

The art of mathematical physics



The art of mathematical physics



“Il faut traiter la nature par le cylindre, la sphère et le cône” . . .

The art of mathematical physics



*Il faut traiter la nature par le cylindre, la sphère et le **tore***

The art of mathematical physics

A Portrait of the Artist as a Young Man

J.-B. Zuber

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The art of mathematical physics

A Portrait of the Artist as a Young Man





A few reminiscences and a review of some papers in the period
1985–2000.

Large numbers . . .

- > 260 publications
- > 100 coauthors
- # students $\gg 1$, # postdocs $\gg 1$,
- 15163 citations according to Google Scholar

Large numbers . . .

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... (making the life of the reviewer (or the future biographer)
quite miserable :-)

How it got started

1985-1986 First papers with B. Derrida, on polymers, percolation, finite size scaling, transfer matrix calculations. . .

– March 1986, first joint paper with C. Itzykson and me,

EUROPHYSICS LETTERS
Europhys. Lett., 2 (2), pp. 91-96 (1986)

15 July 1986

Conformal Invariance of Nonunitary 2d-Models.

C. ITZYKSON, H. SALEUR and J.-B. ZUBER

Service de Physique Théorique, CEN-Saclay - 91191 Gif-sur-Yvette Cedex, France

(received 6 March 1986; accepted 25 April 1986)

PACS. 05.20. – Statistical mechanics.
PACS. 11.30. – Symmetry and conservation laws.
PACS. 64.60. – General studies of phase transitions.

Abstract. – Finite-size effects for the free energy as well as the spectrum of the transfer matrix are modified in a specific way for nonunitary conformal invariant 2d-systems. We illustrate this behaviour in the case of the Lee-Yang edge singularity.

The central charge c of the Virasoro algebra of conformal generators is a major characteristic of two-dimensional critical models in statistical mechanics and field theory as was discussed by BELAVIN, POLYAKOV and ZAMOLODCHIKOV [1]. For the so-called minimal degenerate models, *i.e.* those including a finite number of fundamental covariant conformal operators, these authors have obtained the values

$$c = 1 - \frac{6(p-p')^2}{pp'}, \quad c_{\text{eff}} = c - 24 h_{\min} \quad (1)$$

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1987–1989 : Hubert's anni mirabiles ?

several important papers

- $N \gg 1$ papers on “exact” critical exponents in polymers, percolation, [B. Duplantier]
 - 3 papers on Coulomb gas picture in stat. mech. and CFT (with P. Di Francesco and me)
 - BCFT and integrability [Saleur–Bauer]
 - $U_q(sl_2)$ quantum group with V. Pasquier
- and “*en même temps*” (at the same time), lectures on Yang–Baxter Integrability, etc
and **PhD thesis** (1987), and a **book** (1988)...

**CONFORMAL INVARIANCE
AND
APPLICATIONS TO
STATISTICAL MECHANICS**

Editors
**Claude Itzykson
Hubert Saleur
Jean-Bernard Zuber**

World Scientific

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and

1988 : **Les Houches Summer School . . .**

champs, cordes et
phénomènes critiques

Les Houches 1988



Session XLIX

fields, strings and
critical phenomena

E. Brézin and J. Zinn-Justin

Editors

North-Holland



PDF, HS and JBZ on top of the Prarion (alt. 1969 m), summer 1988

[Courtesy of Philippe DF]

A few selected papers

- * Polymers and percolation
- * Coulomb gas picture
- * $U_qsl(2)$ quantum group
- * BCFT $\leftrightarrow$ Integrability

important papers with a long filiation . . .

A few selected papers

* Polymers (SAW), and percolation : [Duplantier]

From half-integer indices in Kac' formula (1987) ... to $N = 2$ supersymmetry (1991)

In Polymers or Percolation: magn. exponents $x_{H_t} = 2h_{\frac{3}{2}, t+\frac{1}{2}}$, [Dotsenko–Fateev],
thermal exp. $x_{T_t} = 2h_{1, t+1}$ with $h_{r,s} = ((3r - 2s)^2 - 1)/24$

but **also** the *exact* fractal dimension of the external perimeter (or “hull”) of the infinite percolation cluster: $D_H = 7/4$ or of the Ising clusters $D_F = 198/96$,
and the *exact* values of the tricritical exponents of a collapsing polymer in two dimensions, (“ Θ point”)

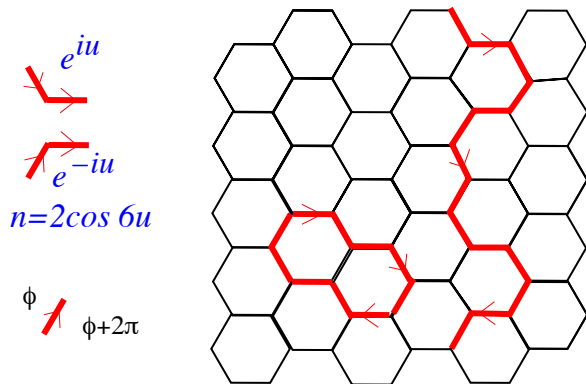
and *exact* critical properties of two-dimensional *dense* self-avoiding walks,
and *exact* surface or wedge exponents, etc etc.

1991 Show that a large class of geometrical critical systems including dilute polymers, polymers at the Θ -point, percolation etc, are described by a **twisted** $N=2$ supersymmetric theory.

A few selected papers

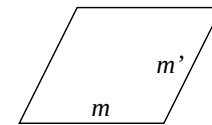
- * Polymers, SAW, and percolation
- * **Coulomb gas picture** [Di Francesco-Saleur-Z] 2D-lattice (vertex or height) models $\rightarrow$ loop models $\rightarrow$ Coulomb gas and free field [Kadanoff, Nienhuis,...]
special weights to non contractible loops $\rightarrow$ background charges e_0 (“charges at infinity” [Dotsenko-Fateev] or “floating charges”).

$O(n)$ model on honeycomb lattice



For a field ϕ compactified on a circle

$$Z = \sum_{m,m'} Z_{mm'} \quad (c = 1)$$



$$\rightarrow Z = \sum_{m,m'} \cos e_0 \langle m, m' \rangle Z_{mm'} \quad (c < 1, \quad 2 \cos \pi e_0 = n)$$

and this extends to all minimal models

Attaching different weights to different classes of loops : a recurrent theme in Hubert’s work,
see [Read-S ’01], [Dubail-Jacobsen-S ’08-’09] etc

A few selected papers

- * Polymers, SAW, and percolation
- * Coulomb gas picture
- * $U_qsl(2)$ quantum group [Pasquier-Saleur '88-89]

The XXZ chain Hamiltonian with free b.c. and a special boundary field

$$H = \sum_1^{N-1} \left(\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1} + \frac{q+q^{-1}}{2} \sigma_z^i \sigma_z^{i+1} \right) + \frac{q-q^{-1}}{2} (\sigma_z^1 - \sigma_z^N) = \sum_1^{N-1} e_i + \text{const.}$$

under action of $U_qsl(2)$.

(Temperley-Lieb alg = commutant of $U_qsl(2)$)

- q not a root of unity ... ~ classical ($q = 1$) case
- $q = e^{i\pi/p}$: new phenomena : indecomposable repns appear in tensor product $(\frac{1}{2})^{\otimes N}$ etc, while irreps have a spin $0 \leq j \leq (p-2)/2$.

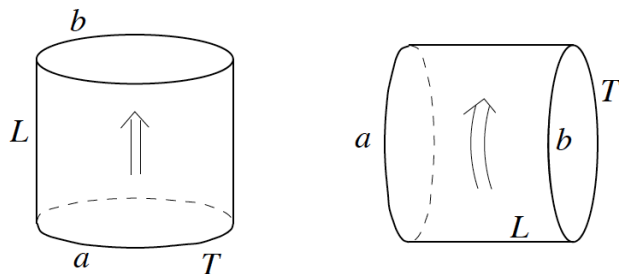
Spectrum of H (by Bethe Ansatz) reproduces in the continuum limit the conformal towers!

Thus a new connection between CFT and Integrable Models...

... and a long and diversified posterity of those papers...

A few selected papers

- * Polymers, SAW, and percolation
- * Coulomb gas picture
- * $U_qsl(2)$ quantum group
- * **BCFT $\leftrightarrow$ Integrability** [Saleur–Bauer '88; Affleck–Oshikawa–Saleur '98]



$$Z_{ab} = \sum_i n_{ia}^b \chi_i(q), \quad q = e^{-\pi T/L}$$

The n_{ia}^b determined for several models (Ising, Potts, ADE Pasquier models) and some b.c., by Bethe Ansatz, Coulomb gas, or transfer matrix calculations. Observe the matrices n_i form a representation of the Fusion Algebra ([PDF-JBZ '89] $\rightarrow$ [Behrend-Pearce-Petkova-Z '98] !)

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- 1987–1989 Hubert’s anni mirabiles

- 1991 exile to the US (Yale and USC) . . .

a great loss for Saclay but. . .

1991–2000: a very productive decade

Representation theory: quantum groups, SCFT, Virasoro, Temperley–Lieb and avatars [Martin–S, Koo–S] . . .

susy $N\mathcal{L}\sigma\mathcal{M}$ [Read–S] and conformal properties; $U(1,1)$ supersymmetric WZW and Chern–Simons theories and knot theory [Rozansky–S '92] . . .

Boundary (conformal) field theory – Integrability away from criticality, Massless and massive flows, S-matrices.

with [Leclair, Mussardo, Skorik, Warner, Zamolodchikov. . .]

Applications to cond. matt., Kondo problem, FQHE and conductance problems, . . . [Fendley–Lesage–Ludwig–S]

. . .

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Representation theory: quantum groups, SCFT, Virasoro, Temperley–Lieb and avatars [Martin–S, Koo–S] . . .

susy $N\bar{L}\sigma M$ [Read–S] and conformal properties; $U(1,1)$ supersymmetric WZW and Chern–Simons theories and knot theory [Rozansky–S '92] (indecomposable representations. . .) . . .

Boundary (conformal) field theory – Integrability away from criticality, Massless and massive flows, S-matrices.

with [Leclair, Mussardo, Skorik, Warner, Zamolodchikov. . .]

Applications to cond. matt., Kondo problem, FQHE and conductance problems, . . . [Fendley–Lesage–Ludwig–S]

. . . **and also** Scaling in Earthquake physics ! [Saleur–Sornette et al]

[From Wiki]

In Chinese culture, 60 years marks one full cycle of life and the 60th birthday is regarded as a very important point as the beginning of a new life cycle. Traditionally, it is the first birthday in a person's life to be marked with a big celebration. And after that, big birthday celebrations are held every 10 years.

[. . .]

Bon Anniversaire, Hubert and Many happy returns



avec toute mon amitié