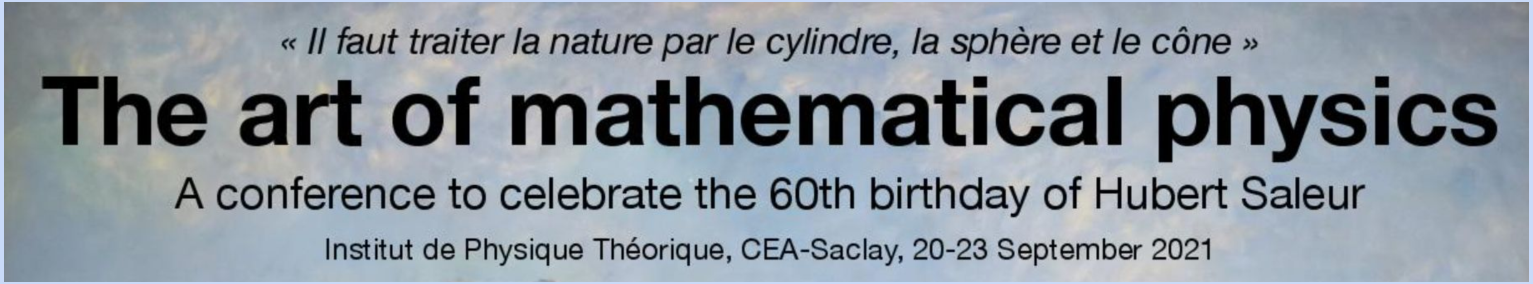
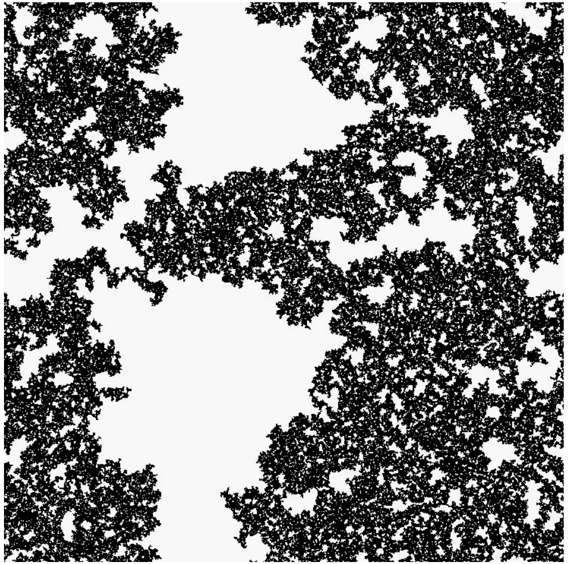
Four-point cluster connectivities in 2d percolation and LCFT

Yifei He

ENS Paris



Cluster connectivity in percolation



geometrical phase transition

described by **non-unitary** conformal field theory

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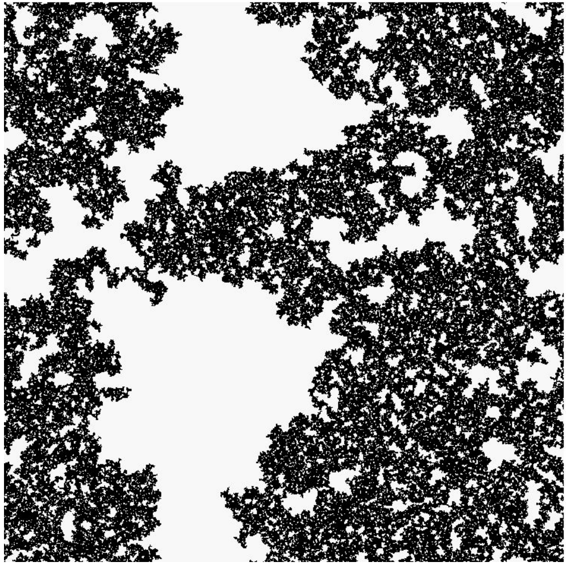

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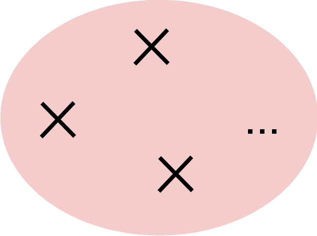

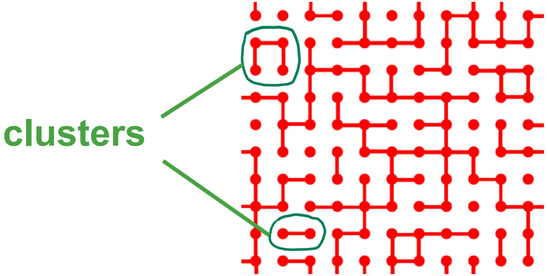

Q-state Potts model

$$Z_{\text{spin}} \longrightarrow Z_{\text{cluster}}$$ [Fortuin, Kasteleyn, 1972] $$Q \in \mathbb{R}$$

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Potts spin: order parameter

cluster connectivities ⟷ CFT correlator of spin operator $\Phi_{\frac{1}{2},0}$

[Delfino, Viti, 2011]

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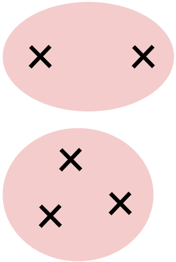


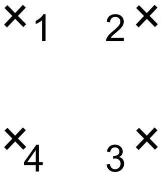
two-point
&
three-point

connectivities are understood

[Delfino, Viti, 2010]
[Picco, Santachiara, Viti, Delfino, 2013]
[Ikhlef, Jacobsen, Saleur, 2015]

Four-point connectivities

highly non-trivial, probe the spectrum of the CFT



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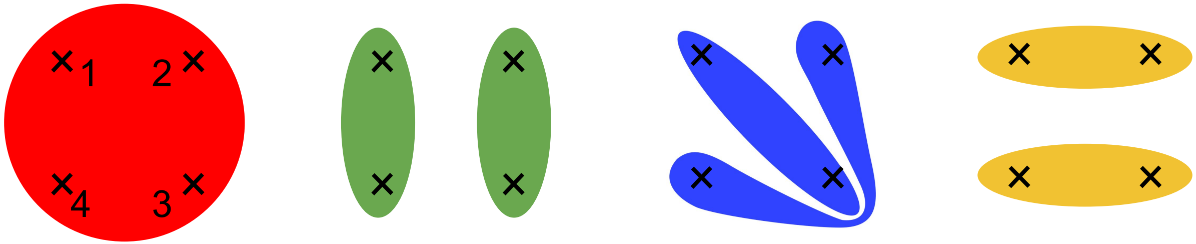

P_{aaaa} P_{aabb} P_{abab} P_{abba}

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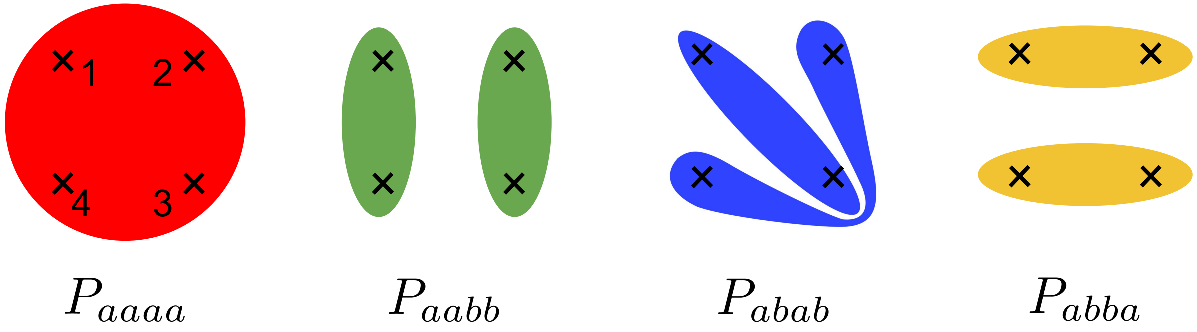

Potts CFT: $\left\langle \Phi_{\frac{1}{2},0}\Phi_{\frac{1}{2},0}\Phi_{\frac{1}{2},0}\Phi_{\frac{1}{2},0} \right\rangle$

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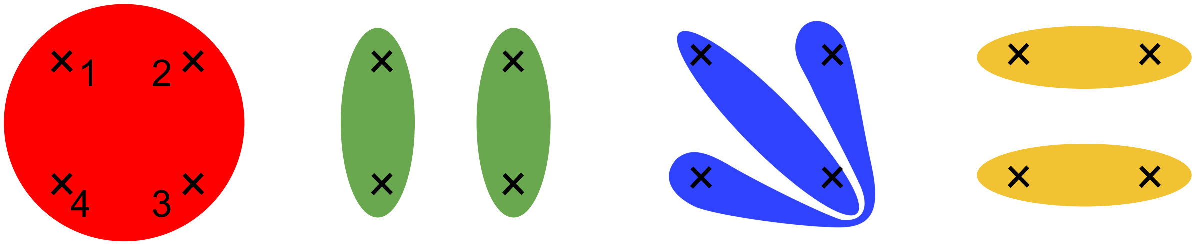

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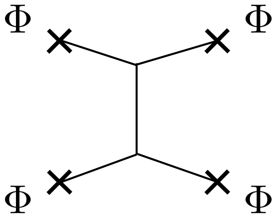
Potts CFT: $\langle \Phi_{\frac{1}{2},0} \Phi_{\frac{1}{2},0} \Phi_{\frac{1}{2},0} \Phi_{\frac{1}{2},0} \rangle$

fractional Kac indices

cannot use BPZ

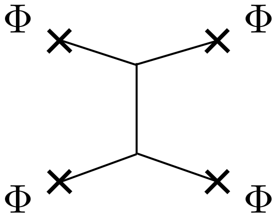
Conformal bootstrap approach



$$\sum_{(h,\bar{h})\in s-\text{channel}} A(h,\bar{h})\mathcal{F}^{(s)}$$

conformal block

Conformal bootstrap approach



amplitudes

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spectrum

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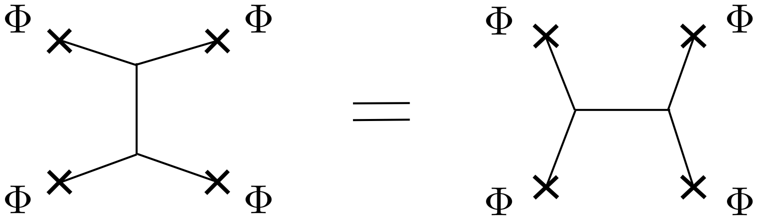

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$$\sum_{(h,\bar{h})\in s-\text{channel}} A(h,\bar{h})\mathcal{F}^{(s)} = \sum_{(h,\bar{h})\in t-\text{channel}} A(h,\bar{h})\mathcal{F}^{(t)}$$

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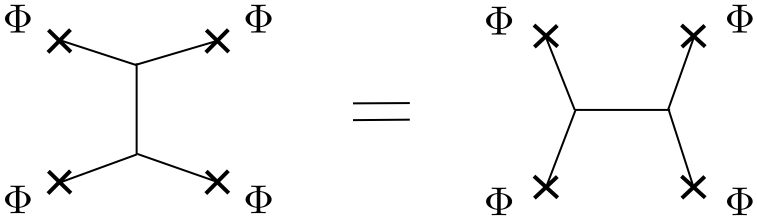

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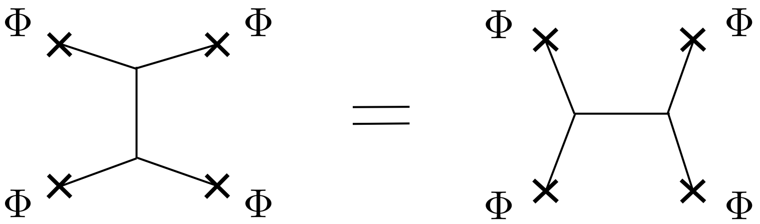

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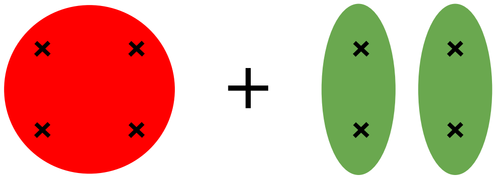
first attempt to
bootstrap connectivity:

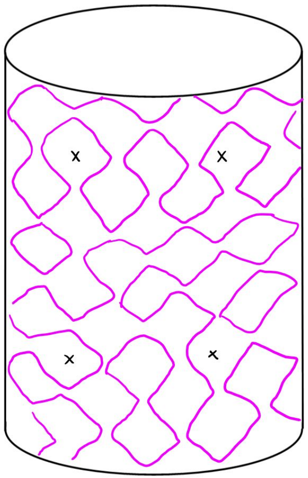


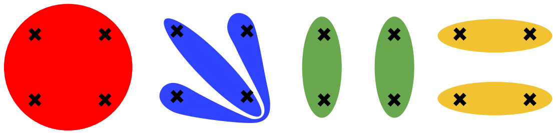
[Picco, Ribault, Santachiara, 2016]

Spectrum of connectivities

[Jacobsen, Saleur, 2018]

equivalent loop representation





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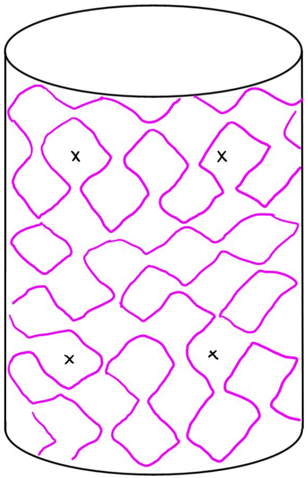
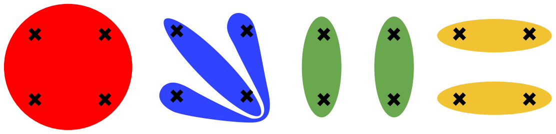


eigenvalues λ_i of transfer matrix

continuum limit

scaling dimensions $(h, \bar{h})$ in the spectrum



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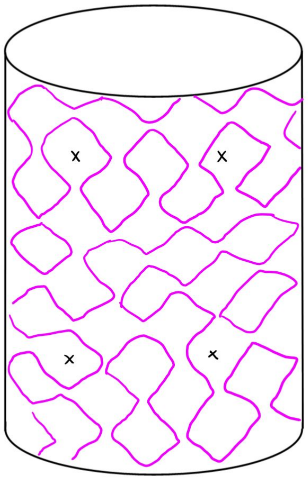
equivalent loop representation

irreducible module of affine Temperley-Lieb algebra



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$\mathcal{W}$

infinite tower of Virasoro conformal family

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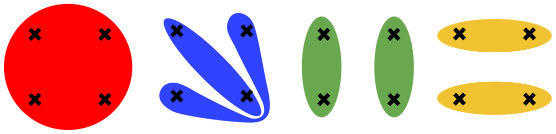

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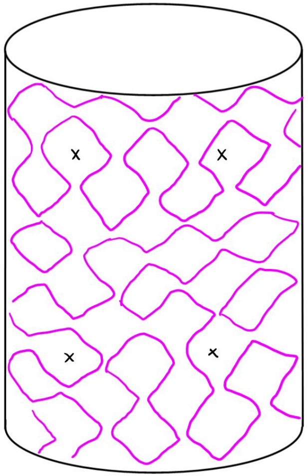

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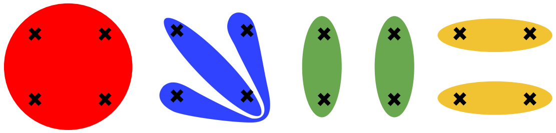

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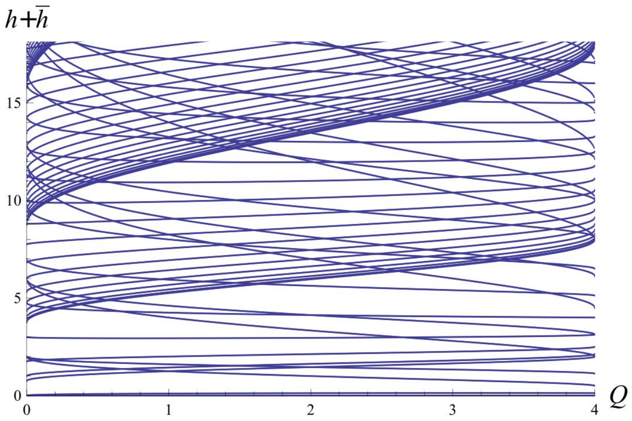
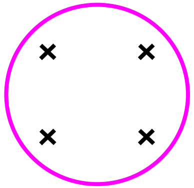
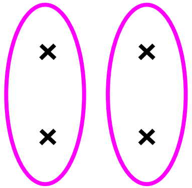


Minimal models on the lattice

$$\langle \Phi_\Delta \Phi_\Delta \Phi_\Delta \Phi_\Delta \rangle \underset{\textit{Monte-Carlo}}{\propto}$$



$$+ \frac{2}{Q-2}$$



[Picco, Ribault, Santachiara, 2016]

$\Delta = \Delta_{\frac{1}{2},0}$

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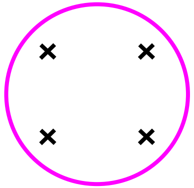

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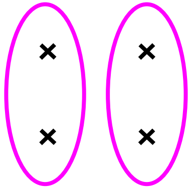

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Bootstrap: simple spectrum (subset of Potts)

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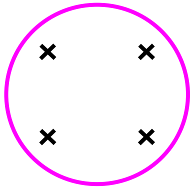
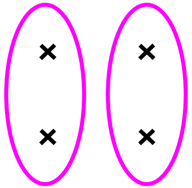
$$\langle \Phi_\Delta \Phi_\Delta \Phi_\Delta \Phi_\Delta \rangle \underset{\text{Monte-Carlo}}{\propto}$$  $$+ \frac{2}{Q-2}$$ 

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Bootstrap: simple spectrum (subset of Potts) ⟶ *analytic continuation of MM Kac table to generic c*

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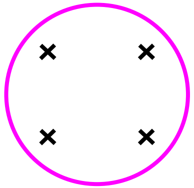
Monte-Carlo



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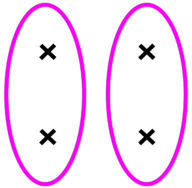

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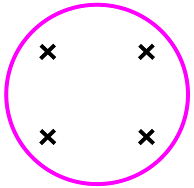
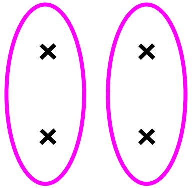
$$\langle \Phi_\Delta \Phi_\Delta \Phi_\Delta \Phi_\Delta \rangle \propto$$ *Monte-Carlo*  $$+ \frac{2}{Q-2}$$  [Picco, Ribault, Santachiara, 2016]

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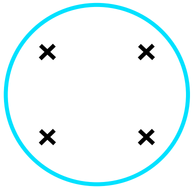
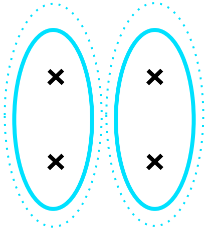
$$\langle \Phi_\Delta \Phi_\Delta \Phi_\Delta \Phi_\Delta \rangle \propto$$  $$+$$ 

[YH, Grans-Samuelsson, Jacobsen, Saleur, 2020]

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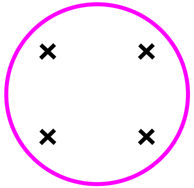
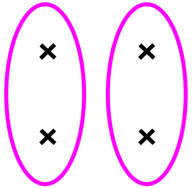
$$\langle \Phi_\Delta \Phi_\Delta \Phi_\Delta \Phi_\Delta \rangle \propto$$  $$+ \frac{2}{Q-2}$$  [Picco, Ribault, Santachiara, 2016]

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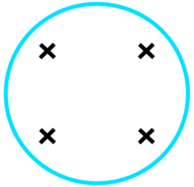
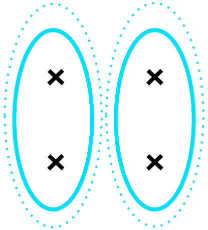
$$\langle \Phi_\Delta \Phi_\Delta \Phi_\Delta \Phi_\Delta \rangle \propto$$  $$+$$ 

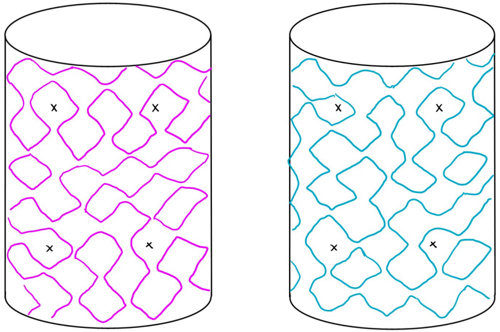
different weights depending on topology

[YH, Grans-Samuelsson, Jacobsen, Saleur, 2020]

pseudo-probabilities $\tilde{P}$

Universal amplitude ratios on the lattice

[YH, Grans-Samuelsson, Jacobsen, Saleur, 2020]



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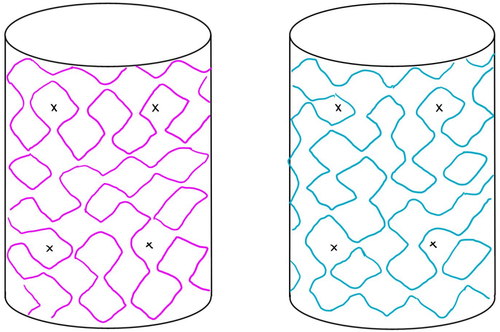

$\tilde{P}$ **spectrum** $\sim$ P **spectrum**

affine Temperley-Lieb modules $\mathcal{W}$

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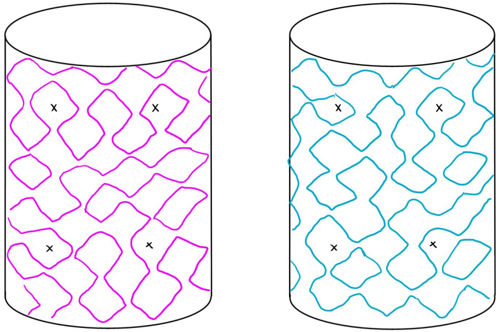

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$$\frac{A_{abab}}{A_{aaaa}}(\lambda_i), \quad \frac{\tilde{A}_{abab}}{A_{abab}}(\lambda_i), \quad \ldots$$

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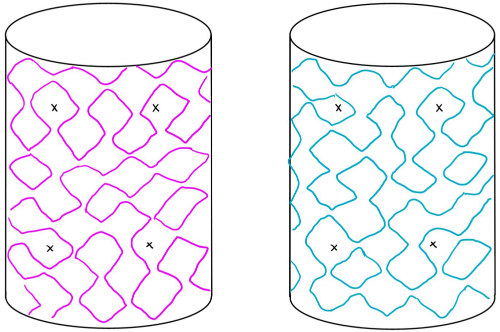

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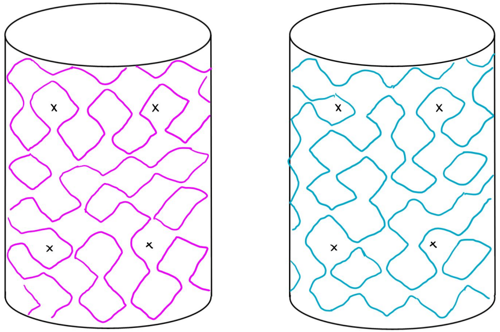

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do not depend on lattice size

$$\frac{\tilde{A}_{abab}}{A_{abab}}(\mathcal{W}_{2,-1}) = \frac{2}{Q-2} \qquad \frac{\tilde{A}_{abab}}{A_{abab}}(\mathcal{W}_{4,-1}) = -\frac{4}{(Q-1)(Q-2)(Q^2-4Q+2)}$$

$$\frac{A_{aabb}}{A_{aaaa}}(\mathcal{W}_{2,1}) = \frac{1}{1-Q} \qquad \frac{A_{abab}}{A_{aaaa}}(\mathcal{W}_{4,-1}) = \frac{(Q-1)(Q-4)}{4}$$

$$\frac{A_{abab}}{A_{aaaa}}(\mathcal{W}_{2,1}) = \frac{2-Q}{2} \qquad \frac{A_{aabb}}{A_{aaaa}}(\mathcal{W}_{4,1}) = -\frac{Q^5-7Q^4+15Q^3-10Q^2+4Q-2}{2(Q^2-3Q+1)}$$

$$\frac{A_{aabb}}{A_{aaaa}}(\mathcal{W}_{4,-1}) = \frac{2-Q}{2} \qquad \frac{A_{abab}}{A_{aaaa}}(\mathcal{W}_{4,1}) = -\frac{(Q^2-4Q+2)(Q^2-3Q-2)}{4}$$

Interchiral conformal blocks

[YH, Jacobsen, Saleur, 2020]

$$\lambda_i \xrightarrow{\text{continuum}} (h, \bar{h})$$

organize the CFT states according to $\mathcal{W}$

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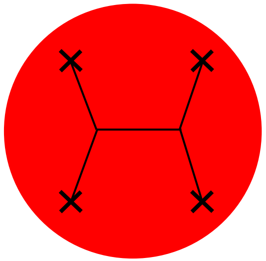

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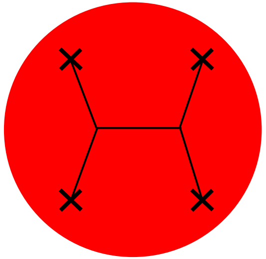
organize the CFT states according to $\mathcal{W}$



P_{aaaa}

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Interchiral conformal blocks

[YH, Jacobsen, Saleur, 2020]

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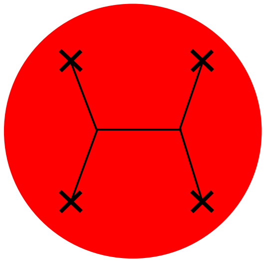

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interchiral conformal block

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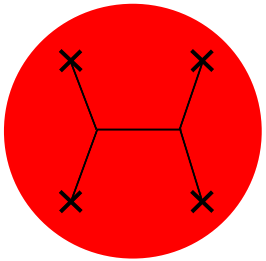

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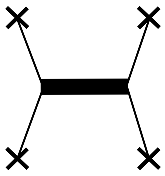

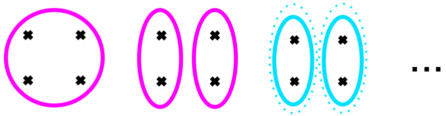

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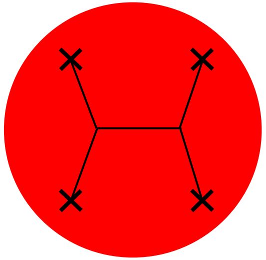

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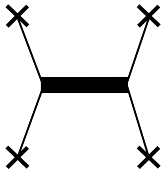
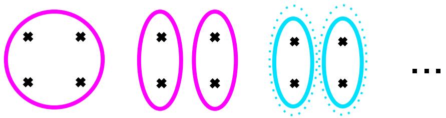
interchiral conformal block

interchiral algebra

[Gainutdinov, Read, Saleur, 2012]





Interchiral conformal bootstrap

[YH, Jacobsen, Saleur, 2020]

construct $\mathbb{F}_{\mathcal{W}}$: degeneracy $\longrightarrow$ recursion

technique in Liouville bootstrap

[Zamolodchikov^2, 1995] [Teschner, 1995]

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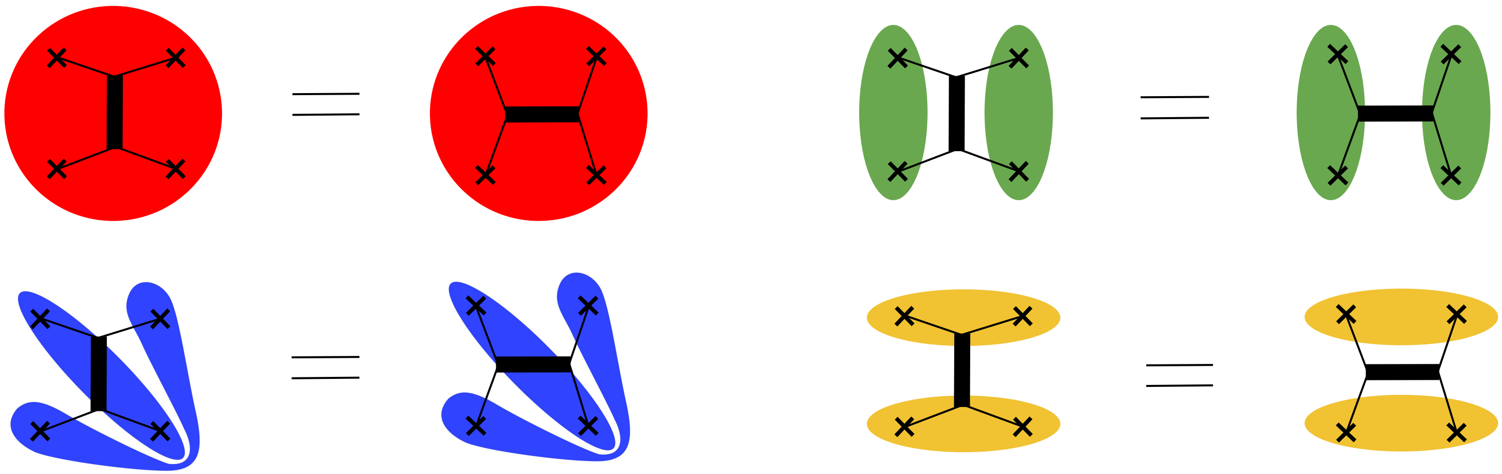

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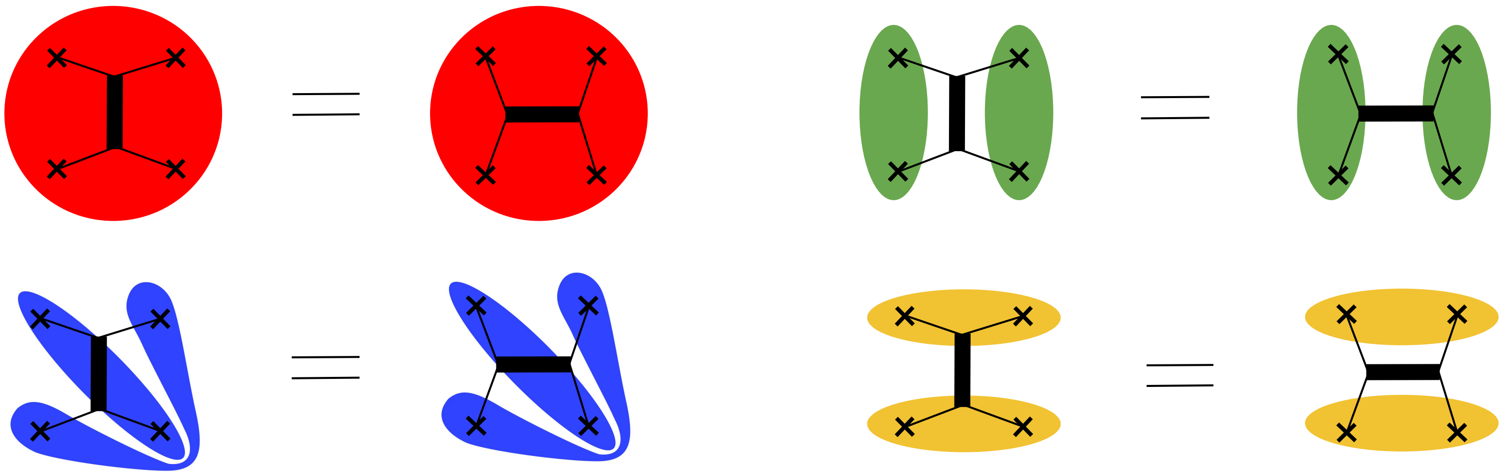
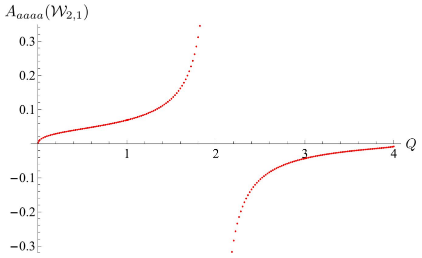
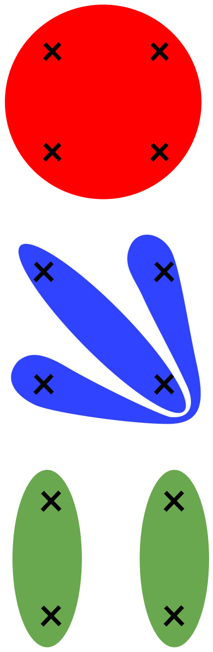
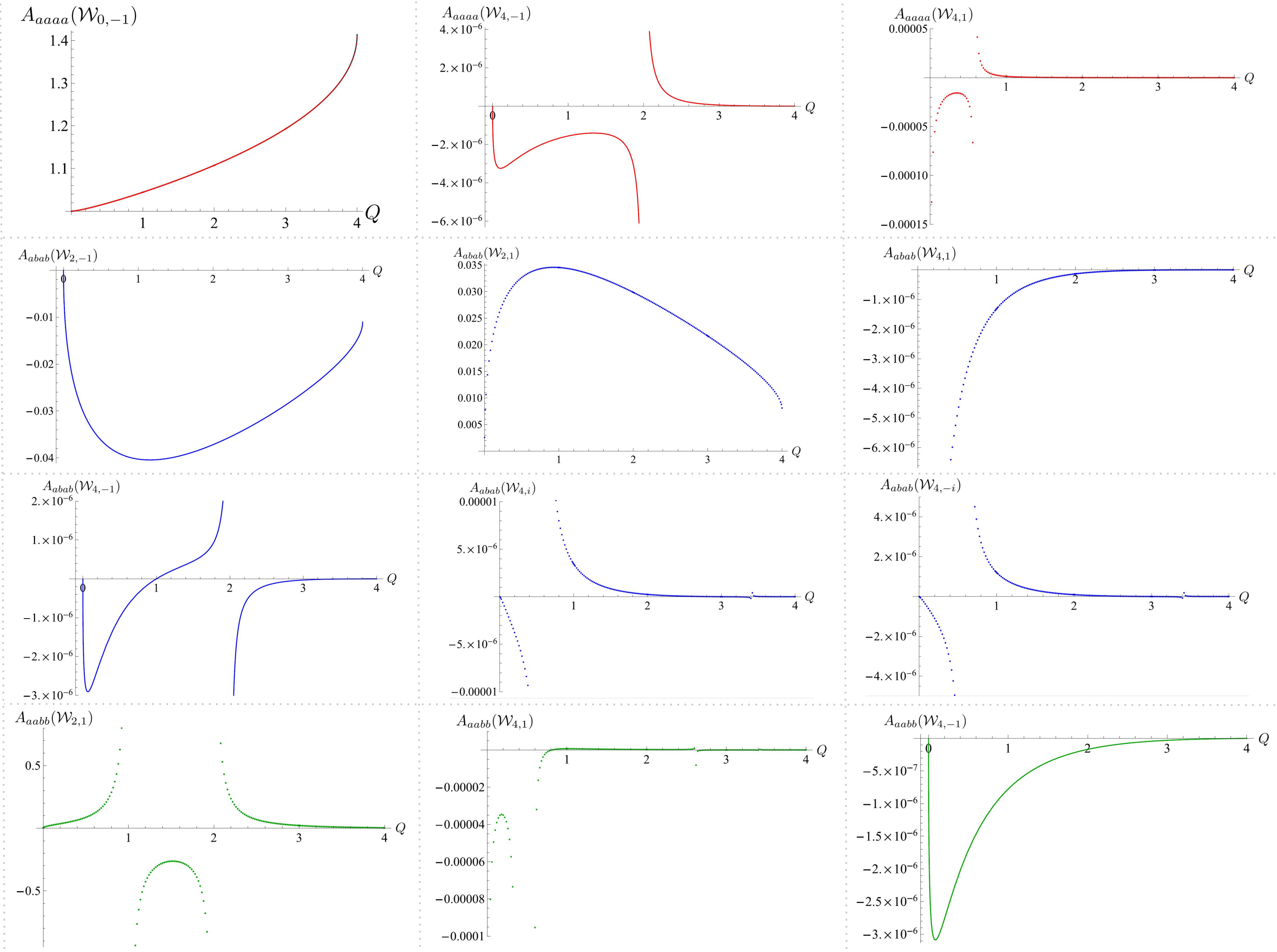


solve $A_{aaaa}(\mathcal{W}),\ A_{abab}(\mathcal{W}),\ A_{aabb}(\mathcal{W}),\ A_{abba}(\mathcal{W})$







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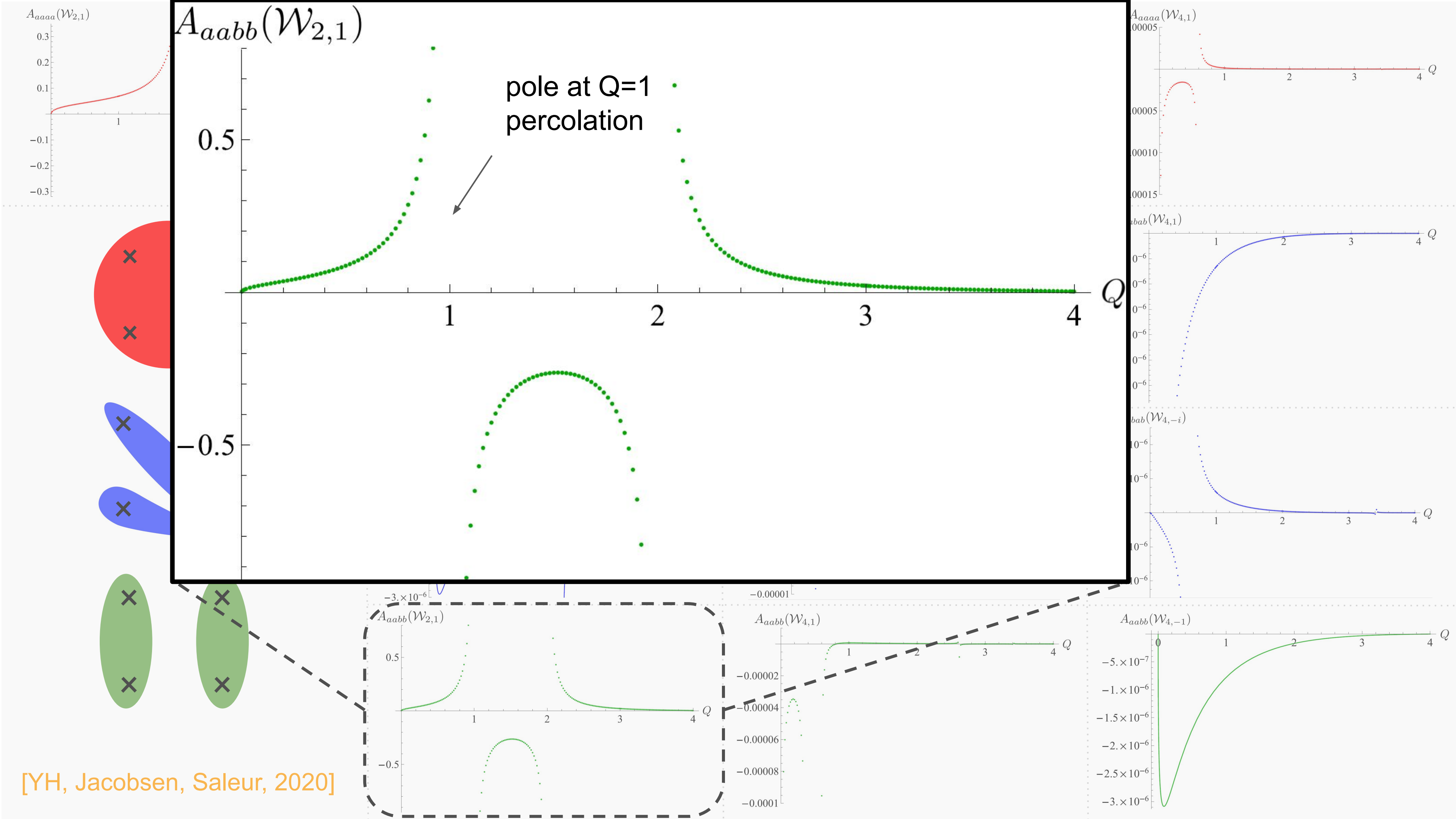
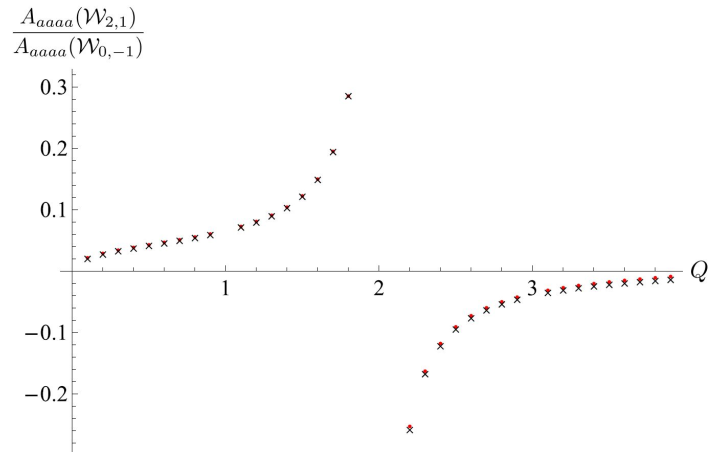


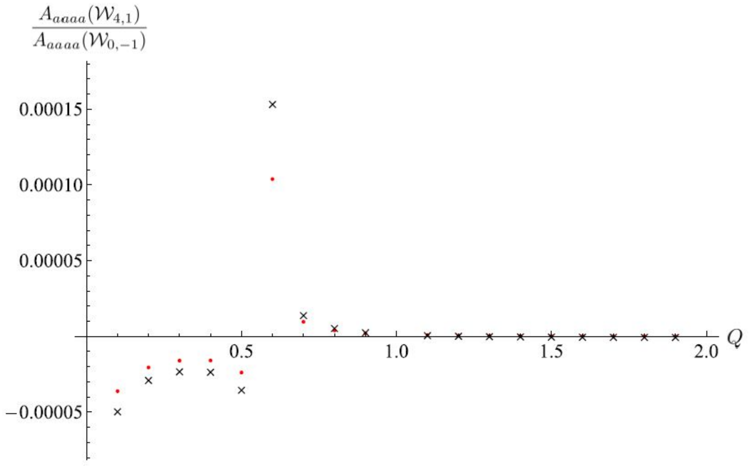
$A_{aabb}(\mathcal{W}_{2,1})$
pole at Q=1
percolation
Q
$A_{aaaa}(\mathcal{W}_{2,1})$
$A_{aaaa}(\mathcal{W}_{4,1})$
$A_{aabb}(\mathcal{W}_{4,1})$
$A_{aabb}(\mathcal{W}_{4,-1})$
[YH, Jacobsen, Saleur, 2020]

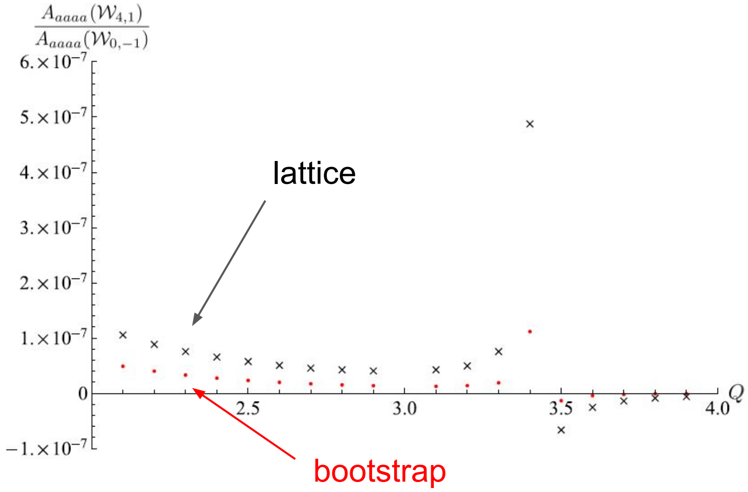
Comparison with lattice

[YH, Jacobsen, Saleur, 2020]

- *order of magnitude*
- *behavior as a function of Q*
- *analytic structure*







“Renormalized” Liouville recursion

[YH, Jacobsen, Saleur, 2020]

in Liouville
(and its non-diagonal generalization)

degenerate

$\Phi_{1,2}, \Phi_{2,1}$

→

amplitude recursions
analytic bootstrap solution

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Potts: only $\Phi_{2,1}$ **degenerate (energy operator)**

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Liouville recursion
IF $\Phi_{1,2}$ is degenerate

“renormalization” factors -- rational functions of Q

“$c \to 0$ catastrophe”

$$\Phi_\Delta(z,\bar{z}) \times \Phi_\Delta(0,0) \sim 1 + \frac{2\Delta}{c}\left(z^2 T + \bar{z}^2 \bar{T}\right) + \dots$$

[Gurarie, 1998]

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logarithmic partner t

$$\begin{aligned}
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\langle T(z) t(0,0)\rangle &= \frac{b}{z^4} \\
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\end{aligned}$$

b-number

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$$(h_X, \bar h_X)|_{c=0} = (2,0)$$

2-leg operator in O(n)
4-leg operator in Potts

Potts and O(n) logarithmic CFT

Potts and O(n) CFT is logarithmic at generic c

[Estienne, Ikhlef, 2015] [Gorbenko, Zan, 2020]
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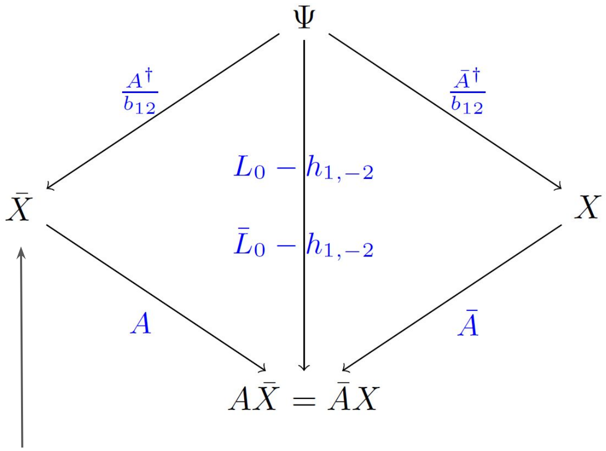

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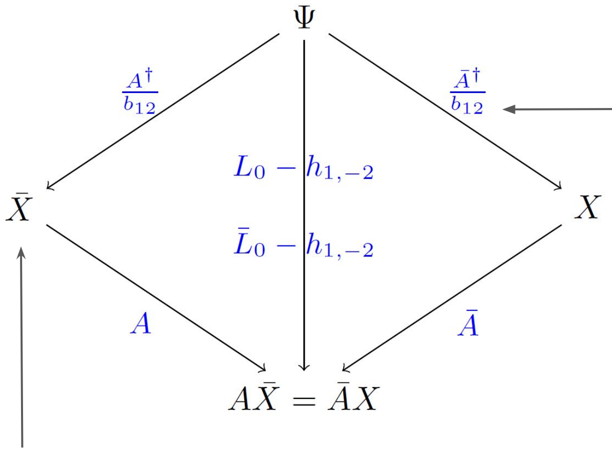
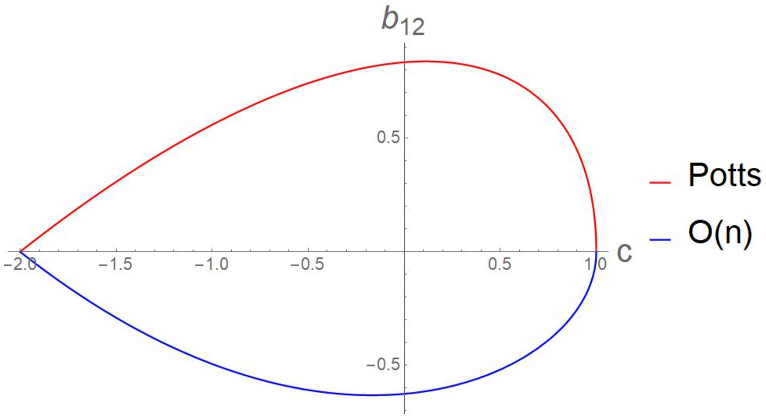


log coupling b_{12}

2-leg operator in O(n)
4-leg operator in Potts



c=0 log OPE and rank-3 Jordan cell

[YH, Saleur, 2021]

a generic c OPE in Potts or O(n):

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$$\Psi_0 = T\bar T$$

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$$a_0 = -\frac{25}{48}$$

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identical for percolation and polymers

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singularity cancellations

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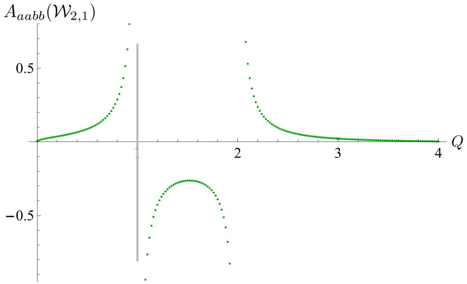
singularity cancellations $\Delta = \Delta_{\frac{1}{2},0}$ ✓



$$a_0 = -\frac{25}{48}$$

rank-3 Jordan cell

$$\Psi_0 = T\bar T$$

identical for percolation and polymers

Virasoro structure [YH, Saleur, 2021]

based on the log OPE: **conformal invariance + self-duality**

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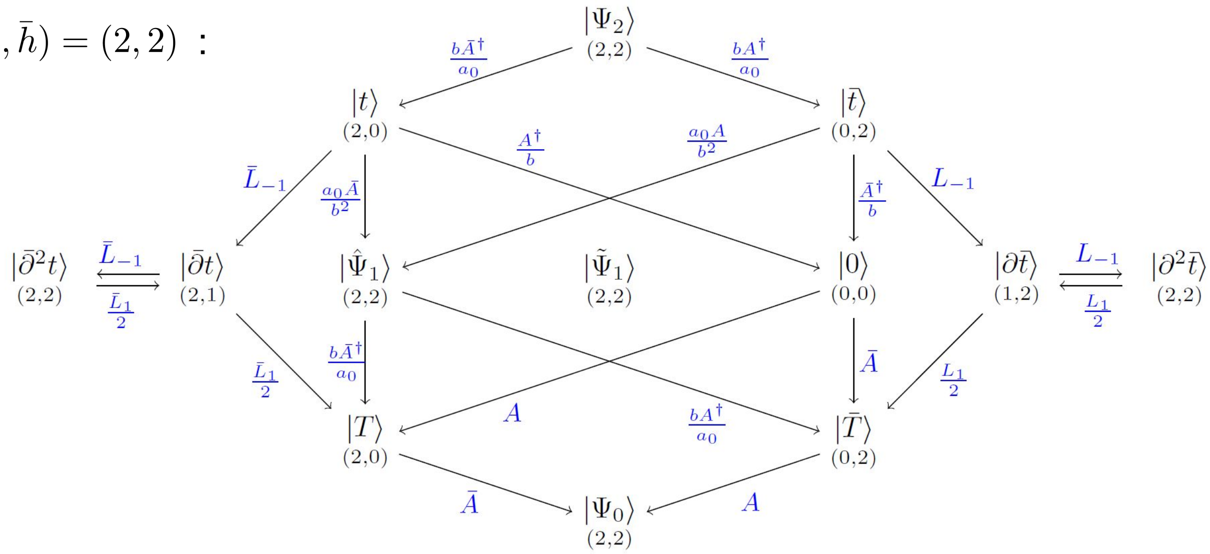

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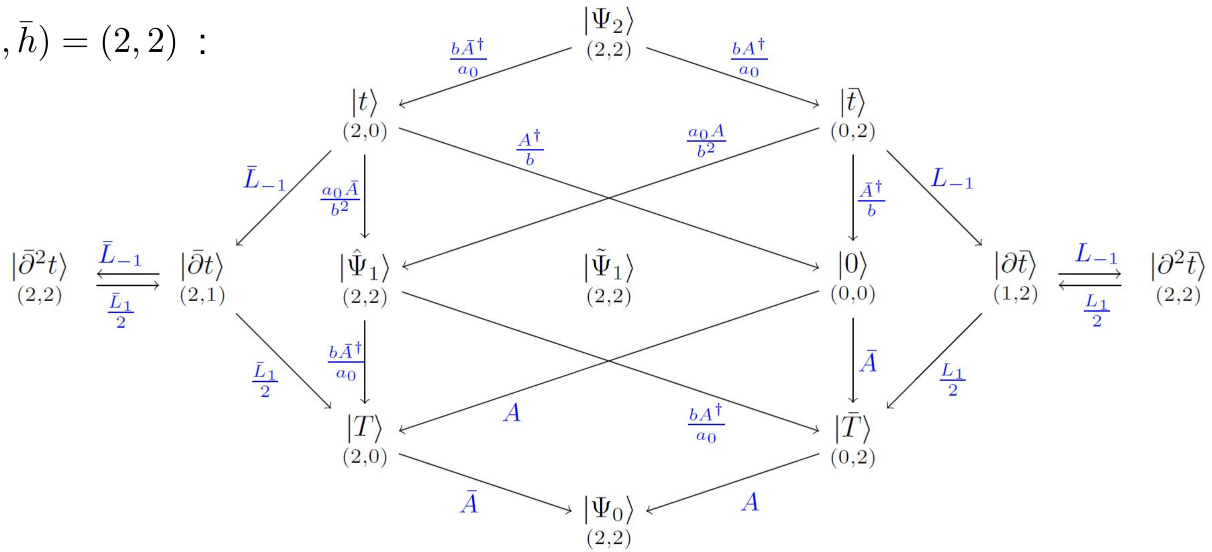

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different fields and log structures at generic c → *identical structure for percolation and polymers at c=0*

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  numerically determined four-point cluster connectivities

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  much more to understand about c=0 LCFT!


Champagne
Bon anniversaire Hubert!