Exact Luttinger correlation prefactors from integrability

Workshop on CFT, topology and information
IHP, Paris, 3 November 2011

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Universiteit van Amsterdam

Work done in collaboration with:

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H. Konno, M. Sorrell, R. Weston
A. Klauser, J. Mossel, G. Palacios

Plan of the talk

- Quick review: correlations from integrability
- Contact with (nonlinear) Luttinger liquid theory
- Another exact approach: quantum groups/vertex operators
- New applications
- Conclusions

Quick review: correlations from integrability

Bethe Ansatz (1931)

July 2, 1906 – March 6, 2005

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Integrable Hamiltonian:

$$H = \int_0^L dx \; \mathcal{H}(x)$$

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... made up of free waves ...

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... with specified relative amplitudes...

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... parametrized by rapidities...

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... and obeying some form of Pauli principle

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- Heisenberg spin-1/2 chain

$$H = \sum_{j=1}^{N} \left[J (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

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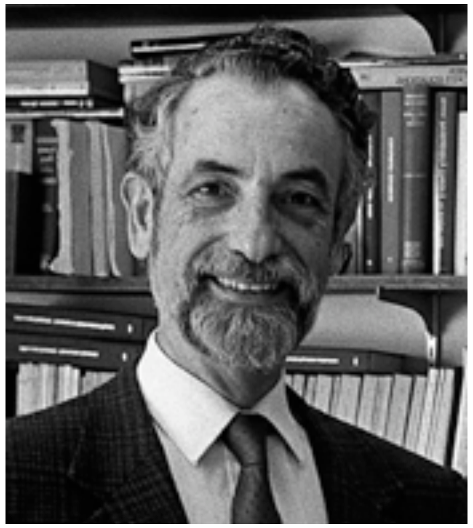

- Interacting Bose gas (Lieb-Liniger)

$$\mathcal{H}_N = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1 \le j < l \le N} \delta(x_j - x_l)$$

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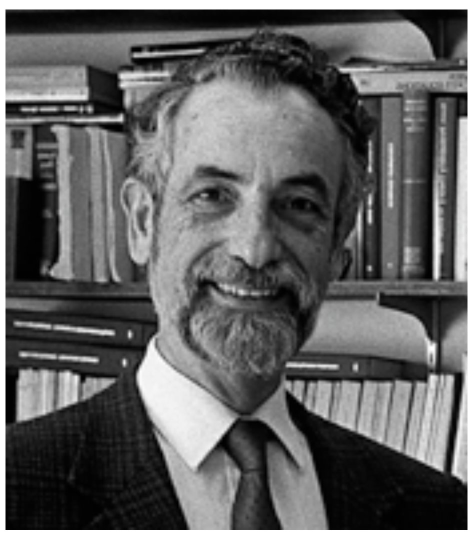

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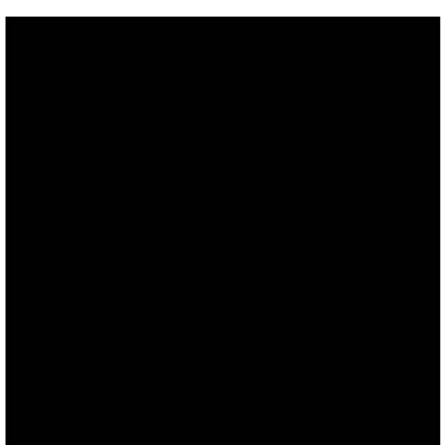

Richardson model (+ Gaudin magnets)

$$H_{BCS} = \sum_{\substack{\alpha=1\\ \sigma=+,-}}^{N}\frac{\varepsilon_\alpha}{2}c^\dagger_{\alpha\sigma}c_{\alpha\sigma} - g\sum_{\alpha,\beta=1}^{N}c^\dagger_{\alpha+}c^\dagger_{\alpha-}c_{\beta-}c_{\beta+}$$

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→ ***inelastic neutron scattering***

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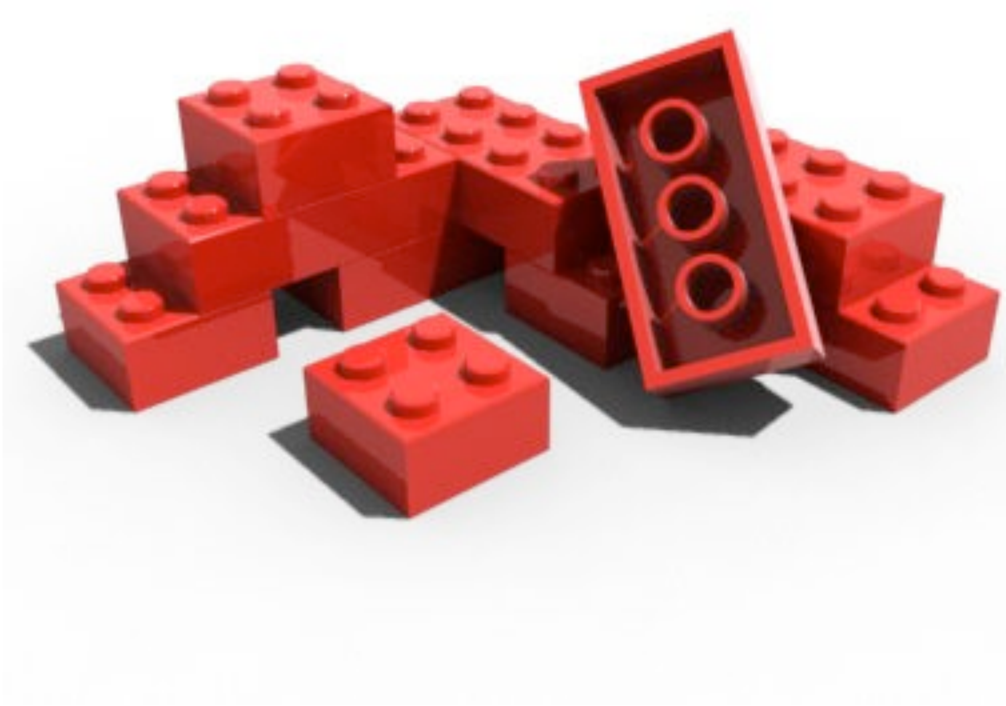
→ ***Bragg spectroscopy, interference experiments, ...***

Building correlation functions piece by piece



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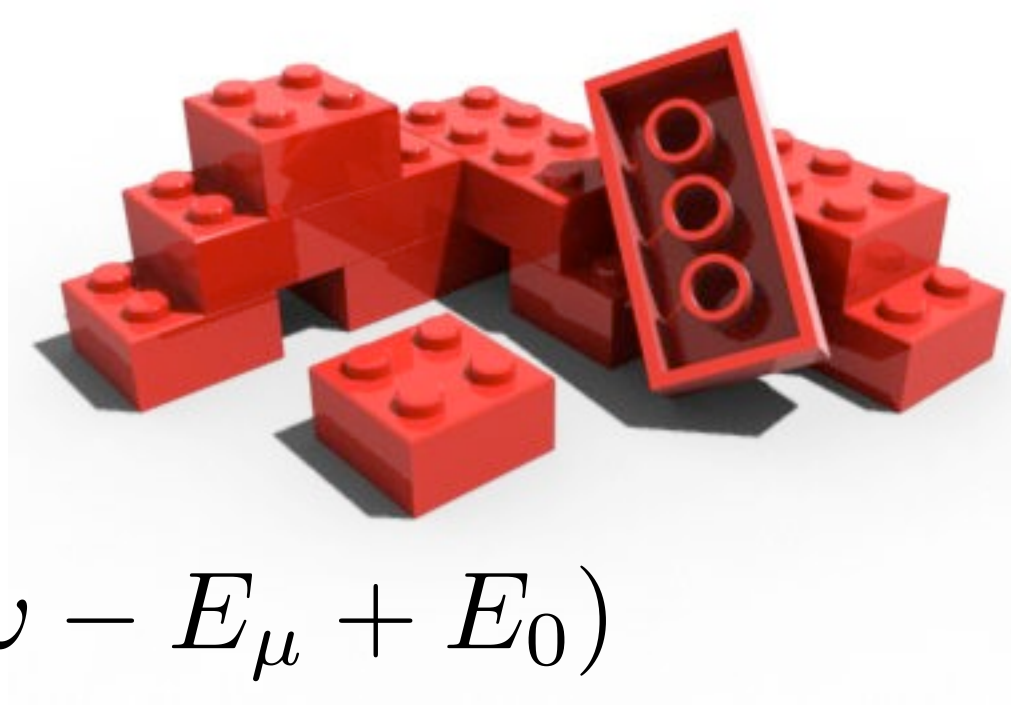

Our needed building blocks are:

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→ ***Bethe Ansatz; quantum groups***

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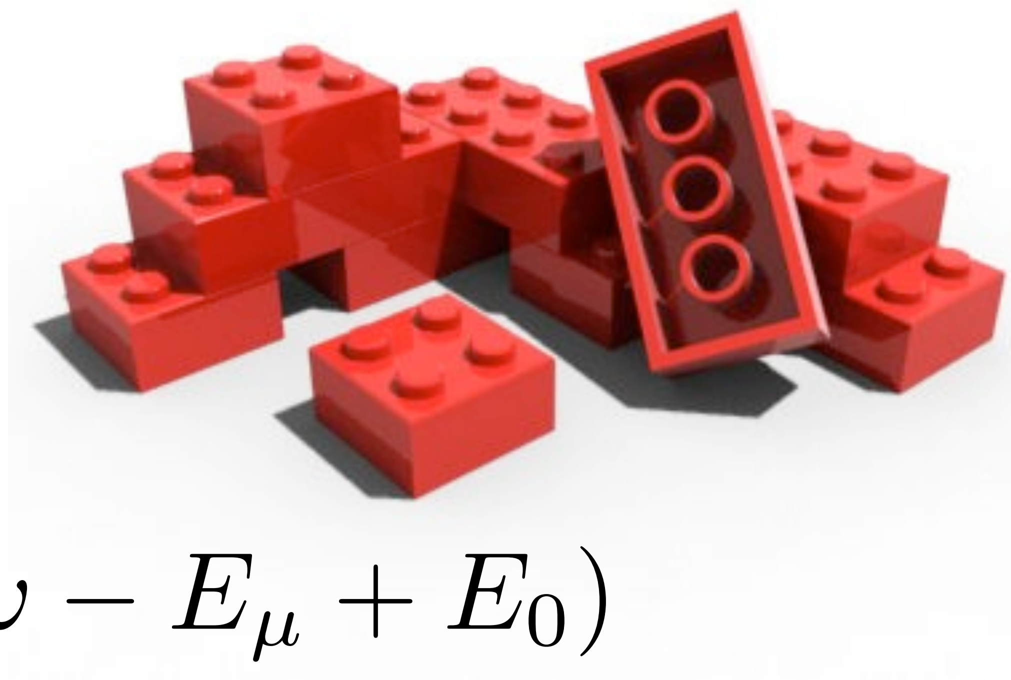

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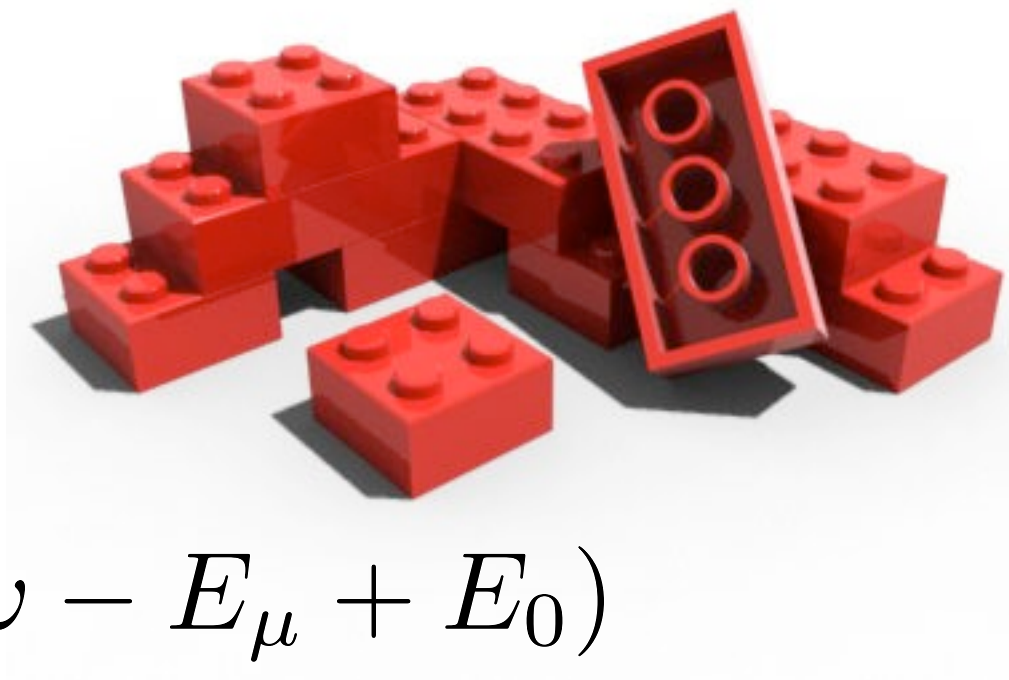

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→ ***Algebraic Bethe Ansatz; q. groups***

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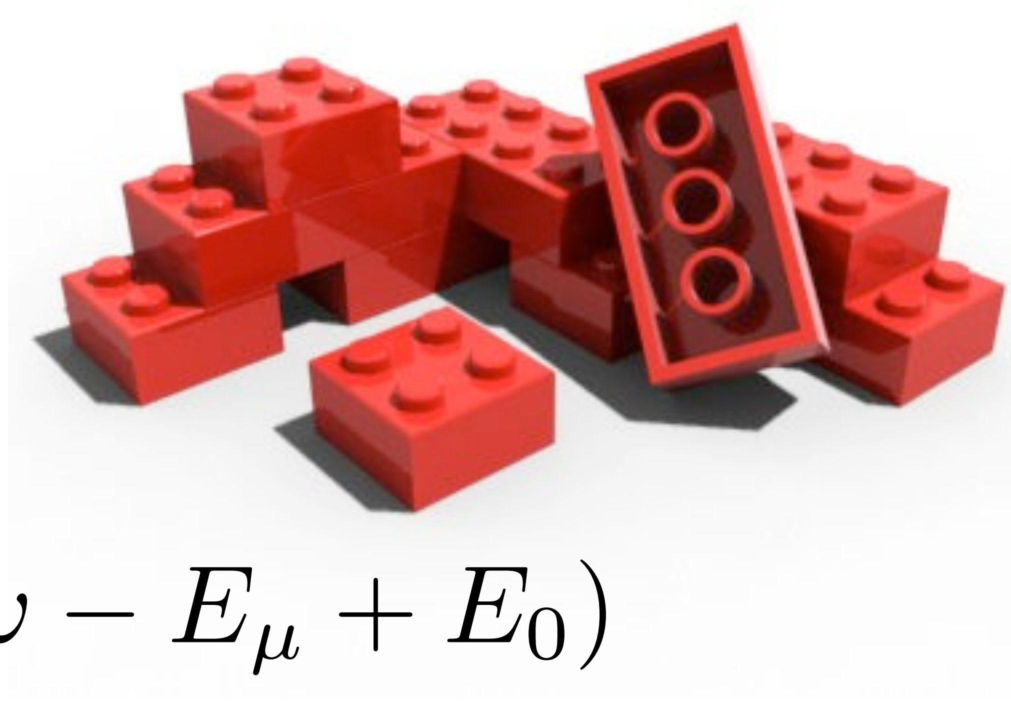

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→ ***Numerics (ABACUS); analytics***

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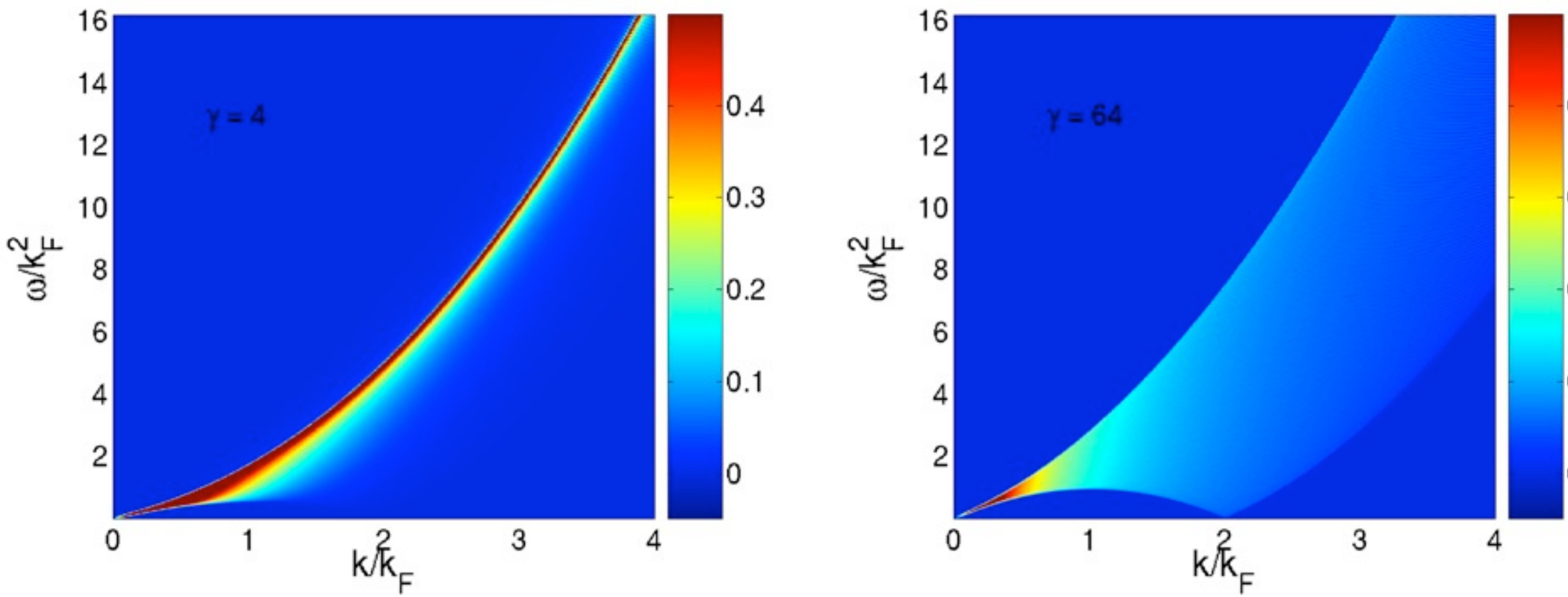
Lieb-Liniger Bose gas

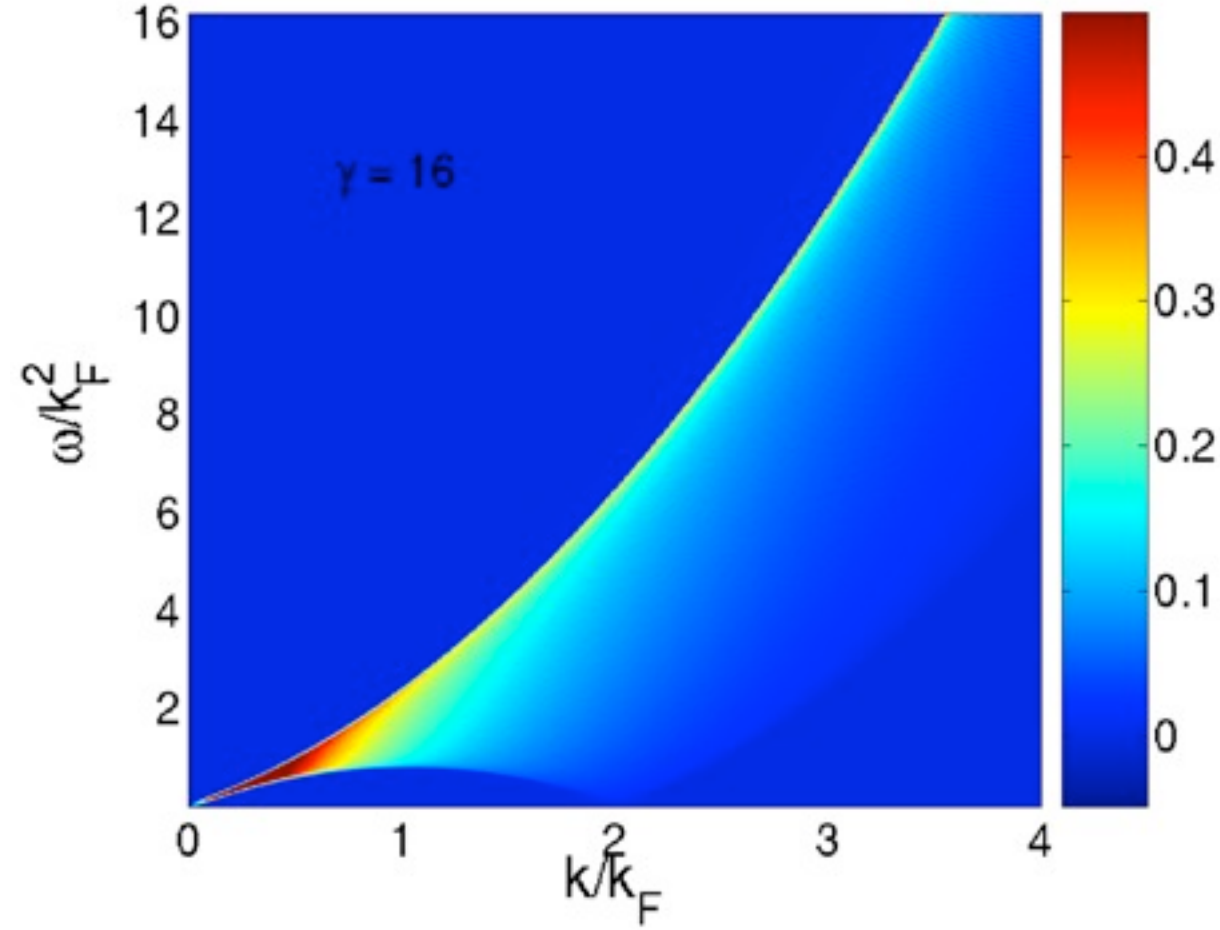
Density-density (dynamical SF)

(J-S C & P Calabrese, PRA 2006)

$$S(k,\omega) = \frac{2\pi}{L} \sum_{\alpha} |\langle 0|\rho_k|\alpha\rangle|^2 \delta(\omega - E_\alpha + E_0)$$



Correspondence with excitations



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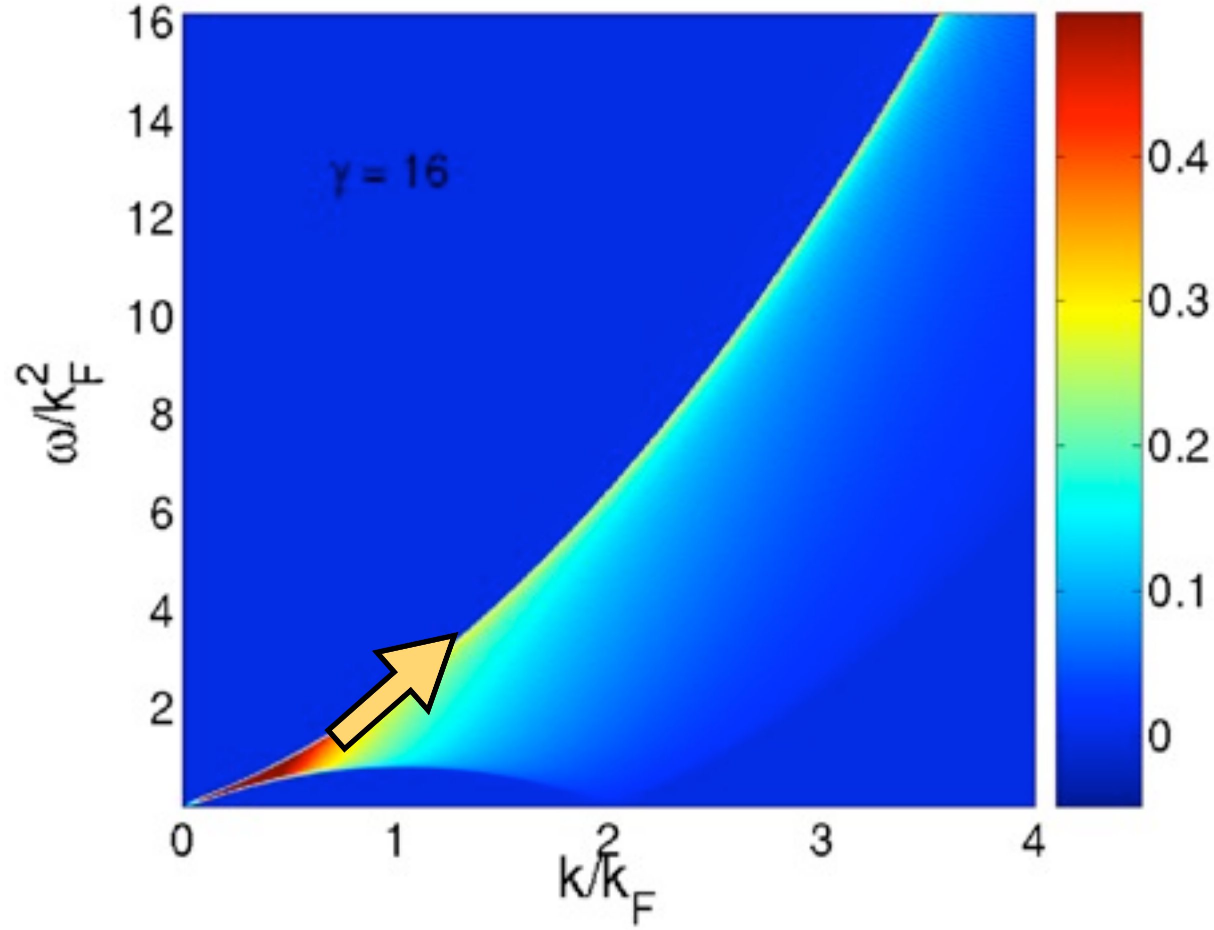
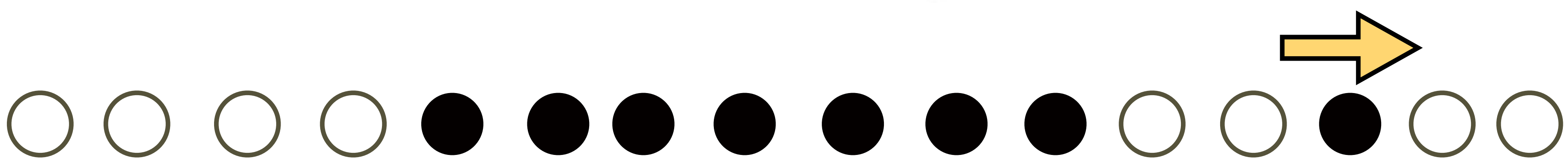


Particle-like



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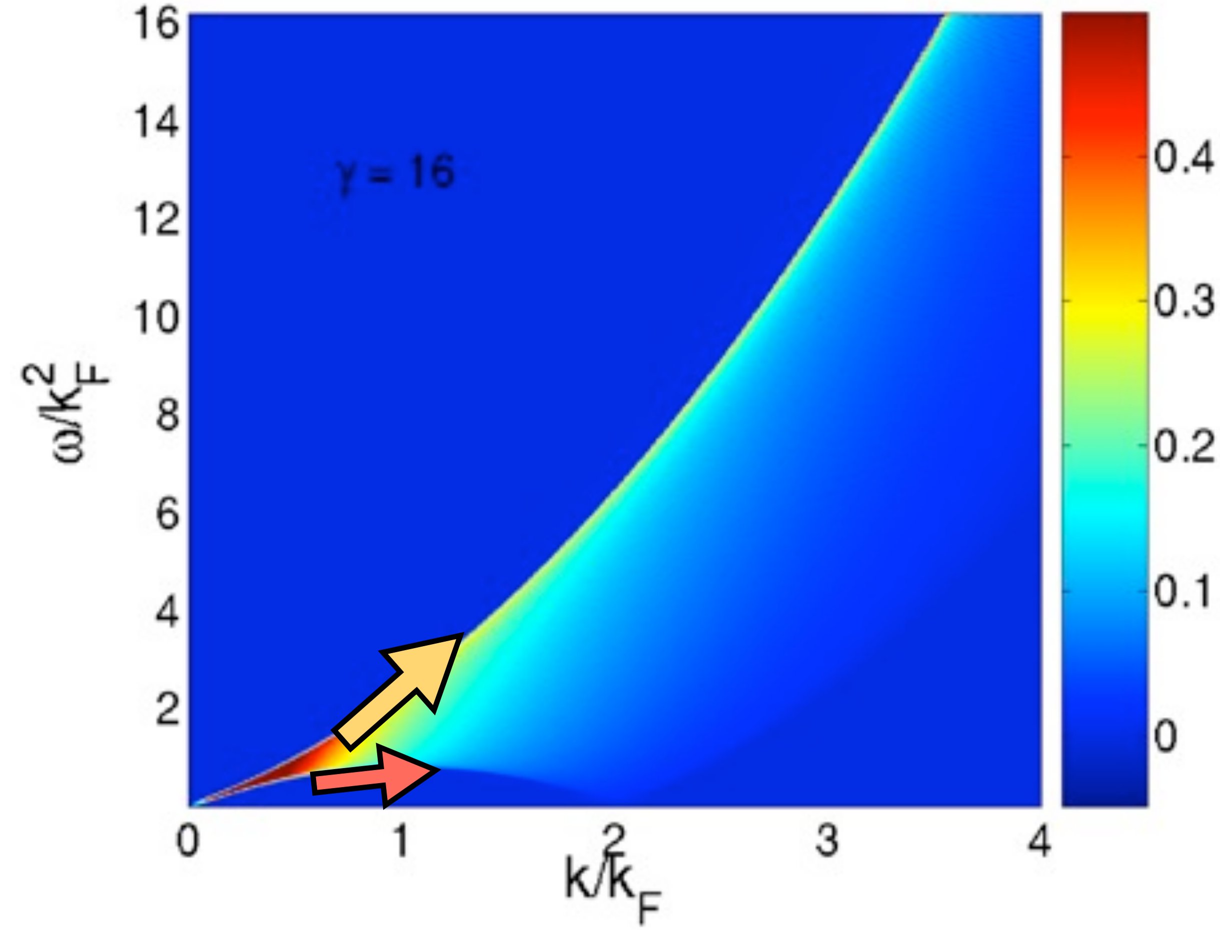
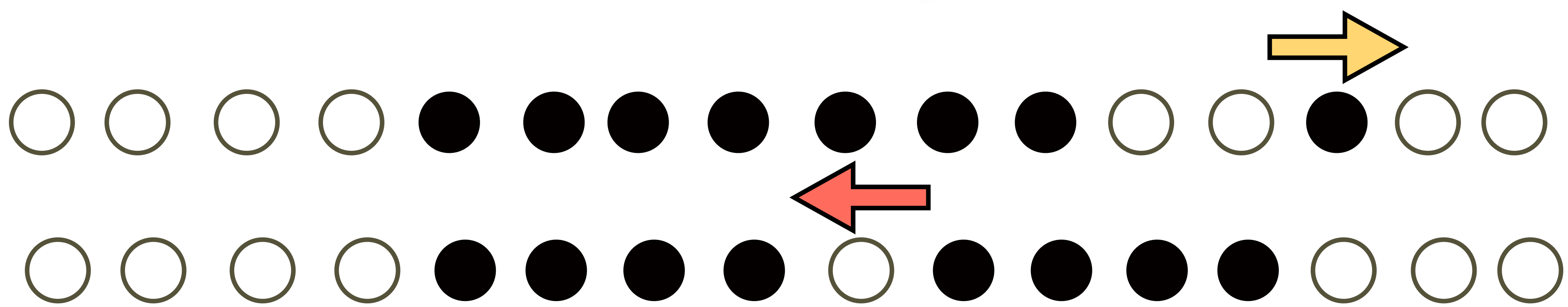


Particle-like

Hole-like



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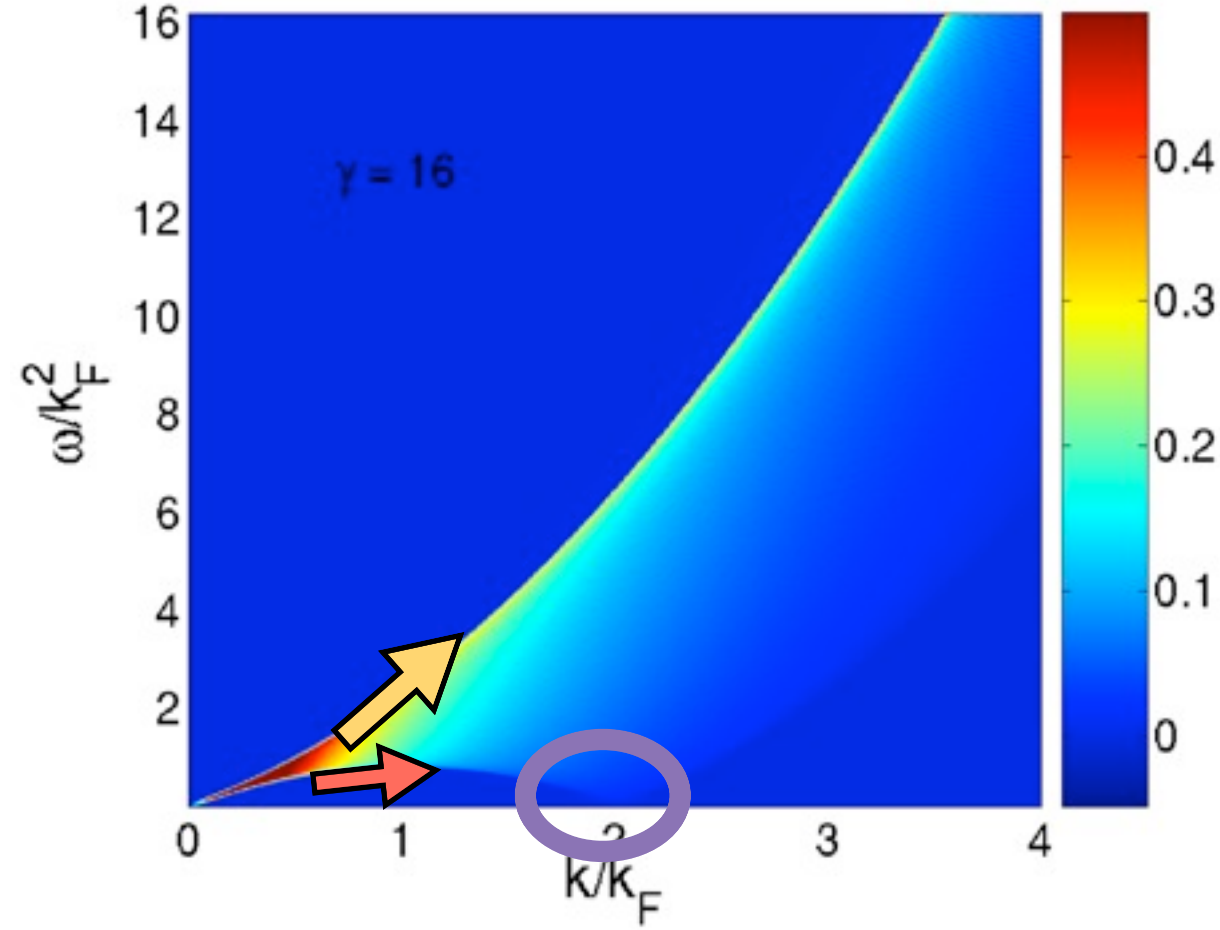

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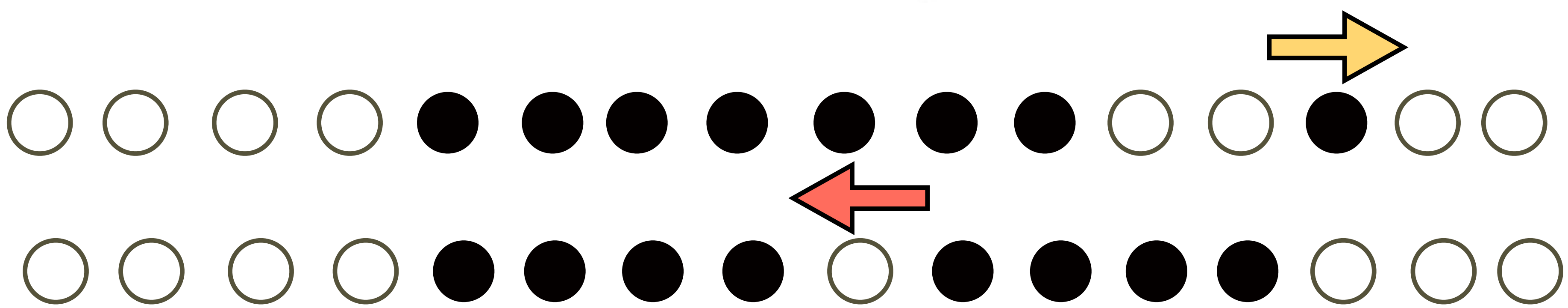
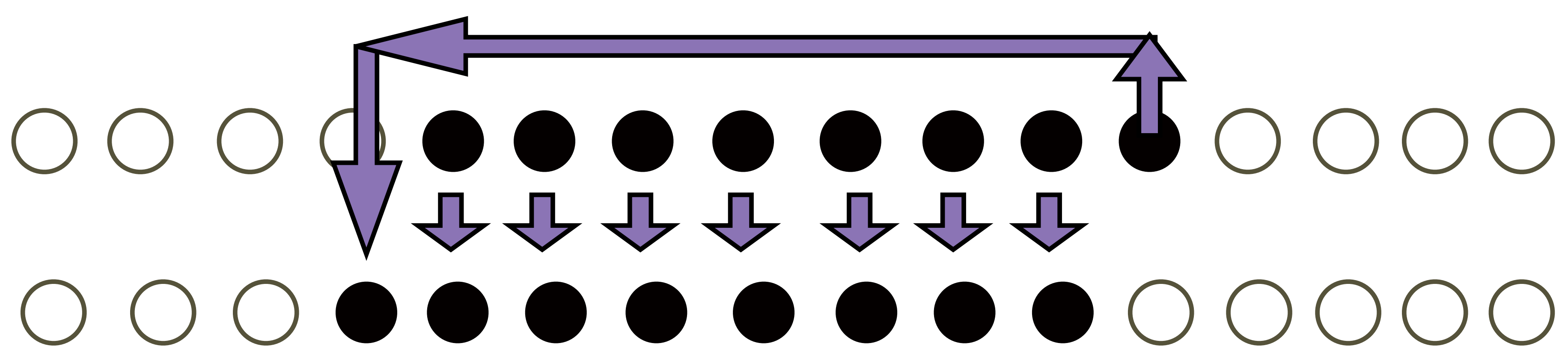


Umklapp



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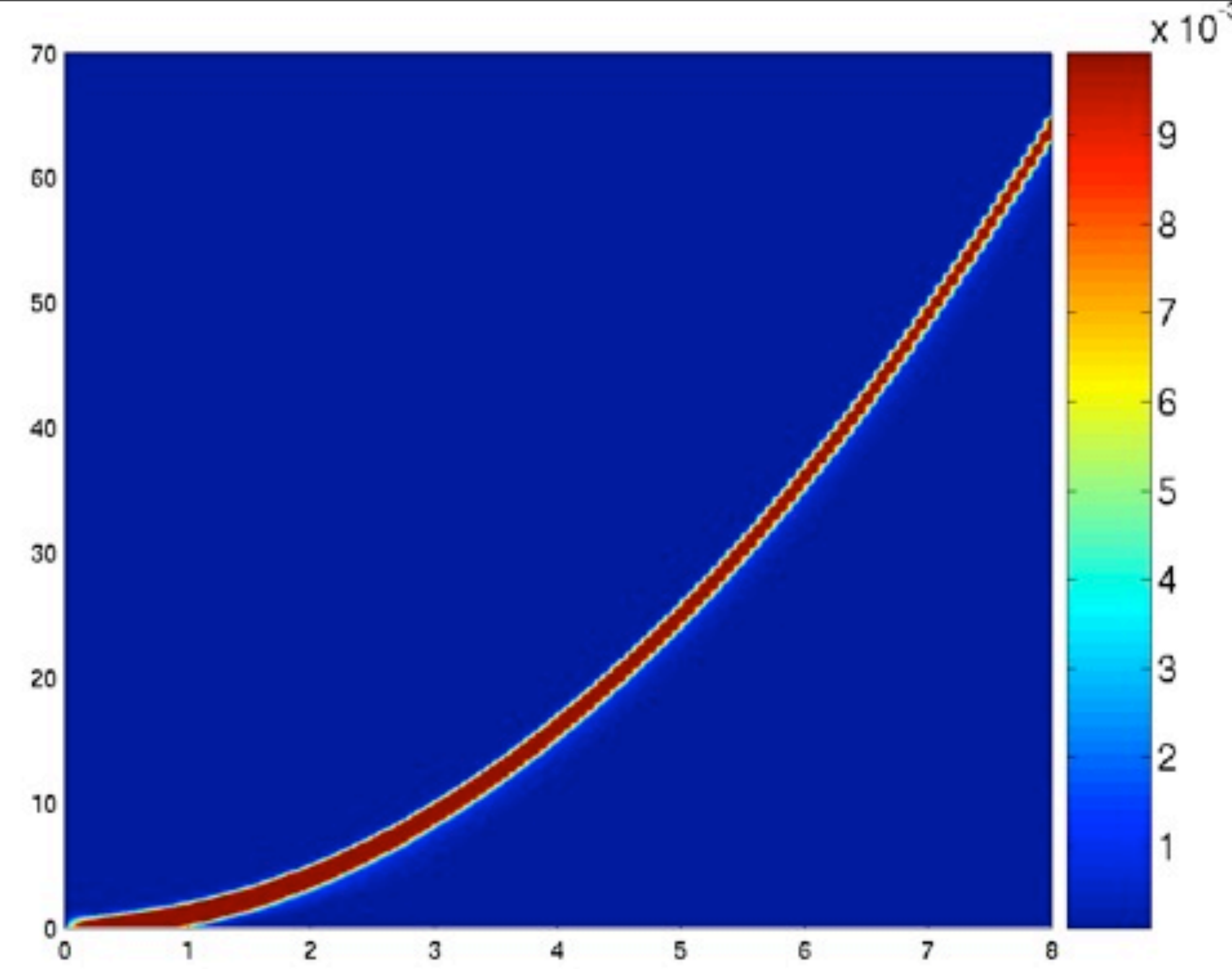

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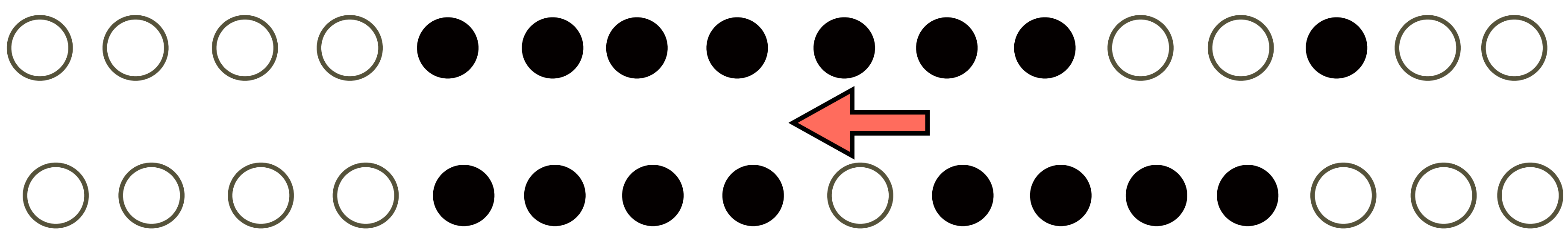

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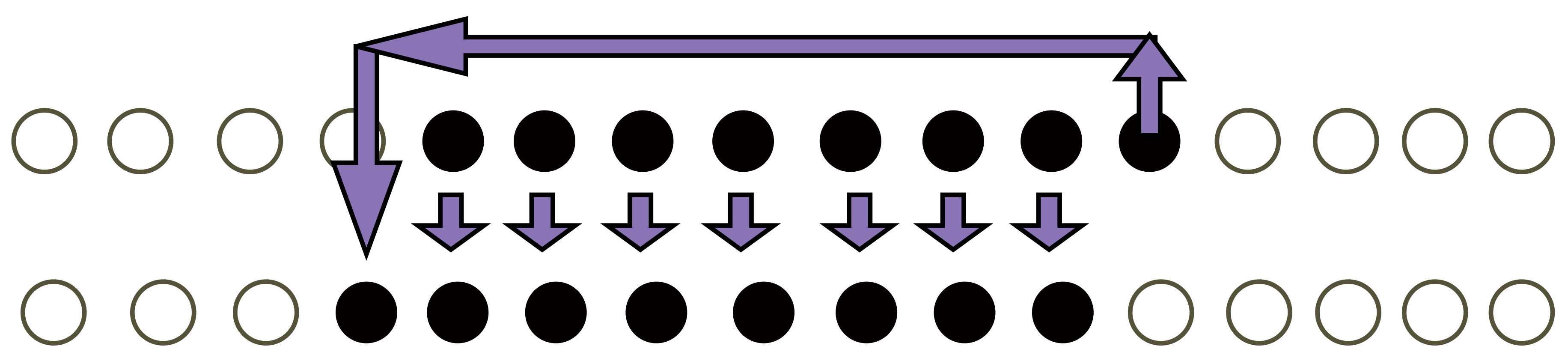
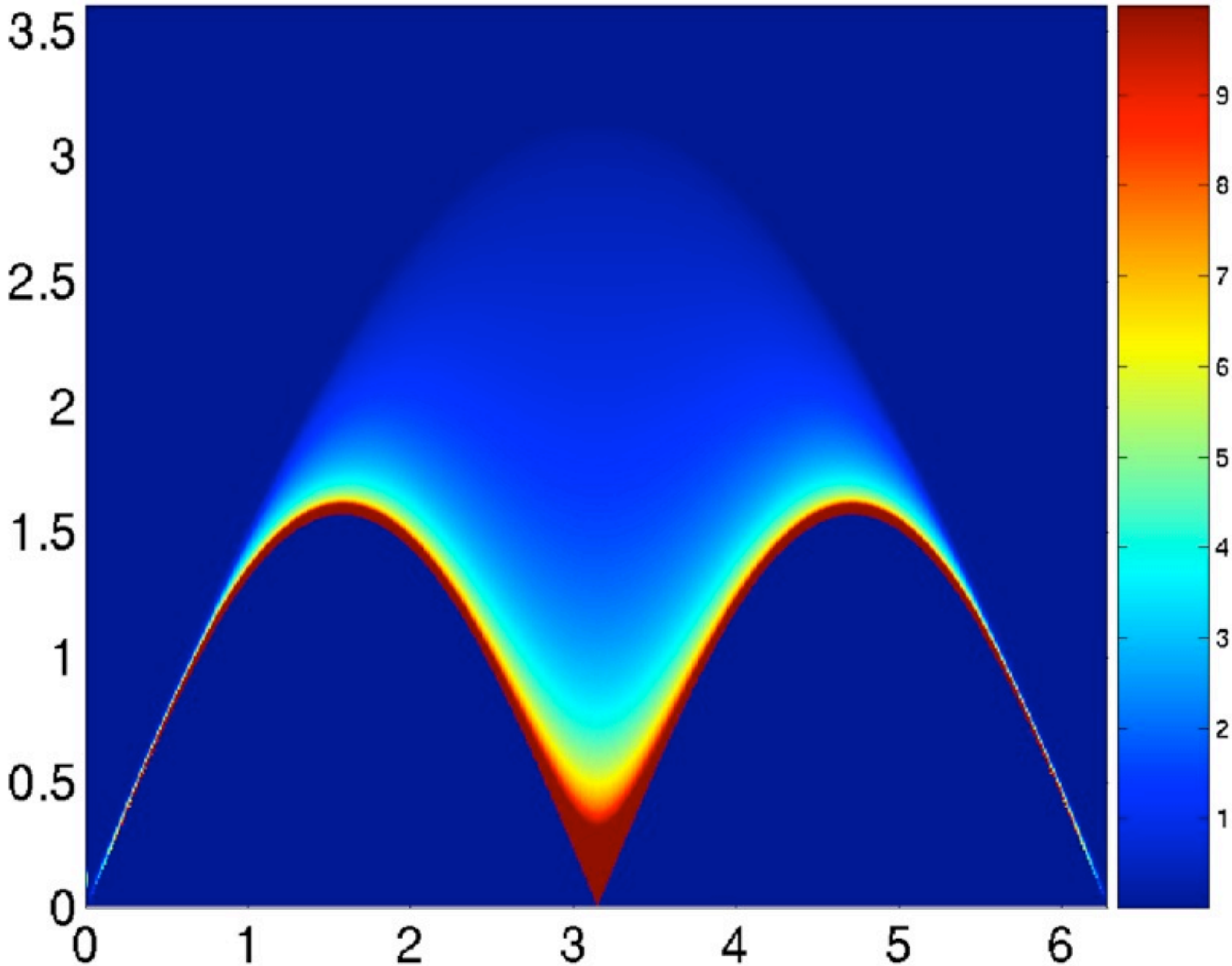
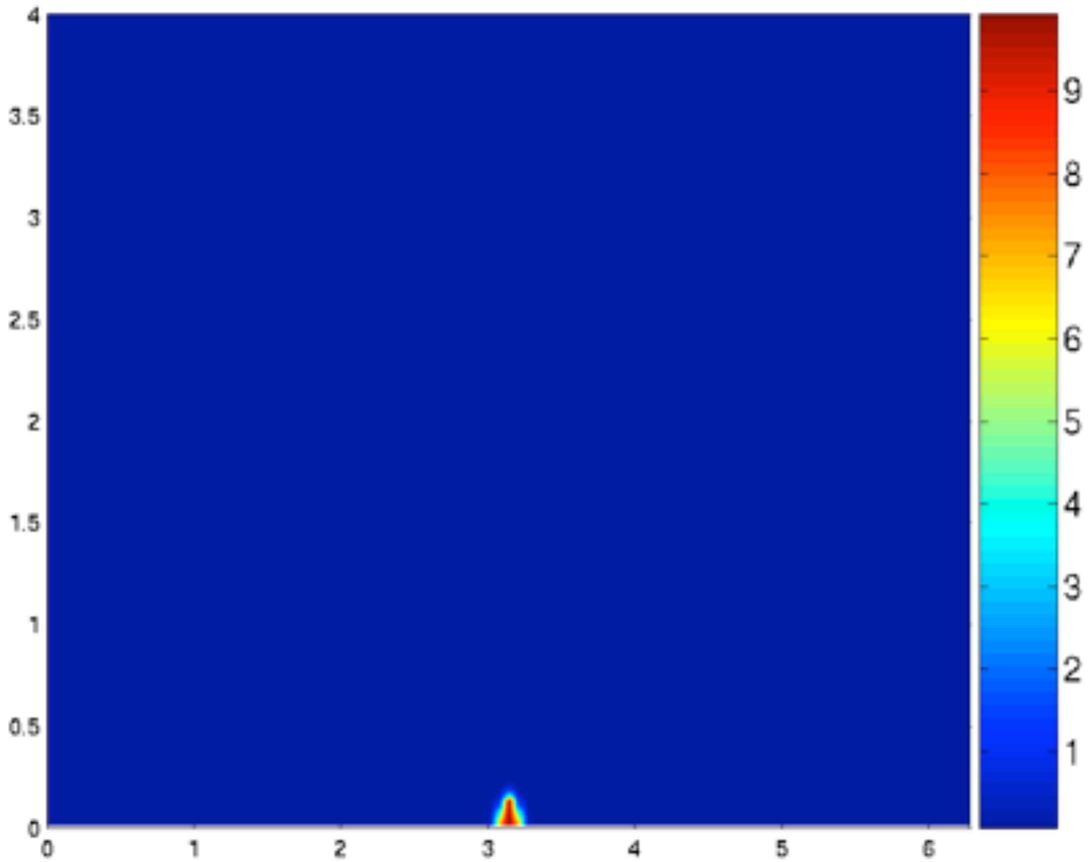


Heisenberg chains

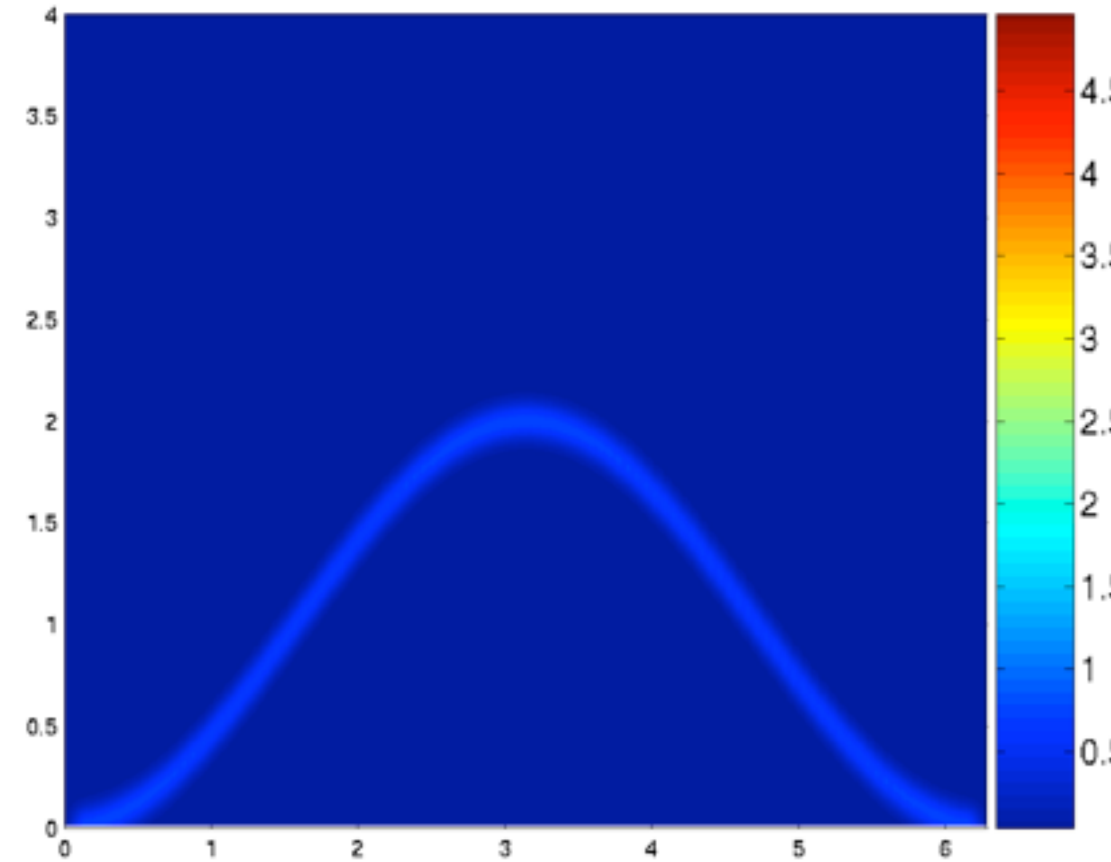
$$S(k,\omega), \quad \Delta = 1, \quad h = 0$$



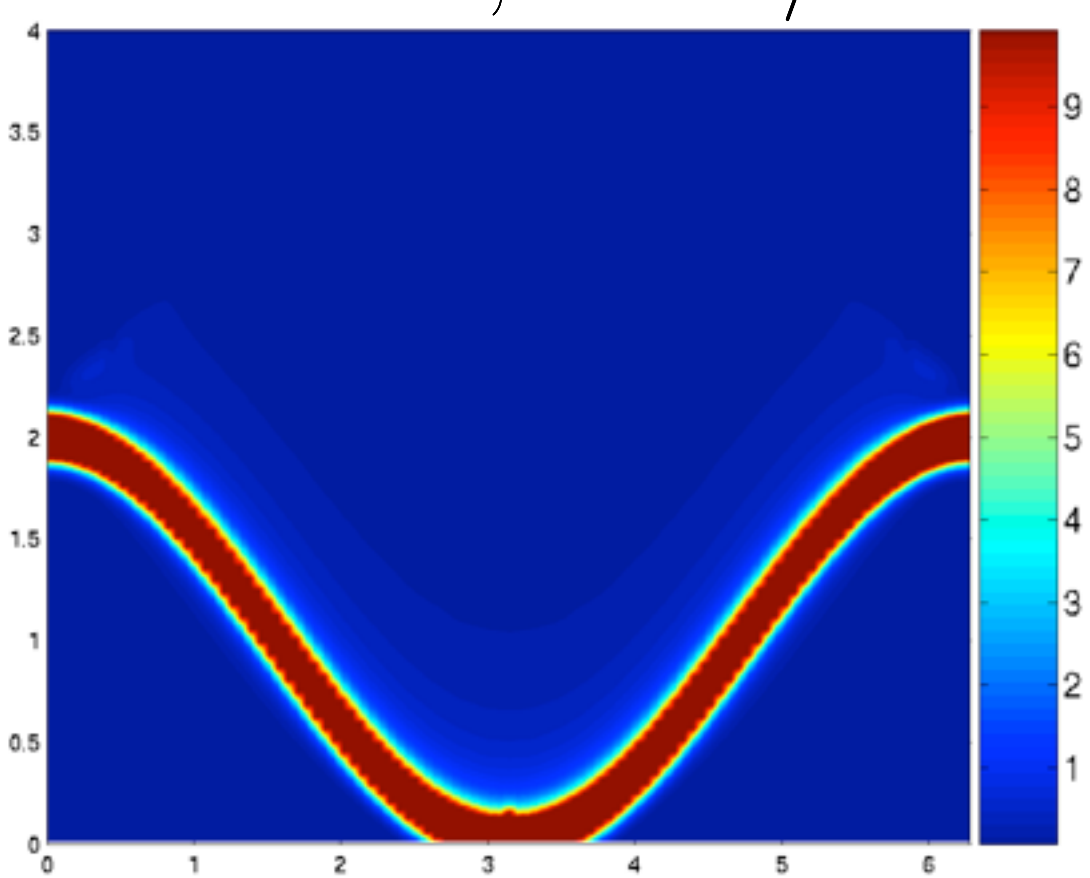
$S^{-+}, \Delta = 1/4$



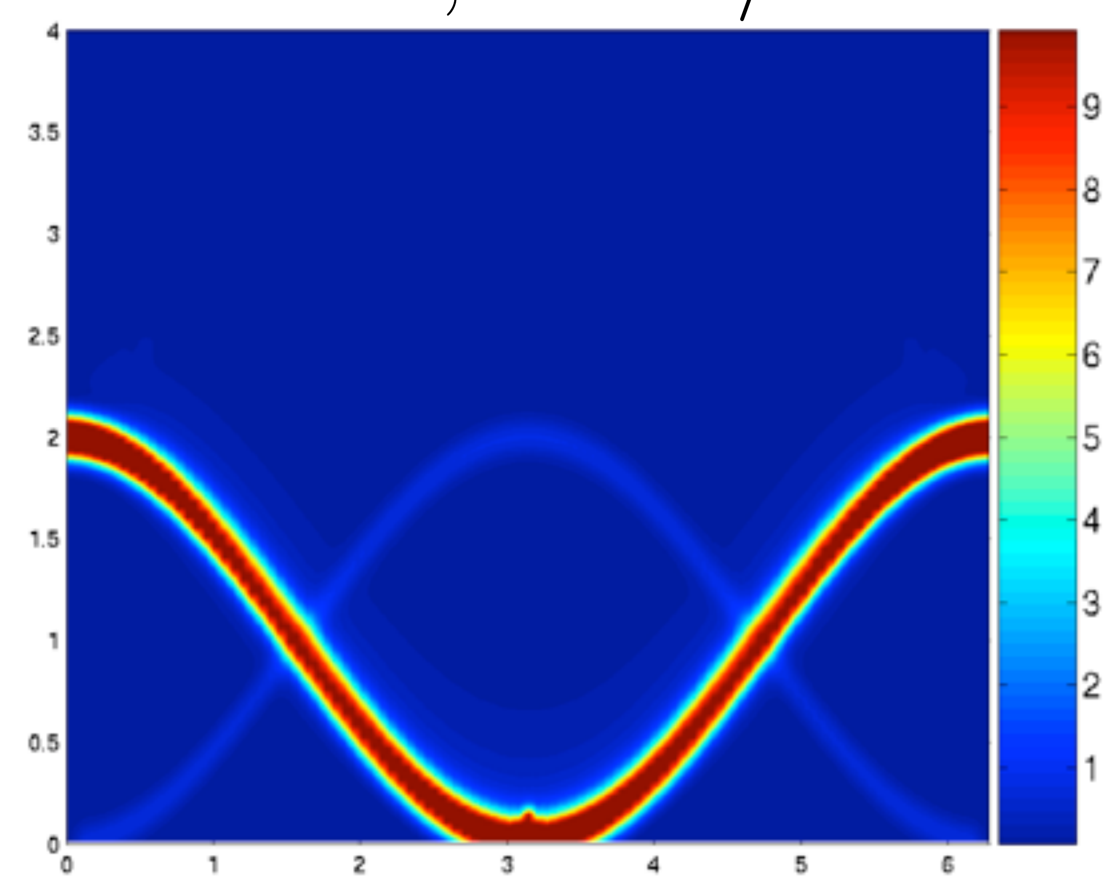
$S^{zz}, \Delta = 1/4$

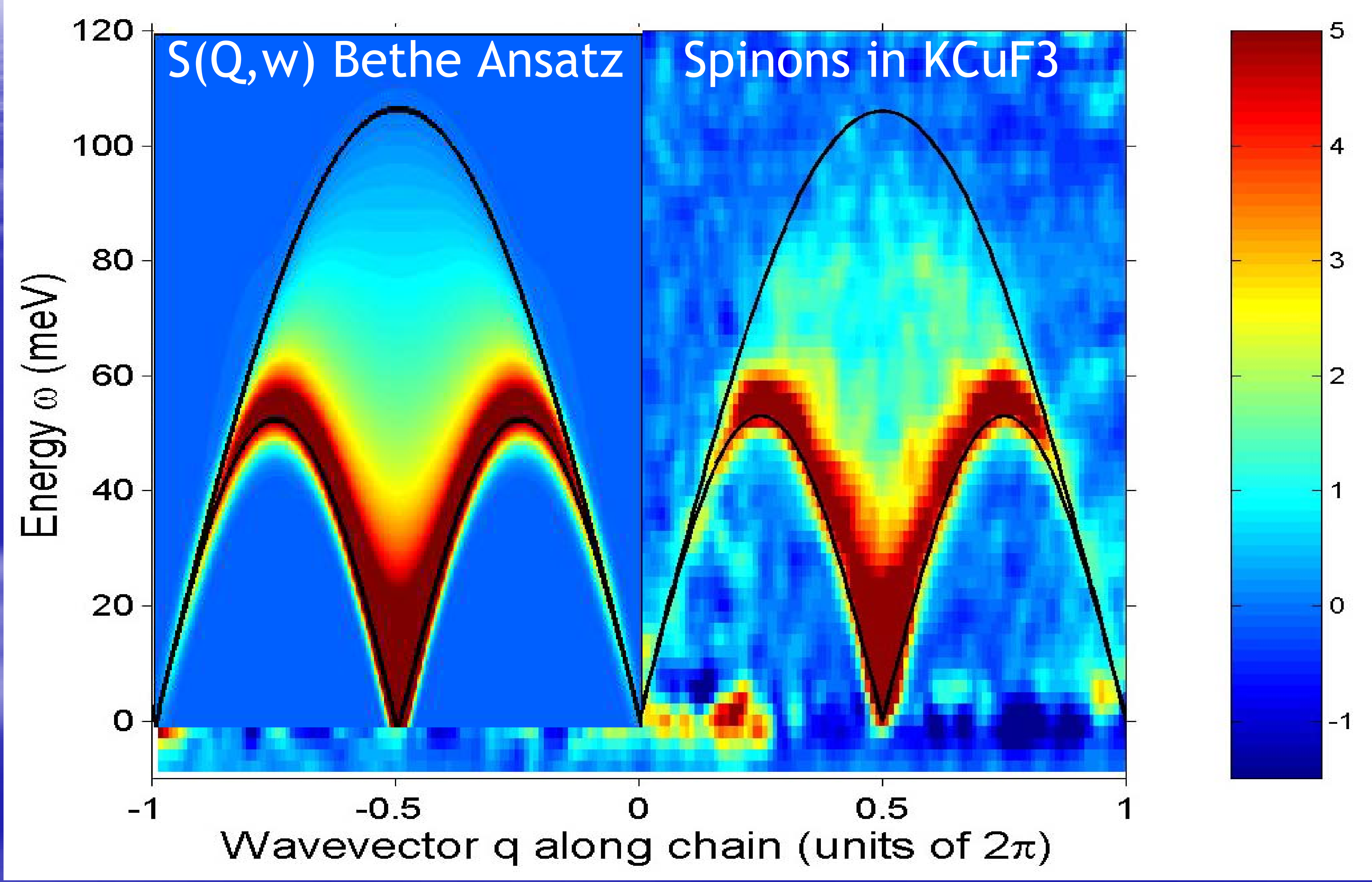


$S^{+-}, \Delta = 1/4$



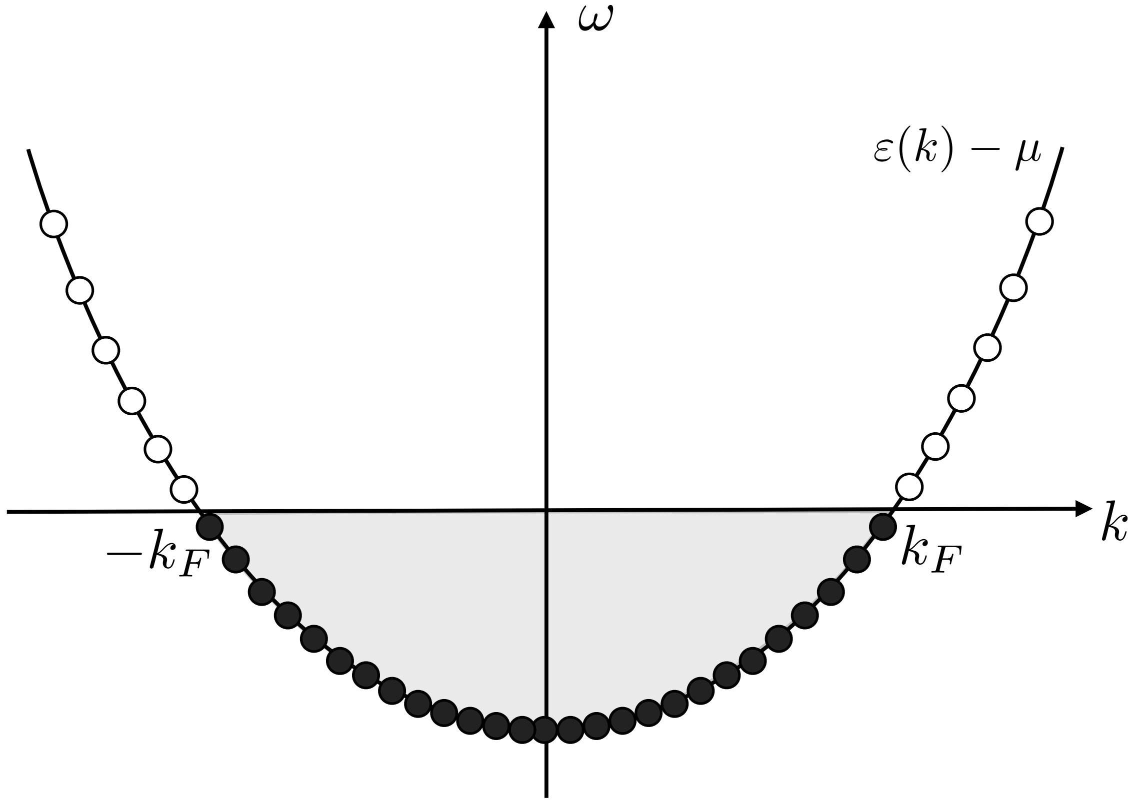
$S^{tot}, \Delta = 1/4$




S(Q,w) Bethe Ansatz
Spinons in KCuF3
120
100
80
60
40
20
0
Energy ω (meV)
-1
-0.5
0
0.5
1
Wavevector q along chain (units of 2π)
5
4
3
2
1
0
-1

Contact with field theory and (nonlinear) Luttinger liquids

Bosonization: 1d fermions



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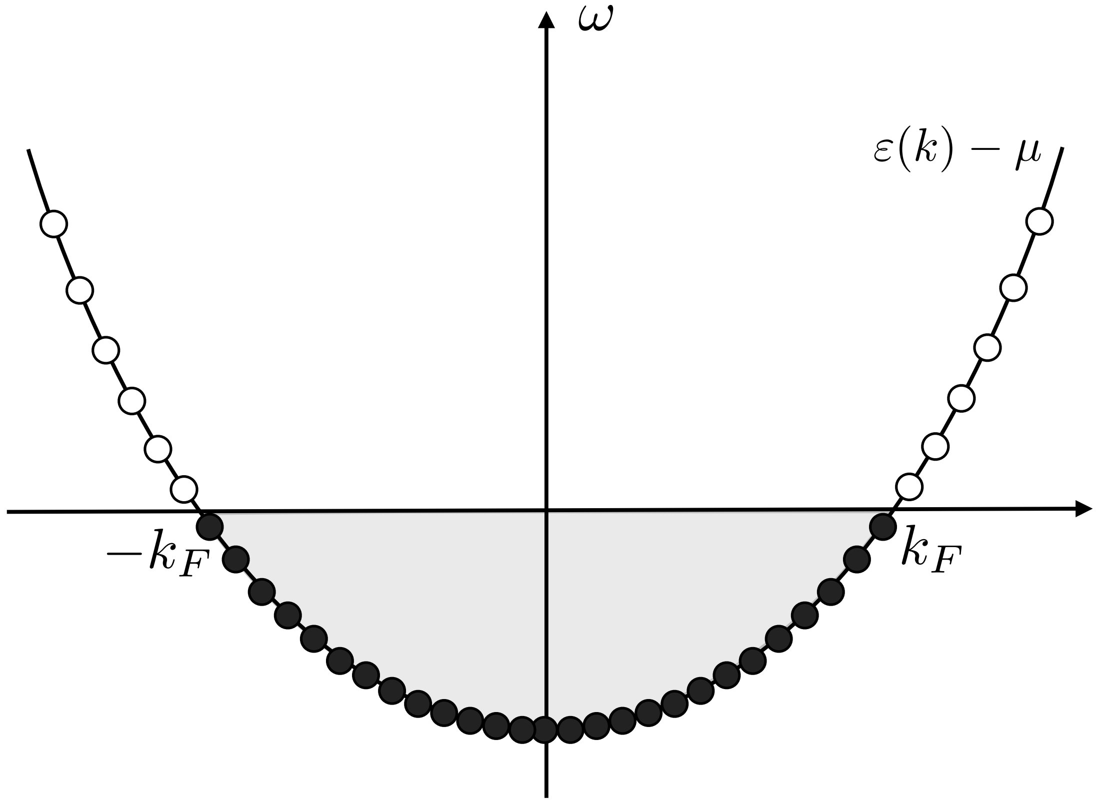
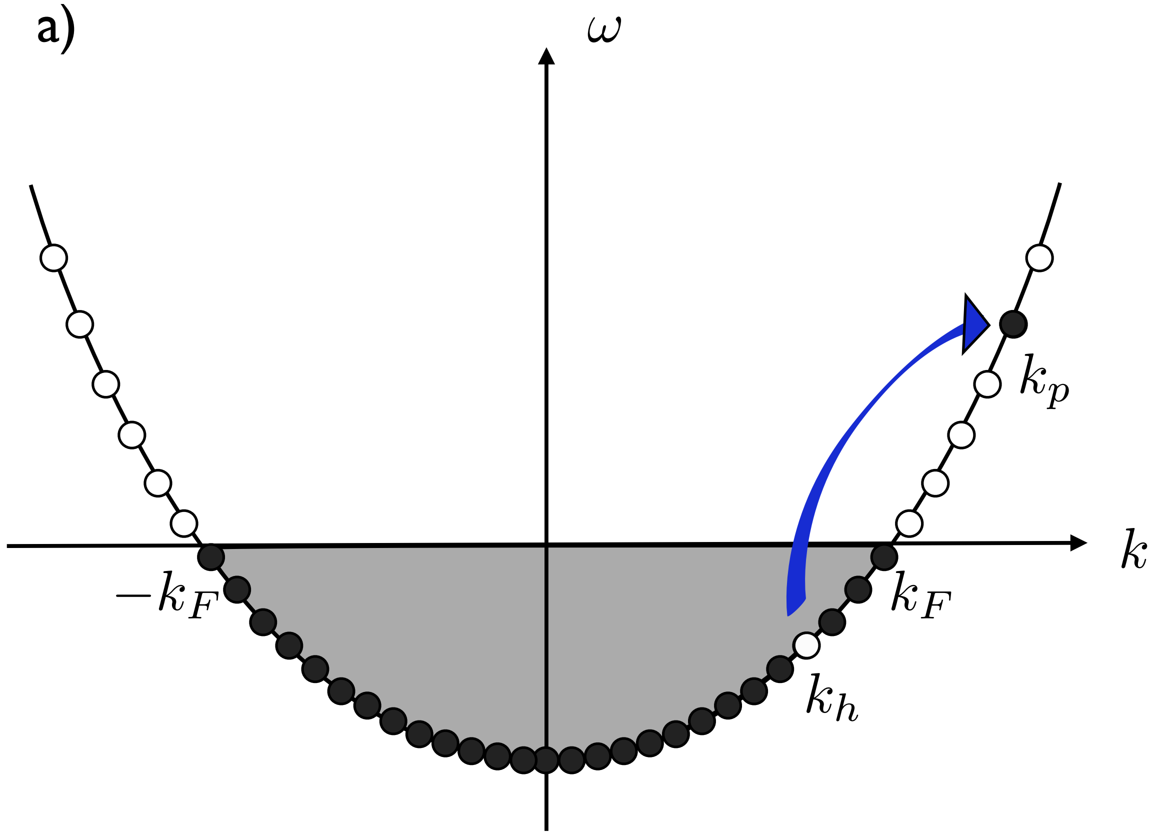


a)



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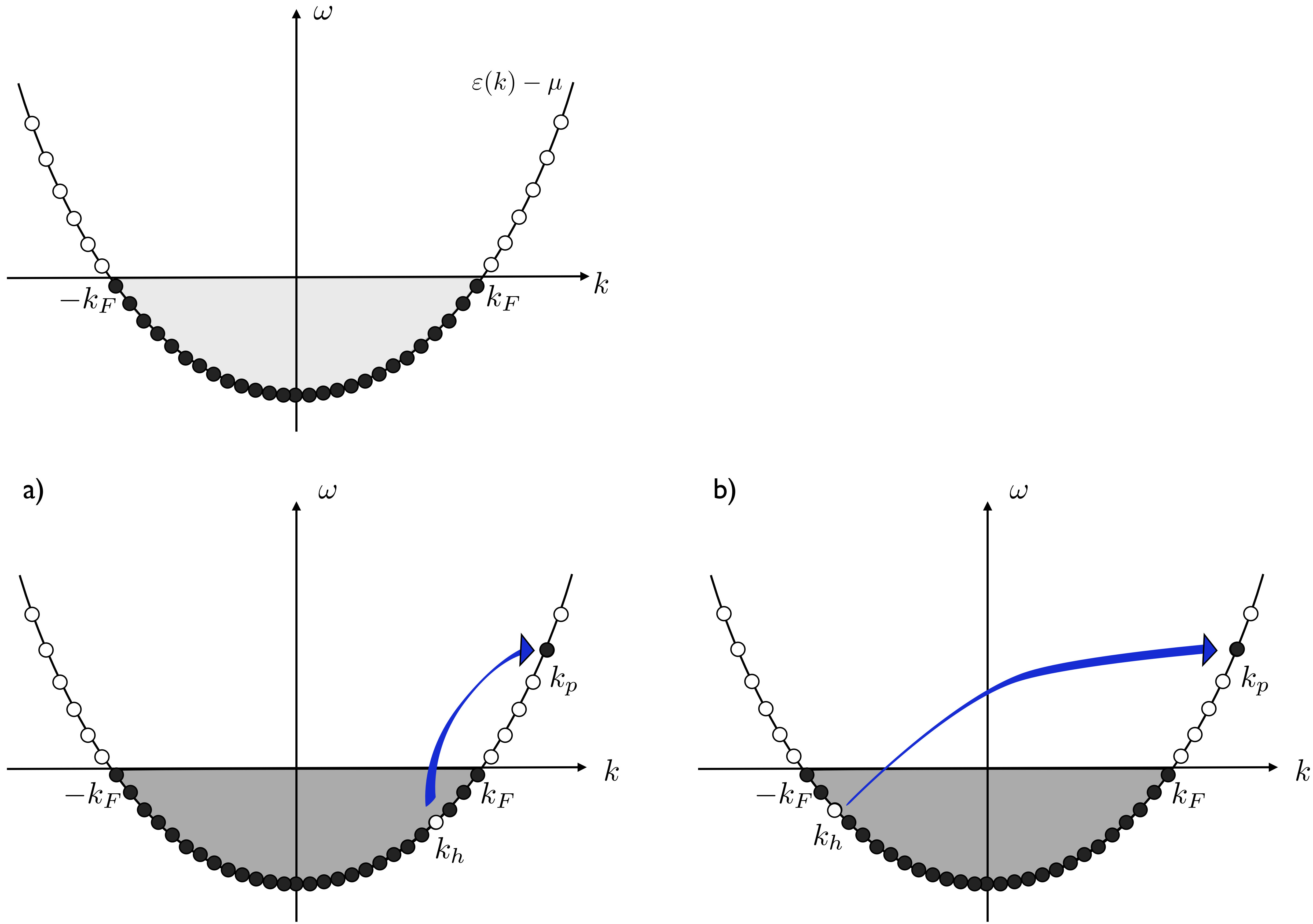

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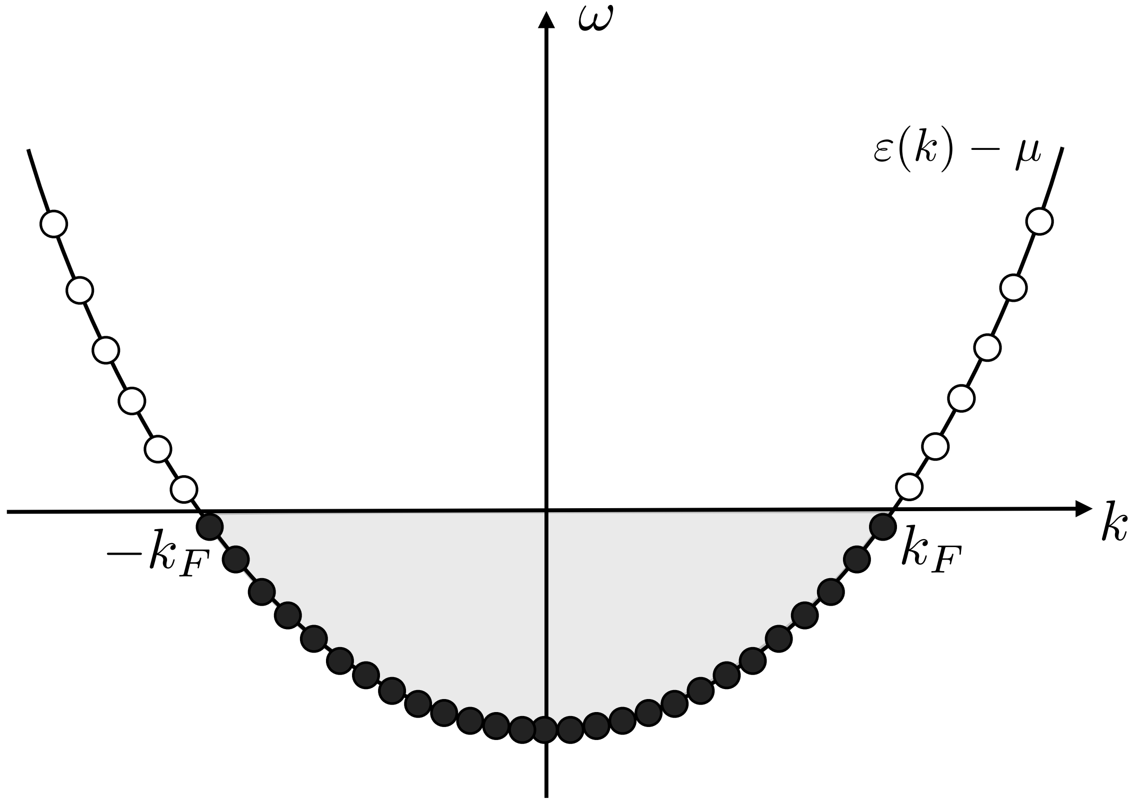
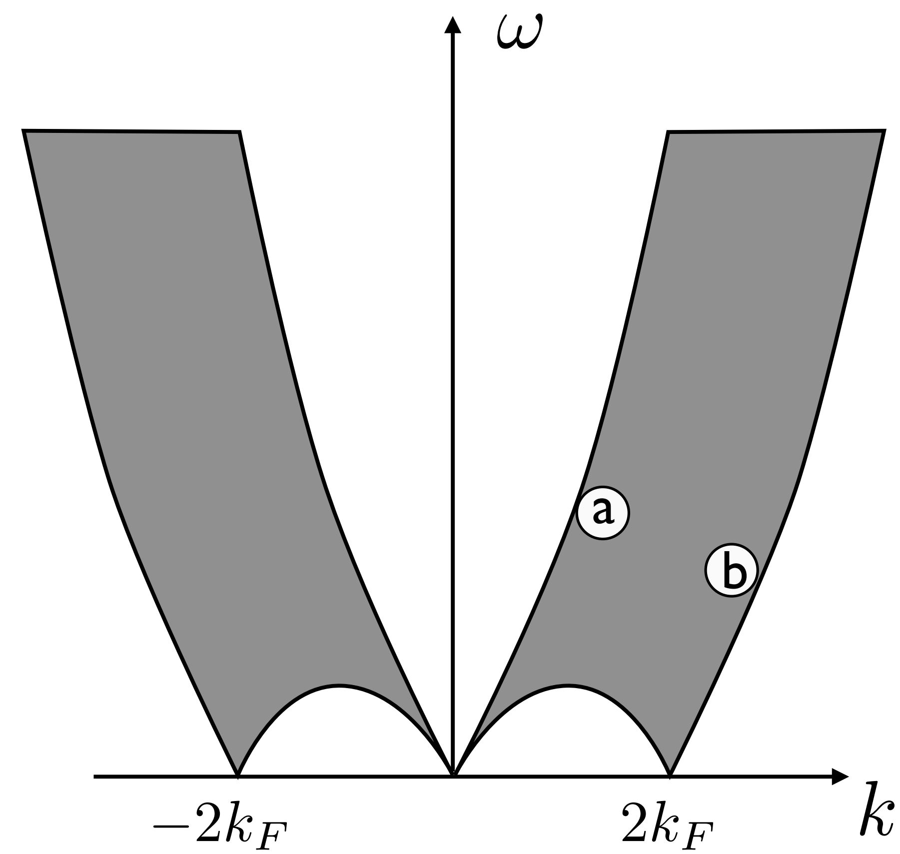
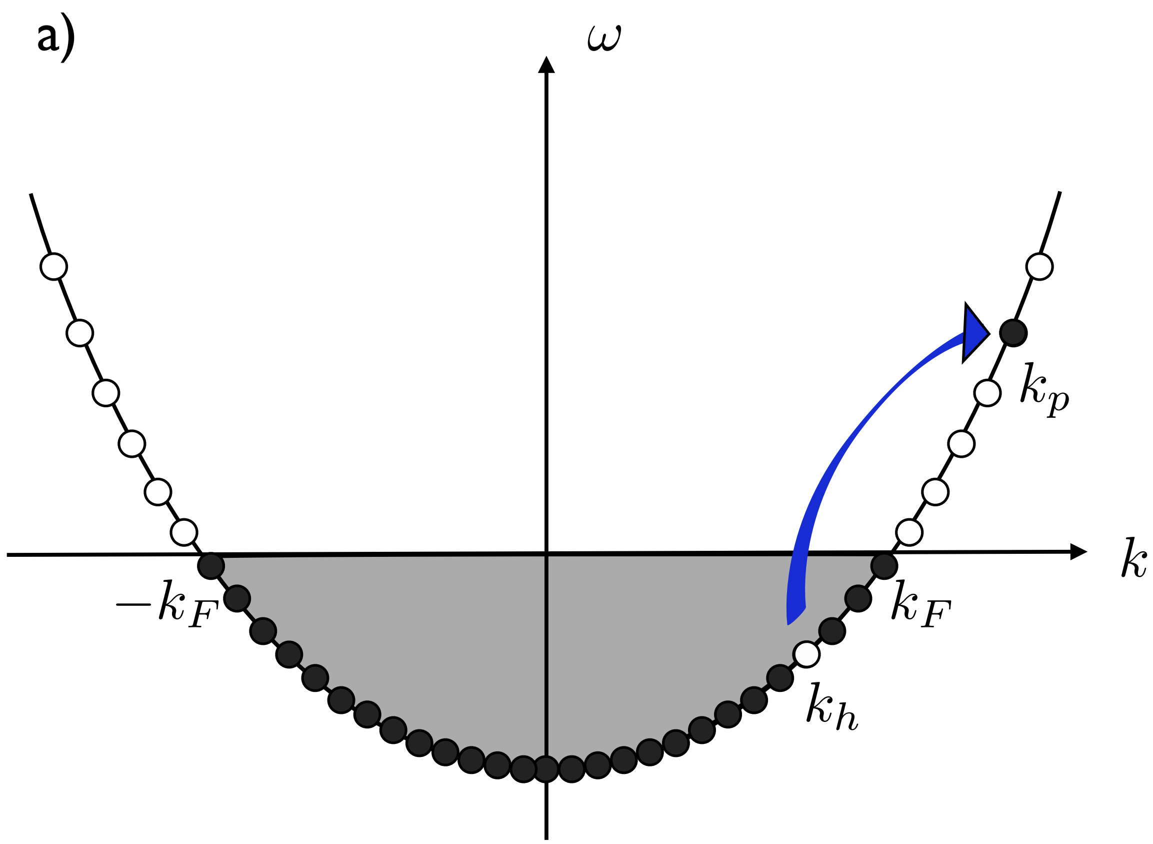
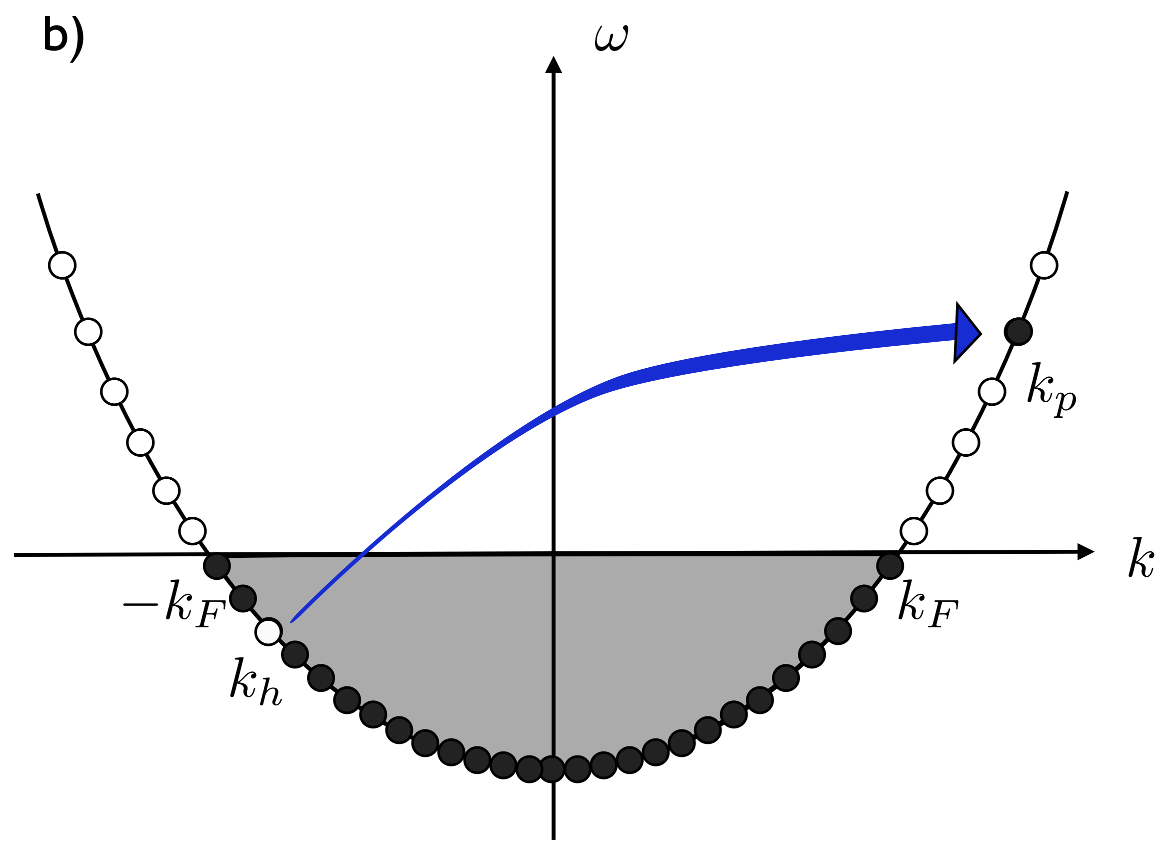


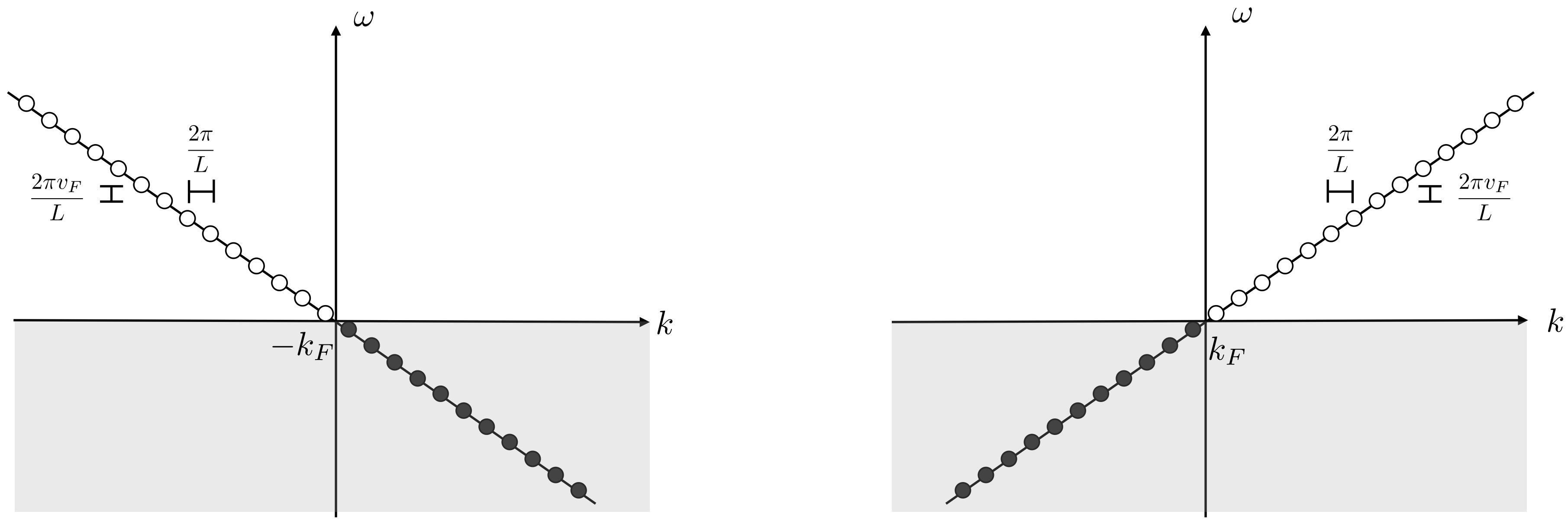






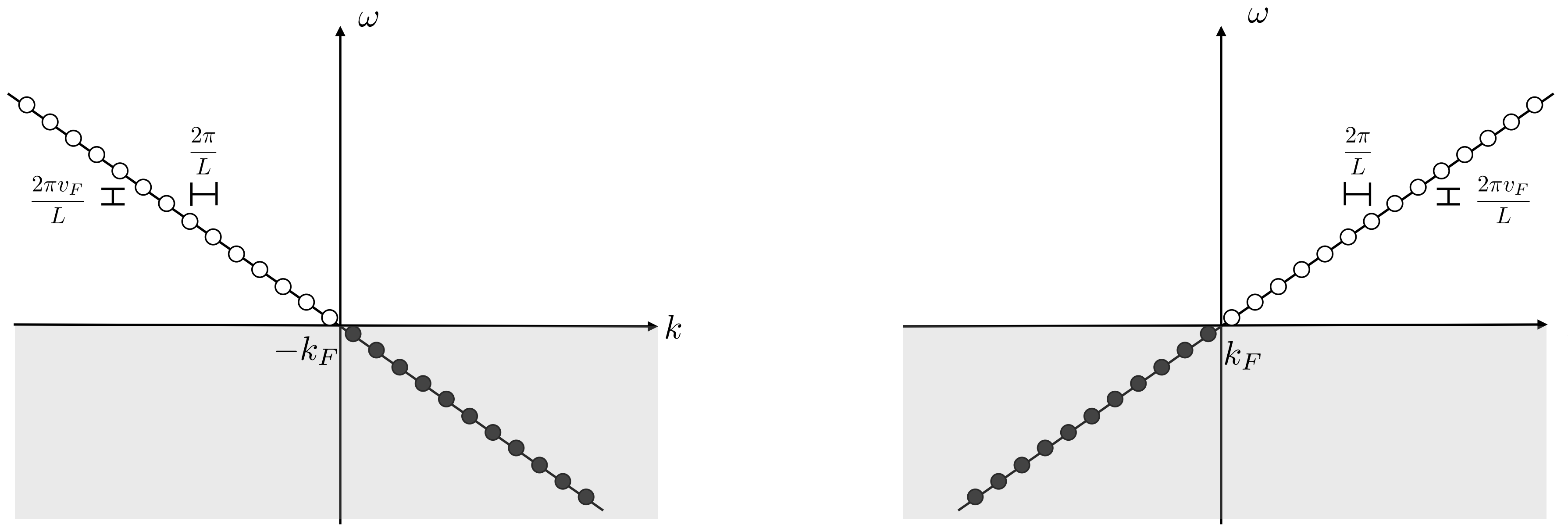
Tomonaga-Luttinger model

linearized
spectrum
(up to infinity)



Tomonaga-Luttinger model

linearized
spectrum
(up to infinity)



Luttinger liquid phenomenology (Haldane 1981)

$$H_0 = \frac{v}{2\pi}\int dx\,\left(K(\nabla\theta)^2 + \frac{1}{K}(\nabla\phi)^2\right)$$

$$[\phi(x), \nabla\theta(x')] = i\pi\delta(x - x')$$

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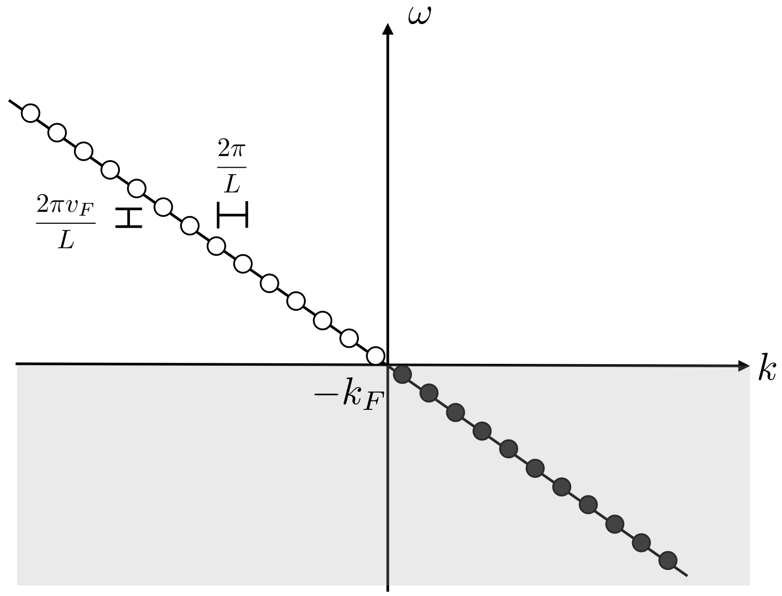

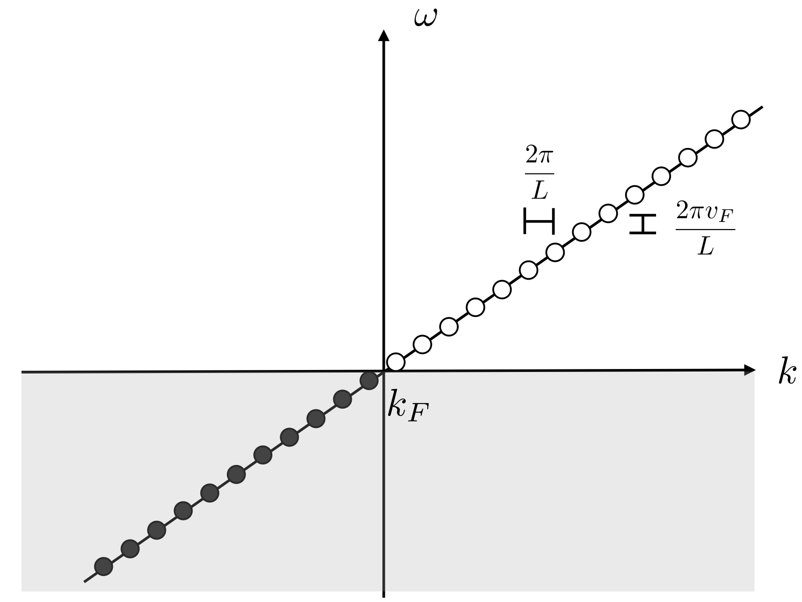

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correlation functions at low energies

Around momentum $(2m + 1/2 \pm 1/2)k_F$

the fields are represented as

$$\psi_{F(B)}(x,t) \sim e^{i(2m+1/2\pm 1/2)[k_F x - \phi(x,t)] + i\theta(x,t)}$$

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LL theory predicts the asymptotics $\rho_0 x \gg 1$

of correlation functions as

$$\frac{\langle \hat{\rho}(x)\hat{\rho}(0)\rangle}{\rho_0^2} \approx 1 - \frac{K}{2(\pi\rho_0 x)^2} + \sum_{m\geq1} \frac{A_m \cos(2mk_F x)}{(\rho_0 x)^{2m^2 K}}$$

$$\frac{\langle \hat{\psi}_B^\dagger(x)\hat{\psi}_B(0)\rangle}{\rho_0} \approx \sum_{m\geq0} \frac{B_m \cos(2mk_F x)}{(\rho_0 x)^{2m^2 K + 1/(2K)}}$$

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Infinite sets of nonuniversal prefactors

$$\ldots \frac{K}{2(\pi\rho_0 x)^2} + \sum_{m\geq 1} \frac{A_m \cos(2mk_F x)}{(\rho_0 x)^{2m^2K}}$$

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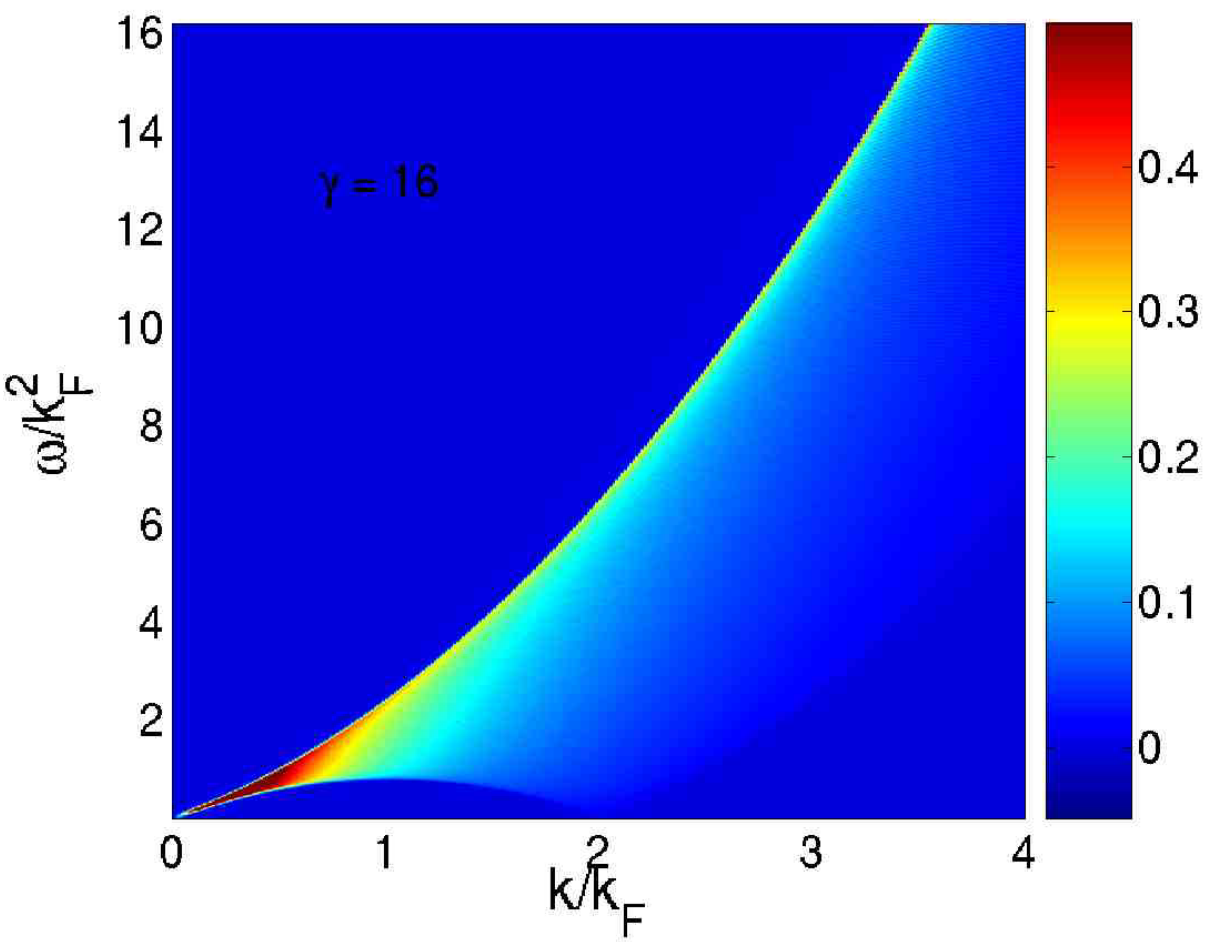
Asymptotes of static function: determined by correlation around Umklapp modes

$S^{\rho\rho}(k,\omega)$



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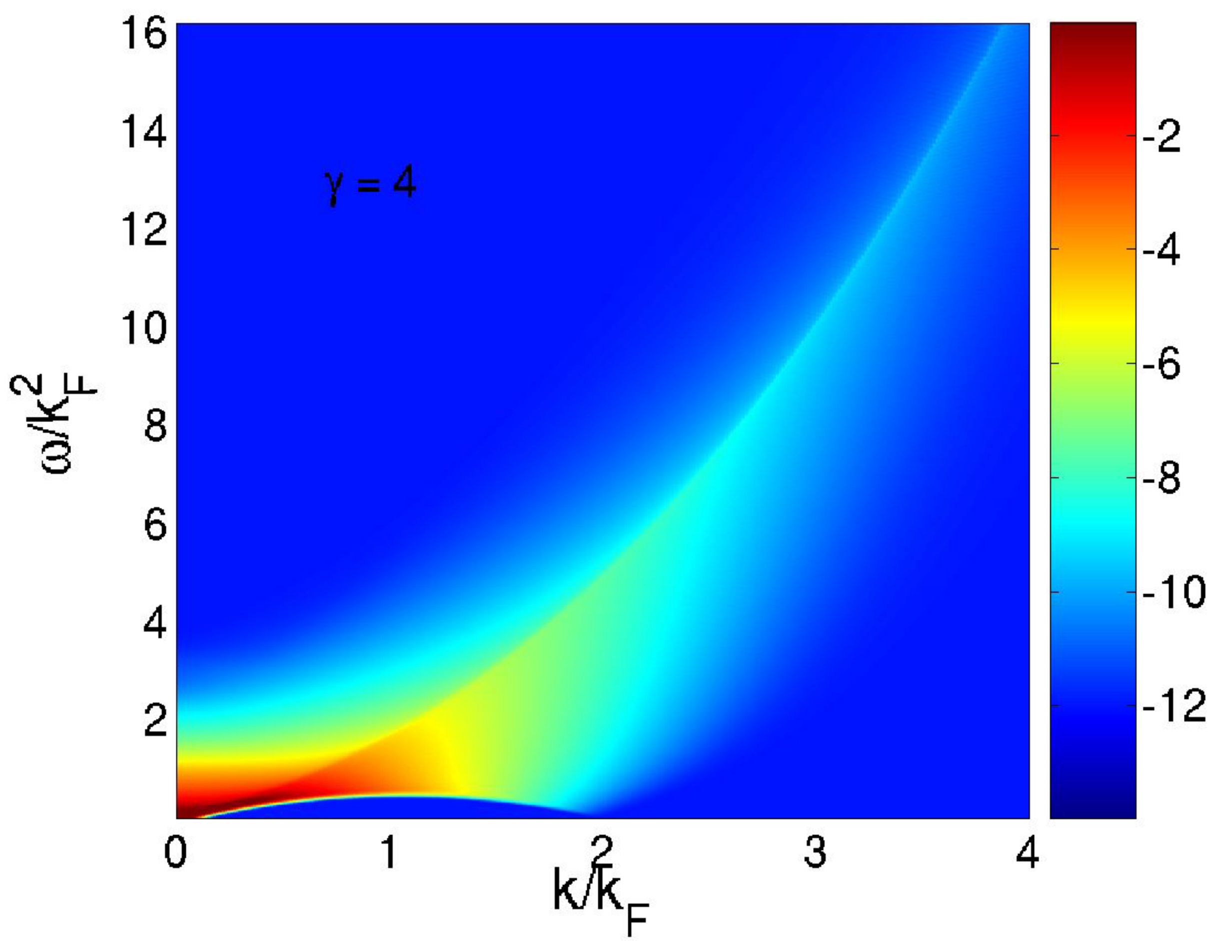

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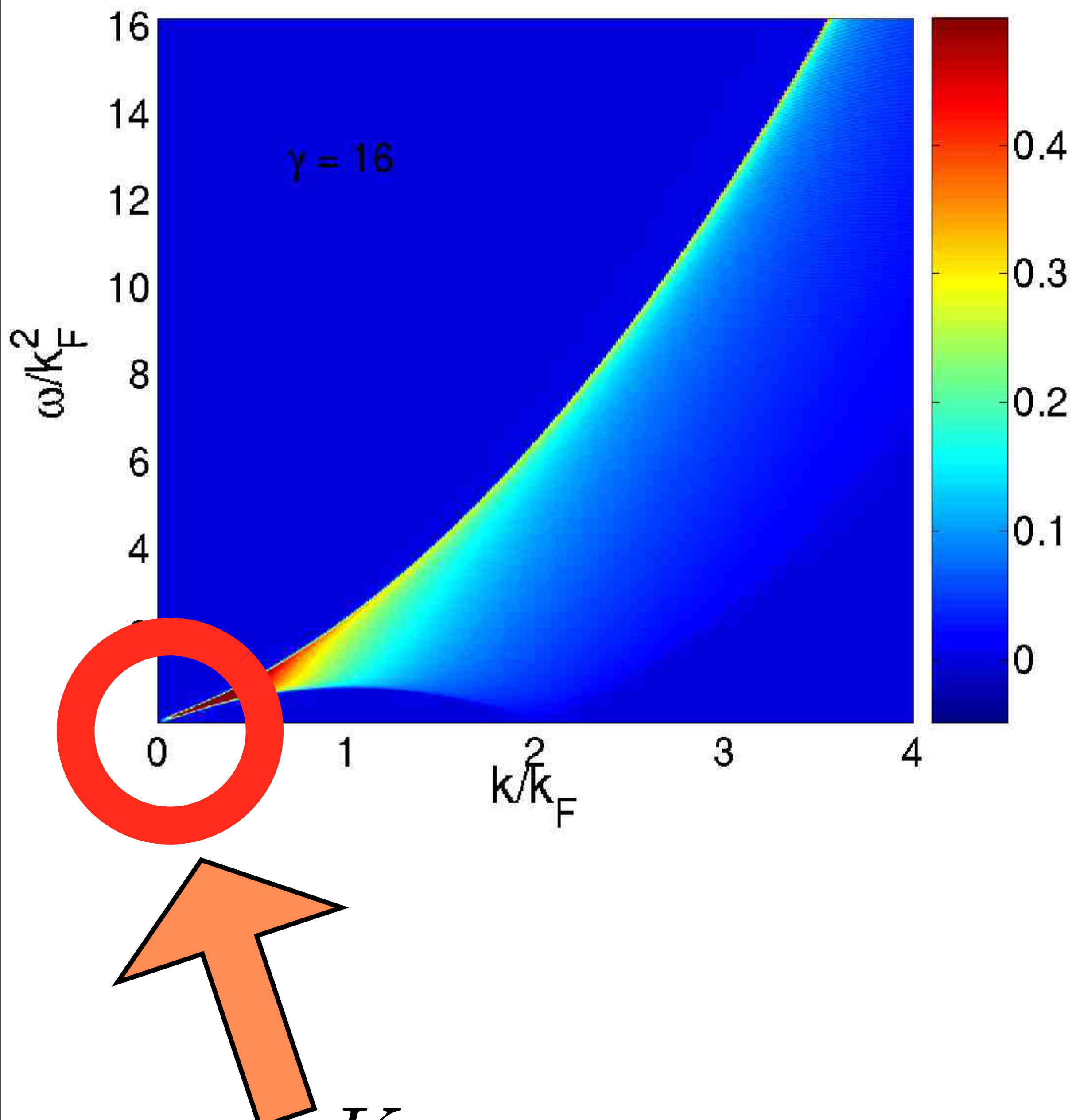

$$-\frac{K}{2\pi^2}$$

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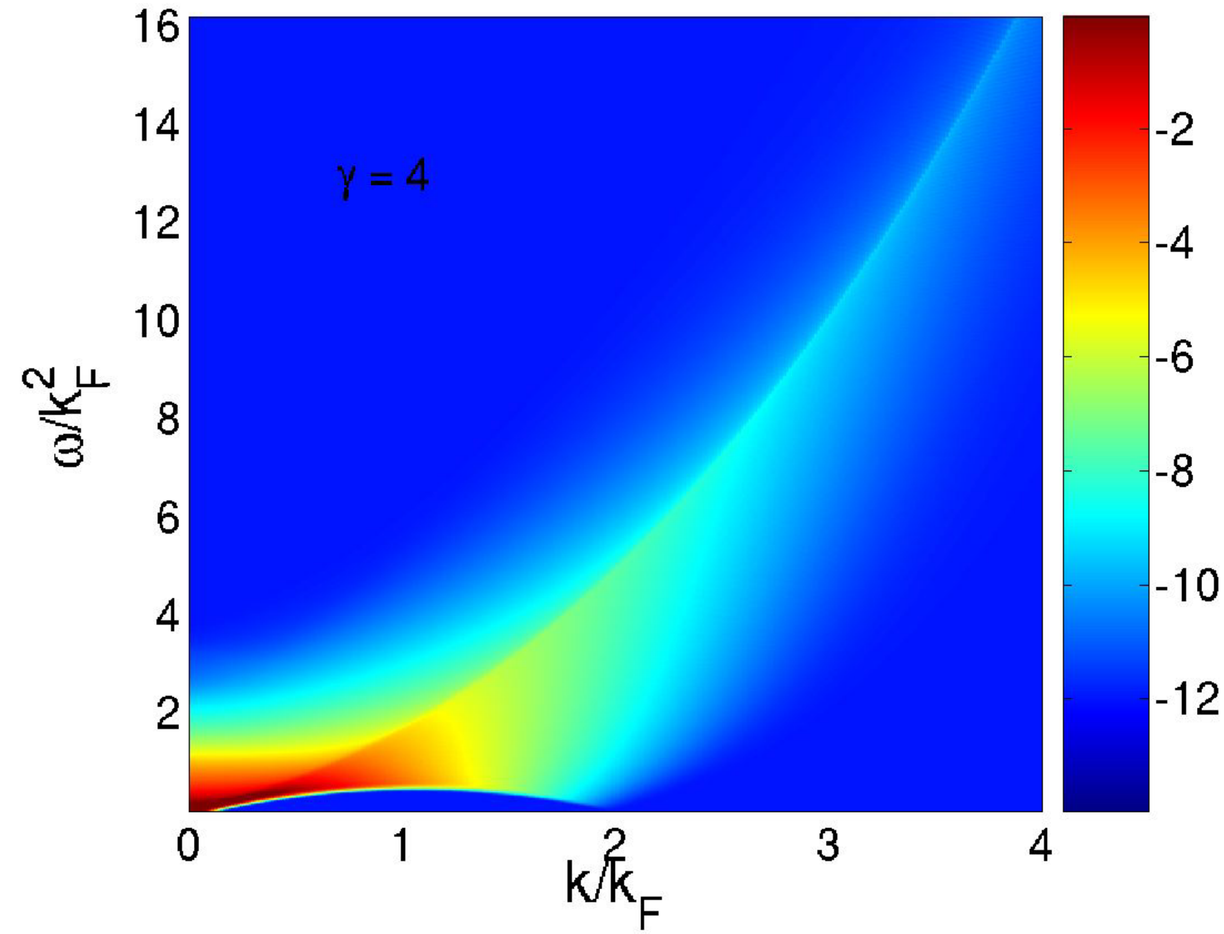

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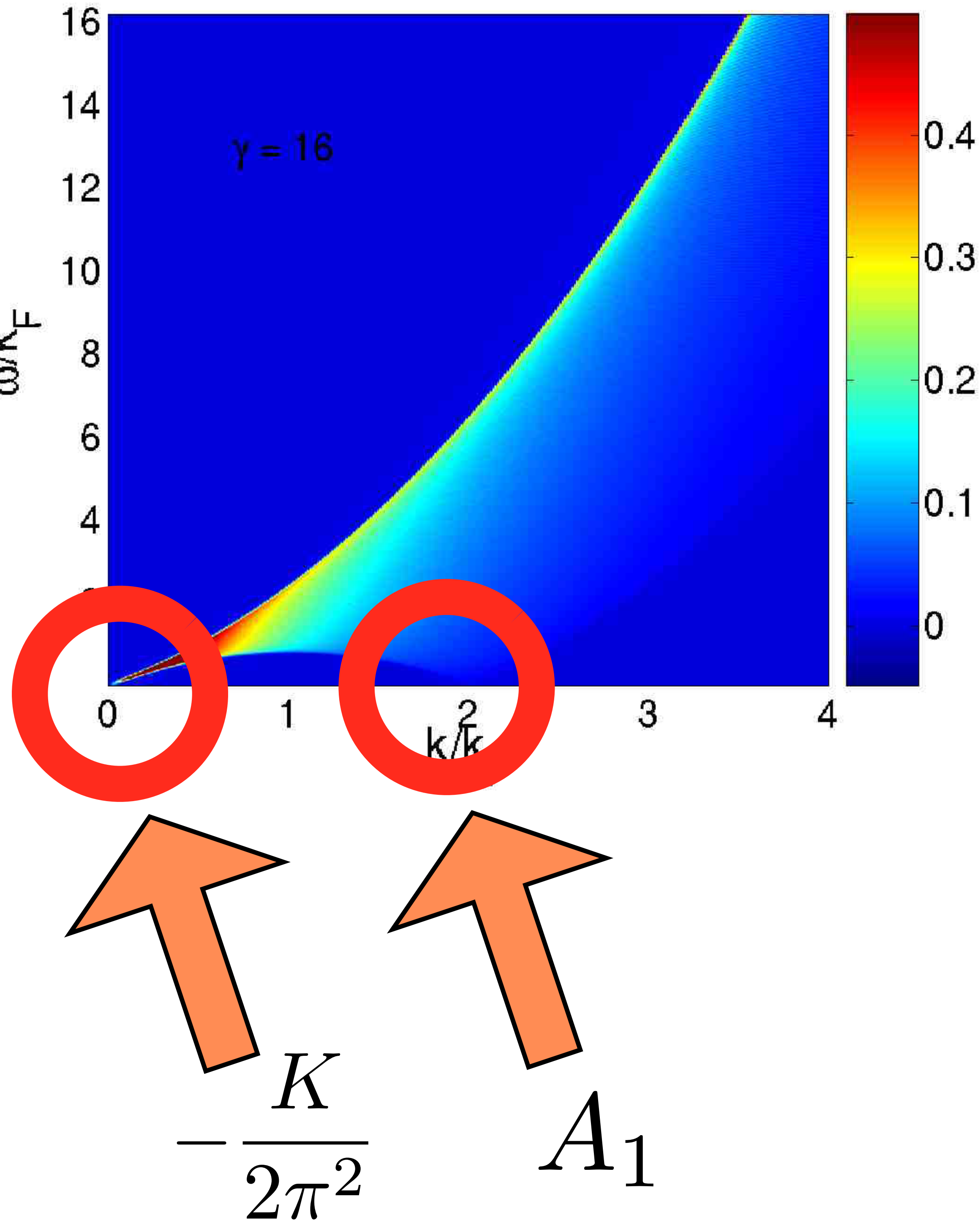

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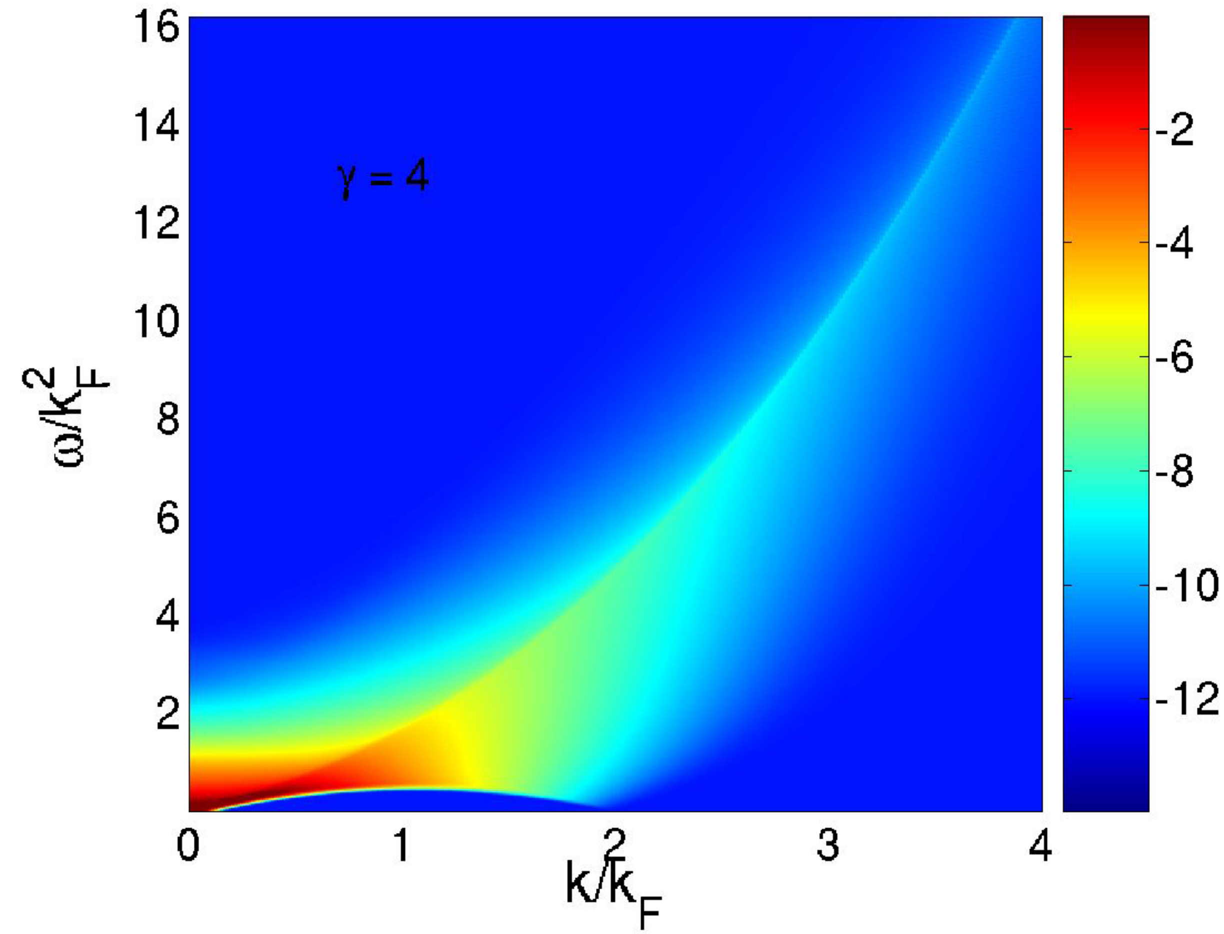

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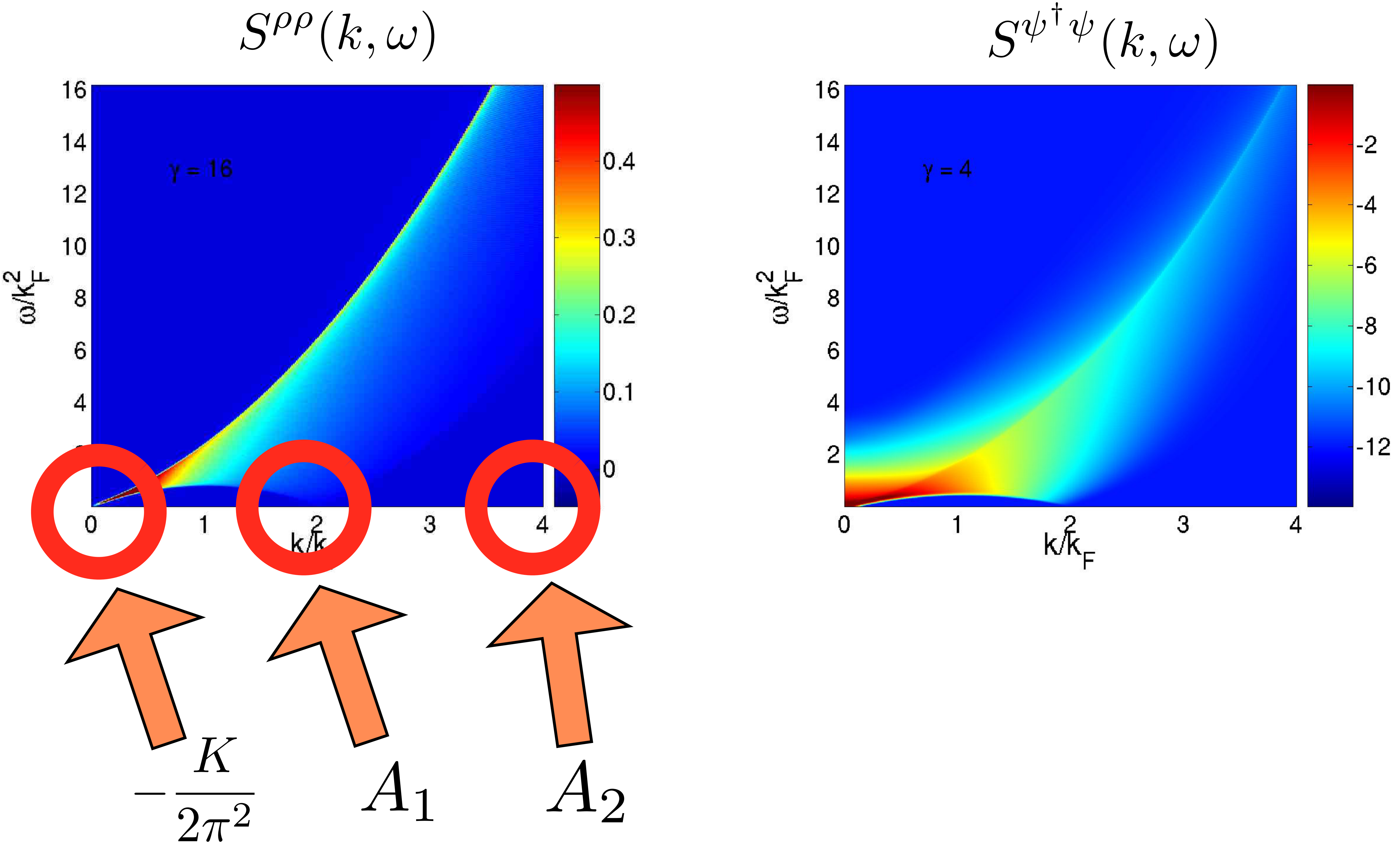

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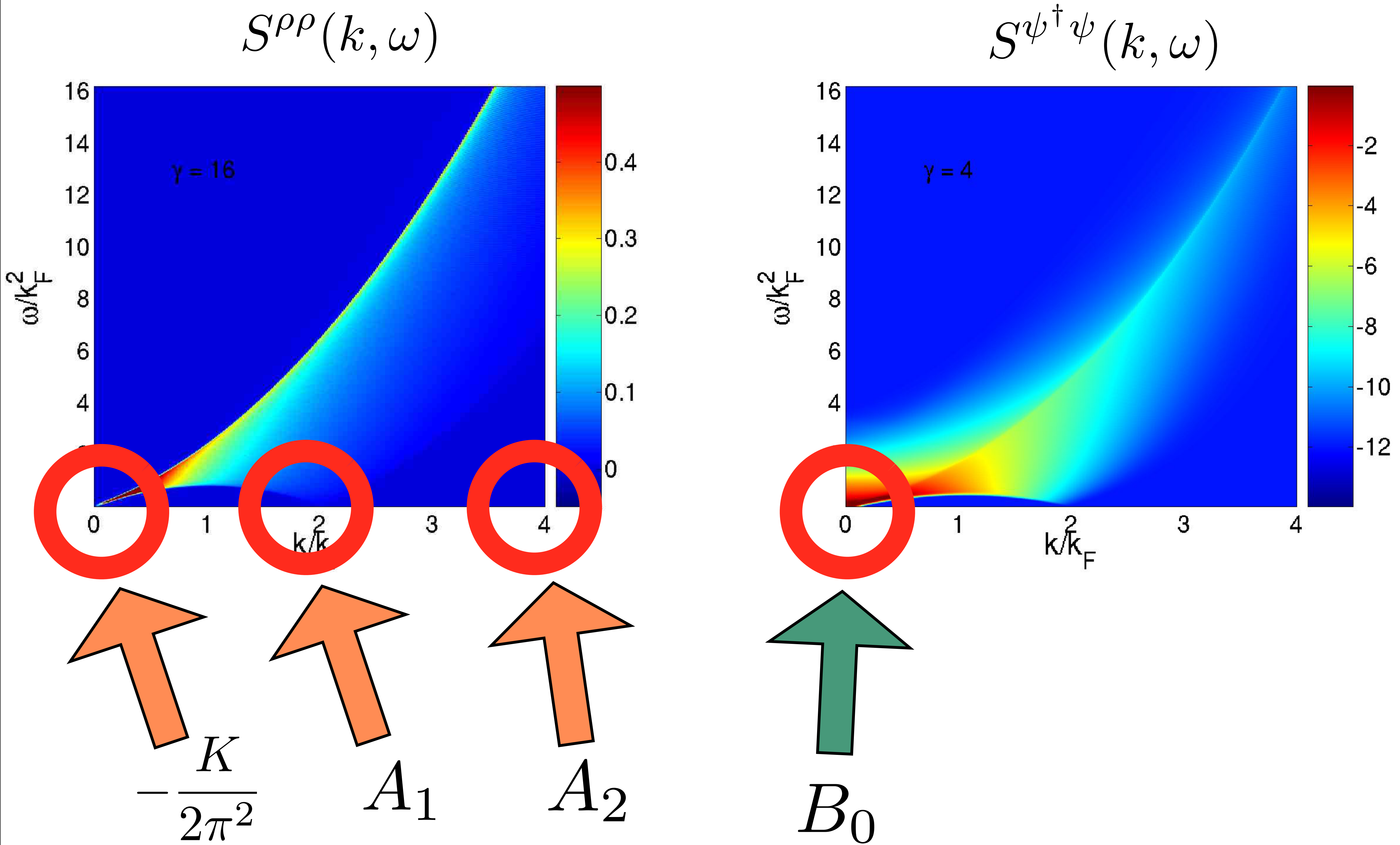

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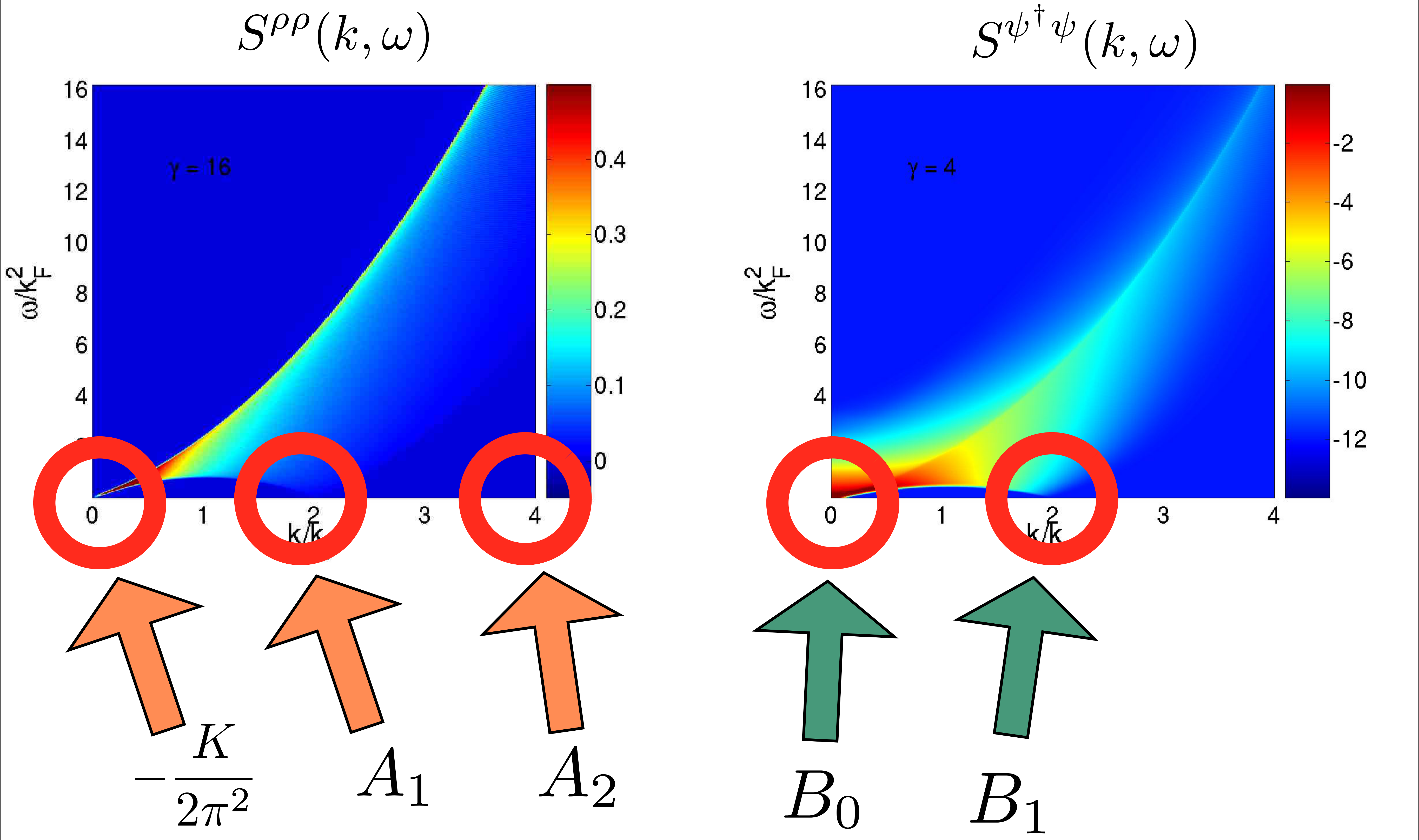

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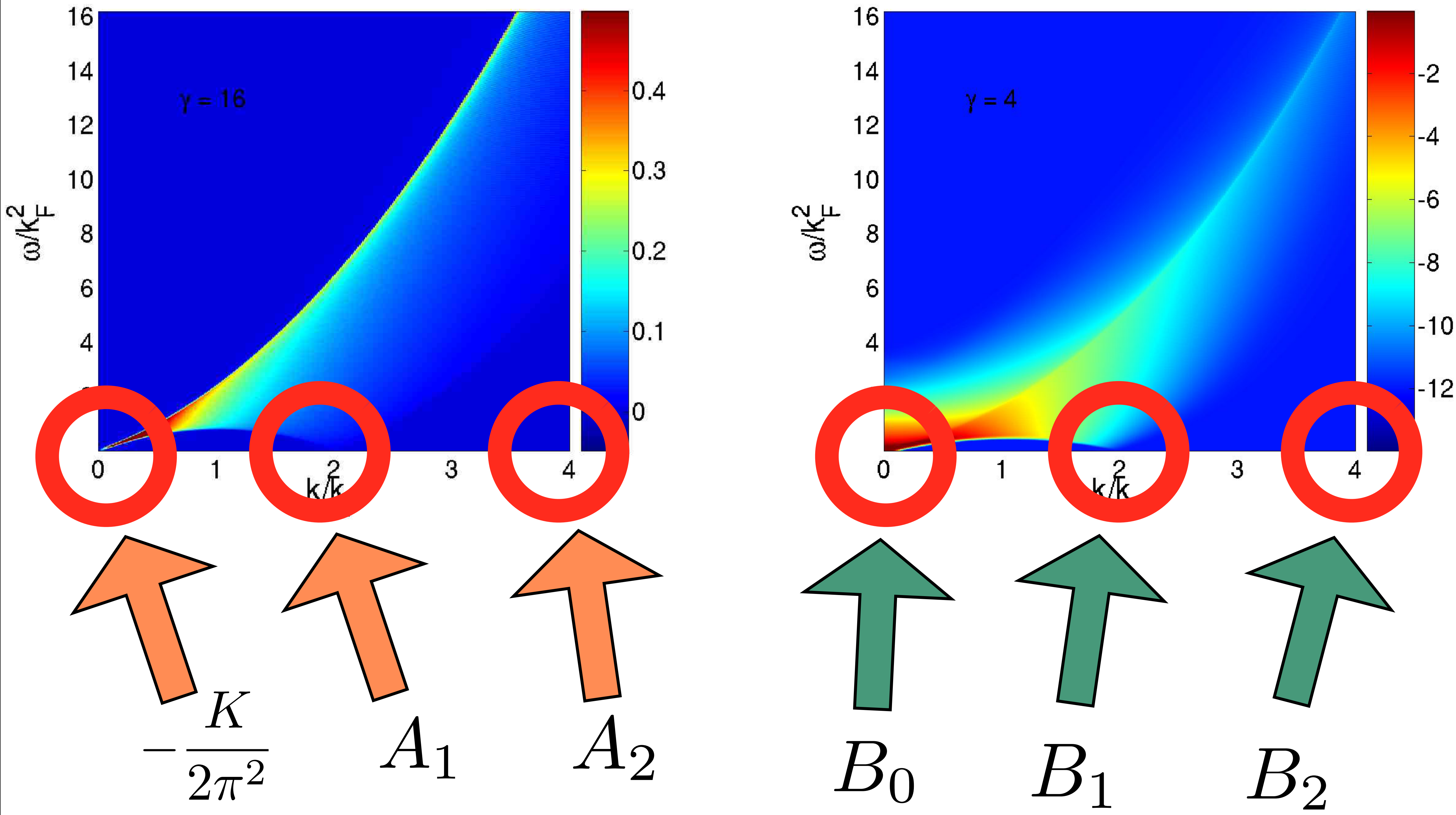
$S^{\rho\rho}(k,\omega)$ $S^{\psi^\dagger\psi}(k,\omega)$



For the XXZ chain, bosonization similary gives

$$S^z(x,t) \sim s_z - \frac{\nabla\phi}{\pi} + e^{i2m[(s_z+1/2)\pi x - \phi(x,t)]}$$

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with long-distance behaviour of static functions

$$\langle S^z(x)S^z(0)\rangle = s_z^2 - \frac{K}{2(\pi x)^2} + \sum_{m\geq 1} \frac{D_m cos(2m(s_z+1/2)\pi x)}{x^{2m^2K}}$$

$$\langle S^+(x)S^-(0)\rangle = (-1)^x \sum_{m\geq 0} \frac{E_m cos(2m(s_z+1/2)\pi x)}{x^{2m^2K+1/(2K)}}$$

given in terms of non-universal prefactors D and E

Asymptotes of static function: determined by correlation around Umklapp/pi modes

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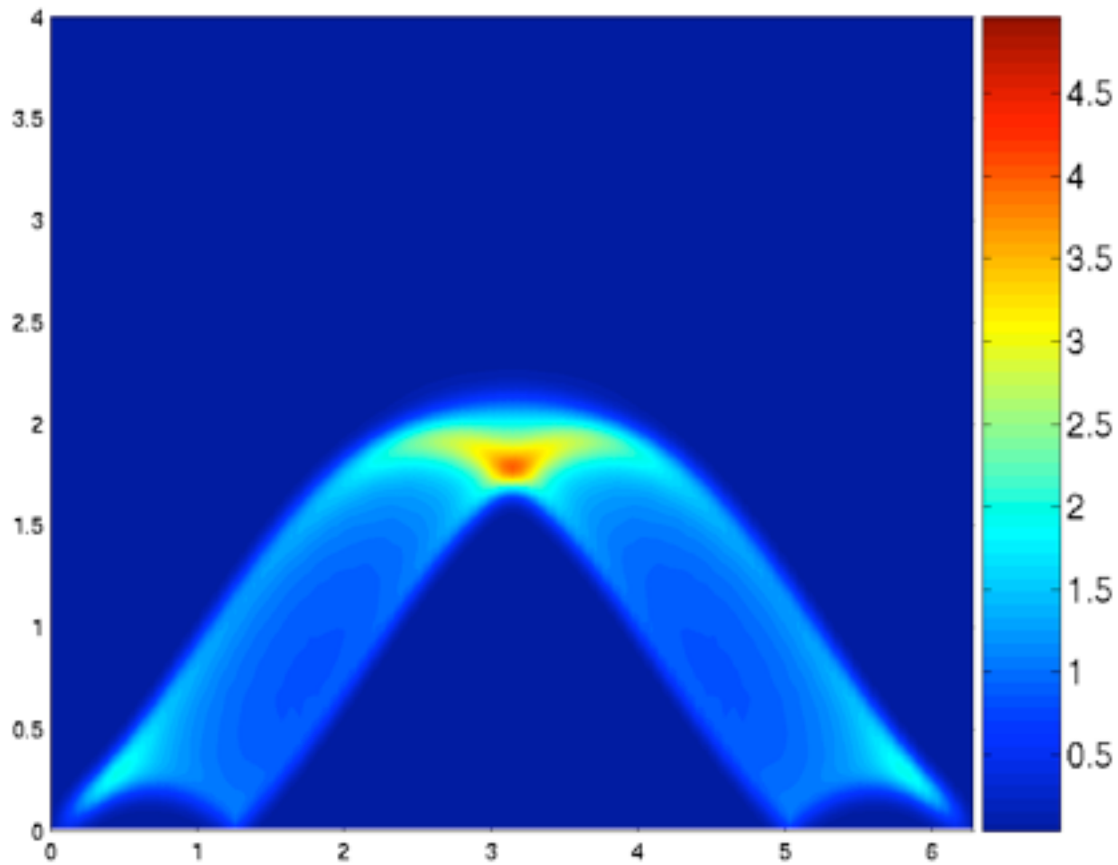

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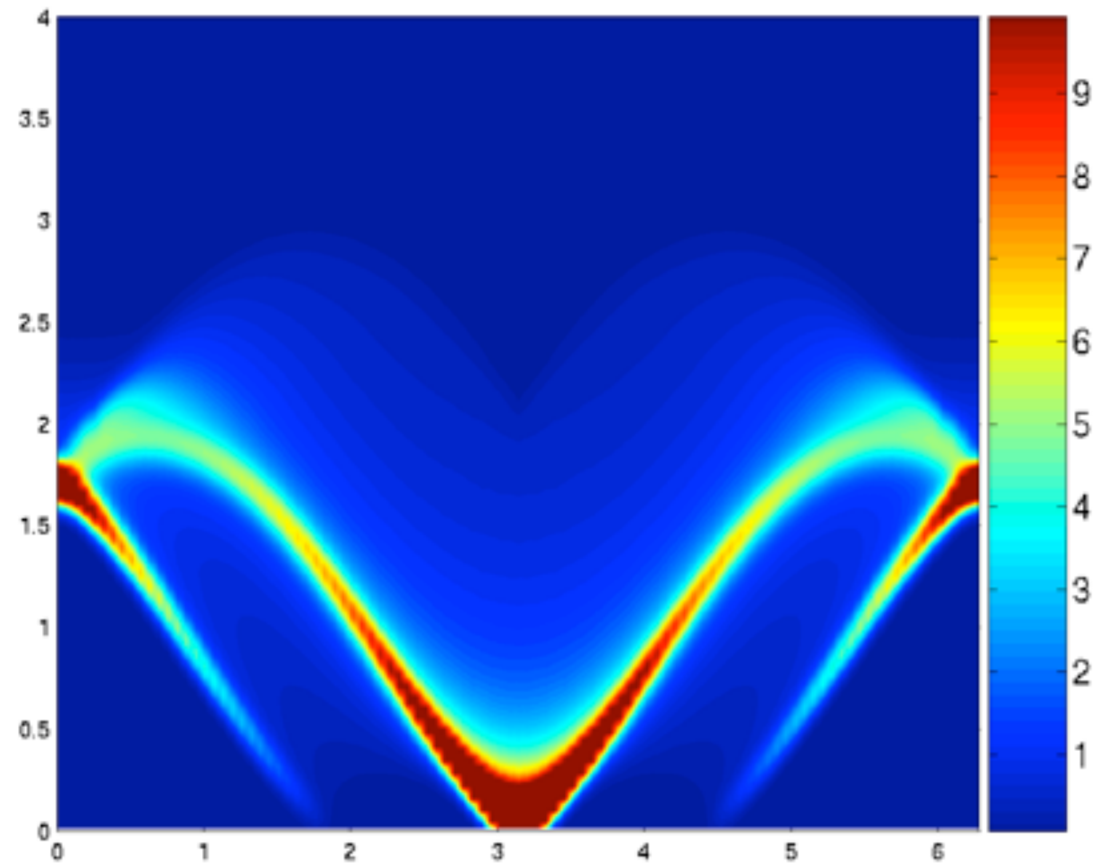

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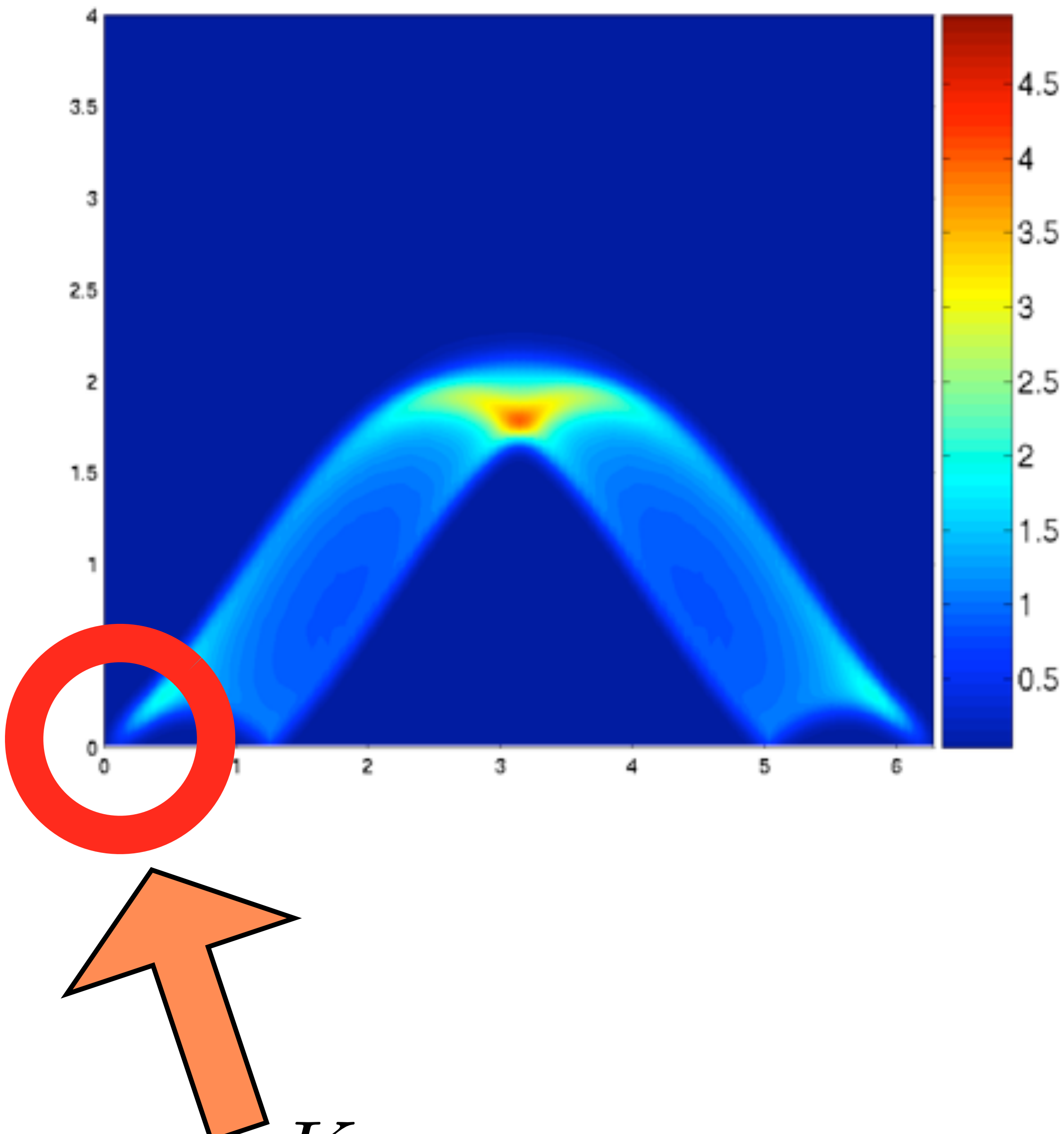

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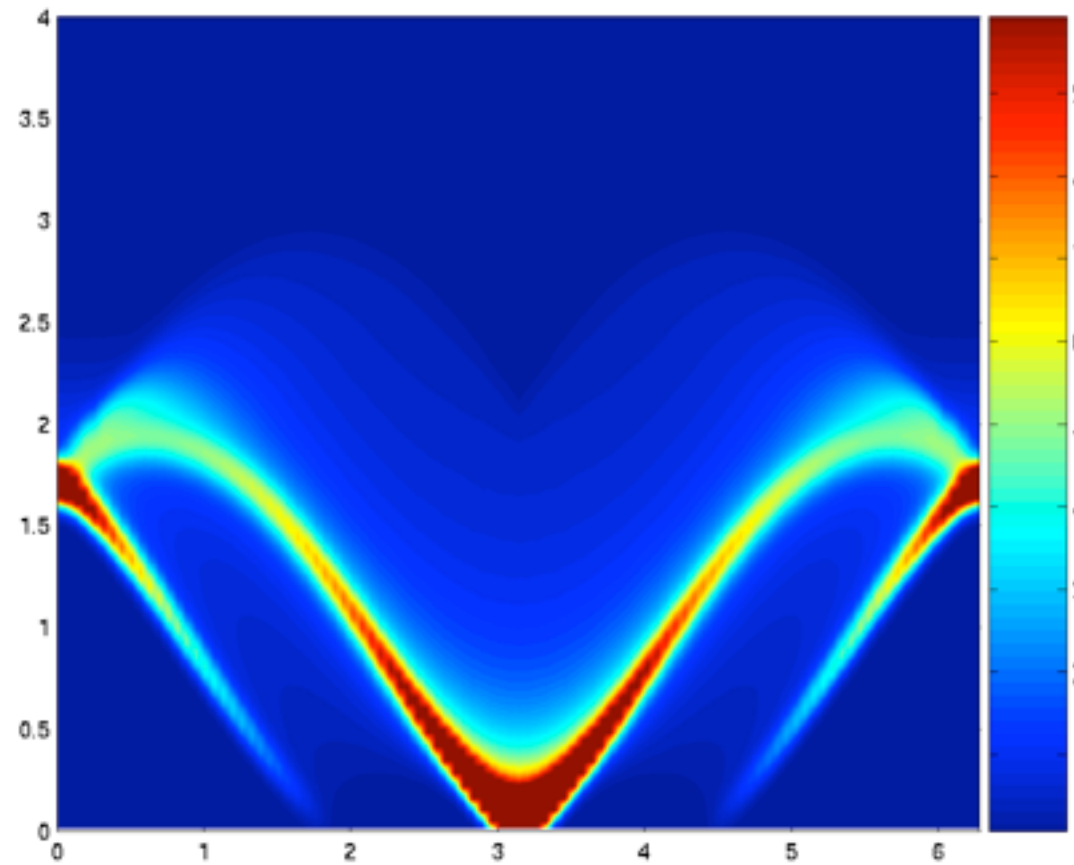

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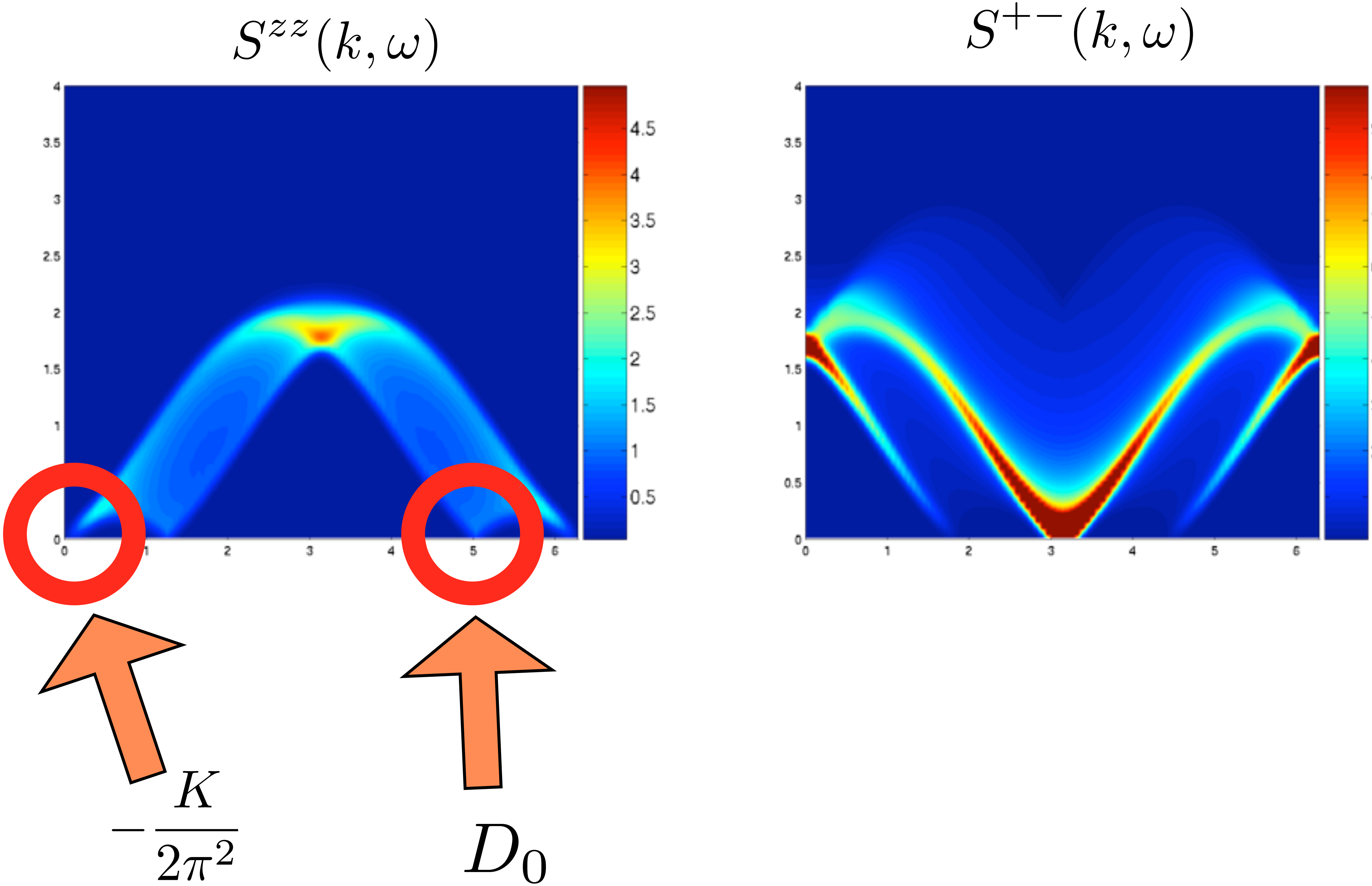

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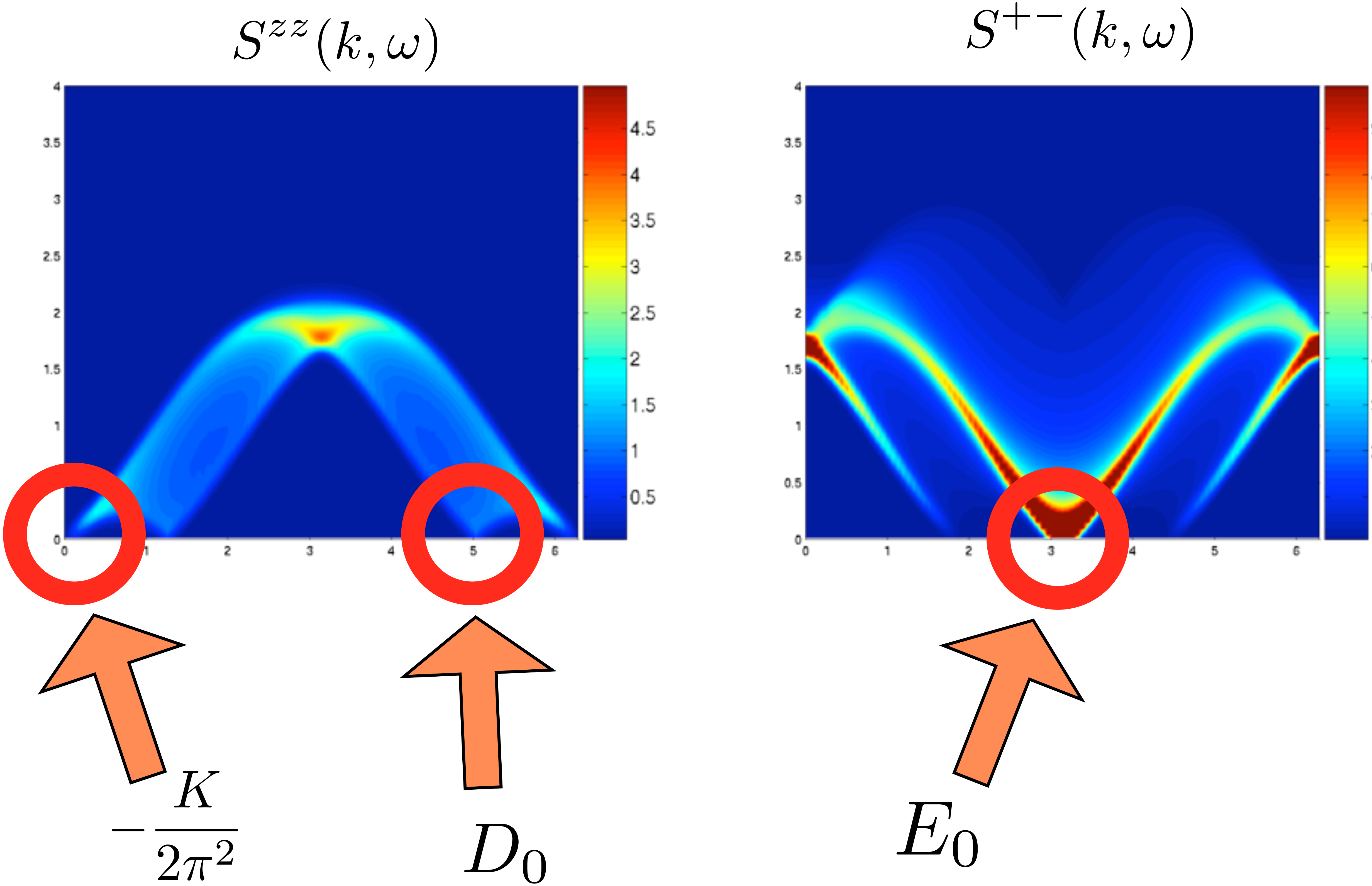

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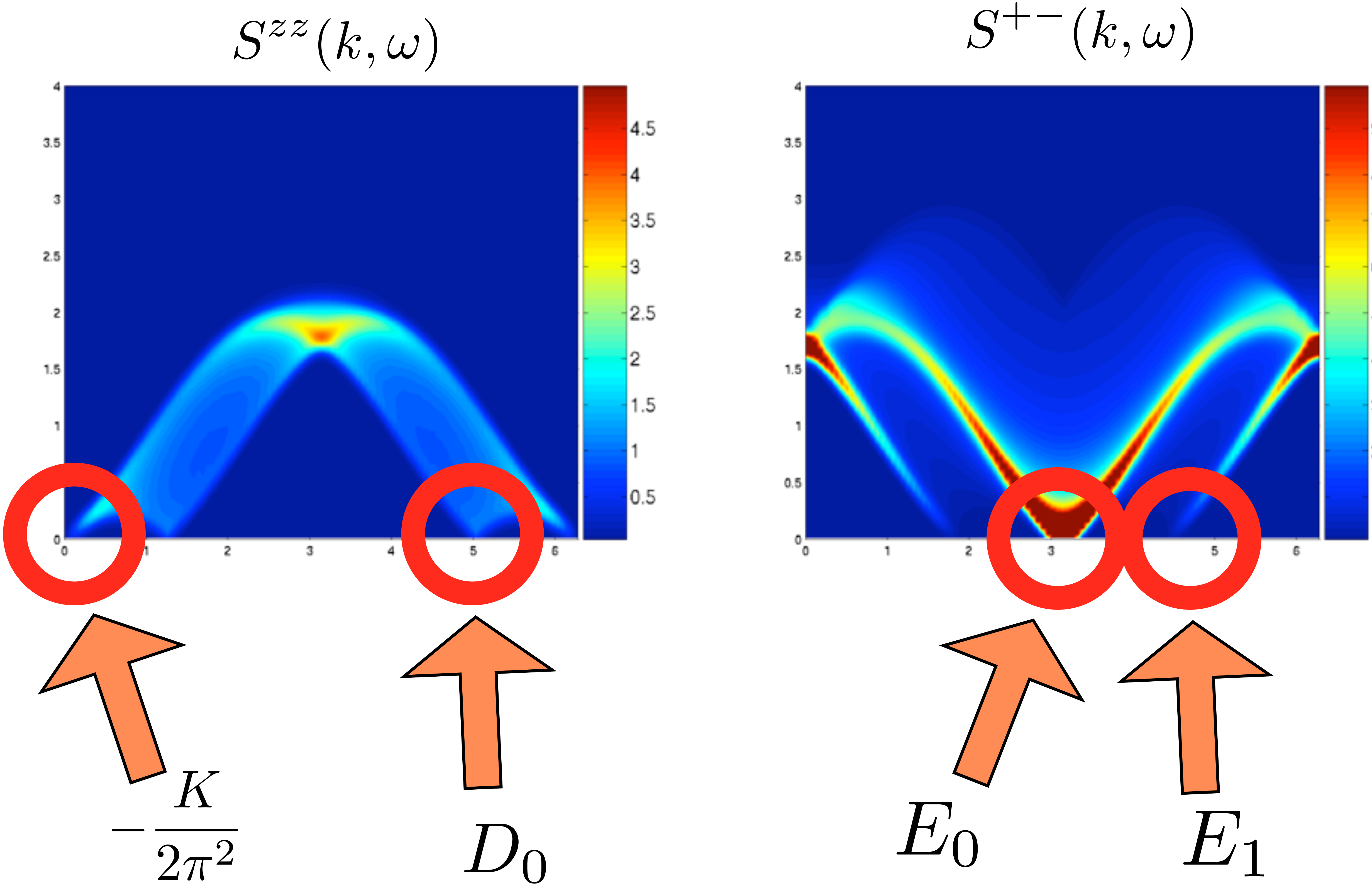

Relating prefactors to form factors

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Start from Lehmann representation

$$\langle \hat{\psi}_B^\dagger(x,t)\hat{\psi}_B(0)\rangle = \sum_{k,\omega} e^{i(kx-\omega t)} \left| \langle k,\omega|\hat{\psi}_B|N\rangle \right|^2$$

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Field separates into left- and right-moving components

$$\varphi_{L(R)} = \theta\sqrt{K} \pm \varphi/\sqrt{K}$$

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The time-dep. m-th Umklapp part can be written

$$\frac{(-1)^m B_m \rho_0^{-2m^2K-1/2K} \cos\left(2mk_F x\right)}{\left(i(vt+x)+0\right)^{\mu_L}\left(i(vt-x)+0\right)^{\mu_R}}$$

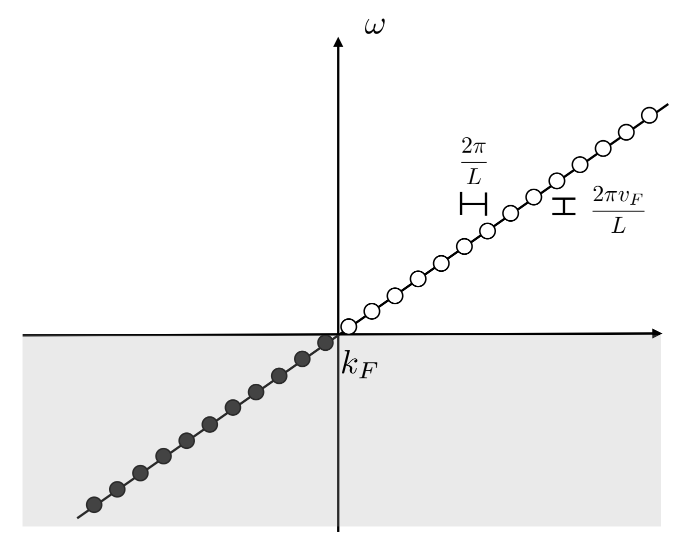
in which $\mu_{L(R)} = m^2K \pm m + 1/4K$

At finite size, the right-moving component becomes

$$\left(\frac{\pi e^{i\pi(vt-x)/L}}{iL\sin\frac{\pi(vt-x)}{L}+0}\right)^{\mu_R} = \sum_{n_r\geq 0} C(n_r,\mu_R)\frac{e^{2i\pi n_r(x-vt)/L}}{(L/2\pi)^{\mu_R}}$$

in which $$C(n_r,\mu_R) = \frac{\Gamma(\mu_R+n_r)}{\Gamma(\mu_R)\Gamma(n_r+1)}$$

and $\frac{2\pi}{L}n_{l,r}$ represents the total momentum of excitations created around the left, right Fermi points



(similarly for the left-moving component)

Considering the n = 0 term gives the scaling law

$$\left|\langle m, N-1|\hat{\psi}_B|0, N\rangle\right|^2 = \frac{(-1)^m B_m \rho_0}{2-\delta_{0,m}} \left(\frac{2\pi}{\rho_0 L}\right)^{\frac{4m^2K^2+1}{2K}}$$

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Finite size scaling of FF ⟺ ***correlation prefactor***

Similarly, for other operators/models, we get

$$\left|\langle m, N-1|\hat{\psi}_F|N\rangle\right|^2 \approx \frac{C_m \rho_0}{2(-1)^m} \left(\frac{2\pi}{\rho_0 L}\right)^{\frac{(2m+1)^2 K^2+1}{2K}}$$

$$|\langle m, N|\hat{\rho}|N\rangle|^2 \approx \frac{A_m \rho_0}{2} \left(\frac{2\pi}{\rho_0 L}\right)^{2m^2 K}$$

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But the problem to be faced is:

how to compute the form factors?

Form factors: from integrability (Algebraic Bethe Ansatz)

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State norms: Gaudin-Korepin formula

Scalar products: Slavnov's formula

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Arbitrary

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where $T_{ab} = \dfrac{\partial}{\partial\lambda_a}\tau(\mu_b,\{\lambda\})$ (N.Slavnov, 1988)

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gives (at least in principle) all matrix elements needed

After a long calculation*, one obtains expressions like...

$$A_m = 2\gamma^2 \left(\frac{q\sqrt{K}}{\rho_0}\right)^{-2m^2K} \left(\frac{4q^2+c^2}{4c^2}\right)^{m^2} \left(\frac{G(1+m\sqrt{K})^2 G(m+1-m\sqrt{K})}{\Gamma(m-m\sqrt{K})^m \Gamma(1-m+m\sqrt{K})^m G(1-m+m\sqrt{K})}\right)^2$$
$$\exp\left[4mq\int_{-q}^{q} d\lambda \frac{F(\lambda)}{((\lambda+ic)^2-q^2)} - \int_{-q}^{q} d\mu \int_{-q}^{q} d\lambda \frac{F(\lambda)F(\mu)}{(\lambda-\mu+ic)^2}\right]$$
$$\exp\left[2P_\pm \int_{-1}^{1} dx \frac{F^2(qx)-2mF(qx)}{x^2-1} - \frac{1}{2}\int_{-q}^{q}\int_{-q}^{q} d\lambda d\mu \left(\frac{F(\lambda)-F(\mu)}{\lambda-\mu}\right)^2\right] \frac{\mathrm{Det}^2(1+\hat{G})}{\mathrm{Det}^2\left(1-\frac{\hat{K}}{2\pi}\right)}$$

for Lieb-Liniger, and for XXZ in a field:

$$D_m = \frac{N}{2}\left(2\pi q\rho(q)\right)^{-2m^2K} \left(\frac{2q}{\sinh(2q)}\right)^{2m^2} \left(\frac{\sinh^2(2q)+\sin^2\zeta}{4\sin^2\zeta}\right)^{m^2}$$
$$\times \left(\frac{G^2(1+m\sqrt{K})G(1+m-m\sqrt{K})}{\Gamma^m(m-m\sqrt{K})\Gamma^m(1-m+m\sqrt{K})G(1-m+m\sqrt{K})}\right)^2 \frac{\det^2\left(1+\hat{G}^z\right)}{\det^2(1+\hat{a}_2)}$$
$$\times \exp\left(-\int_{-q}^{q} d\mu d\lambda \frac{F(\lambda)F(\mu)}{\sinh^2(\lambda-\mu-i\zeta)} - \frac{1}{2}\int_{-q}^{q} d\lambda\, d\mu \left(\frac{F(\lambda)-F(\mu)}{\sinh(\lambda-\mu)}\right)^2\right)$$
$$\times \exp\left(P_+\int_{-1}^{1} dx \frac{q(F^2(qx)-2mF(qx))}{\tanh(q(x-1))} - P_-\int_{-1}^{1} dx \frac{q(F^2(qx)-2mF(qx))}{\tanh(q(x+1))}\right)$$
$$\times \exp\left(-\int_{-q}^{q} d\lambda \frac{2mF(\lambda)\sinh(2(q-\lambda))}{\cosh(2(q-\lambda)-\cos(2\zeta)}\right)$$

After a long calculation*, one obtains expressions like...

$$A_m = 2\gamma^2 \left(\frac{q\sqrt{K}}{\rho_0}\right)^{-2m^2 K} \left(\frac{4q^2+c^2}{4c^2}\right)^{m^2} \left(\frac{G(1+m\sqrt{K})^2 G(m+1-m\sqrt{K})}{\Gamma(m-m\sqrt{K})^m \Gamma(1-m+m\sqrt{K})^m G(1-m+m\sqrt{K})}\right)^2$$
$$\exp\left[4mq\int_{-q}^{q} d\lambda \frac{F(\lambda)}{((\lambda+ic)^2-q^2)} - \int_{-q}^{q} d\mu \int_{-q}^{q} d\lambda \frac{F(\lambda)F(\mu)}{(\lambda-\mu+ic)^2}\right]$$
$$\exp\left[2P_{\pm}\int_{-1}^{1} dx \frac{F^2(qx)-2mF(qx)}{x^2-1} - \frac{1}{2}\int_{-q}^{q}\int_{-q}^{q} d\lambda d\mu \left(\frac{F(\lambda)-F(\mu)}{\lambda-\mu}\right)^2\right] \frac{\mathrm{Det}^2(1+\hat{G})}{\mathrm{Det}^2\left(1-\frac{\hat{K}}{2\pi}\right)}$$

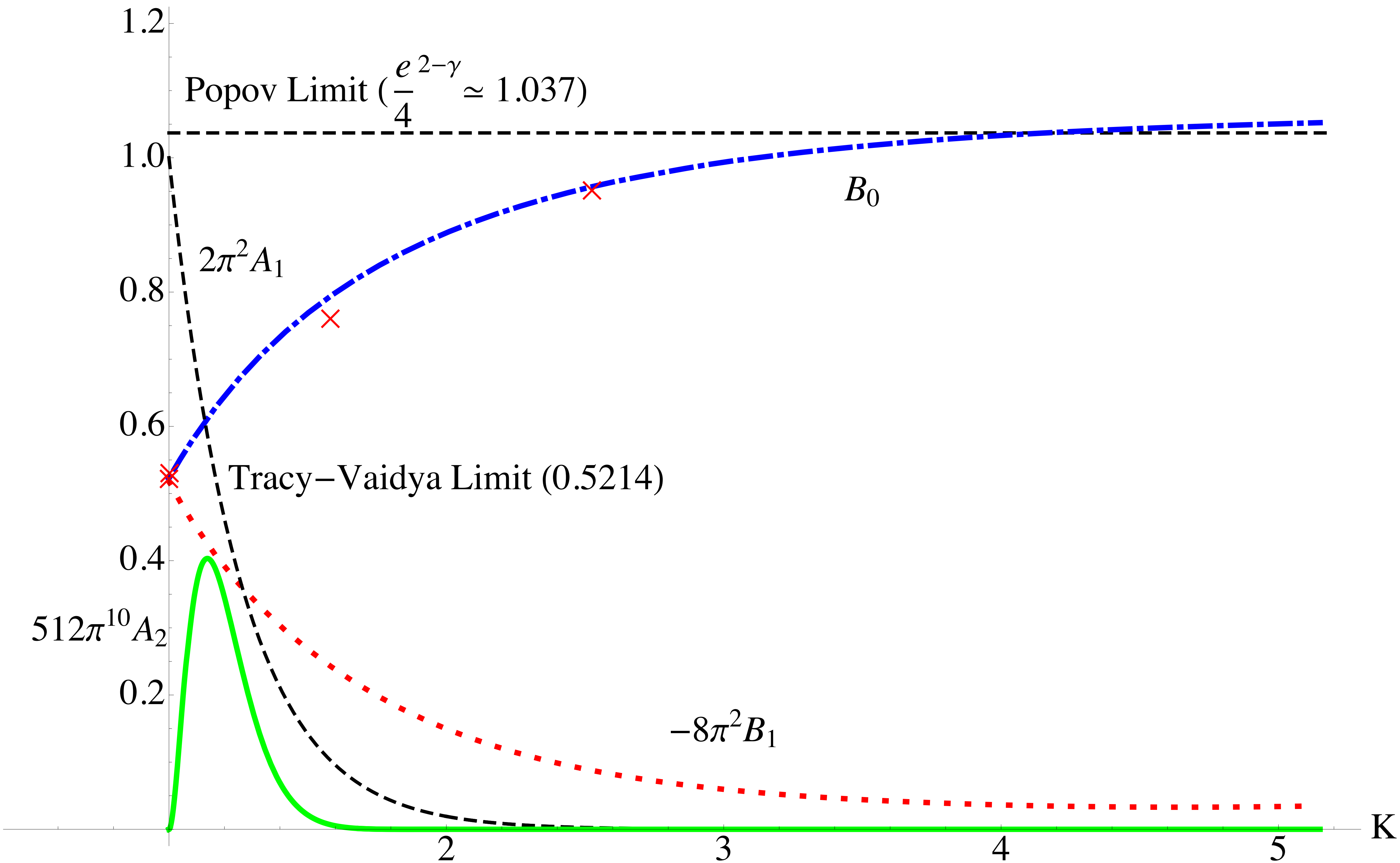
for Lieb-Liniger, and for XXZ in a field:

$$D_m = \frac{N}{2}\left(2\pi q\rho(q)\right)^{-2m^2 K}\left(\frac{2q}{\sinh(2q)}\right)^{2m^2}\left(\frac{\sinh^2(2q)+\sin^2\zeta}{4\sin^2\zeta}\right)^{m^2}$$
$$\times\left(\frac{G^2(1+m\sqrt{K})G(1+m-m\sqrt{K})}{\Gamma^m(m-m\sqrt{K})\Gamma^m(1-m+m\sqrt{K})G(1-m+m\sqrt{K})}\right)^2 \frac{\det^2\left(1+\hat{G}^z\right)}{\det^2(1+\hat{a}_2)}$$
$$\times\exp\left(-\int_{-q}^{q} d\mu d\lambda \frac{F(\lambda)F(\mu)}{\sinh^2(\lambda-\mu-i\zeta)} - \frac{1}{2}\int_{-q}^{q} d\lambda\, d\mu \left(\frac{F(\lambda)-F(\mu)}{\sinh(\lambda-\mu)}\right)^2\right)$$
$$\times\exp\left(P_+\int_{-1}^{1} dx \frac{q(F^2(qx)-2mF(qx))}{\tanh(q(x-1))} - P_-\int_{-1}^{1} dx \frac{q(F^2(qx)-2mF(qx))}{\tanh(q(x+1))}\right)$$
$$\times\exp\left(-\int_{-q}^{q} d\lambda \frac{2mF(\lambda)\sinh(2(q-\lambda))}{\cosh(2(q-\lambda)-\cos(2\zeta)}\right)$$

*(A. Shashi, L. Glazman, J.-S. Caux and A. Imambekov, arXiv 1010.2268, 1103.4176)

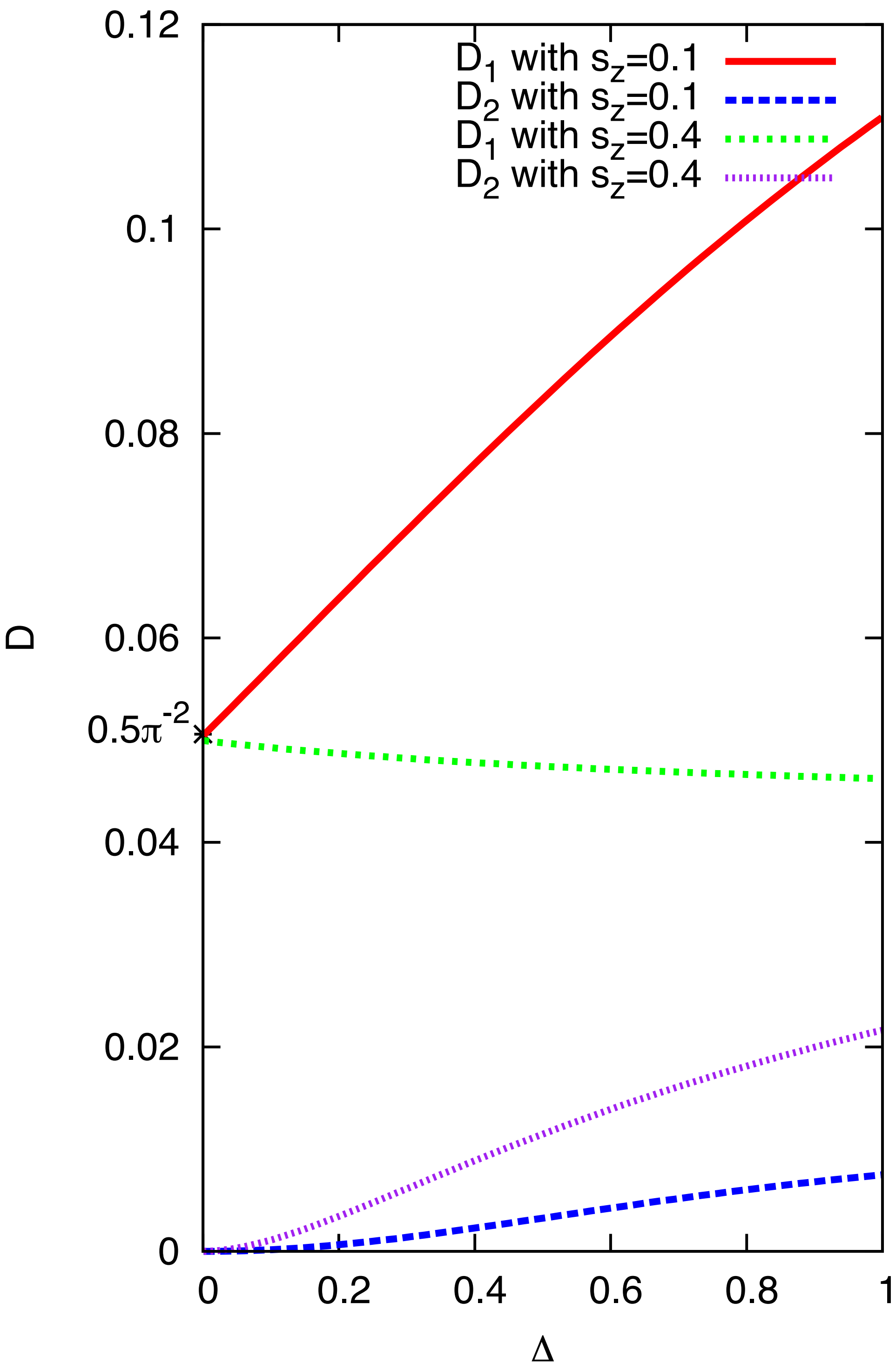
(Kozlowski and Terras 2011; Kitanine et al 2009)

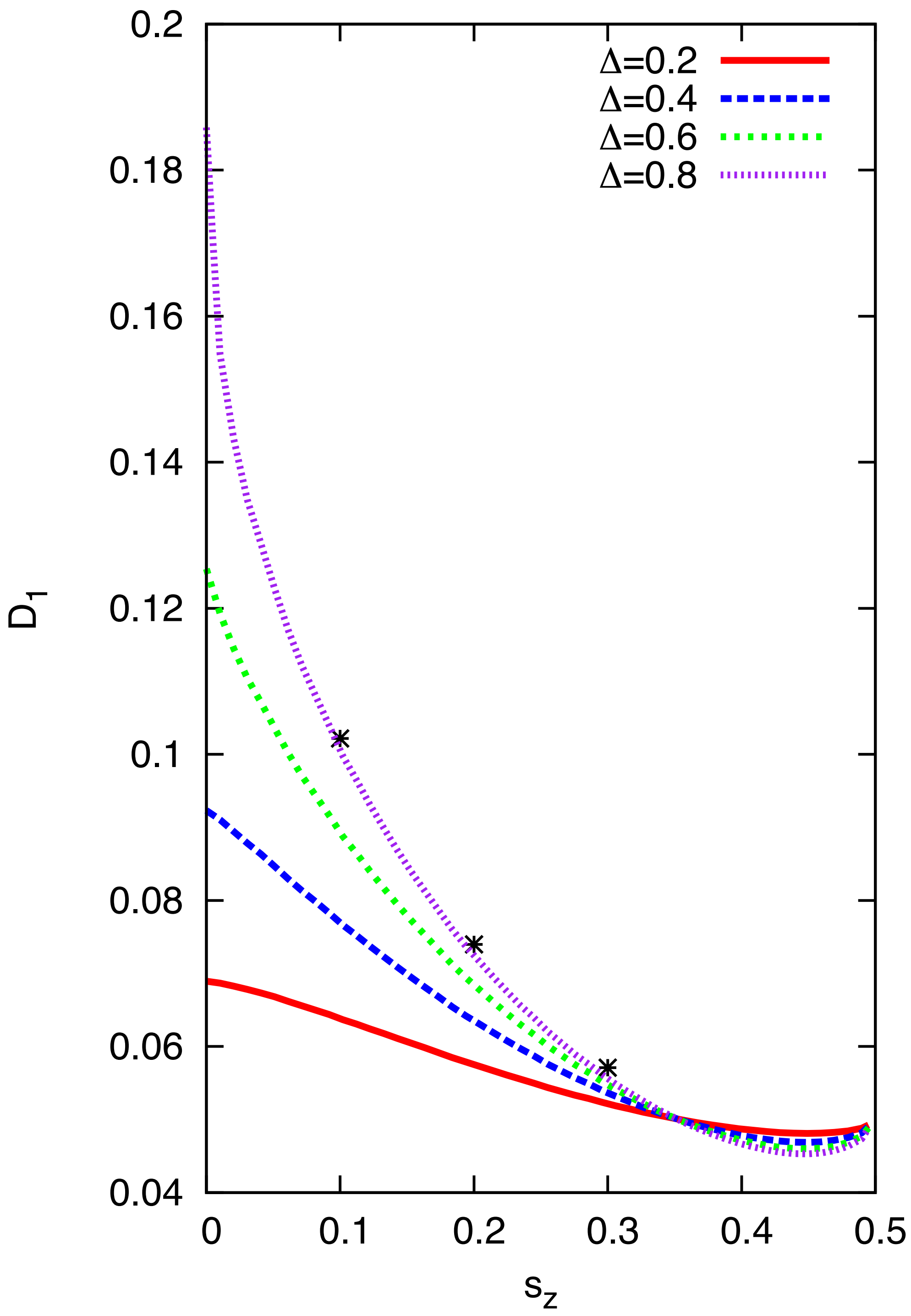
... or in more readable plots, for Lieb-Liniger



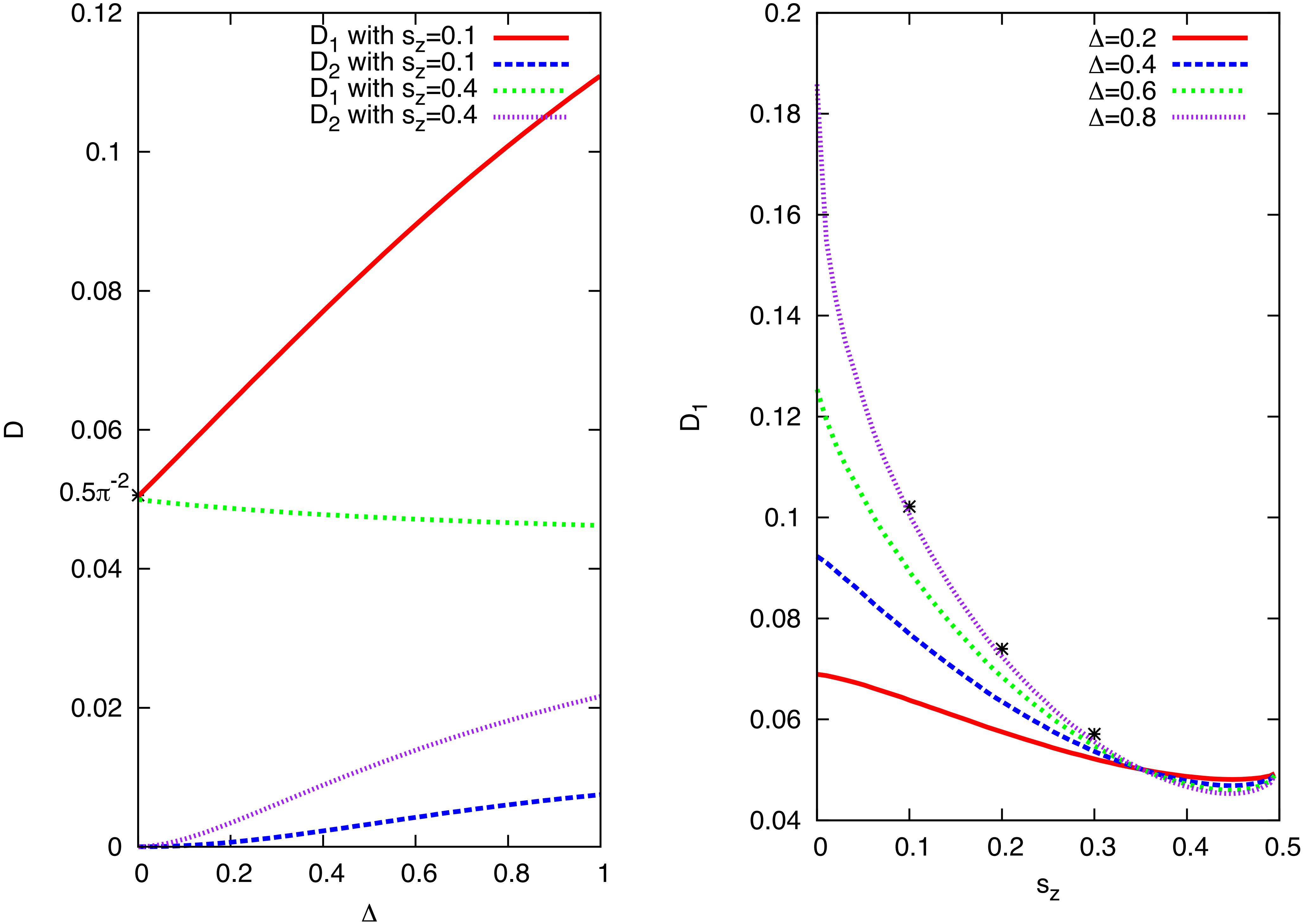
... and for XXZ (longitudinal correlation)

... and for XXZ (longitudinal correlation)

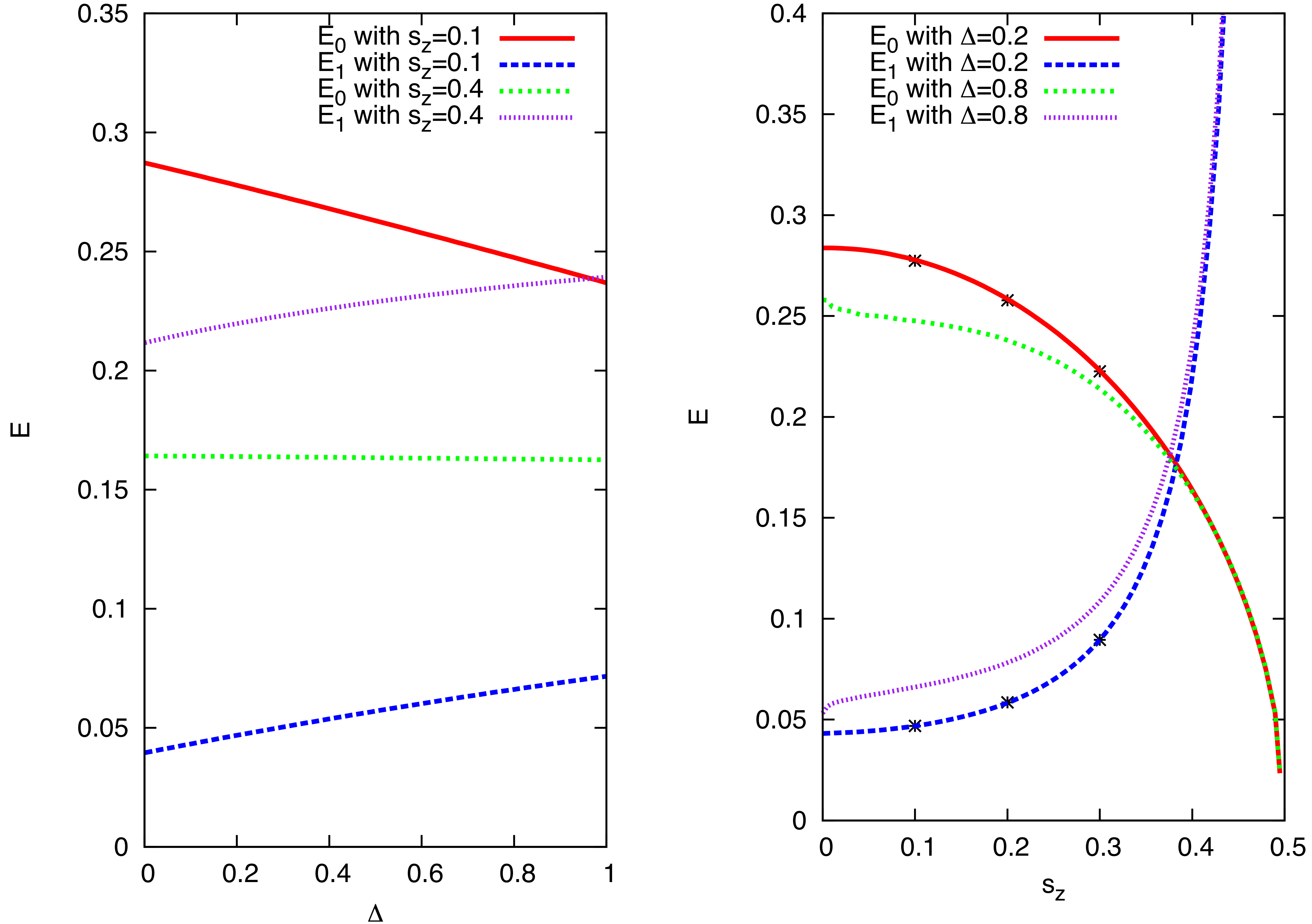




... and for XXZ (longitudinal correlation)

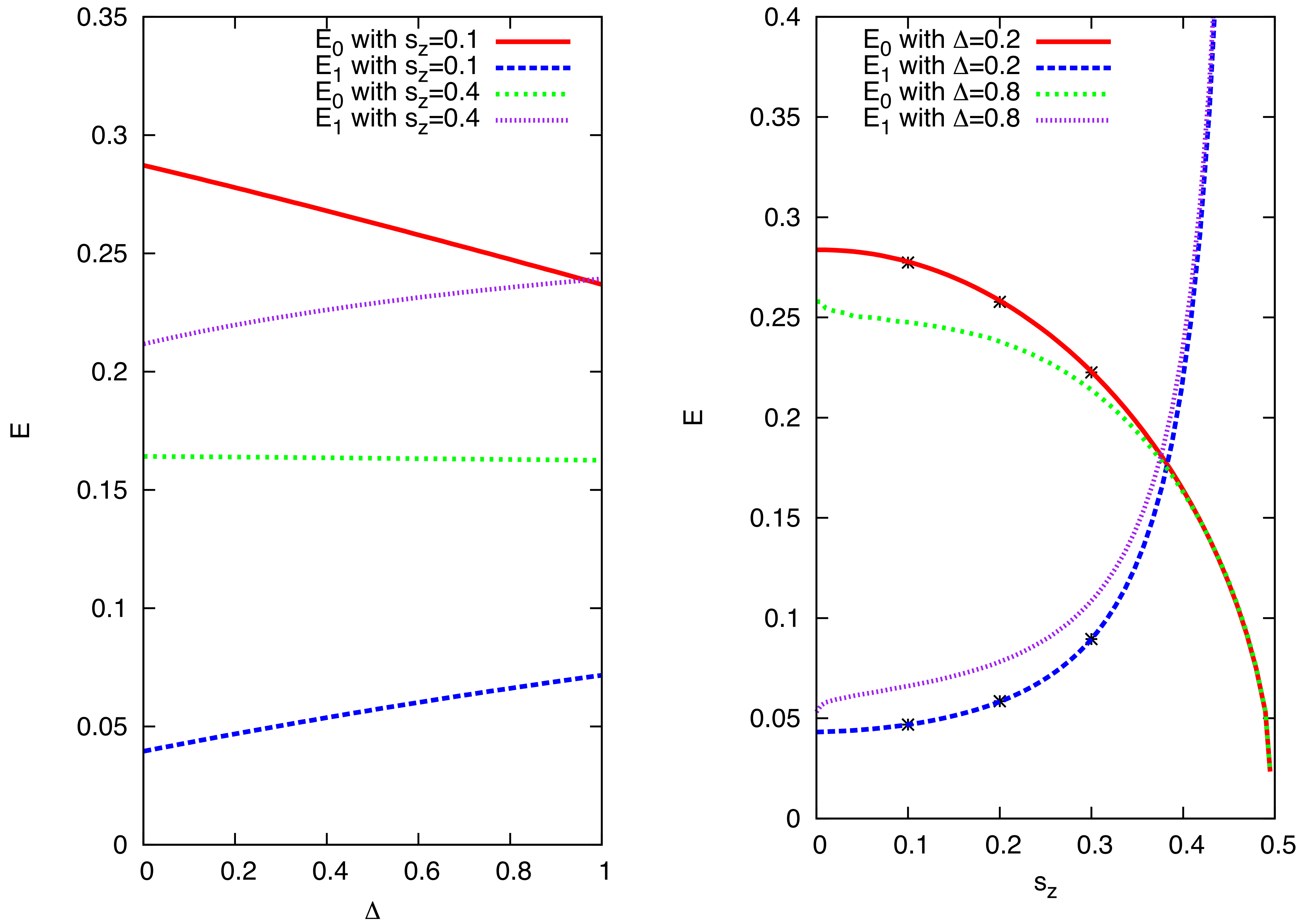


Fits with DMRG results of Hikihara & Furusaki



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
E
0
0.2
0.4
0.6
0.8
1
Δ
E0 with sz=0.1
E1 with sz=0.1
E0 with sz=0.4
E1 with sz=0.4
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
E
0
0.1
0.2
0.3
0.4
0.5
sz
E0 with Δ=0.2
E1 with Δ=0.2
E0 with Δ=0.8
E1 with Δ=0.8

... and for XXZ (transverse correlation)



Dynamical correlators: Nonlinear Luttinger Liquid Theory

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Glazman, Imambekov, Khodas, Kamenev, Cheianov, Pustilnik, Affleck, Pereira, Sirker, JSC, ...

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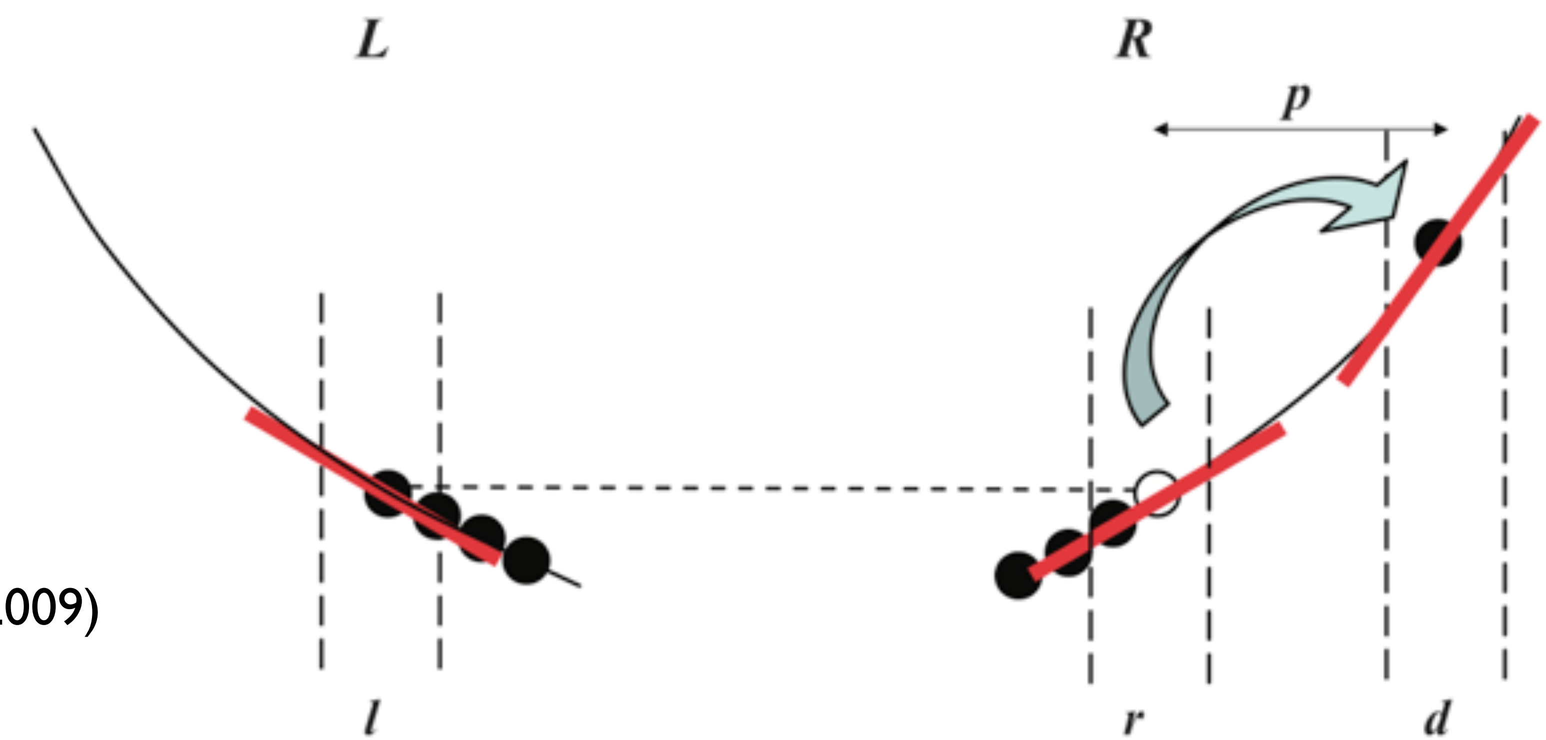
Three subband model



From Imambekov & Glazman, SCIENCE 323 (2009)

Dynamical correlators: Nonlinear Luttinger Liquid Theory

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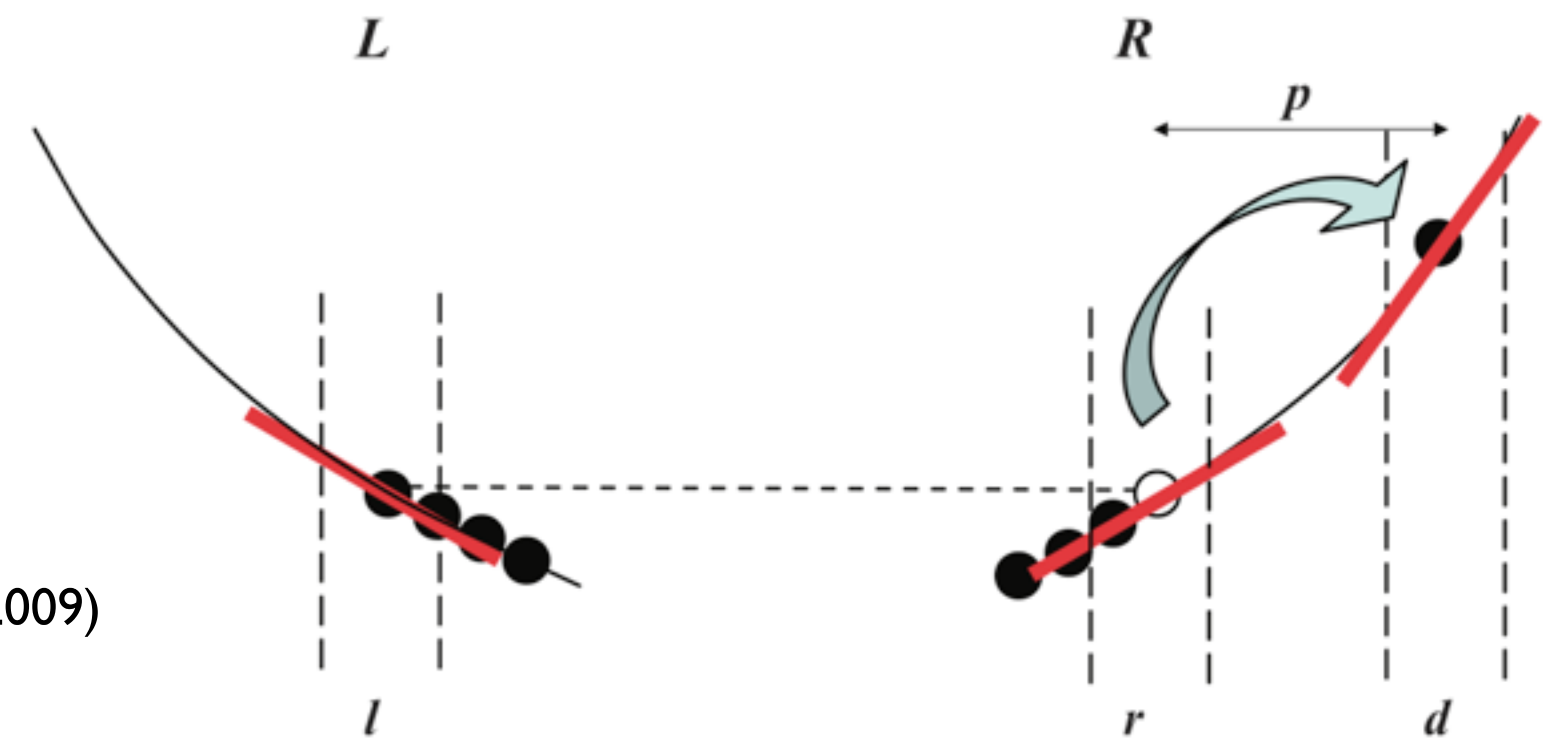

$$\tilde{H}_{l,r} = iv \int dx \left[: \tilde{\Psi}_l^\dagger(x) \nabla \tilde{\Psi}_l(x) : - : \tilde{\Psi}_r^\dagger(x) \nabla \tilde{\Psi}_r(x) : \right]$$

$$\tilde{H}_d = \int dx \ \tilde{d}^\dagger(x) \left[vp + \frac{p^2}{2m^*} - i \left(v + \frac{p}{m^*} \right) \nabla \right] \tilde{d}(x)$$

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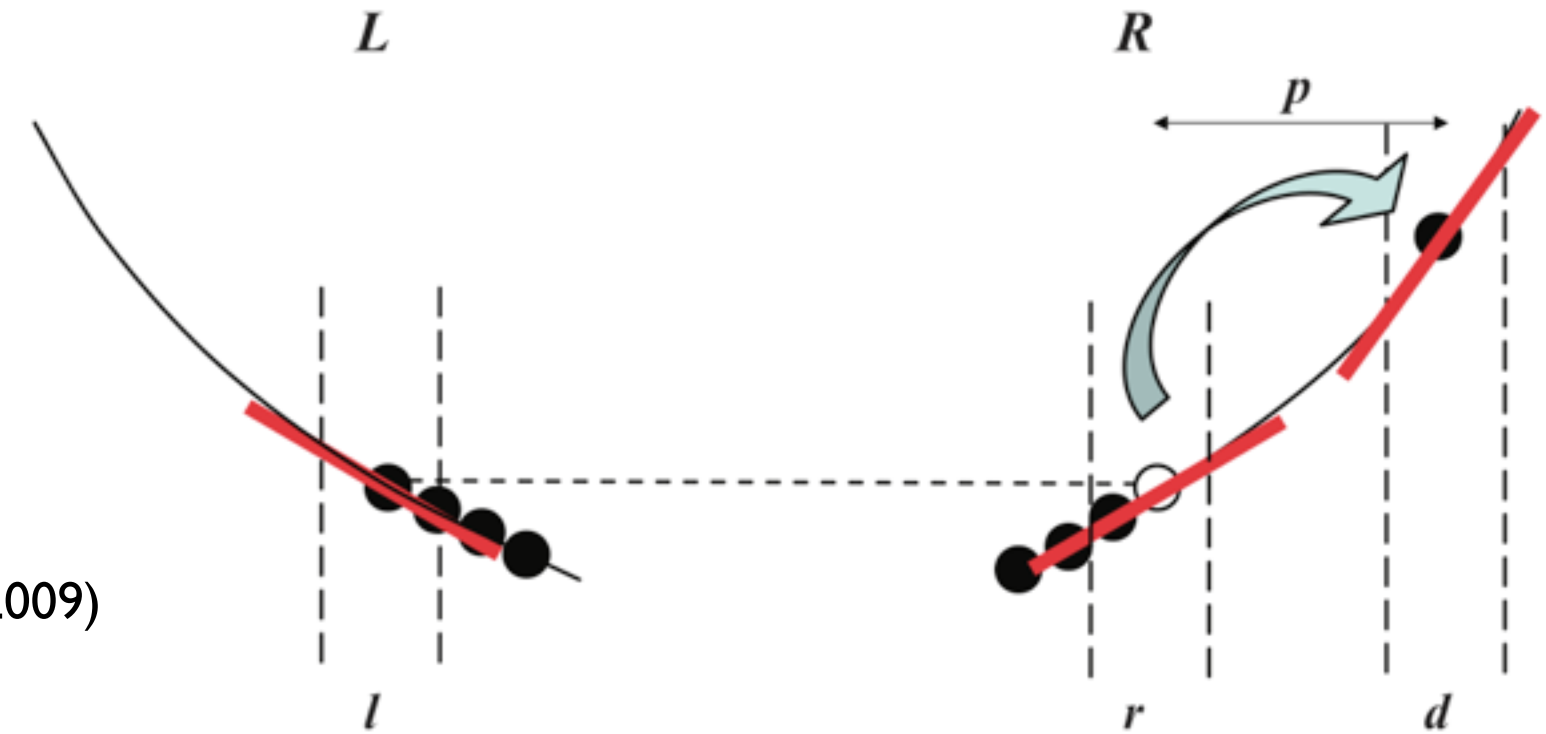


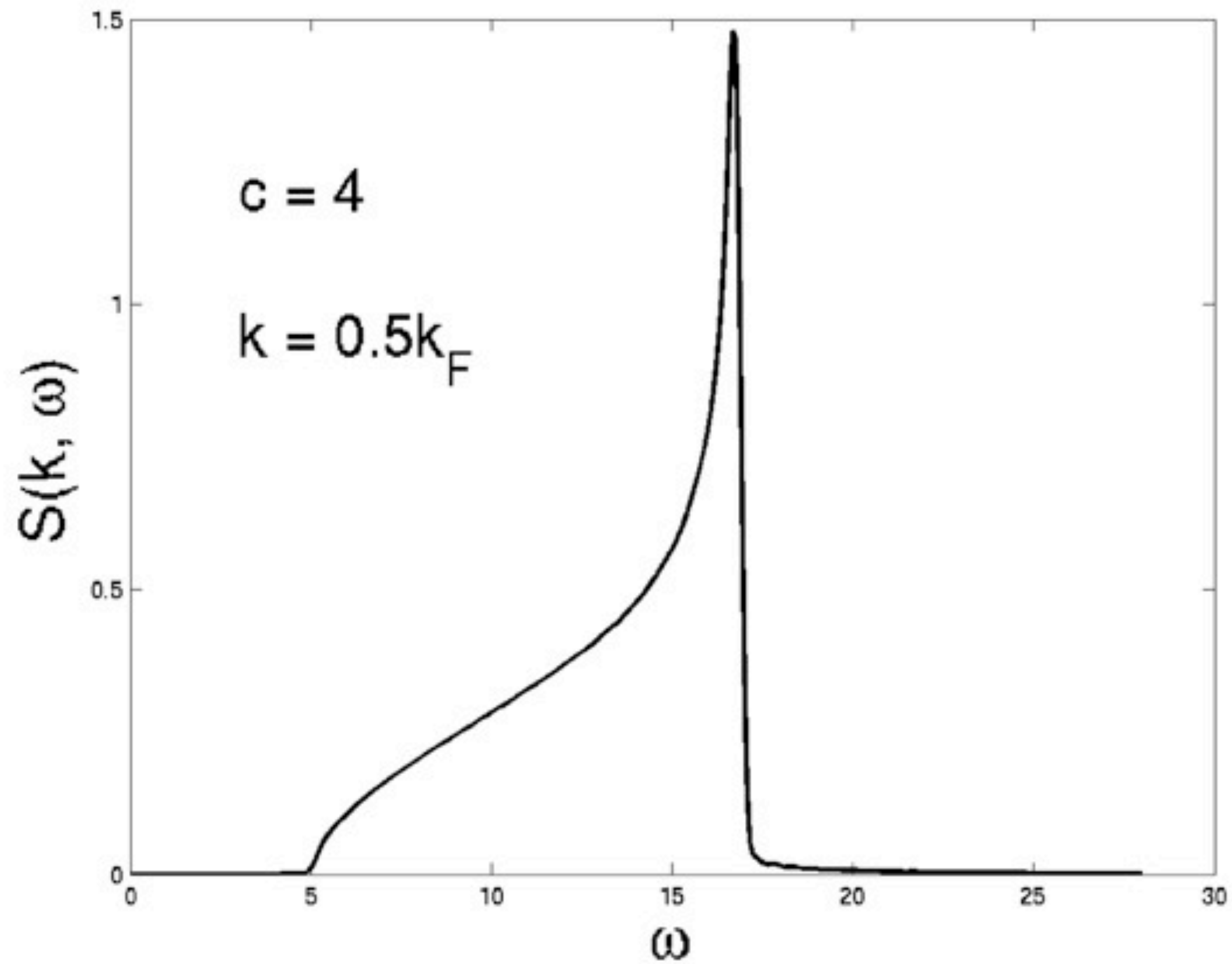
$$\tilde{H}_{l,r} = iv \int dx \left[: \tilde{\Psi}_l^\dagger(x) \nabla \tilde{\Psi}_l(x) : - : \tilde{\Psi}_r^\dagger(x) \nabla \tilde{\Psi}_r(x) : \right]$$

$$\tilde{H}_d = \int dx \, \tilde{d}^\dagger(x) \left[vp + \frac{p^2}{2m^*} - i \left(v + \frac{p}{m^*} \right) \nabla \right] \tilde{d}(x)$$

Effective mass:

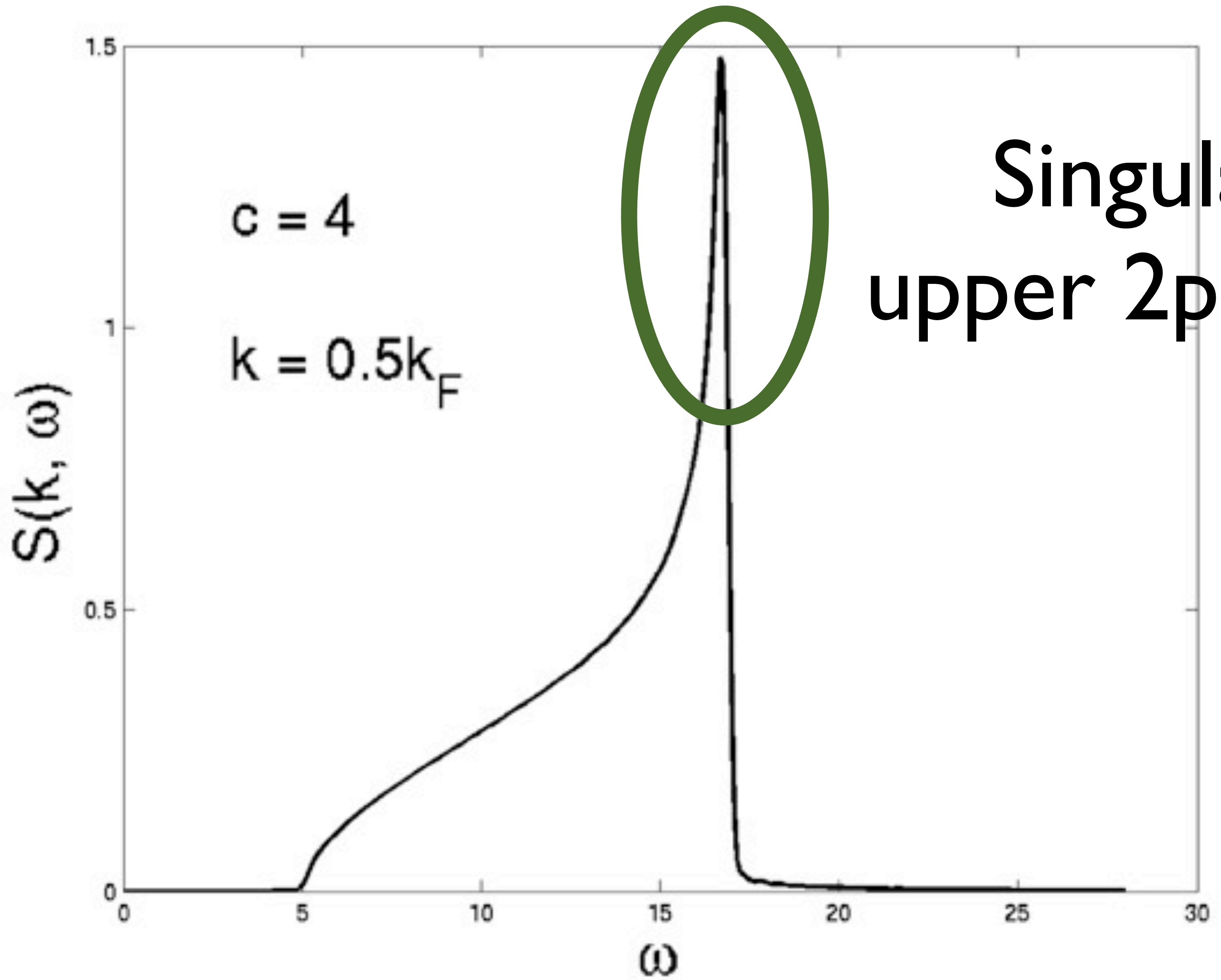
$$\frac{1}{m^*} = \frac{v}{K^{1/2}} \frac{\partial v}{\partial h} + \frac{v^2}{2K^{3/2}} \frac{\partial K}{\partial h}$$

Singularity structure (Khodas, Pustilnik, Kamenev, Glazman; Imambekov & Glazman)



Singularity structure (Khodas, Pustilnik, Kamenev, Glazman; Imambekov & Glazman)

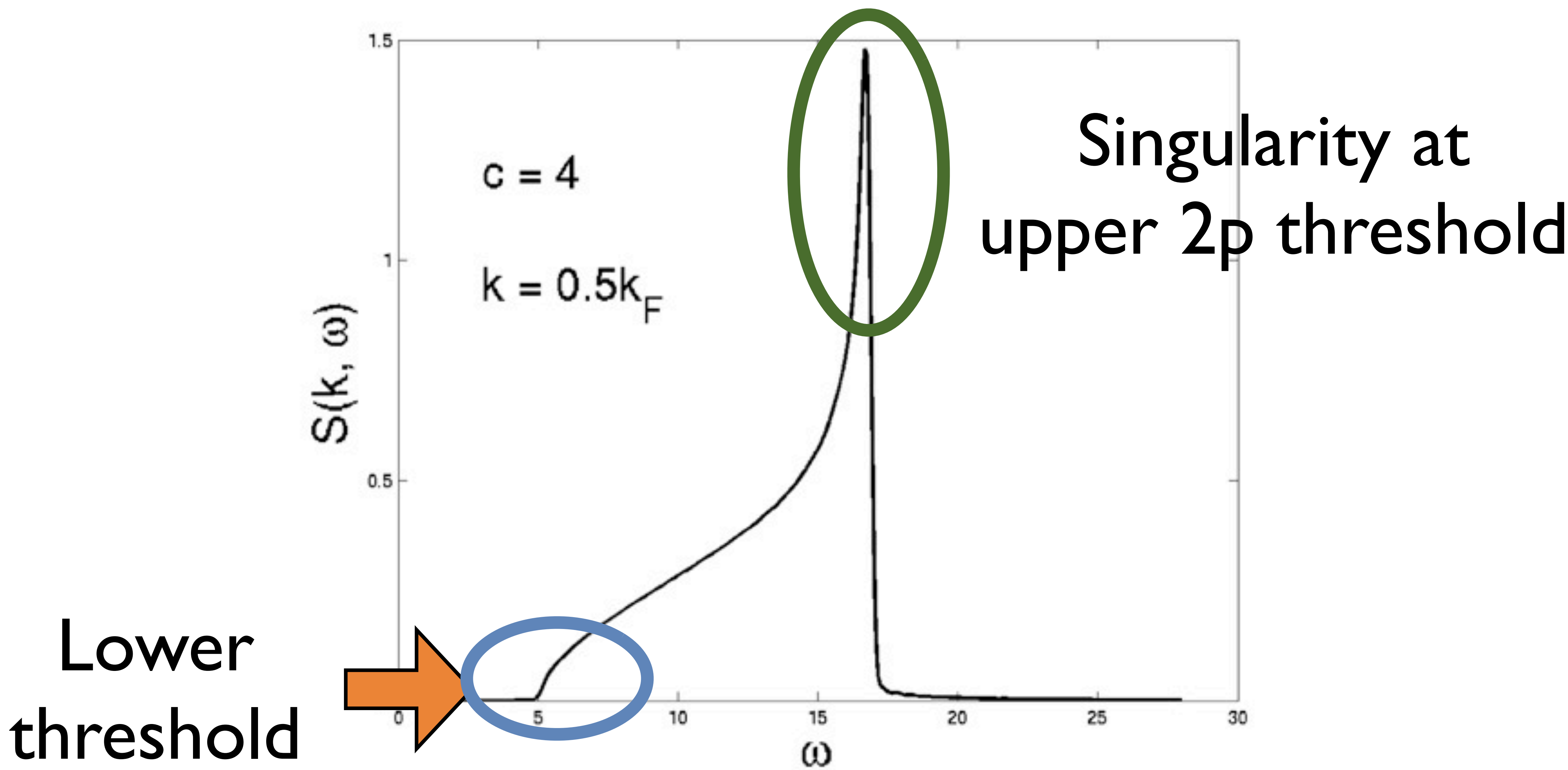
$$S \sim \frac{1}{|\omega - \epsilon_I|^{\mu_1}} \left(\theta(\epsilon_I - \omega) + \nu_1 \theta(\omega - \epsilon_I) \right)$$



Singularity at upper 2p threshold

Singularity structure (Khodas, Pustilnik, Kamenev, Glazman; Imambekov & Glazman)

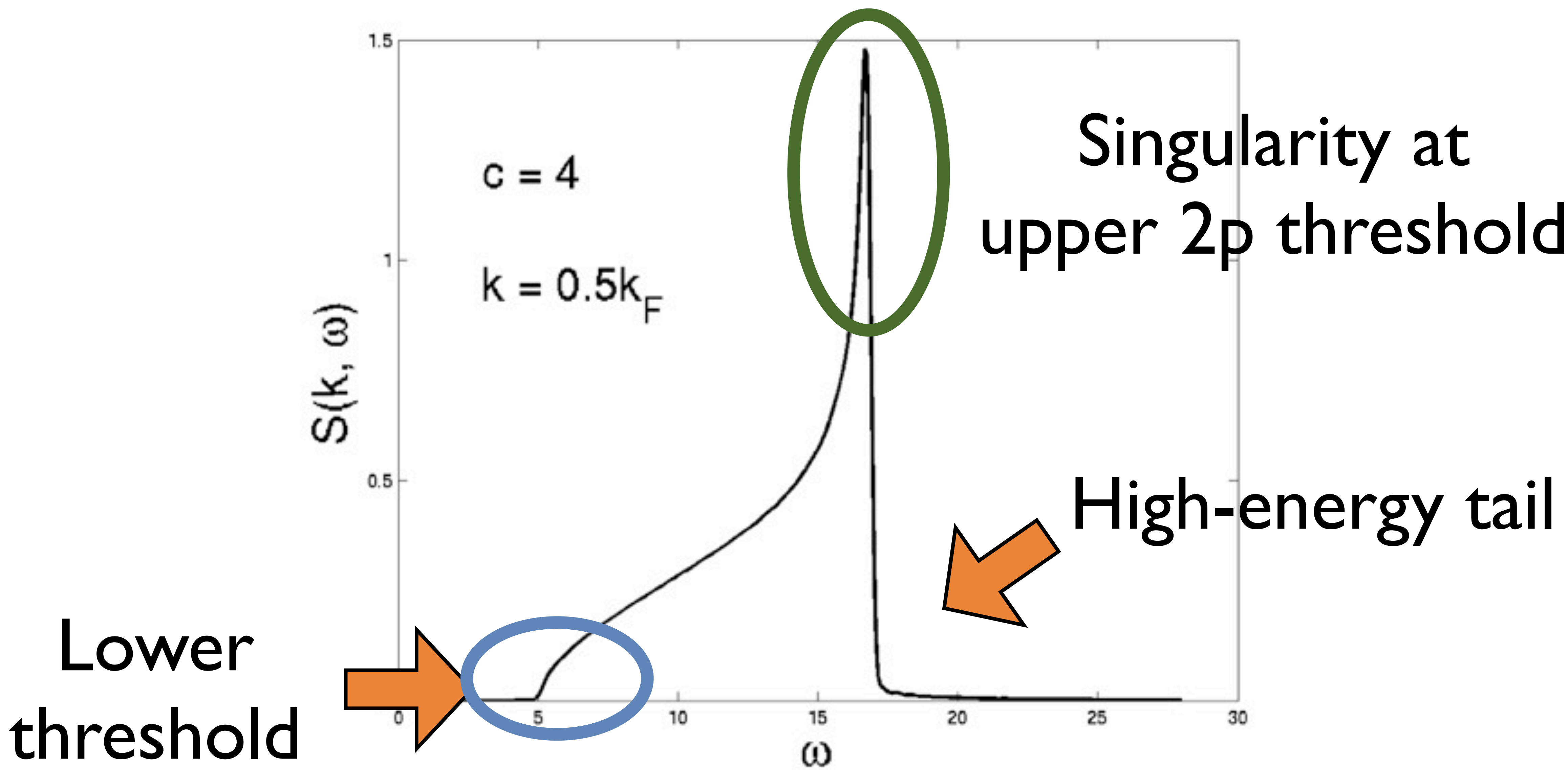
$$S \sim \frac{1}{|\omega - \epsilon_I|^{\mu_1}} (\theta(\epsilon_I - \omega) + \nu_1 \theta(\omega - \epsilon_I))$$



$$S \sim (\omega - \epsilon_{II})^{\mu_2}$$

Singularity structure (Khodas, Pustilnik, Kamenev, Glazman; Imambekov & Glazman)

$$S \sim \frac{1}{|\omega - \epsilon_I|^{\mu_1}}(\theta(\epsilon_I - \omega) + \nu_1 \theta(\omega - \epsilon_I))$$



$$S \sim (\omega - \epsilon_{II})^{\mu_2}$$

Similarly to the case of prefactors, a single form factor completely determines the correlation near the singularities: e.g. for density density in Lieb-Liniger,

a) near $\varepsilon_2(k)$

$$S(k,\omega) = \theta(\delta\omega)\frac{2\pi S_2(k)\delta\omega^{\tilde{\mu}_R+\tilde{\mu}_L-1}}{\Gamma(\tilde{\mu}_R+\tilde{\mu}_L)(v+v_d)^{\tilde{\mu}_L}|v-v_d|^{\tilde{\mu}_R}}$$

b) around $\varepsilon_1(k)$

$$S(k,\omega) = \frac{\sin\pi\tilde{\mu}_L\theta(\delta\omega)+\sin\pi\tilde{\mu}_R\theta(-\delta\omega)}{\sin\pi(\tilde{\mu}_L+\tilde{\mu}_R)}\frac{2\pi S_1(k)\delta\omega^{\tilde{\mu}_R+\tilde{\mu}_L-1}}{\Gamma(\tilde{\mu}_R+\tilde{\mu}_L)(v+v_d)^{\tilde{\mu}_L}|v-v_d|^{\tilde{\mu}_R}}$$

in which

$$|\langle k;N|\hat{\rho}|N\rangle|^2 \approx \frac{S_{1(2)}(k)}{L}\left(\frac{2\pi}{L}\right)^{\tilde{\mu}_R+\tilde{\mu}_L} \qquad \tilde{\mu}_{R(L)} = \left(\frac{\sqrt{K}}{2}\pm\frac{1}{2\sqrt{K}}+\frac{\delta_\pm(k)}{2\pi}\right)^2$$

Another exact approach towards correlations

Gapless XXZ AFM: analytics using vertex operator approach

JSC, H. Konno, M. Sorrell and R. Weston, PRL 106, 217203 (2011), arxiv 1110.6641

We consider the XXZ in zero field,

$$H = J \sum_{j=1}^{N} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right)$$

$$0 \le \Delta \le 1$$

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$$H = J \sum_{j=1}^{N} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right) \qquad 0 \leq \Delta \leq 1$$

Longitudinal structure factor:

$$S^{zz}(k,\omega) = \frac{1}{N} \sum_{j,j'} e^{-ik(j-j')} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle S_j^z(t) S_{j'}^z(0) \rangle$$

Longitudinal structure factor

Separates into $$S^{zz}(k,\omega) = \sum_{m=1}^{\infty} S^{zz}_{(2m)}(k,\omega)$$

Longitudinal structure factor

Separates into $$S^{zz}(k,\omega) = \sum_{m=1}^{\infty} S^{zz}_{(2m)}(k,\omega)$$

Matrix elements: from vertex operator approach

Jimbo, Miwa, Lashkevich, Pugai, Kojima, Konno, Weston, JSC

Longitudinal structure factor

Separates into $$S^{zz}(k,\omega) = \sum_{m=1}^{\infty} S^{zz}_{(2m)}(k,\omega)$$

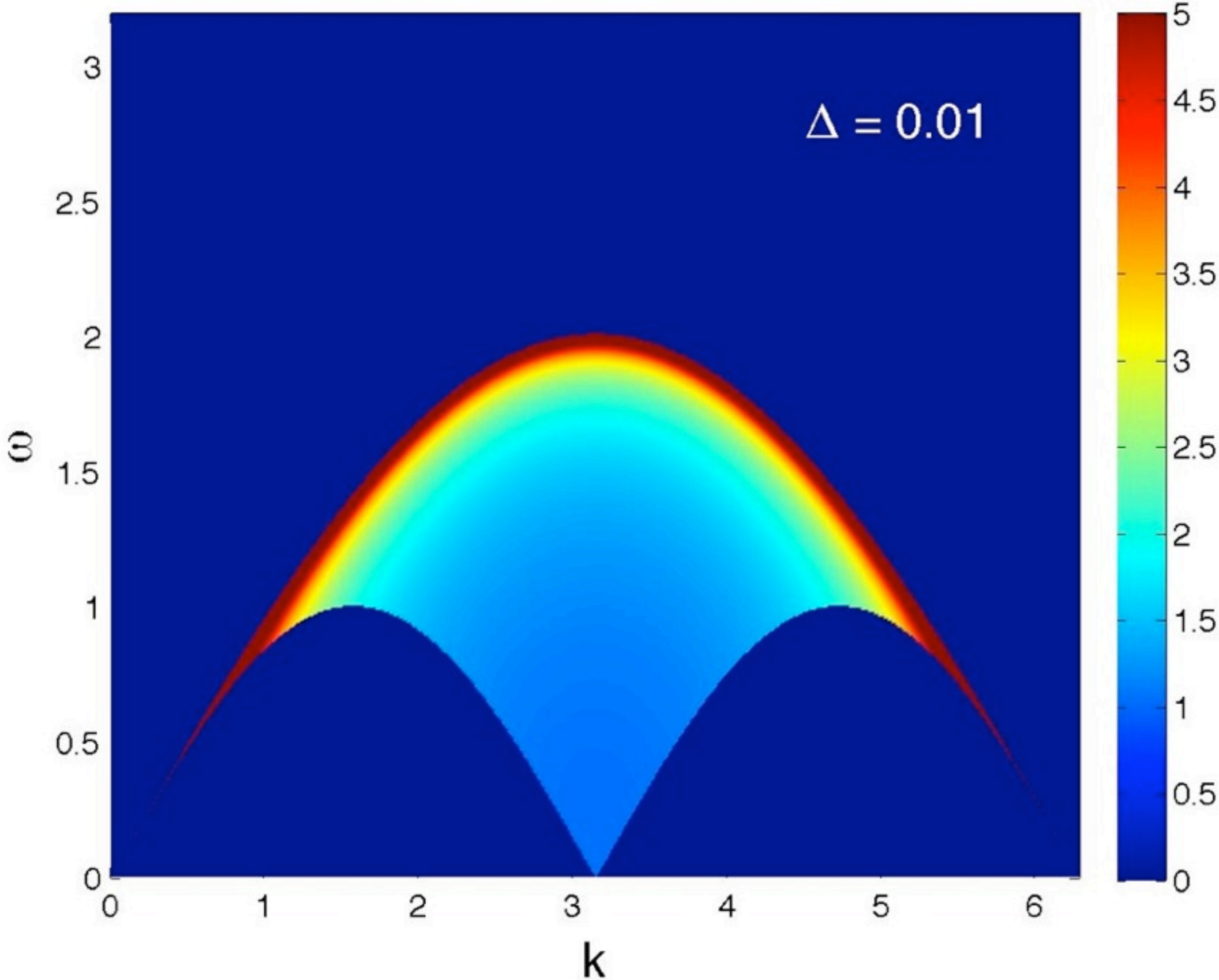
Matrix elements: from vertex operator approach

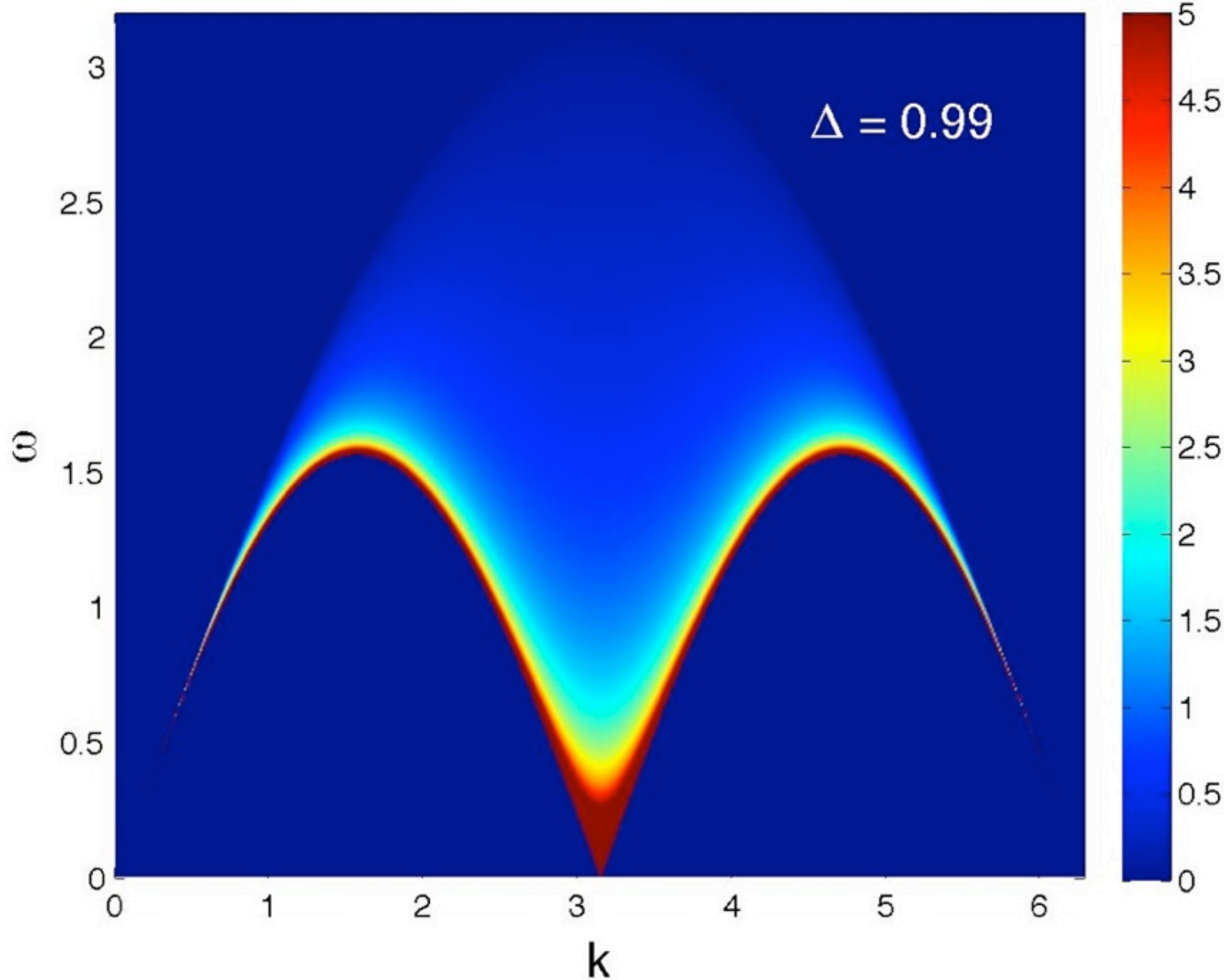
Jimbo, Miwa, Lashkevich, Pugai, Kojima, Konno, Weston, JSC

$$S_2^{zz}(k,\omega) = \frac{\Theta(\omega_{2,u}(k)-\omega)\Theta(\omega-\omega_{2,l}(k))}{\sqrt{\omega_{2,u}^2(k)-\omega^2}} \times (1+1/\xi)^2 \frac{e^{-I_\xi(\rho(k,\omega))}}{\cosh\frac{2\pi\rho(k,\omega)}{\xi}+\cos\frac{\pi}{\xi}}$$

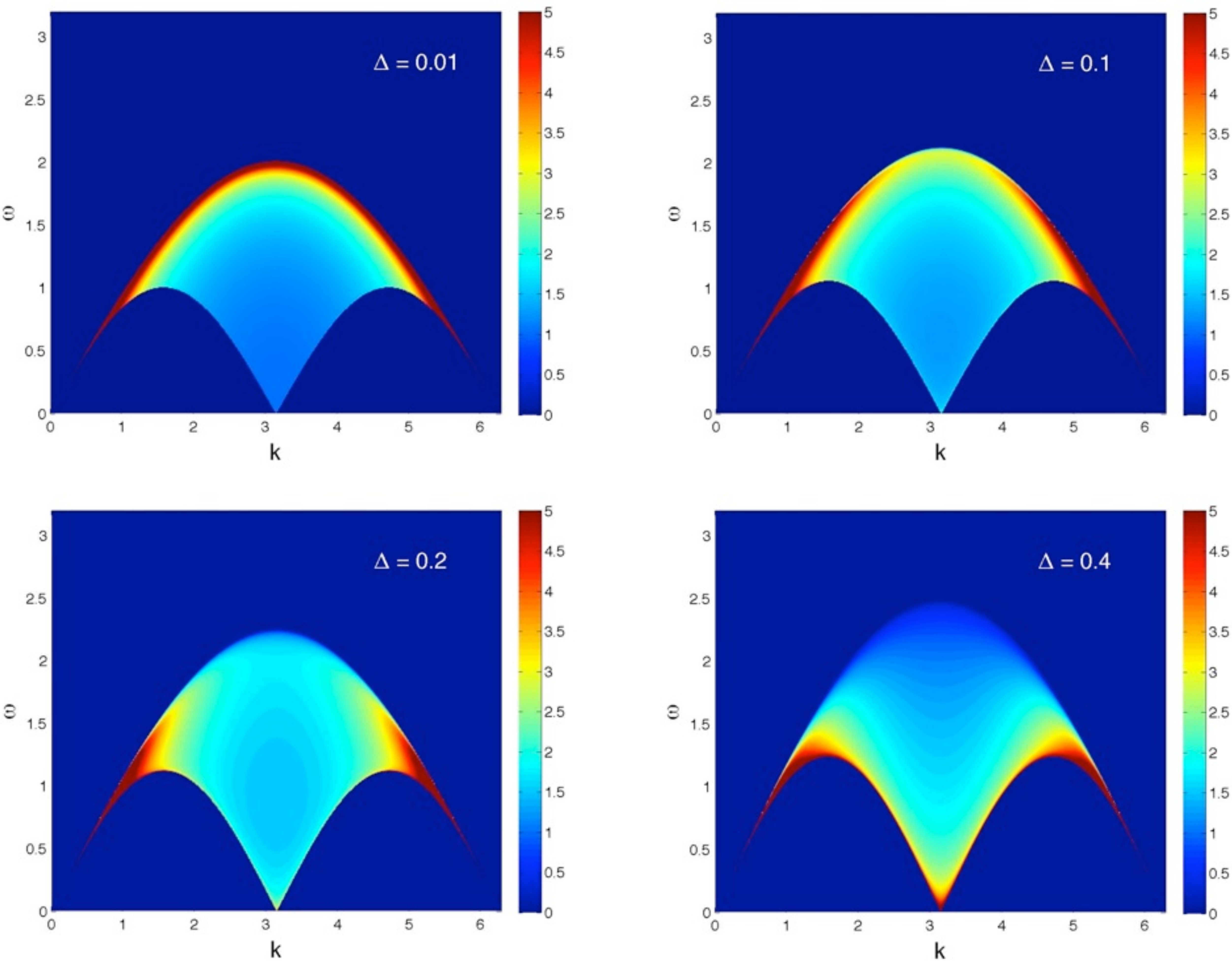
where $\xi = \frac{\pi}{\text{acos}\Delta} - 1$ $\qquad \cosh(\pi\rho(k,\omega)) = \sqrt{\frac{\omega_{2,u}^2(k)-\omega_{2,l}^2(k)}{\omega^2-\omega_{2,l}^2(k)}}$

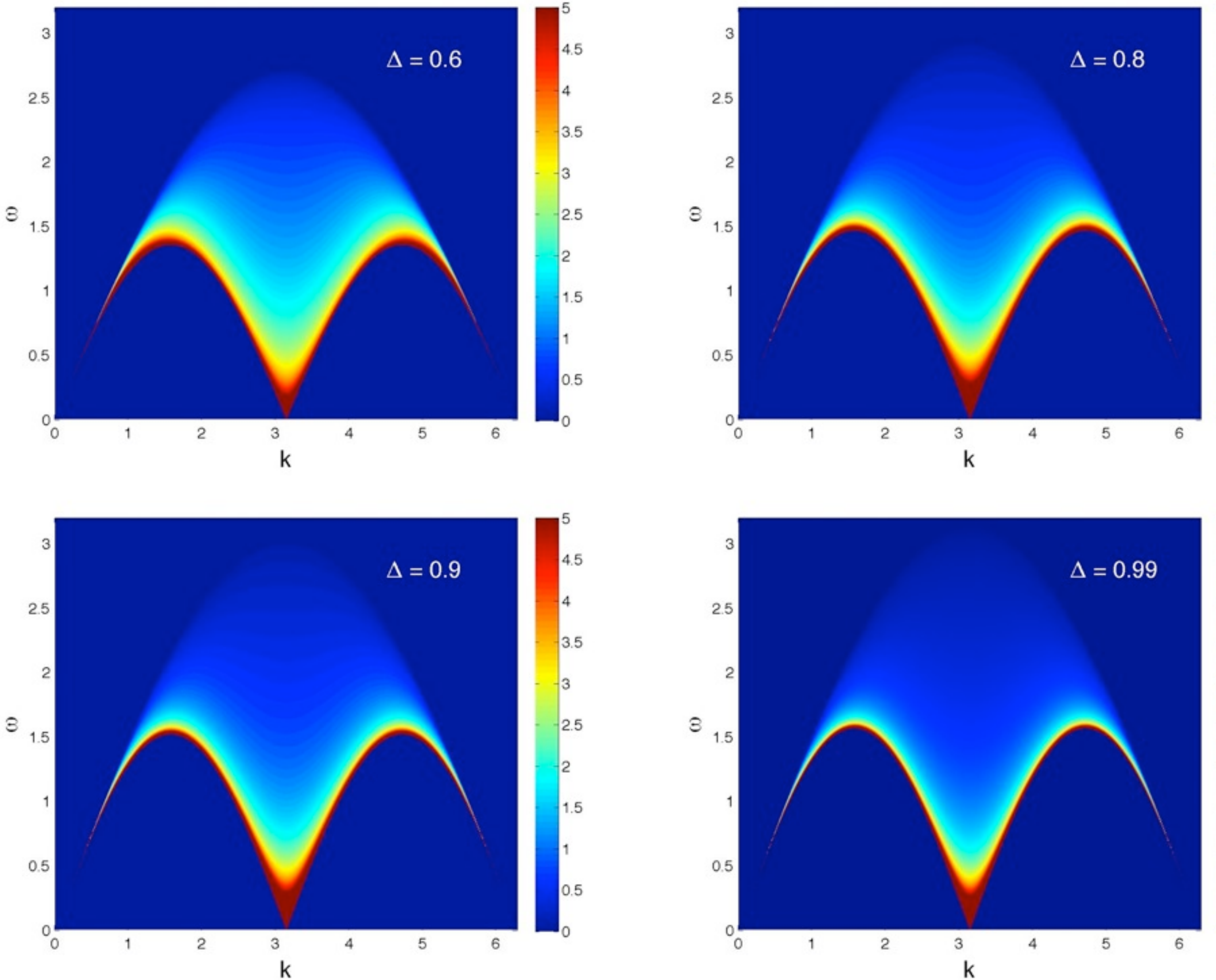
$$I_\xi(\rho) \equiv \int_0^\infty \frac{dt}{t} \frac{\sinh(\xi+1)t}{\sinh\xi t} \frac{\cosh(2t)\cos(4\rho t)-1}{\cosh t \sinh(2t)}$$


Δ = 0.01
ω
k
0
1
2
3
4
5
6
0.5
1.5
2.5
4.5
3.5



Δ = 0.99
ω
k
0
0.5
1
1.5
2
2.5
3
0
1
2
3
4
5
6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Δ = 0.01
Δ = 0.1
Δ = 0.2
Δ = 0.4
ω
k

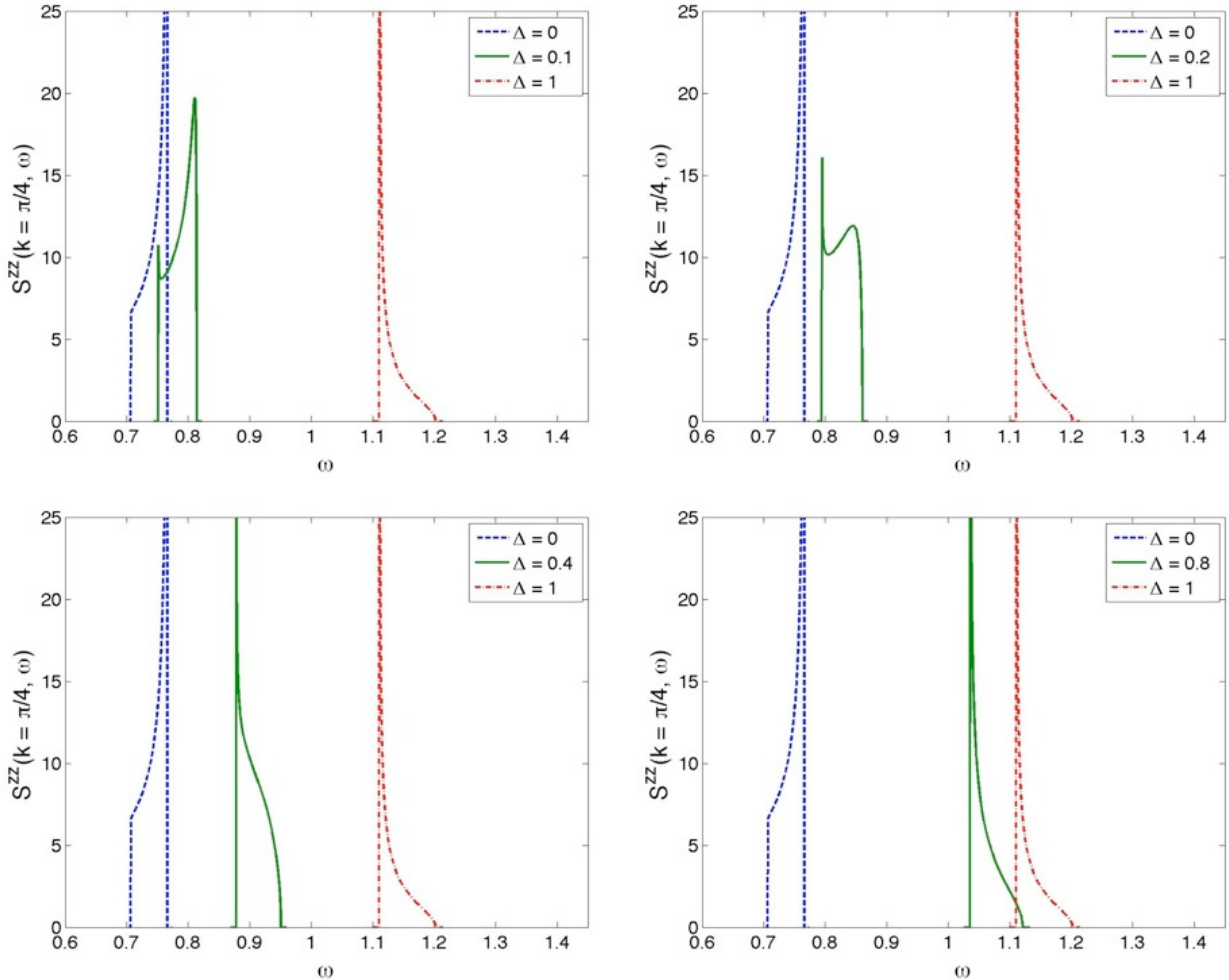

Δ = 0.6
Δ = 0.8
Δ = 0.9
Δ = 0.99
ω
k

Sum rule saturations from two-spinon states

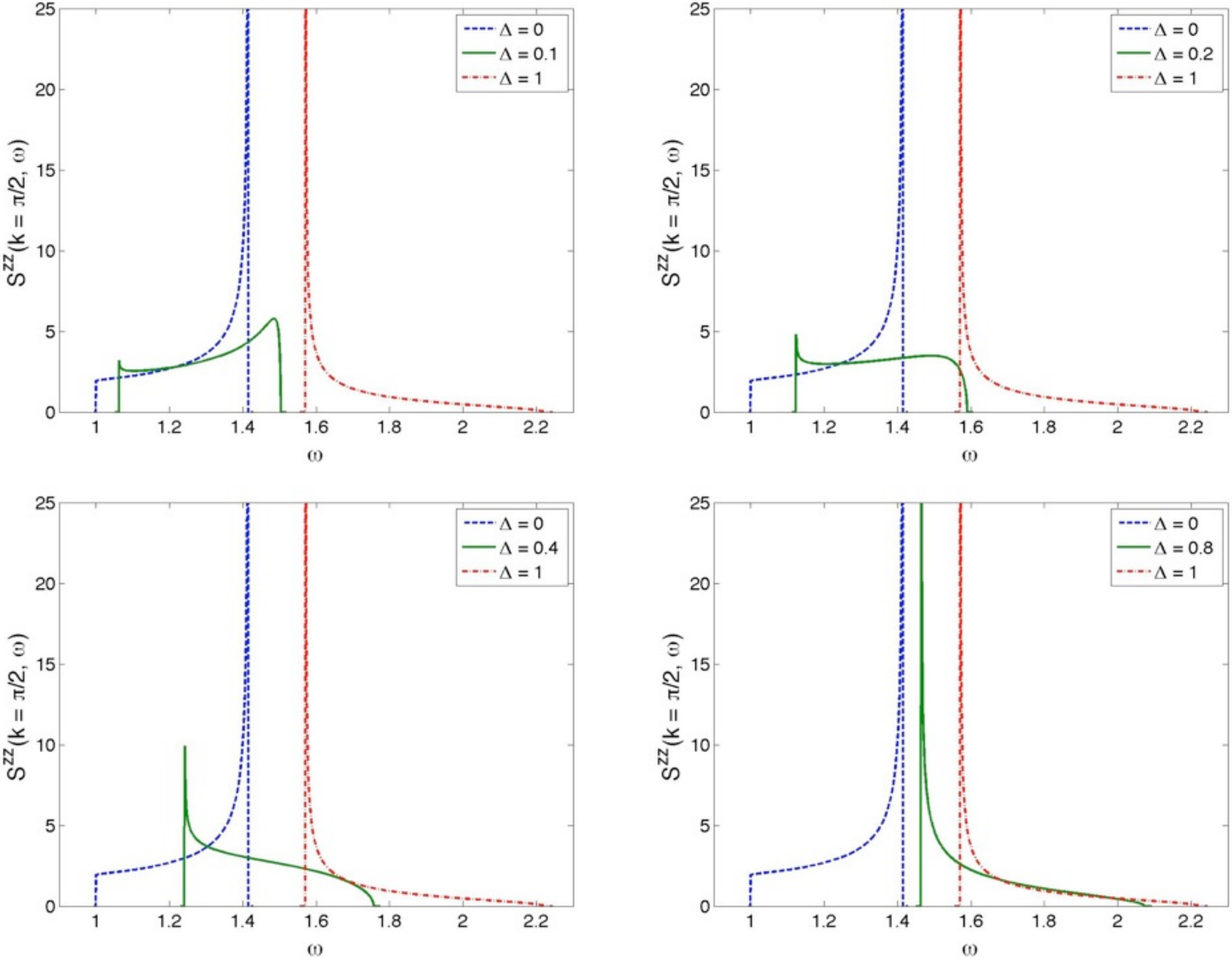
Integrated intensity $I^{zz} = \int_0^{2\pi} \frac{dk}{2\pi} \int_0^{\infty} \frac{d\omega}{2\pi} S(k,\omega) = 1/4$

f-sumrule $I_1^{zz}(k) = \int_0^{2\pi} \frac{d\omega}{2\pi} \omega S(k,\omega) = -2X^x(1-\cos k)$ $\quad X^x \equiv \langle S_j^x S_{j+1}^x \rangle$

Δ	I_{2sp}^{zz}/I^{zz}	$I_{1,2sp}^{zz}/I_1^{zz}$		Δ	I_{2sp}^{zz}/I^{zz}	$I_{1,2sp}^{zz}/I_1^{zz}$
0	1	1		0.6	0.9778	0.9743
0.1	0.9997	0.9997		0.7	0.9637	0.9578
0.2	0.9986	0.9984		0.8	0.9406	0.9314
0.3	0.9964	0.9959		0.9	0.8980	0.8844
0.4	0.9927	0.9917		0.99	0.7918	0.7748
0.5	0.9869	0.9849		0.999	0.7494	0.7331

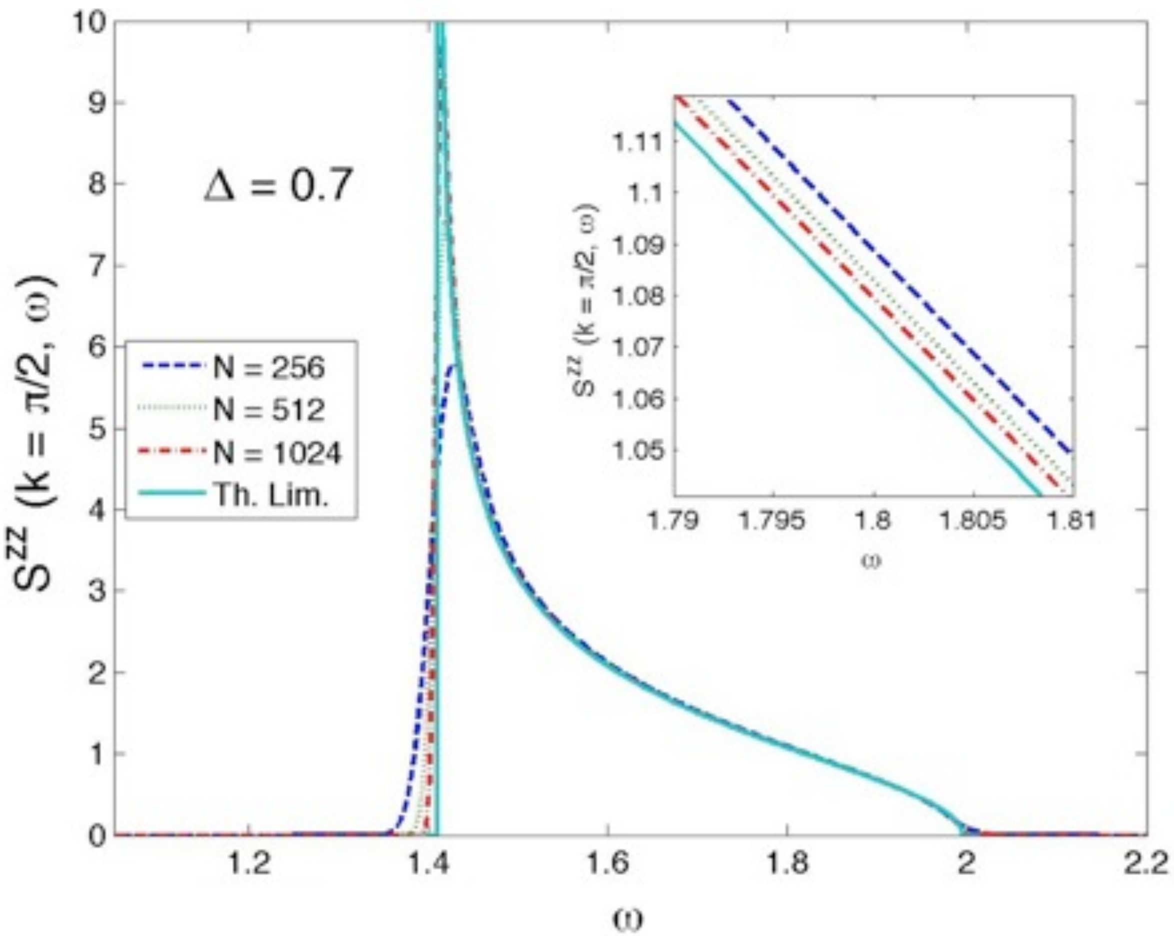


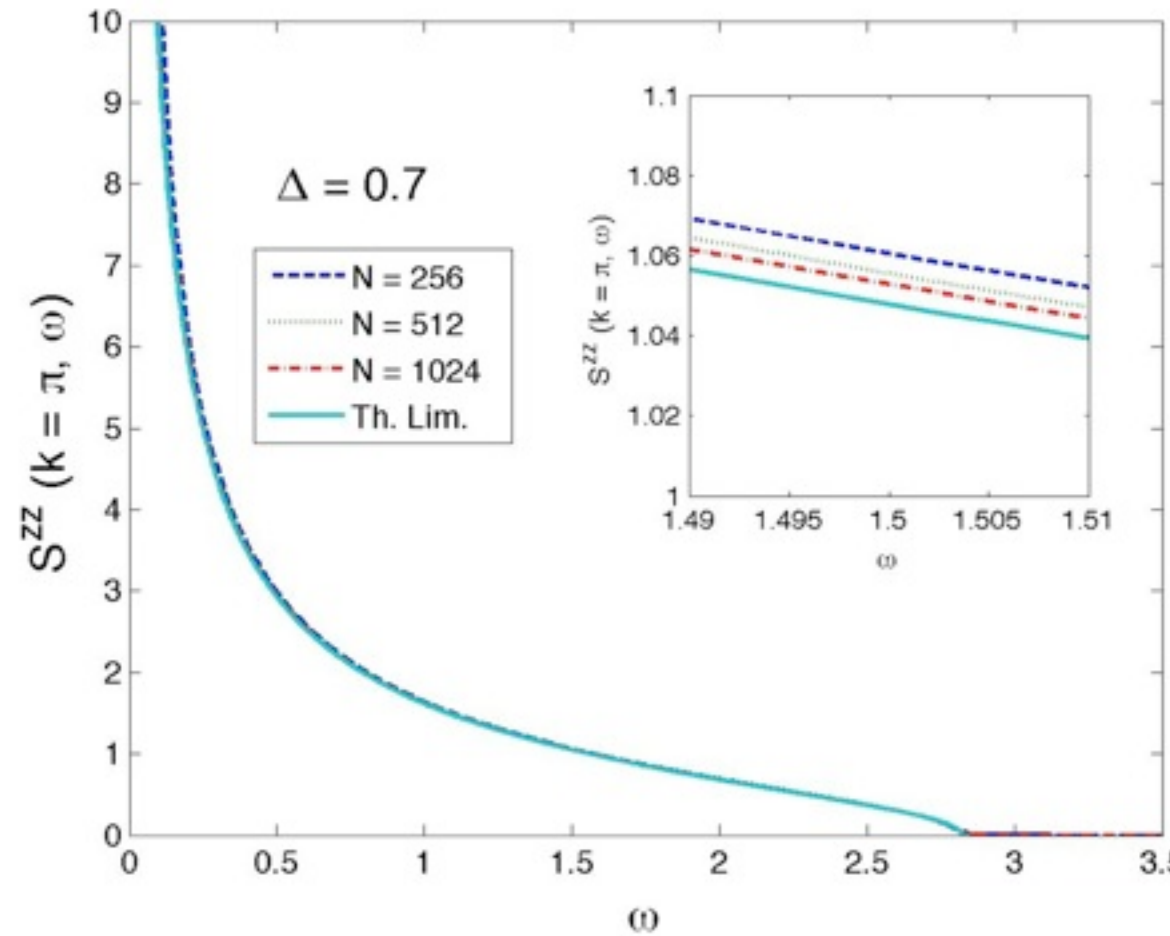
Δ = 0
Δ = 0.1
Δ = 1
Δ = 0.2
Δ = 0.4
Δ = 0.8
S^zz(k = π/4, ω)
ω



S^zz(k = π/2, ω)
ω
Δ = 0
Δ = 0.1
Δ = 1
Δ = 0
Δ = 0.2
Δ = 1
Δ = 0
Δ = 0.4
Δ = 1
Δ = 0
Δ = 0.8
Δ = 1

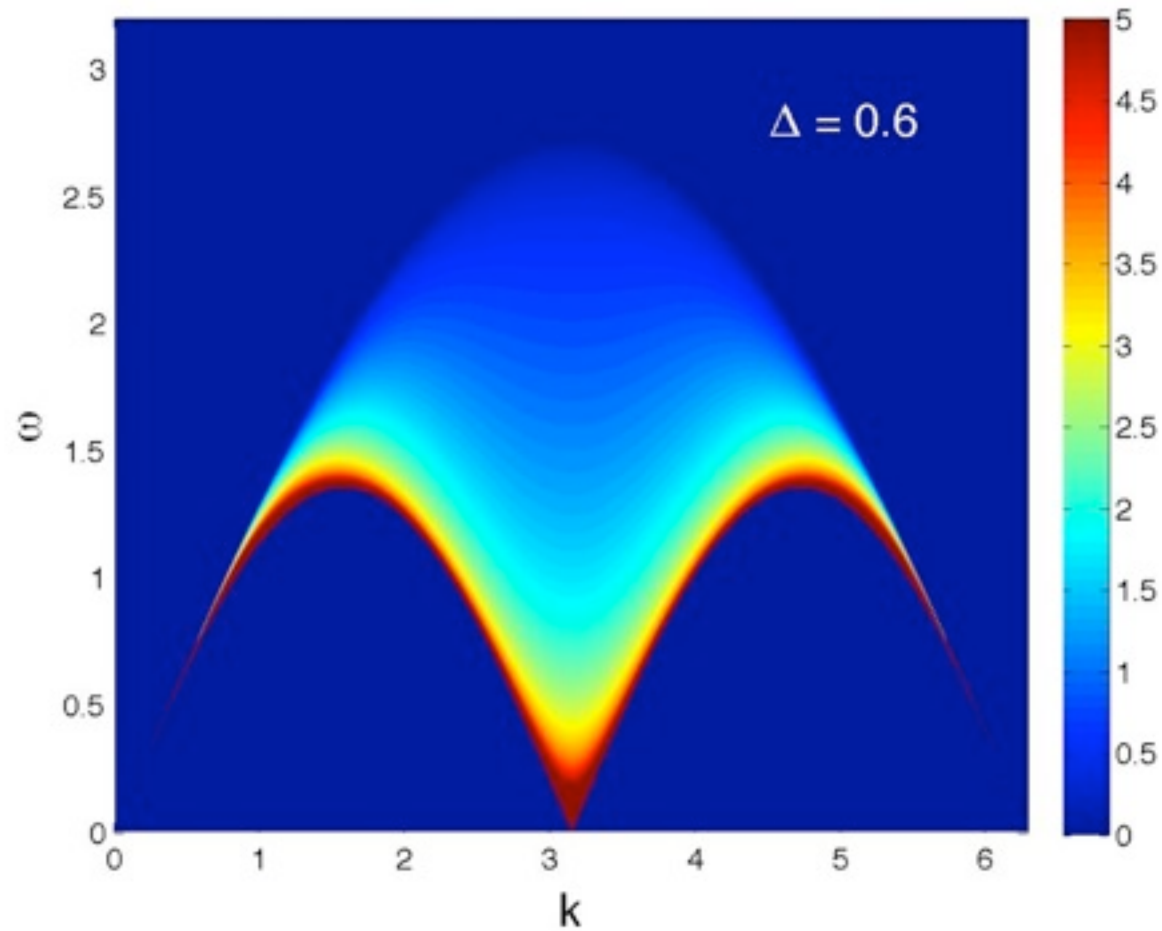
Check against finite-size results





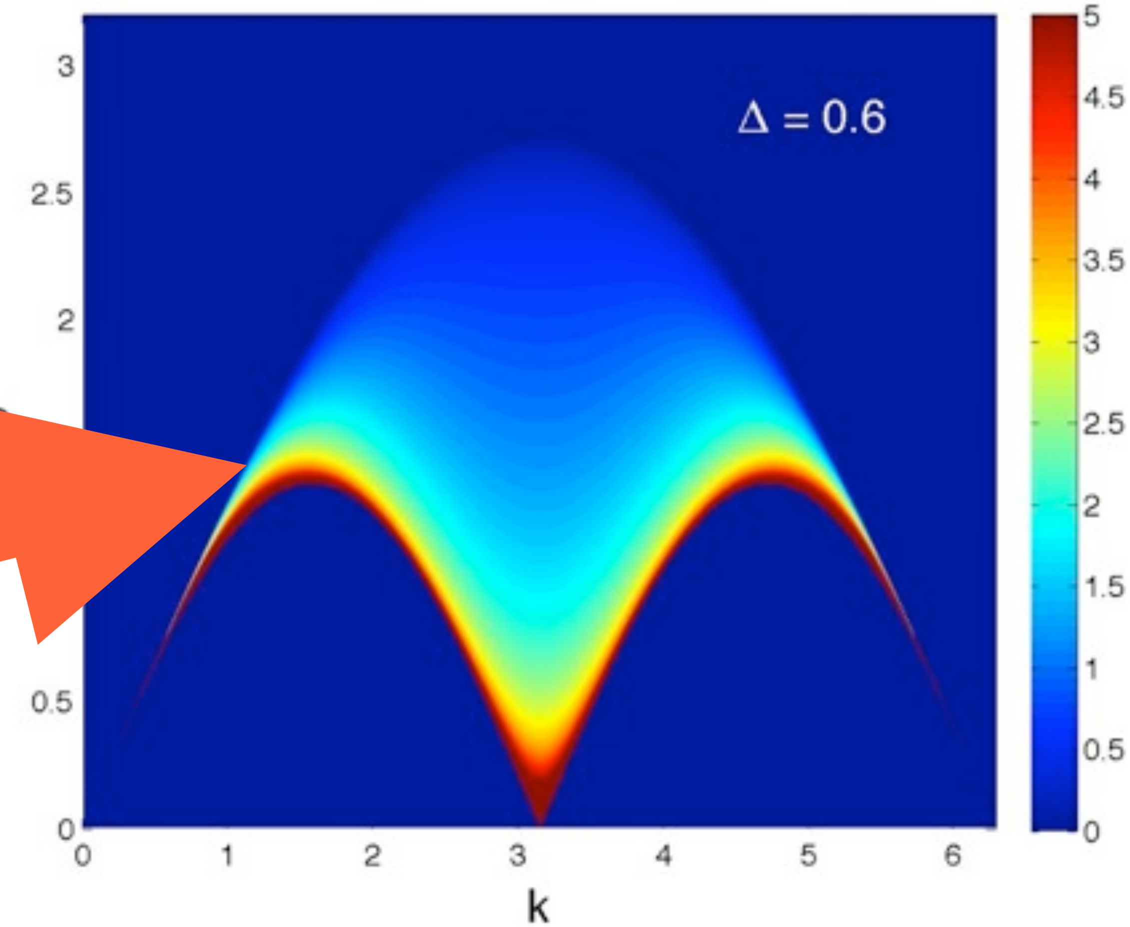
N	I^{zz}_{2sp}/I^{zz}	$I^{zz}_{1,2sp}(\pi/2)/I^{zz}_1$	$I^{zz}_{1,2sp}(\pi)/I^{zz}_1$
64	0.9893	0.9825	0.9852
128	0.9843	0.9778	0.9776
256	0.9796	0.9733	0.9713
512	0.9756	0.9695	0.9668
1024	0.9724	0.9664	0.9636
extrap	0.963(2)	0.957(4)	0.957(4)
∞	0.9637	0.9578	0.9578

Threshold behaviour

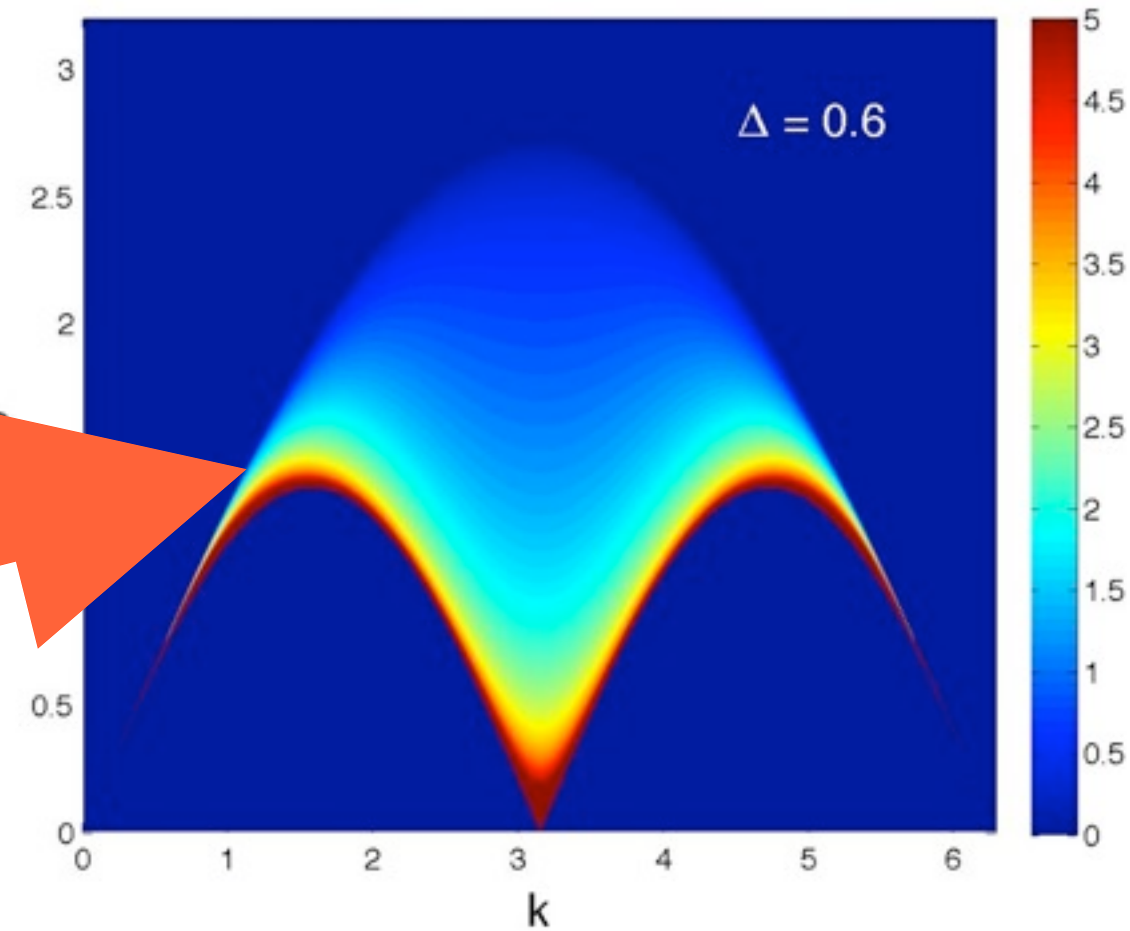


Threshold behaviour

Near upper threshold:



Threshold behaviour

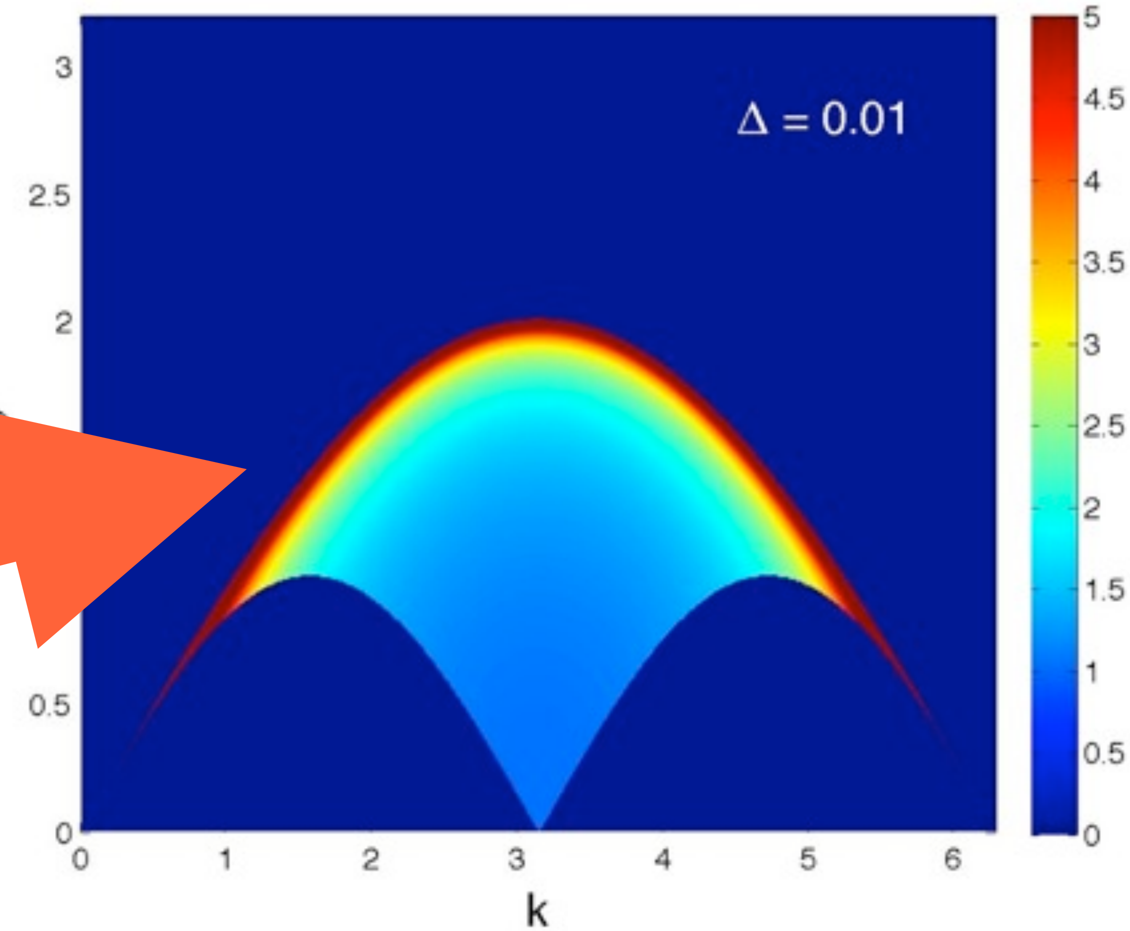


Near upper threshold:

For $0 < \Delta \leq 1$:

$$S_2^{zz}(k,\omega) \xrightarrow[\omega \to \omega_{2,u}(k)]{} f_u(\xi)(\sin\frac{k}{2})^{-7/2}\sqrt{\omega_{2,u}(k) - \omega}$$

Threshold behaviour



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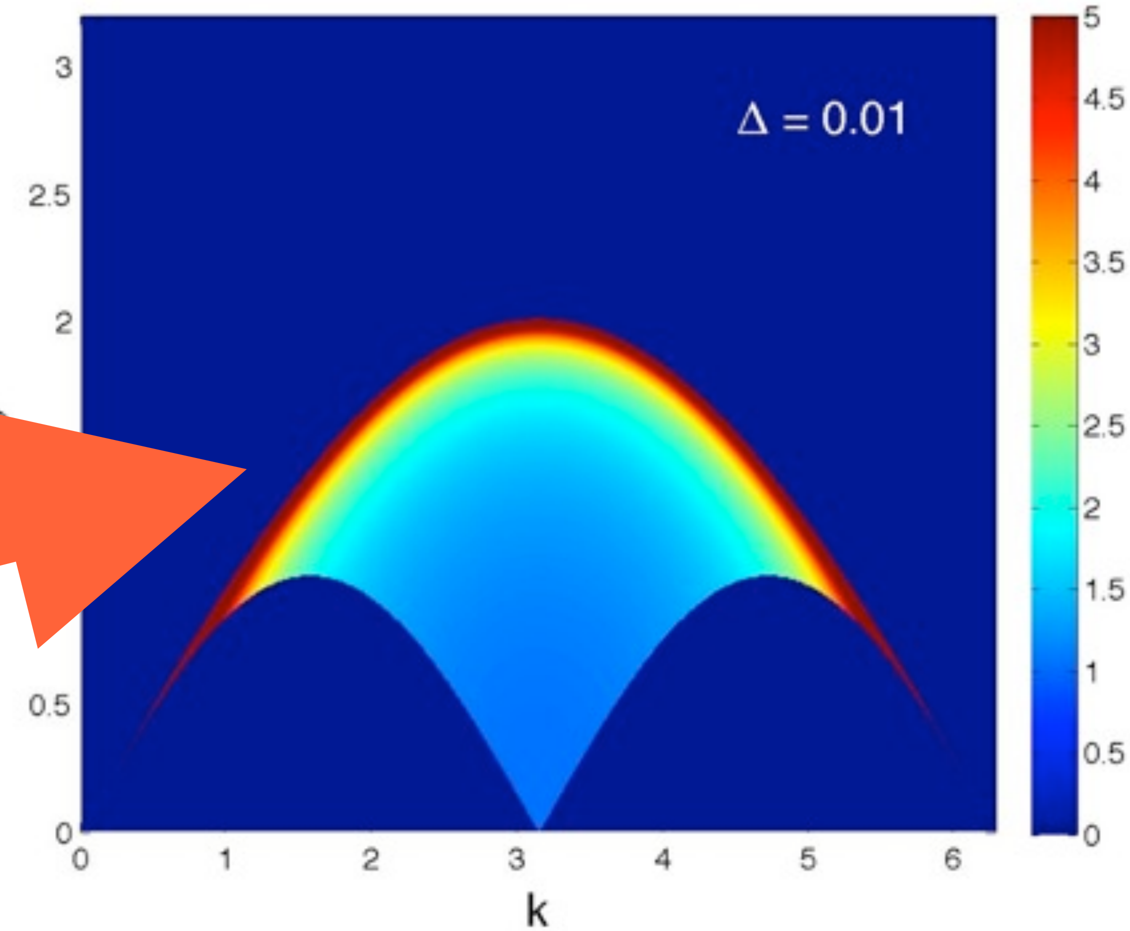
$$S_2^{zz}(k,\omega) \xrightarrow[\omega \to \omega_{2,u}(k)]{} f_u(\xi)(\sin\frac{k}{2})^{-7/2}\sqrt{\omega_{2,u}(k) - \omega}$$

For $\Delta \to 0$:

$$S_2^{zz}(k,\omega) \xrightarrow[\omega \to \omega_{2,u}(k)]{} f_u(1)\frac{(\sin\frac{k}{2})^{-1/2}}{\sqrt{\omega_{2,u}(k) - \omega}}$$

Threshold behaviour



Near upper threshold:

For $0 < \Delta \leq 1$:

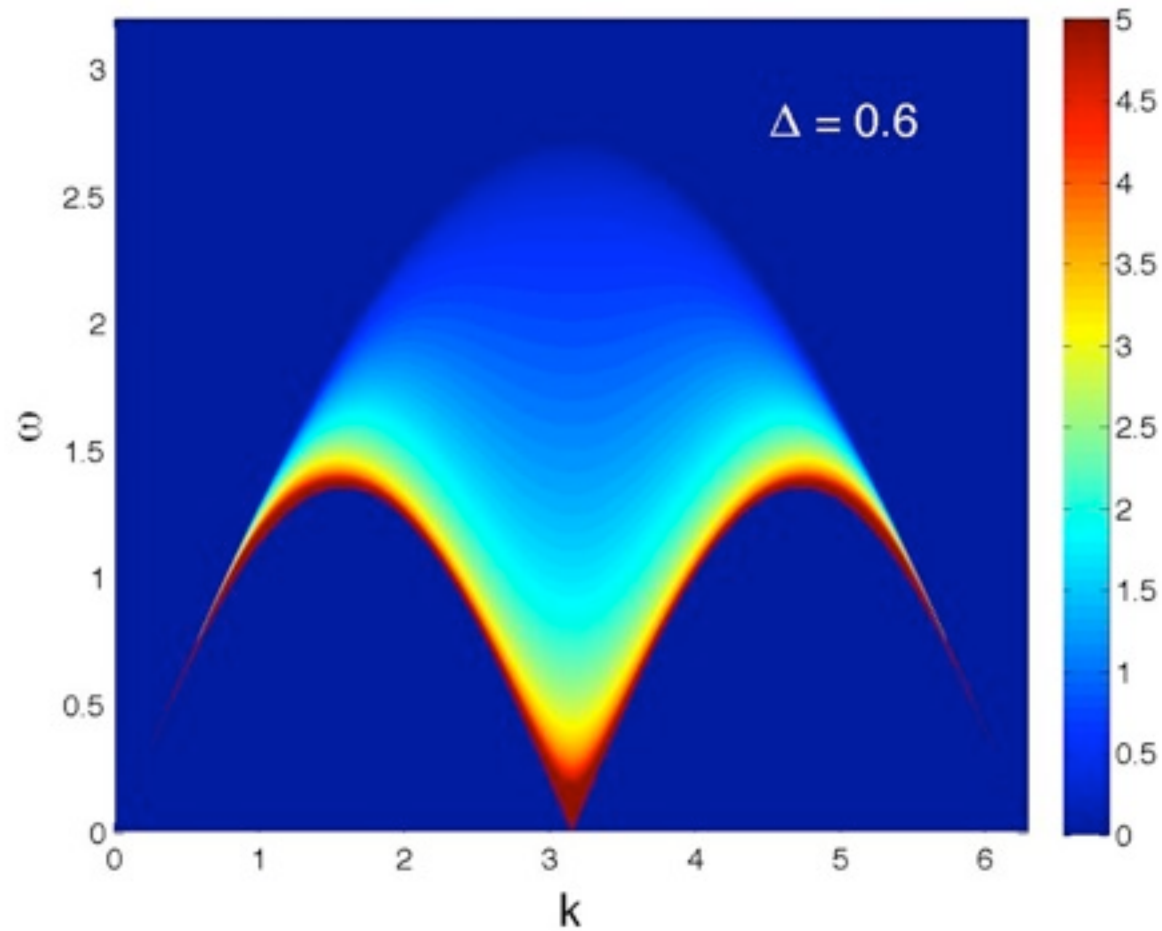
$$S_2^{zz}(k,\omega) \xrightarrow[\omega\to\omega_{2,u}(k)]{} f_u(\xi)(\sin\frac{k}{2})^{-7/2}\sqrt{\omega_{2,u}(k)-\omega}$$

For $\Delta \to 0$:

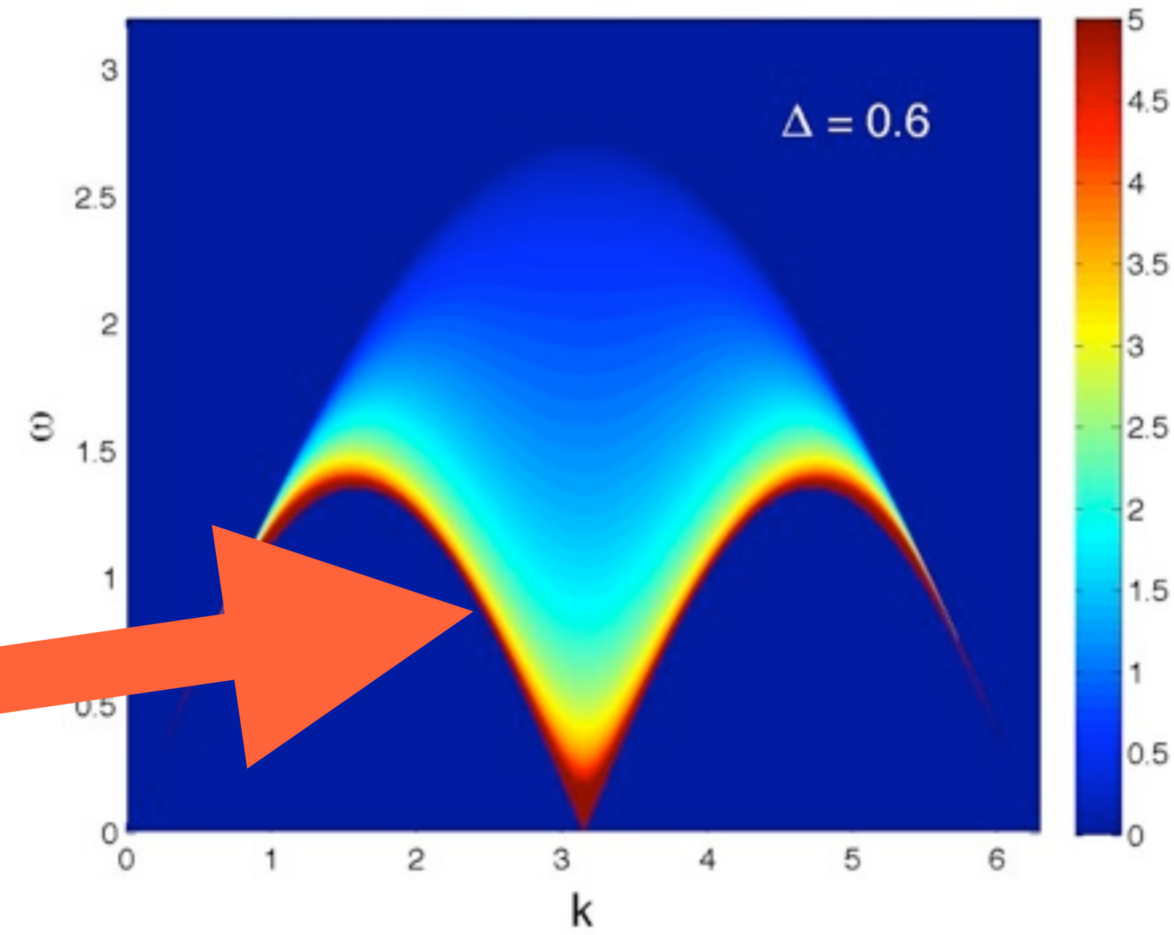
$$S_2^{zz}(k,\omega) \xrightarrow[\omega\to\omega_{2,u}(k)]{} f_u(1)\frac{(\sin\frac{k}{2})^{-1/2}}{\sqrt{\omega_{2,u}(k)-\omega}}$$

Agrees with Nonlinear LL predictions (Imambekov, Glazman, Pereira, Affleck, ...)
Adds momentum-dependent prefactors

Threshold behaviour

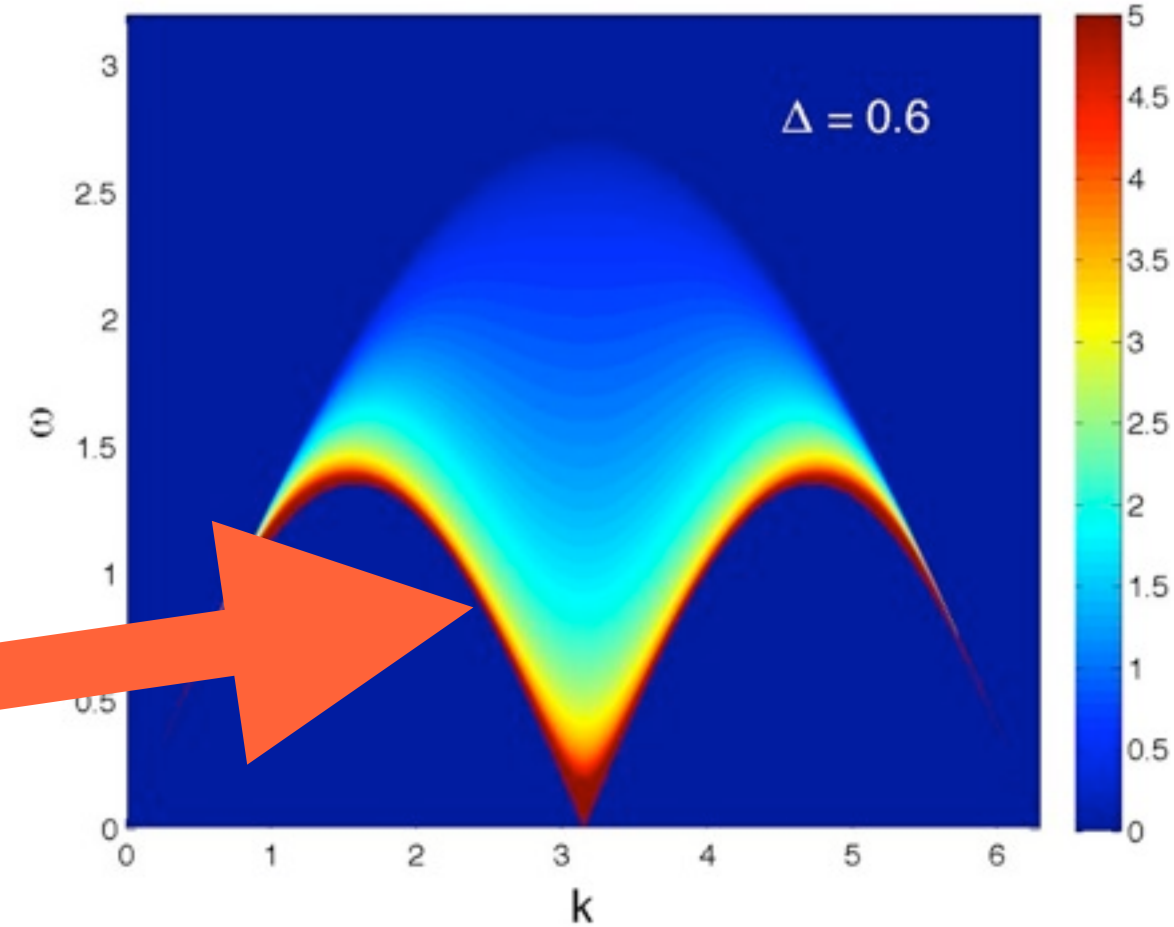


Threshold behaviour



Near lower threshold:

Threshold behaviour

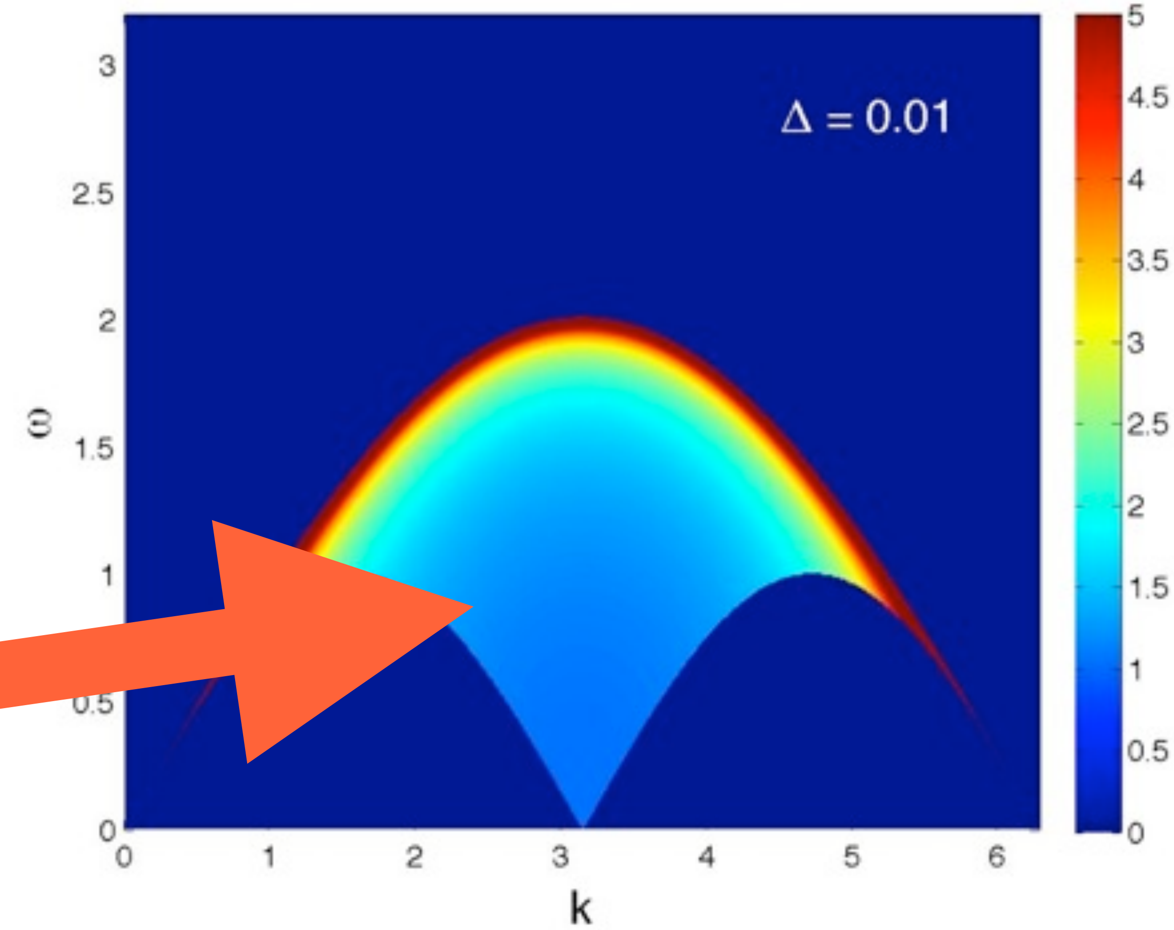


Near lower threshold:

For $0 < \Delta \leq 1$:

$$S_2^{zz}(k,\omega) \xrightarrow[\omega \to \omega_{2,l}(k)]{} f_l(\xi) \frac{|\sin k|^{-\frac{1}{2}(1-\frac{1}{\xi})} (\sin \frac{k}{2})^{-\frac{2}{\xi}}}{[\omega - \omega_{2,l}(k)]^{\frac{1}{2}(1-\frac{1}{\xi})}}$$

Threshold behaviour



Near lower threshold:

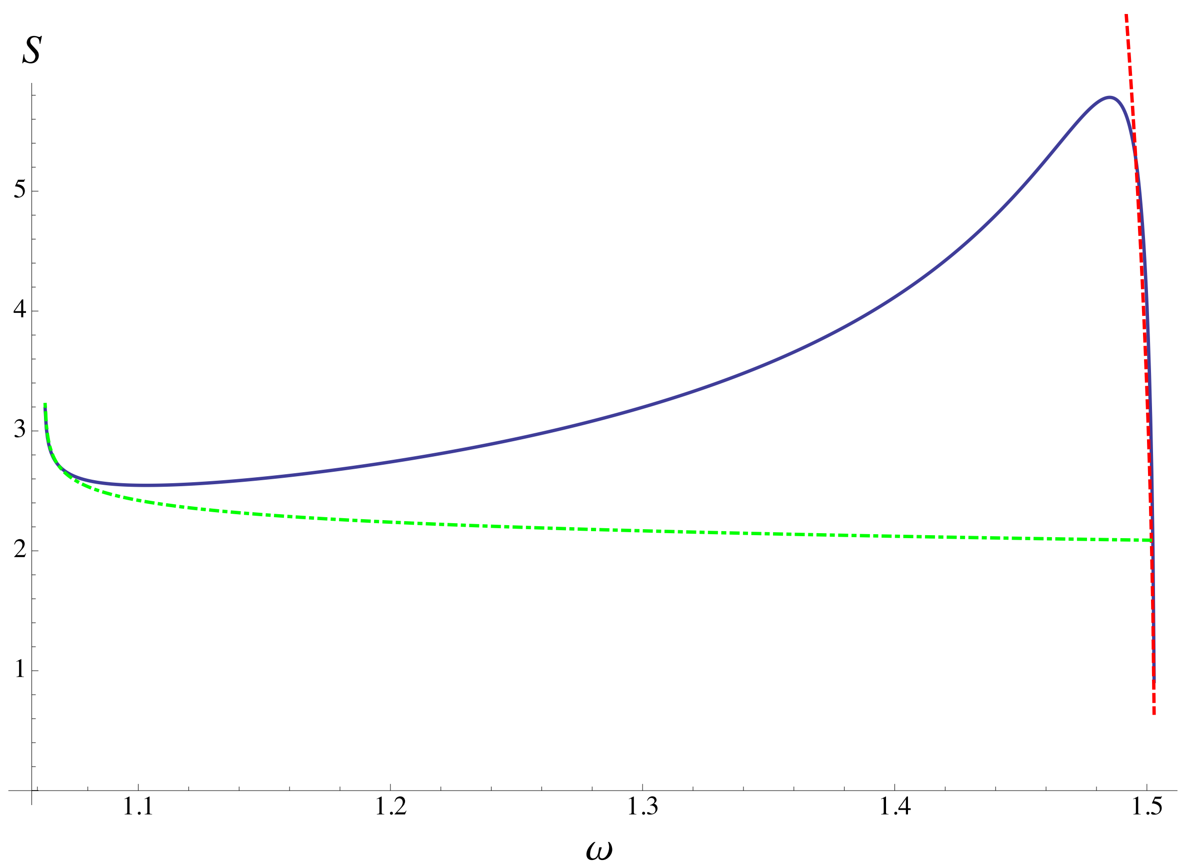
For $0 < \Delta \leq 1$:

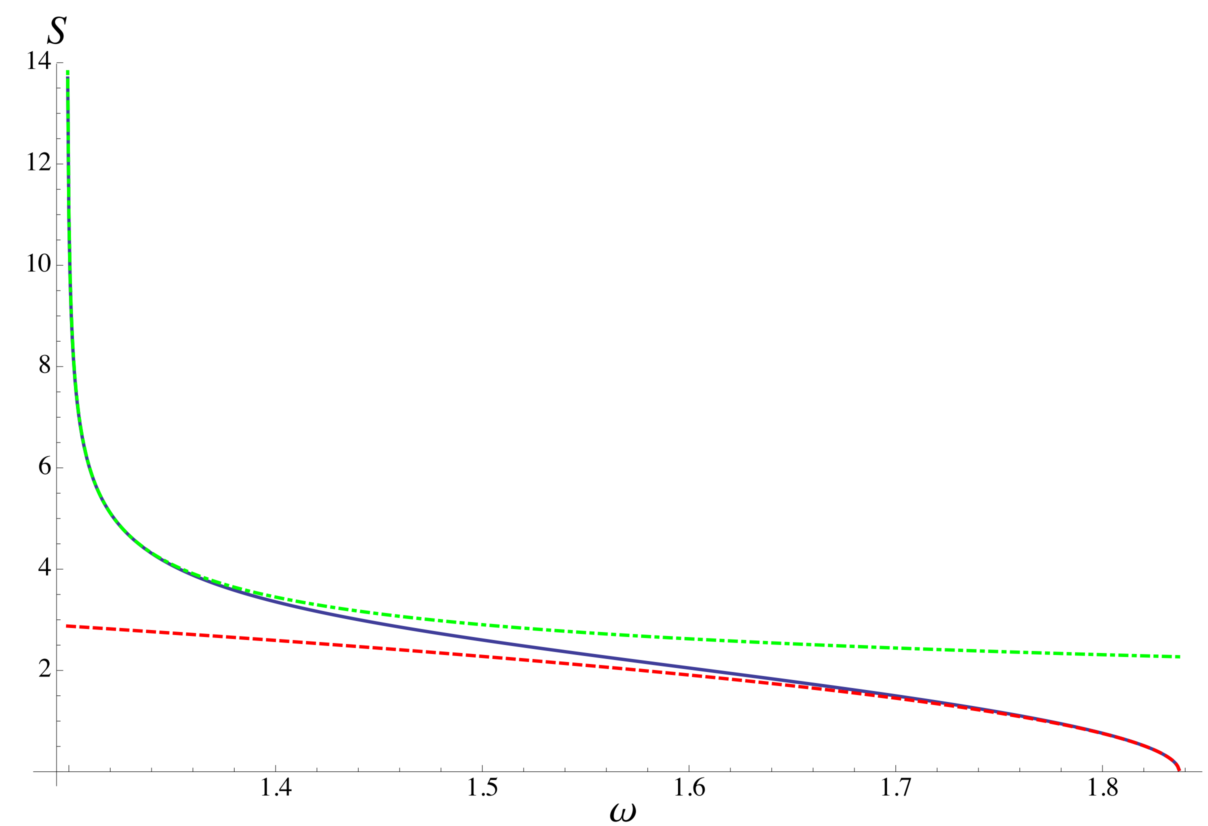
$$S_2^{zz}(k,\omega) \xrightarrow[\omega\to\omega_{2,l}(k)]{} f_l(\xi) \frac{|\sin k|^{-\frac{1}{2}(1-\frac{1}{\xi})} (\sin\frac{k}{2})^{-\frac{2}{\xi}}}{[\omega - \omega_{2,l}(k)]^{\frac{1}{2}(1-\frac{1}{\xi})}}$$

For $\Delta \to 0$:

$$S_2^{zz}(k,\omega) \xrightarrow[\omega\to\omega_{2,l}(k)]{} O(1)$$

Region of validity of threshold behaviour





$$\Delta = 0.1, k = \pi/4$$

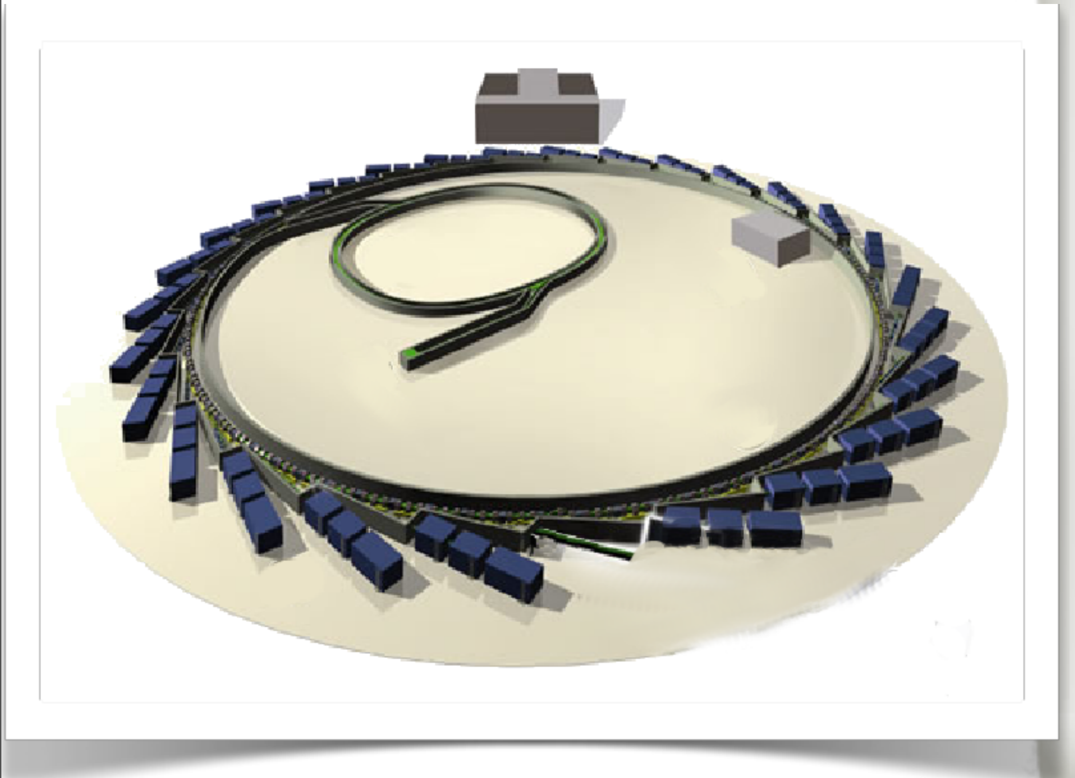
$$\Delta = 0.9, k = \pi/4$$

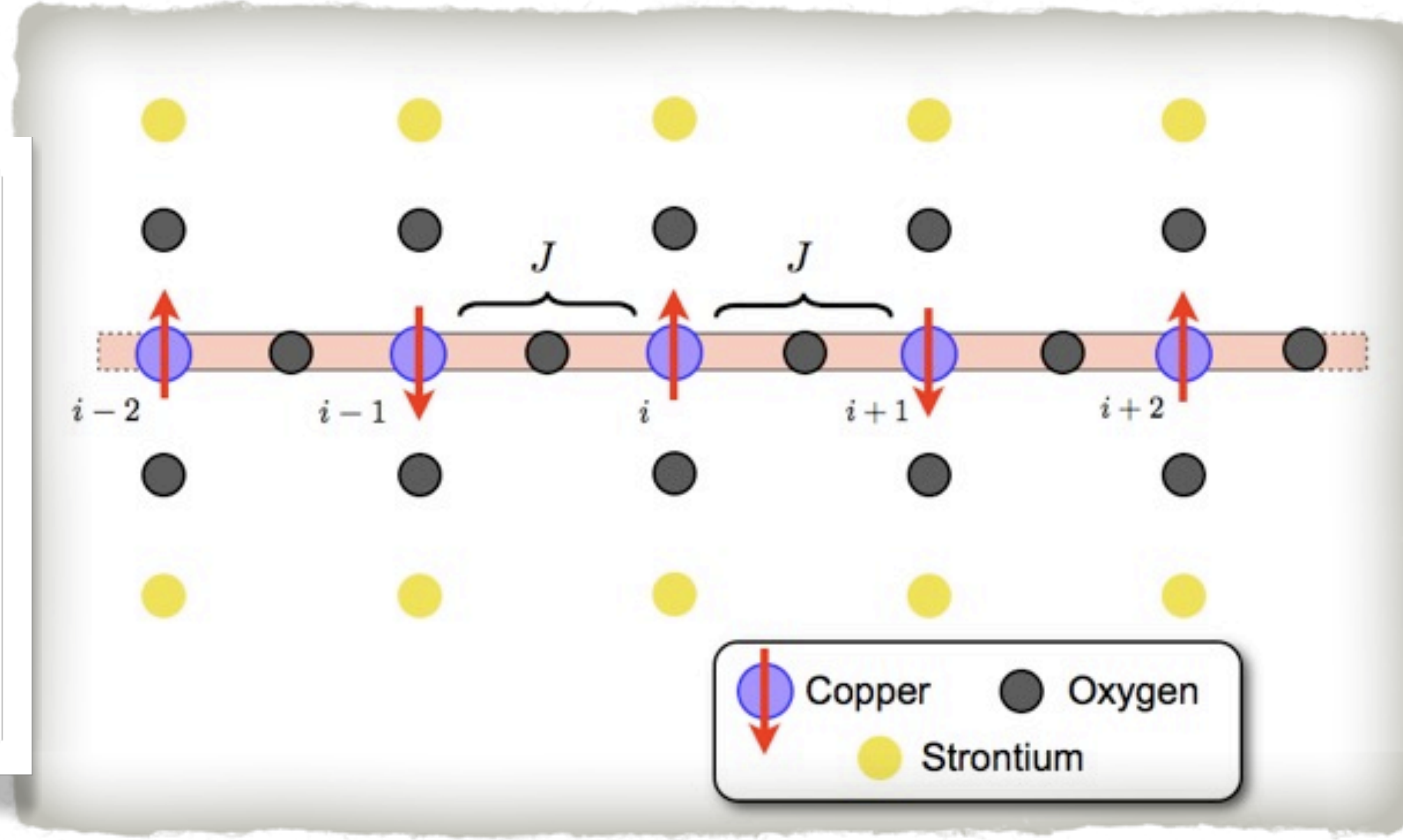
New applications

Another experimental method: RIXS

(Resonant Inelastic X-ray Scattering)

Synchrotron





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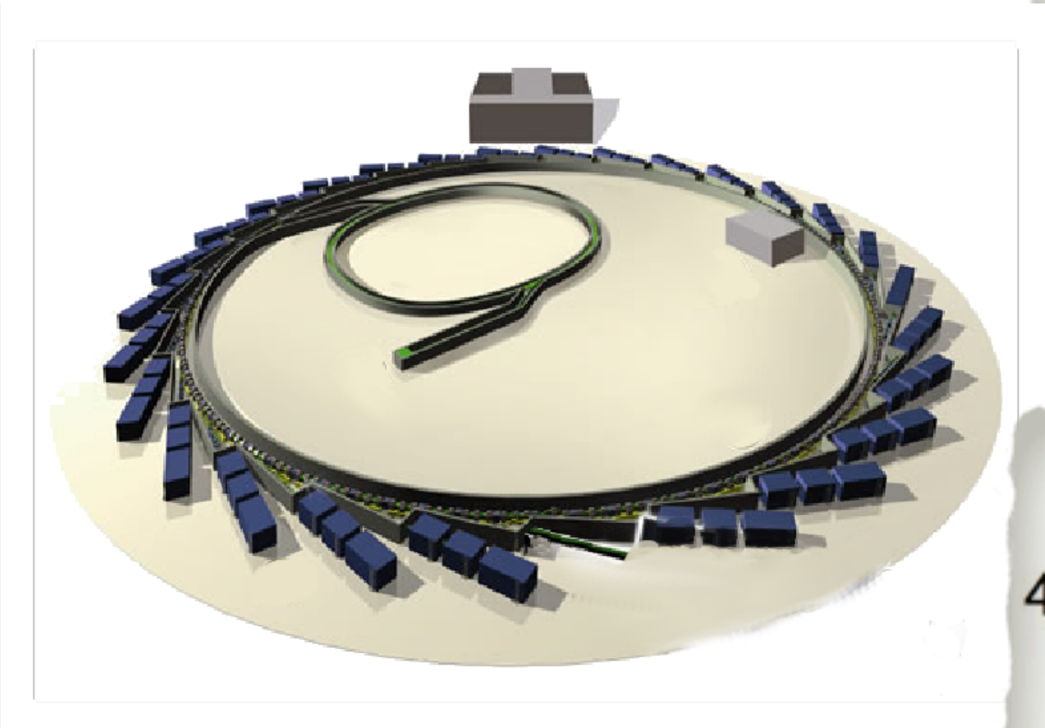
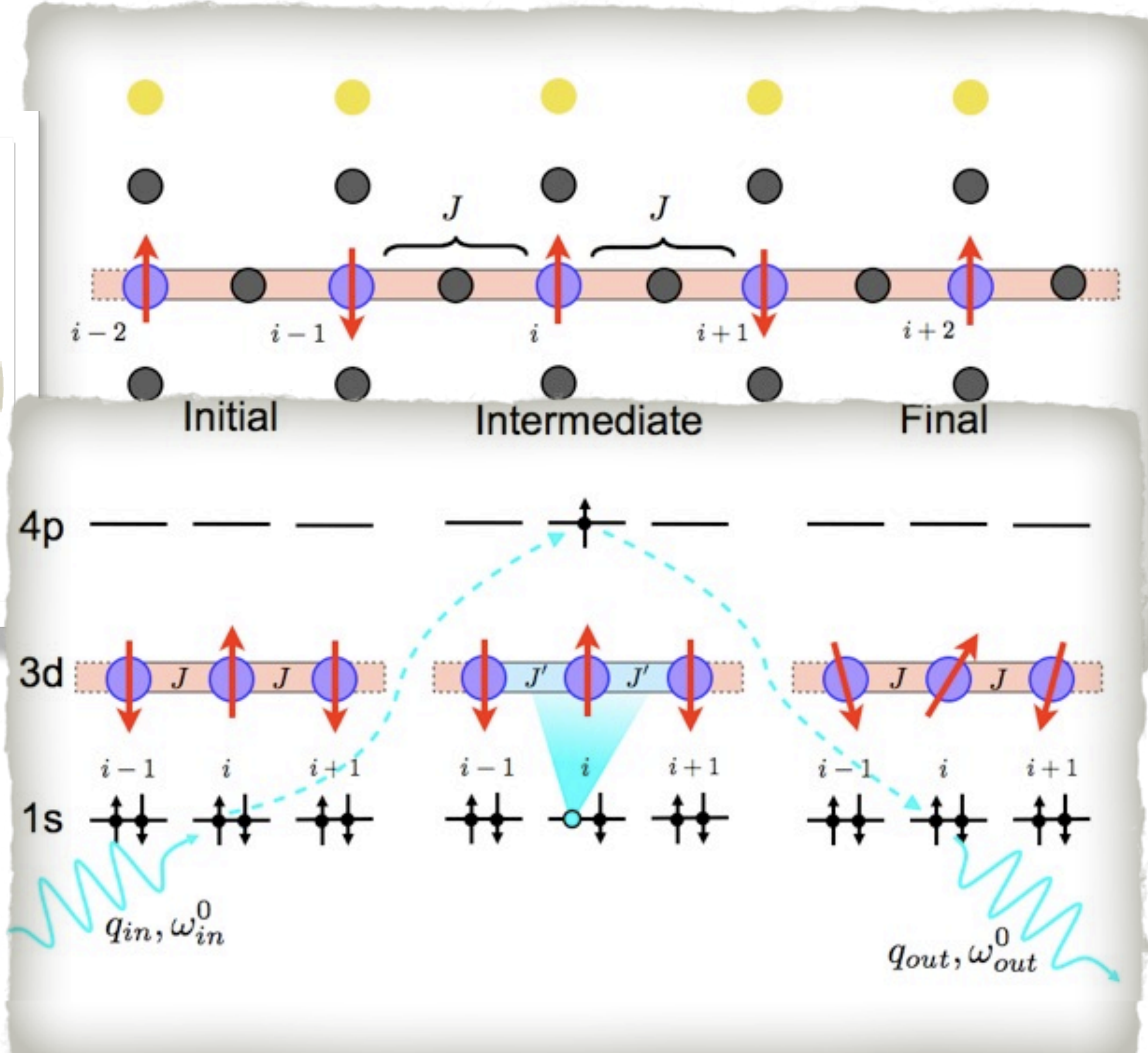


X-ray induces a 1s-4p transition on copper, modifying exchange term



RIXS response from ABACUS

A. Klauser, J. Mossel, JSC and J. van den Brink, Phys. Rev. Lett. 106, 157205 (2011)

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in which $$X_q \equiv \frac{1}{\sqrt{N}} \sum_j e^{iqj} (\mathbf{S}_{j-1} \cdot \mathbf{S}_j + \mathbf{S}_j \cdot \mathbf{S}_{j+1})$$

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Kitanine & al.; A. Klauser, J. Mossel and JSC, to be published

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Resummed by ABACUS

Using SU(2) symmetry:

$$S^{\mathrm{exch}}(q,\omega) = \cos^2(q/2)\frac{72\pi}{N} \times \sum_{\alpha \in S_{tot}=0} \left| \sum_j e^{iqj} \langle 0| S_j^z S_{j+1}^z |\alpha\rangle \right|^2 \delta(\omega - \omega_\alpha)$$

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Sum rules:

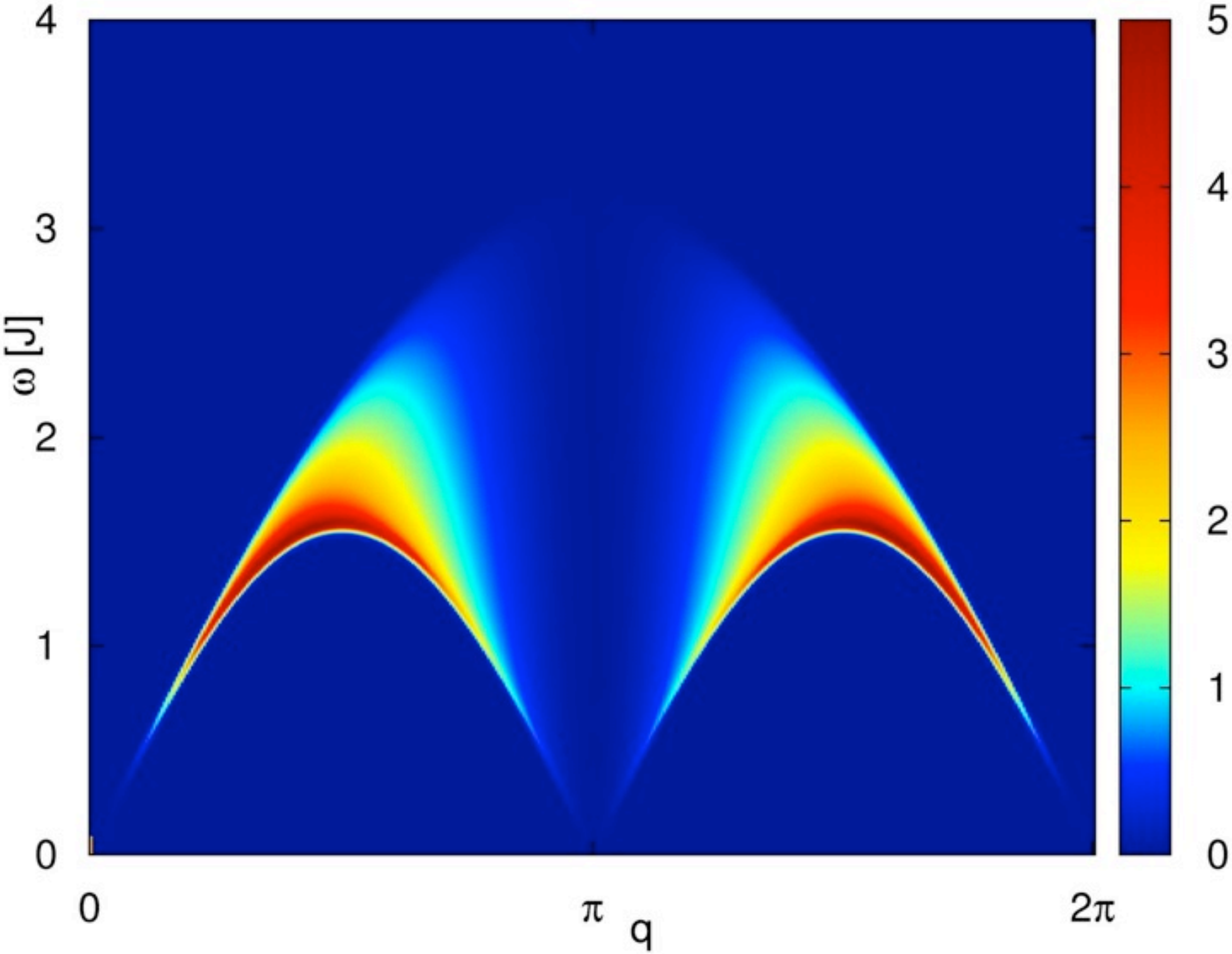
Integrated intensity $\displaystyle\int \frac{d\omega}{2\pi}\frac{1}{N}\sum_q S^{\mathrm{exch}}(q,\omega) = \frac{1}{4} - \ln(2) + \frac{9}{8}\zeta(3)$

f-sumrule

$$\int \frac{d\omega}{2\pi}\omega S^{\mathrm{exch}}(q,\omega) = 6\sin^2(q)\left\{(x_1 - x_2)\left(1 - 4\cos^2(q/2)\right) + \frac{3\zeta(3) - 4\ln(2)}{8}\right\}$$

in which $x_i \equiv \langle S^z_j S^z_{j+i}\rangle$ K. Sakai, M. Shiroishi, Y. Nishiyama, and M. Takahashi (2003)

RIXS response from ABACUS



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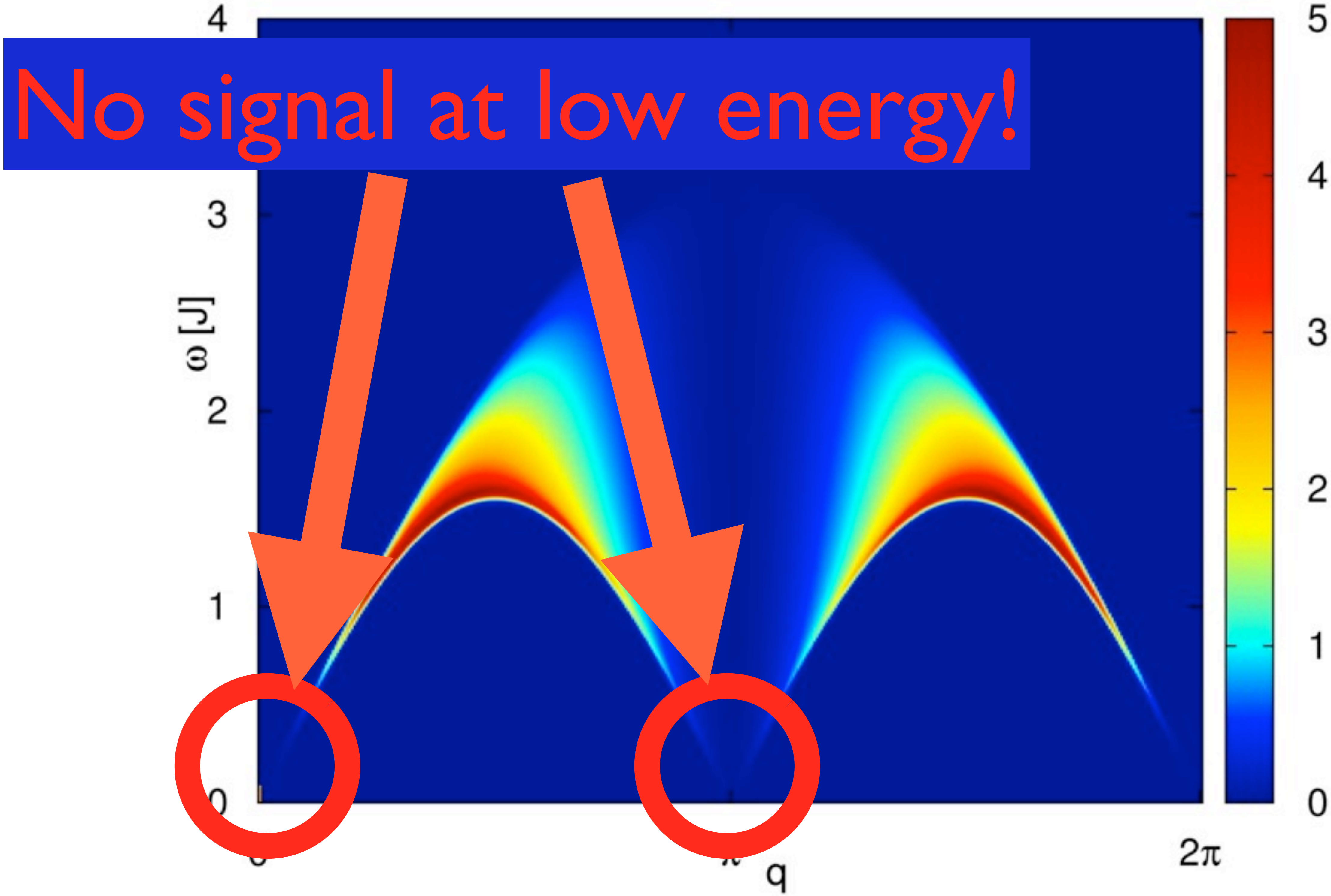

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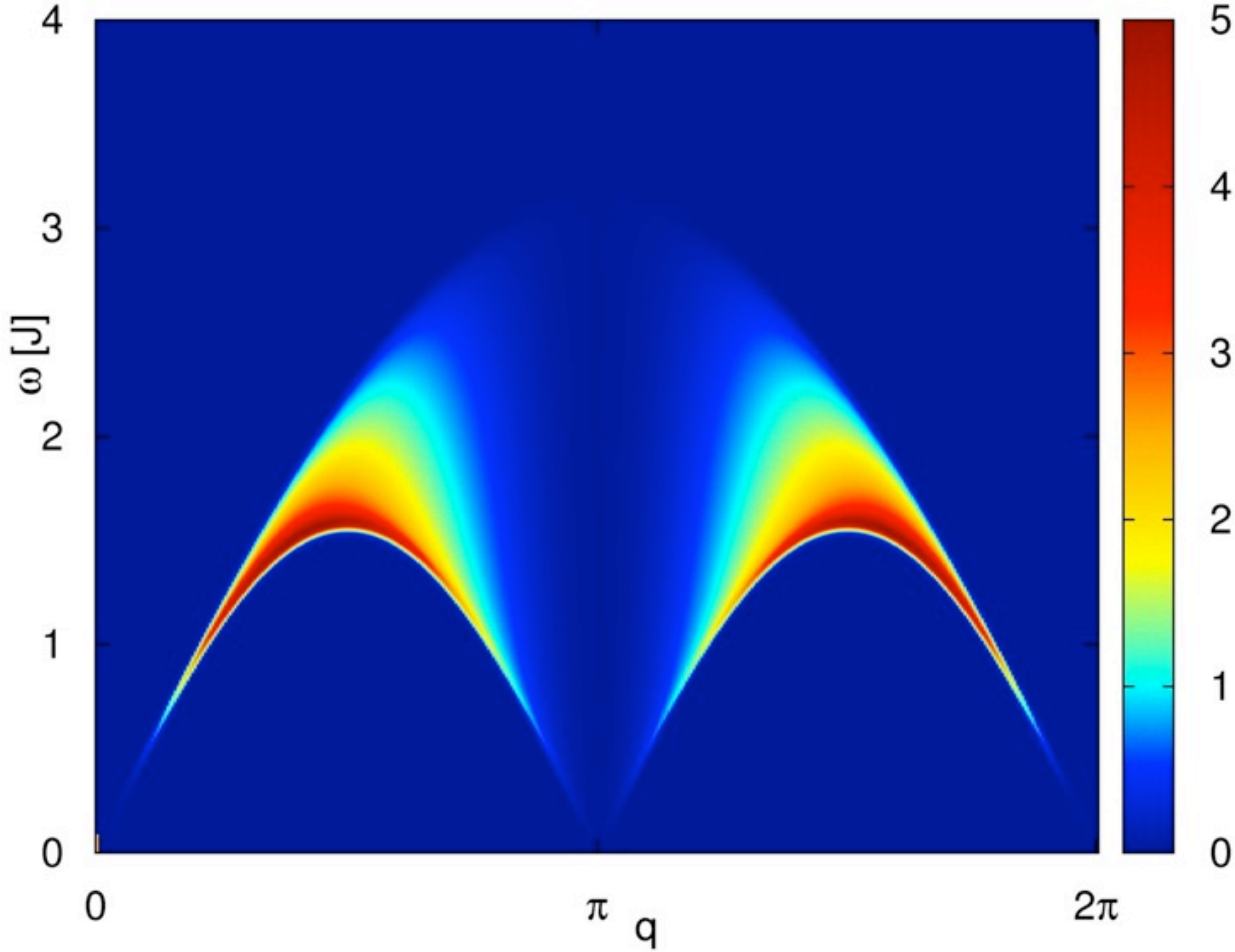

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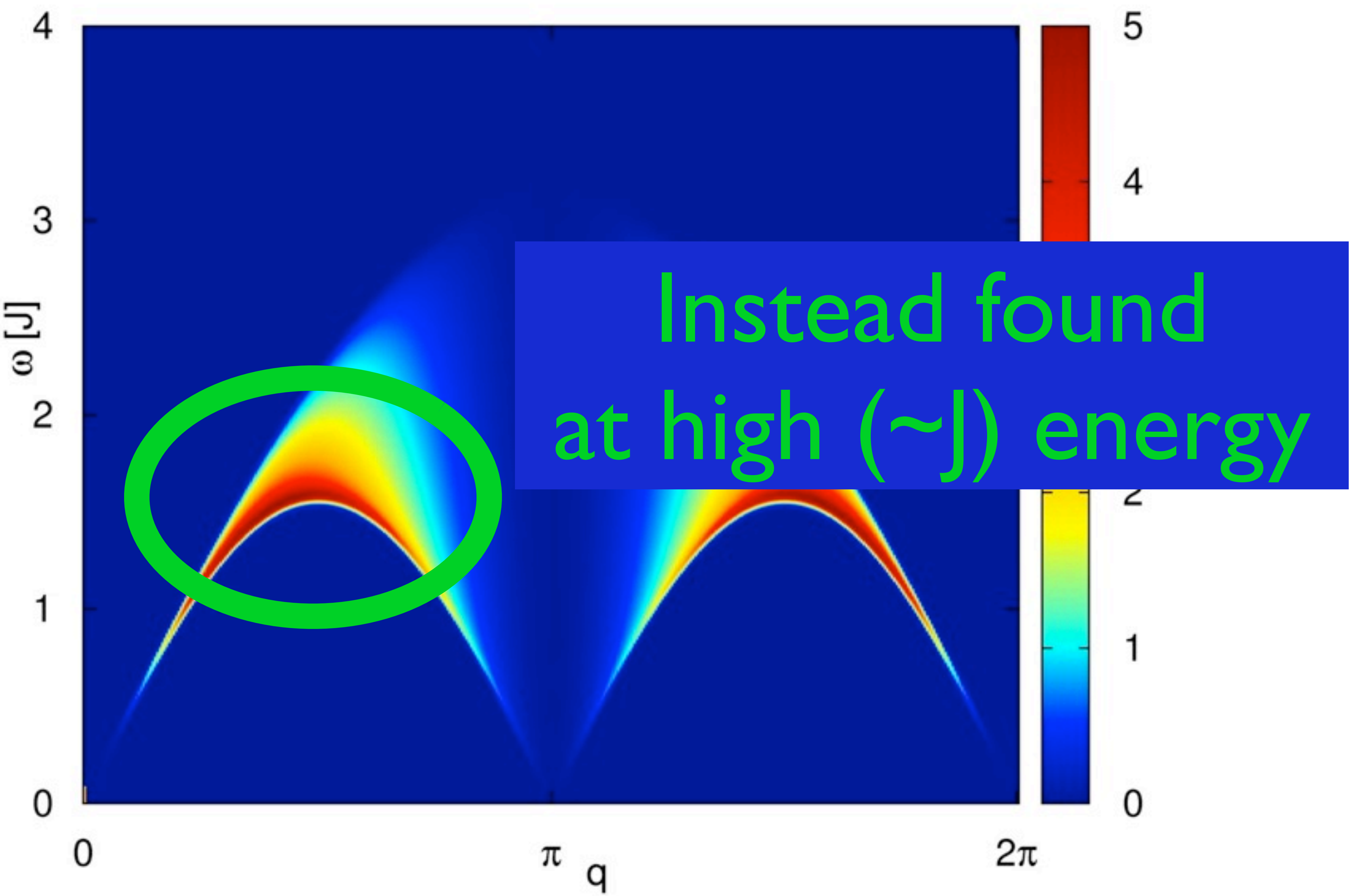

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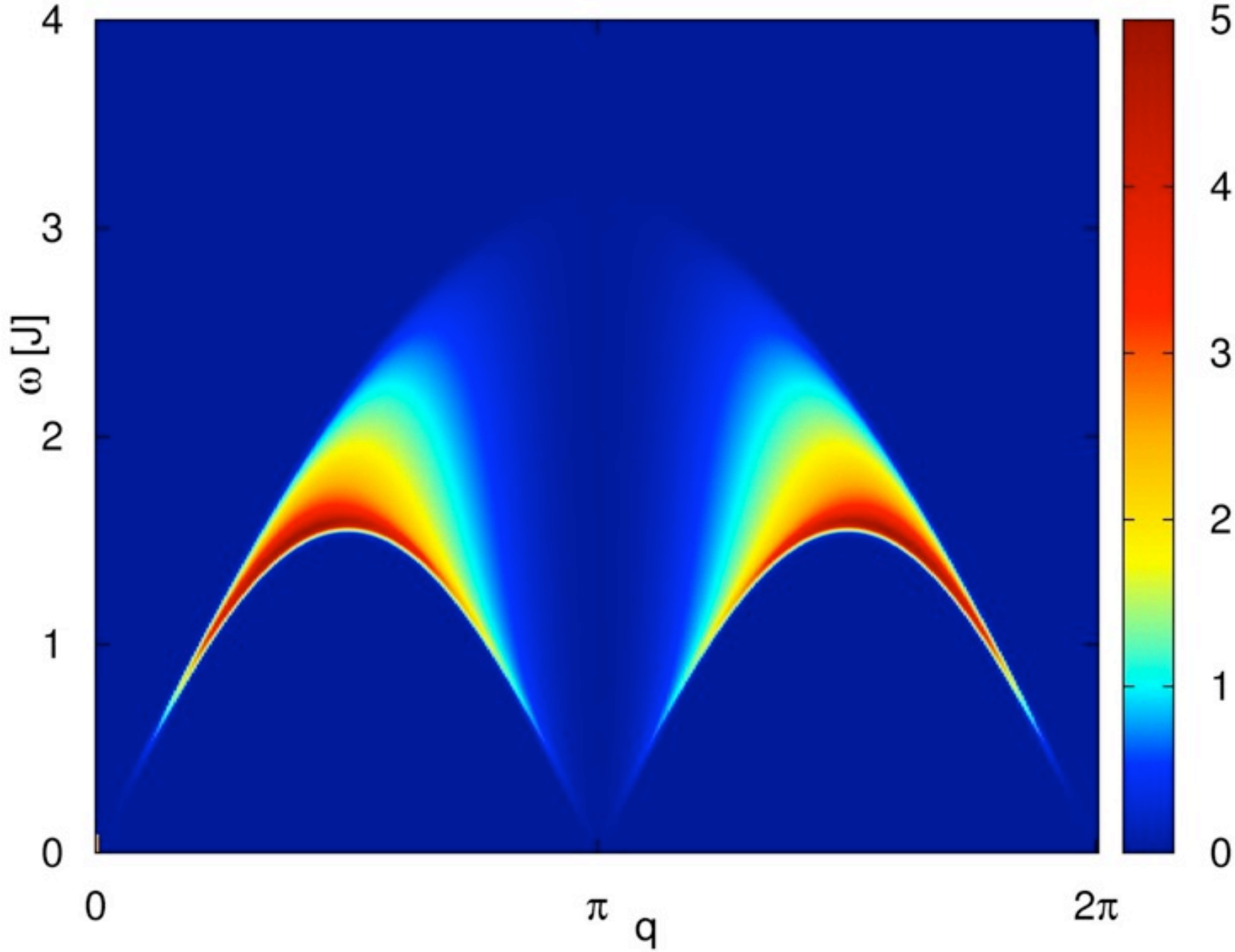

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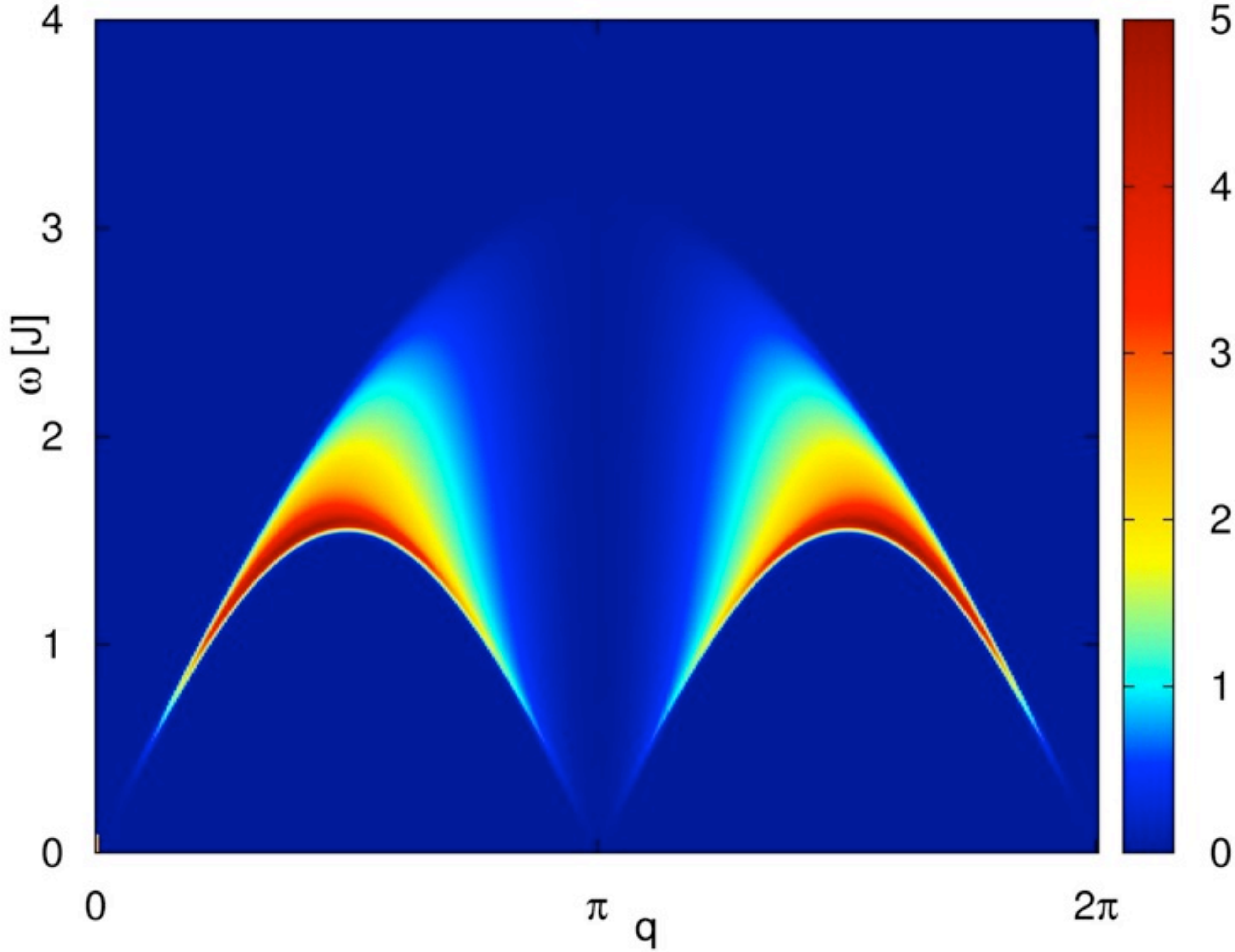
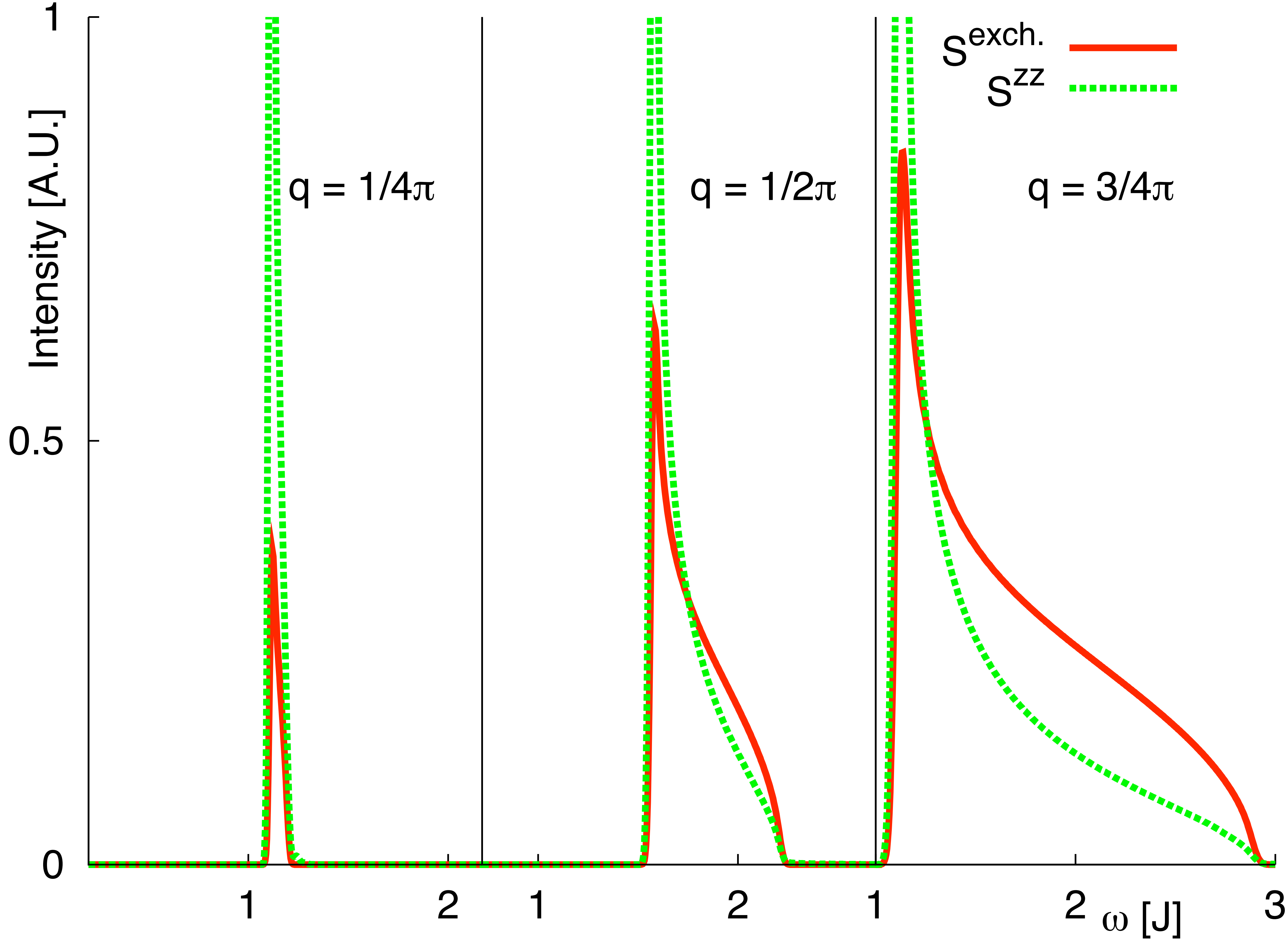


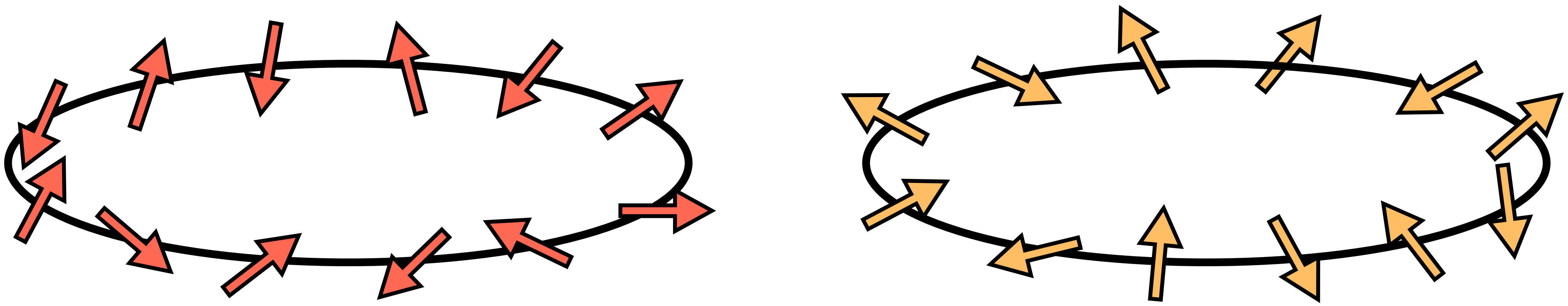
Waiting for experimental correspondence (ongoing)



Intensity [A.U.]
1
0.5
0
q = 1/4π
q = 1/2π
q = 3/4π
S^exch.
S^zz
1
2
1
2
1
2
ω [J]
3

Coupled chains: spin conductivity

(G. Palacios and JSC, unpublished)



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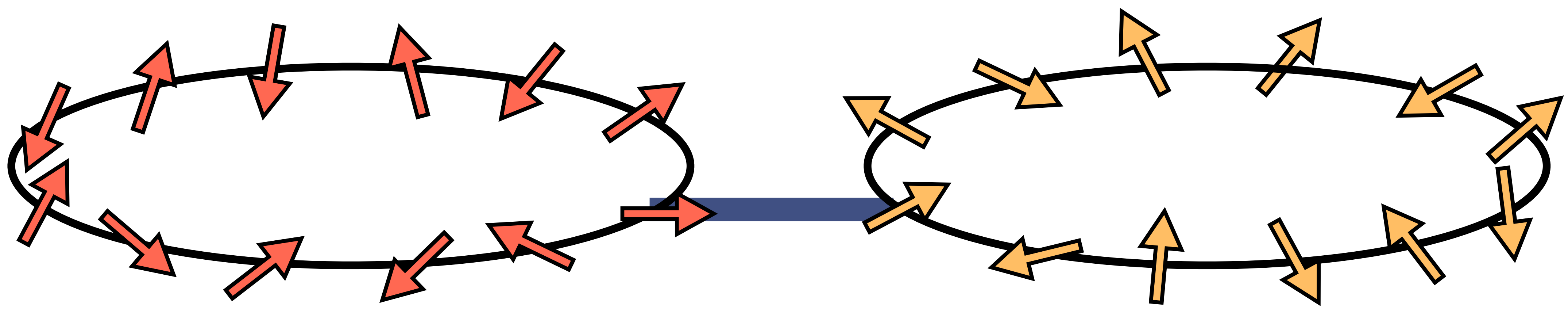

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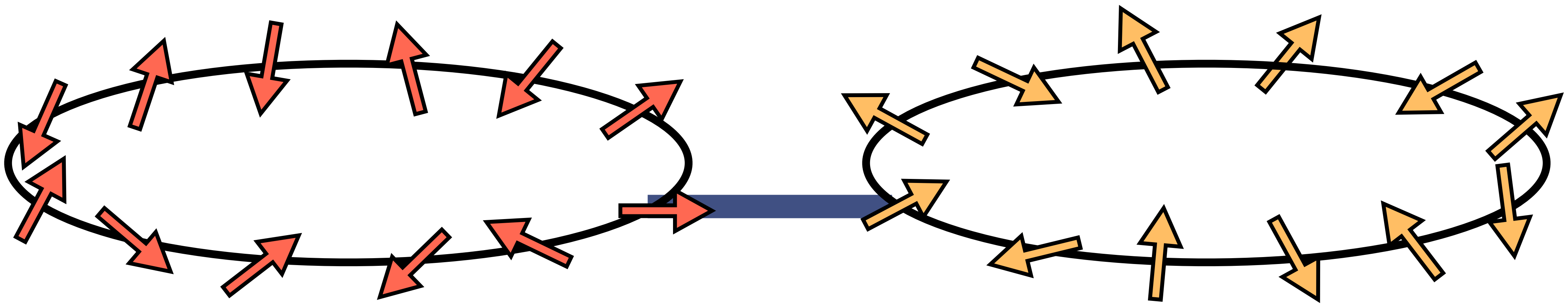

$$\sigma(\omega) = \frac{2\pi}{\omega} \sum_{\alpha_l, \alpha_r} \delta(\omega - \omega_{\alpha_l} - \omega_{\alpha_r})(|\langle 0_l|S_l^-|\alpha_l\rangle|^2 |\langle 0_r|S_r^+|\alpha_r\rangle|^2 + (r \leftrightarrow l))$$

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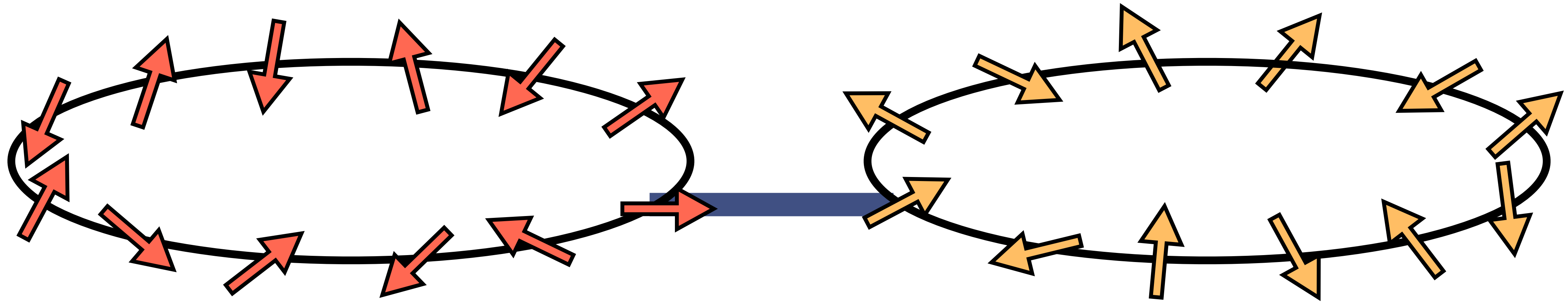

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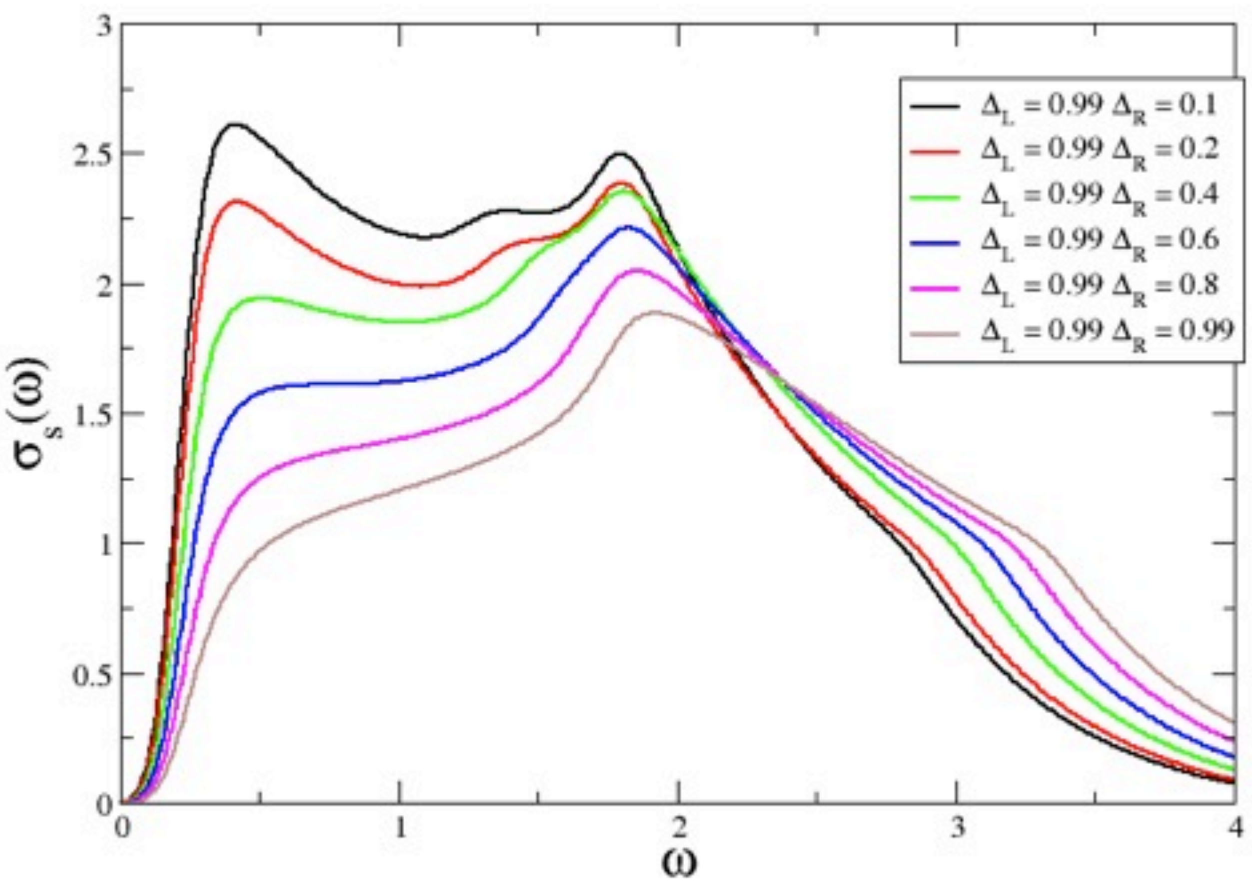

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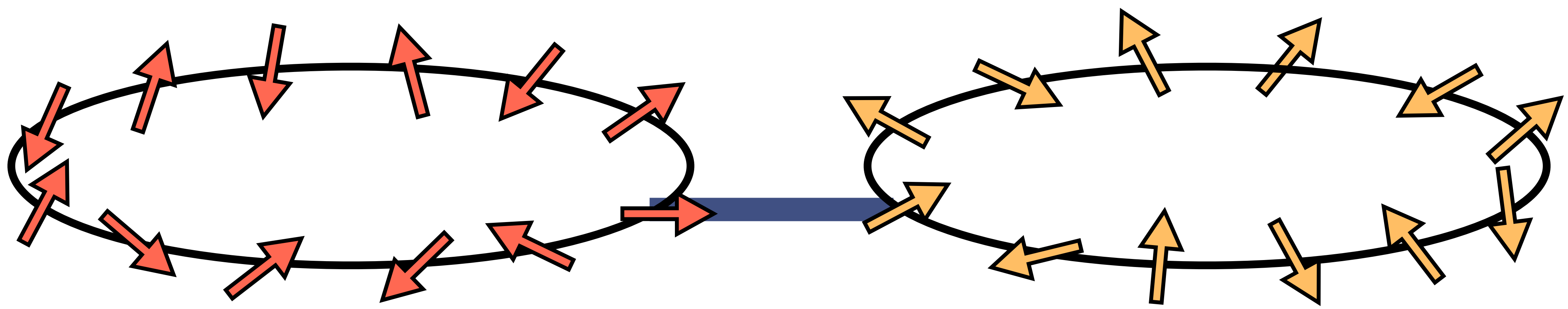

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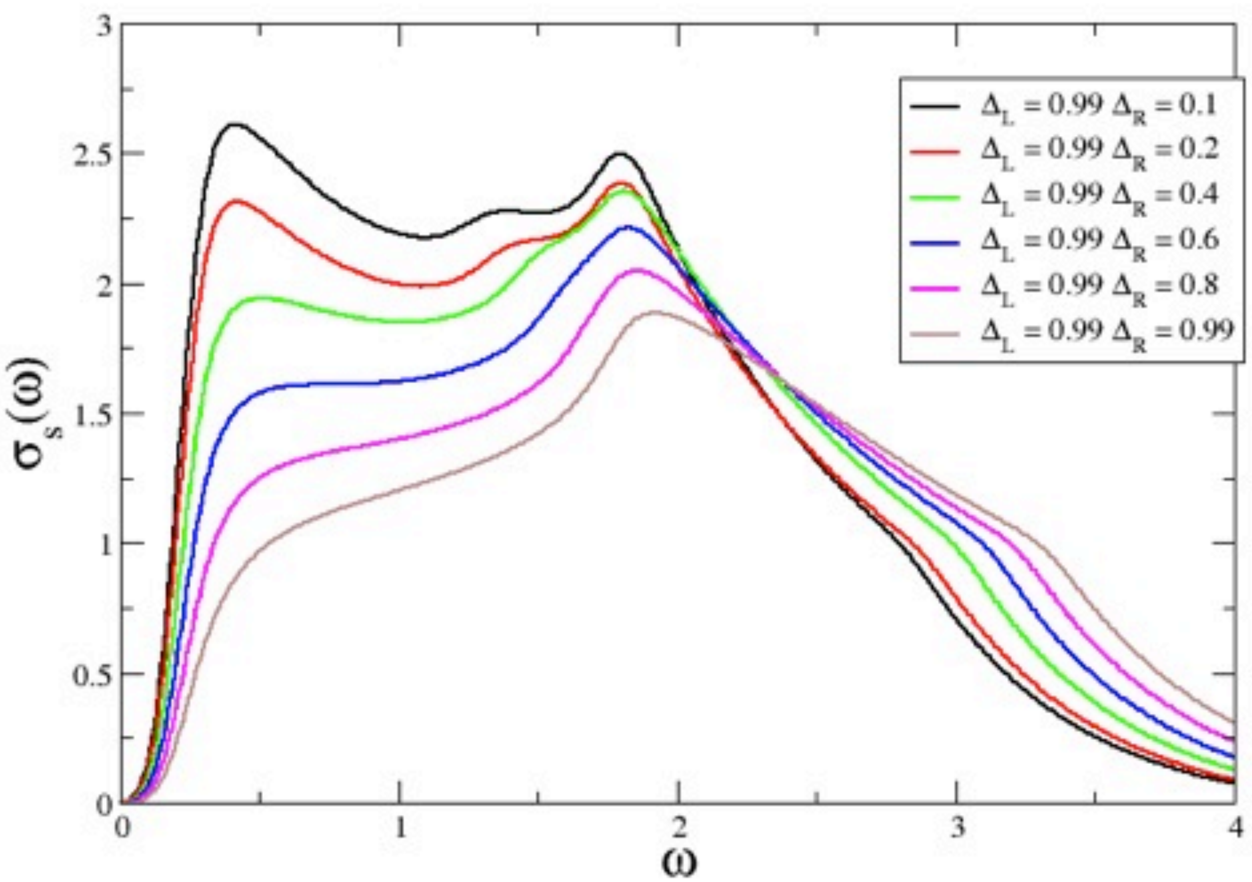

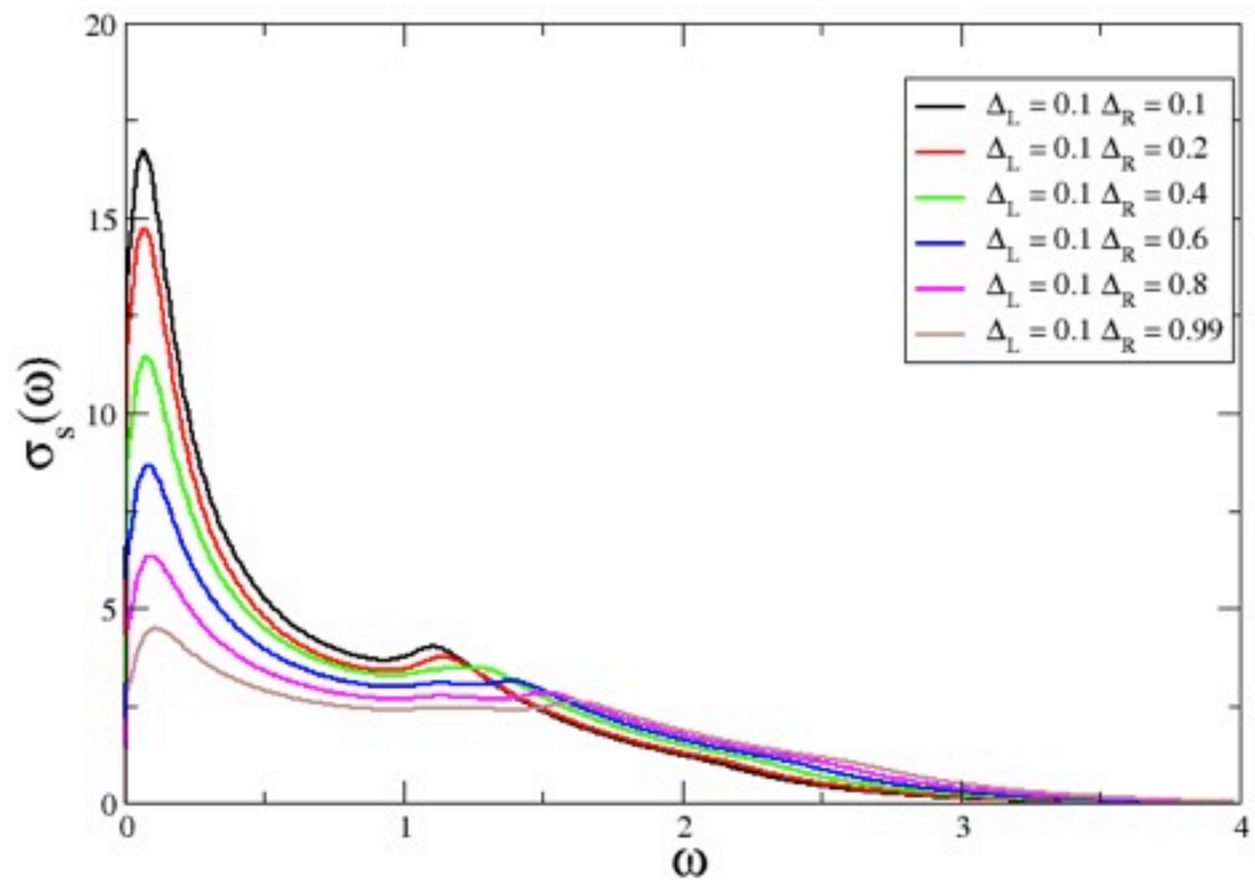

Conclusions

- Correlations from integrability: real usefulness for experiment, and theory
- New developments in field theory: exact prefactors, correlators away from low-energy limit

Not detailed here:

- Applications: neutron scattering, atomic gases, ...
- Out-of-equilibrium from integrability
- Renormalization from integrable points

*** ***PhD and postdoc positions available*** ***