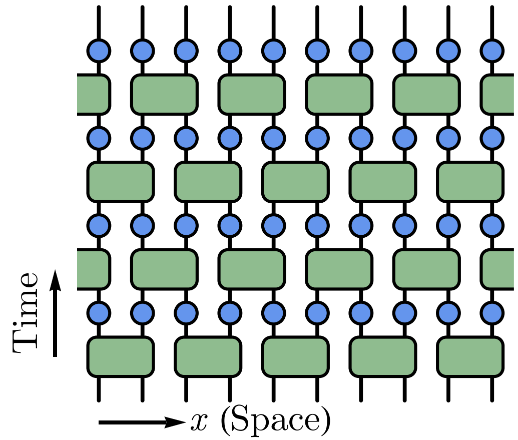
Measurement-induced criticality and c=0 LCFTs

Romain Vasseur

(UMass Amherst)

The Art of Mathematical Physics



RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig, PRB ‘19
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig, PRB ‘20
Zabalo, Gullans, Wilson, RV, Ludwig, Gopalakrishnan, Huse, Pixley, 2107.03393
Agrawal, Zabalo, Chen, Wilson, Potter, Pixley, Gopalakrishnan, RV, 2107.10279

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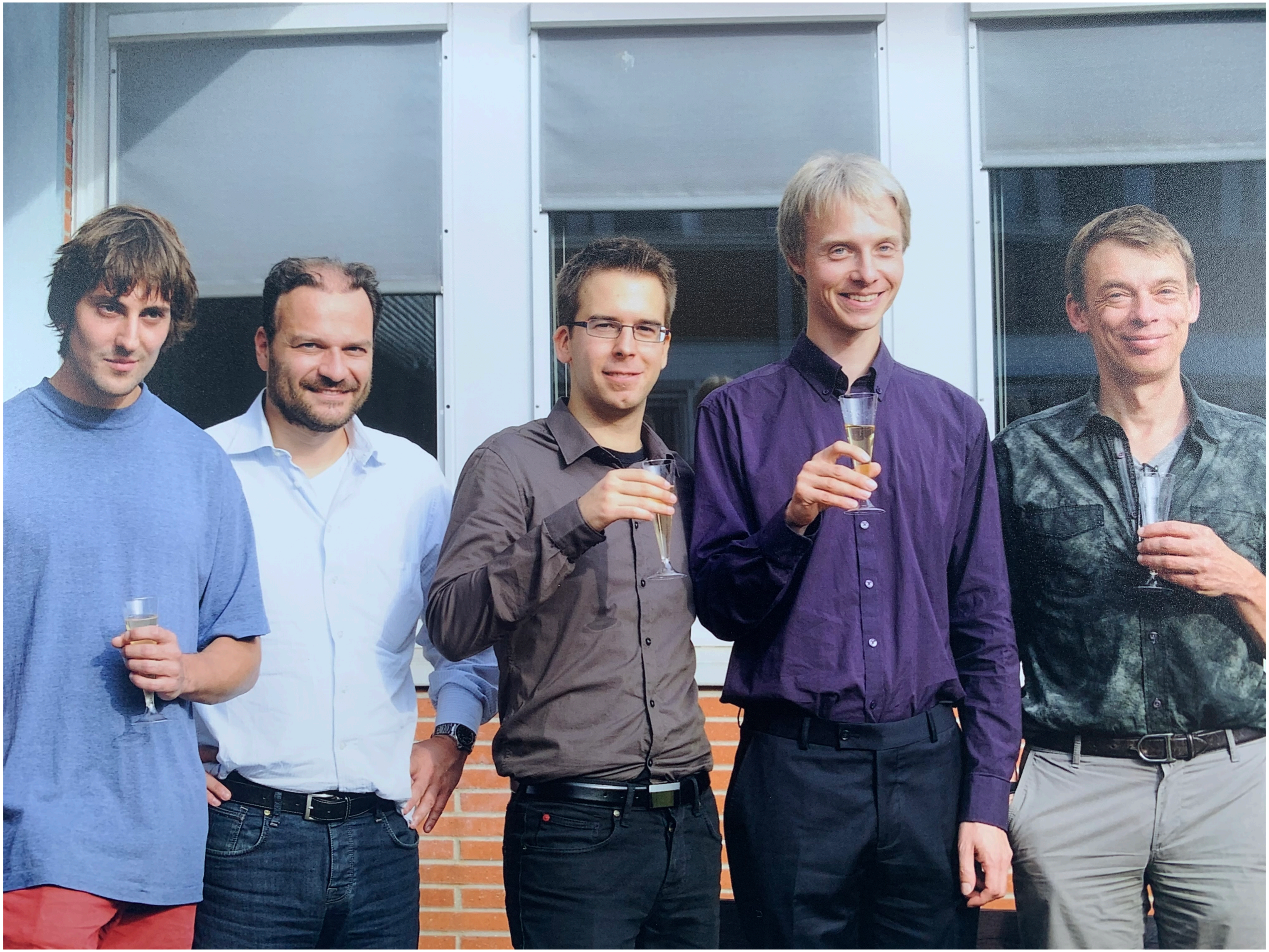
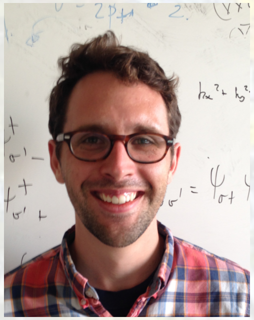
Happy Birthday Hubert!



Collaborators



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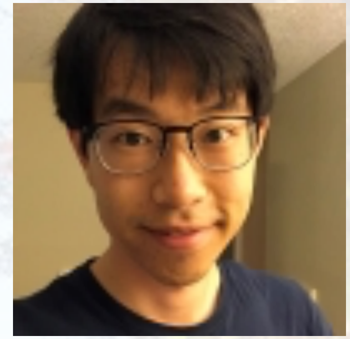
A.W.W. Ludwig
(UCSB)



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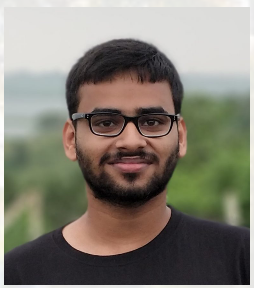

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M. Gullans
(UMD/NIST)

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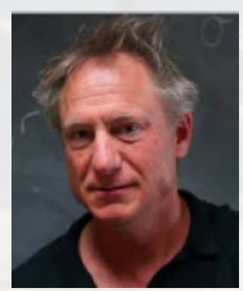
(Rutgers)



M. Fisher
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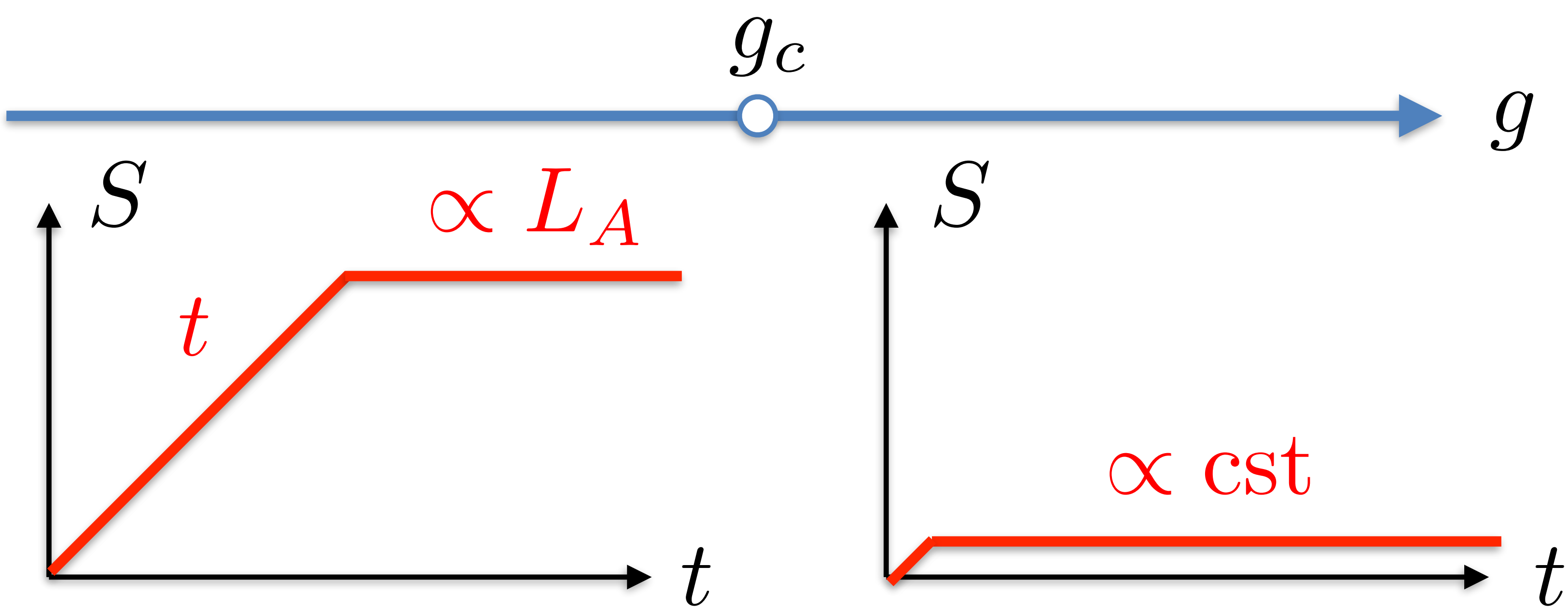
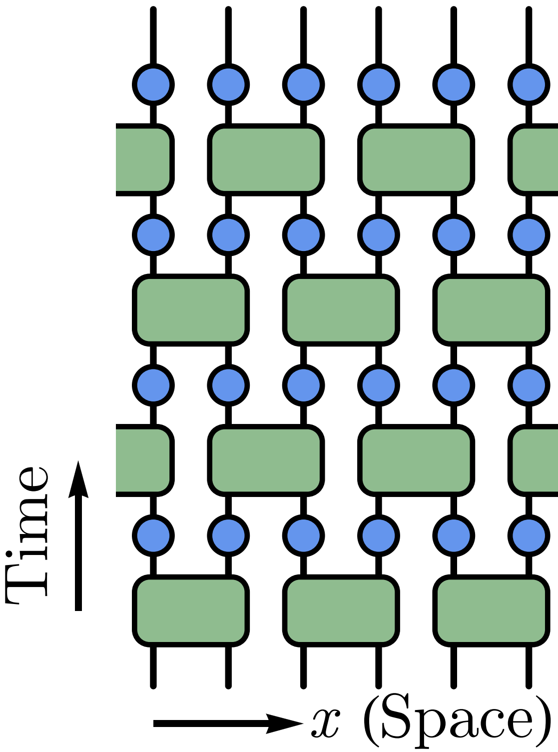
Dynamical and eigenstate "entanglement transitions"

- Can we slow down entanglement growth?
- Entanglement "phase transitions"? In eigenstates or dynamics



Measurement-induced transition

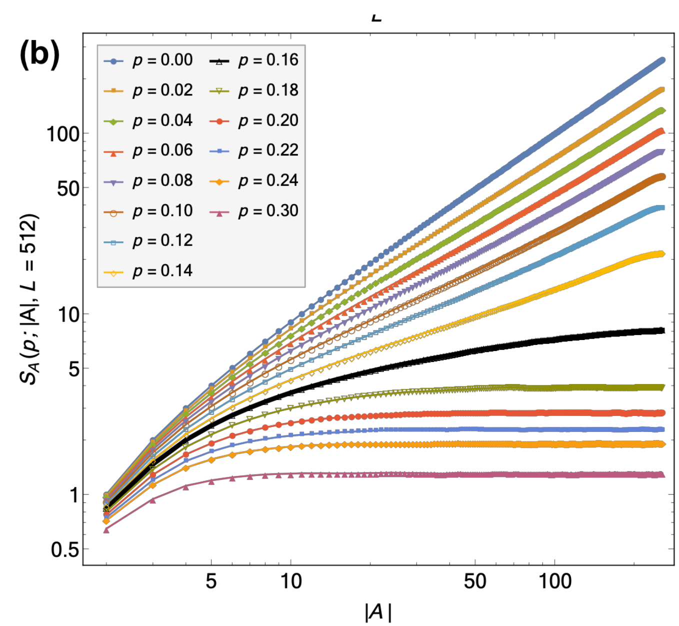


- Chaotic dynamics: Random unitary circuits (Haar or Clifford)

  Nahum, Vijay & Haah ’17, …

- Local projective measurements with probability “p”

Skinner, Ruhman & Nahum ’19
Li, Chen, Fisher ’19



Competition between scrambling/chaotic dynamics and disentangling measurements

Related to quantum error correction problem:

Gullans and Huse, Bao, Choi, Altman, …

Measurements: numerical results

- Second order transition in all Renyi entropies

$$S_n = \frac{1}{1-n} \log \mathrm{tr} \rho_A^n$$

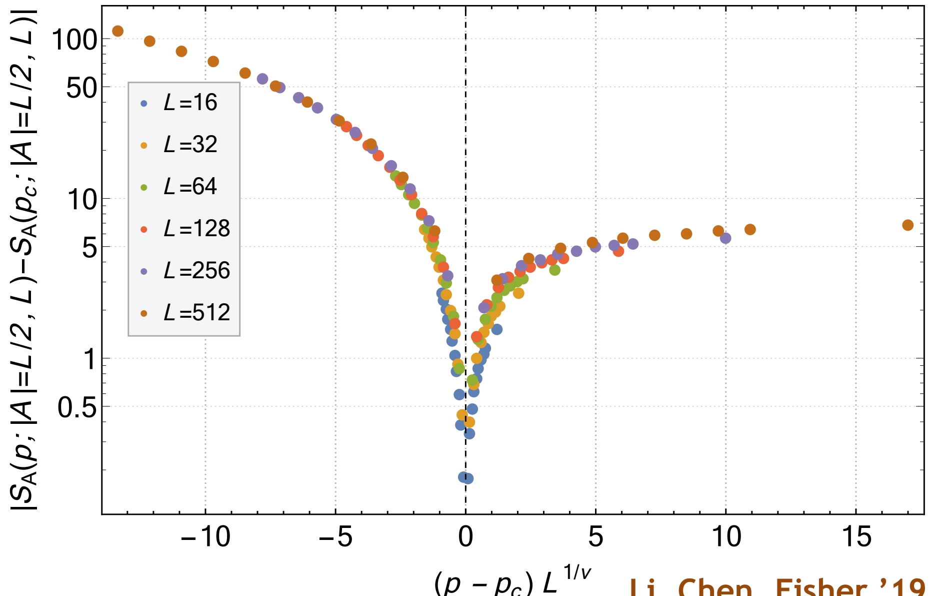
- Critical exponents (universality!)

$$\nu \simeq 1.3 \quad z = 1$$

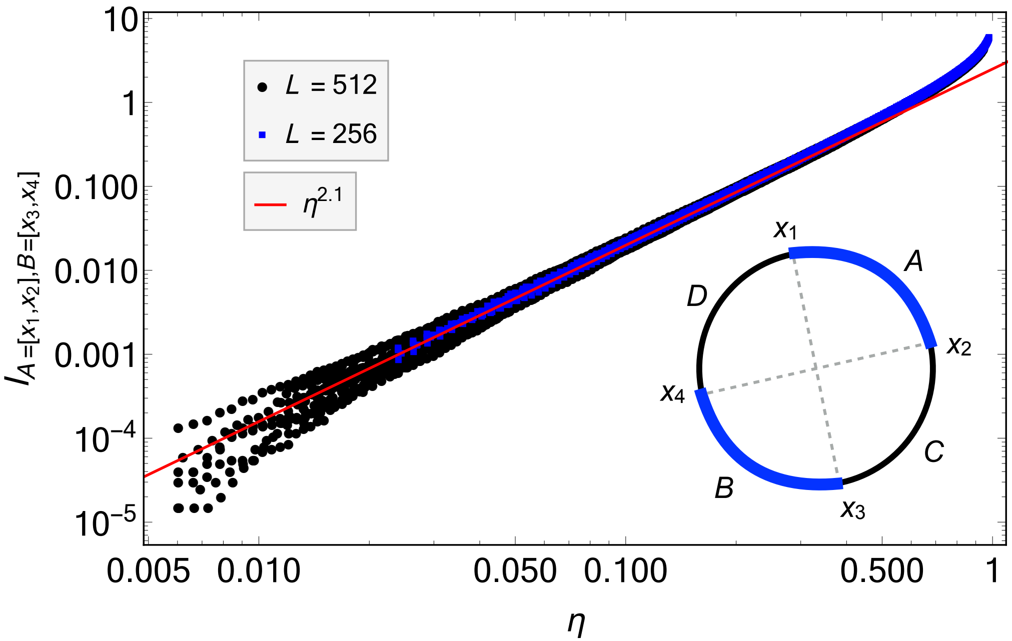
- conformal invariance (!)

Li, Chen, Ludwig, Fisher '19

$$I = S_A + S_B - S_{AB}$$
$$= f(\eta)$$



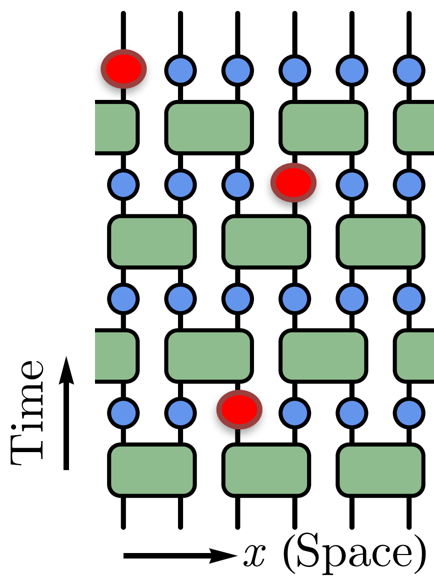
Li, Chen, Fisher '19



Replica trick and stat mech model

- How can we analyze this transition? Seems like a hard, non-linear problem…
- Need to compute entanglement entropy, averaged over Haar gates, measurement locations, and measurement outcomes.

For each circuit: quantum trajectories (with different measurement outcomes) are weighted by $p_m = \mathrm{tr}\rho$

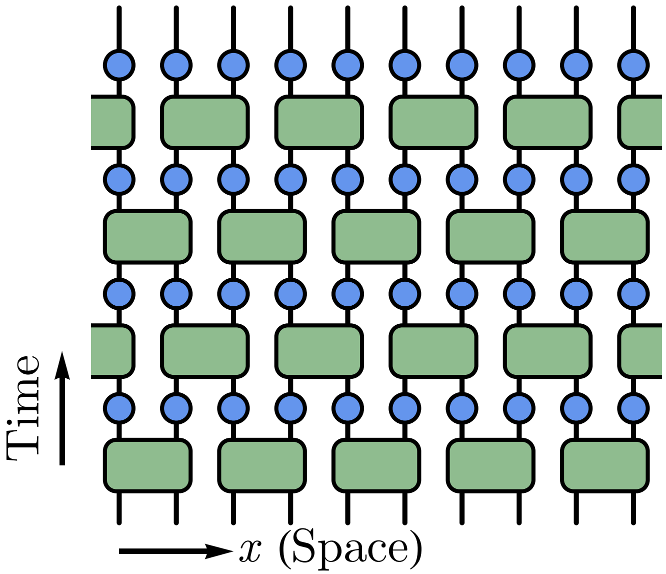


 $=$ Projective measurement

$$\sum_m p_m = 1 \quad \text{(Born rule)}$$

$$S_n = \left\langle \sum_m (\mathrm{tr}\rho) \frac{1}{1-n} \log \frac{\mathrm{tr}\rho_A^n}{(\mathrm{tr}\rho)^n} \right\rangle_{\mathrm{circuits}}$$

Replica trick and stat mech model

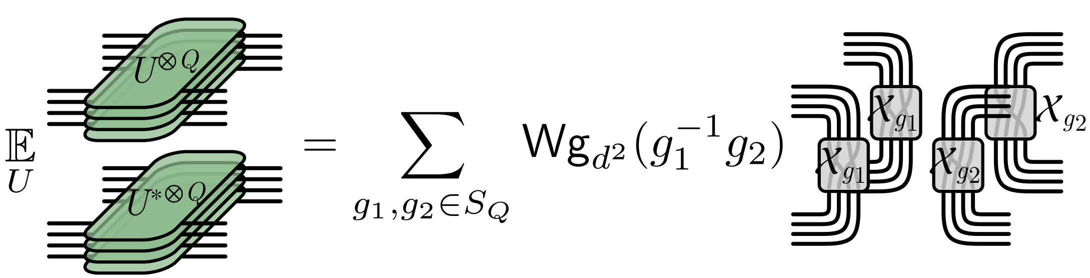


Replica trick:

$$\log \mathrm{tr}\rho_A^n = \lim_{k \to 0} \frac{\partial}{\partial k} (\mathrm{tr}\rho_A^n)^k$$

For n and k integers, average of $(\mathrm{tr}\rho_A^n)^k$

can be mapped onto the partition function of a 2D classical stat mech model defined on the circuit!



spins =

$$g_i \in S_{Q=nk+1}$$

Schur-Weyl duality: "spins" belong to commutant

Generalization to Clifford group: Li, RV, Fisher & Ludwig '21 (to appear)

Replica trick and stat mech model

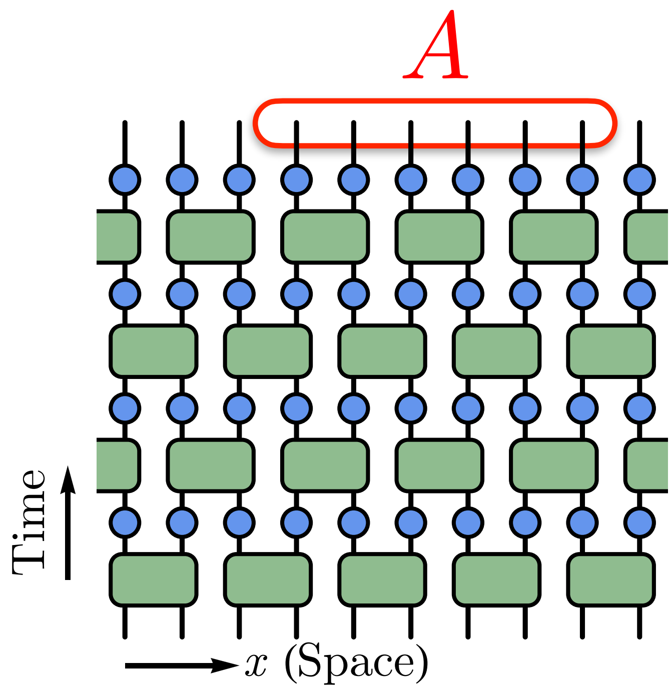
Partition function:

$$Z_0 = \left\langle \sum_m (\mathrm{tr}\rho)^Q \right\rangle_{\text{circuits}} = \sum_{\{g_i \in S_Q\}} \mathrm{e}^{-\mathcal{H}}$$

Entanglement entropy = free energy cost of boundary domain wall:

$$S_n = \frac{1}{n-1} \lim_{k\to 0} \frac{\partial}{\partial k}(F_A - F_0)$$

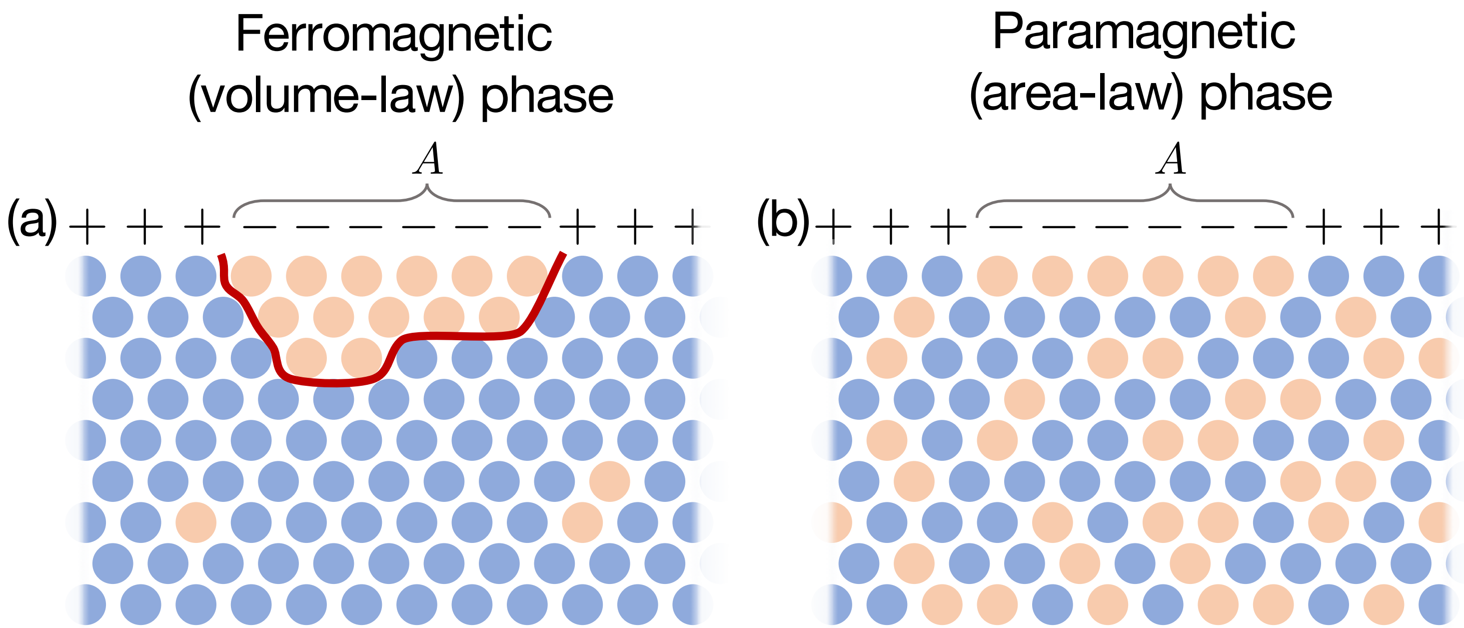
Z_A has boundary fields acting on A



Classical domain wall picture

RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig '19
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig '20
Bao, Choi & Altman '20

Figure from Bao, Choi and Altman



"Minimal cut", RT formula

p

$$F_A - F_0 \sim L_A \qquad\qquad F_A - F_0 \sim \mathrm{cst}$$

$$S_n = \frac{1}{n-1} \lim_{k \to 0} \frac{\partial}{\partial k} (F_A - F_0)$$

Entanglement ~ Free energy cost of domain wall

Consequences for scaling

- Entanglement transition —> simple ordering transition in 2D

- Naturally explains scaling properties near criticality:

$$F_A - F_0 = -\log\langle \phi_{\mathrm{BCC}}(L_A)\phi_{\mathrm{BCC}}(0)\rangle$$

two-point function of a boundary condition changing operator

$$S_n = \alpha_n \log L_A + f_n\left(\frac{L_A}{\xi}\right) + \ldots$$

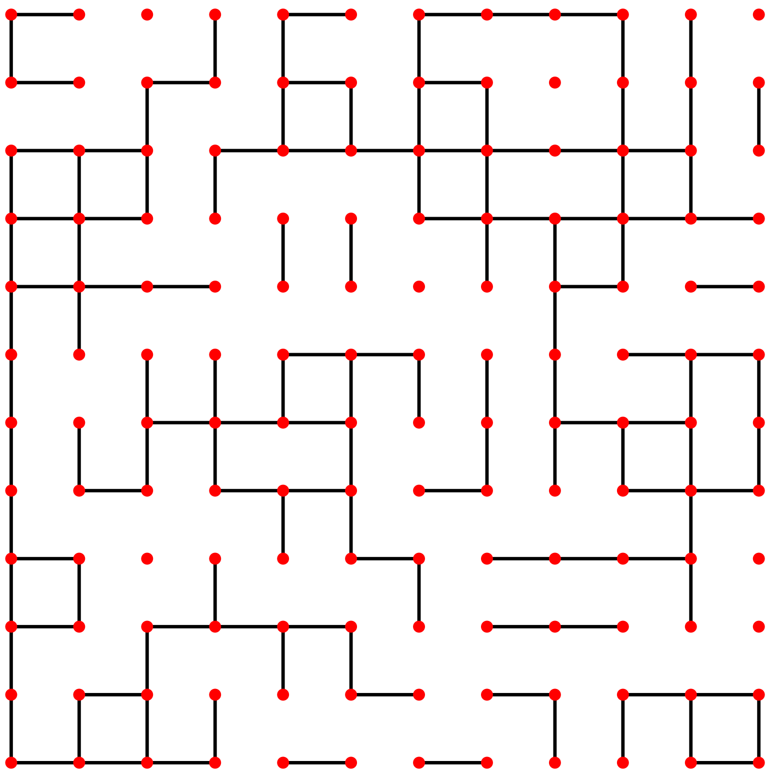
logarithmic critical behavior

- Mutual-information = 4-point function. Explains conformal invariance at criticality

Large on-site Hilbert space: Percolation

Especially simple for infinite on-site Hilbert space:

$$Z_0 = \sum_{\{g_i \in S_Q\}} \prod_{<ij>} \left(p + (1-p)\delta_{g_i,g_j}\right)$$



$Q!$ -state Potts model

Replica limit: $Q \to 1$

bond percolation

$$\xi \sim |p - p_c|^{-4/3}$$

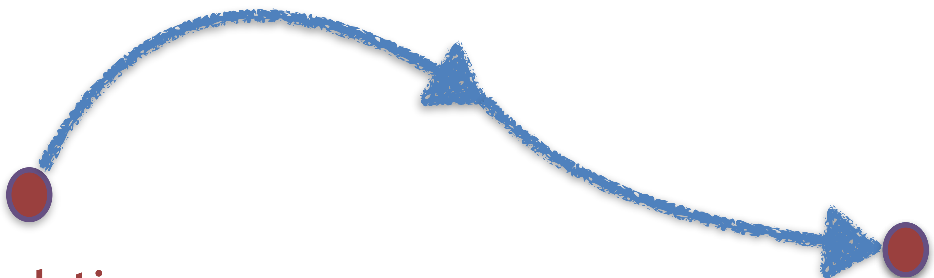
$$\bar{S}_{n,A} = \frac{1}{3} \log L_A + \ldots,$$

Percolation in S_0: Skinner, Ruhman & Nahum '19

Finite d universality class?

Infinite Hilbert space limit has accidental enlarged symmetry:

$$S_{Q!} \to S_Q \times S_Q$$



Percolation
$c = 0$

Finite Hilbert space transition
$c = 0$

CFT = IR fixed point of percolation + relevant perturbation

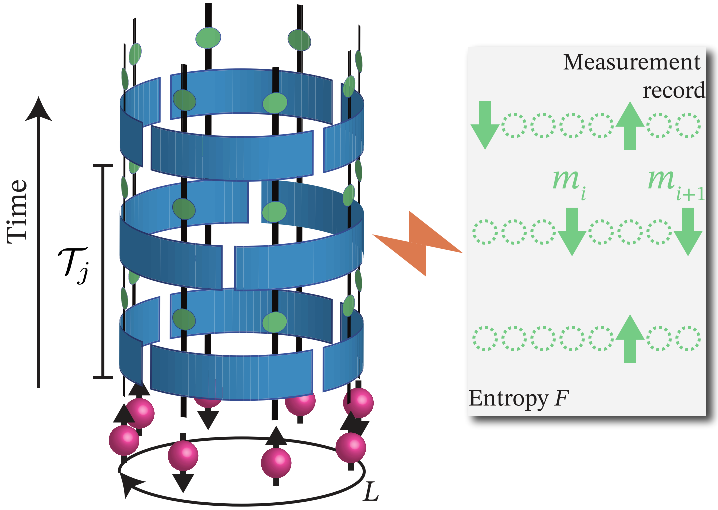
- **Field Theory**: $\mathcal{L} = \sum_{a=1}^{Q!} \frac{1}{2}(\partial \phi_a)^2 + \frac{m^2}{2} \sum_a \phi_a^2 + g \sum_a \phi_a^3 + \sum_{a,b \in S_Q} W(a^{-1}b)\phi_a \phi_b$

With $Q \to 1$

RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig, 1807.07082
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig, 1908.08051

Transfer matrix

Zabalo, Gullans, Wilson, RV, Ludwig, Gopalakrishnan, Huse, Pixley, 2107.03393

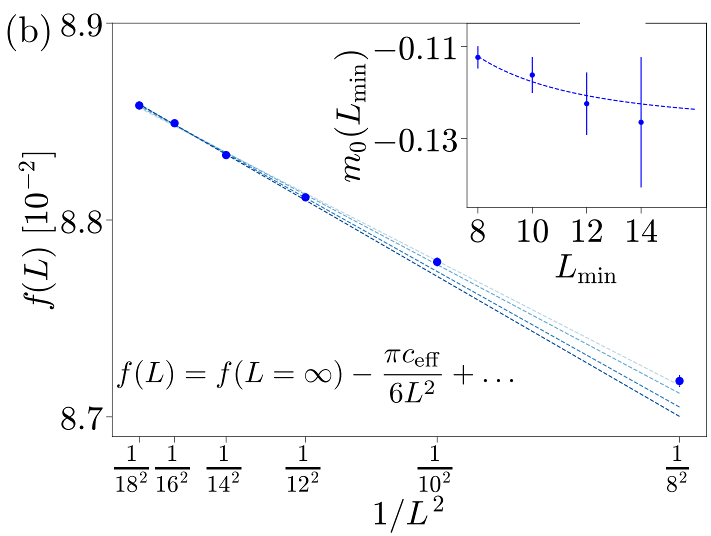


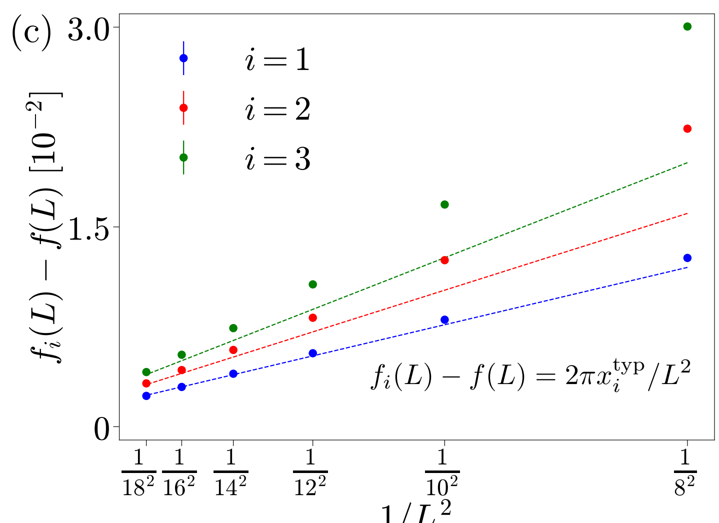
Free energy = entropy of measurement record:

$$F = -\sum_{\mathbf{m}} p_{\mathbf{m}} \ln p_{\mathbf{m}}$$

Higher Lyapunov exponents can also be extracted

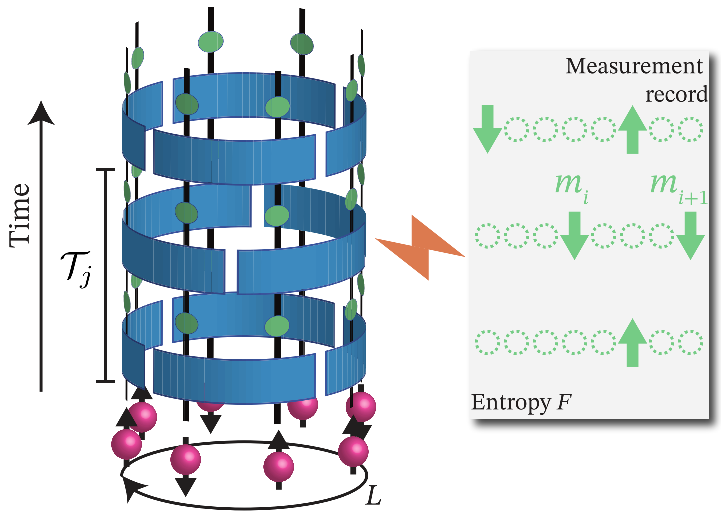
	Haar	Dual Unitary	Clifford	Dual Clifford	$d=\infty$ Haar/Clifford
c_{eff}	0.25(3)	0.24(2)	0.37(1)		0.2914/0.3652
x_1	$0.14(2)^{\dagger}$	$0.122(1)^{\dagger}$	0.120(5)	0.111(1)	0.1042
MF	✓	✓	×	×	×





Multifractality

Zabalo, Gullans, Wilson, RV, Ludwig, Gopalakrishnan, Huse, Pixley, 2107.03393



$$\overline{G_1(t)^n} \sim \exp\left[-2\pi t x_1(n)/L\right]$$

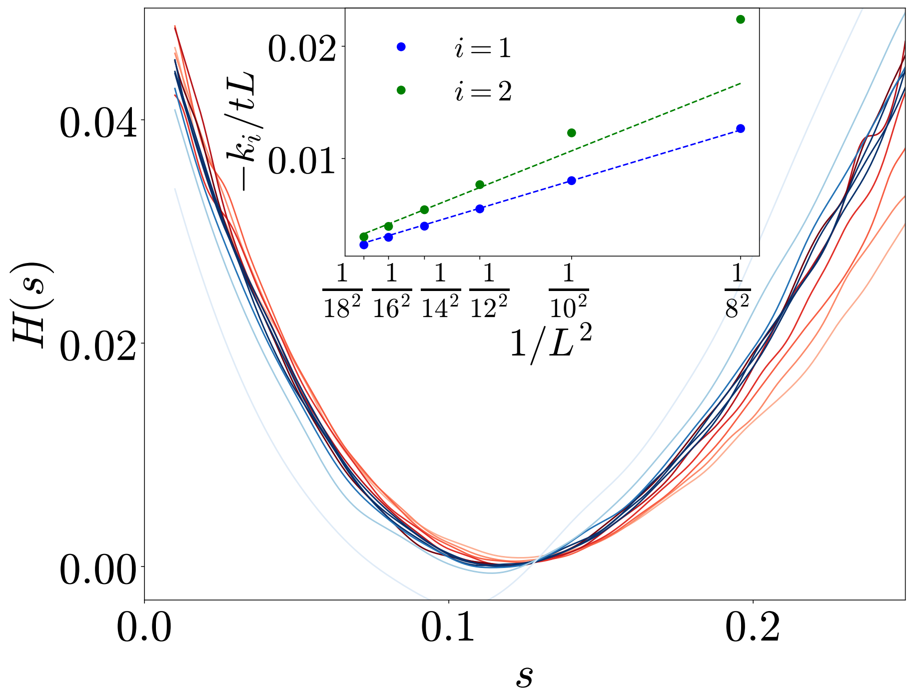
$$\ln \overline{G_1(t)^n} = n\overline{\ln G_1(t)} + \frac{n^2}{2!}\overline{\left(\ln G_1(t) - \overline{\ln G_1(t)}\right)^2} + \ldots,$$

Jacobsen, Cardy NPB '98

$$Y(t) \equiv -\ln G_1(t)$$

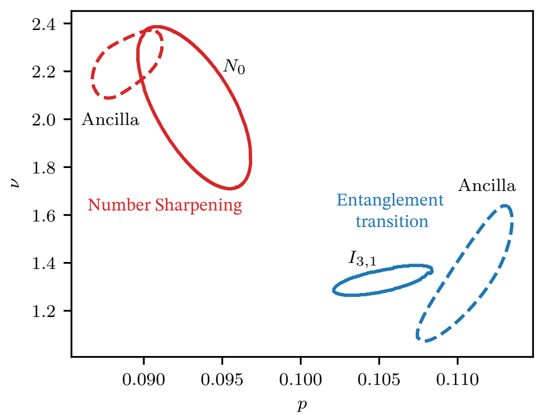
$$P\left[Y(t)\right] \sim \left(\frac{2\pi\alpha t}{L}\right)^{-1/2} \exp\left[-\frac{2\pi\alpha t}{L} H\left(\frac{Y(t)}{2\pi\alpha t/L}\right)\right]$$

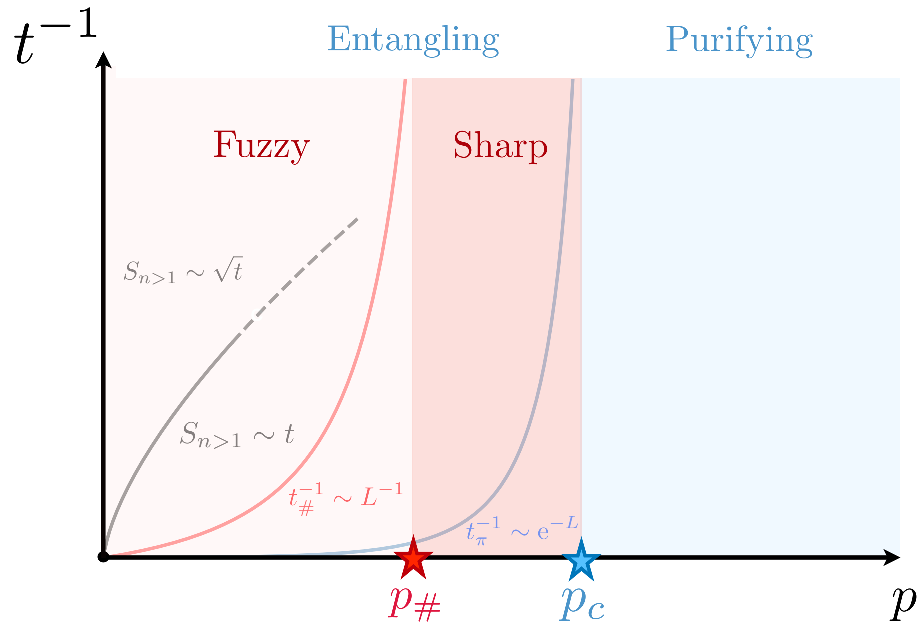
A.W.W Ludwig, NPB '90

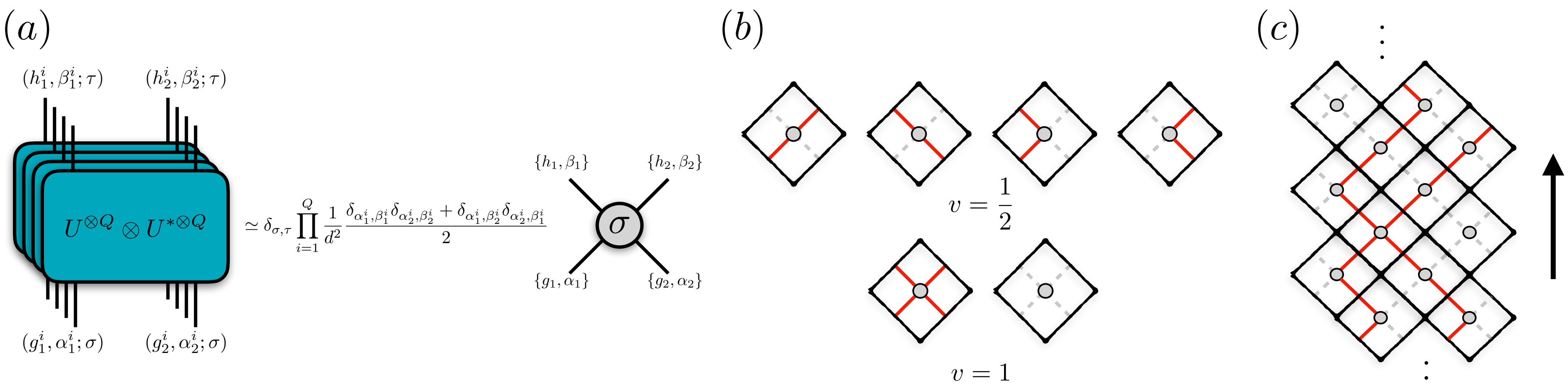


U(1) symmetry and charge sharpening

$$U_{i,i+1} = \begin{pmatrix} U^0_{d^2\times d^2} & 0 & 0 \\ 0 & U^1_{2d^2\times 2d^2} & 0 \\ 0 & 0 & U^2_{d^2\times d^2} \end{pmatrix}$$





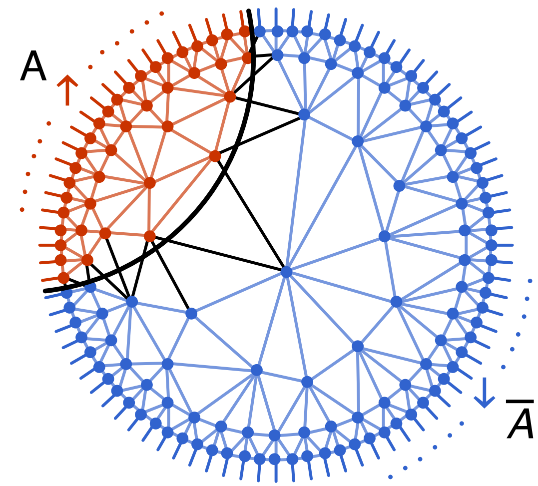


Agrawal, Zabalo, Chen, Wilson, Potter, Pixley, Gopalakrishnan, RV, 2107.10279

Conclusion

- New class of "entanglement transitions"
- Exact mapping onto classical stat mech model
- Analytic handle on field theory description of such entanglement transitions (can Hubert solve this c=0 LCFT?)
- Classification? Universality class? Experiments:

C. Noel et al (Monroe group), arXiv '21



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