

Rejoyce, the artist comes of age!

Hubert in the Third Millennium

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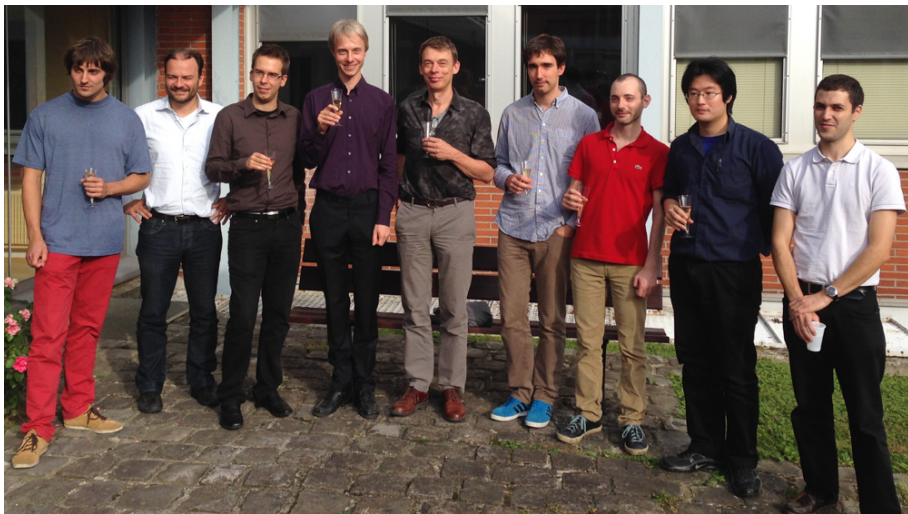
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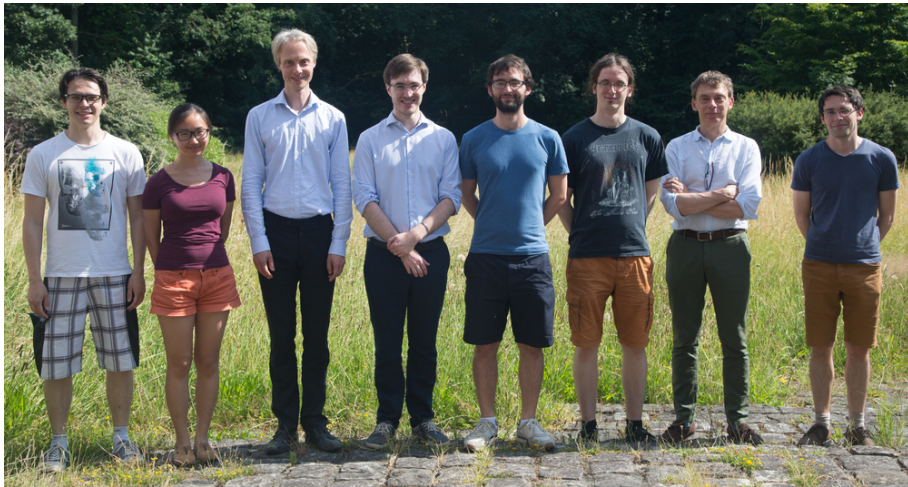
The Art of Mathematical Physics, 20 September 2021

- About 130 research papers published since 2000
- Moves back to France in 2003
- PhD students:
Francesco Siano (2001), Andrei Koutouza (2003),
Yacine Ikhlef (2007), Constantin Candu (2008),
Jérôme Dubail (2010), Roberto Bondesan (2012),
Romain Vasseur (2013), Eric Vernier (2015),
Romain Couvreur (2019), Etienne Granet (2019),
Niall Robertson (2020)
- Ongoing PhD students:
Linnéa Gräns Samuelsson (2022), Dmitry Chernyak (2023).
- Many postdocs (grants from USC, DOE, ANR, ERC)

27 September 2013



3 September 2019



The year 2001

- Wikipedia, Mac OSX, Windows XP, iPod, Xbox
- First Harry Potter movie
- Buddhas of Bamiyan destroyed by the Taliban
- September 11 attacks, US wages war in Afghanistan



- 2D non-linear σ -models with supermanifold target spaces
- Cosets of $U(n + m|n)$ and $OSp(2n + m|2n)$
- Bosonic and fermionic fields with “norm-one” constraint
- Lagrangian with kinetic and topological θ -term
- Phase transition at $\theta = \pi$, second order for $m \leq 2$
- Exact spectra (with multiplicities) obtained by using lattice models
 - Loop models for Q -state Potts and $O(m)$ vector models
 - Cancelling boson/fermion loops gives n -independent physics
- Coulomb gas methods, continuing [Di Francesco-Saleur-Zuber 86]
- Bonus: Number theoretical expression for multiplicities on torus

Integrability of supersymmetric spin chains (2000–07)

HS, *The continuum limit of $sl(N|K)$ integrable super spin chains*, Nucl. Phys. B **578**, 552–576 (2000)

- Bethe equations for these models with fermionic representations
- With full supersymmetry, the models are not even relativistic
- Maybe look at q -deformations, or alternating representations?

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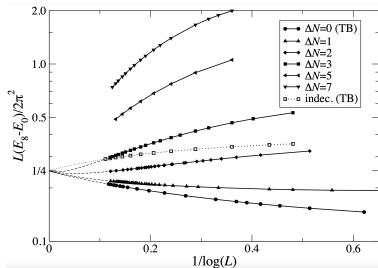
HS and B. Wehefritz-Kaufmann, *Integrable quantum field theories with $OSP(m|2n)$ symmetries*, Nucl. Phys. B **628**, 407–441 (2002)

- OSP models of Gross-Neveu and supersphere type
- Cf. $O(N)$ scattering matrix $\check{S} = \sigma_1 E + \sigma_2 P + \sigma_3 I$
- Birman-Wenzl algebras with SUSY generators
- In the graded case the S-matrix is **not unitary**
- Write and analyse TBA equation

JJ, N. Read and HS, *Dense loops, supersymmetry, and Goldstone phases in two dimensions*, Phys. Rev. Lett. **90**, 090601 (2003)

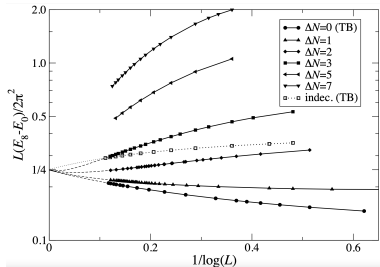
- Geometric interpretation of OSP models using **crossing loops**
- $O(N)$ model can have Goldstone broken-symmetry phase for $|N| < 2$, different from low-temperature phase without crossings
- An infinite family of critical exponents is **zero**, what is going on?

- Is the integrable $(3 \otimes \bar{3})^{\otimes N}$ model in the same universality class as the $(3 \otimes \bar{3})^{\otimes N}$ chain describing the spin quantum Hall effect?
- **No**, rather as the $SU(2|1)_1$ WZW model.
- It has a **continuous spectrum** of critical exponents.
- Also appearance of **arbitrarily large conformal weights**.
Non-normalisable states?



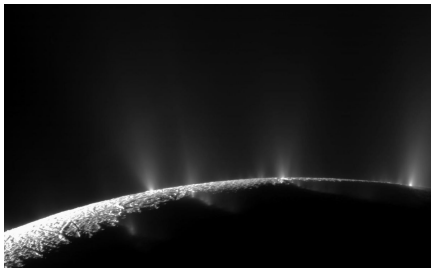
HS and V. Schomerus, *On the $SU(2|1)$ WZNW model and its statistical mechanics applications*, Nucl. Phys. B **775**, 312–340 (2007)

- Construct the WZNW model (state space and partition function)
- Draws on repr. theory of $SU(2|1)$ and their earlier work for $GL(1|1)$
- **Reducible yet indecomposable** reps. typical of logarithmic CFT
- Resolve several paradoxes found in the study of the $3\bar{3}$ chain: CFT involves a non-compact boson of radius $R \sim \sqrt{\log L}$



The year 2006

- Twitter, WikiLeaks, Nintendo Wii
- Sequencing of human genome completed
- Saddam Hussein sentenced and hanged
- Cassini-Huygens discovers water geysers on Saturn's Enceladus



Non-compact continuum limit in statphys (2006–now)

JJ and HS, *The antiferromagnetic transition for the square-lattice Potts model*, Nucl. Phys. B **743**, 207–248 (2006)

- Non-local definition $Z = \sum_{A \subseteq E} Q^{k(A)} (e^K - 1)^{|A|}$ for generic Q
- Cornucopia of different reps. (spins, cluster, height, vertex, ...)
- Two-boson continuum limit of which one is **non-compact**

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Y. Ikhlef, JJ and HS, *A staggered six-vertex model with non-compact continuum limit*, Nucl. Phys. B **789**, 483–524 (2007)

- Staggered model mapped to homogeneous 38-vertex model
- Study of numerical Bethe eqs. and a special $OSP(2|2)$ point
- Confirms non-compact limit with exponent gaps $\sim (\log L)^{-2}$

Y. Ikhlef, JJ and HS, *An integrable spin chain for the $SL(2, R)/U(1)$ black hole sigma model*, Phys. Rev. Lett. **108**, 081601 (2012)

- Wiener-Hopf analysis of the Bethe eqs. of staggered 6V model
- Confirms non-compact limit; gives access to **density of states**.
- Identification with Witten's Euclidean black-hole sigma model
- Improvement using NLIE by Ikhlef and Candu (2013)
- Confirmed by extensive study by Bazhanov, Kotousov, Koval, Lukyanov (2021), up to subtle detail about the signature

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More on non-compact continuum limits

- Model of polymer collapse (Izergin-Korepin model in regime III): **Same** continuum limit. [E. Vernier, JJ and HS (2014)]
- Integrable boundary conditions in AF Potts with non-compact limit: [N. Robertson, M. Pawelkiewicz, JJ and HS (2020–21)]
- Thermodynamic limit of $SL(2, \mathbb{C})$ non-compact spin chain: [E. Granet, JJ and HS (2020–22)]

Boundary conditions in loop models

JJ and HS, *Conformal boundary loop models*, Nucl. Phys. B **788**, 137–166 (2007)

- Bijection between bulk fields and conformal bcs in rational CFT
- In non-rational case, get a **continuous family of conformal bcs**
- Give specific weight to each boundary-touching loops
- Relation to “blob algebra” of Martin and Saleur (1994)

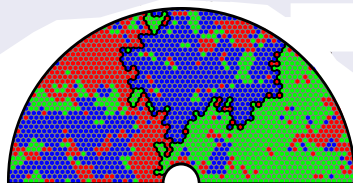
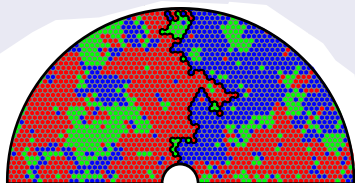
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Nice geometrical applications [J. Dubail, JS and HS (2009–10)]

- Two-boundary loop model. Thin and thick domain walls.



The year 2007

- iPhone, Windows Vista, Tumb
- Virginia Tech shootings
- Bulgaria and Romania join the European Union
- French TGV high-speed train runs at record 574.8 km/hour



Log-CFT and indecomposability in lattice models

James Bond: “Shaken, not stirred”

HS: “Reducible, yet indecomposable”

Log-CFT and indecomposability in lattice models

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N. Read and HS, *Associative algebraic approach to logarithmic conformal field theories*, Nucl. Phys. B **777**, 316–351 (2007)

- Enlarged symmetries and non semi-simple associative algebras in lattice models.
- Continuum limit of XXZ spin-1/2 chain with open (free) boundary conditions.
- Dense polymers ($c = -2$) with $gl(n|n)$ symmetry
- Percolation clusters ($c = 0$) with $gl(n+1|n)$ symmetry
- Bimodule picture: interplay between Temperley-Lieb and quantum group $U_q(sl_2)$
- Submodule structures of indecomposable Vir representations derived from lattice model

Giving flesh to this set of ideas:

- Numerical measure of indecomposability parameter b from lattice:
[J. Dubail, JJ and HS (2010)]
- Systematic derivation of such parameters in many theories:
[R. Vasseur, JJ and HS (2011)]
- Indecomposability in *bulk* percolation, also in higher dimension:
[R. Vasseur, JJ and HS (2012)]
- Measured numerically for $d = 3, 4, 5$ percolation.
- Rank-two Jordan cells in Q -state Potts models for *any* Q :
[L. Gräns-Samuelsson, L. Liu, Y. He, JJ and HS (2020)]

The year 2013

- Human embryonic stem cells by cloning
- Edward Snowden disclosures
- Euromaidan pro-EU demonstrations in Ukraine



Bulk LCFT from SUSY spin chains (2013–now)

A.M. Gainutdinov, N. Read and HS, *Continuum limit and symmetries of the periodic $gl(1|1)$ spin chain*, Nucl. Phys. B **871**, 245–288 (2013)

- TL replaced by bigger Jones-Temperley-Lieb algebra
- $U_q(sl_2)$ replaced by smaller $U_q^{\text{odd}}(sl_2)$
- Analysis of spin chain as bimodule over those two
- Identification of JTL elements that are precursors of $L_n, \bar{L}_n$

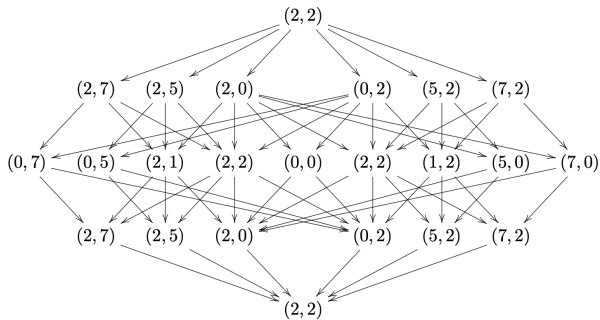
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A.M. Gainutdinov, N. Read, HS and R. Vasseur, *The periodic $sl(2|1)$ alternating spin chain and its continuum limit as a bulk logarithmic conformal field theory at $c = 0$* , JHEP 114 (2015)

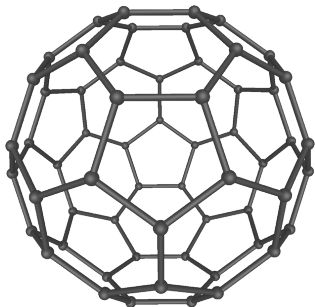
- Idem, using repr. theory for alternating $sl(2|1)$ chain
- Full structure of vacuum module
- Jordan cells of arbitrarily high rank



Correlation functions in loop models (2016–now)

- Three-point functions of “electric” operators in loop models.
Link to DOZZ structure constant from $c < 1$ Liouville theory.
[Y. Ikhlef, JJ and HS (2016)]
- Four-point functions of “order parameter” operator in Potts model.
Conjecture for spectrum of “exchanged” fields.
Rich spectrum need to cancel singularities for rational Q .
[JJ and HS (2019)]
- Interchiral conformal bootstrap
[Y. He, JJ and HS (2020)]

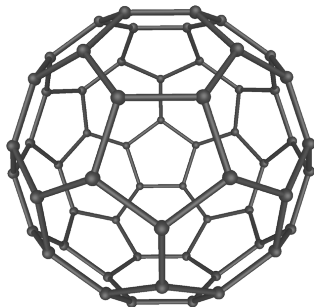
The magical number 60



1	11	21	31	41	51
2	12	22	32	42	52
3	13	23	33	43	53
4	14	24	34	44	54
5	15	25	35	45	55
6	16	26	36	46	56
7	17	27	37	47	57
8	18	28	38	48	58
9	19	29	39	49	59
10	20	30	40	50	



The magical number 60



𐤀 1	𐤁𐤀 11	𐤁𐤁 21	𐤁𐤂 31	𐤁𐤃 41	𐤁𐤄 51
𐤂 2	𐤂𐤀 12	𐤂𐤁 22	𐤂𐤂 32	𐤂𐤃 42	𐤂𐤄 52
𐤃 3	𐤃𐤀 13	𐤃𐤁 23	𐤃𐤂 33	𐤃𐤃 43	𐤃𐤄 53
𐤄 4	𐤄𐤀 14	𐤄𐤁 24	𐤄𐤂 34	𐤄𐤃 44	𐤄𐤄 54
𐤅 5	𐤅𐤀 15	𐤅𐤁 25	𐤅𐤂 35	𐤅𐤃 45	𐤅𐤄 55
𐤆 6	𐤆𐤀 16	𐤆𐤁 26	𐤆𐤂 36	𐤆𐤃 46	𐤆𐤄 56
𐤇 7	𐤇𐤀 17	𐤇𐤁 27	𐤇𐤂 37	𐤇𐤃 47	𐤇𐤄 57
𐤈 8	𐤈𐤀 18	𐤈𐤁 28	𐤈𐤂 38	𐤈𐤃 48	𐤈𐤄 58
𐤉 9	𐤉𐤀 19	𐤉𐤁 29	𐤉𐤂 39	𐤉𐤃 49	𐤉𐤄 59
𐤊 10	𐤊𐤀 20	𐤊𐤁 30	𐤊𐤂 40	𐤊𐤃 50	

