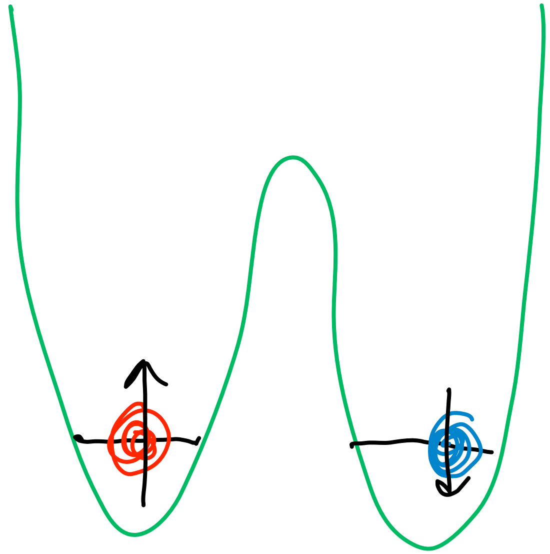
2nd order perturbation theory:



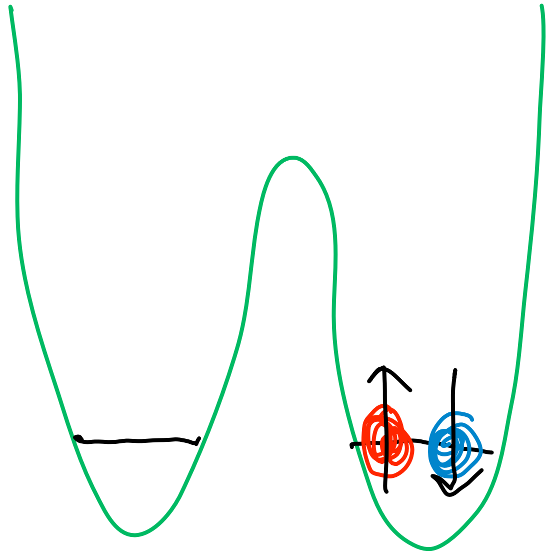
$$\Delta E = -\frac{|\langle g | M | e \rangle|^2}{E_e - E_g}$$

Double well potential:

$$\Delta E = -\frac{|\langle g|M|e\rangle|^2}{E_e - E_g}$$



$\hat{=} \; |g\rangle$

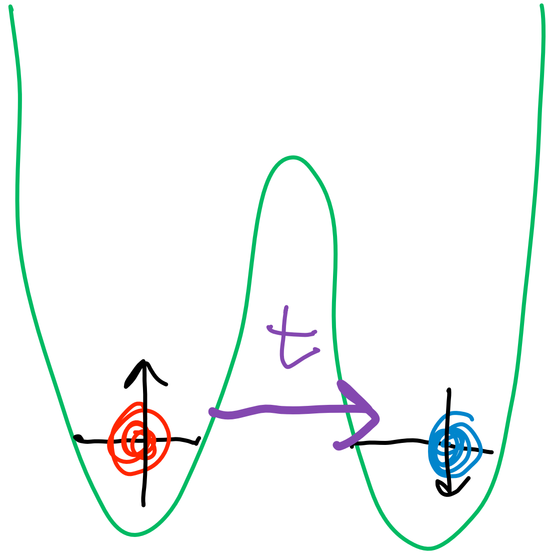


$\hat{=} \; |e\rangle$

$E_e - E_g \simeq U$

Double well potential:

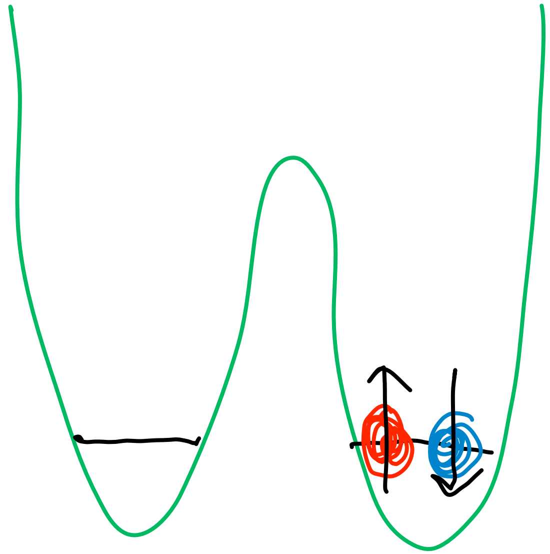
$$\Delta E = -\frac{|\langle g|M|e\rangle|^2}{E_e - E_g}$$



$\hat{=}\ |g\rangle$

$$\langle g|M|e\rangle = t$$

tunneling

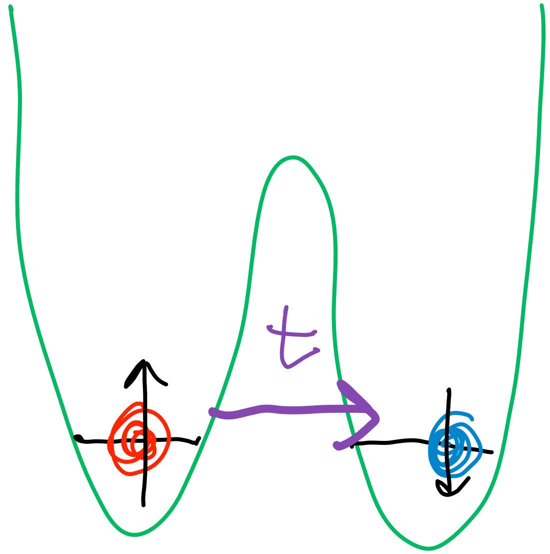


$\hat{=}\ |e\rangle$

$E_e - E_g = U$

Double well potential:

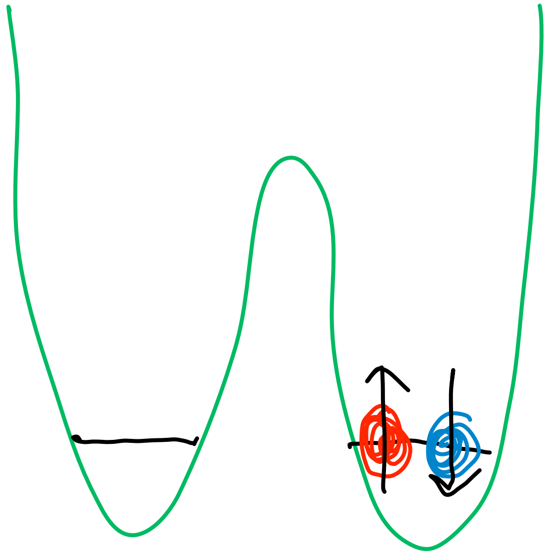
$$\Delta E = -\frac{|\langle g|M|e\rangle|^2}{E_e - E_g}$$



$\hat{=} \; |g\rangle$

$\langle g|M|e\rangle = t$

tunneling



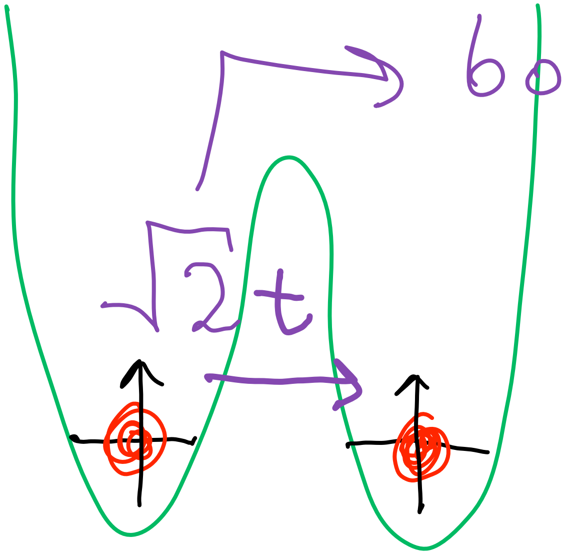
$\hat{=} \; |e\rangle$

$E_e - E_g = U$

$$\Delta E = -\frac{t^2}{U}$$

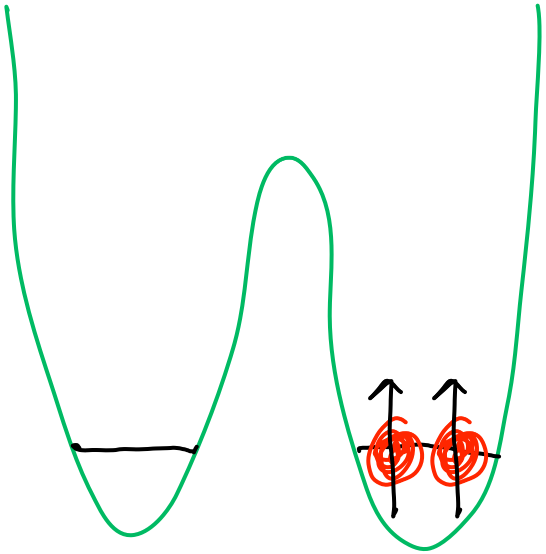
Double well potential:

$$\Delta E = -\frac{|\langle g|M|e\rangle|^2}{E_e - E_g}$$

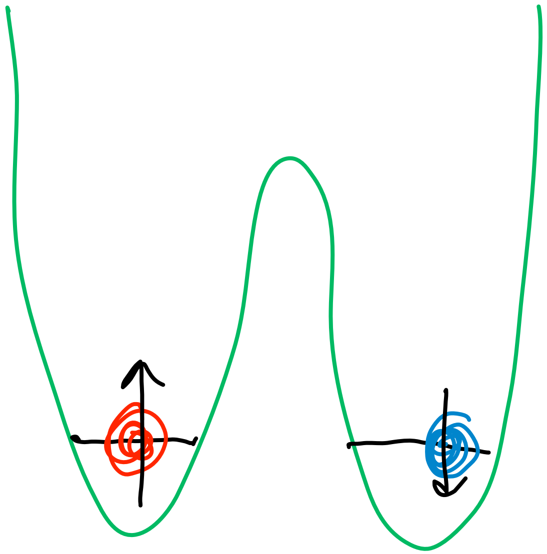


bosonic stimulation $\sqrt{n+1}$

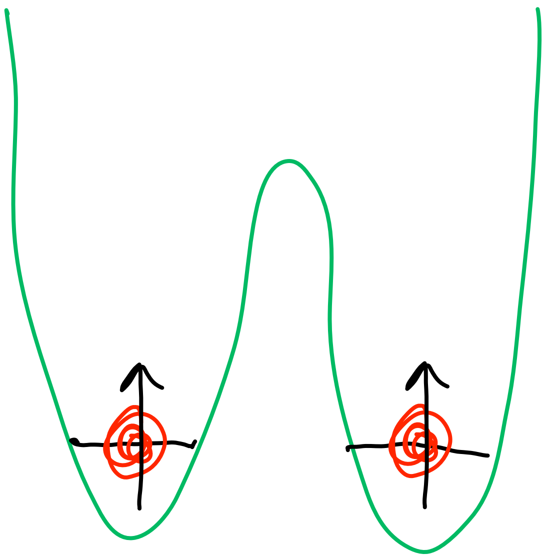
$\hat{=} \; |g\rangle$



$$\Delta E = -\frac{2t^2}{U}$$

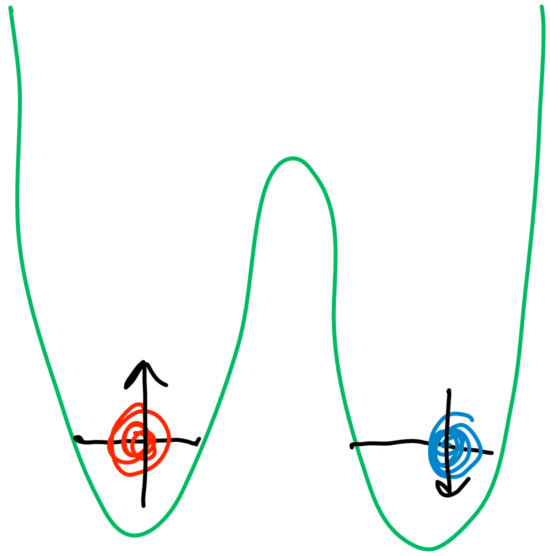


$$\Delta E = -\frac{t^2}{u}$$

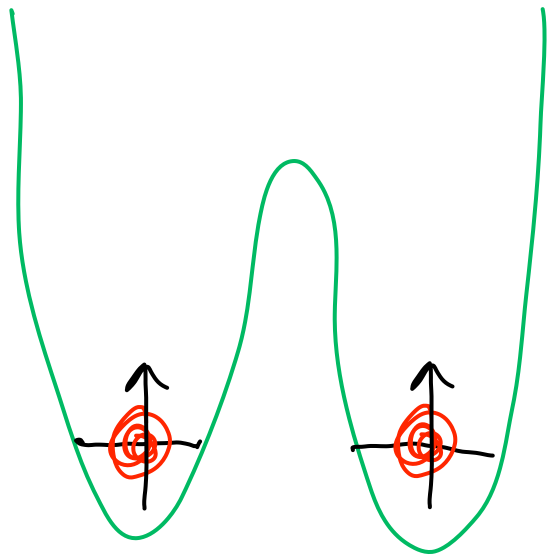


$$\Delta E = -\frac{2t^2}{u}$$

Bosons want to be ferromagnetic unless ….



$$\Delta E = -\frac{t^2}{U_{\uparrow\downarrow}}$$



$$\Delta E = -\frac{2t^2}{U_{\uparrow\uparrow}}$$

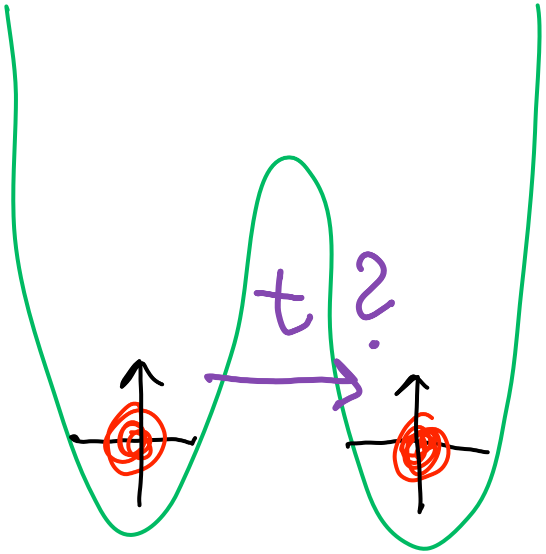
$$U_{\uparrow\downarrow} < U_{\uparrow\uparrow}/2$$

but we have to consider xy ferromagnetism and anti-ferromagnetism

Fermions

Double well potential:

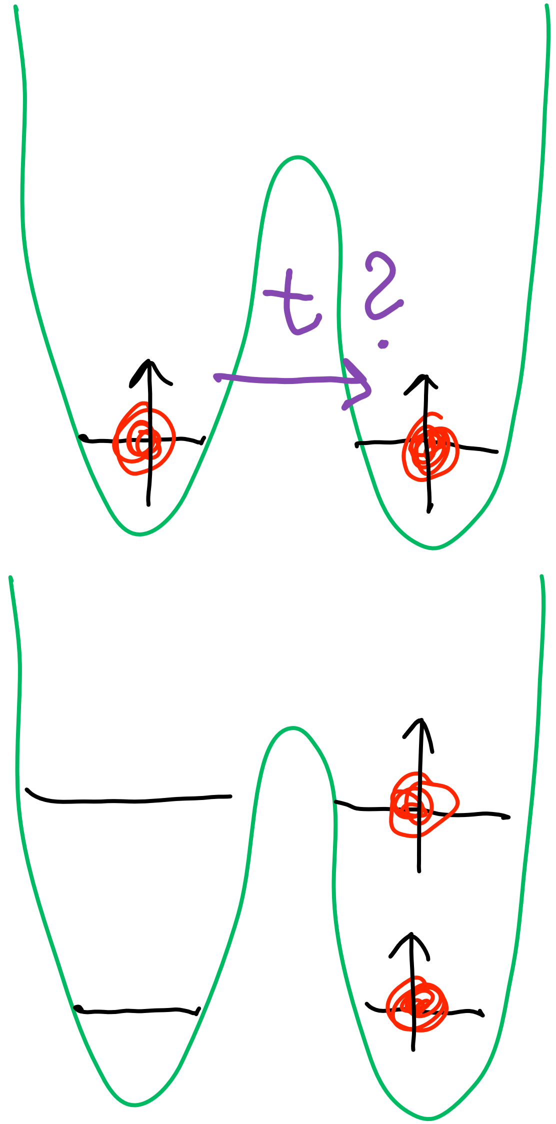
$$\Delta E = -\frac{|\langle g | M | e \rangle|^2}{E_e - E_g}$$



$\hat{=} \; |g\rangle$

Double well potential:

$$\Delta E = -\frac{|\langle g | M | e \rangle|^2}{E_e - E_g}$$



$\hat{=}\ |g\rangle$

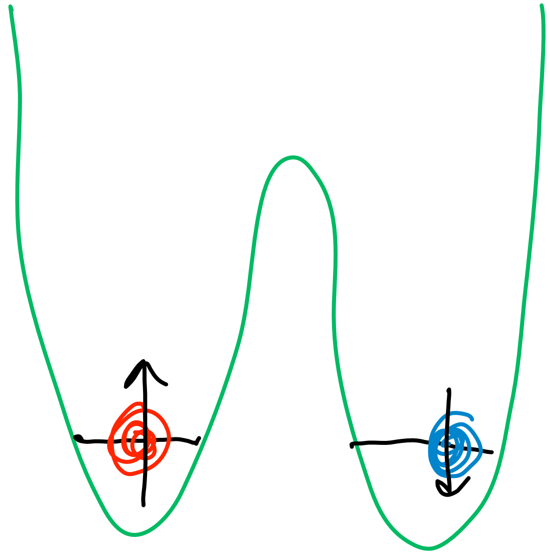
$\hat{=}\ |e\rangle$

$$E_e - E_g = \Delta_{GAP} \gg U$$

$$\Delta E = -\frac{t^2}{\Delta_{GAP}} \approx 0$$

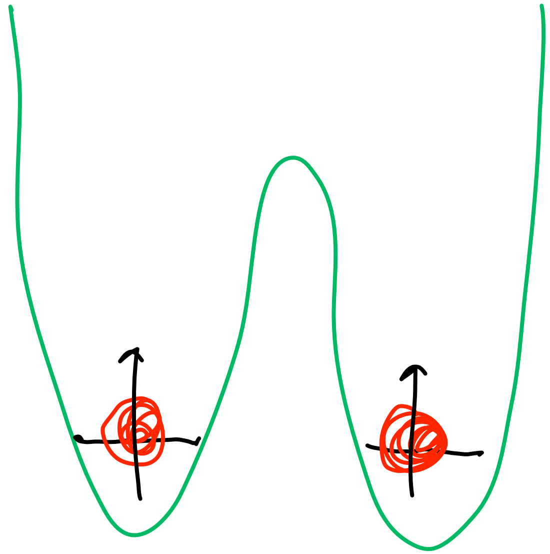
Double well potential:

$$\Delta E = -\frac{|\langle g|M|e\rangle|^2}{E_e - E_g}$$



$$\Delta E = -\frac{t^2}{U}$$

Fermions want to have opposite spin as neighbors!



$$\Delta E \approx 0$$

VOLUME 91, NUMBER 9 PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2003

Controlling Spin Exchange Interactions of Ultracold Atoms in Optical Lattices

L.-M. Duan,[1] E. Demler,[2] and M. D. Lukin[2]

Bosonic or fermionic Hubbard Hamiltonian

$$H = -\sum_{\langle ij\rangle\sigma}(t_{\mu\sigma}a_{i\sigma}^{\dagger}a_{j\sigma} + \mathrm{H.c.}) + \frac{1}{2}\sum_{i,\sigma}U_{\sigma}n_{i\sigma}(n_{i\sigma}-1) + U_{\uparrow\downarrow}\sum_{i}n_{i\uparrow}n_{i\downarrow},$$

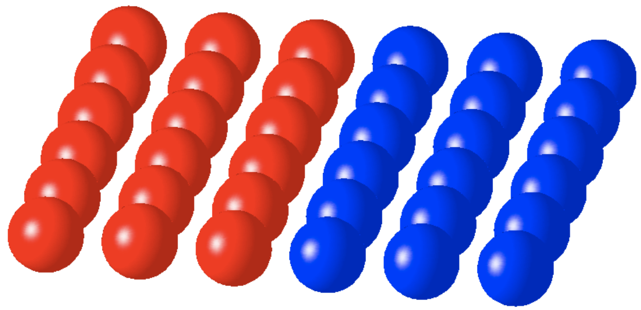
is equivalent to spin Hamiltonian (for localized particles)

$$H = \sum_{\langle i,j\rangle}[\lambda_{\mu z}\sigma_i^z\sigma_j^z \pm \lambda_{\mu\perp}(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)]$$

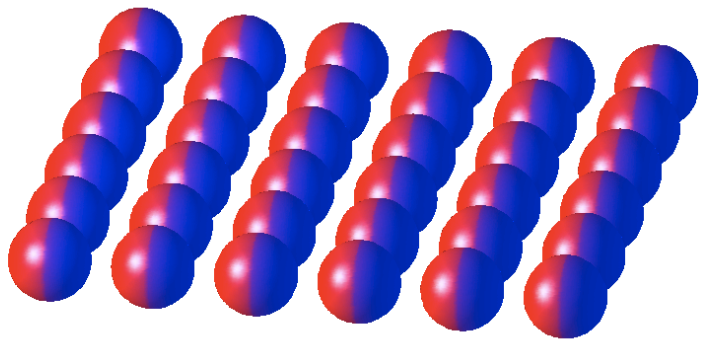
$$\lambda_{\mu z} = \frac{t_{\mu\uparrow}^2 + t_{\mu\downarrow}^2}{2U_{\uparrow\downarrow}} - \frac{t_{\mu\uparrow}^2}{U_{\uparrow}} - \frac{t_{\mu\downarrow}^2}{U_{\downarrow}}, \qquad \lambda_{\mu\perp} = \frac{t_{\mu\uparrow}t_{\mu\downarrow}}{U_{\uparrow\downarrow}}$$

Magnetic Ground States

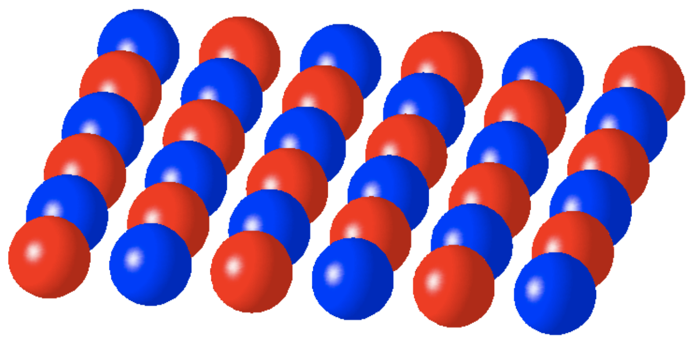
Z-Ferromagnet:



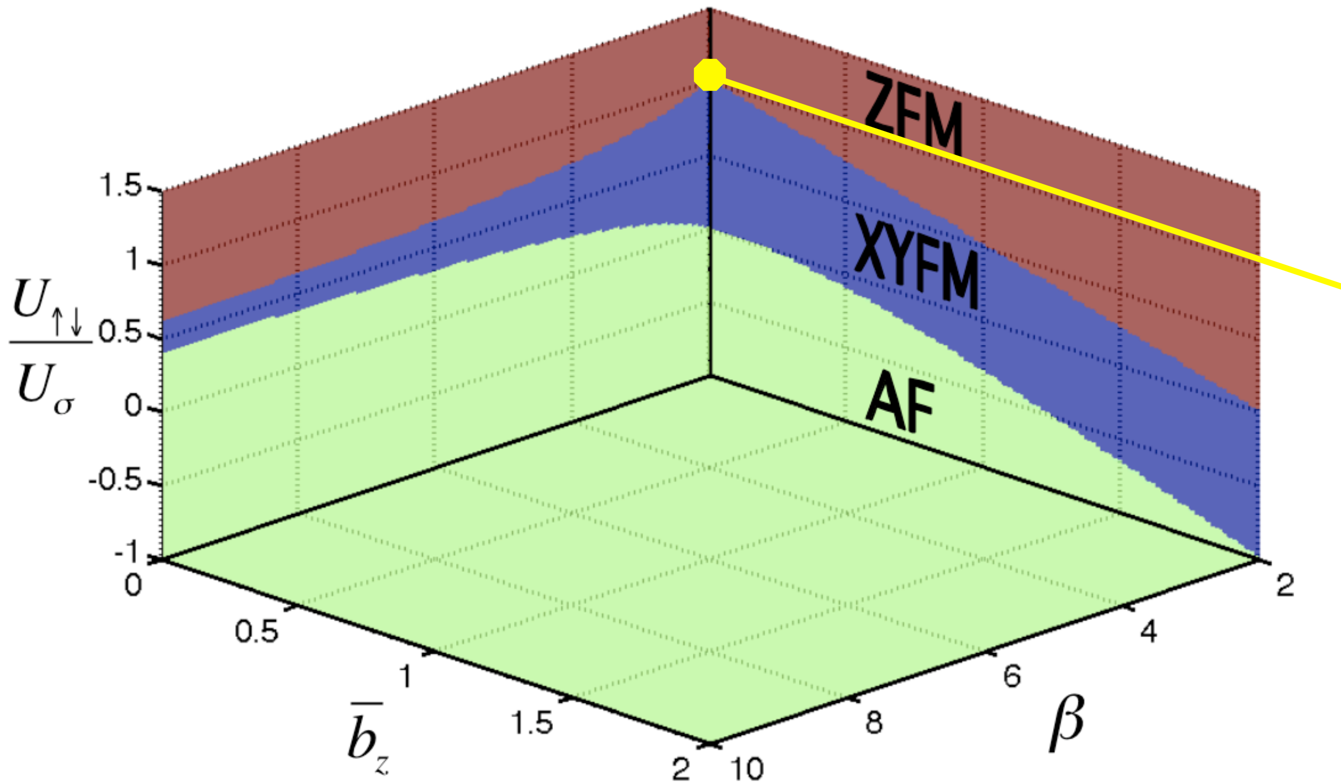
XY-Ferromagnet:



Antiferromagnet:



using Spin dependent lattices



Phase diagram of two-component bosons in a spin-dependent lattice.
x axis (ß) is the tunneling asymmetry
z axis is the ratio between the interspin and intraspin interaction energies
y axis is the normalized magnetic field

Altman, Hofstetter, Lukin, Demler (2003)

$$T_{Neel} \approx \frac{t^2}{U}$$

$$\approx 100 \text{ pK}$$

For Rb atoms in
0.5μm optical Lattice

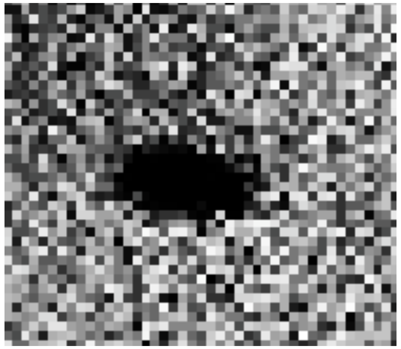
Two component Bosons for Quantum Magnetism

Before we can mix them, we have to control their separation

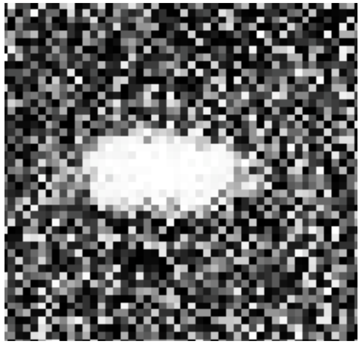
Two component Mott insulator

- equal mix of (2,-2) and (2, 2) states (or (1,-1))
- Variable B-field gradient
- Polarization contrast imaging (or two absorption pictures):

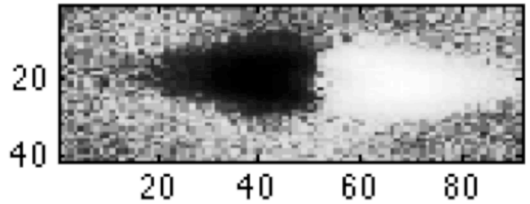
2,+2 (or (1,-1) black, 2,-2 white



2,+2



2,-2



Mixture

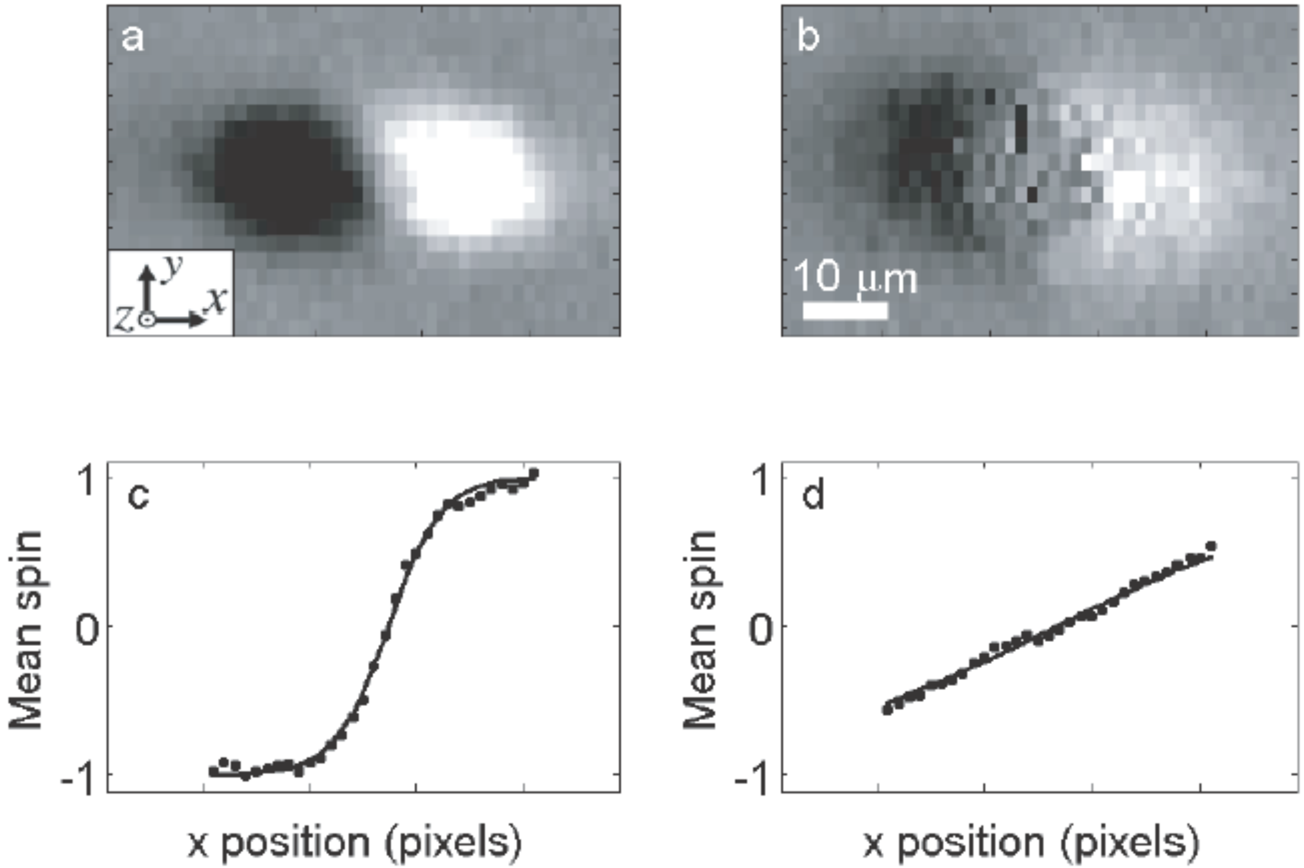
Spin Gradient Thermometry in a 2-component Mott insulator

- Spins separate in a magnetic field gradient (1 G/cm typical)
- At zero temperature: sharp boundary
- At non-zero temperatures: finite boundary region

$$\langle s \rangle = 2\tanh\left(\beta\mu_B \frac{d|B|}{dx} x \right)$$

Spin Gradient Thermometry in the Mott Insulator

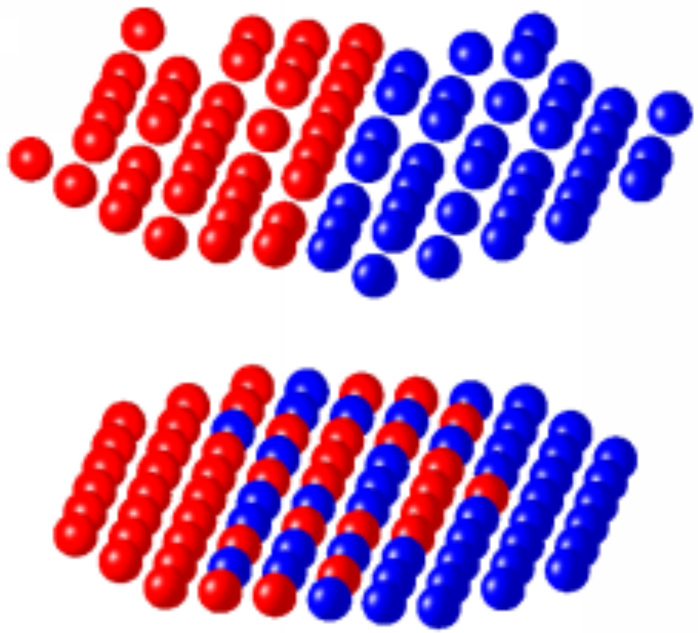
Cold **Hot**

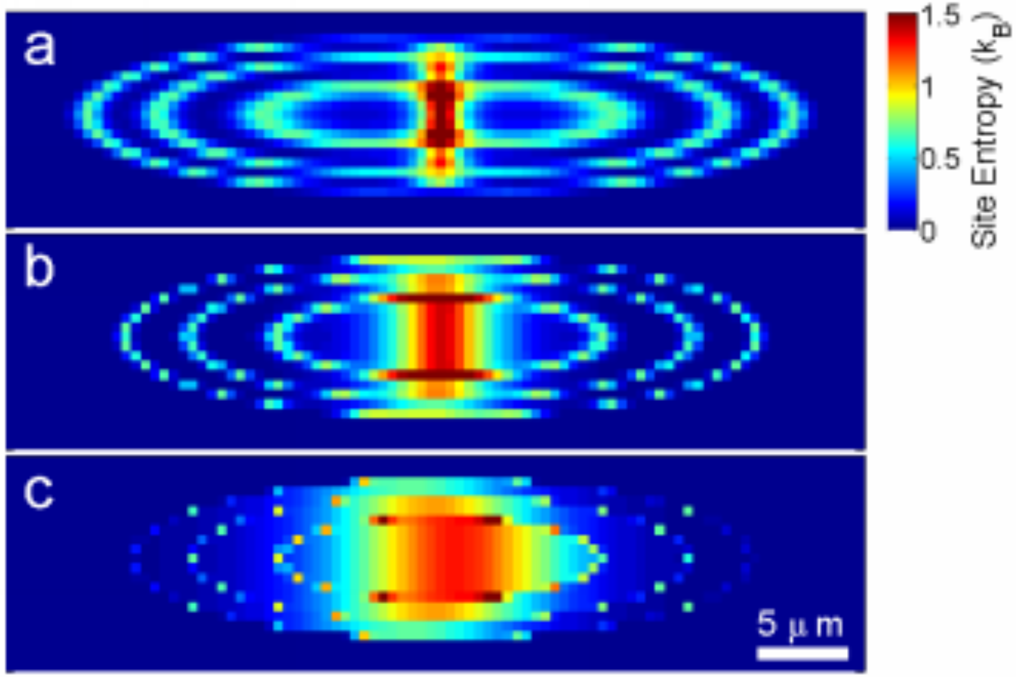


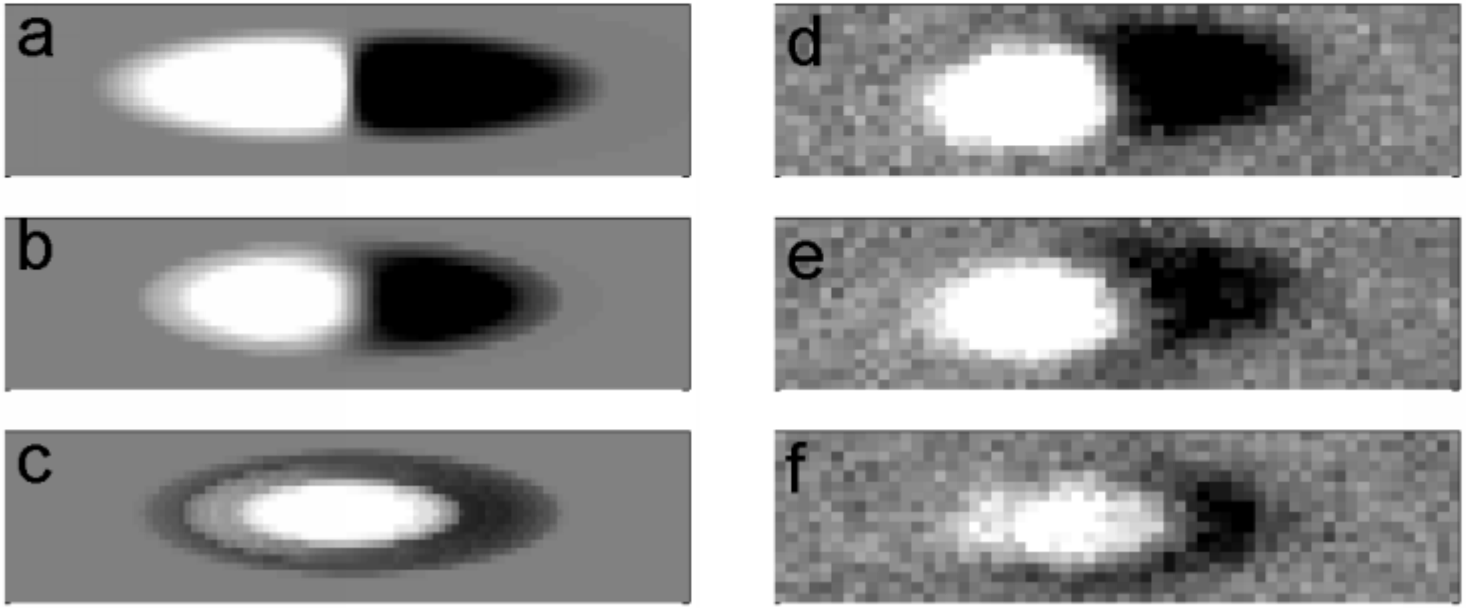
52 nK 296 nK

D.M. Weld, P. Medley, H. Miyake, D. Hucul, D.E. Pritchard, W. K., Phys. Rev. Lett. 103, 245301 (2009).

Spin gradient demagnetization cooling: adiabatic cooling by “mixing” two spin states







0.7 G/cm
6 nK

0.06 G/cm
2 nK

0.0024 G/cm
0.4 nK

Simulation Experiment

Simulation includes buoyancy effect due to slightly different scattering lengths of the two states

D.M. Weld, H. Miyake, P. Medley, D.E. Pritchard, W. K.
Phys. Rev. A 82, 051603 (2010)

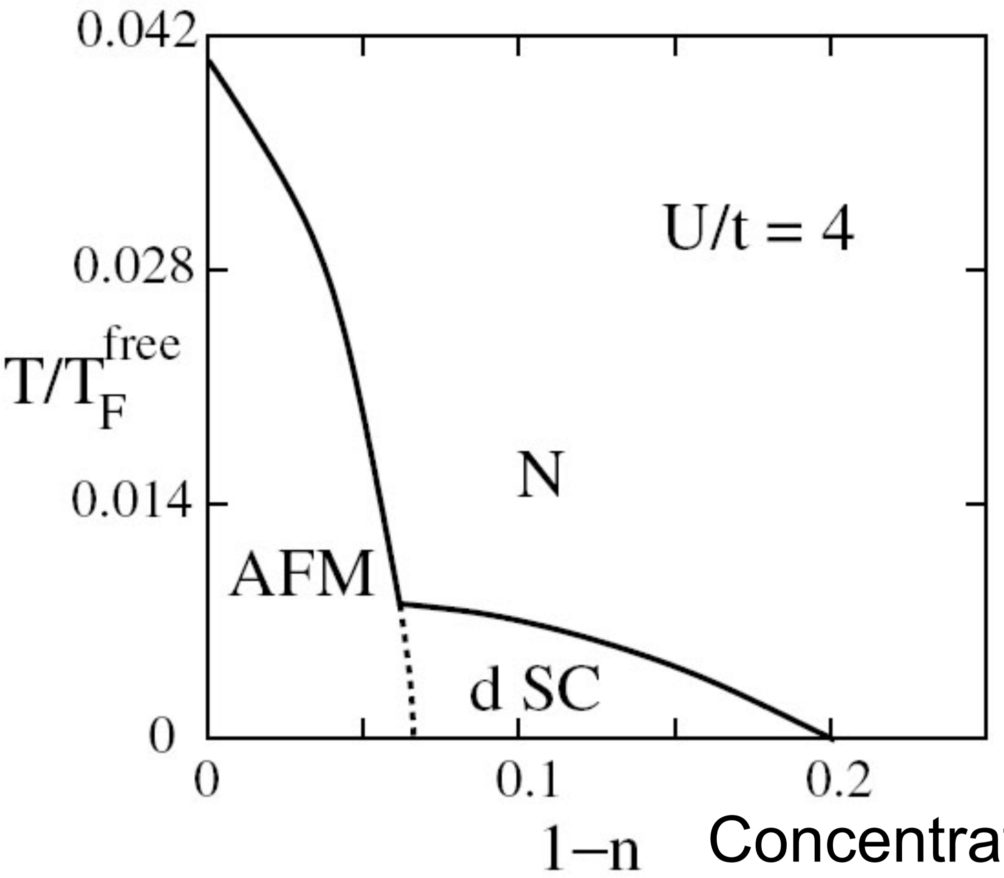
Lowest temperatures reached with spin gradient demagnetization cooling:

spin temperature 50 pK
partially equilibrated Mott insulator 350 pk

P. Medley, D.M. Weld, H. Miyake, D.E. Pritchard, W.K.,
Phys. Rev. Lett. **106**, 195301 (2011).

Long term goal for fermions

Suggested phase diagram for positive U Fermi Hubbard model (Scalapino 1995)



Concentration of holes

Needed: Lattice cooling methods