Plasma Analogy and non-Abelian Braiding Statistics in Ising-type Quantum Hall States

Parsa Bonderson

Microsoft Station Q

CFT, Topology, and Information Workshop

Institut Henri Poincare

November 4, 2011

work done in collaboration with:

V. Gurarie and C. Nayak, Phys. Rev B **83**, 075303 (2011) [arXiv:1008.5194]

E. Herland, E. Babaev, V. Gurarie, C. Nayak, and A. Sudbo [arXiv:1111.0135]

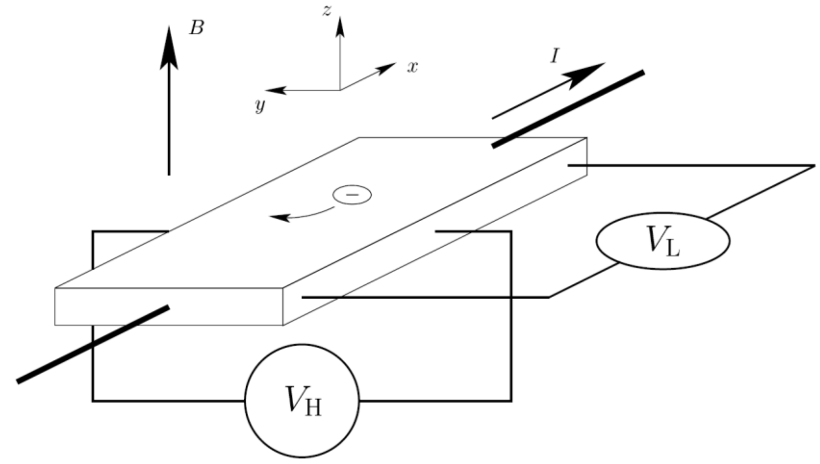
Introduction

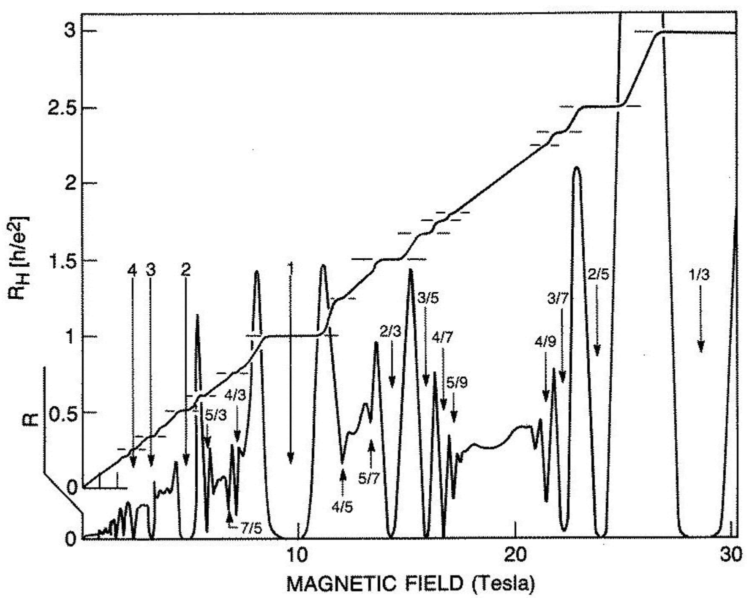
- Quantum Hall systems are topologically ordered!
- ν=5/2, 12/5,… may be non-Abelian!
- Non-Abelian states are a new phase of matter and may have technological application as a topologically-protected media for quantum information processing!!
- But why do we believe this?

Fractional Quantum Hall Effect

- 2DEG
- large B field (~ 10T)
- low temp (< 1K)
- gapped (incompressible)
- quantized (rational) filling fraction plateaus:

$$\nu = \frac{n}{m}, \quad R_{xy} = \frac{1}{\nu}\frac{h}{e^2}, \quad R_{xx} = 0$$

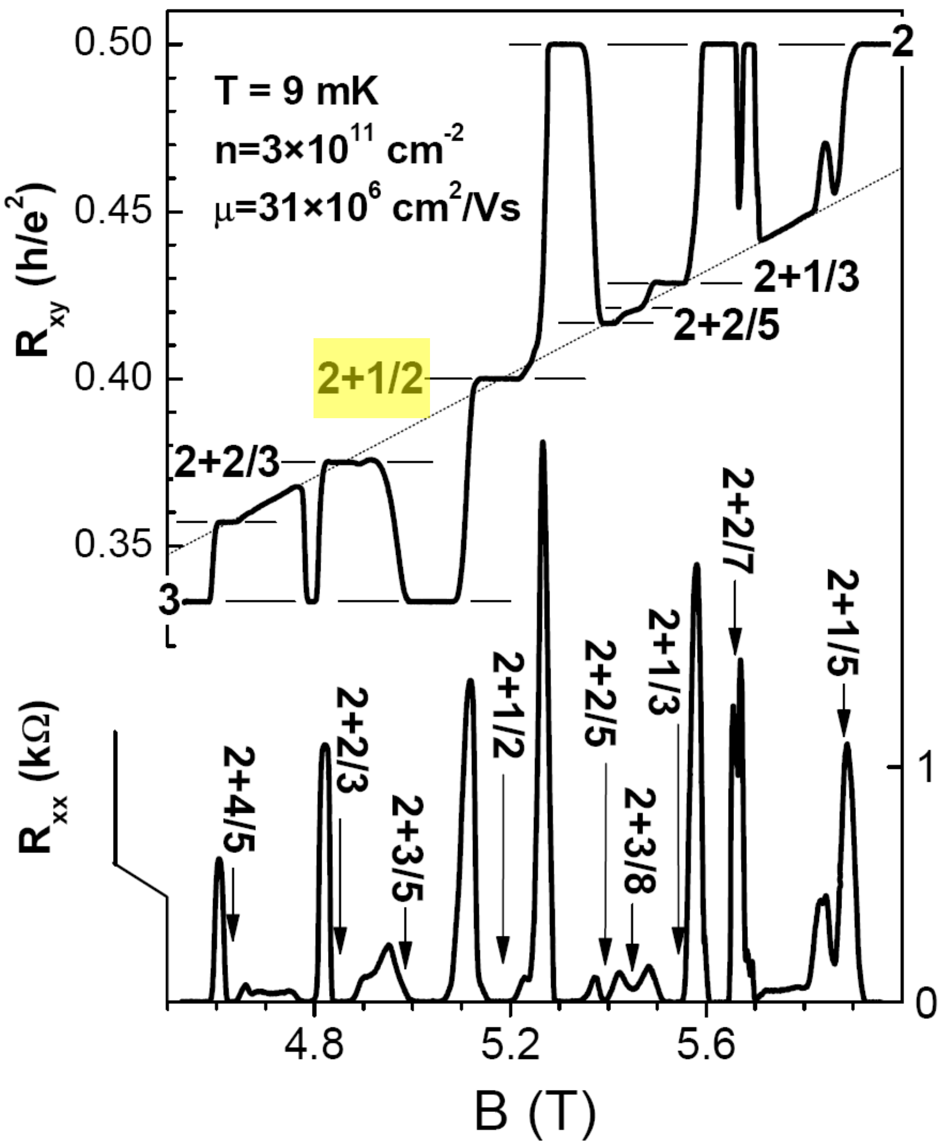


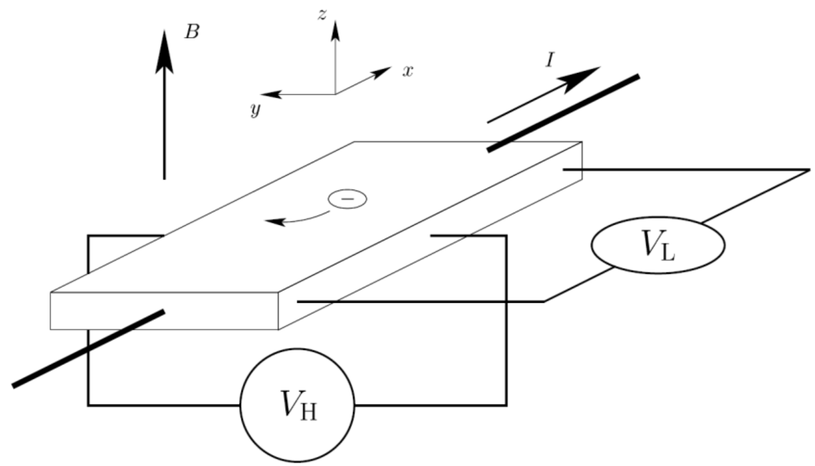


- fractionally charged quasiparticles
- Abelian anyons at most filling fractions $\theta = \pi \frac{p}{m}$
- non-Abelian anyons in 2^{nd} Landau level, e.g. ν= 5/2, 12/5, …?

Fractional Quantum Hall Effect

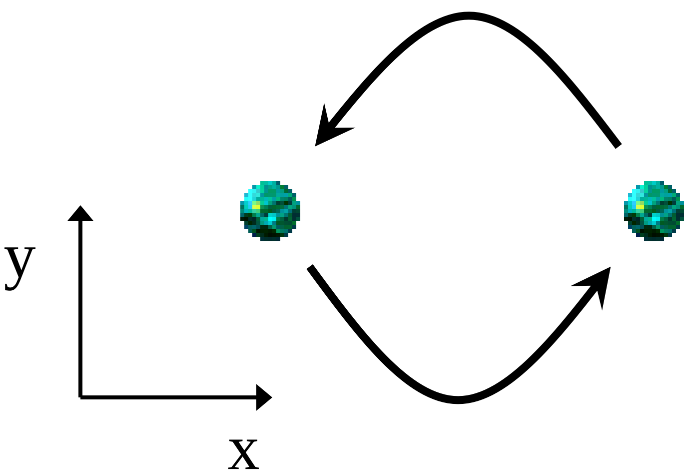
Xia et al, Fig.1





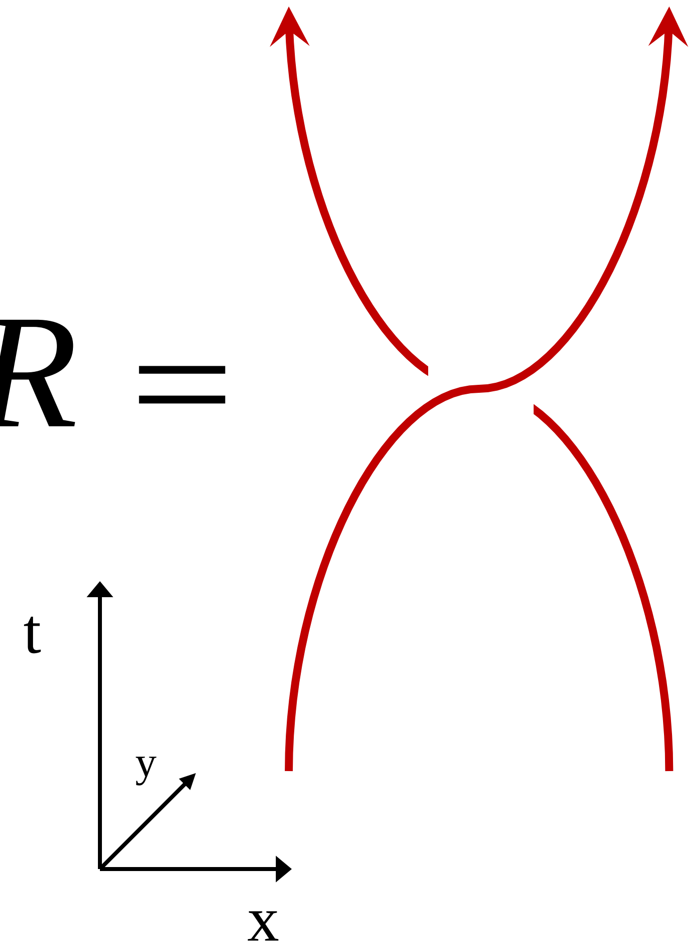
- fractionally charged quasiparticles
- Abelian anyons at most filling fractions $\theta = \pi \frac{p}{m}$
- non-Abelian anyons in 2^{nd} Landau level, e.g. ν= 5/2, 12/5, …?

Particle Exchange “Statistics”

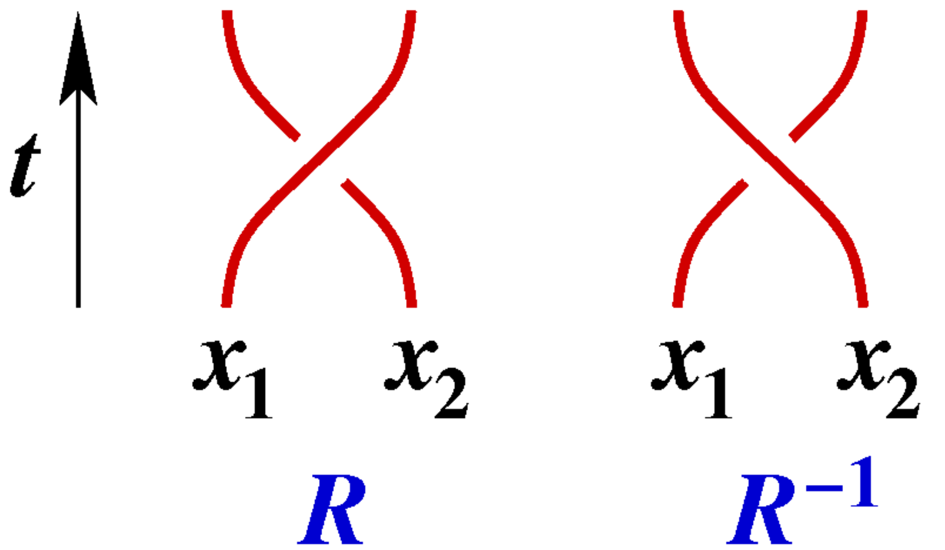


$$|\Psi\rangle \mapsto U[R]|\Psi\rangle$$

$R =$



Particle Exchange "Statistics"

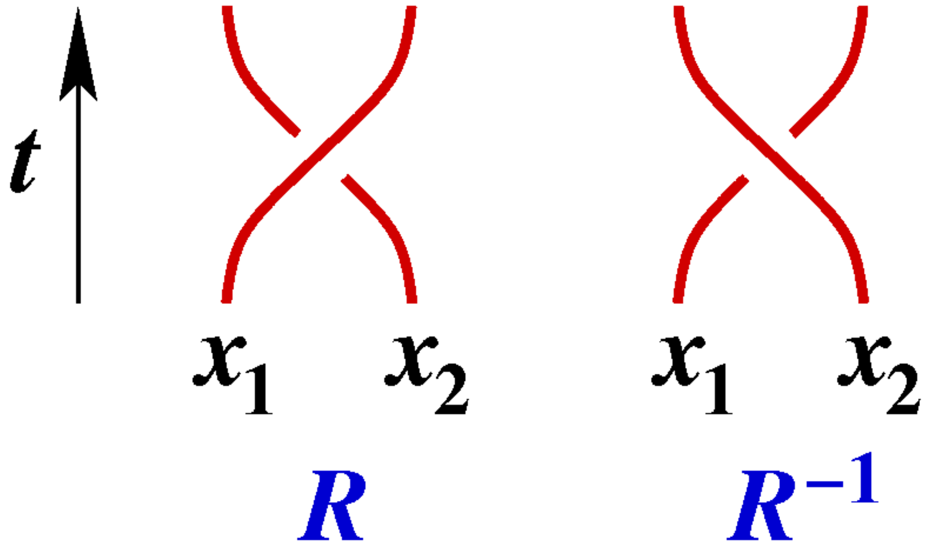


3 (and higher) spatial dimensions:

$$R = R^{-1} \quad \text{and} \quad R^2 = \mathbf{1}$$

- Only initial and final positions are topologically distinguished
- Statistics characterized by permutation group S_n
- Bosons and Fermions

Particle Exchange "Statistics"

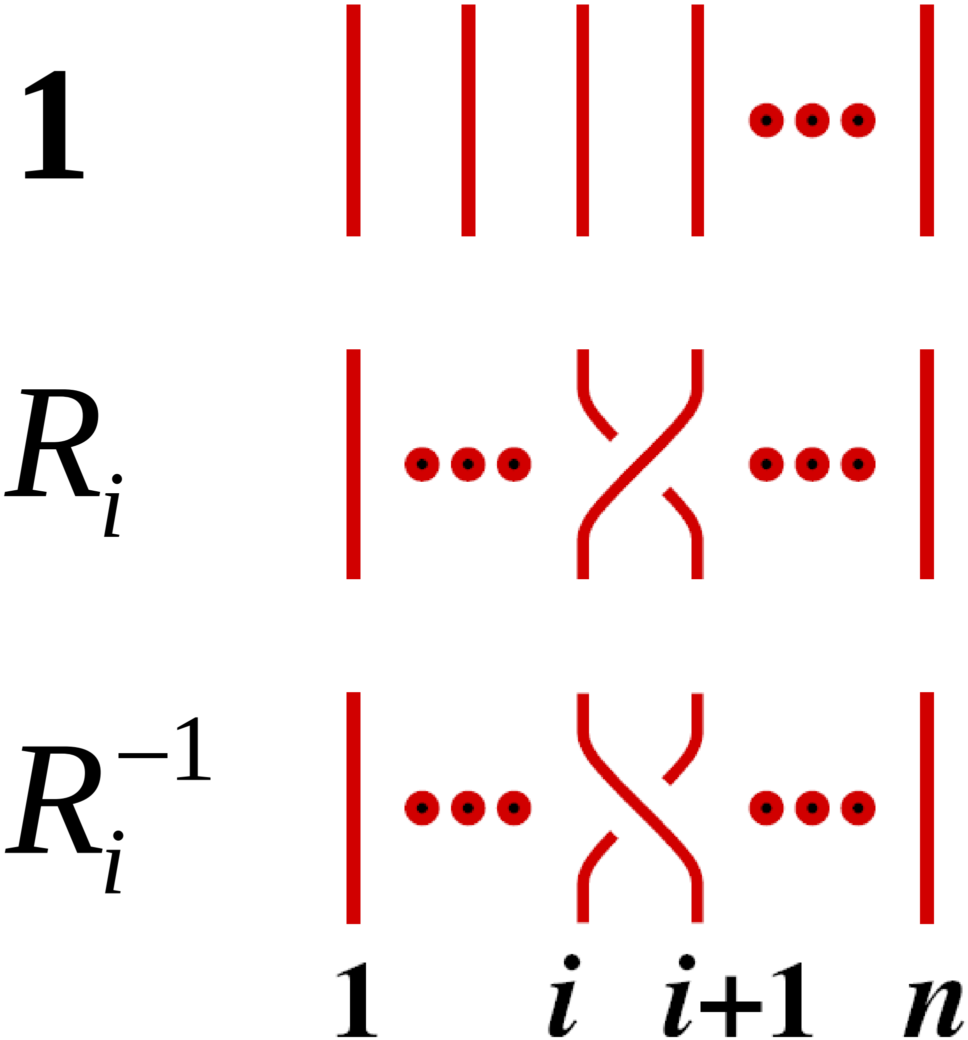


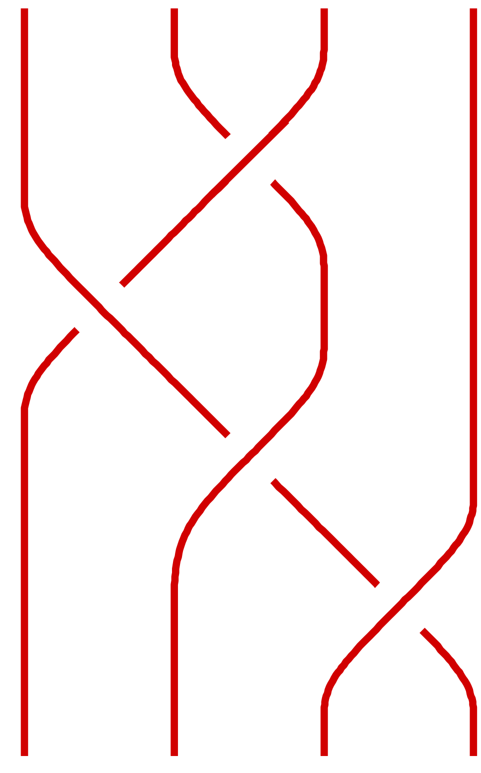
2 spatial dimensions:

$$R \neq R^{-1}$$

- Worldlines form topologically distinct braid configurations
- Statistics characterized by braid group B_n

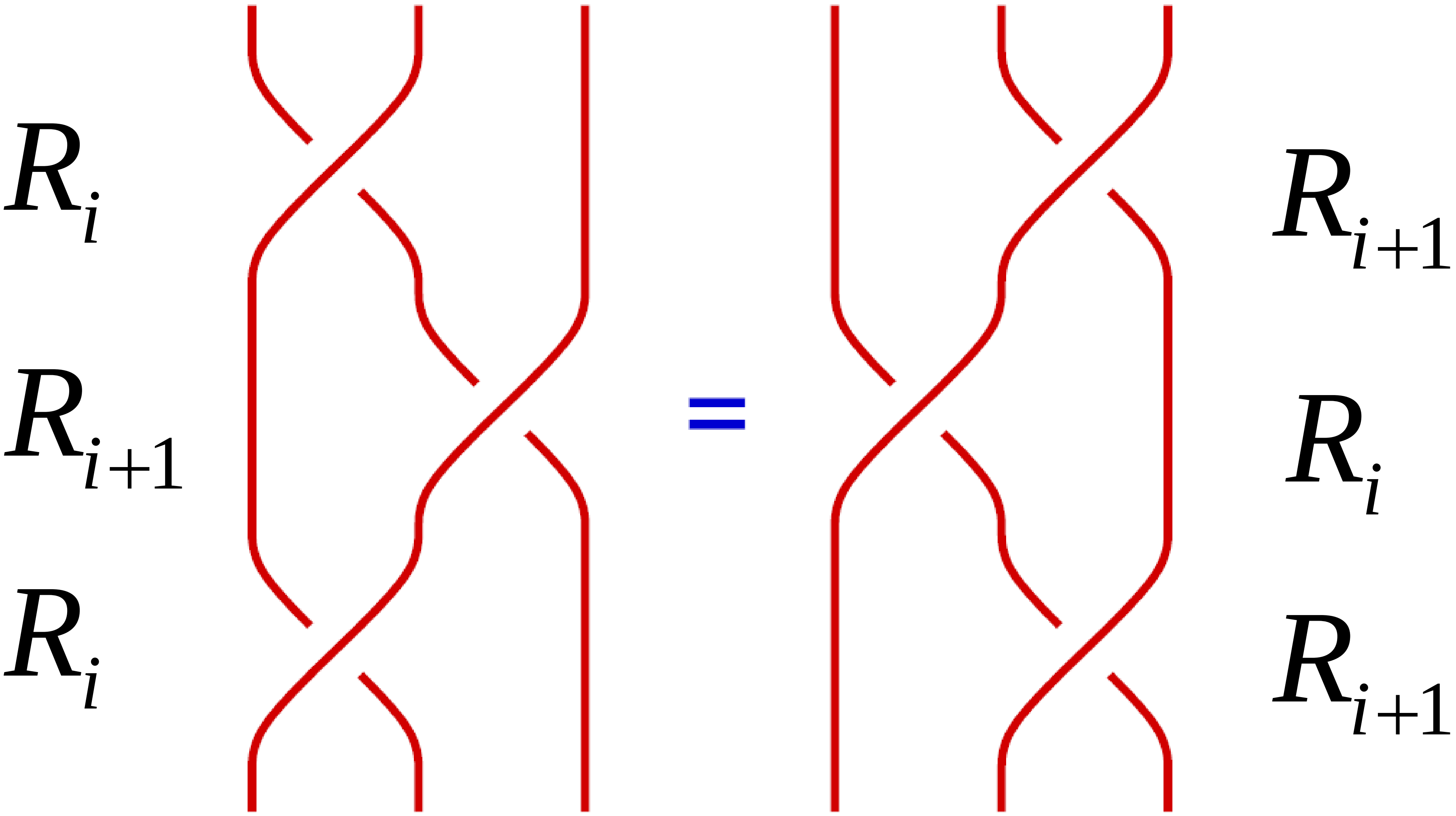
(n strand) braid group B_n





$R_2 R_1^{-1} R_2 R_3$

Yang - Baxter constraint : $R_i R_{i+1} R_i = R_{i+1} R_i R_{i+1}$



Braiding "Statistics"

One dim unitary reps of B_n assign a phase to each braid generator:

$$U[R_i]\left|\Psi\right\rangle = e^{i\theta}\left|\Psi\right\rangle \quad \Rightarrow \quad \text{Abelian anyons}$$

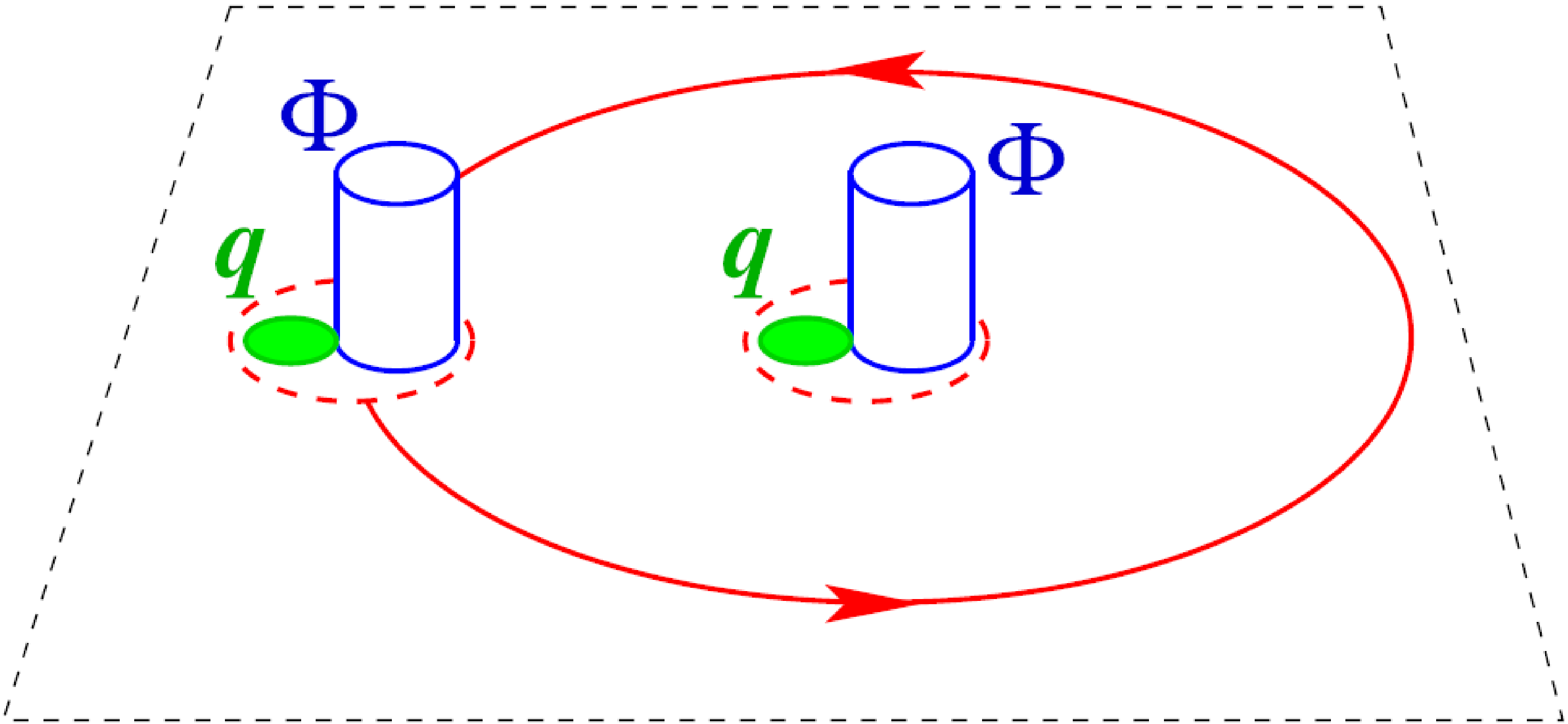
$(\text{bosons}: \ \theta = 0, \ \text{fermions}: \ \theta = \pi)$

Higher dim reps of B_n mean Hilbert space is multi-dimensional, and unitary **matrices** are assigned to braid generators:

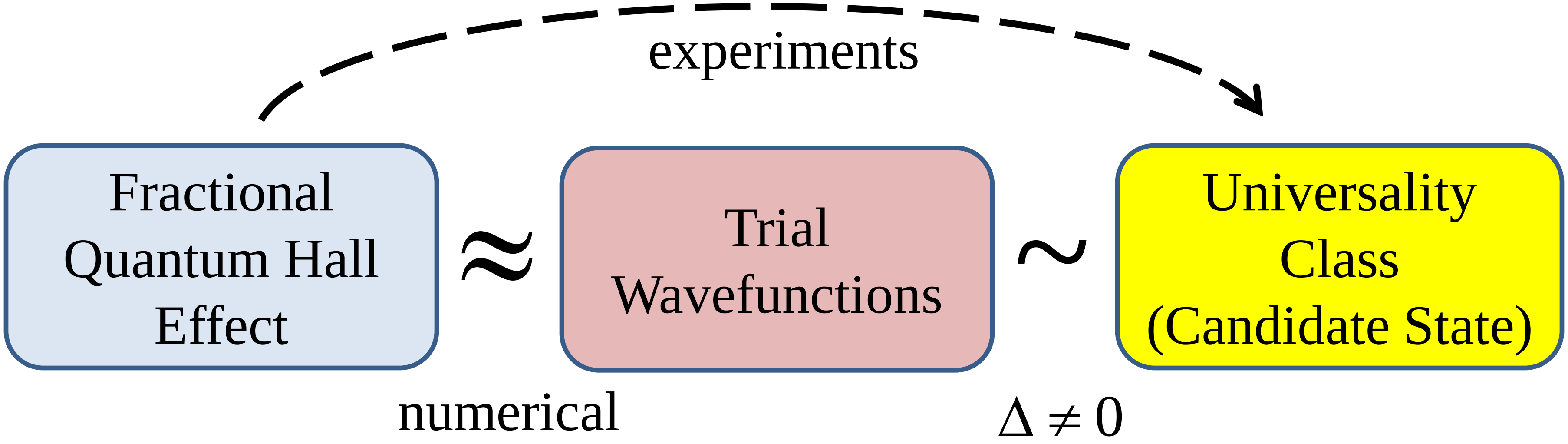
$$U[R_i]\left|\Psi_\alpha\right\rangle = \sum_\beta U_{\alpha\beta}\left|\Psi_\beta\right\rangle \Rightarrow \text{non-Abelian anyons!}$$

Toy model of Abelian Anyons:

charge q - flux Φ composites



Aharonov - Bohm effect : $\theta = q\Phi$



- Trial wavefunctions are adiabatically connected to a universality class (deform Hamiltonian without closing the gap).
- Experiments are compared against universal properties of candidate states, e.g. quasiparticle charge and statistics.
- There is some experimental evidence confirming the nature of FQH states.
- **How do we determine the universal properties from trial wavefunctions?**

Adiabatic transport

- **Adiabatic theorem**: States in a (possibly degenerate) energy subspace separated from others by a gap remain in the subspace when the system is changed adiabatically without closing the gap.

- Adiabatic transport (holonomy) = combination of Berry's phase/matrix and (instantaneous energy) eigenstate transformation under transport

- Holonomy is invariant, but Berry's phase/matrix and eigenstate transformation depend on choice of gauge (and can be shifted from one to the other)

- Question: How can one compute the eigenstate transformation for quantum Hall wavefunctions under transport of quasiparticles?

- Answer: Analytic continuation of the wavefunctions.

- Question: How can one compute the Berry’s phase/matrix for quantum Hall wavefunctions?

- Answer: Plasma analogy.

We have accomplished this for the Moore-Read state!

Bonderson-Gurarie-Nayak `11

Laughlin states

Laughlin `83

$$\Psi_{\frac{1}{M}}(z_1, \ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j)^M e^{-\frac{1}{4}\sum_{i=1}^{N} |z_i|^2}$$

electron coordinates $z_i = x_i + iy_i$
(disk geometry)

$$\nu = \frac{1}{M} = 1, \frac{1}{3}, \frac{1}{5}, \ldots$$

M is odd for electrons (fermions)

M=1 is one filled Landau level, i.e. wavefunction for non-interacting free electrons in a background magnetic field

Laughlin states

Laughlin `83

$$\left\|\Psi_{\frac{1}{M}}\right\|^2 = \int \prod_{k=1}^{N} d^2 z_k \, \bar{\Psi}_{\frac{1}{M}} \Psi_{\frac{1}{M}} = \int \prod_{k=1}^{N} d^2 z_k \prod_{i<j}^{N} |z_i - z_j|^{2M} e^{-\frac{1}{2}\sum_{i=1}^{N}|z_i|^2}$$

$$= \int \prod_{k=1}^{N} d^2 z_k \, e^{-V_1/T}$$

$$V_1 = -\sum_{i<j}^{N} Q^2 \log|z_i - z_j| - \frac{\pi}{2} Q\rho \sum_{i=1}^{N} |z_i|^2$$

$$Q = \sqrt{2MT}, \qquad \rho = -\frac{Q}{2\pi M}$$

Coulomb Interaction in 2D

Potential energy between two point charges q_1 and q_2 at positions z_1 and z_2:

$$V = -q_1 q_2 \log\left|z_1 - z_2\right|$$

Potential energy of a point charge q in a disk of uniform charge density ρ centered at the origin

$$V = -\tfrac{\pi}{2} q\rho\left|z\right|^2$$

Laughlin states

Laughlin `83

$$\left\|\Psi_{\frac{1}{M}}\right\|^2 = \int \prod_{k=1}^{N} d^2 z_k \, \bar{\Psi}_{\frac{1}{M}} \Psi_{\frac{1}{M}} = \int \prod_{k=1}^{N} d^2 z_k \prod_{i<j}^{N} |z_i - z_j|^{2M} e^{-\frac{1}{2}\sum_{i=1}^{N}|z_i|^2}$$

$$= \int \prod_{k=1}^{N} d^2 z_k \, e^{-V_1/T}$$

$$V_1 = -\sum_{i<j}^{N} Q^2 \log|z_i - z_j| - \frac{\pi}{2} Q\rho \sum_{i=1}^{N} |z_i|^2$$

$$Q = \sqrt{2MT}, \qquad \rho = -\frac{Q}{2\pi M}$$

V_1 is the 2D Coulomb potential of N charges Q in a uniform background charge density ρ

F_1 is the free energy of a plasma at temperature T!

Laughlin states

Laughlin `83

$$\left\| \Psi_{\frac{1}{M}} \right\|^2 = \int \prod_{k=1}^{N} d^2 z_k \, \bar{\Psi}_{\frac{1}{M}} \Psi_{\frac{1}{M}} = \int \prod_{k=1}^{N} d^2 z_k \prod_{i<j}^{N} |z_i - z_j|^{2M} e^{-\frac{1}{2} \sum_{i=1}^{N} |z_i|^2}$$

$$= \int \prod_{k=1}^{N} d^2 z_k \, e^{-V_1/T} = e^{-F_1/T}$$

This 2D one-component plasma screens for $\Gamma = Q^2 / T < 140$
i.e. for M<70.

Caillol *et al.* `82

This tells us the ground state wavefunction has no long range correlations (i.e. the pair correlation function is constant, up to exponentially suppressed corrections), which is necessary for it to describe a gapped state.

Laughlin states

Laughlin `83

Trial wavefunction for N electrons at positions z_i
with n quasiholes (flux 1 vortices) at positions η_μ:

$$\Psi_{\frac{1}{M}}(\eta_\mu; z_i) \;=\; \prod_{\mu=1}^{n}\prod_{i=1}^{N}(\eta_\mu - z_i)\prod_{i<j}^{N}(z_i - z_j)^M e^{-\frac{1}{4}\sum_{i=1}^{N}|z_i|^2}$$

Laughlin states

Laughlin `83

Trial wavefunction for N electrons at positions z_i
with n quasiholes (flux 1 vortices) at positions η_μ:

$$\Psi_{\frac{1}{M}}(\eta_\mu; z_i) = \prod_{\mu=1}^{n}\prod_{i=1}^{N}(\eta_\mu - z_i)\prod_{i<j}^{N}(z_i - z_j)^M e^{-\frac{1}{4}\sum_{i=1}^{N}|z_i|^2}$$
$$\times \prod_{\mu<\nu}^{n}(\eta_\mu - \eta_\nu)^{\frac{1}{M}} e^{-\frac{1}{4M}\sum_{\mu=1}^{n}|\eta_\mu|^2}$$

This gives the "correct" normalization prefactor of the electron wavefunction with quasiholes (up to an overall constant).

Exchanging two quasiholes give a phase of $e^{i\pi/M}$
from analytic continuation (eigenstate transformation).

Laughlin states

Laughlin `83

$$\left\| \Psi_{\frac{1}{M}} \right\|^2 = \int \prod_k d^2 z_k \, e^{-V_1/T}$$

$$V_1 = -\sum_{\mu<\nu} \left(\frac{Q}{M}\right)^2 \log|\eta_\mu - \eta_\nu| - \sum_{\mu,i} \frac{Q}{M} Q \log|\eta_\mu - z_i| - \sum_{i<j} Q^2 \log|z_i - z_j|$$

$$-\frac{\pi}{2}\frac{Q}{M}\rho \sum_\mu |\eta_\mu|^2 - \frac{\pi}{2} Q\rho \sum_i |z_i|^2$$

$$Q = \sqrt{2MT}, \qquad \rho = -\frac{Q}{2\pi M}$$

V_1 is the 2D Coulomb potential of:
N charges Q and
n charges Q/M
in a disk of uniform background charge density ρ

Laughlin states

Laughlin `83

$$\left\|\Psi_{\frac{1}{M}}\right\|^2 = \int \prod_k d^2 z_k \, e^{-V_1/T} = e^{-F_1/T} = C_1 + O(e^{-|\eta_\mu - \eta_\nu|/\ell_1})$$ for $M < 70$

$$V_1 = -\sum_{\mu<\nu} \left(\frac{Q}{M}\right)^2 \log|\eta_\mu - \eta_\nu| - \sum_{\mu,i} \frac{Q}{M} Q \log|\eta_\mu - z_i| - \sum_{i<j} Q^2 \log|z_i - z_j|$$
$$-\frac{\pi}{2}\frac{Q}{M}\rho \sum_\mu |\eta_\mu|^2 - \frac{\pi}{2} Q\rho \sum_i |z_i|^2$$

$$Q = \sqrt{2MT}, \qquad \rho = -\frac{Q}{2\pi M}$$

F_1 is the free energy of a 2D one-component plasma at temperature T, with n fixed test charges at the quasihole coordinates.

This is in the screening plasma phase for M<70, in which case the free energy is independent of the positions of the test charges.

Caillol *et al*. `82

Berry's phase

Arovas-Schrieffer-Wilczek `84
Laughlin `87

$$\Psi_{\frac{1}{M}}(\eta_\mu; z_i) = \prod_{\mu<\nu}^{n}(\eta_\mu - \eta_\nu)^{\frac{1}{M}} \prod_{\mu=1}^{n}\prod_{i=1}^{N}(\eta_\mu - z_i)\prod_{i<j}^{N}(z_i - z_j)^M e^{-\frac{1}{4M}\sum_{\mu=1}^{n}|\eta_\mu|^2 - \frac{1}{4}\sum_{i=1}^{N}|z_i|^2}$$

$$\left\|\Psi_{\frac{1}{M}}\right\|^2 = C_1 + O(e^{-|\eta_\mu - \eta_\nu|/\ell_1})$$

$$\mathcal{A}^{\bar\eta_\mu} = i\int\prod_{k=1}^{N} d^2 z_k \frac{\bar\Psi}{\|\Psi\|}\frac{\partial}{\partial\bar\eta_\mu}\left(\frac{\Psi}{\|\Psi\|}\right)$$

The only dependence on $\bar\eta_\mu$ is in the Gaussian term!

The only dependence on η_μ is in the Gaussian term!

$$\mathcal{A}^{\eta_\mu} = i\int\prod_{k=1}^{N} d^2 z_k \frac{\bar\Psi}{\|\Psi\|}\frac{\partial}{\partial\eta_\mu}\left(\frac{\Psi}{\|\Psi\|}\right)$$

$$= i\frac{\partial}{\partial\eta_\mu}\left(\|\Psi\|^{-2}\int\prod_{k=1}^{N} d^2 z_k\,\bar\Psi\Psi\right) - i\int\prod_{k=1}^{N} d^2 z_k \frac{\partial}{\partial\eta_\mu}\left(\frac{\bar\Psi}{\|\Psi\|}\right)\frac{\Psi}{\|\Psi\|}$$

0

Berry's phase

Arovas-Schrieffer-Wilczek `84
Laughlin `87

$$\mathcal{P}\exp\left[i\int_0^{t_f} dt\mathcal{A}(t)\right] = \exp\left[-\frac{1}{4M}\sum_{\mu=1}^{n}\int_0^{t_f} dt\left(\bar{\eta}_\mu\frac{d\eta_\mu}{dt}-\eta_\mu\frac{d\bar{\eta}_\mu}{dt}\right)\right]$$

$$= \exp\left(-i\frac{e}{M}\frac{BA}{\hbar c}\right)$$

(Stokes's Theorem)

This is just the Aharonov-Bohm phase of a charge $q = \frac{e}{M}$ particle encircling flux of $\Phi = -BA$

"braiding statistics gauge"

Thus, using the "correct" normalization prefactor :

- Berry's phase = Aharonov-Bohm phase (geometric)
- eigenstate transformation (analytic continuation) = braiding statistics (topological) of the quasiholes!

Berry's phase

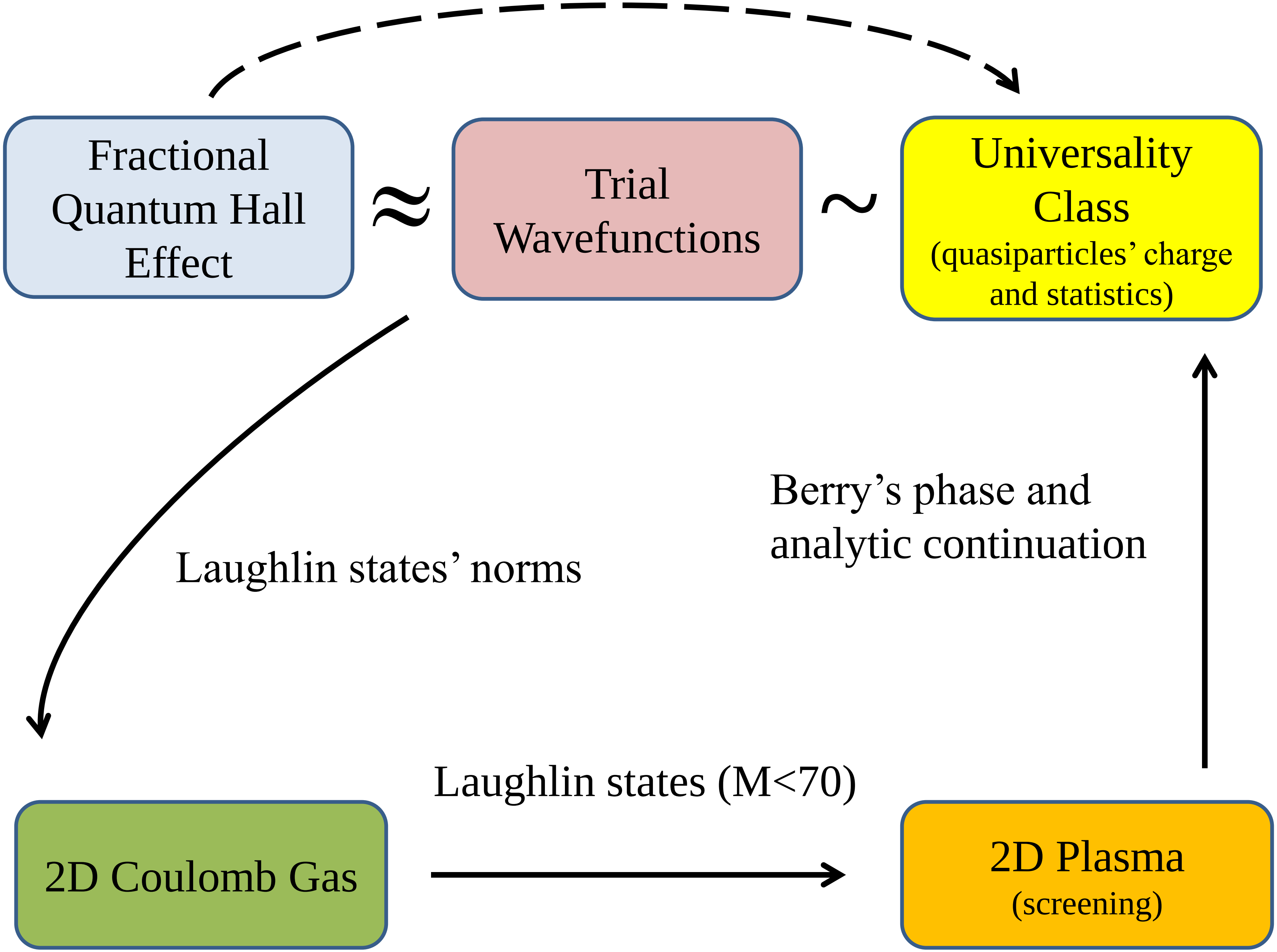
In contrast, one could use a different "gauge," such as:

$$\Psi_{\frac{1}{M}}(\eta_\mu; z_i) = \prod_{\mu=1}^{n} \prod_{i=1}^{N} (\eta_\mu - z_i) \prod_{i<j}^{N} (z_i - z_j)^M e^{-\frac{1}{4}\sum_{i=1}^{N}|z_i|^2}$$

$$\times \prod_{\mu<\nu}^{n} |\eta_\mu - \eta_\nu|^{\frac{1}{M}} e^{-\frac{1}{4M}\sum_{\mu=1}^{n}|\eta_\mu|^2}$$

The plasma mapping still applies, but in this case, we find:

- Berry's phase = Aharonov-Bohm phase + braiding statistics
- eigenstate transformation (analytic continuation) = trivial


Fractional Quantum Hall Effect
≈
Trial Wavefunctions
~
Universality Class
(quasiparticles' charge and statistics)
Laughlin states' norms
Berry's phase and analytic continuation
Laughlin states (M<70)
2D Coulomb Gas
2D Plasma
(screening)

CFT in FQH

Moore-Read `91

Laughlin wavefunctions can be obtained from correlation functions of a $U(1)_M$ CFT

$$\Psi_{\frac{1}{M}}(\eta_\mu; z_i) = \prod_{\mu<\nu}^{n} (\eta_\mu - \eta_\nu)^{\frac{1}{M}} \prod_{\mu=1}^{n} \prod_{i=1}^{N} (\eta_\mu - z_i) \prod_{i<j}^{N} (z_i - z_j)^M e^{-\frac{1}{4M} \sum_{\mu=1}^{n} |\eta_\mu|^2 - \frac{1}{4} \sum_{i=1}^{N} |z_i|^2}$$

$$= \left\langle e^{i\frac{1}{\sqrt{2M}}\varphi(\eta_1)} \ldots e^{i\frac{1}{\sqrt{2M}}\varphi(\eta_n)} e^{i\sqrt{\frac{M}{2}}\varphi(z_1)} \ldots e^{i\sqrt{\frac{M}{2}}\varphi(z_N)} e^{-i\frac{1}{2\pi\sqrt{2M}}\int d^2z\, \varphi(z)} \right\rangle$$

Why not try using other CFTs?

Moore-Read state

Moore-Read `91

Ising x $U(1)_M$

$$\Psi(z_1,\ldots,z_N) = \langle\psi(z_1)\ldots\psi(z_N)\rangle \times \left\langle e^{i\sqrt{\frac{M}{2}}\varphi(z_1)}\ldots e^{i\sqrt{\frac{M}{2}}\varphi(z_N)} e^{-i\frac{1}{2\pi\sqrt{2M}}\int d^2z\,\varphi(z)}\right\rangle$$
$$= \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \times \prod_{i<j}^{N}(z_i - z_j)^M\, e^{-\frac{1}{4}\sum_{i=1}^{N}|z_i|^2}$$

$$\mathrm{Pf}(A_{i,j}) \equiv \frac{1}{N!!}\sum_{\sigma\in S_N}\mathrm{sgn}(\sigma)\prod_{k=1}^{N/2}A_{\sigma(2k-1),\sigma(2k)}$$

ψ are fermionic, so M must be even (to describe FQH electrons)

$\nu = \frac{1}{M} = \frac{1}{2}, \frac{1}{4}, \ldots$ Perhaps this describes $\nu = 5/2 = 2+½$

(There is numerical support and some experimental evidence in favor of this!)

Moore-Read state

Moore-Read `91

Ising x $U(1)_M$

Add n quasiholes (flux 1/2 vortices carrying σ Ising charge):

$$\Psi_\alpha(\eta_\mu; z_i) = \langle \sigma(\eta_1)\ldots\sigma(\eta_n)\,\psi(z_1)\ldots\psi(z_N)\rangle_\alpha$$
$$\times \left\langle e^{i\frac{1}{2\sqrt{2M}}\varphi(\eta_1)}\ldots e^{i\frac{1}{2\sqrt{2M}}\varphi(\eta_n)} e^{i\sqrt{\frac{M}{2}}\varphi(z_1)}\ldots e^{i\sqrt{\frac{M}{2}}\varphi(z_N)} e^{-i\frac{1}{2\pi\sqrt{2M}}\int d^2z\,\varphi(z)}\right\rangle$$

$\alpha = 0,\ldots,q-1$ labels the conformal blocks

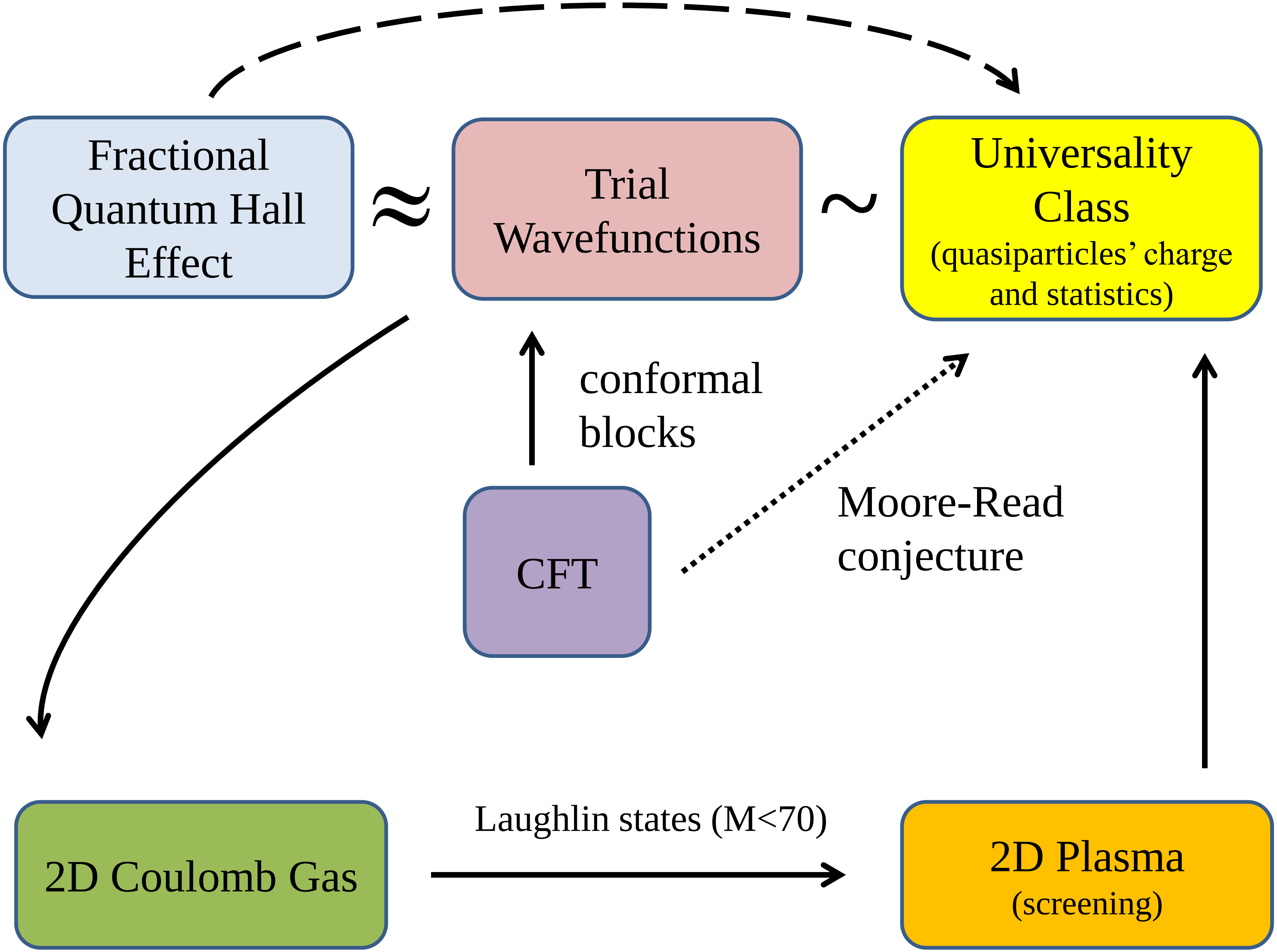
where $q = 2^{\frac{n}{2}-1}$ for Ising

Analytic continuation of the quasihole wavefunctions (conformal blocks) exhibits **non-Abelian** statistics!

Moore-Read Conjecture

Using the CFT generated wavefunctions gives:

- Berry's matrix = Aharonov-Bohm phase (geometric)
- eigenstate transformation (analytic continuation) = braiding statistics of the quasiholes (topological)



Fractional Quantum Hall Effect
≈
Trial Wavefunctions
~
Universality Class
(quasiparticles' charge and statistics)
conformal blocks
CFT
Moore-Read conjecture
2D Coulomb Gas
Laughlin states (M<70)
2D Plasma
(screening)

Partial Results

Nayak-Wilczek `96
Gurarie-Nayak `97
Read-Green `00
Ivanov `01
Tserkovnyak-Simon `03
Stern-von Oppen-Mariani `04
Stone-Chung `06
Seidel `08
Read `09
Baraban *et al.* `09
Prodan-Haldane `09

Berry's matrix?

We want to demonstrate that the Berry's matrix for the MR wavefunctions (with the CFT normalization) gives only the Aharonov-Bohm phase.

Similar to the Laughlin states, this holds if we can show:

$$G_{\alpha,\beta}(\bar{\eta}_\mu, \eta_\mu) \equiv \int \prod_{k=1}^{N} d^2 z_k \, \bar{\Psi}_\alpha(\bar{\eta}_\mu; \bar{z}_i) \Psi_\beta(\eta_\mu; z_i) = C\delta_{\alpha\beta} + O\left(e^{-|\eta_\mu - \eta_\nu|/\ell}\right)$$

i.e. if the conformal blocks give rise to orthogonal wavefunctions, with constant norm (independent of η_μ)

Berry's matrix?

Specifically, this gives:

$$\mathcal{A}^{\eta_\mu}_{\alpha,\beta} \ = \ i \int \prod_{k=1}^{N} d^2 z_k \, \frac{\bar{\Psi}_\alpha}{G^{1/2}_{\alpha,\alpha}} \frac{\partial}{\partial \eta_\mu} \left(\frac{\Psi_\beta}{G^{1/2}_{\beta,\beta}} \right) = i \frac{\bar{\eta}_\mu}{8M} \delta_{\alpha\beta} \ + \ O\big(e^{-|\eta_\mu - \eta_\nu|/\ell}\big)$$

$$\mathcal{A}^{\bar{\eta}_\mu}_{\alpha,\beta} \ = \ i \int \prod_{k=1}^{N} d^2 z_k \, \frac{\bar{\Psi}_\alpha}{G^{1/2}_{\alpha,\alpha}} \frac{\partial}{\partial \bar{\eta}_\mu} \left(\frac{\Psi_\beta}{G^{1/2}_{\beta,\beta}} \right) = -i \frac{\eta_\mu}{8M} \delta_{\alpha\beta} \ + \ O\big(e^{-|\eta_\mu - \eta_\nu|/\ell}\big)$$

$$\mathcal{P} \exp \left[i \int_0^{t_f} dt \mathcal{A}(t) \right] = \exp \left(-i \frac{e}{2M} \frac{BA}{\hbar c} \right) \mathbb{1}$$

This is the Aharonov-Bohm phase of a charge $q = \frac{e}{2M}$ particle encircling flux of $\Phi = -BA$

Plasma Analogy?

Can general CFTs be described as a 2D Coulomb gas?

Yes! (for minimal models and some others) Dotsenko-Fateev `84, Felder `89

BUT, this involves screening operators (screening charges integrated around certain contours), so it is not a conventional Coulomb gas.

Can we salvage this? Yes! (to some extent) Mathur `92

Bonderson-Gurarie-Nayak `11

We can exchange holomorphic-antiholomorphic pairs of screening charge contour integrals for 2D integrals by summing over conformal blocks appropriately.

In particular, this works best for the Ising CFT!

The correlation function of ψ fields:

$$
\begin{aligned}
\mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) &= \langle V_{31}^{20}(z_1) V_{31}^{00}(z_2) \ldots V_{31}^{20}(z_{N-1}) V_{31}^{00}(z_N) \rangle \\
&= \Bigg\langle \oint_{C_{z_1}} dw_1 \oint_{C_{z_1}} dw_2 \, e^{i\alpha_{31}\varphi(z_1)} e^{i\alpha_-\varphi(w_1)} e^{i\alpha_-\varphi(w_2)} e^{i\alpha_{31}\varphi(z_2)} \times \ldots \\
&\qquad \ldots \times \oint_{C_{z_{N-1}}} dw_{N-1} \oint_{C_{z_{N-1}}} dw_N \, e^{i\alpha_{31}\varphi(z_{N-1})} e^{i\alpha_-\varphi(w_{N-1})} e^{i\alpha_-\varphi(w_N)} e^{i\alpha_{31}\varphi(z_N)} \Bigg\rangle
\end{aligned}
$$

combine to give:

$$
\left|\mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\right|^2 = \int \prod_k^N d^2 w_k \prod_{i<j}^N |w_i - w_j|^3 \prod_{i,j}^N |w_i - z_j|^{-3} \prod_{i<j}^N |z_i - z_j|^3
$$

Plasma Analogy

If we ignore the $U(1)_M$ sector of the MR state, we have

$$\left\| \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \right\|^2 \equiv \int \prod_{k=1}^{N} d^2 z_k \left| \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \right|^2$$

$Q = \sqrt{3T}$

$$= \int \prod_{k=1}^{N} d^2 z_k \, d^2 w_k \, e^{-V_2/T} = e^{-F_2/T}$$

$$V_2 = -\sum_{i<j}^{N} Q^2 \log|w_i - w_j| + \sum_{i,j}^{N} Q^2 \log|w_i - z_j| - \sum_{i<j}^{N} Q^2 \log|z_i - z_j|$$

V_2 is the 2D Coulomb potential of N charges Q and N charges -Q

F_2 is the free energy of a two-component plasma at temperature T, which screens for $\Gamma = Q^2 / T < 4$

May `67, Kosterlitz-Thouless `73

Plasma Analogy

Including the $U(1)_M$ sector of the MR state, we have:

$$||\Psi(z_i)||^2 \equiv \int \prod_{k=1}^{N} d^2 z_k \, |\Psi(z_i)|^2 = \int \prod_{k=1}^{N} d^2 z_k \, d^2 w_k \, e^{-(V_1+V_2)/T} = e^{-F/T}$$

V_1 and V_2 are the previously defined 2D Coulomb potentials

F is the free energy of a 2D plasma at temperature T of:
N particles with charge (Q_1, Q_2)
N particles with charge $(0, -Q_2)$
and uniform background charge density $(\rho_1, 0)$

This plasma is expected to be in the screening phase when plasmas 1 and 2 are independently in their screening phases. This has been numerically verified for M=2.

Herland *et al.* `11
arXiv:1111.0135

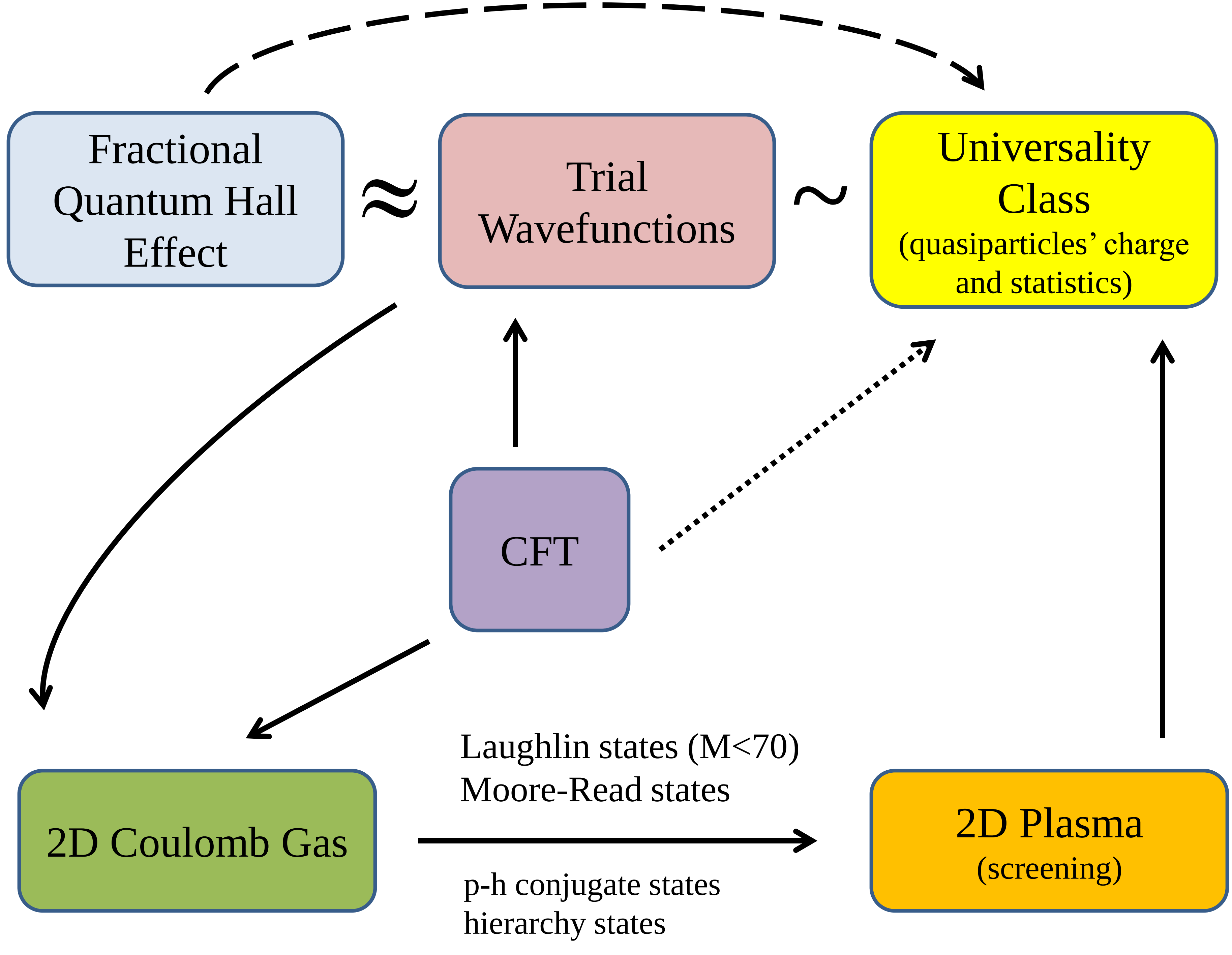
Plasma Analogy

For MR wavefunctions with n quasiholes, we map to this plasma with n test particles that may carry both electric charge and magnetic charge.

This proves:

$$G_{\alpha,\beta}(\bar{\eta}_\mu, \eta_\mu) \equiv \int \prod_{k=1}^{N} d^2 z_k \, \bar{\Psi}_\alpha(\bar{\eta}_\mu; \bar{z}_i) \Psi_\beta(\eta_\mu; z_i) = C\delta_{\alpha\beta} + O\left(e^{-|\eta_\mu - \eta_\nu|/\ell}\right)$$

and consequently also that the braiding statistics of the MR quasiparticles are non-Abelian and given by analytic continuation of the wavefunctions.


Fractional Quantum Hall Effect
≈
Trial Wavefunctions
~
Universality Class (quasiparticles' charge and statistics)
CFT
Laughlin states (M<70)
Moore-Read states
p-h conjugate states
hierarchy states
2D Coulomb Gas
2D Plasma (screening)

Conclusion

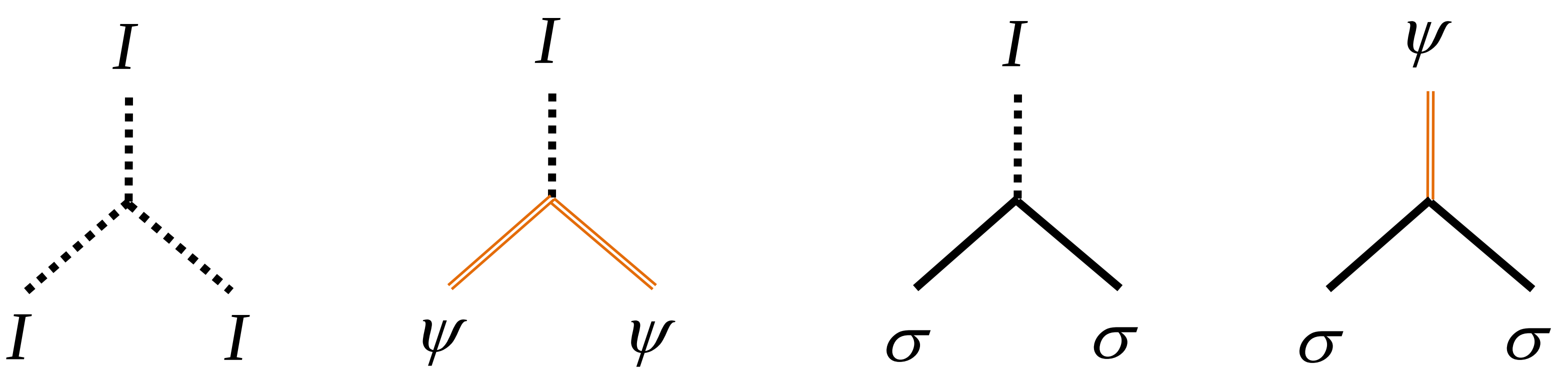
- We have constructed a plasma analogy (which includes magnetic charges) for the Ising-type FQH states, i.e. the MR state, the anti-Pfaffian state (MR's p-h conjugate), and the BS hierarchy states built on these.
- This finally establishes the non-Abelian braiding statistics of the quasiparticles of these states.
- We can use this construction to rule out certain other CFT based candidate wavefunctions.
- We also have constructed (with B. Estienne) a plasma analogy for the Read-Rezayi $\mathbf{Z}_k$–Parafermion states! These have additional interactions and species and their screening properties are not yet established (in progress).

Moore-Read state
Braiding
(from analytic continuation of conformal blocks)

Ising anyons

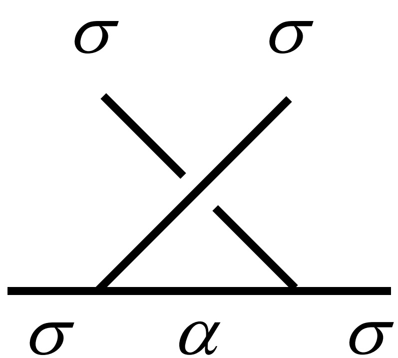
Particle types: $I,\ \sigma,\ \psi$

Fusion rules:

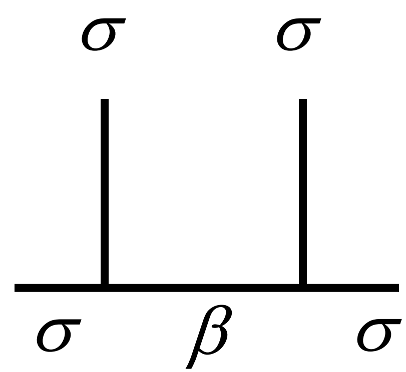


Moore-Read state
Braiding

(from analytic continuation of conformal blocks)



$$= \sum_{\beta} B^{(2)}_{\alpha\beta}$$

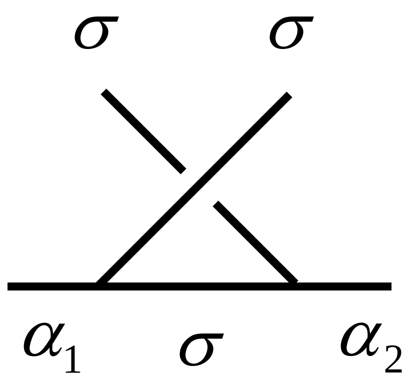


$\alpha, \beta = I \text{ or } \psi$

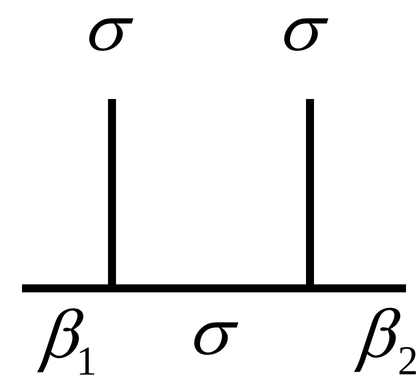
$$B^{(2)} = \frac{e^{i\left(\frac{\pi}{8}+\frac{\pi}{4M}\right)}}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$$

Moore-Read state
Braiding

(from analytic continuation of conformal blocks)



$$= \sum_{\beta_1,\beta_2} B^{(4)}_{(\alpha_1\alpha_2)(\beta_1\beta_2)}$$



$\alpha, \beta = I \text{ or } \psi$

$$B^{(4)} = e^{i\left(-\frac{\pi}{8}+\frac{\pi}{4M}\right)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$