

Lecture II: Precision quantum metrology

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ICAP Summer School, Paris, July 20, 2012

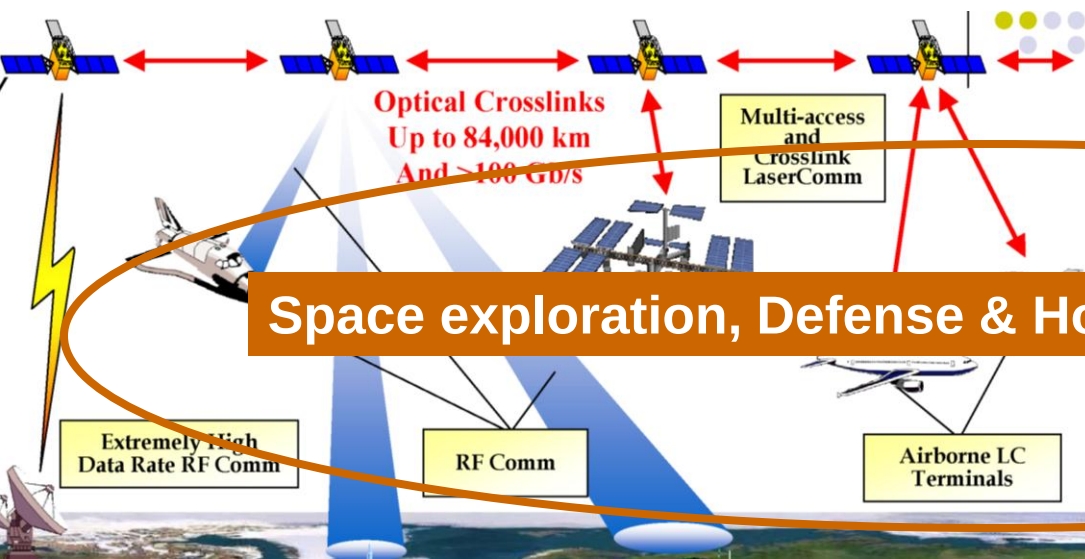
\$ Funding \$

NIST, NSF,
AFOSR, DARPA
DOE, ONR



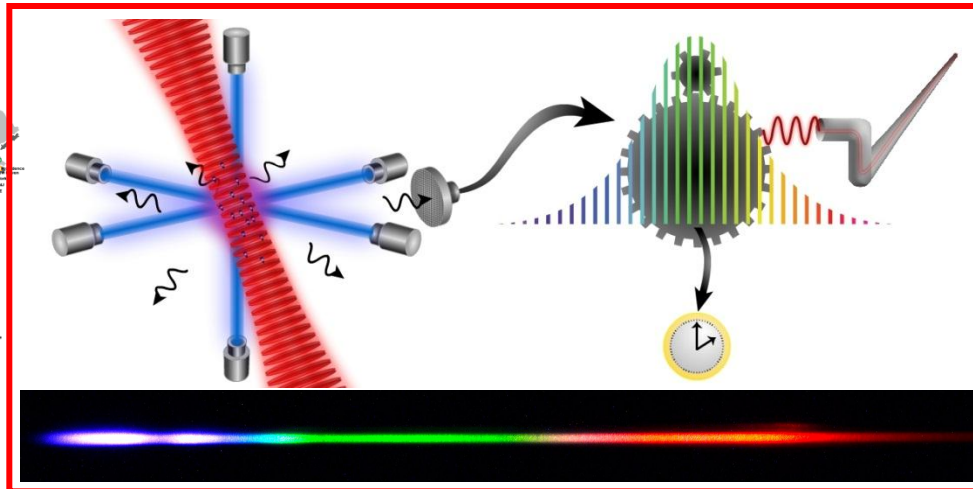
Clocks are everywhere

Space exploration, Defense & Homeland security

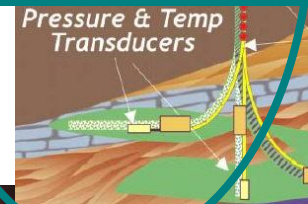


Broadwing All-Optical Switched Network

**Tele-
communications**



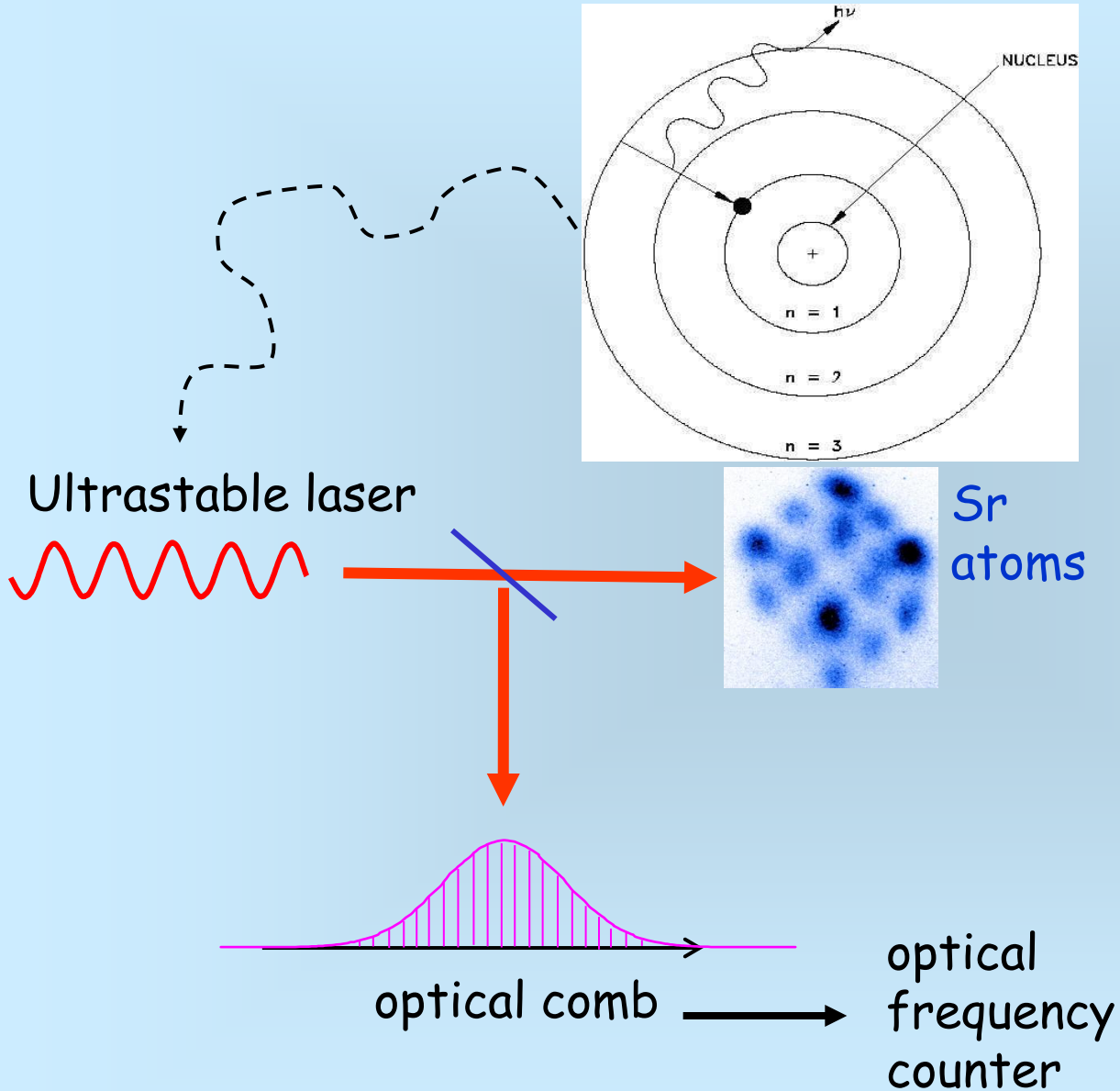
**Standards for
Industry**



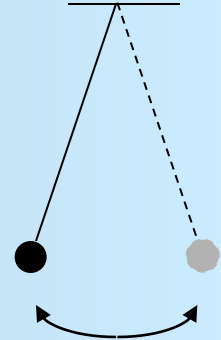
Fundamental and Applied Science



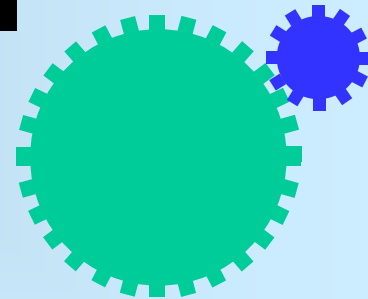
Optical atomic clocks



Oscillator



Counter



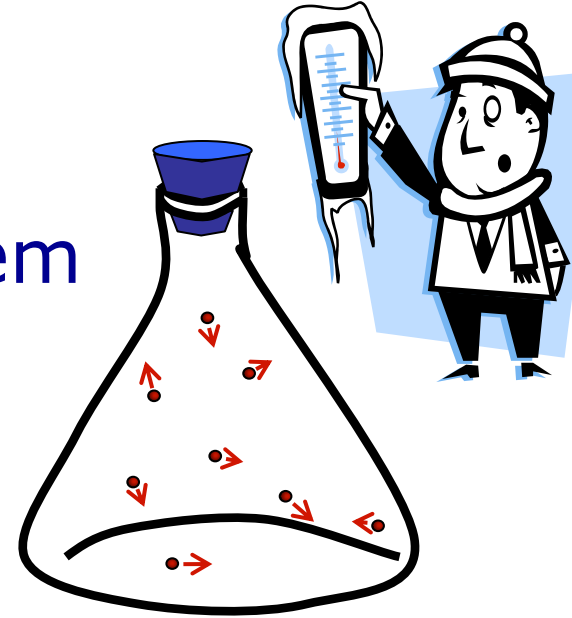
Ultracold Matter

Precise control of a quantum system

The most precise measurements, e.g., clocks

Quantum information

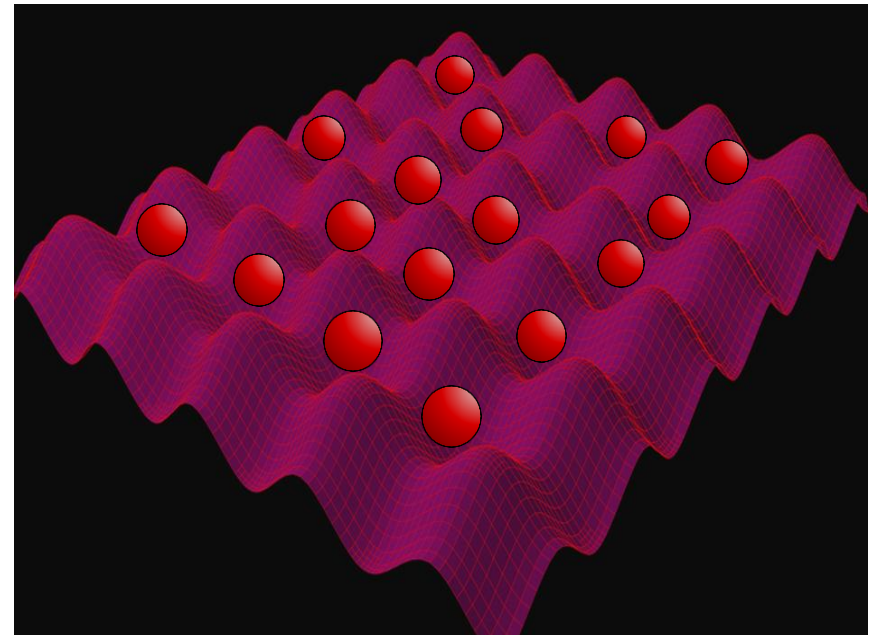
Quantum sensors



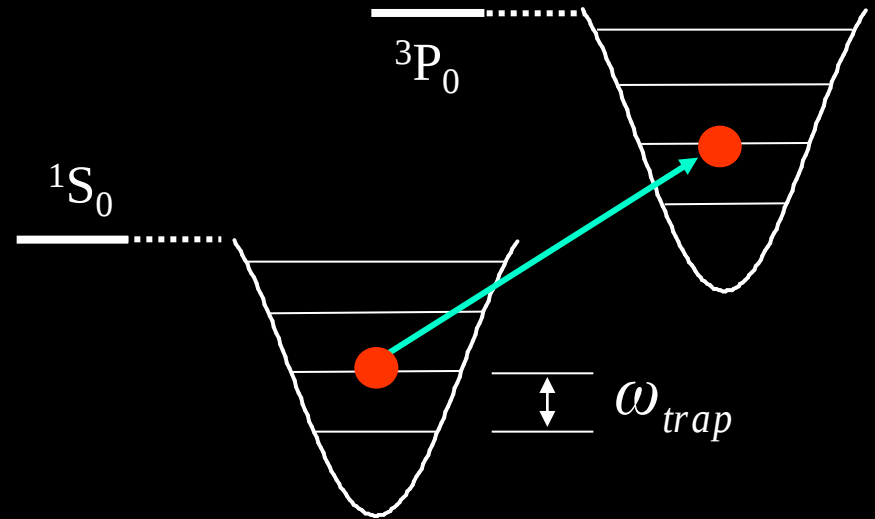
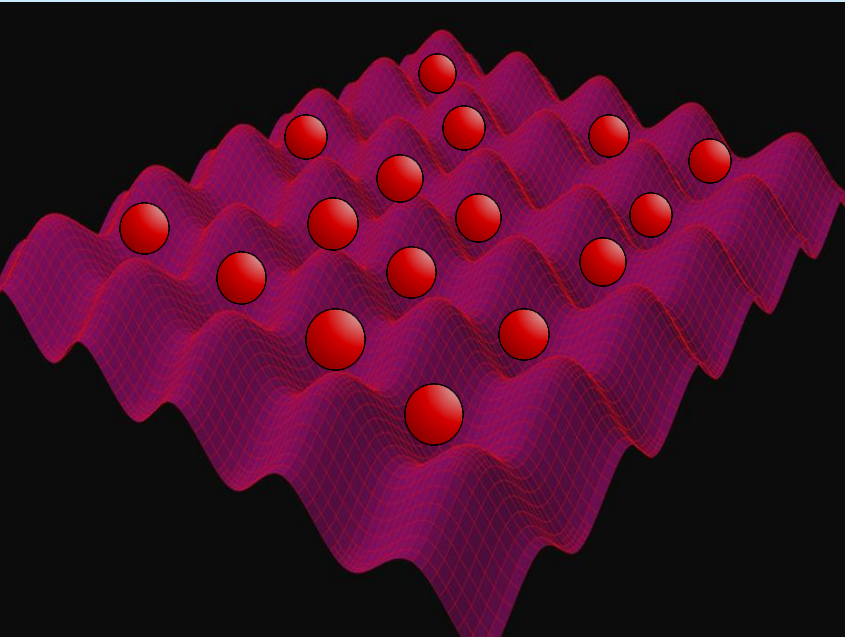
Control: A tool for understanding complexity

Strongly correlated many-body quantum systems

- Superfluidity & Superconductivity
- Quantum magnetism
- (Fully-)Quantum chemistry

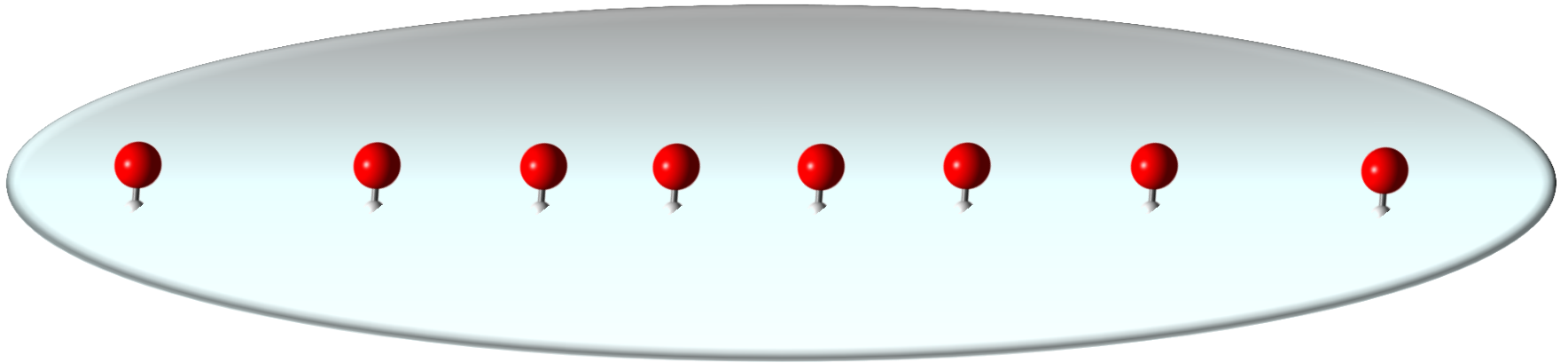


Quantum metrology in optical lattice



- Atomic confinement $\ll \lambda$ ($e^{ikx} \sim 1$, $k = 2\pi/\lambda_{probe}$)
- Trap potential identical for 1S_0 and 3P_0
- Precision improvement by $N^{1/2}$
$$\frac{\delta\omega(\tau)}{\omega_0} = \frac{1}{Q} \times \frac{1}{\sqrt{N}} \times \sqrt{\frac{T_c}{\tau}}$$
- Long coherence time; Zero Doppler shift, Zero recoil shift
- No light shift from the trap; but, Interaction effects? Accuracy?

An interacting many-body quantum system

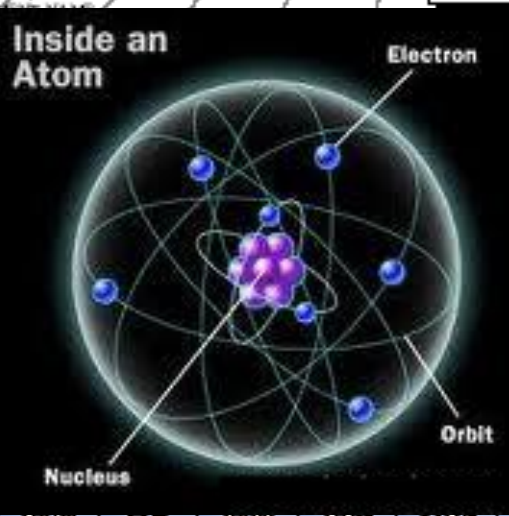


$$U \sim 1 \text{ Hz} \sim 50 \times 10^{-12} \text{ K}$$

PERIODIC TABLE OF THE ELEMENTS

GROUP	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
PERIOD	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1.0079 H HYDROGEN																	
2	3 6.941 Li LITHIUM	4 9.0122 Be BERYLLIUM																
3	11 22.990 Na SODIUM	12 24.305 Mg MAGNESIUM																
4	19 39.098 K POTASSIUM	20 40.078 Ca CALCIUM	21 44.956 Sc SCANDIUM	22 47.867 Ti TITANIUM														
5	37 85.468 Rb RUBIDIUM	38 87.62 Sr STRONTIUM	39 88.906 Y YTRIUM	40 91.224 Zr ZIRCONIUM														
6	55 132.91 Cs CAESIUM	56 137.33 Ba BARIUM	57-71 La-Lu Lanthanide	72 178.49 Hf HAFNIUM														
7	87 (223) Fr FRANCIUM	88 (226) Ra RADIUM	89-103 Ac-Lr Actinide	104 (261) Rf RUFORFIUM														

Strontium



Quality factor
> 10^{17}

Once set, it swings during
the entire age of the universe

18	VIII
4.0026	
He	
HELIUM	
0 20.180	
Ne	
NEON	
8 39.948	
Ar	
ARGON	
6 83.80	
Kr	
KRYPTON	
4 131.29	
Xe	
XENON	
6 (222)	
Rn	
RADON	

GROUP	13	14	15	16	17	18
PERIOD	7	8	9	10	11	12
7	113 (288) Nh NIHONIUM	114 (289) Fl FLEROVIUM	115 (290) Mc MOSCOVIUM	116 (291) Lv LIVERMORIUM	117 (292) Ts TENNESSIUM	118 (293) Og OGANESSONIUM

GROUP	3	4	5	6	7	8	9	10	11	12
PERIOD	7	8	9	10	11	12	13	14	15	16
7	101 (257) Db DUBNIUM	102 (258) Sg SEABORGIUM	103 (259) Bh BOHRNIUM	104 (261) Rf RUFORFIUM	105 (262) Db DUBNIUM	106 (263) Sg SEABORGIUM	107 (264) Bh BOHRNIUM	108 (265) Hs HASSIUM	109 (266) Mt MEITNERIUM	110 (267) Ds DARMSTADTIUM

Alkaline Earth versus Alkali

Alkali

Alkaline Earth

All-Optical Cooling to Ultra-Low Temperatures

Polarization Gradient
Stimulated Raman
VSCPT

Limited to Low Densities

High Density

Intercombination line sub-recoil
Sideband Cooling in Dipole Traps

Low Transfer Efficiency

Cold / Ultra-Cold Collisions

High Collision Rates
Feshbach Resonances
Hyperfine structure

Quantitative Studies
Diverse polar molecules
Low Collision Rates

BEC / FDG

Ground-State Magnetic Traps
Tunable Interactions
Only two Fermions: ^{40}K , ^6Li

Diversity of Bose, Fermi Isotopes
Optical Feshbach resonance
Structure Free Ground State

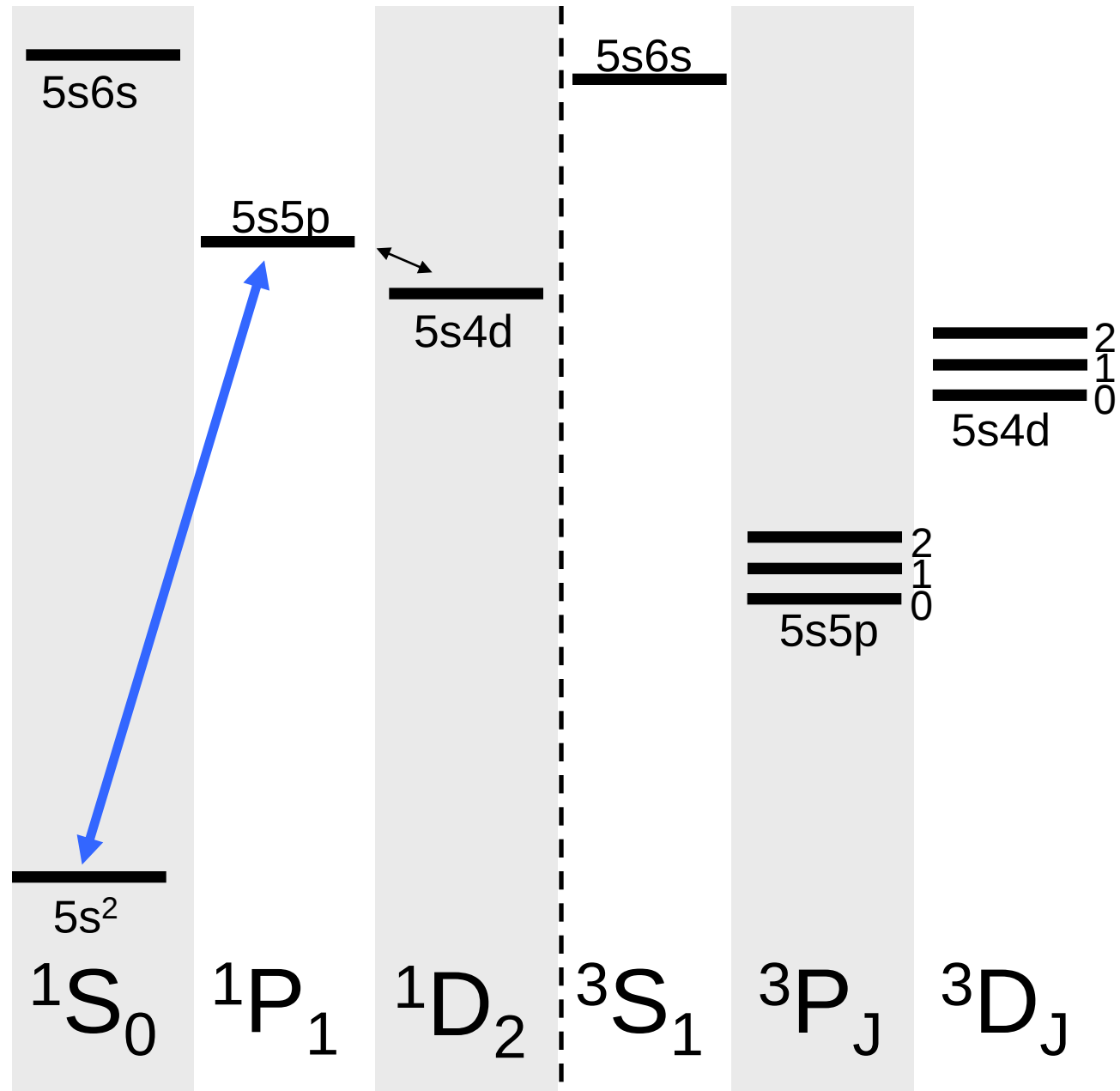
Time / Frequency Metrology

Microwave Clocks
Small Optical Line Q

High Optical Line Q
Second-Stage Cooling Required

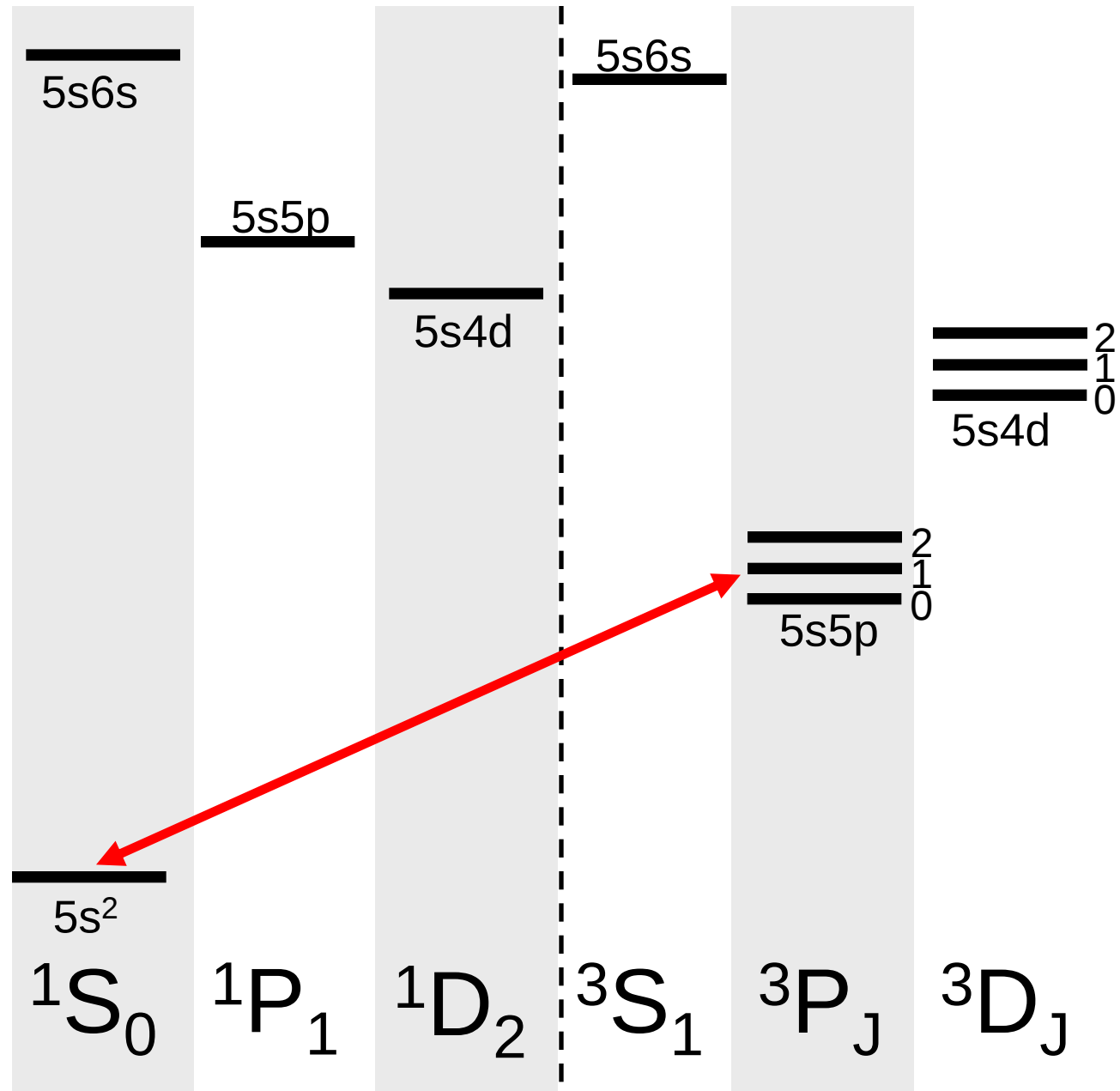
Strontium: first stage cooling

- large dipole moment
- mostly closed transition
- $J=0$ to $J=1$
- diode laser with frequency doubling

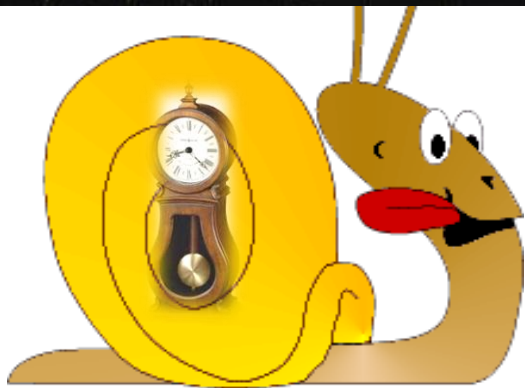
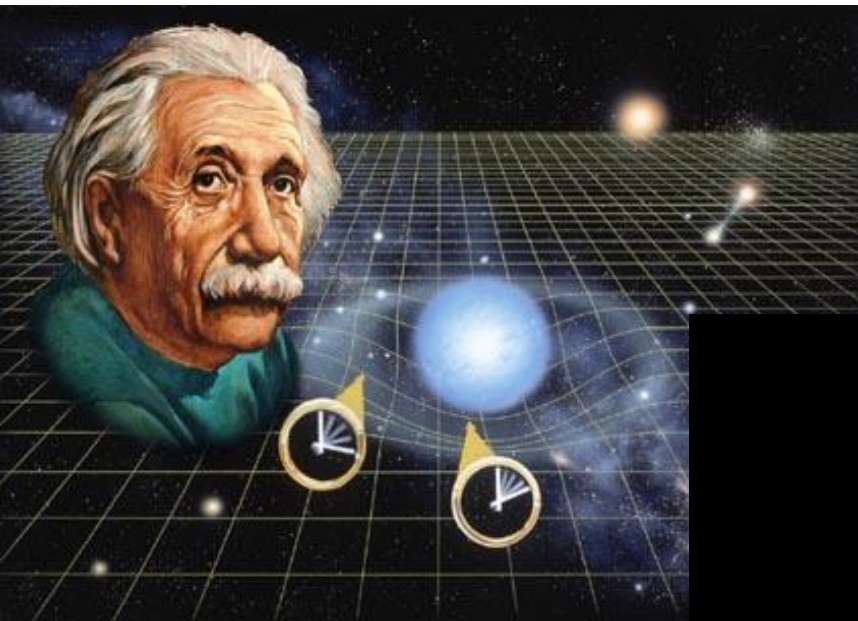


Strontium: Narrow Line Laser Cooling

- smaller dipole moment
- closed transition
- $J=0$ to $J=1$
- diode laser accessible



"Bad" things about motion - Short observations; Relativity (Doppler) shifts



Stationary Clock



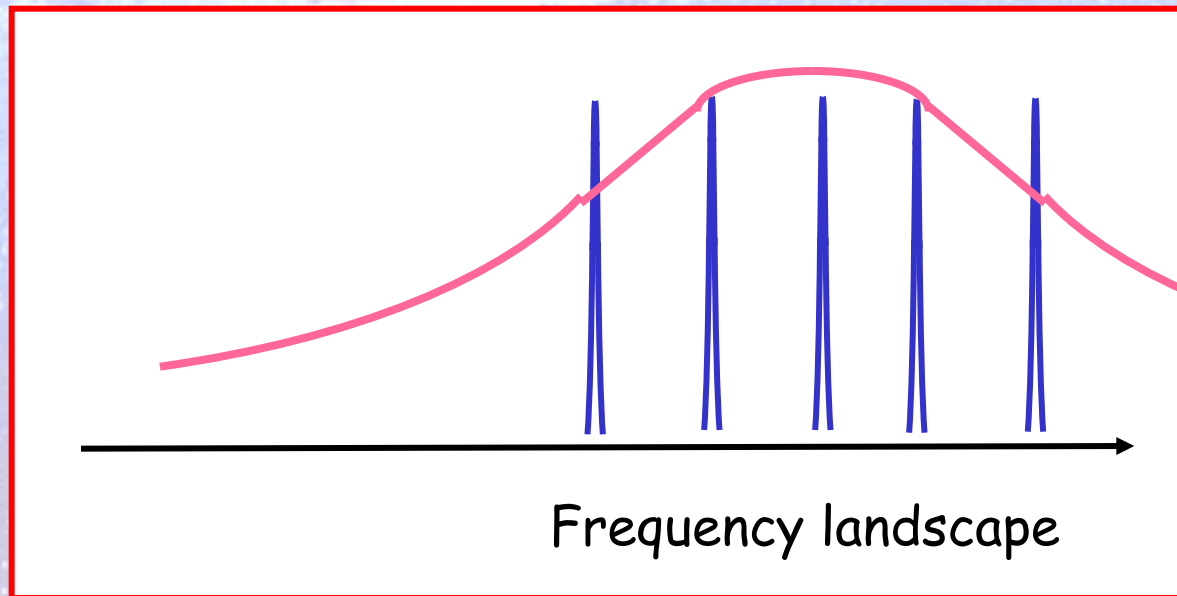
Moving Clock



87% speed of light

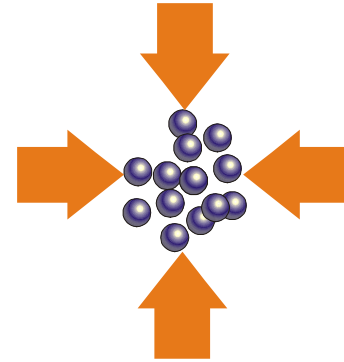
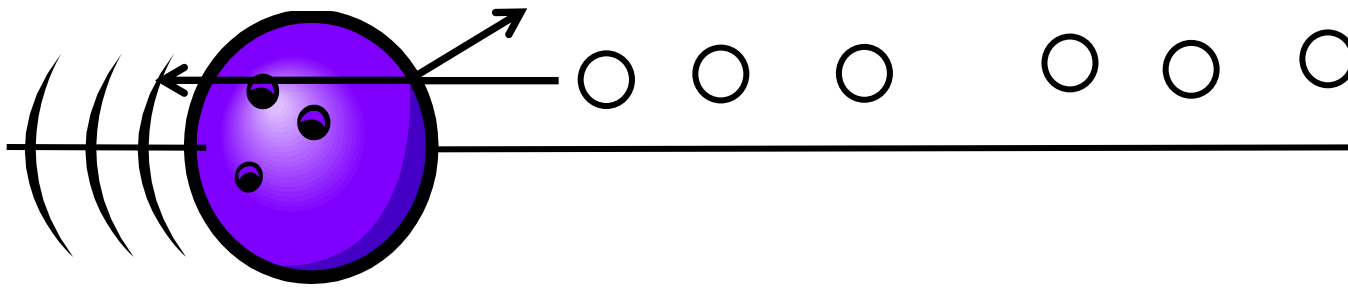


The Doppler Problem



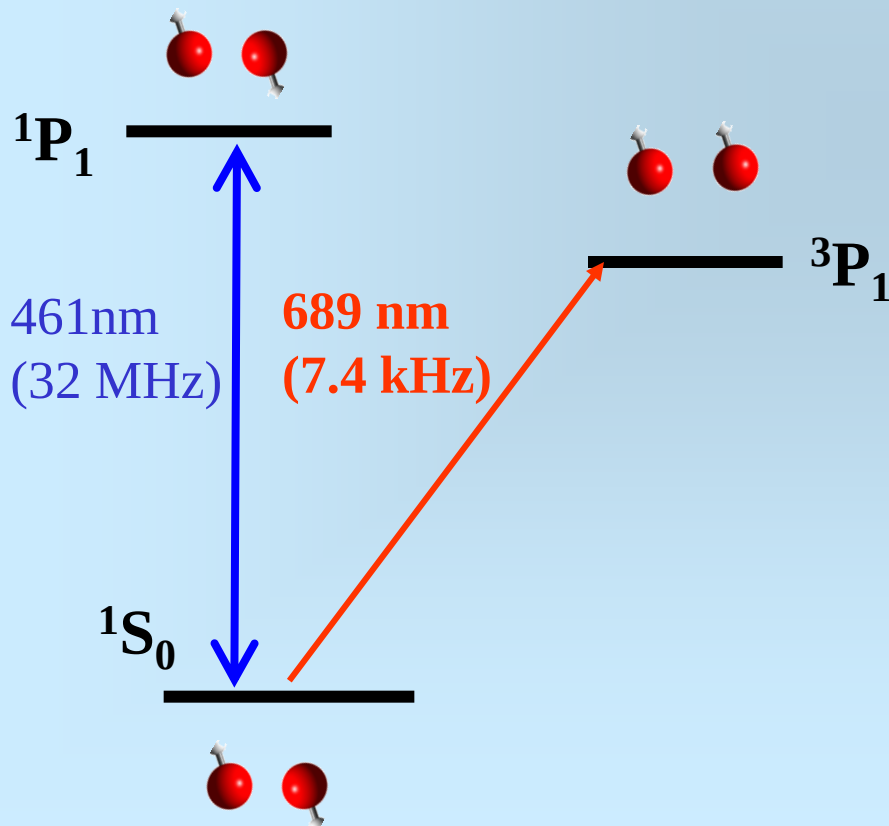
Cooling atoms using lasers

Always pushing atoms
against their motions

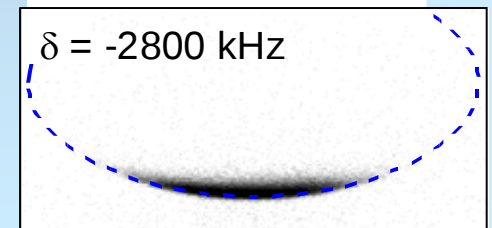
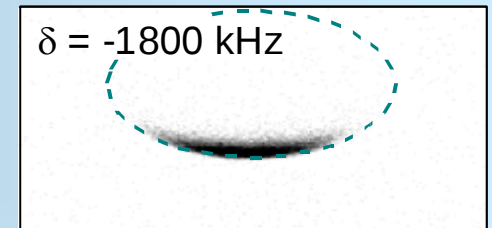
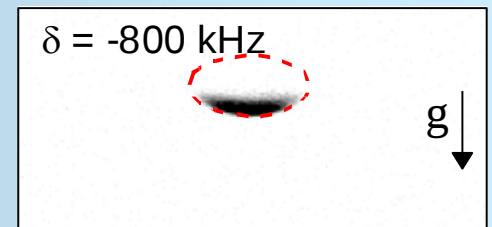


Stop the moving atom

1 billionth (10^{-9}) of room temperature



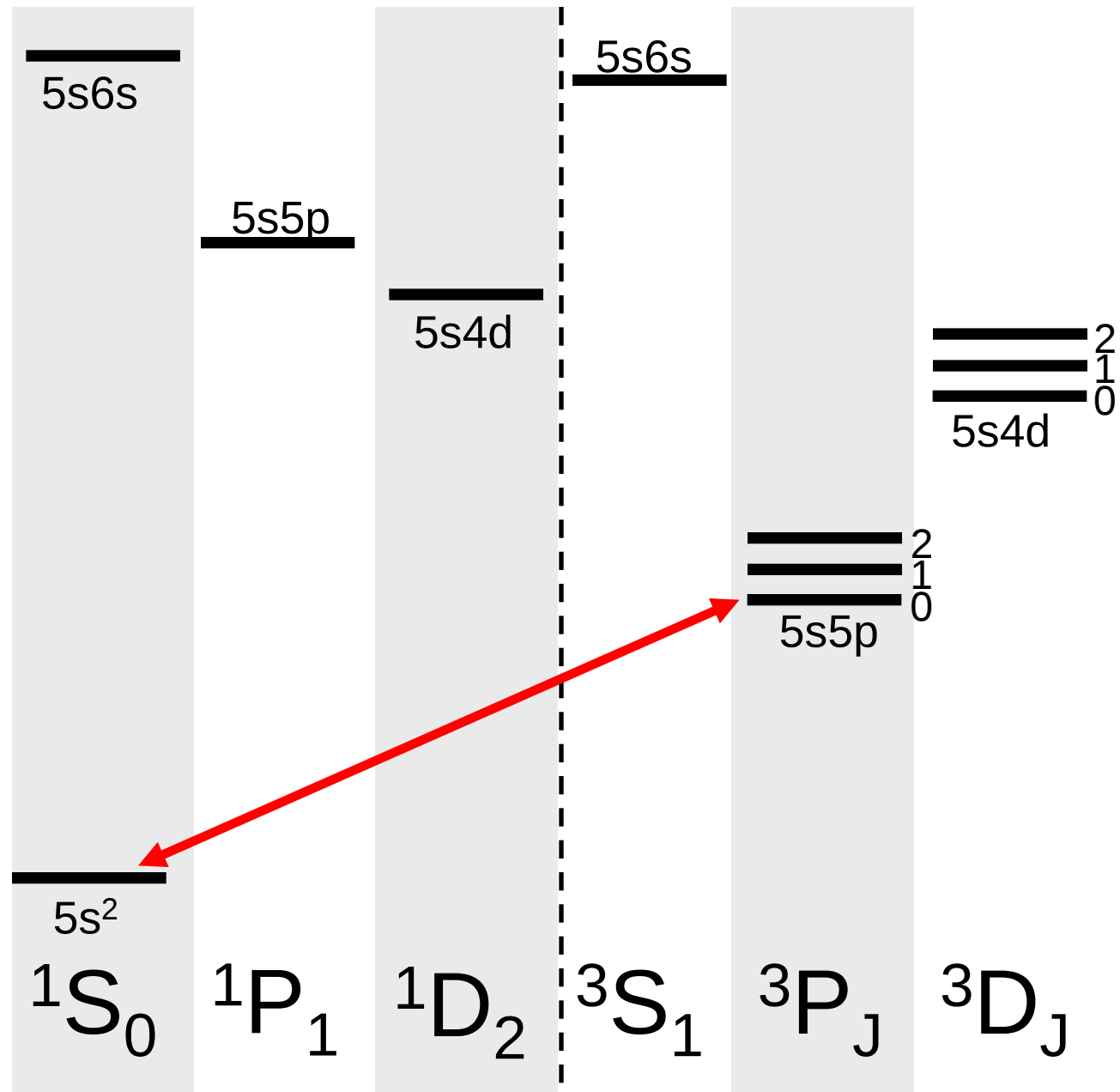
T ~ 0.5 photon recoil
~ 220 nK



5.6 mm

Strontium: Clock Transition

- HFI provides 1S_0 - 3P_0 clock ($\sim 1\text{mHz}$)
- field insensitive states
- diode laser
- accessible Stark-free confinement wavelength
- clock states $J=0$



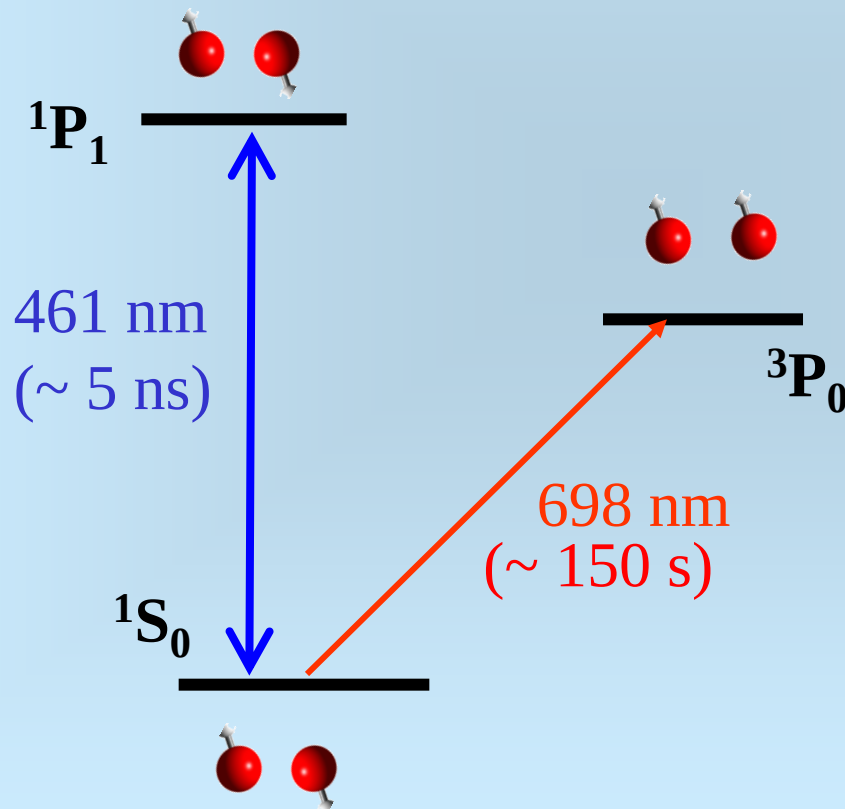
Alkaline earth - A tale of Twin Electrons

JILA, Tokyo, Paris, NIST, PTB, Florence, NICT, NPL, NIM, NRC

Science 314, 1430 (2006).

PRL 98, 083002 (2007).

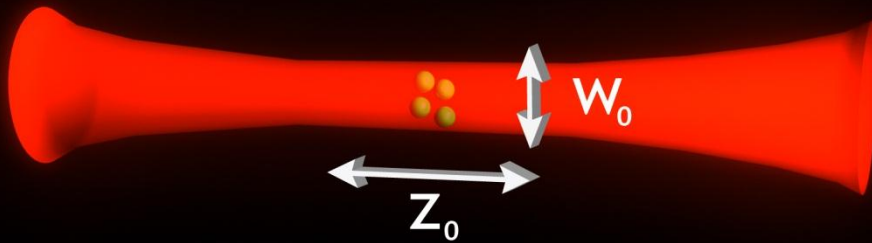
Science 319, 1805 (2008).



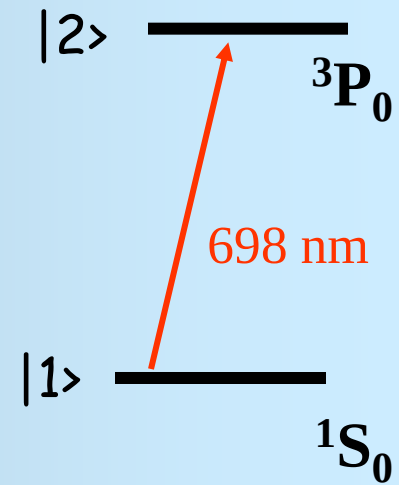
Metastable state
quality factor

$$Q > 10^{17}$$

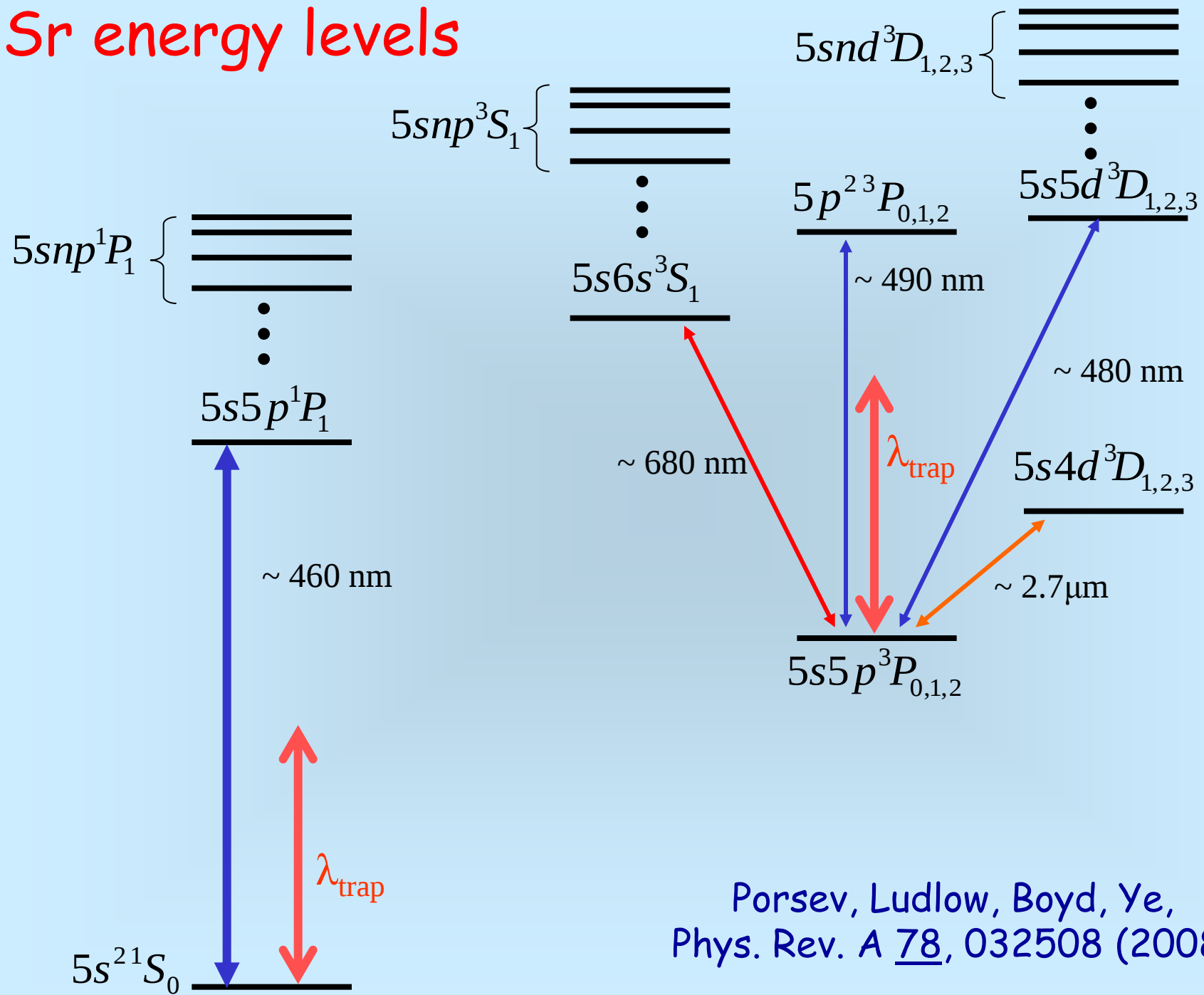
Trapping atom with light



Ye, Kimble, & Katori,
Science **320**, 1734 (2008).

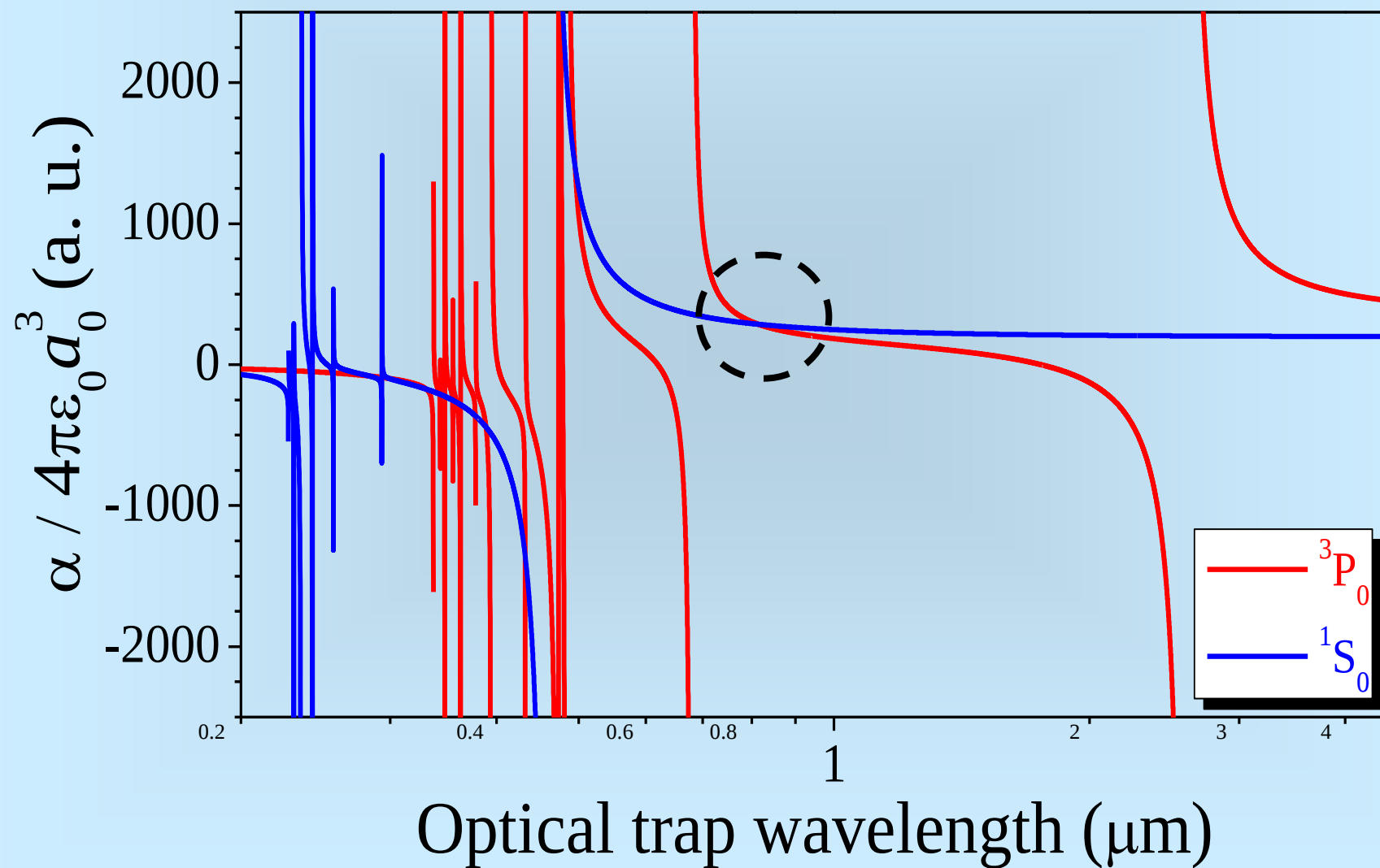


Sr energy levels

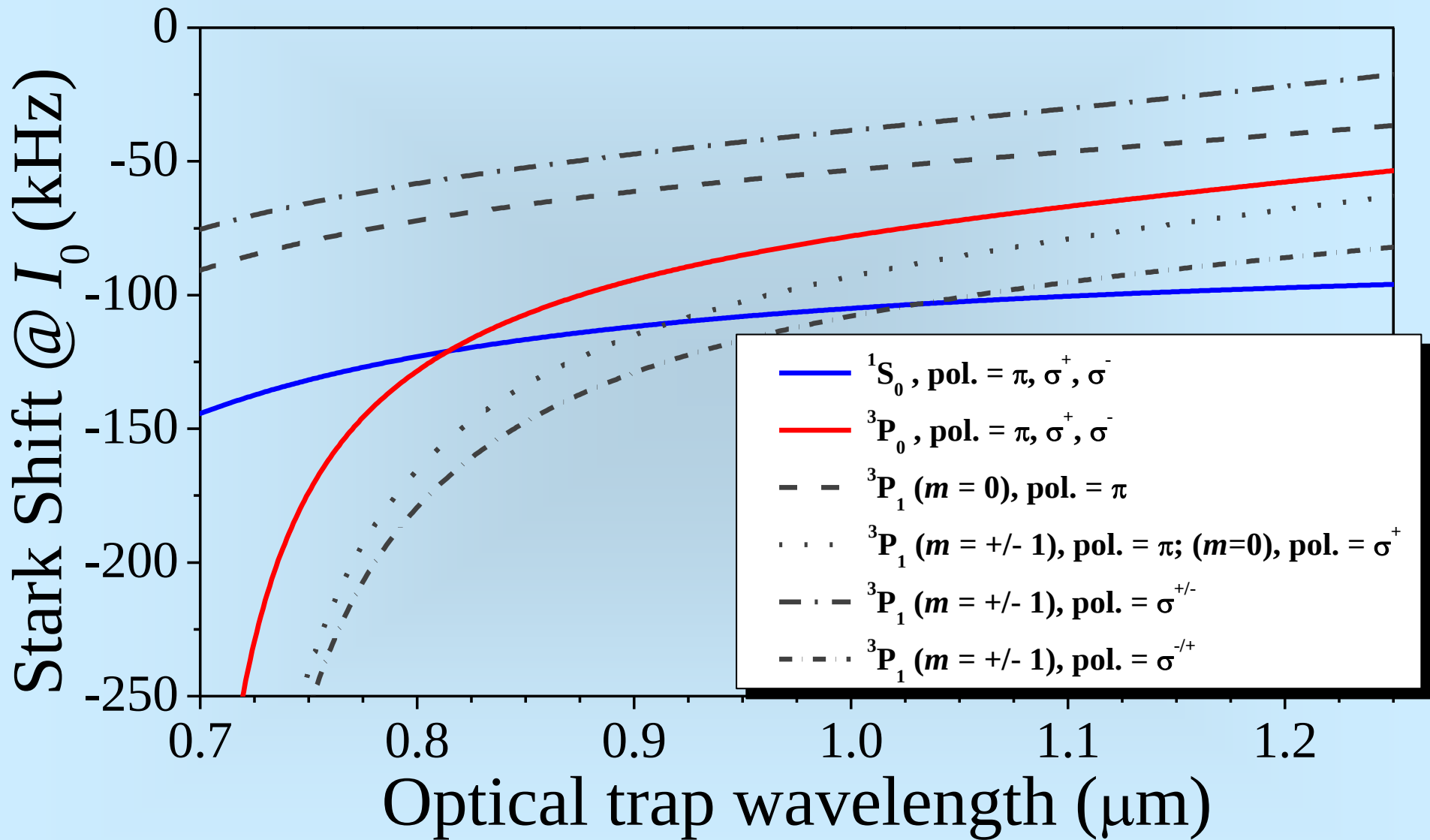


Porsev, Ludlow, Boyd, Ye,
Phys. Rev. A 78, 032508 (2008).

Crossing of polarizabilities



It's a mess if $J \neq 0$

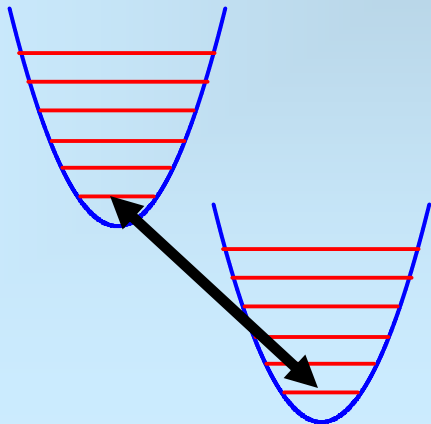


Important Regimes for Spectroscopy

$\omega_{trap} \gg \Gamma$ well-resolved sideband

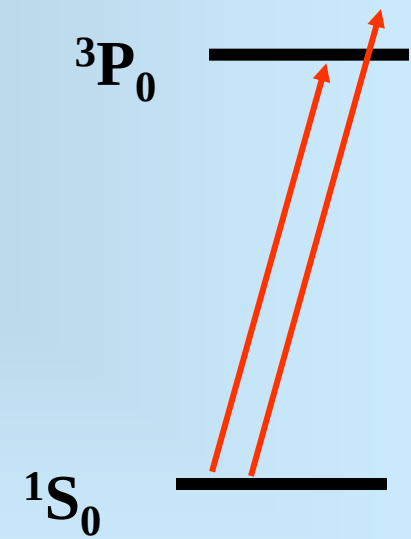
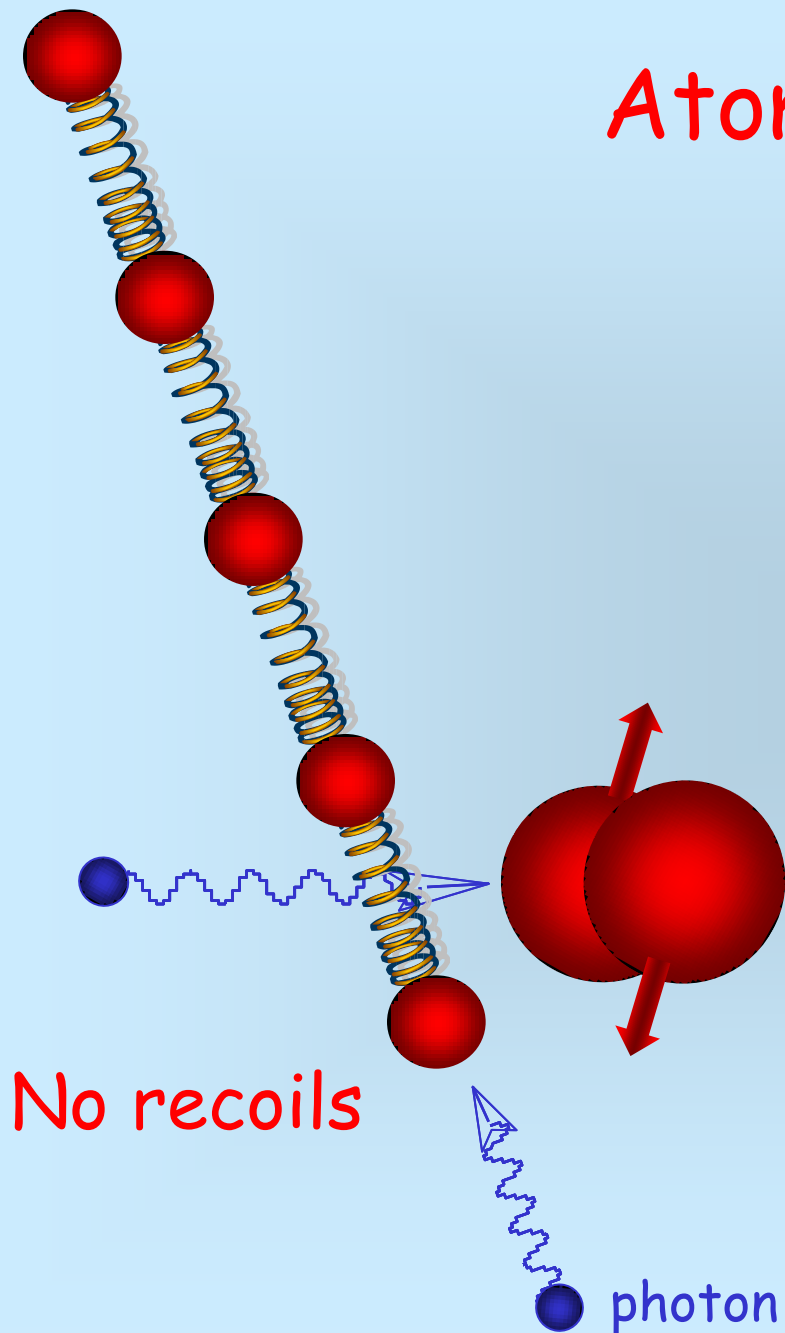
Lamb-Dicke

$\omega_{trap} \gg \omega_{recoil}$



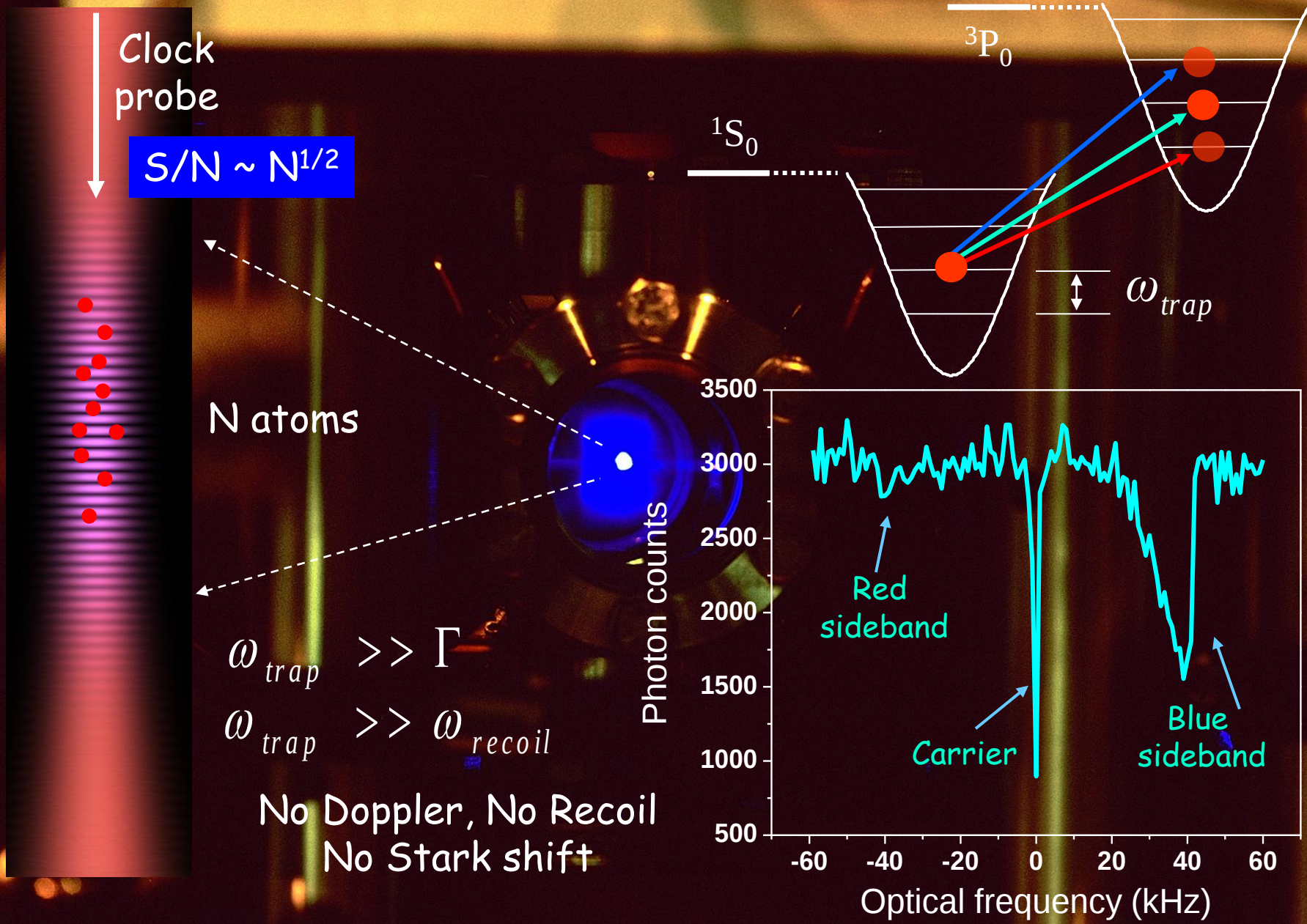
uniform confinement

Atomic recoil



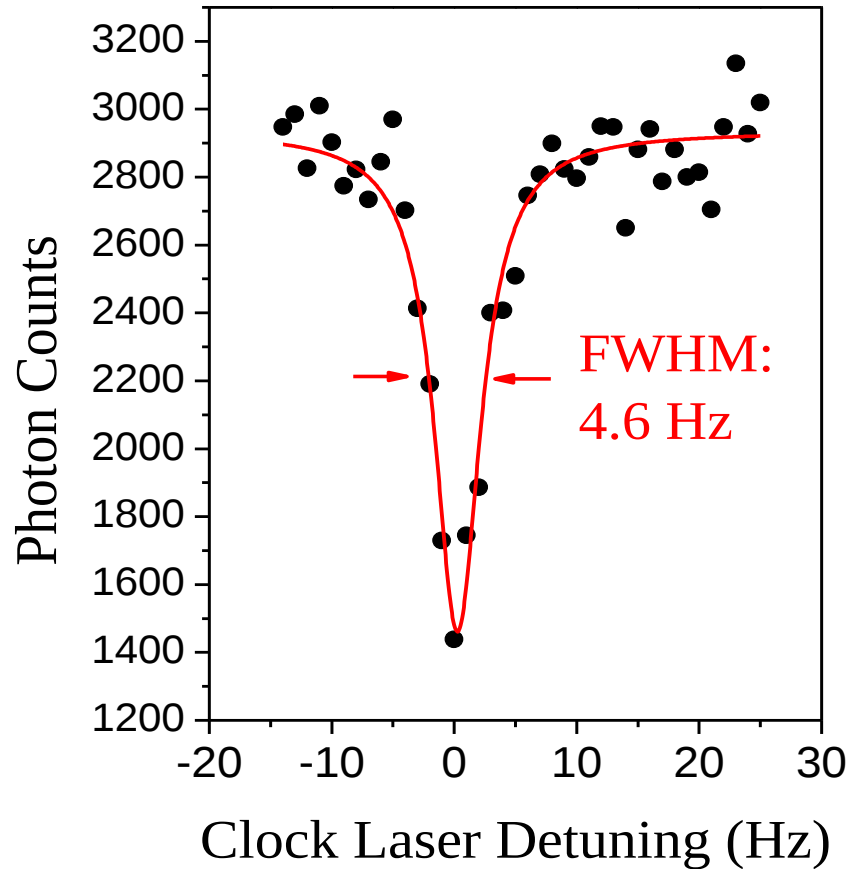
Spectroscopy at the magic wavelength

Ludlow et al., Phys. Rev. Lett. 96, 033003 (2006).



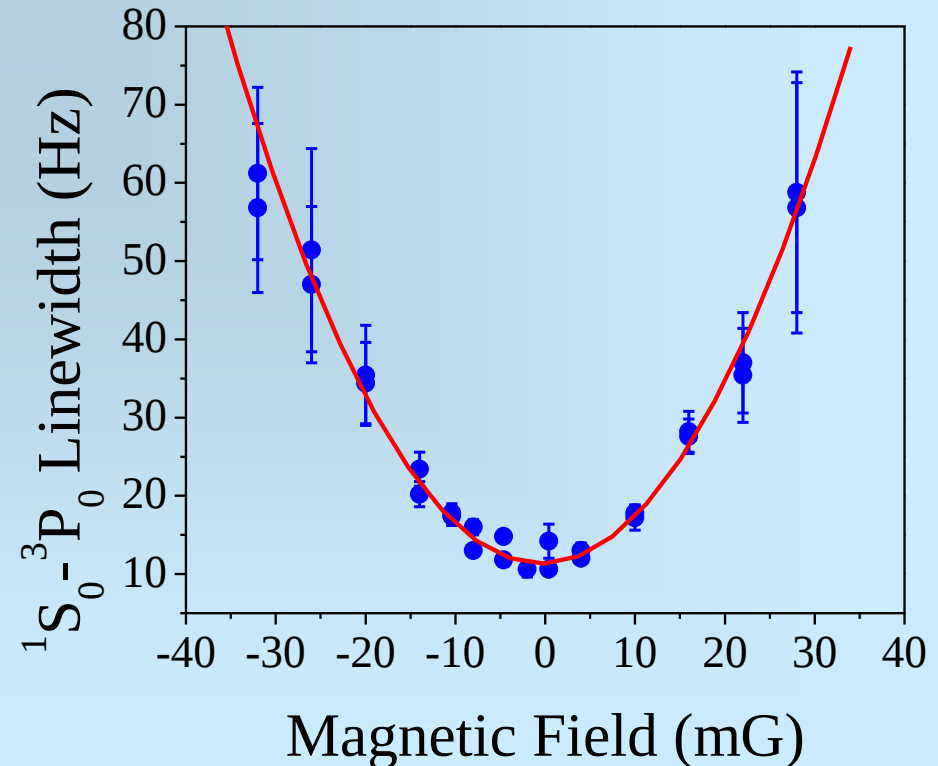
Zoom into the carrier of ^{87}Sr $^1\text{S}_0 - ^3\text{P}_0$

$Q \sim 1 \times 10^{14}$



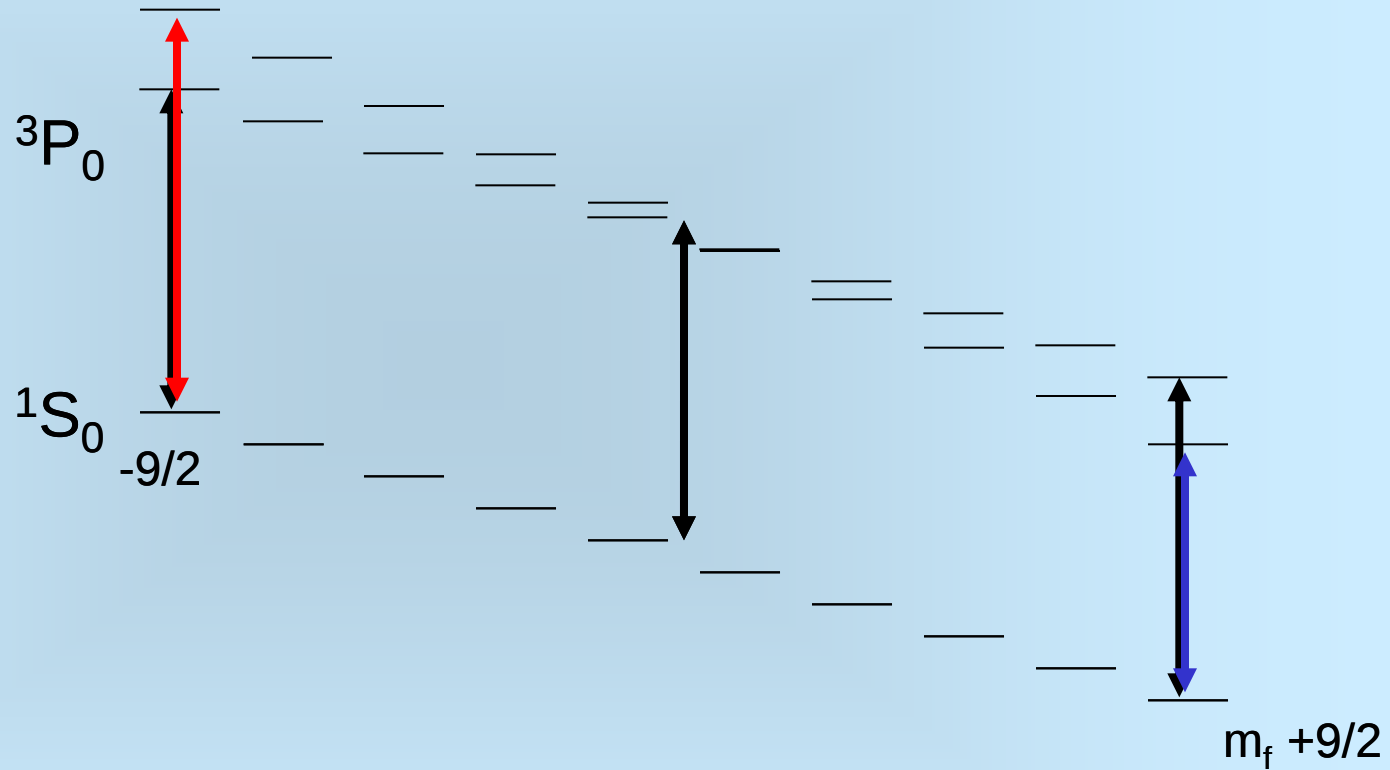
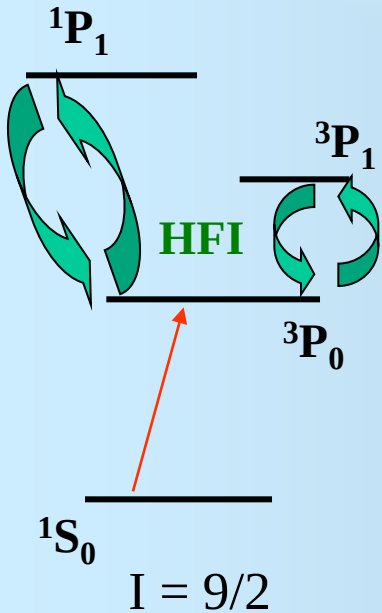
• Single trace without averaging

Magnetic Broadening



Differential Landé g -factor

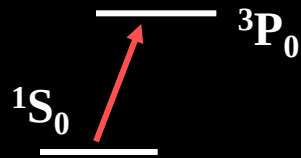
$$|{}^3P_0'\rangle = |{}^3P_0\rangle + c_1|{}^1P_1\rangle + c_2|{}^3P_1\rangle$$



- 3P_0 g-factor different from 1S_0 due to hyperfine
- Zeeman-shift; vector & tensor light shifts
- All are determined with high-resolution measurement

Optical Measurement of Nuclear g -factor

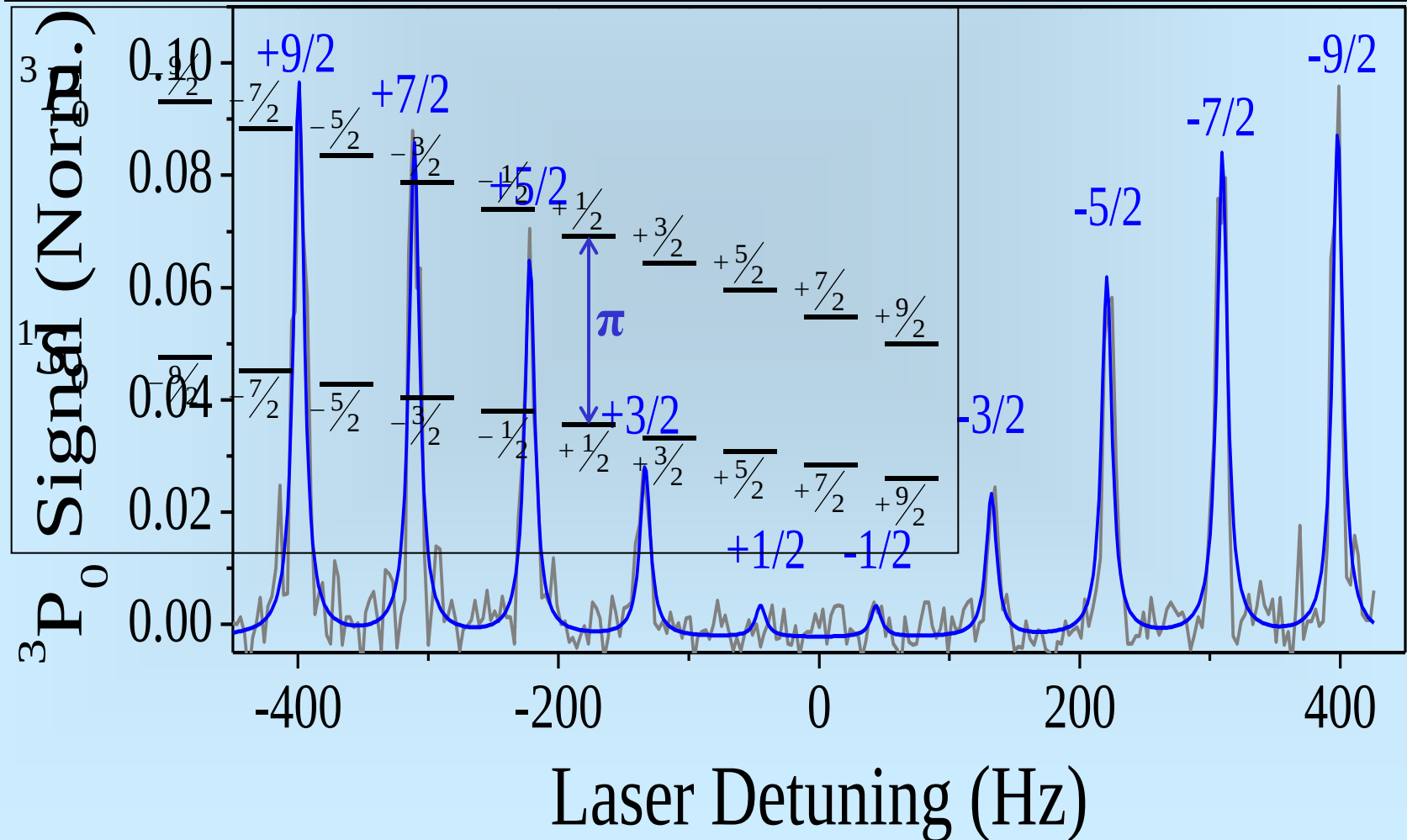
Boyd, Zelevinsky, Ludlow, Foreman, Blatt, Ido, & Ye, Science 314, 1430 (2006).



No net electronic angular momentum

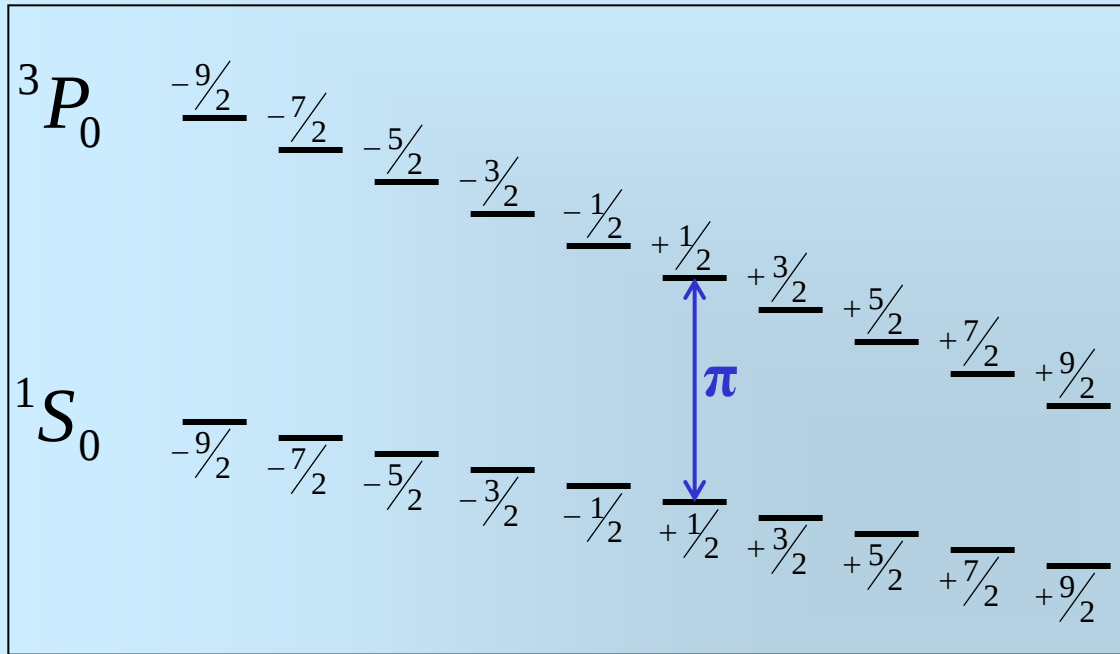
$$\Delta g = -108.4(4) \text{ Hz}/(\text{G m}_F)$$

$3P_0$ lifetime 140(40) s



Scalar, vector, tensor polarizabilities

Boyd *et al.*, PRA 76, 022510 (2007). Westergaard et al. PRL 106, 210801 (2011).



Hyperpolarizability
 $\sim (I_{\text{trap}})^2 : < 1\text{E-17}$
 P. Lemonde, SYRTE

$\Delta\alpha$: differential polarizability

ξ : polarization ellipticity

$$\begin{aligned}
 \nu_{\pi_{mF}} = & \nu_0 \\
 & - \Delta g \ m_F \ \mu_0 \ B \\
 & - (\Delta\alpha^S - \Delta\alpha^T F(F+1)) I_{\text{trap}} \\
 & - (\Delta\alpha^V \xi \ m_F + \Delta\alpha^T 3m_F^2) I_{\text{trap}}
 \end{aligned}$$

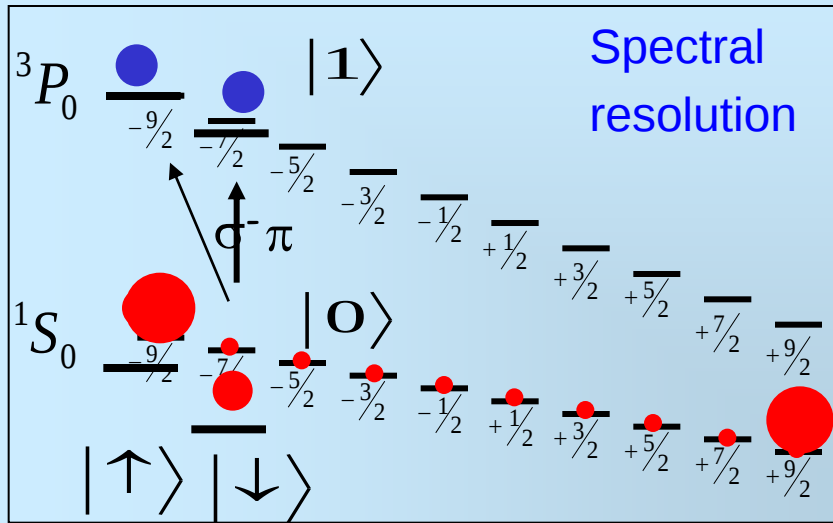
Clock frequency

1st order Zeeman

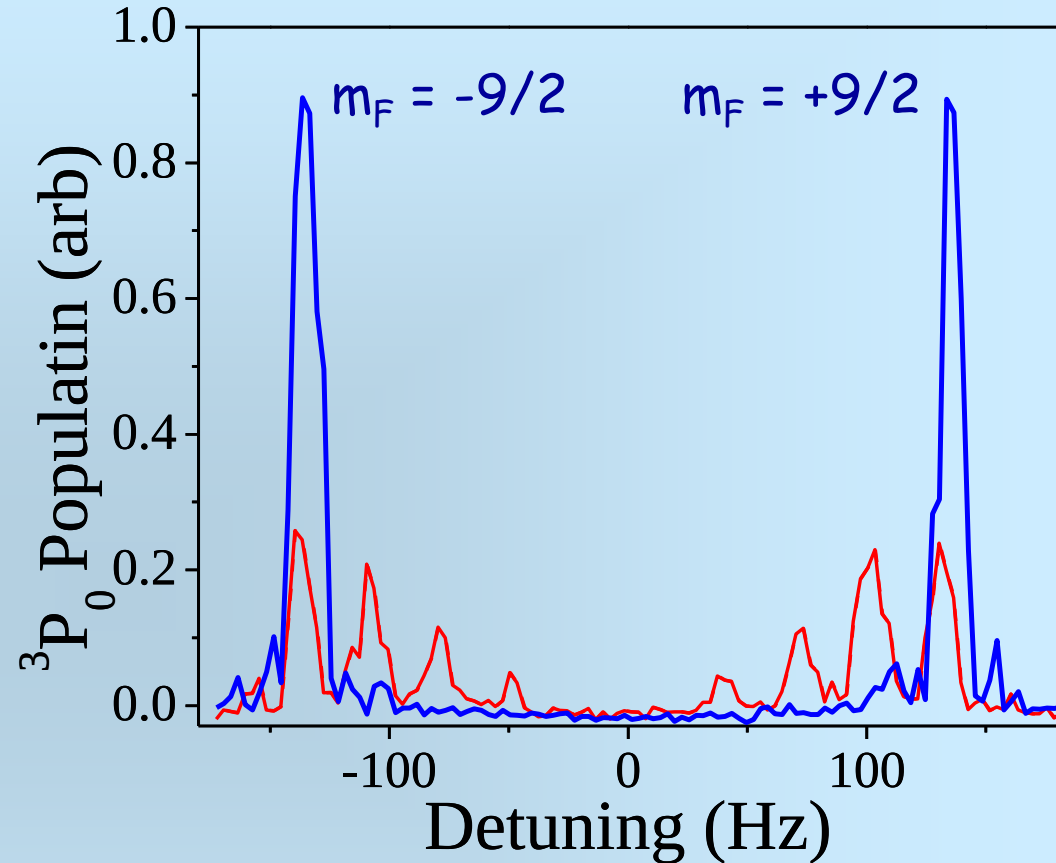
Scalar + Tensor
polarizability

Vector + Tensor
polarizability

Coherent optical manipulation of nuclear spins



Quantum computing
Quantum register
Quantum simulations

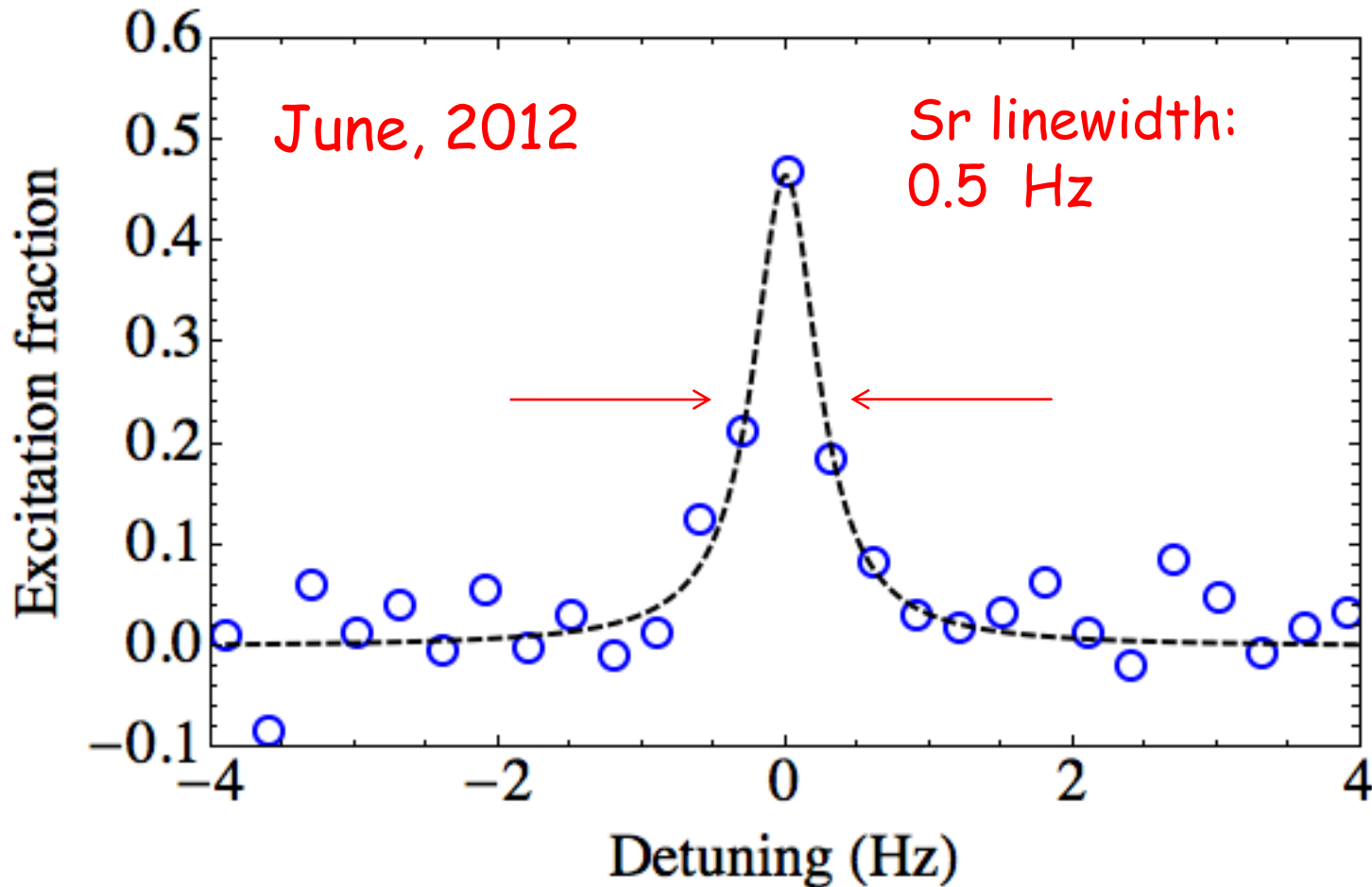


- Two stable spin systems, separately manipulated or entangled.
- Nuclear spins for information storage & accessed via electronic qubits.
- Electronic states for state-dependent lattice & control.
- $SU(N)$ symmetry; spin – orbital coupling dynamics.

Deutsch *et al.*, PRL **98** (2007); PRL **99**, (2007). Daley *et al.*, PRL **101** (2008).
Gorshkov *et al.*, PRL. **102** (2009); Nature Phys. **6**, 289 (2010).

Coherent spectroscopy $Q \sim 1 \times 10^{15}$

Boyd *et al.*, Science 314, 1430 (2006); PRL 98, 083002 (2007).



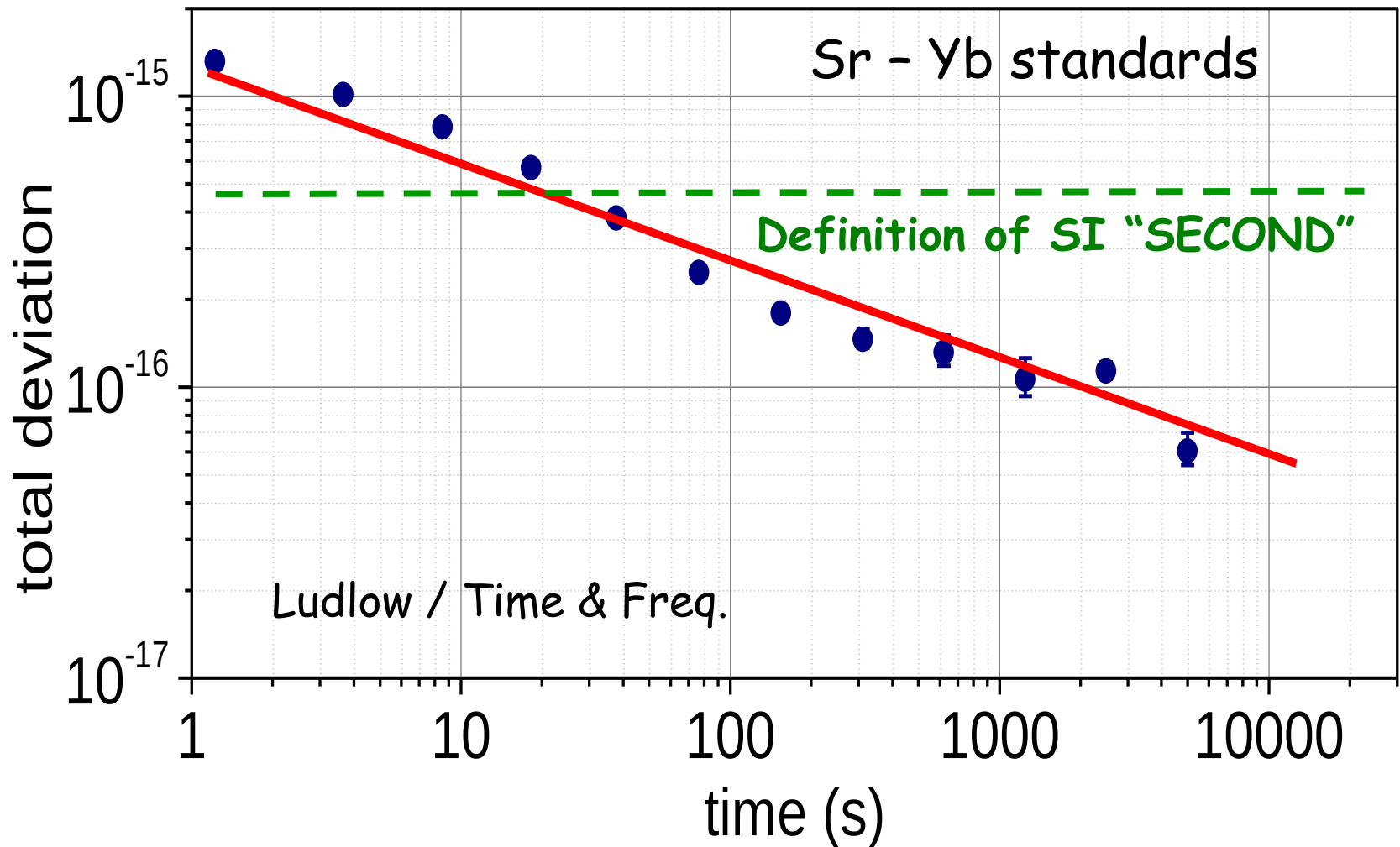
Instability $\sim 2 \times 10^{-16} / \sqrt{\tau}$

Near the quantum projection noise

JILA Sr atomic clock

Science **314**, 1430 (2006); Science **319**, 1805 (2008); Science **320**, 1734 (2008); Science **324**, 360 (2009); Science **331**, 1043 (2011).

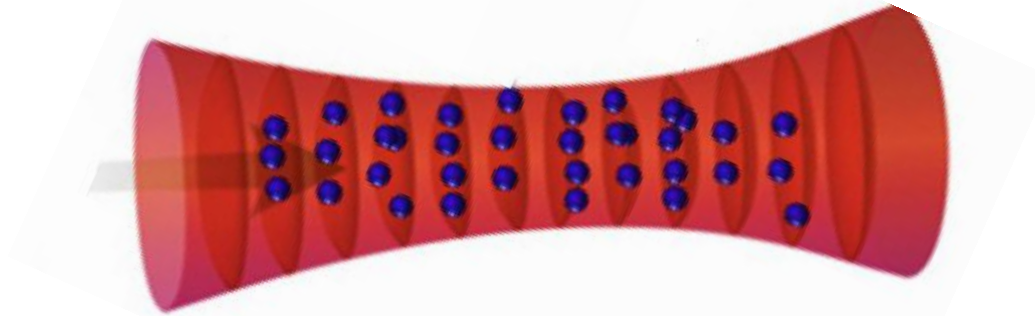
$10,000,000,000,000,000 \pm 1 \quad (10^{-16})$



Collision is a big deal for Sr Systematics

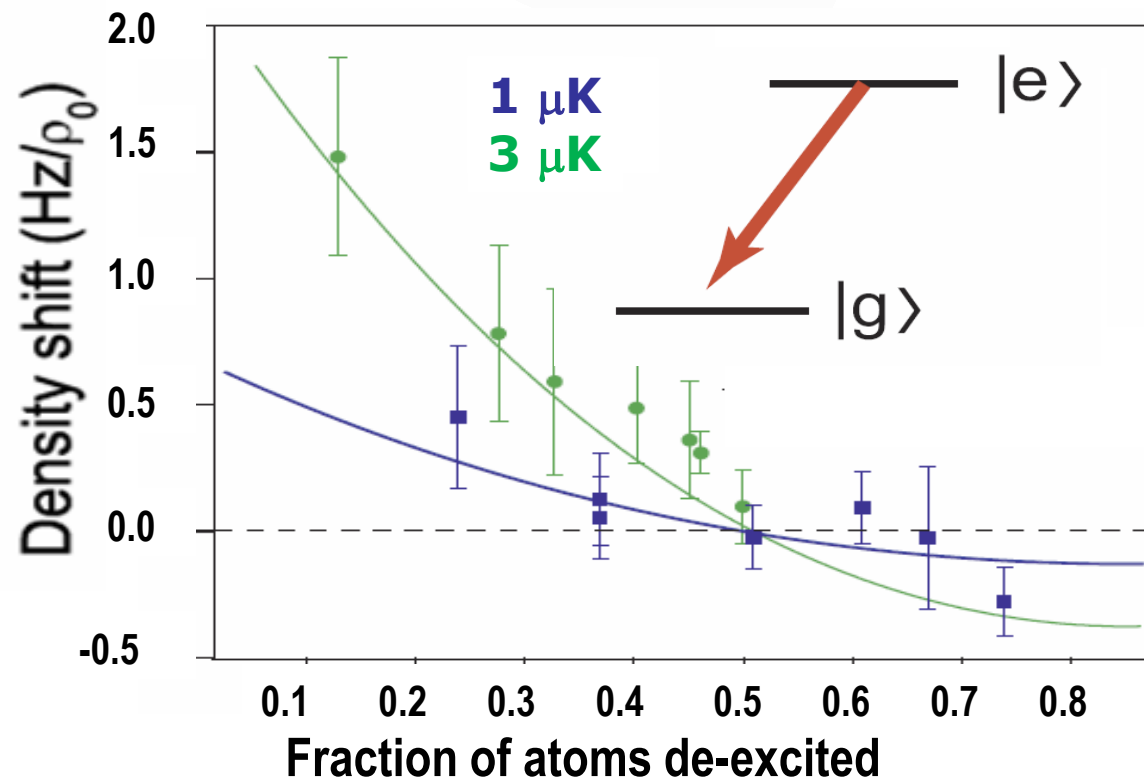
Contributor	Correction (10^{-16})	Uncertainty (10^{-16})
Lattice Stark (scalar/tensor)	-6.5	0.5
Hyperpolarizability (lattice)	-0.2	0.2
BBR Stark	52.1	1.0
ac Stark (probe)	0.2	0.1
First-order Zeeman	0.2	0.2
Second-order Zeeman	0.2	0.02
Density	8.9	0.8
Line pulling	0	0.2
Servo error	0	0.5
Second-order Doppler	0	<<0.01
Systematic total	54.9	1.5

1D lattice



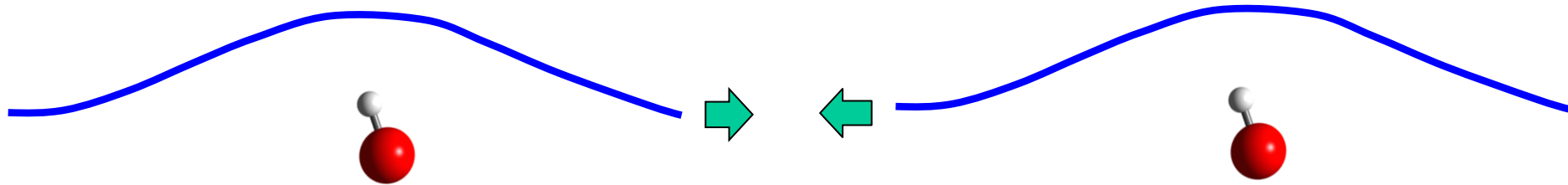
Measurement of frequency shift (10^{-16}) for spin-polarized fermions

G. Campbell et al, Science 324, 360 (2009).

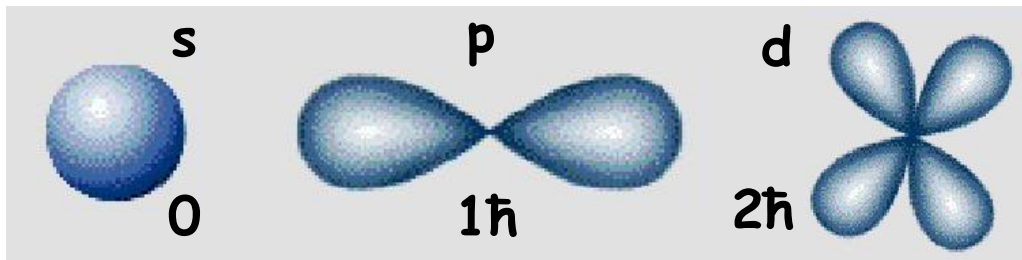


Interactions between identical Fermions

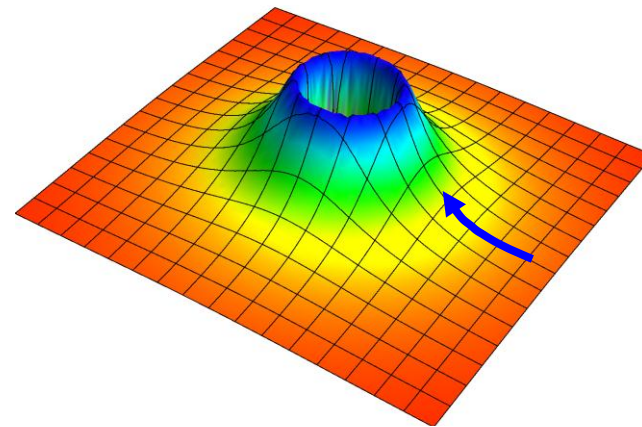
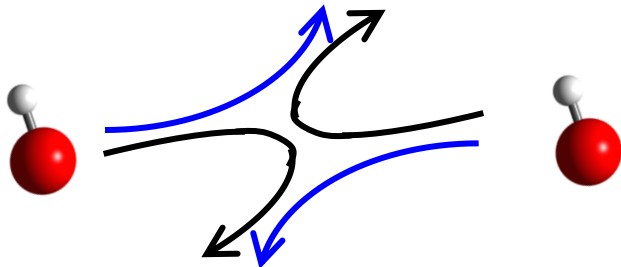
(1) Particles behave like waves ($T \rightarrow 0$)



(2) Angular momentum is quantized



(3) Quantum statistics matter



$$|\psi_0\rangle|\psi_1\rangle - |\psi_1\rangle|\psi_0\rangle$$

Fermions $\Rightarrow L = 1$, p -wave collisions

Lattice clock: an interacting quantum system

Singlet State

Interaction

$$U \propto a_{eg}^-$$

$$|s\rangle = \frac{|\text{blue red}\rangle - |\text{red blue}\rangle}{\sqrt{2}} \frac{|n_1 n_2\rangle + |n_2 n_1\rangle}{\sqrt{2}}$$

Triplet States

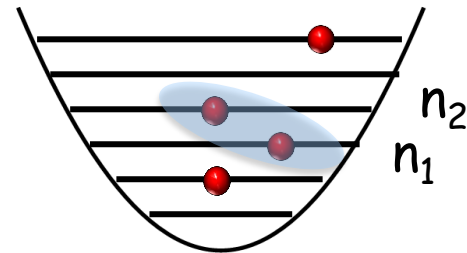
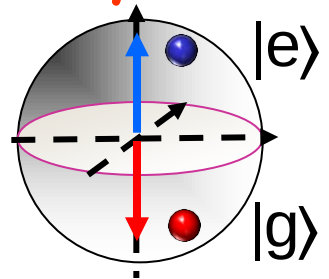
No s-wave collisions

$$U = 0$$

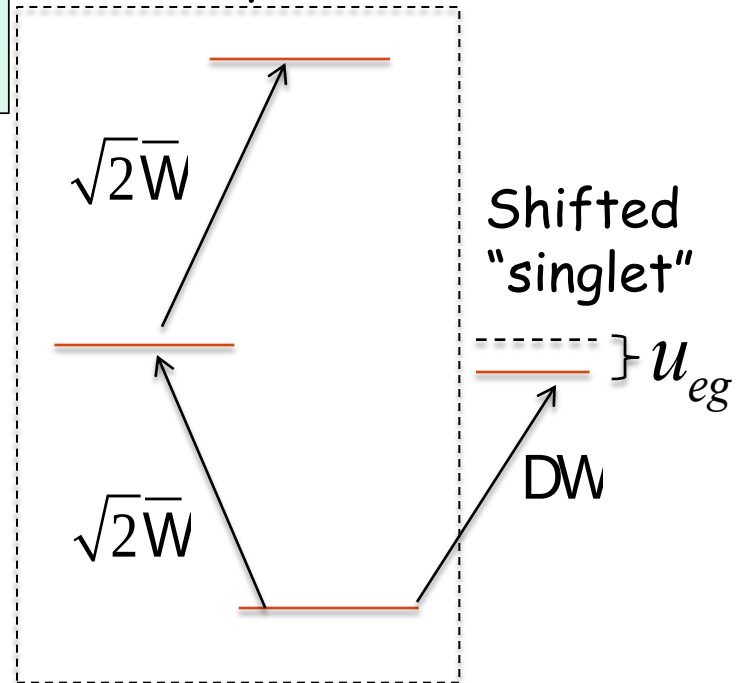
$$|t^+\rangle = |\text{red red}\rangle \frac{|n_1 n_2\rangle - |n_2 n_1\rangle}{\sqrt{2}}$$

$$|t^-\rangle = |\text{blue blue}\rangle \frac{|n_1 n_2\rangle - |n_2 n_1\rangle}{\sqrt{2}}$$

$$|t^0\rangle = \frac{|\text{blue red}\rangle + |\text{red blue}\rangle}{\sqrt{2}} \frac{|n_1 n_2\rangle - |n_2 n_1\rangle}{\sqrt{2}}$$



Triplet



Singlet state $\rightarrow$ s-wave interaction shift.

Triplet states can also have p-wave interactions.

Rey *et al* PRL (2009), Gibble PRL (2009),
Yu & Pethick, PRL (2010).

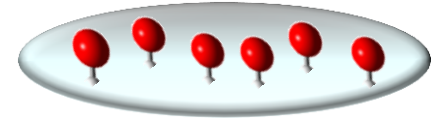
Swallows *et al.*, Science **331**, 1043 (2011).

p-wave interactions

Two-particle:

Lemke *et al.* PRL **107** (2011);
Bishof *et al.* PRA **84**, 052716 (2011).

Many-particle: $J=N/2$

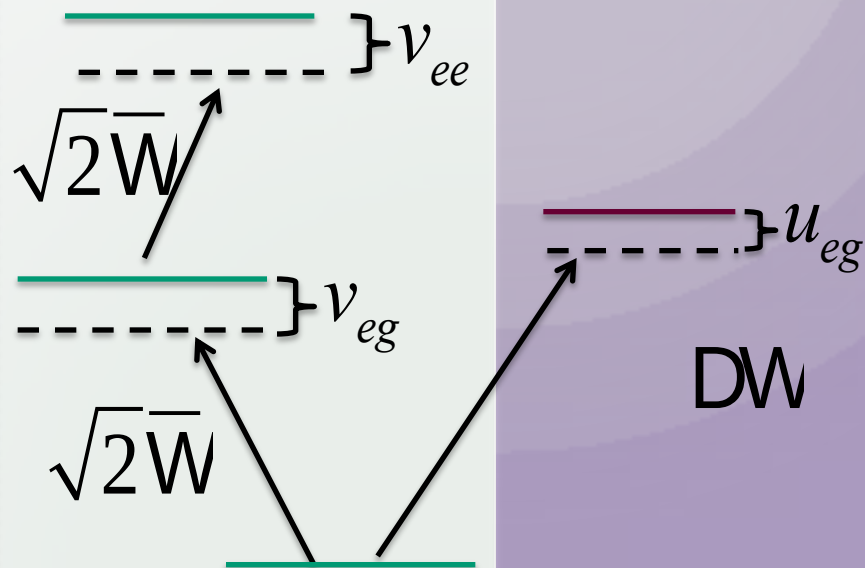


$J=1$

$J=0$

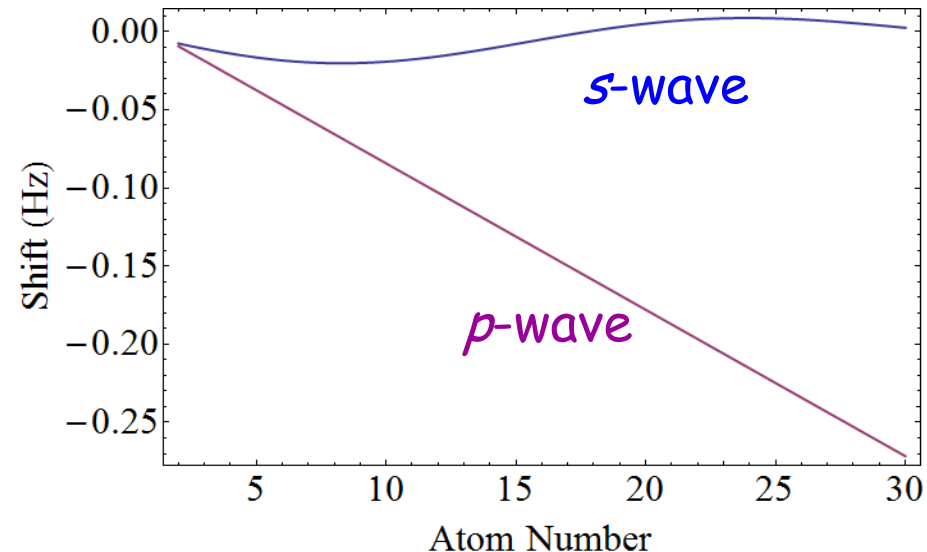
Triplet
p-wave

Singlet
s-wave



In a strongly interacting system:

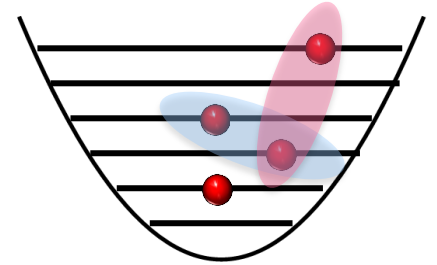
- *s*-wave suppressed (contributing as sidebands)
- *p*-wave dominant (acting on the carrier)



Spin model

(Ana Maria Rey *et al.*)

Pseudo-spin $S = N/2$



$$\hat{H}/\hbar = -\delta S^z - \Omega S^x + \frac{J^\perp}{2} \vec{S} \cdot \vec{S} + \chi (S^z)^2 + C (N-1) S^z$$

$$C = (V_{ee} - V_{gg})/2$$

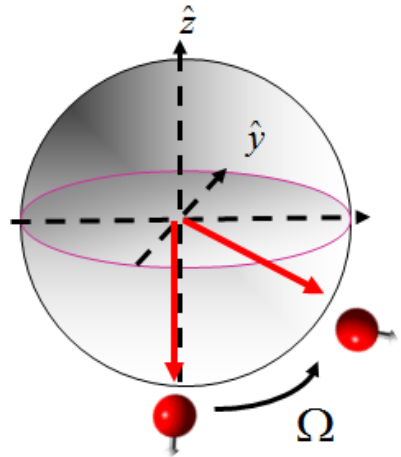
$$\chi = (V_{ee} + V_{gg} - 2V_{eg})/2$$

$$J^\perp = V_{eg} - U_{eg}$$

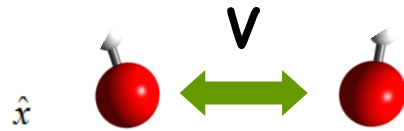
δ and Ω : laser detuning and Rabi freq.

Mapping interacting Fermions to Bosons

Competition between Ω and V

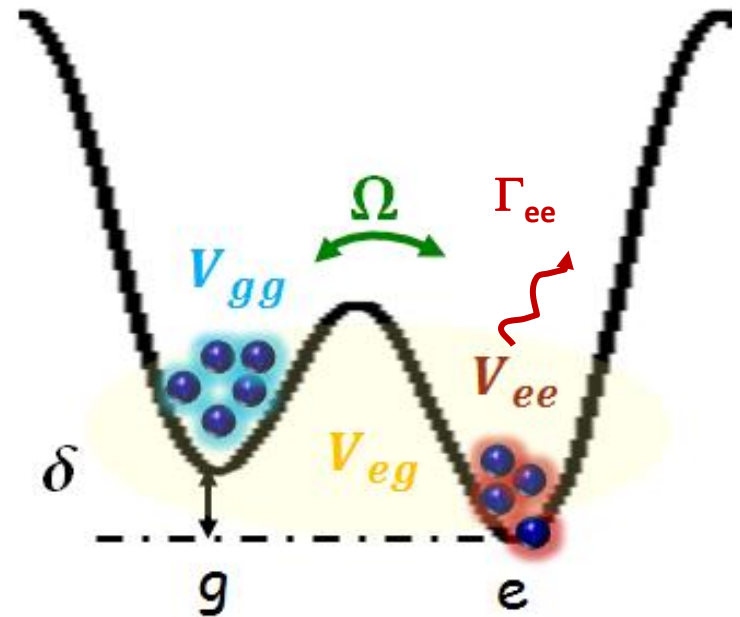


Rabi frequency

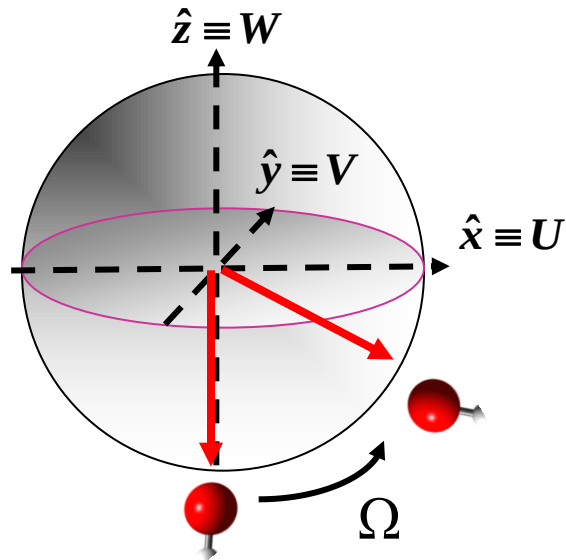


Interaction energy:
Elastic & Inelastic

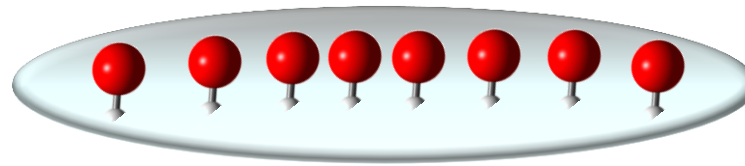
mapping to bosons
GPE (mean field + loss)



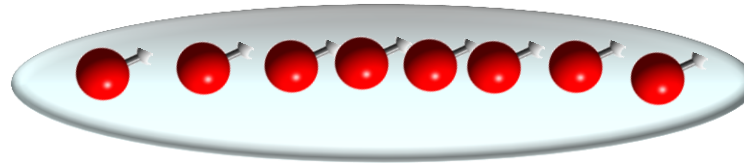
Competition between Ω and V



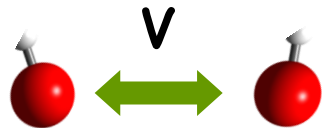
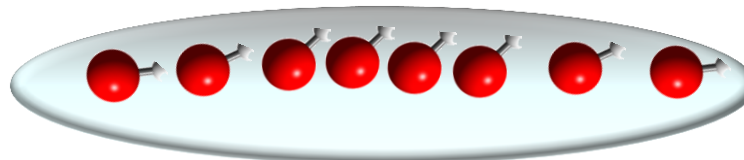
Rabi frequency



(1) $\Omega \gg V$

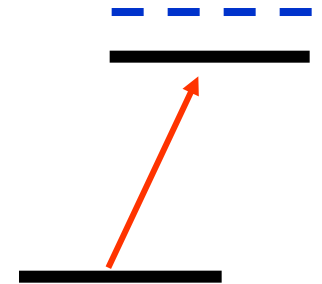


(2) $\Omega < V$

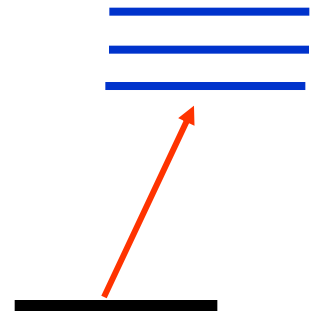


Interaction energy

mean-field
shift

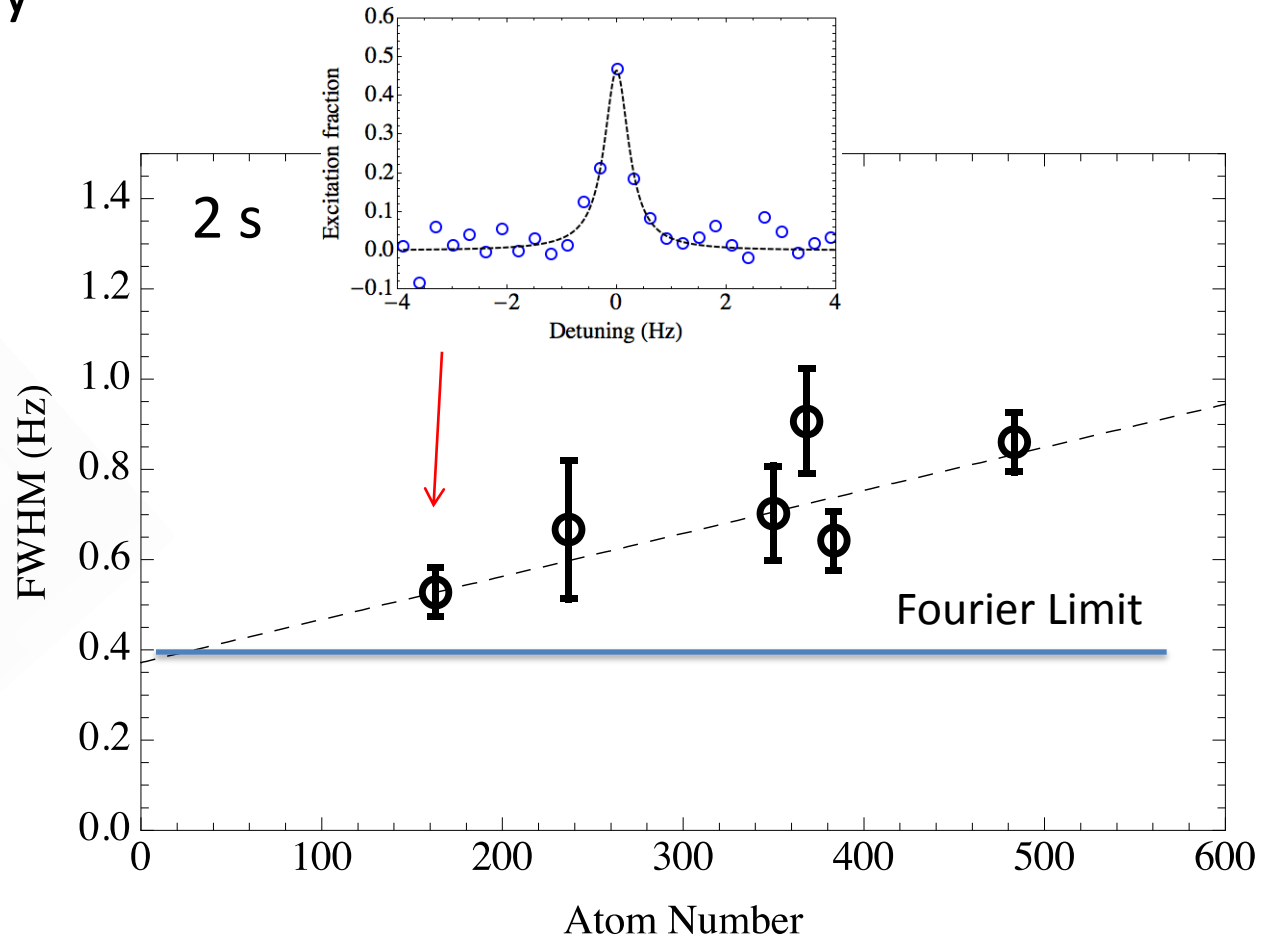
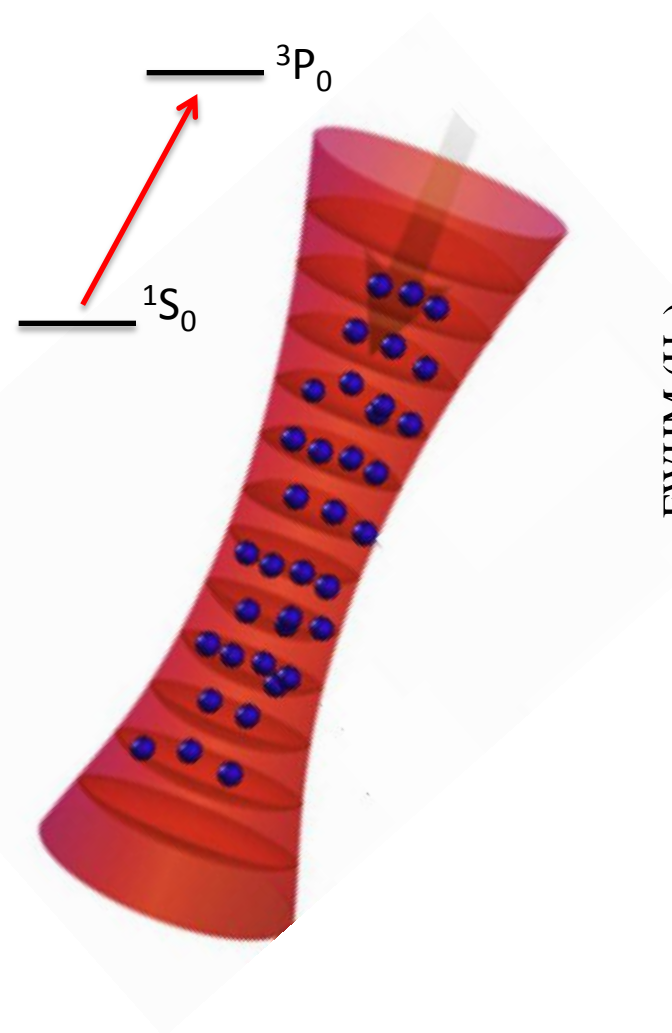


Correlated
spin spectrum



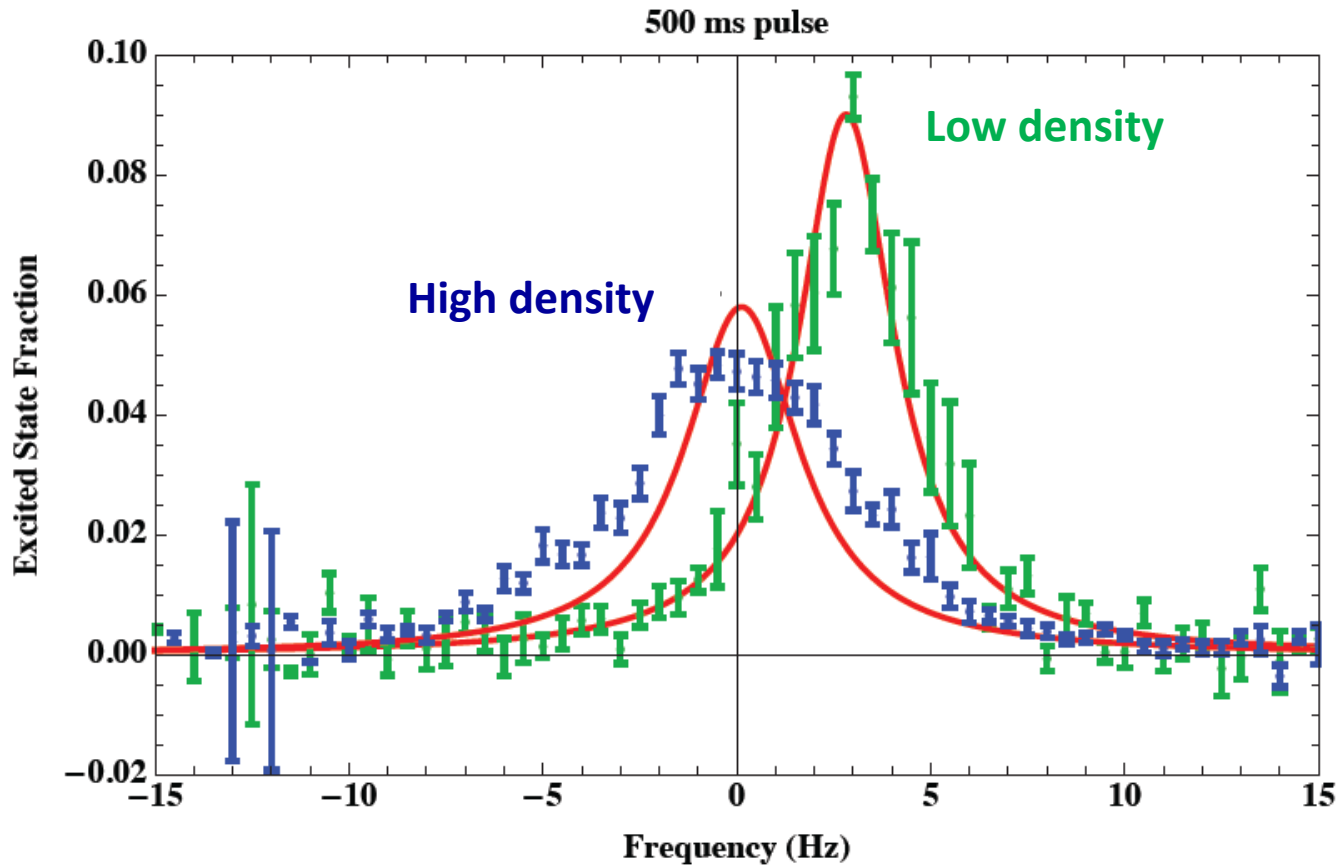
2-s optical/atomic coherence

1D lattice spectroscopy



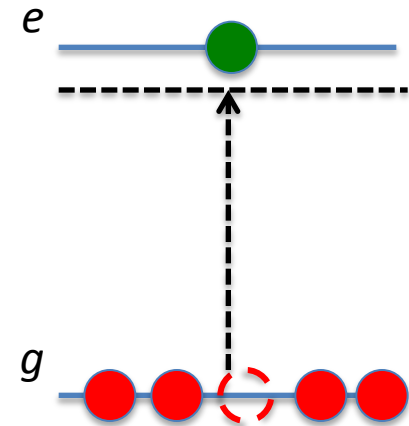
Linear response regime (Rabi spectroscopy)

2 uK sample temperature

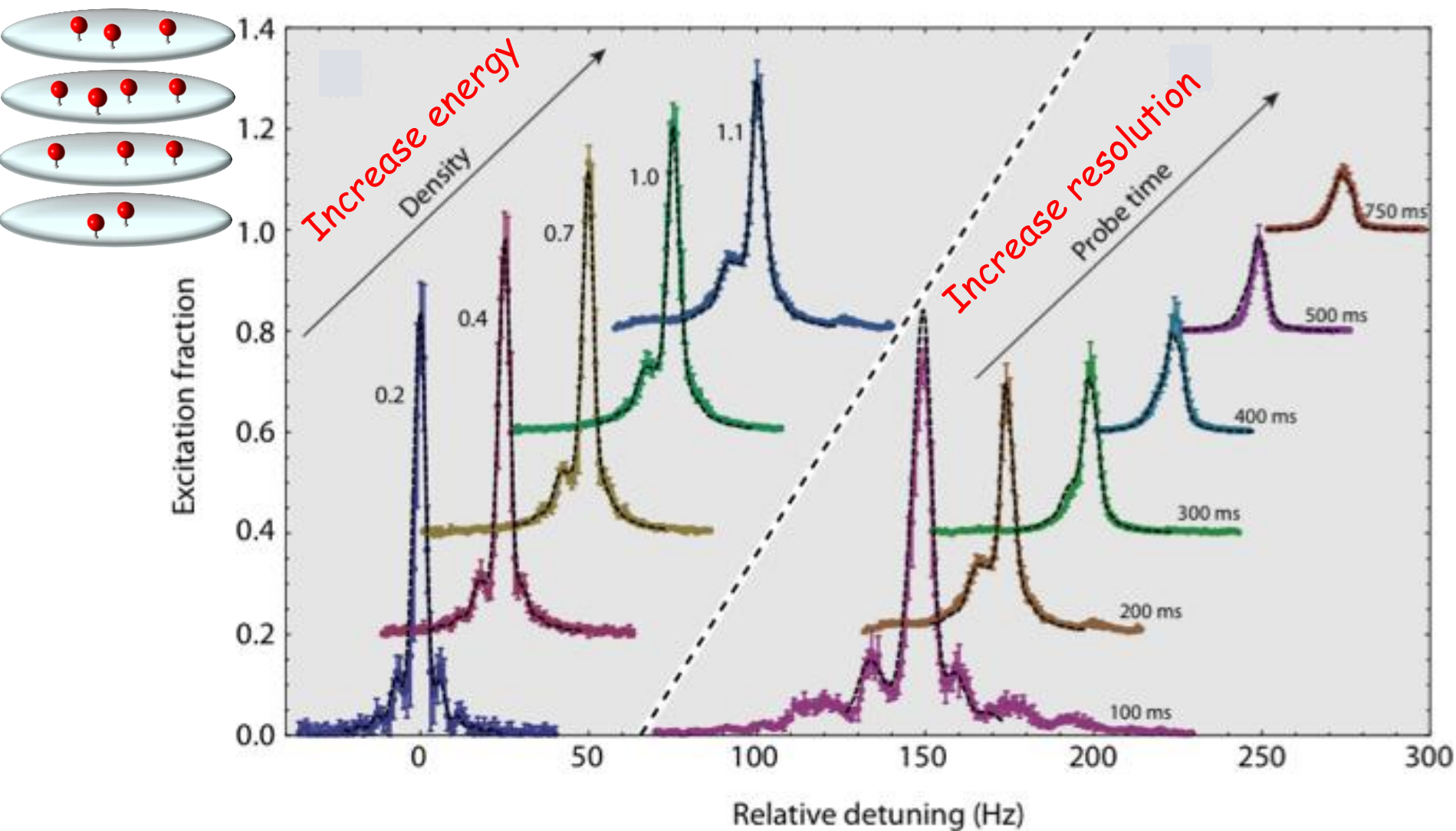


Experiences shift

$$DE \propto v_{eg} \frac{(N-1)}{V}$$



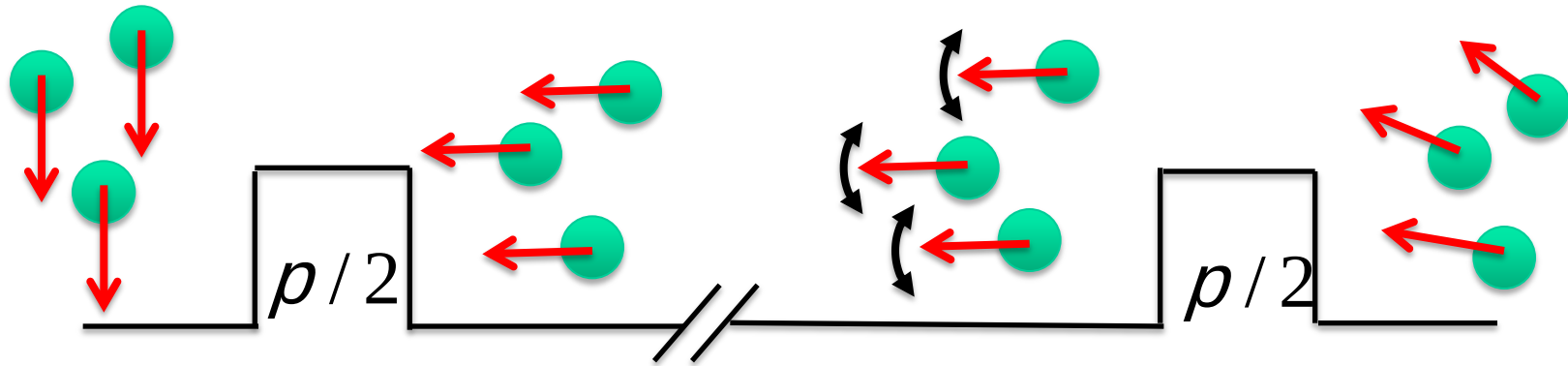
Rabi spectroscopy with strong interactions



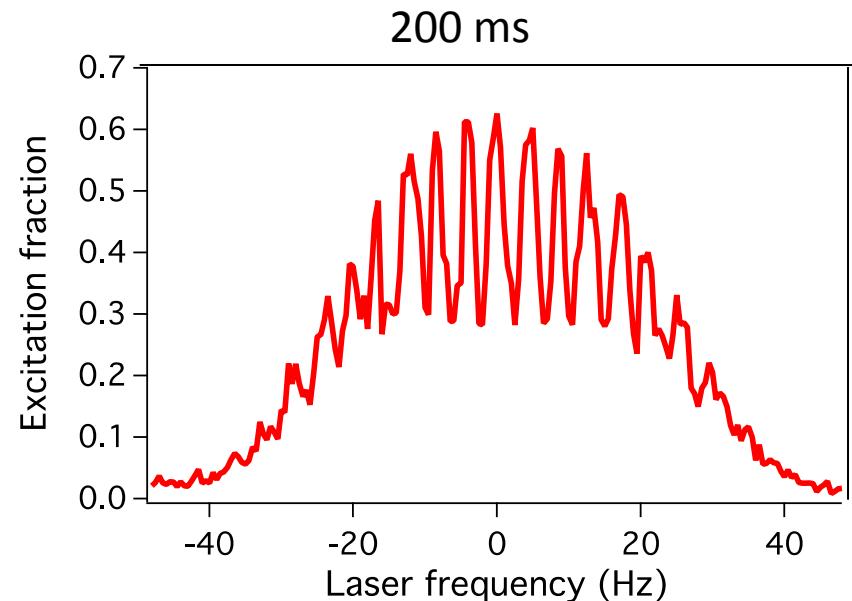
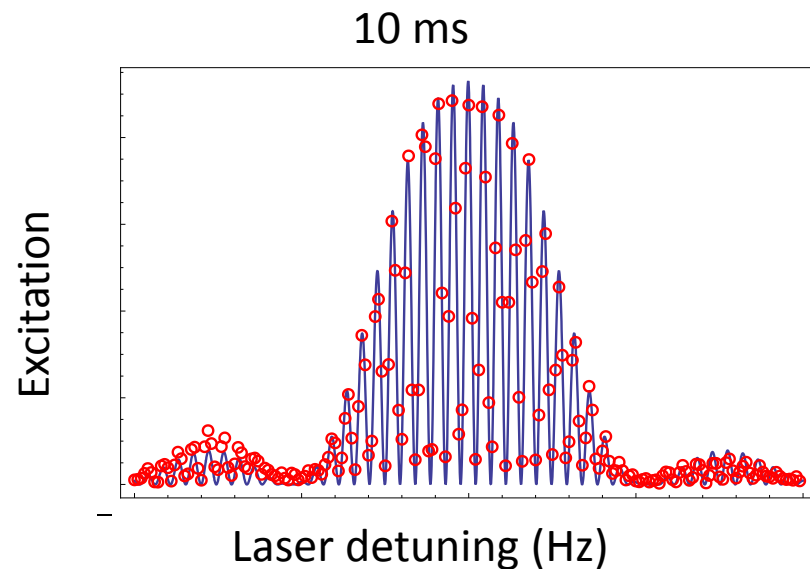
Structure due to interactions (both elastic & inelastic).
– excitation blockade at increasing density or probe time.

Ramsey spectroscopy under interactions

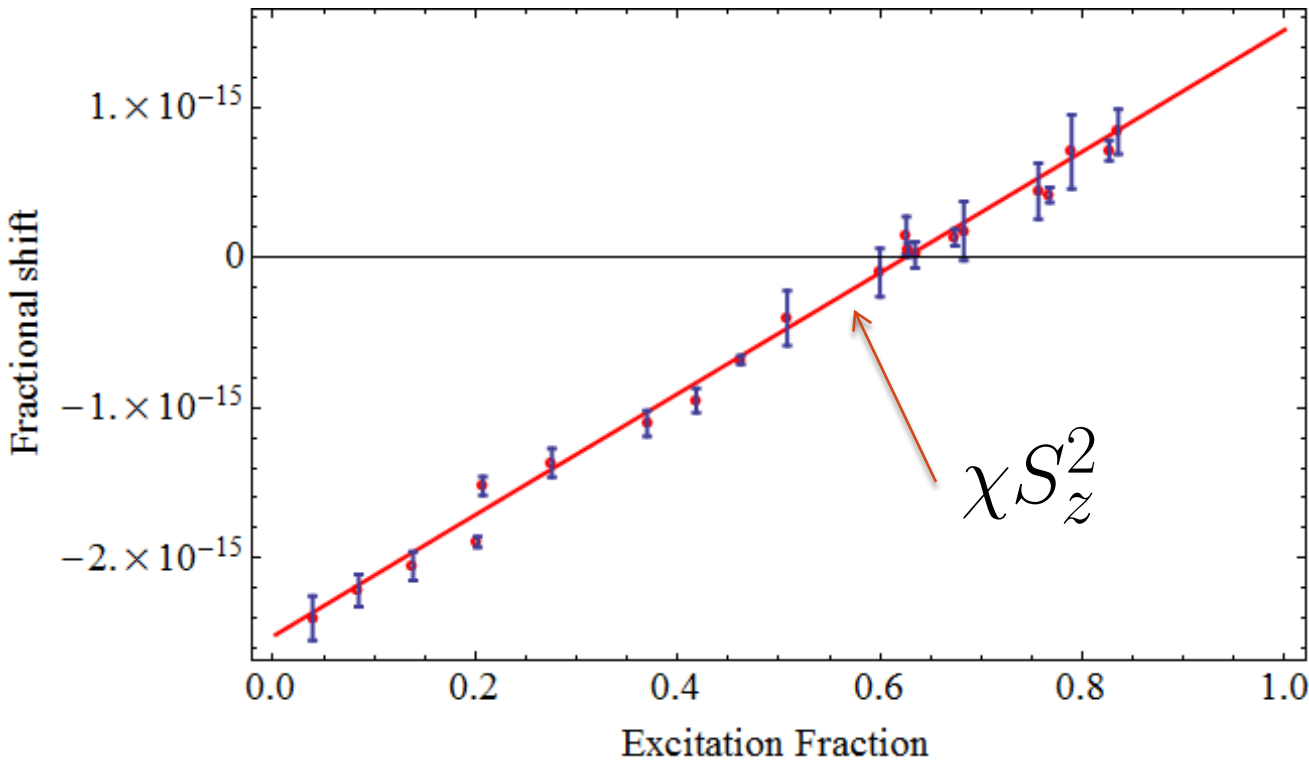
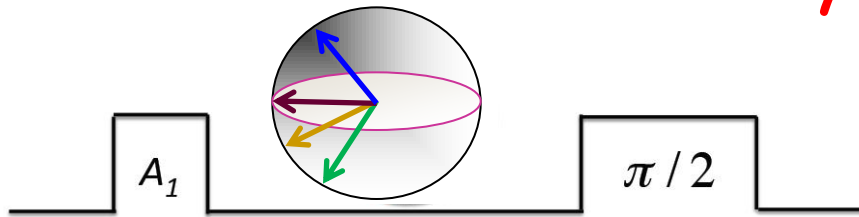
$$\hat{H}/\hbar = \chi (S^z)^2 + C (N - 1) S^z$$



Interaction dominates over optical dephasing



Density shift in Ramsey



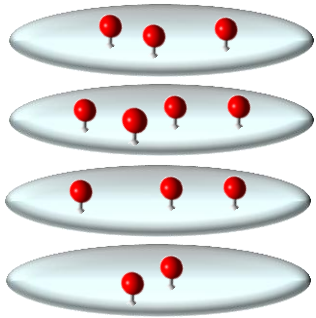
$$\frac{V_{ee} - V_{gg}}{V_{eg} - V_{gg}} = 0.4$$

Collisional shift has a linear dependence on the tipping angle.

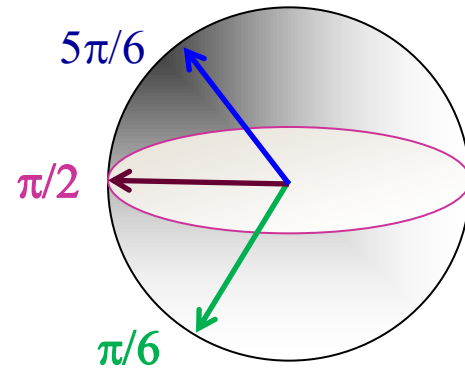
$$2\pi\Delta\nu \sim \frac{V_{ee} - V_{gg}}{2} + \cos(\bar{\Omega}t) \left[\frac{2V_{eg} - V_{ee} - V_{gg}}{2} \right] \chi S_z^2$$

Beyond mean field

Interaction & Poissonian distribution $\rightarrow$ Ramsey fringe decay



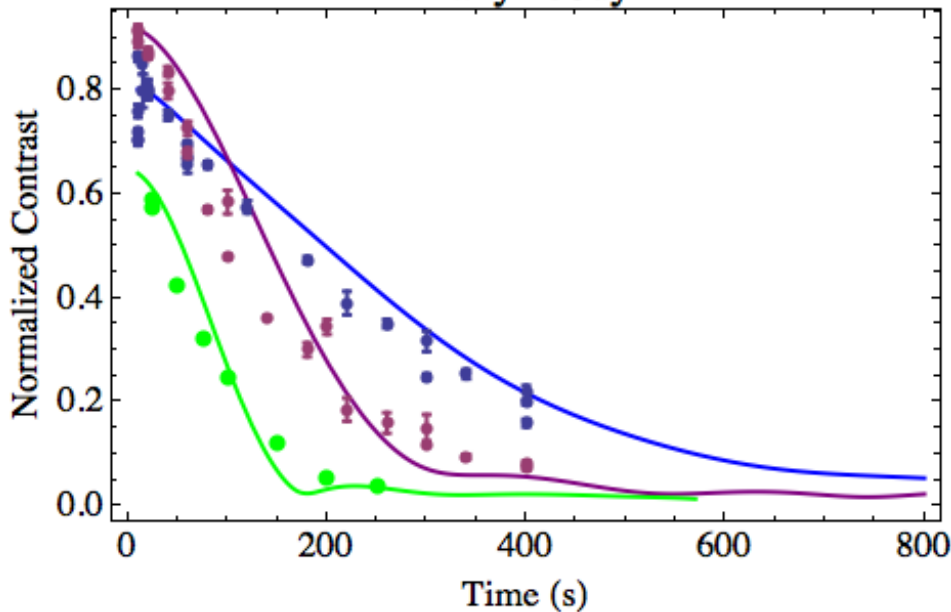
$$N_e = \sum_N N_e^N P(N)$$



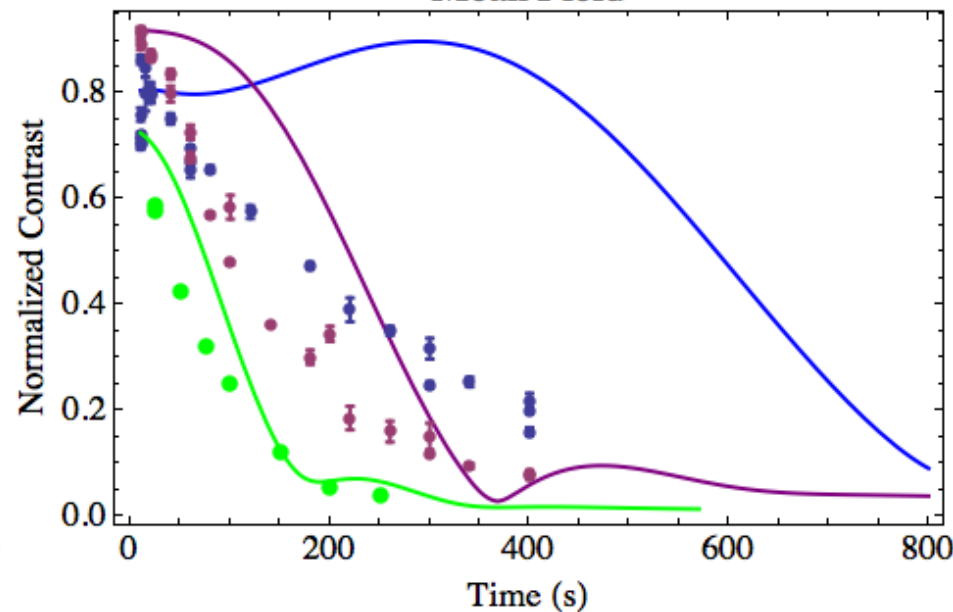
Tipping angle dependence

Ramsey fringe decay vs. the spin tipping angle

many-body

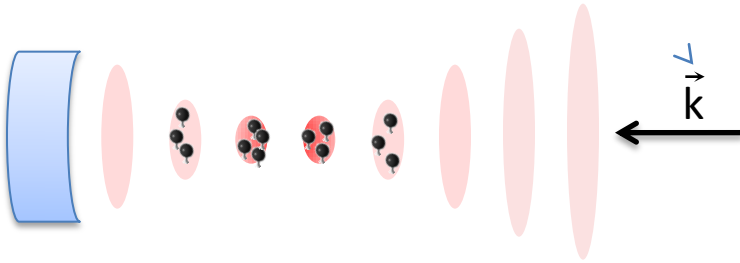


Mean Field

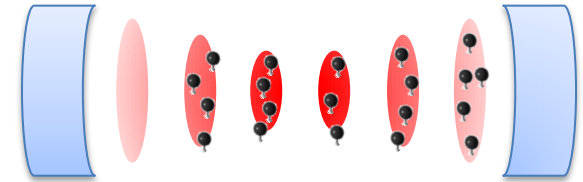


Cavity-Enhanced Lattice System

Retroreflected Lattice:

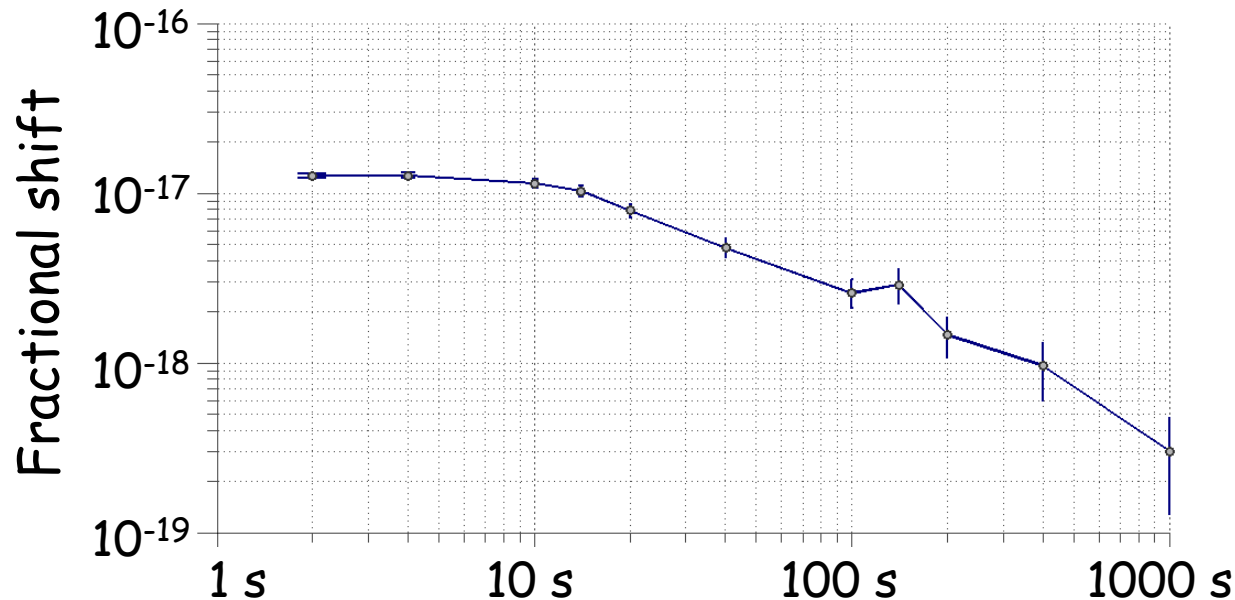


Cavity Lattice:



	ν_{axial}	ν_{radial}	N
Retro-Refl.	80 kHz	450 Hz	10^3
Cavity	90 kHz	100 Hz	10^5

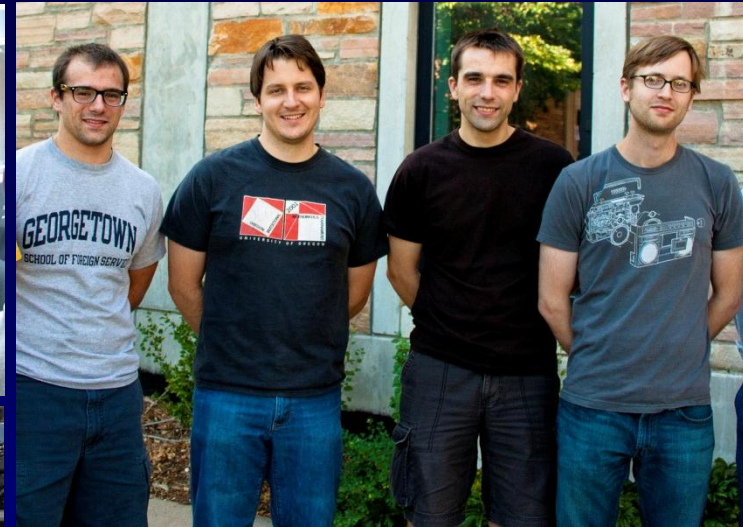
Pierre Lemonde
SYRTE



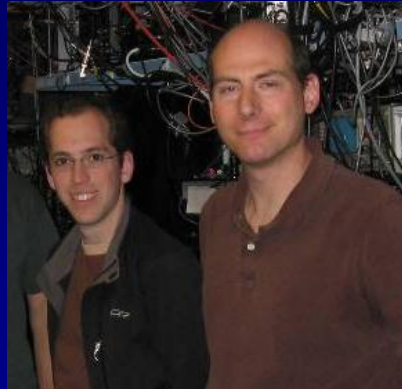
Density shift: 10^{-17}
Uncertainty: 5×10^{-19}

Special thanks

M. Swallows T. Nicholson
M. Martin B. Bloom
M. Bishof J. Williams



M. Swallows (JPL/Caltech)
S. Blatt (Harvard)
A. Ludlow (NIST)
Y. Lin (NIM, Beijing)
G. Campbell (U. Maryland)
M. Boyd (AO Sense)
J. Thomsen (U. Copenhagen)
T. Zelevinsky (Columbia U.)
T. Zanon (Univ. Paris 13)
S. Foreman (Stanford U.)
X. Huang (WIPM)
T. Ido (Tokyo NICT)
T. Loftus (AO Sense)
X. Xu (ECNU)



Ana M. Rey
(theory)

