

# Statistical Physics, Inference and Applications to Biology

Physics Department, Ecole Normale Supérieure, Paris, France.

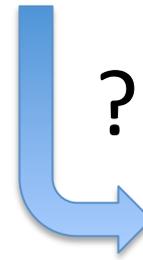
Simona Cocco

Office:GH301 mail:cocco@lps.ens.fr

# Deriving Protein Structure and Function from Sequence

Amino-Acid Sequence

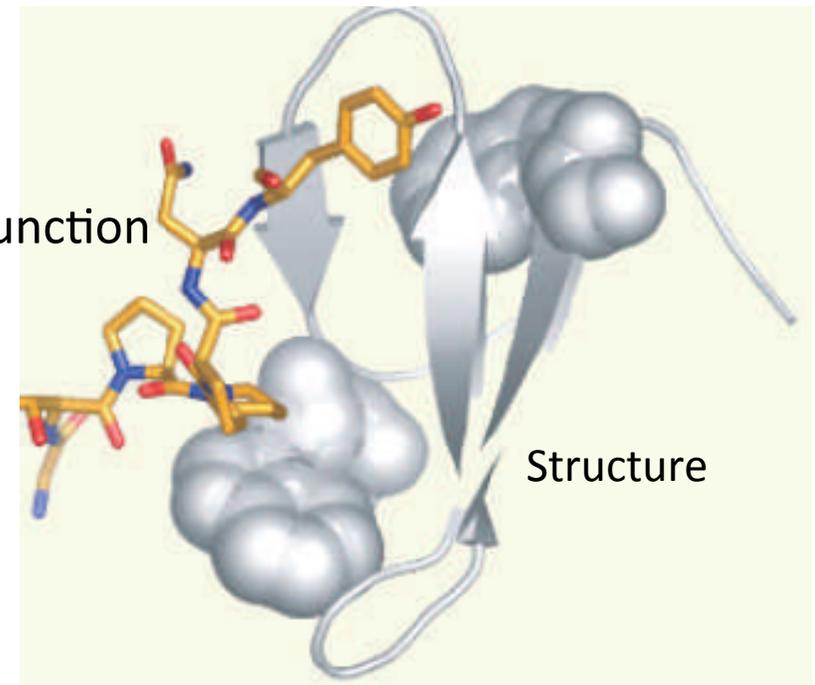
PLPPGWEERIHLD-GRTFYIDHNSKITQWEDPRLQ



?

Function

WW protein



Structure

One of the major unsolved problems in biology.

# Deriving Protein Structure and Function from Sequence

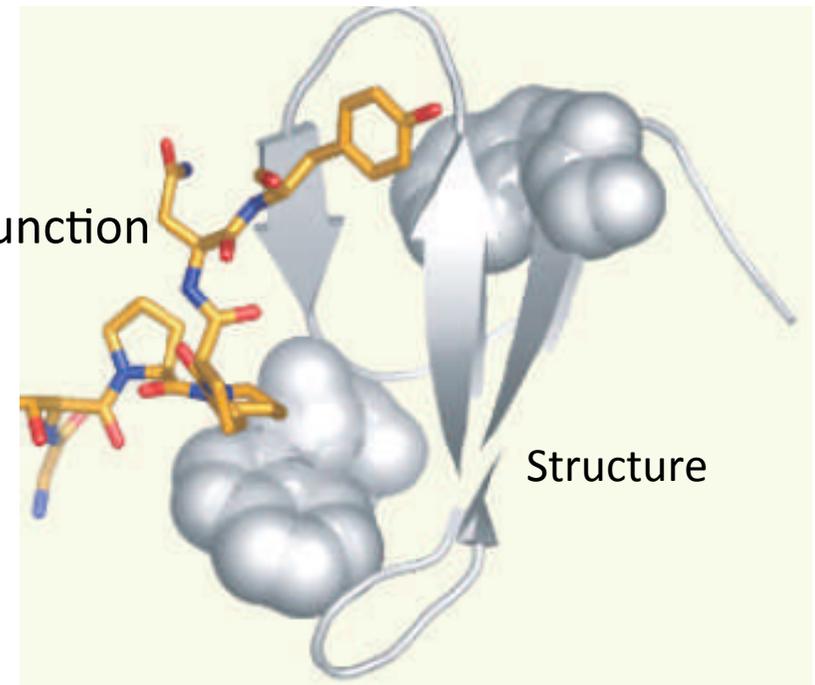
Amino-Acid Sequence

**A**LPPGWEERIHLD-GRTFYIDHNSKITQWEDPRLQ

↓  
mutation



WW protein



Mutations can be:

- **neutral:** no effect
- **detrimental:** functionality decreases, e.g. gives a cancer
- **beneficial:** functionality increases. Dangerous for us if the organism is a bacterium e.g. mutation confers resistance to antibiotics

# Multi-sequence alignments (MSA)



ACSLPKVQGPCSGKHSYFFNSANQQCETFVYGGCLGNTNRFATIEECNARC-  
VCLLPKSAGPCTGFTKKWYFDVDRNRCEEFYGGCYGTNNRFDLSLEQCQGTC-  
VCAMPPDAGVCTNYTPRFFFNSQTGQCEQFAYGSCGGNENFFDRNTCERKCM  
TCSLSPSPGTCGPGVFKYHYNPQTQECESFEYLGCDGNSNTFASRAECENYCG  
-CHTEHSSGACPGAVTMFYHDPRTKKCTPFTFLGCGGNSNKFDTRPQCERFCK  
PCMLPSDKGNCQDILTRWYFDSQKHQCRAFLYSGCRGNANNFLTKTDCRNACM  
-----RLVGYCSPYLRRYFFNRTTEKCVLFIPERCEKDGNNFNRKVCMTKCM  
PCSLKEDYGIGRAYYERWYFNTTTANCTRFIWGGNHKEWQQER-----  
PCKQDLQGHGKTLQARYYFNKYAKVCEQFDYRGIDGNRNNFESLQEGQQQC-  
-CFLKPDEGVGRAILKAFYYPKNRRCEEFEYGGGLGGNENFEETMEKCEEECK  
-CSQPAASGHGEQYLSRYFYSPEYRQCLHFIYSGERGNLNNFESLTDCLETCV  
LCNLKYDSGVGGEKSDKYFWVPKYTTQMRFSFYGTLGNANNFPNYNSCMATCG  
-----RGADTIQRWYWDNDLTCRTHKYHGQGGNFNFGDKOGCLDFC-  
PCEQAIEEGIGNVLLBRWYFDPATRLCQPFYKGFKGNQNNFMSFDYQNRACG  
PCGQPLDRGVGGSQLSRWYWNQSQCCLPFSYCGGKSTQNNFLTQKQDCBRTC-  
VCIQPLESGD-EPVPRWYNSATGTCVQFMWDFDTTANNFRTAEHCESYCR  
TCVQPTATGP-NPTEPRWYNSITGMCQQFLWDPTASGPNNFRTVEHCESFCR  
-CDQQLMLGVGGASMERFYDYDTDDACLVFNYSVGVGQNNNFLTKAECQIAC-  
PCSVPLAPGTGNAGLARYYYNPDDRQCLPFQYNGKRGNQNNFENQADCERTC-  
-----PESEGVTGAPTSRWYYDQTDQCKQFTYNGRRGNQNNFLTQEDGAATC-  
ACKMPLSVGIGGAPANRWYYDAAASTCKTFEYNGRKGNNQNNFISEADCAATC-  
VCNLPMTSEGNANLDRFYDQDSKTCPVYNGLEKGNQNNFISLRACQISCS-  
ICQQPMAVGTGGATLPRWYNAQTMQCVQENYAGRMGNQNNFQSQQAQCEQTL-  
PCSLPMFSGEGTGNLTRWYADSCSRQCKSFTYNGSKGNQNNFLTQKQDESKCK  
PCEEEMTQGECSAALTRFYDALQRKCLAFNYLGLKGNRNNFQSKPHCEESTC-  
TCELPMTKGYGNSHLTRWHFDKNLNKCVKFIYSGEGGNQNMFLTQEDCLTVC-  
TCELMTKGYGNSHLTRWHFDKNLNKCVKFIYSGEGGNQNMFLTQEDCLSVC-  
RCHLPPAVGYGKQRMRRFYFDWKTACHELQYSGIGGNENIFMDYEQCERVCR  
-CMESLDRGSCEAMSNRYFFNKRARQCKGFHYTGCGKSGNNFLTKEECQTKC-  
PCQQPLQRGNCQRIPLFYNIHNHKCRKFMYRGCNGNENRFSNRRQCQAKCG

Information on:

- Structure
- Evolutionary Dynamics

**Big data** bases!

*PFAM*

*15,000 protein families  
thousands of sequences  
for each...*

**Model Probability for  
Sequences in the MSA**

$P(A_1, \dots, A_N)$  ?

# State of the art on Protein Sequences

PLPPGWEERIHLD-GRTFYIDHNSKITQWEDPRLQ

Genetic code



P L P  
CCUCUUCGCCA....

# State of the art on Protein Sequences

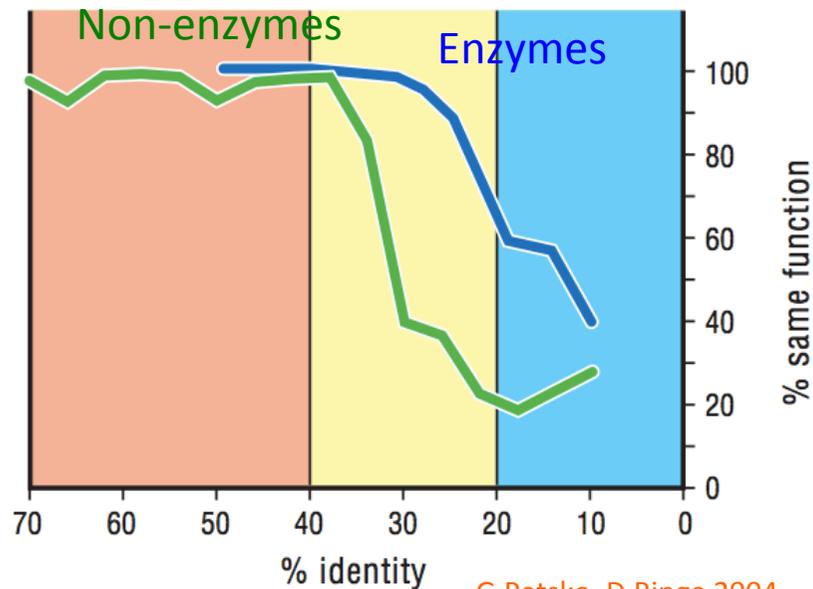
## Theory

Classify sequences corresponding to different proteins (bioinformatics)

(Uniprot, PFAM, genbank)

Extract structural information from protein sequence data (statistical physics)

PLPPGWEERIHLD-GRTFYIDHNSKITQWEDPRLQ  
GLPKPWIVKISRSRNRPYFFNTETHESLWEPPAAT



G.Petsko, D.Ringe 2004

CCUCUUCGCCA....

## Sequencing

1995

~2000

~2010

2015

Cost of sequencing (\$ per base): 1

Cost of sequencing (\$ per base):  $10^{-6}$

Nb. of protein sequences in databases  $\sim 10^8$

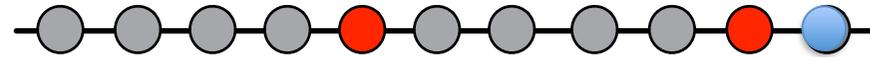
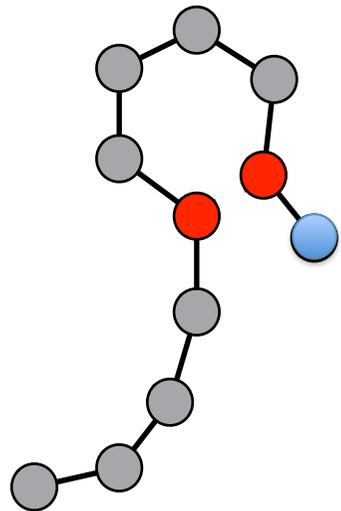
## Genome Sizes of Representative Organisms

Organism	Genome size (base pairs)	Number of genes
<i>Mycoplasma genitalium</i>	$45.8 \times 10^5$	483
<i>Methanococcus jannaschii</i>	$1.6 \times 10^6$	1,783
<i>Escherichia coli</i>	$4.6 \times 10^6$	4,377
<i>Pseudomonas aeruginosa</i>	$6.3 \times 10^6$	5,570
<i>Saccharomyces cerevisiae</i>	$1.2 \times 10^7$	6,282
<i>Caenorhabditis elegans</i>	$1.0 \times 10^8$	19,820
<i>Drosophila melanogaster</i>	$1.8 \times 10^8$	13,601
<i>Arabidopsis thaliana</i>	$1.2 \times 10^8$	25,498
<i>Homo sapiens</i>	$3.3 \times 10^9$	~30,000 (?)

1998-2003



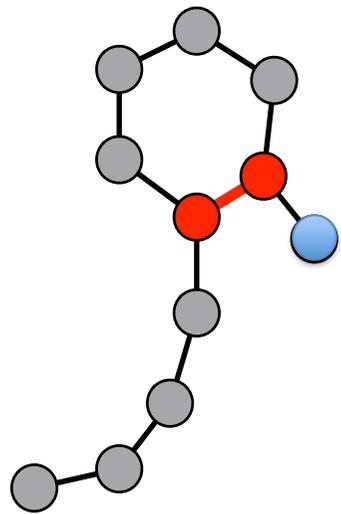
# Evolution reflects functional and structural constraints



R	I	D	H	R	L	K	N	T	D	H
F	L	N	G	R	L	R	D	T	D	H
H	E	R	Q	E	T	G	E	L	K	H
K	Y	R	T	R	L	T	D	L	D	H
R	R	A	M	E	V	G	N	L	K	H
T	Q	K	E	E	L	A	N	L	K	H
K	Q	Q	S	E	V	E	N	A	K	H
R	L	N	Q	R	A	D	D	L	D	H

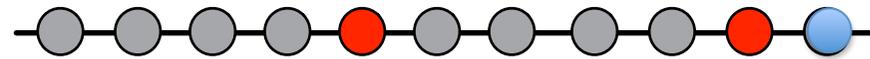
↑  
conservation

# Evolution reflects functional and structural constraints



contact in 3D

co-evolution



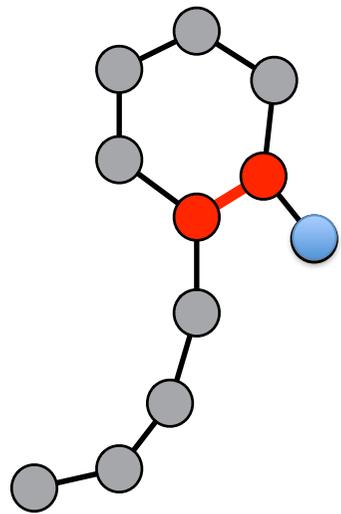
R	I	D	H	R	L	K	N	T	D	H
F	L	N	G	R	L	R	D	T	D	H
H	E	R	Q	E	T	G	E	L	K	H
K	Y	R	T	R	L	T	D	L	D	H
R	R	A	M	E	V	G	N	L	K	H
T	Q	K	E	E	L	A	N	L	K	H
K	Q	Q	S	E	V	E	N	A	K	H
R	L	N	Q	R	A	D	D	L	D	H



correlation

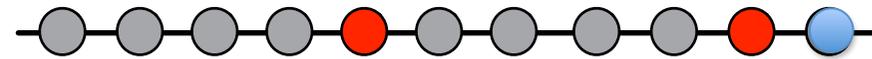
conservation

# Evolution reflects functional and structural constraints



contact in 3D

co-evolution  
 ←→  
 statistical analysis



R	I	D	H	R	L	K	N	T	D	H
F	L	N	G	R	L	R	D	T	D	H
H	E	R	Q	E	T	G	E	L	K	H
K	Y	R	T	R	L	T	D	L	D	H
R	R	A	M	E	V	G	N	L	K	H
T	Q	K	E	E	L	A	N	L	K	H
K	Q	Q	S	E	V	E	N	A	K	H
R	L	N	Q	R	A	D	D	L	D	H



correlation

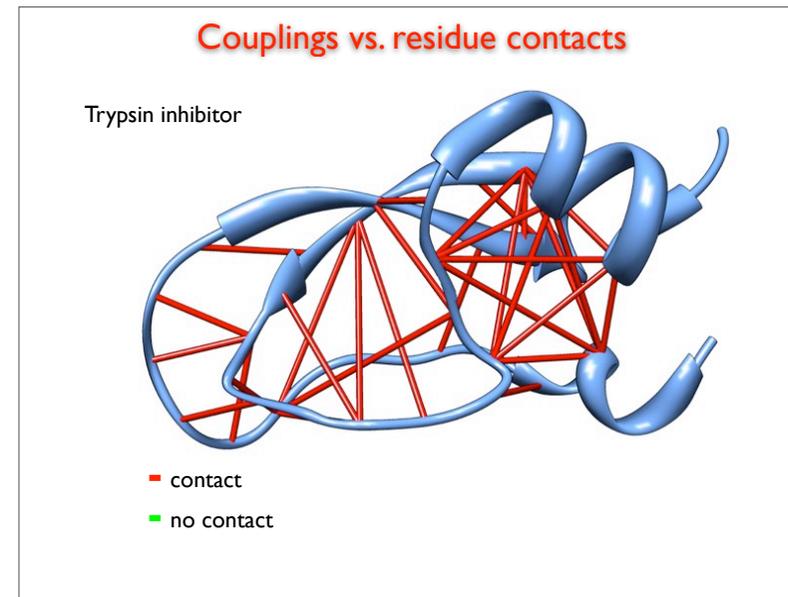
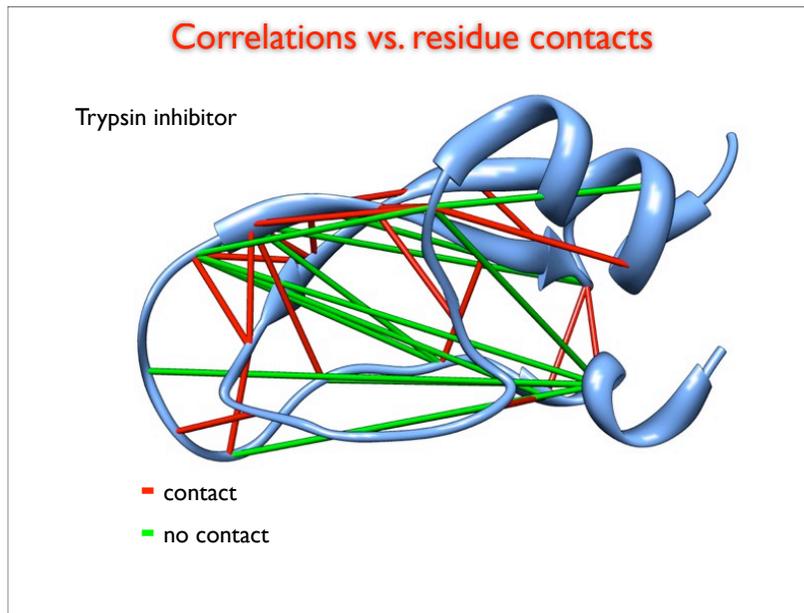
conservation

Inverse questions :

- Are sequence correlations indicative for residue-residue contacts?

# Statistical Couplings are better estimators of contacts than correlations ...

Trypsin inhibitor (small protein, N=52 residues, contact if distance < 0.8 nm)



[Gobel et al., Proteins 1994]

$$C_{ij} = \underbrace{\bullet \text{---} \bullet}_{J_{ij}} + \underbrace{\bullet \text{---} \bullet \text{---} \bullet}_{\sum_k J_{ik} J_{kj}} + \underbrace{\bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet}_{\sum_{k,l} J_{ik} J_{kl} J_{lj}} + \dots$$

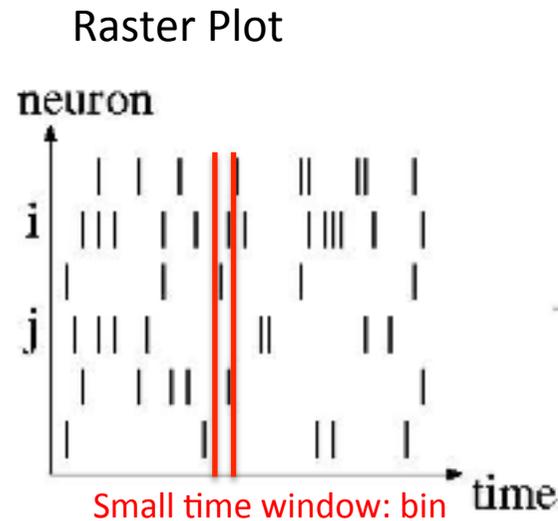
[Weigt et al. PNAS 2009, Structural prediction:  
Hopf et al. Cell 2012, Nugent, Jones 2012 (psicov)  
Protein-protein interactions.. ]

For Gaussian variables  $J = -C^{-1}$

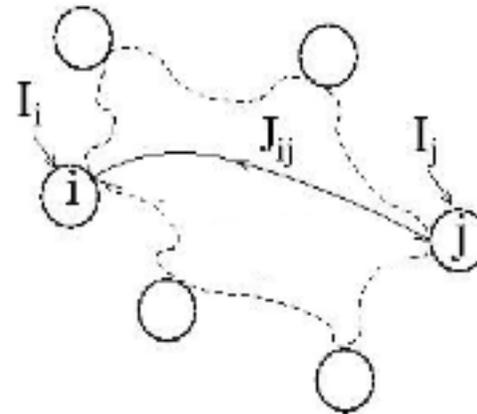
but C is affected by noise, need for regularization [S.C., Weigt, Monasson PloS Comp. Biol. 2013]

# Inference of functional interactions from the spiking activity of a neural population

10-100  
neurons



?



time: 1hour,  $10^4$ - $10^6$  spikes

Retina, Cortex,...

*Correlations*  $\longrightarrow$  *Interactions*

Aim: to find an **Ising** model  $P(s_1 \dots s_N)$  which reproduces and interprets the data

# Multi-sequence alignments

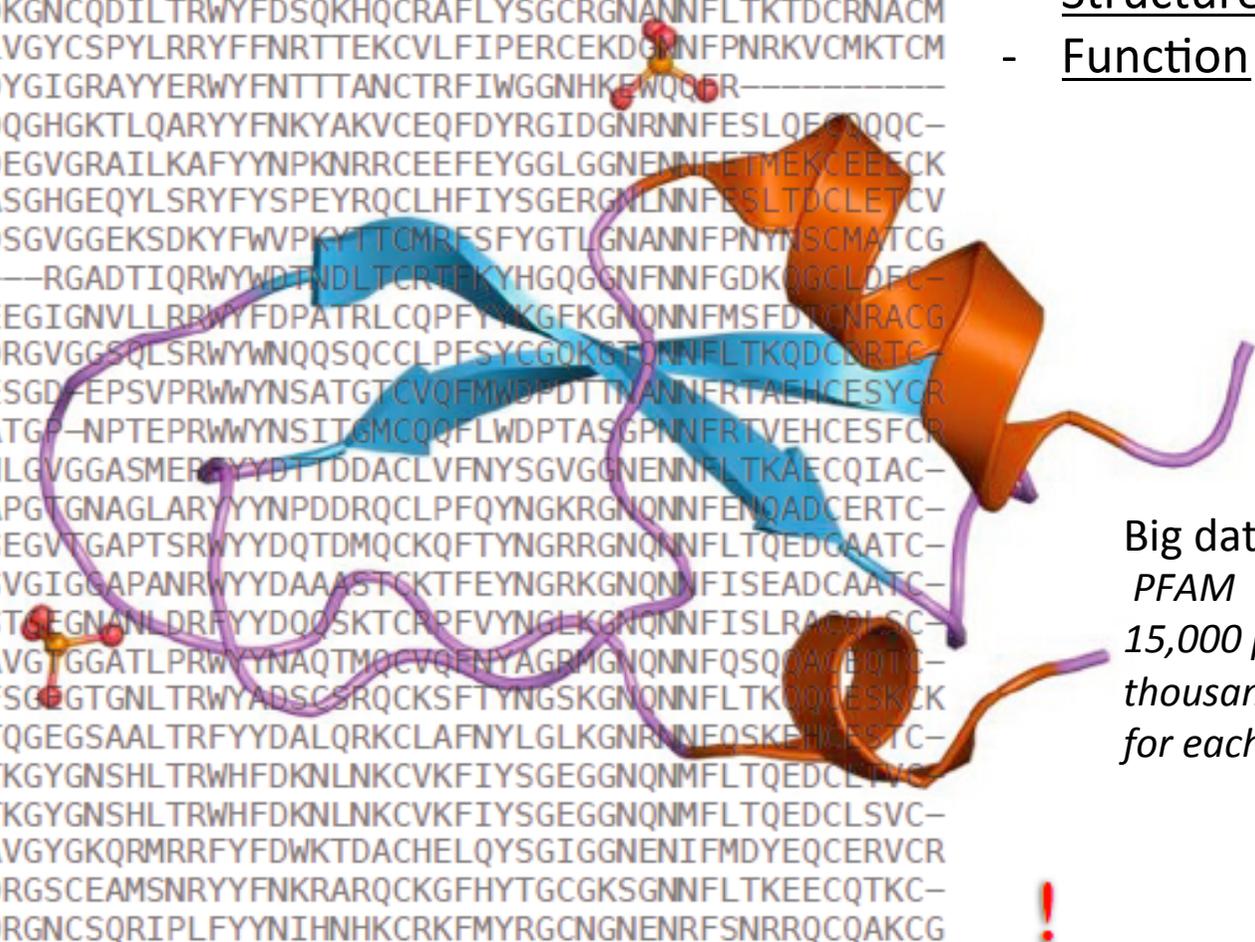


```

ACSLPKVQGPCSGKHSYYYFNSANQQCETFVYGGCLGNTNRFATIEECNARC-
VCLLPKSAGPCTGFTKKWYFDVDRNRCEEFOYGGCYGTNNRFDLSLECOGTC-
VCAMPPDAGVCTNYTPRWFNSQTGQCEQFAYGSCGGNENFFDRNTCERKCM
TCSLSPSPGTCGPGVFKYHYNPQTQECESFEYLGCDGNSNTFASRAECENYCG
-CHTEHSSGACPGAVTMFYHDPRTKKCTPFTFLGCGGNSNKFDTRPQCERFCK
PCMLPSDKGNCQDILTRWYFDSQKHQCRAFLYSGCRGNANNFLTKTDCRNACM
-----RLVGYCSPYLRRYFFNRTTEKCVLFIPERCEKDGNNFPNRKVCMTKCM
PCSLKEDYGIGRAYYERWYFNTTTANCTRFIWGGNHKEWQQR-----
PCKQDLQGHGKTLQARYYFNKYAKVCEQFDYRGIDGNRNNFESLQEQQQC-
-CFLKPDEGVGRAILKAFYYPKNRRCEEFEYGGGLGGNENNFETMEKCEEECK
-CSQPAASGHGEQYLSRYFYSPYRQCLHFIYSGERGNLNNFESLTDCLCTCV
LCNLKYDSGVGGEKSDKYFWVPKYTTCMRFSFYGTLGNANNFPNYNSCMATCG
-----RGADTIQRWYWDTNDLTCRTEFKYHGQGGNFNFGDKQGCCLDFC-
PCEQAIIEEGIGNVLLBRNYFDPATRLCQPFYKGFKGNONNFMSFDTCNRACG
PCGQPLDRGVGGSQLSRWYWNQQSQCCLPESYCGQKGTNNFLTQDCBRTG-
VCIQPLESGD-EPSVPRWWYNSATGTCVQFMWDPDTMANNFRTAEHCESYCR
TCVQPTATGP-NPTEPRWWYNSITGMCOQFLWDPTASGPNNFRTVEHCESFCR
-CDQQLMLGVGGASMERFYDYDTDDACLVFNYSGVGGNENNFLTKAECQIAC-
PCSVPLAPGTGNAGLARYYYPDDRQCLPFQYNGKRGNONNFENQADCERTC-
-----PESEGVTGAPTSRWYYDQTD MQCKQFTYNGRRGNQNNFLTQEDCAATC-
ACKMPLSVGIGCAPANRWYYDAAASTCKTFEYNGRKGNNQNFISEADCAATC-
VCNLPMSTGEGNANLDRFYDQCSKTCRPFVYNGLEKGNQNNFISLRACQISC-
ICQQPMAVGTGGATLPRWYNAQTMQCVQENYAGRMGNQNNFQSQAQCBQTC-
PCSLPMFSGEGTGNLTRWYADSCSRQCKSFTYNGSKGNQNNFLTQDCESKCK
PCEEEMTQGECSAALTRFYDALQRKCLAFNYLGLKGNRNNFOSKEHCFSTC-
TCELPMTKGYGNSHLTRWHFDKNLNKCVKFIYSGEGGNQNMFLTQEDCLTVC-
TCELMTKGYGNSHLTRWHFDKNLNKCVKFIYSGEGGNQNMFLTQEDCLSVC-
RCHLPPAVGYGKQRMRRFYFDWKTACHELQYSGIGGNENIFMDYEQCERVCR
-CMESLDRGSCEAMSNRYFYNKRARQCKGFHYTGCGKSGNNFLTKEECQTKC-
PCQQPLQRGNCSQRIPLFYNYIHNHKCRKFMYRGCNGNENRFSNRRQCQAKCG
    
```

Information on:

- Structure
- Function



Big data bases!

*PFAM*

*15,000 protein families  
thousands of sequences  
for each...*



Aim: to find a **Potts** model  $P(A_1 \dots A_N)$  which reproduces and interpret the data

# A 20+1 possible state model: Potts model

Name	One letter code	Abbreviation
Alanine	A	Ala
Cysteine	C	Cys
Aspartic acid	D	Asp
Glutamic acid	E	Glu
Phenylalanine	F	Phe
Glycine	G	Gly
Histidine	H	His
Isoleucine	I	Ile
Lysine	K	Lys
Leucine	L	Leu
Methionine	M	Met
Asparagine	N	Asn
<del>Pyrrolysine</del>	<del>O</del>	<del>Pyl</del>
Proline	P	Pro
Glutamine	Q	Gln
Arginine	R	Arg
Serine	S	Ser
Threonine	T	Thr
<del>Selenocysteine</del>	<del>U</del>	<del>Sec</del>
Valine	V	Val
Tryptophan	W	Trp
Tyrosine	Y	Tyr

## Broad Outline: Applications

- Potts model to understand structure of proteins
- Potts model to decode relationship between protein's sequence and function (genotype-phenotype mapping):
  - Design new proteins with the same structure and function of the natural ones .
  - Forecasting Viral Evolution
  - Predicting antibiotic resistance

# Plan

## Theory :

- The coupled 20-letter model for Proteins: The Potts Model
- Pseudo-Likelihood Method

## Applications:

Analysis of Multiple Sequence Alignment in Proteins for:

- Contacts predictions
- Structural prediction
- Design new functional proteins the case of a small protein domain WW
- Predict viral evolution for HIV virus
- Predict antibiotic resistance of TEM 1

## Example 4:

# Multi-sequence alignments



ACSLPKVQGPCSGKHSYYYFNSANQQCETFVYGGCLGNTNRFATIEECNARC-  
VCLLPKSAGPCTGFTKKWYFDVDRNRCEEFYGGCYGTNNRFDLSLEQCQGT-  
VCAMPPDAGVCTNYTPRWFNFNSQTGQCEQFAYGSCGGNENFFDRNTCERKCM  
TCSLSPSPGTCGPGVFKYHYNPQTQECESFEYLGCDGNSNTFASRAECENYCG  
-CHTEHSSGACPGAVTMFYHDPRTKKCTPFTFLGCGGNSNKFDTRPQCERFCK  
PCMLPSDKGNCQDILTRWYFDSQKHQCRAFLYSGRCRGNANFLTKTDCRNACM  
-----RLVGYCSPYLRRYFFNRRTTEKCVLFIPERCEKDGIFNPNRKVCMKTCM  
PCSLKEDYGIGRAYYERWYFNTTTANCTRFIWGGNHKEWQQR-----  
PCKQDLQGHGKTLQARYYFNKYAKVCEQFDYRGIDGNRNNFESLQEEQQQC-  
-CFLKPDEGVGRAILKAFYYPKNRRCFEFEYGGGLGGNENNFETMEKCEEECK  
-CSQPAASGHGEQYLSRYFYSPYRQCLHFIYSGERGNLNNFESLTDCLETCV  
LCNLKYDSGVGGGEKSDKYFWVPKYTTTCMRFSFYGTLGNANFPNYNSCMATCG  
-----RGADTIQRWYWDTNDLTCRTFKYHGQGGNFNNFGDKOGCLDFC-  
PCEQAIEEGIGNVLLBRWYFDPATRLCQPFYKGFKGNQNNFMSFDTONRACG  
PCGQPLDRGVGGSQLSRWYWNQSQCCLPFSYCGQKSTONNFLTQKDCBRTG-  
VCIQPLESGD-EPSVPRWWYNSATGTCVQFMWDPDTIMANNFRTAEHCESYCR  
TCVQPTATGF-NPTEPRWWYNSITGMCQQFLWDPTASGPNNFRTVEHCESFCR  
-CDQQLMLGVGGASMERFYDITDDACLVFNYSGVGGNENNFLTKAECQIAC-  
PCSVPLAPGTGNAGLARYYYPDDRQCLPFQYNGKRGNNQNNFENQADCERTC-  
-----PESEGVTGAPTSRWYYDQTD MQCKQFTYNGRRGNQNNFLTQEDCAATC-  
ACKMPLSVGIGGAPANRWYYDAAASTCKTFEYNGRKGNNQNNFISEADCAATC-  
VCNLPMTSEGNANLDRFYDQOSKTCRPFVYNGLEKGNQNNFISLRAQQLSC-  
ICQQPMAVGIGGATLPRWYNAQTMQCVQENYAGRMGNQNNFQSQAQCBQTC-  
PCSLPMFSGEGTGNLTRWYADSCSRQCKSFTYNGSKGNQNNFLTQKQCESKCK  
PCEEEMTQGECSAALTRFYDALQRKCLAFNYLGLKGNRNNFESKEHHEESTC-  
TCELPMTKGYGNSHLTRWHFDKNLNKCVKFIYSGEGGNQNMFLTQEDCLTVC-  
TCELMTKGYGNSHLTRWHFDKNLNKCVKFIYSGEGGNQNMFLTQEDCLSVC-  
RCHLPPAVGYGKQRMRRFYFDWKTACHELQYSGIGGNENIFMDYEQCERVCR  
-CMESLDRGSCEAMSNRYFFNKRARQCKGFHYTGCGKSGNNFLTKEECQTKC-  
PCQQPLQRGNCSQRIPLFYNIHNNHKCRKFMRYRGCNGNENRFSNRRQCQAKCG

Information on:

- Structure
- Function

*Big data bases!*

*PFAM*

*15,000 protein families  
thousands of sequences  
for each...*



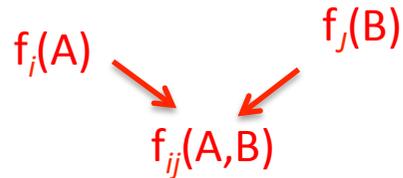
# Inference of couplings and fields for Potts model

Multiple sequence alignment (MSA):  $\{A_i^m \mid i=1,\dots,N ; m=1,\dots,M\}$

```

CSGKHSYYYFNSANQQCETFVYGGCLGN
CTGFTKKWYFDVDRNRCEEFOYGGCYGT
CTNYTPRWF FNSQTGQCEQFAYGSCGGN
CGPGVFKYHYNPQTQECESFEYLGCDGN
CPGAVTMFYHDPRTKKCTPFTFLGCGGN
CQDILTRWYFDSQKHQCRAFLYSGCRGN
CSPYLRRYFFNRTTEKCVLFIPERCEKD
    
```

$i$ 
 $j$



$$\frac{1}{M} \sum_{m=1}^M \delta_{A,A_i} = f_i(A) \quad \Leftarrow$$

$$\frac{1}{M} \sum_{m=1}^M \delta_{A,A_i} \delta_{B,A_j} = f_{ij}(A,B)$$

# Inference of couplings and fields for Potts model

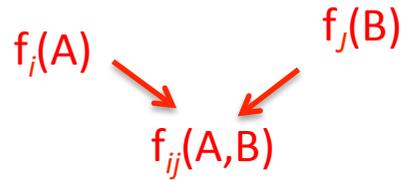
Multiple sequence alignment (MSA):  $\{A_i^m \mid i=1, \dots, N ; m=1, \dots, M\}$

```

CSGKHSYYYFNSANQQCETFVYGGCLGN
CTGFTKKWYFDVDRNRCEEFOYGGCYGT
CTNYTPRWFFNSQTGQCEQFAYGSCGGN
CGPGVFKYHYNPQTQECESFEYLGCDGN
CPGAVTMFYHDPRTKKCTPFTFLGCGGN
CQDILTRWYFDSQKHQCRAFLYSGCRGN
CSPYLRRYFFNRTTEKCVLFIPERCEKD
    
```

i

j



$$\sum_{\{A\}} P(A_1, \dots, A_N) \delta_{A, A_i} = f_i(A) \quad \leftarrow$$

$$\sum_{\{A\}} P(A_1, \dots, A_N) \delta_{A, A_i} \delta_{B, A_j} = f_{ij}(A, B)$$

**Potts Model :**  $P(A_1, \dots, A_N) = \frac{e^{\sum_{i < j} J_{ij} (A_i, A_j) + \sum_i h_i(A_i)}}{Z[\{J_{ij}(A, B), h_i(A)\}]}$

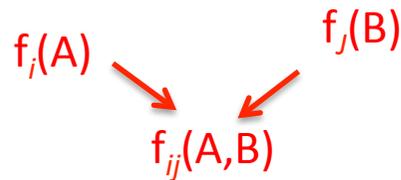
⇒ 21 N+21<sup>2</sup> N (N-1)/2 Coupled equations to solve!!

# Inference of couplings and fields for Potts model

Multiple sequence alignment (MSA):  $\{A_i^m \mid i=1,\dots,N ; m=1,\dots,M\}$

```

CSGKHSYYYFNSANQQCETFVYGGCLGN
CTGFTKKWYFDVDRNRCEEFOYGGCYGT
CTNYTPRWFFNSQTGQCEQFAYGSCGGN
CGPGVFKYHYNPQTQECESFEYLGCDGN
CPGAVTMFYHDPRTKKCTPFTFLGCGGN
CQDILTRWYFDSQKHQCRAFLYSGCRGN
CSPYLRRYFFNRTTEKCVLFIPERCEKD
    
```



$$\sum_{\{A\}} P(A_1, \dots, A_N) \delta_{A, A_i} = f_i(A) \quad \leftarrow$$

$$\sum_{\{A\}} P(A_1, \dots, A_N) \delta_{A, A_i} \delta_{B, A_i} = f_{ij}(A, B)$$

Potts Model : 
$$P(A_1, \dots, A_N) = \frac{e^{\sum_{i < j} J_{ij} (A_i, A_j) + \sum_i h_i(A_i)}}{Z[\{J_{ij}(A_i, A_j), h_i(A_i)\}]}$$

⇒ 21 N+21<sup>2</sup> N (N-1)/2 Coupled equations to solve!!

Problem rewritten as the Minimization of the Cross Entropy

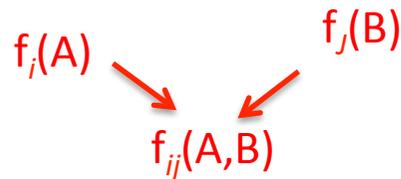
$$S = \log Z[\{J_{ij}(A, B), h_i(A)\}] - \sum_{i < j, A, B} J_{ij} (A, B) f_{ij}(A, B) - \sum_{i, A} h_i(A) f_i(A)$$

# Inference of couplings and fields for Potts model

Multiple sequence alignment (MSA):  $\{A_i^m \mid i=1, \dots, N ; m=1, \dots, M\}$

```

CSGKHSYYYFNSANQQCETFVYGGCLGN
CTGFTKKWYFDVDRNRCEEFOYGGCYGT
CTNYTPRWFFNSQTGQCEQFAYGSCGGN
CGPGVFKYHYNPQTQECESFEYLGCDGN
CPGAVTMFYHDPRTKKCTPFTFLGCGGN
CQDILTRWYFDSQKHQCRAFLYSGCRGN
CSPYLRRYFFNRTTEKCVLFIPERCEKD
    
```



$$\sum_{\{A\}} P(A_1, \dots, A_N) \delta_{A, A_i} = f_i(A) \quad \leftarrow$$

$$\sum_{\{A\}} P(A_1, \dots, A_N) \delta_{A, A_i} \delta_{B, A_i} = f_{ij}(A, B)$$

**Potts Model :** 
$$P(A_1, \dots, A_N) = \frac{e^{\sum_{i < j} J_{ij} (A_i, A_j) + \sum_i h_i(A_i)}}{Z[\{J_{ij}(A_i, A_j), h_i(A_i)\}]}$$

⇒ 21 N+21<sup>2</sup> N(N-1)/2 Coupled equations to solve!!

Problem rewritten as the Minimization of the Cross Entropy

$$S = \log Z[\{J_{ij}(A, B), h_i(A)\}] - \sum_{i < j, A, B} J_{ij} (A, B) f_{ij}(A, B) - \sum_{i, A} h_i(A) f_i(A)$$

Regularization is needed !

1. The Potts model is a generalization of the Ising model (binary variables) for any given number of category or Potts states in each site (21-a.a.)

2. Gauge invariance and overparametrization

It is possible to arbitrarily fix  $h_i(c_i) = 0$ ,  $J_{ij}(c_i, b) = 0$

or other gauges without changing the model probability distribution

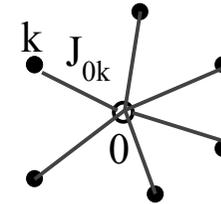
3. Minimising the Cross Entropy allows to find the parameters to reproduce frequencies and correlations

4. Equivalence between minimising the Cross Entropy and maximising the Log-Likelihood of the data given the model

5. Pseudo-Likelihood model

# Pseudo-likelihood method

**Pseudo log-likelihood of the node 0:  $l_0$**   
 =- pseudo cross entropy:  $S_0$



$$S_0 = \frac{1}{B} \sum_{\tau=1}^B \log \left[ \sum_A \exp \left( \sum_j J_{0j}(A, A_j^\tau) + h_0(A) \right) \right] - \sum_A \left[ h_0(A) p_0(A) - \sum_{j, A_j} J_{0j}(A, A_j) p_{ij}(A, A_j) \right]$$

**Idea:** avoid calculation of partition function using the sampled configurations

Total Pseudo-log likelihood: Sum of contributions of each node

**Prior:** increase signal/noise ratio by exploiting the sparsity of  $J_{ij}$

$$\text{cost function} (\{J\}) = \text{pseudo-cross-entropy} + \begin{cases} \Gamma \sum_{ij} |J_{ij}(A, B)| \\ \Gamma \sum_{ij} J_{ij}^2(A, B) \end{cases}$$

# Bibliography

- P. Ravikumar, M. J. Wainwright, and J. D. Lafferty (2010). *High-dimensional Ising model selection using  $\ell_1$ -regularized logistic regression*. J. Ann Stat. 38, 1287.
- M. Ekeberg, C. Lovkvist, Y. Lan, M. Weigt, E. Aurell (2013). *Improved contact prediction in proteins: Using pseudolikelihood to infer Potts models*. Phys. Rev. E 87, 012707.

# State of the art in protein structure prediction

## Improved Contact Predictions Using the Recognition of Protein Like Contact Patterns

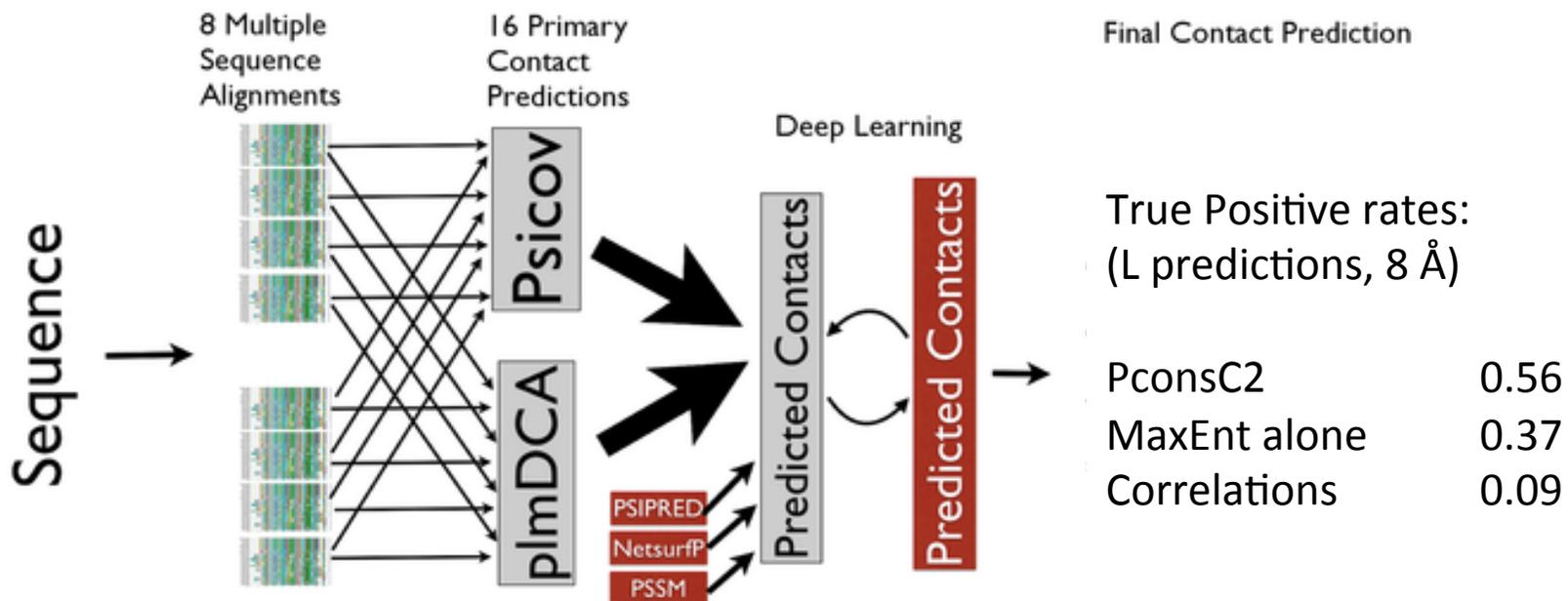
Marcin J. Skwark  Daniele Raimondi , Mirco Michel, Arne Elofsson 

Published: November 6, 2014 • DOI: 10.1371/journal.pcbi.1003889

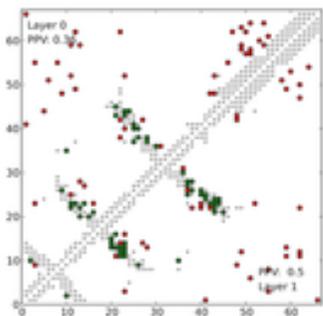
### Author Summary

Here, we introduce a novel protein contact prediction method PconsC2 that, to the best of our knowledge, outperforms earlier methods. PconsC2 is based on our earlier method, PconsC, as it utilizes the same set of contact predictions from plmDCA and PSICOV. However, in contrast to PconsC, where each residue pair is analysed independently, the initial predictions are analysed in context of neighbouring residue pairs using a deep learning approach, inspired by earlier work. We find that for each layer the deep learning procedure improves the predictions. At the end, after five layers of deep learning and inclusion of a few extra features provides the best performance. An improvement can be seen for all types of proteins, independent on length, number of homologous sequences and structural class. However, the improvement is largest for  $\beta$ -sheet containing proteins. Most importantly the improvement brings for the first time sufficiently accurate predictions to some protein families with less than 1000 homologous sequences. PconsC2 outperforms as well state of the art machine learning based predictors for protein families larger than 100 effective sequences. PconsC2 is licensed under the GNU General Public License v3 and freely available from <http://c2.pcons.net/>.

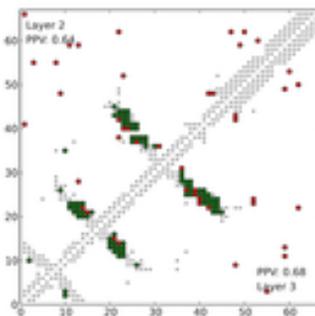
# State of the art in protein structure prediction



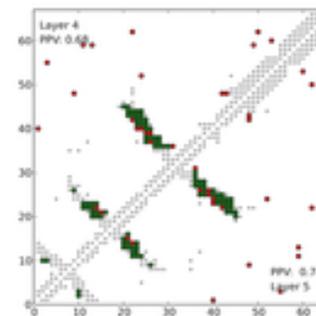
(a) Pipeline



(b) 1pcf:A Layer 0-1



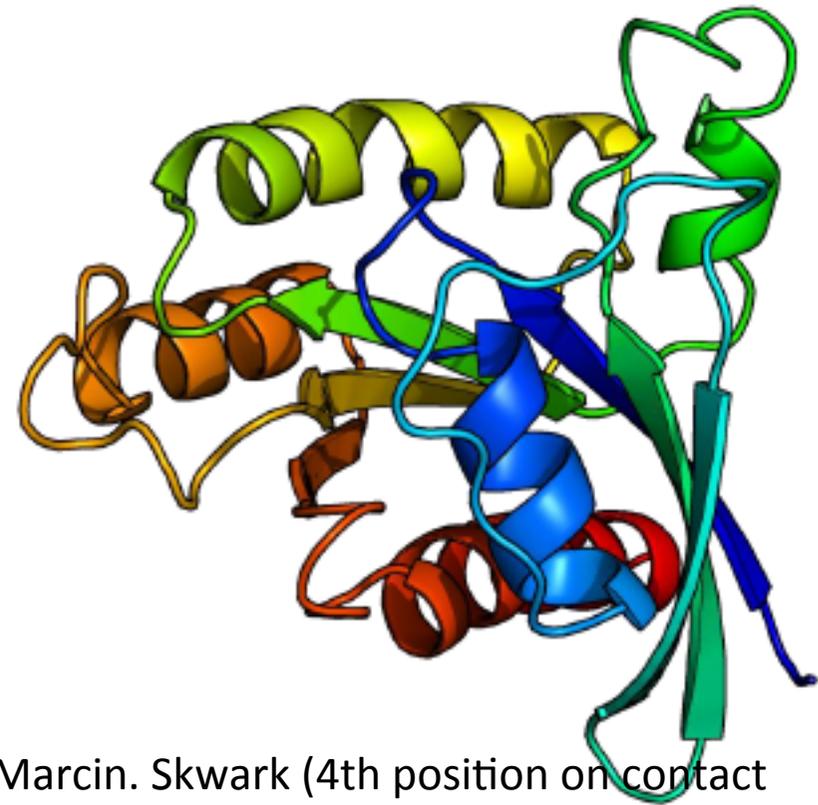
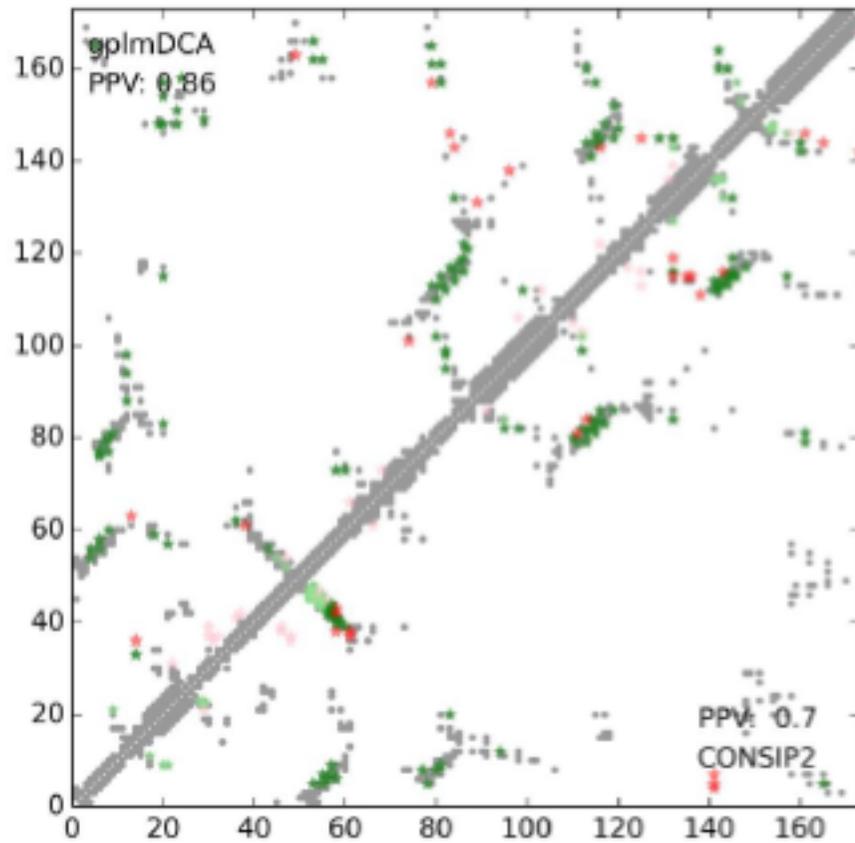
(c) 1pcf:A Layer 2-3



(d) 1pcf:A Layer 4-5

plmDCA: Ekeberg, Aurell (2014)

# Prediction of Contacts by Pseudo-Likelihood in CASP 2015

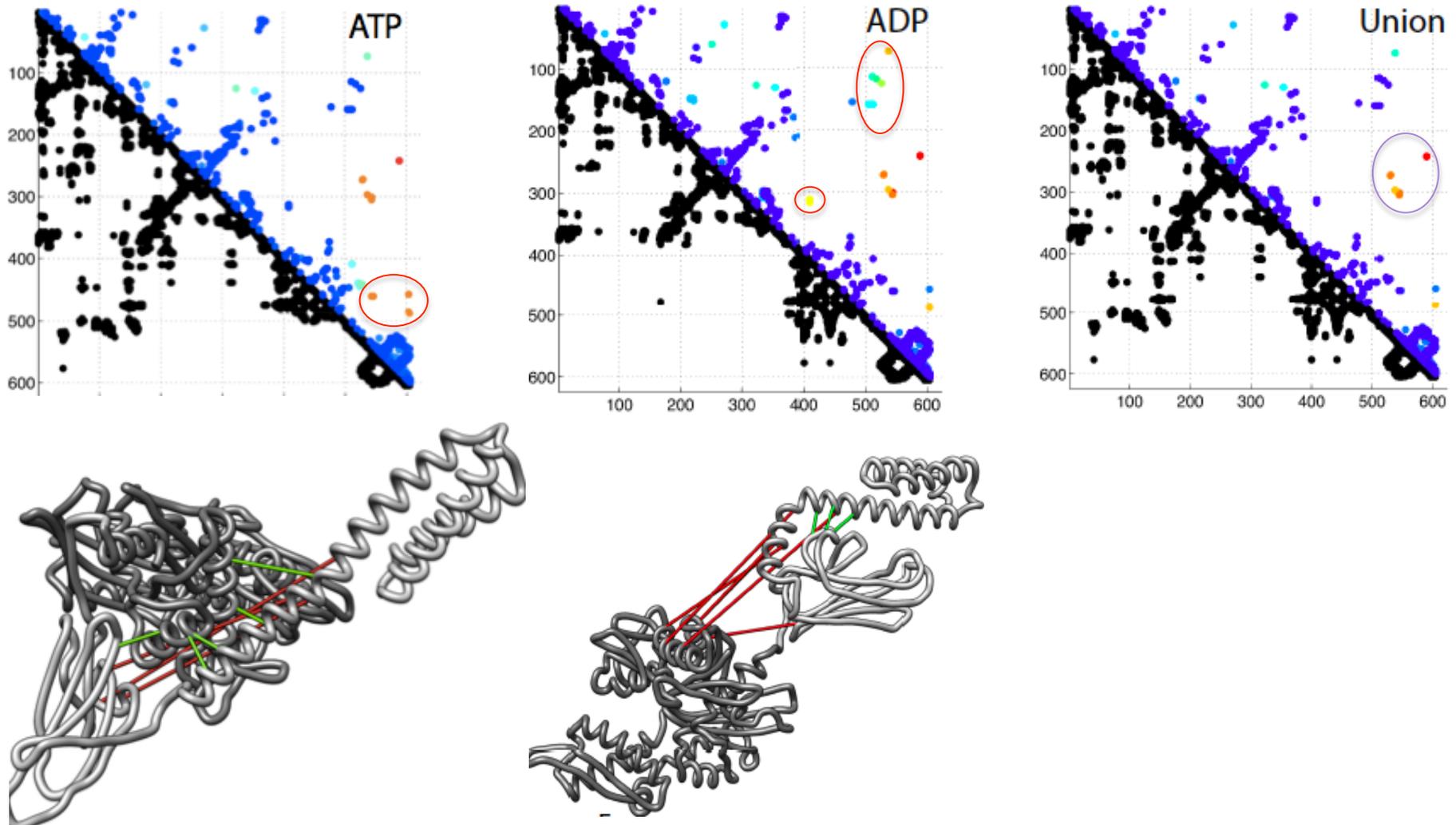


Marcin. Skwark (4th position on contact prediction )

T0798: RAS11B a protein involved in membrane trafficking, 88253 sequences at 90%

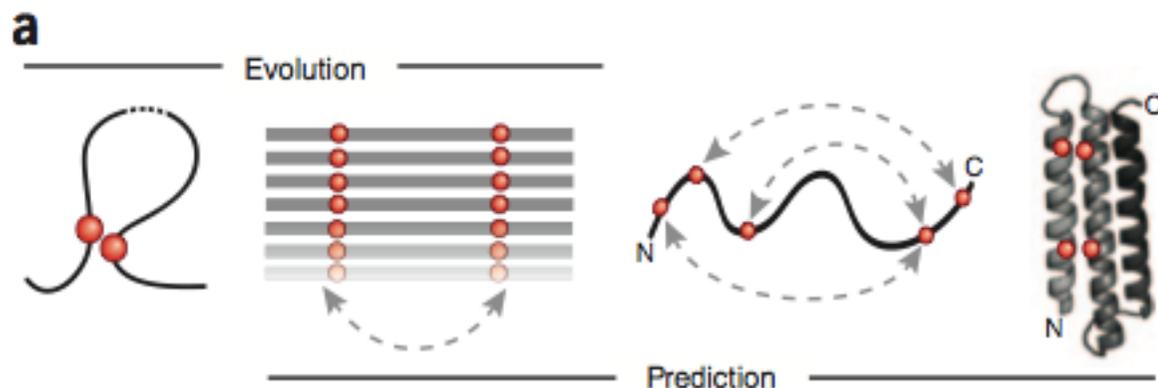
Ekeberg et al Phys Rev E 2013, Monastyrsky et al CASP 2015

# Prediction of Contacts in the Allosteric HSP70 protein



Maliverni D,..., De Los Rios P. HSP70 dimerization predicted by Sequence Variation  
Plos Comp Bio 100426 (2015).

# Contact Prediction and Protein Folding



D. Marks T.Hopf, C. Sanders  
Protein Structure Prediction Via Sequence variation  
Nat. Biotechnology 2012.

-Combine Contact Prediction and classical  
Molecular Dynamic simulations or

Monte Carlo Methods with effective Potentials:  
structures with very high resolution in the range of 2-6 Å

-Blind prediction for transmembranar proteins that are hard to crystallize

Classical Methods:

-All atom Potential:

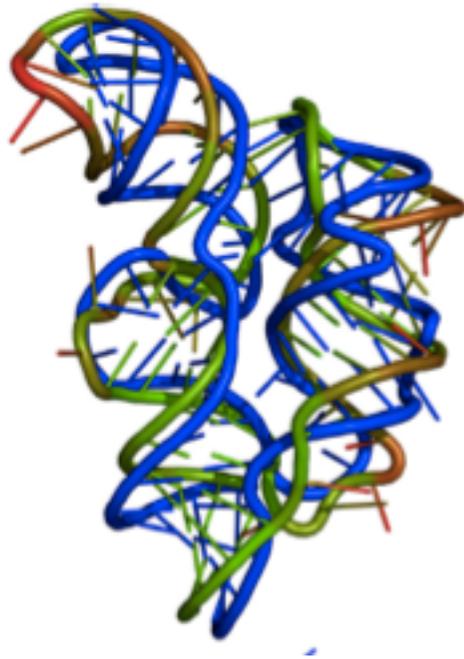
Computationally very expensive!

-Effective Potential

difficult: find right level of  
description, take into account  
context dependence..

# Contact Prediction and RNA Folding

## RNA + Rosetta



Rosetta (Baker group): Monte Carlo Minimization Of Effective Interaction potential between amino acids. The bond angles are chosen among a library of known and possible structures.

+ Lennard Jones attractive Potentials for pairs in contact

Rosetta alone RMSD  $\approx 16$  Å

Rosetta + First 25 Coplings: 7.5 Å

Rosetta+ All contacts: 6.3 Å

Riboswitch:  
2gdi (RF00059)

De Leonardis et al. NAR (2015)