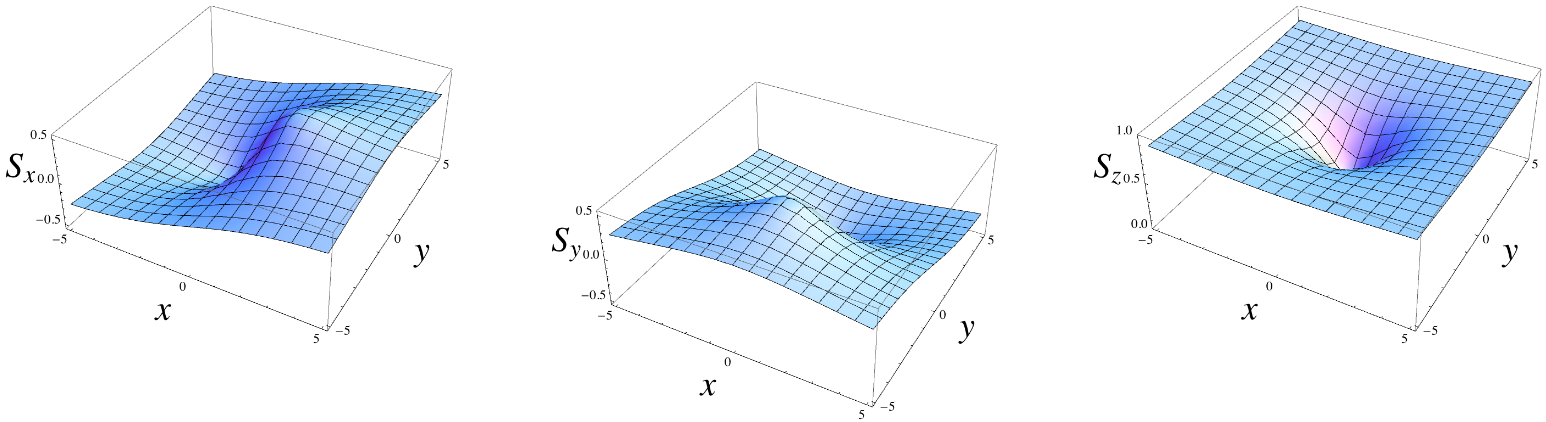
Non-Abelian Spin Textures



Kareljan Schoutens

Institute for Theoretical Physics

University of Amsterdam

ACFTA

IHP, Paris

November 4, 2011

Mathematics Genealogy Project

Henri Poincaré

Biography MathSciNet

Ph.D. Université de Paris 1879

Dissertation: *Sur les propriétés des fonctions définies par les équations différences*

Advisor: Charles Hermite

Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Louis Bachelier		1900	
Théophile De Donder	Université Libre de Bruxelles	1901	524
Mihailo Petrović	University of Belgrade		
Dimitrie Pompeiu	Université de Paris	1905	77
Kyrille Popoff	Université de Paris	1912	

According to our current on-line database, Henri Poincaré has 5 students and 606 descendants.
We welcome any additional information.

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Bernard de Wit

MathSciNet

Ph.D. Universiteit Utrecht 1973

Dissertation: *Field-Theoretical Approach to Symmetry Aspects of the Weak and Electromagnetic Interactions*

Mathematics Subject Classification: 81—Quantum Theory

Advisor: Martinus Justinus Godefridus Veltman

Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Eric Bergshoeff	Universiteit Leiden	1983	9
Jacob Goeree	Universiteit Utrecht	1993	
Kasper Peeters	Universiteit Utrecht	1998	
Kareljan Schoutens	Universiteit Utrecht	1989	
Jan-Willem van Holten	Universiteit Leiden	1980	
Erik Verlinde	Universiteit Utrecht	1988	4

Mathematics Genealogy Project

Martinus Justinus Godefridus Veltman

MathSciNet

Ph.D. Universiteit Utrecht 1963

Dissertation: *Intermediate Particles in S-matrix Theory and Calculation of Higher Order Effects in the Production of Intermediate Vector Bosons*

Advisor: Léon Charles Prudent Van Hove

Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Bernard de Wit	Universiteit Utrecht	1973	19
Michel Lemoine	Universiteit Utrecht	1979	
Christianus Otten	Universiteit Utrecht	1978	
Johannes Reiff	Universiteit Utrecht	1971	
Gerardus 't Hooft	Universiteit Utrecht	1972	24
Jochum van der Bij	Universiteit Utrecht	1983	
Peter van Nieuwenhuizen	Universiteit Utrecht	1971	3

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Please email us with feedback.

Léon Charles Prudent Van Hove

Docteur en Sciences Université Libre de Bruxelles 1946

Dissertation: *Sur les conditions du second ordre du calcul des variations*

Advisor: Théophile De Donder

Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Nicolaas Hugenholtz	Universiteit Utrecht	1957	21
Theodorus Ruijgrok	Universiteit Utrecht	1958	16
Willem van Haeringen	Universiteit Utrecht	1960	12
Martinus Veltman	Universiteit Utrecht	1963	52
Edward Verboven	Universiteit Utrecht	1961	1

According to our current on-line database, Léon Van Hove has 5 students and 107 descendants.
We welcome any additional information.

Mathematics Genealogy Project

Théophile De Donder

MathSciNet

Docteur en Sciences Université Libre de Bruxelles 1901

Dissertation: *Etude sur les invariants intégraux*

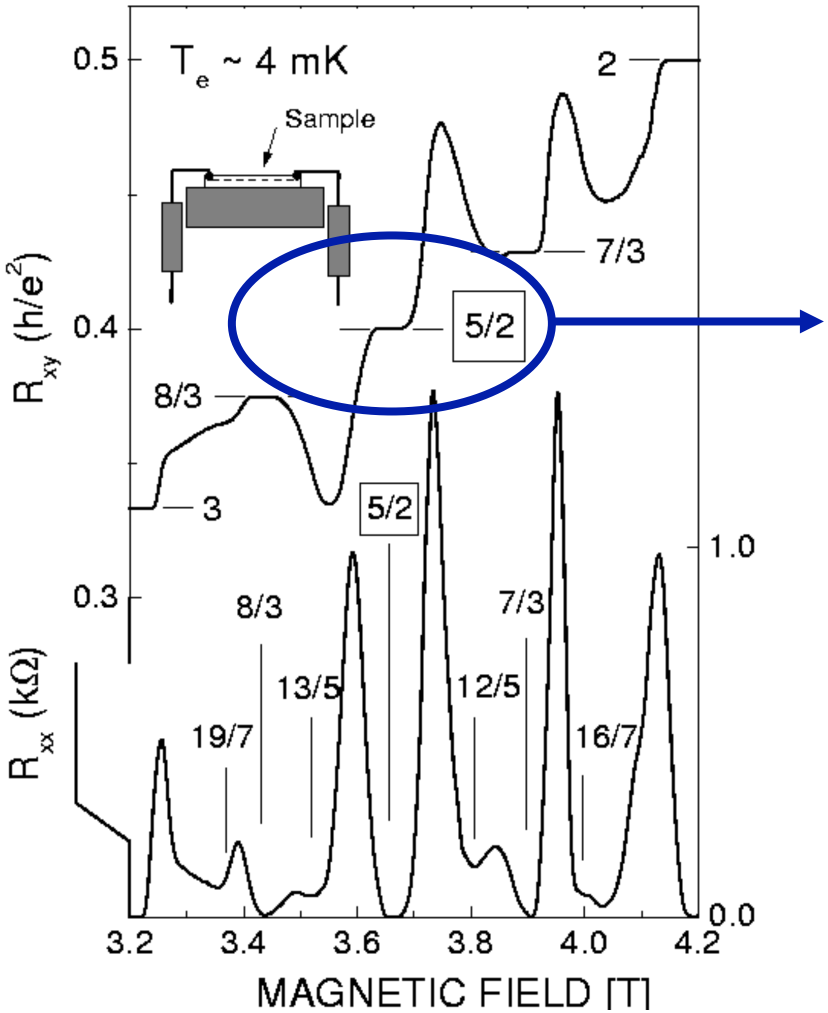
Advisor 1: Henri Poincaré

Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Raymond Coutrez	Université Libre de Bruxelles	1941	1
Jules Géhéniau	Université Libre de Bruxelles	1931, 1938	38
Théophile Lepage	Université Libre de Bruxelles	1929	305
Maurice Nuyens	Université Libre de Bruxelles		15
Ilya Prigogine	Université Libre de Bruxelles	1941	52
Léon Van Hove	Université Libre de Bruxelles	1946	107

According to our current on-line database, Théophile De Donder has 6 students and 524 descendants.
We welcome any additional information.

5/2 state



non-Abelian qH state?

5/2 state

Physics

Physics **3**, 93 (2010)

Viewpoint

Non-Abelian anyons: New particles for less than a billion?

Kirill Shtengel
Department of Physics and Astronomy, University of California, Riverside, CA 92521, USA

Published November 1, 2010

The potential discovery of anyons in a fractional quantum Hall device tests the limits of what is known about particles confined to two dimensions.

Subject Areas: **Semiconductor Physics, Quantum Mechanics, Strongly Correlated Materials**

A Viewpoint on:
Alternation and interchange of e/4 and e/2 period interference oscillations consistent with filling factor 5/2 non-Abelian quasiparticles
R. L. Willett, L. N. Pfeiffer and K. W. West
Phys. Rev. B **82**, 205301 (2010) – Published November 1, 2010

Spin at 5/2

• is the ν=5/2 qH state spin polarized?

If yes:

• do fundamental excitations (charge e/4) involve spin: CST - charged spin textures ?

If yes:

• do these e/4 charged spin textures bind into charge e/2 (anti-)skyrmions?

Dimov, Halperin, Nayak, PRL 2008

Feiguin, Rezayi, Yang, Nayak, Das Sarma, PRB 2009

Wójs, Möller, Simon, Cooper, PRL 2010

Spin at 5/2 – new results

- charged spin textures (CST) over MR state studied through algebraic approach
- fundamental CST identified as polar core vortex

Jesper Romers, Liza Huijse, KjS, NJP 2011
see also: Dimov, Nayak, unpublished

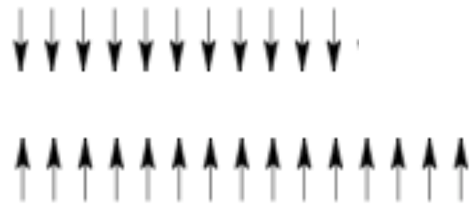
- explicit relation between spin texture of fused CST and underlying fusion path

Jesper Romers, KjS, to appear

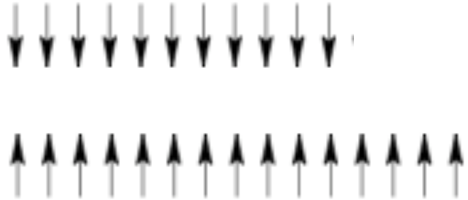
Jesper Romers
PhD thesis 2012

Skyrmions @ ν=1

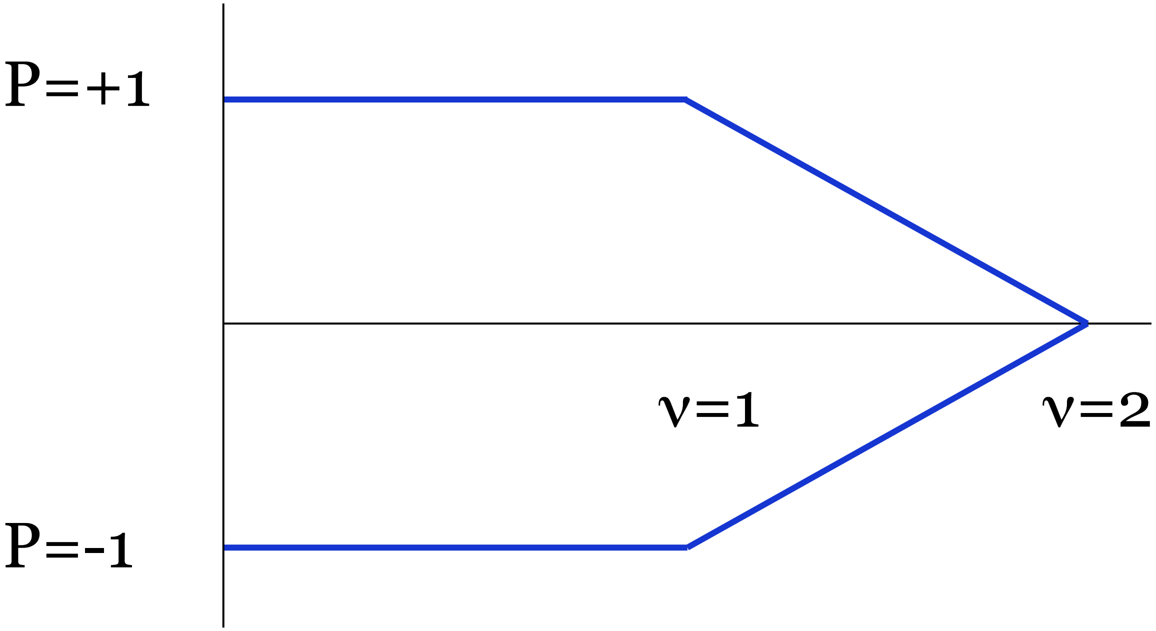
- polarization vs filling in Lowest Landau Level for both spins



Skyrmions @ ν=1



- polarization vs filling in Lowest Landau Level for both spins



Skyrmions @ ν=1

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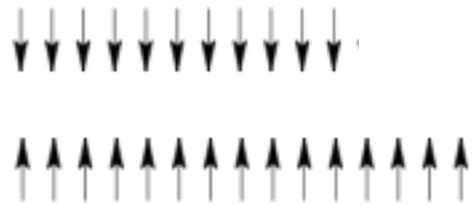

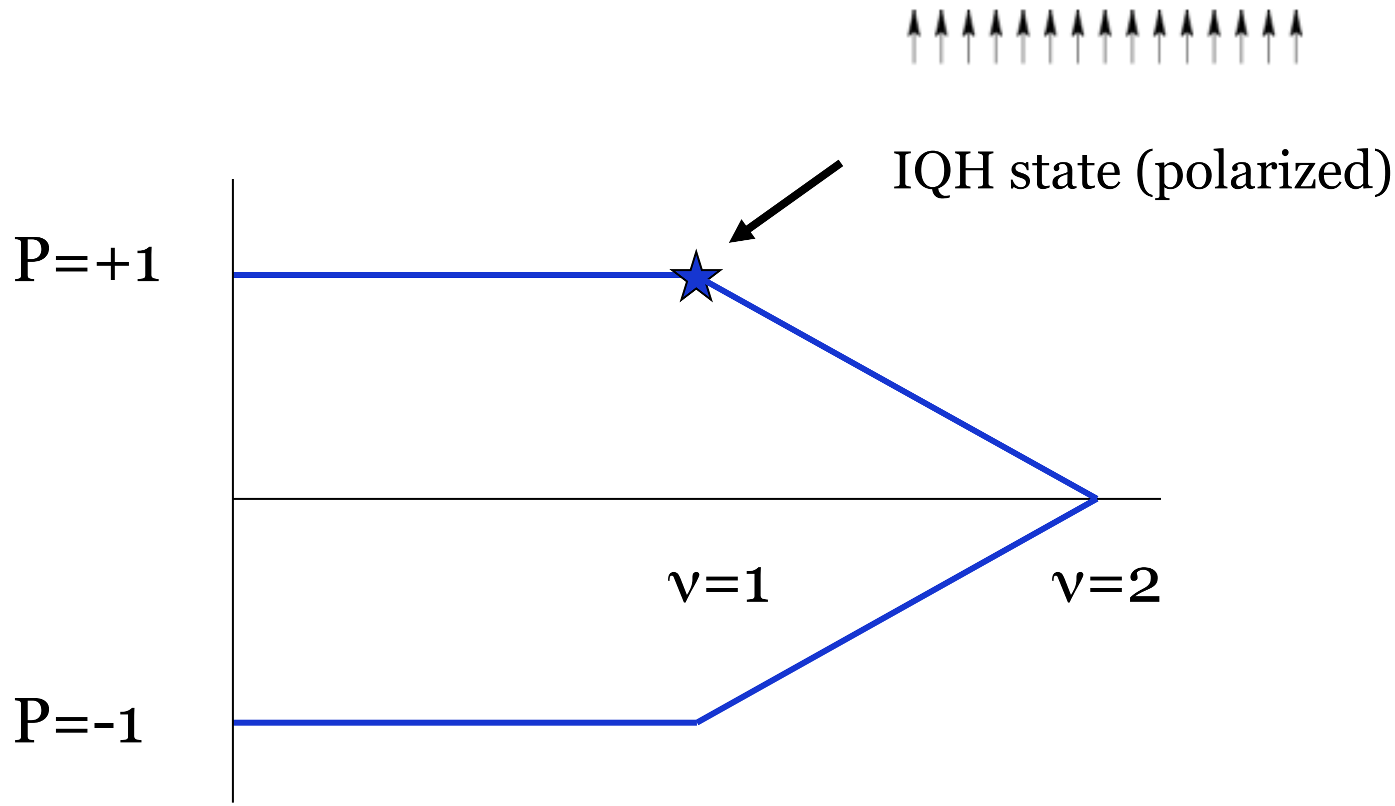

Skyrmions @ ν=1

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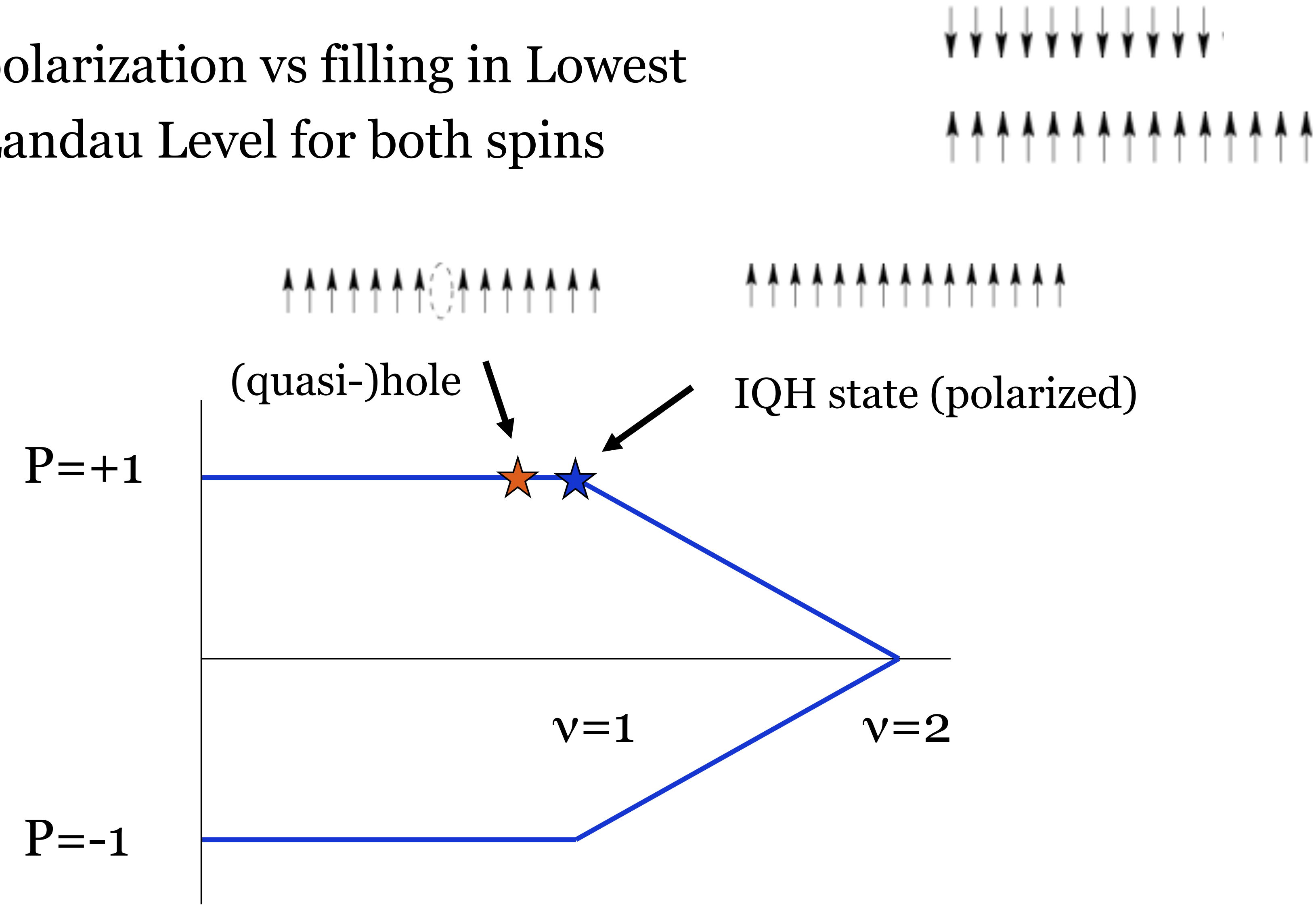
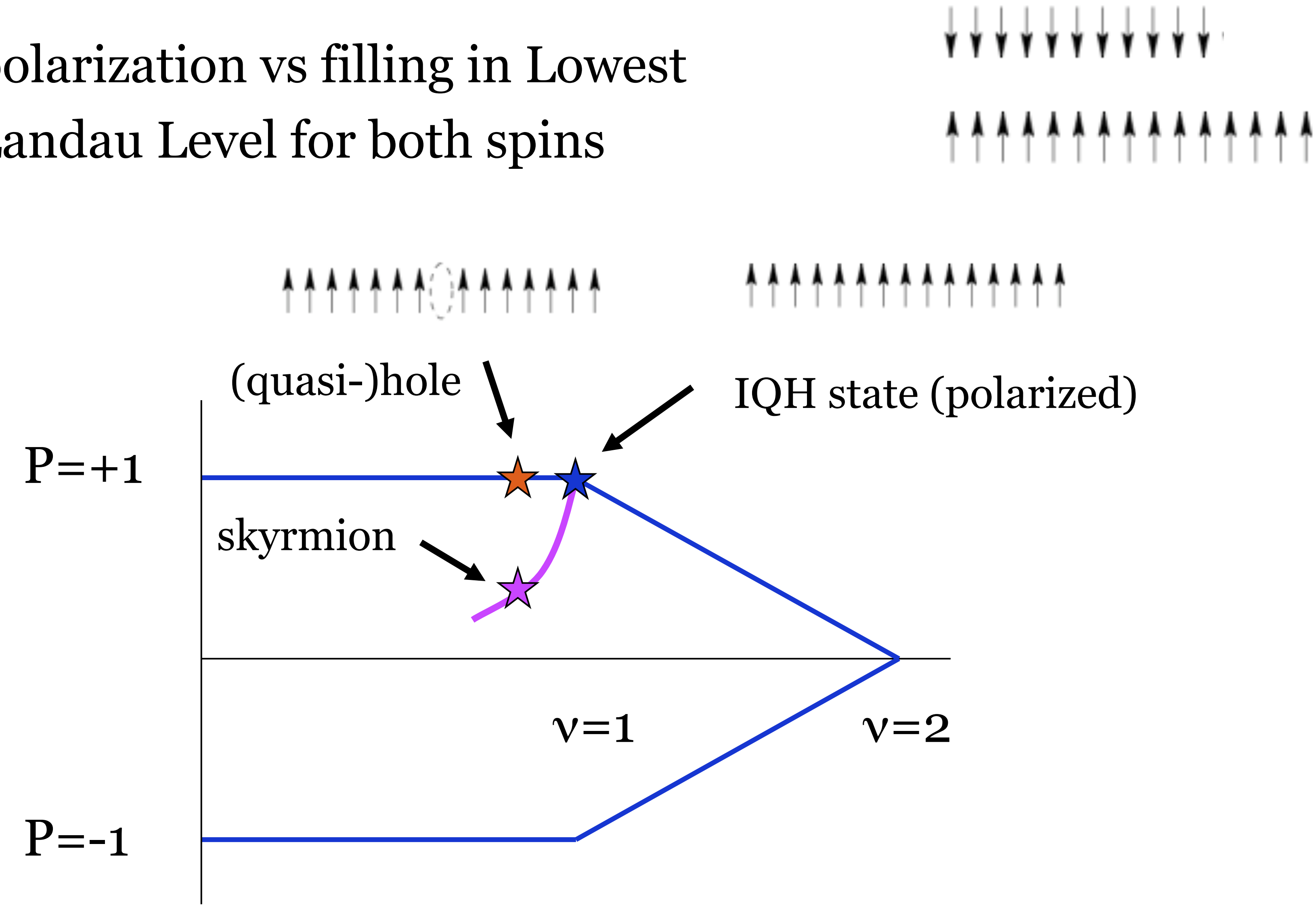


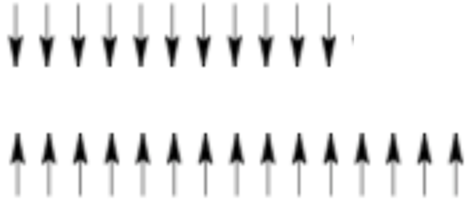
Skyrmions @ ν=1

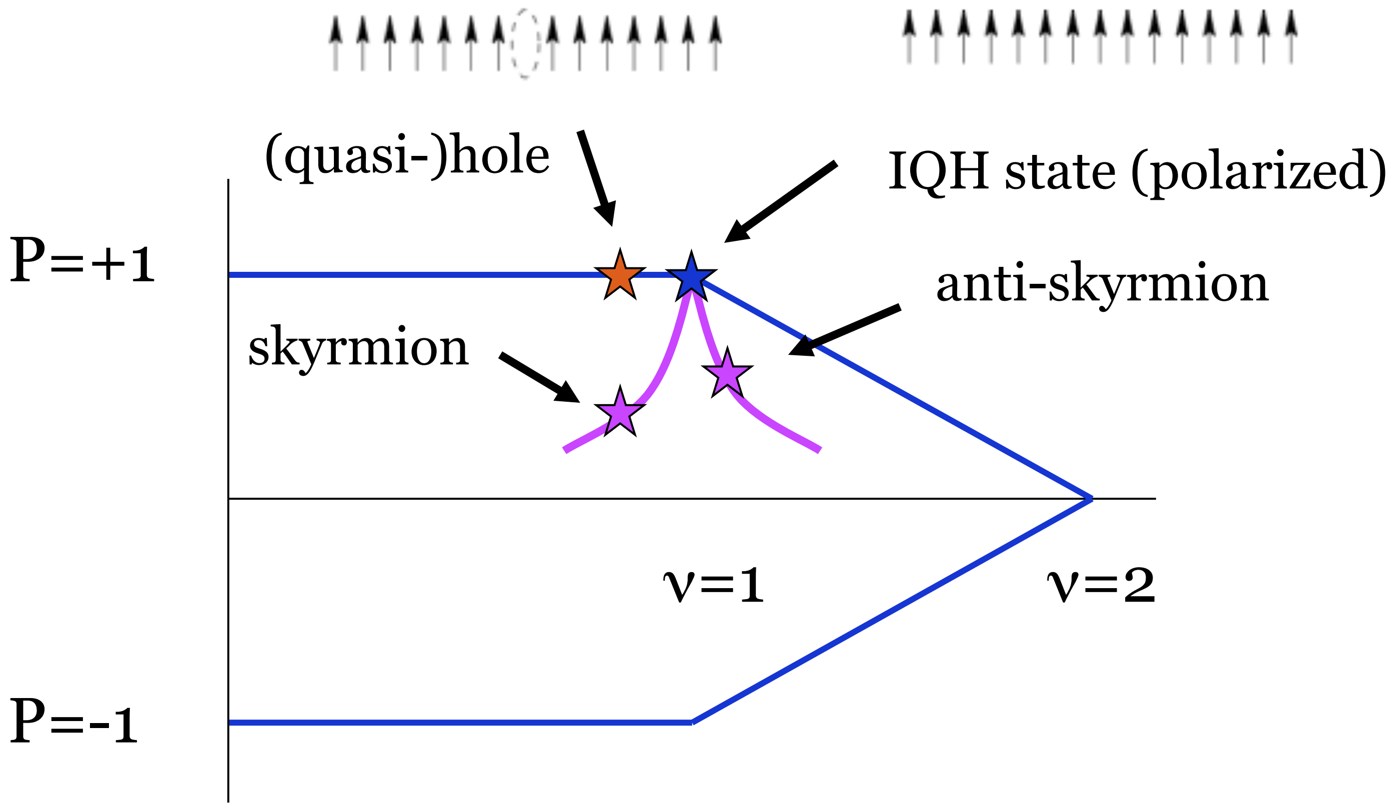
- polarization vs filling in Lowest Landau Level for both spins



Skyrmions @ ν=1

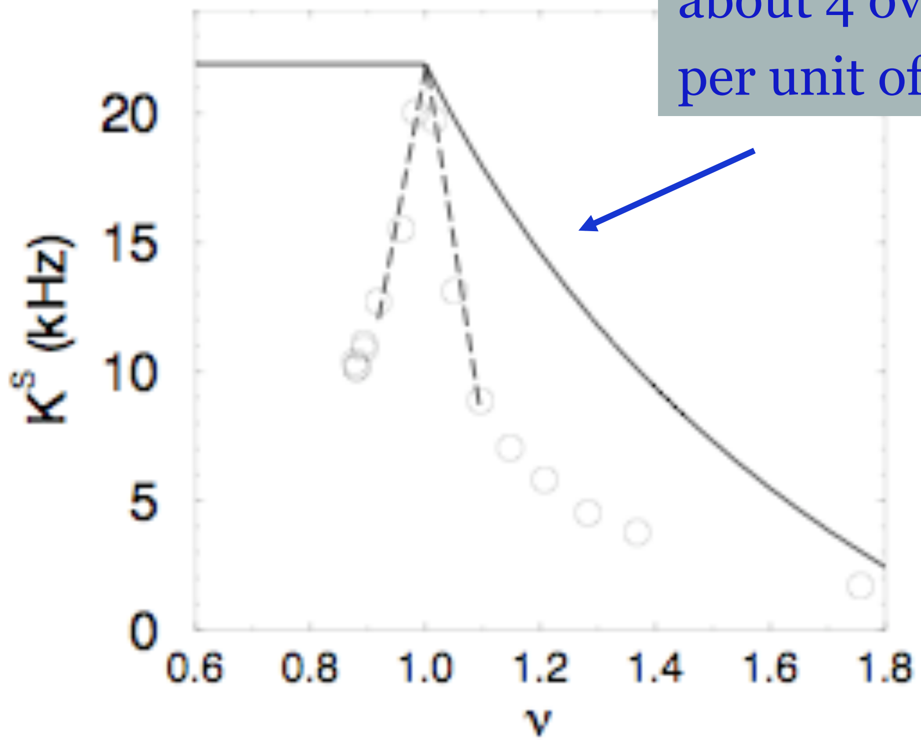
- polarization vs filling in Lowest Landau Level for both spins





Skyrmions @ ν=1

- experiment



slopes of P vs ν indicates about 4 overturned spins per unit of charge

Barrett et al, PRL 1995

Skyrmions over iqH state at ν=1

At $N_\phi = N\text{-}1$: spin polarized iqH state

$$\Psi_{\text{iqH}}^{\nu=1}(z_1,\ldots,z_N) = \prod_{i<j}(z_i - z_j)$$

At $N_\phi = N$: skyrmion wave function

$$\Psi_{\text{Skyrmion}} = \Psi_B \times \Psi_{\text{iqH}}^{\nu=1}$$

View Ψ_B as spin-full wavefunction for N bosonic spin-1/2 particles in two orbitals z^0 and z^1

MacDonald, Fertig, Brey, 1996

Spin-1/2 bosons in two orbitals

In 2nd quantization $(0\uparrow, 0\downarrow, 1\uparrow, 1\downarrow)$

Spin-1/2 bosons in two orbitals

In 2nd quantization $(0\uparrow, 0\downarrow, 1\uparrow, 1\downarrow)$

N=1 particle: 4 states

form *L=1/2, S=1/2*

Spin-1/2 bosons in two orbitals

In 2nd quantization $(0\uparrow, 0\downarrow, 1\uparrow, 1\downarrow)$

N=1 particle: 4 states
form *L=1/2, S=1/2*

N bosons

$$\text{no. states} = \binom{N+3}{3}$$

$$= \sum_{K=0}^{N/2} (L = K, S = K)$$

$$= \text{irrep } (N00) \text{ of SU(4)}$$

Spin-1/2 bosons in two orbitals

In 2nd quantization $(0\uparrow, 0\downarrow, 1\uparrow, 1\downarrow)$

N=1 particle: 4 states

form *L=1/2, S=1/2*

N bosons

$$\text{no. states} = \binom{N+3}{3}$$

$$= \sum_{K=0}^{N/2} (L = K, S = K)$$

$$= \text{irrep } (N00) \text{ of SU(4)}$$

N=4

	S=0	S=1	S=2
L=0	1	0	0
L=1	0	1	0
L=2	0	0	1

no. states = 1 + 9 + 25 = 35

irrep (400) of SU(4)

Spin factor in Skyrmion wavefunction

N bosons

$$\text{no. states} = \binom{\mathrm{N}+3}{3}$$

$$= \sum_{K=0}^{N/2} (L = K, S = K)$$

Spin factor built as weighted sum over diagonal terms

$$\Psi_B = \sum_{K=0}^{N/2} \lambda^K \left| L = \frac{N}{2} - K, S = \frac{N}{2} - K \right\rangle^{\mathrm{HW}}$$

LLL lift and size-λ skyrmions

LLL lift from $N_\phi = 1$ to $N_\phi = N$

$$\Psi_{\text{Skyrmion}} = \Psi_B \prod_{i<j} (z_i - z_j)$$

Size-λ skyrmion

$$\Psi_{\text{Skyrmion}}^{(\lambda)} = \sum_{K=0}^{N/2} \lambda^K \left| L = \frac{N}{2} - K, S = \frac{N}{2} - K \right\rangle^{\text{HW}} \prod_{i<j} (z_i - z_j)$$

$$\cong \prod_{m=0}^{\infty} \left[\lambda \left| \downarrow_m \right\rangle + z_m \left| \uparrow_m \right\rangle \right] \prod_{i<j} (z_i - z_j)$$

iqH skyrmions

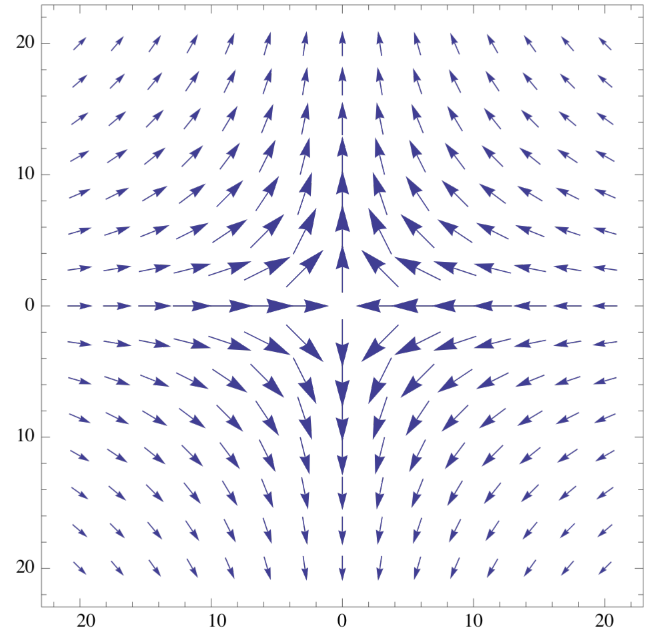
iqH skyrmion

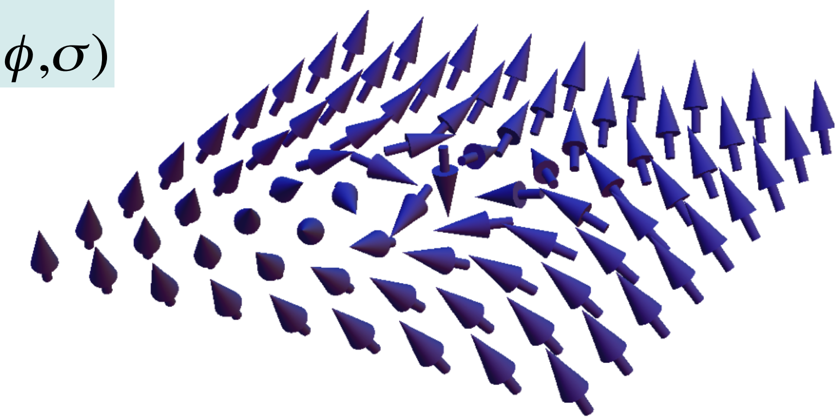
$$\Psi^{(\lambda)}_{\text{Skyrmion}} \cong \prod_{m=0}^{\infty} \left[\lambda \left| \downarrow_m \right\rangle + z_m \left| \uparrow_m \right\rangle \right] \prod_{i<j} (z_i - z_j)$$

Continuum limit agrees with expression from sigma-model

$$\vec{S}(r,\phi) = (\sqrt{1-\sigma^2}\,\cos\phi, -\sqrt{1-\sigma^2}\,\cos\phi, \sigma)$$

$$Q_{\text{topo}} = Q_{\text{electric}} = -1$$



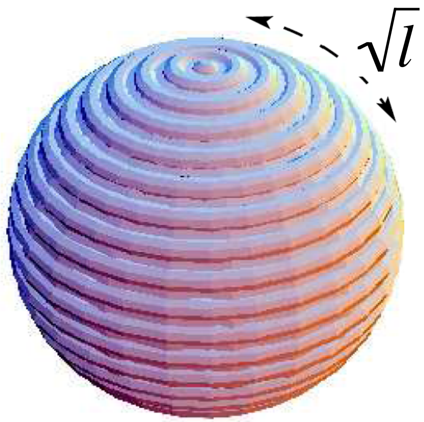


Charged spin textures over MR state

Strategy for numerical analysis (Wójs et al)

- working in spherical geometry, use orbital angular momentum L and spin S to arrange states in (L,S) multiplets with $(2L+1)(2S+1)$ degenerate states; single particle gives $L = N_\phi/2$, $S=1/2$
- hamiltonian: Coulomb w/t corrections for finite thickness w
- zoom in on $N_\phi = N_\phi^{MR} \pm 1$ and for given S find (L,S) multiplet lowest in energy

Spherical geometry

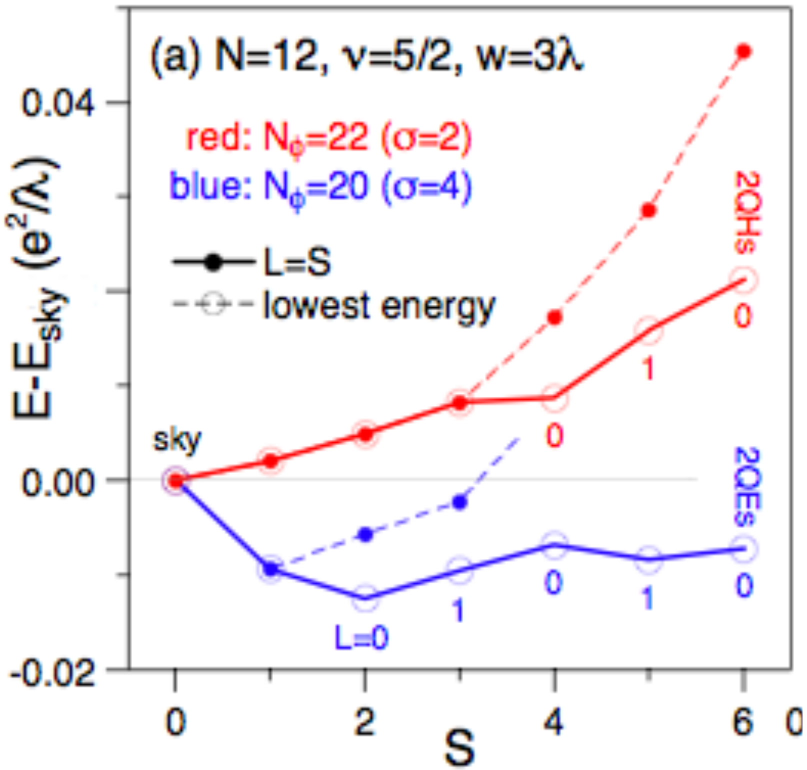


- $N_\uparrow$, $N_\downarrow$ fermions on sphere
- N_ϕ magnetic flux quanta
- LLL orbitals: eigenstates of orbital angular momentum localized on latitude lines,

$$L_z = l - N_\phi/2, \quad l = 0, 1, \ldots, N_\phi$$

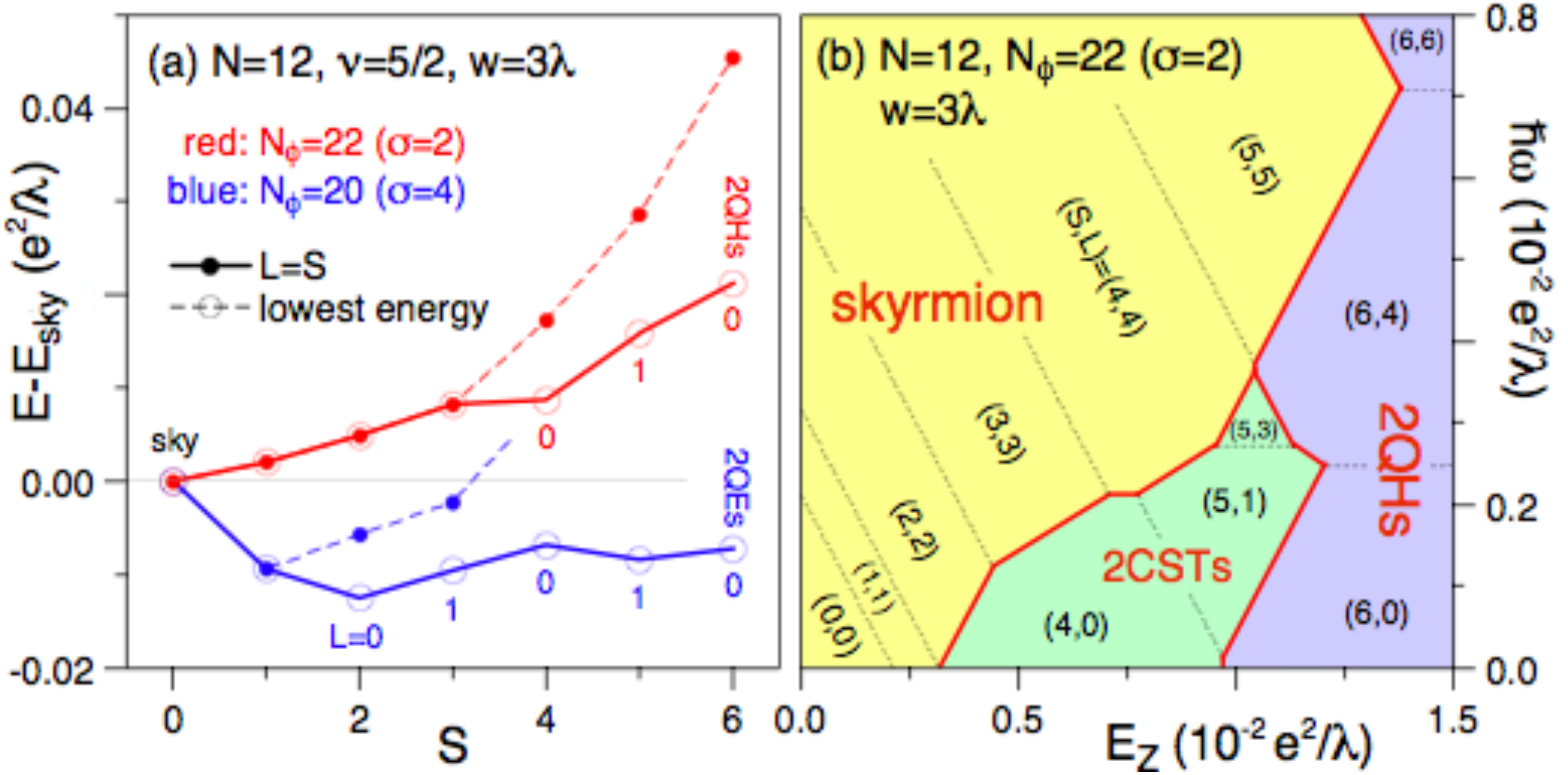
MR: skyrmions and 2CST

Results for N=*12*, where N_ϕ^{MR} =*21*

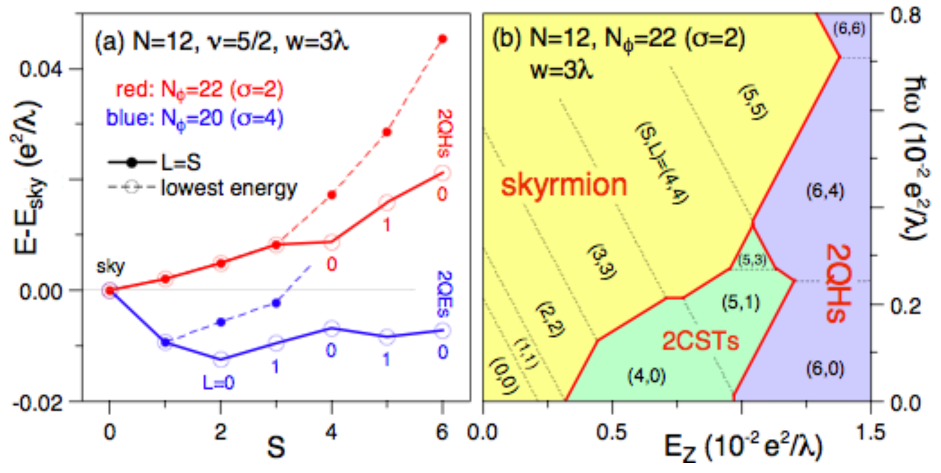


MR: skyrmions and 2CST

Results for $N=12$, where $N_{\phi}^{MR}=21$



MR: skyrmions and 2CST



• *(S=0,L=0)* groundstate at N_ϕ^{MR} *-1* has respectable overlap with standard skyrmion Ansatz

$$\Psi_{\text{skyrmion}} = \Psi_B \times \Psi_{MR}$$

• no such Ansatz available for 2CST states

Skyrmions in spherical geometry

Strategy for analytical study

- restrict analysis to LLL states with MR pairing condition
- simplify further by analyzing bosonic case with MR at filling $\nu = 1$ and NASS at $\nu = 4/3$
- use Ansatz wavefunctions of form

$$\Psi = \underset{\{I,II\}}{\text{Symm}}\left[\Psi_I \Psi_{II}\right]$$

- obtain $\Psi_{I,II}$ via group theoretical analysis at $N_\phi = 1$ and successive lift into the LLL

Moore-Read pairing condition

$$\Psi_{MR}(z_1,..,z_N) = \Psi_{\text{boson}}(z_1,..,z_N)\prod_{i<j}(z_i - z_j)^M \exp\left(-\frac{|z|^2}{4l^2}\right)$$

$$\Psi_{\text{boson}}(z_1,..,z_N) = \text{Pf}\,(\frac{1}{z_i - z_j})\prod_{i<j}(z_i - z_j)$$

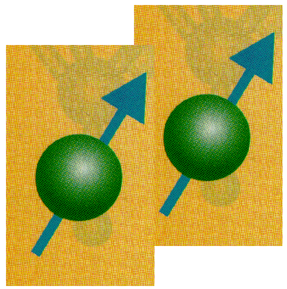
Moore-Read pairing condition

$$\Psi_{MR}(z_1,\ldots,z_N) = \Psi_{\text{boson}}(z_1,\ldots,z_N)\prod_{i<j}(z_i - z_j)^M \exp\left(-\frac{|z|^2}{4l^2}\right)$$

$$\Psi_{\text{boson}}(z_1,\ldots,z_N) = \mathrm{Pf}\,\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)$$

Pairing property

$z_1 = z_2$	$\Psi_{\text{boson}} \neq 0$
$z_1 = z_2 = z_3$	$\Psi_{\text{boson}} = 0$



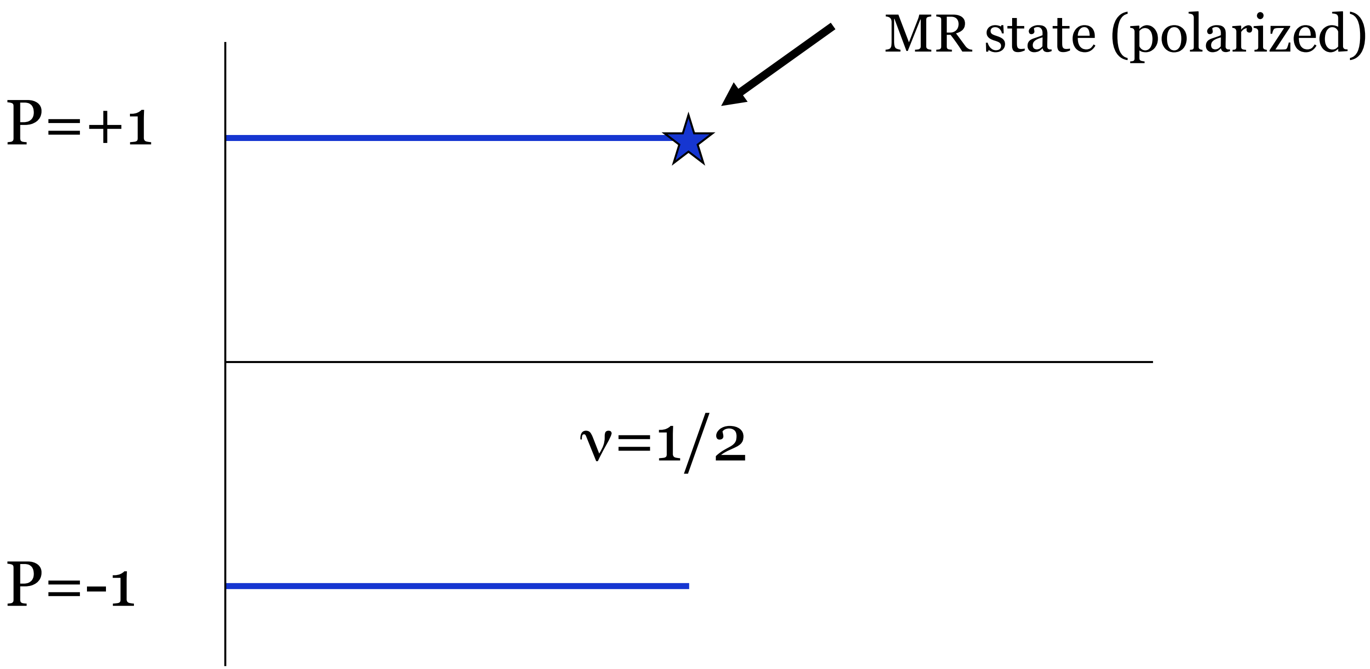
Spin @ ν=5/2

look for spin excitations within subspace of LLL states subject to MR pairing condition

$$H_{\text{pair}} \Psi_{\text{boson}} = 0$$

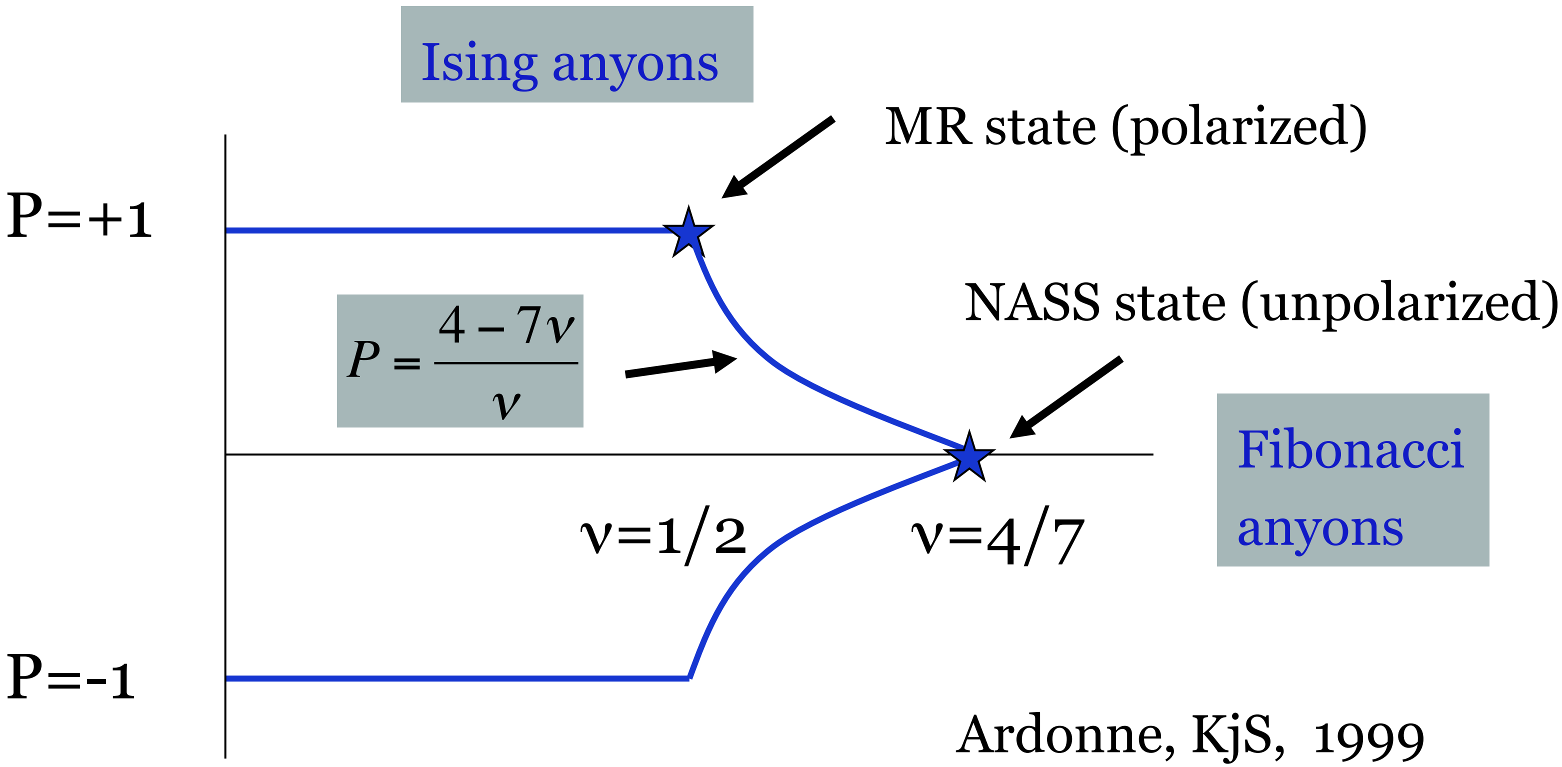
Spin @ ν=5/2

• polarization vs. filling in LLL for fermions with both spins, subject to MR pairing condition



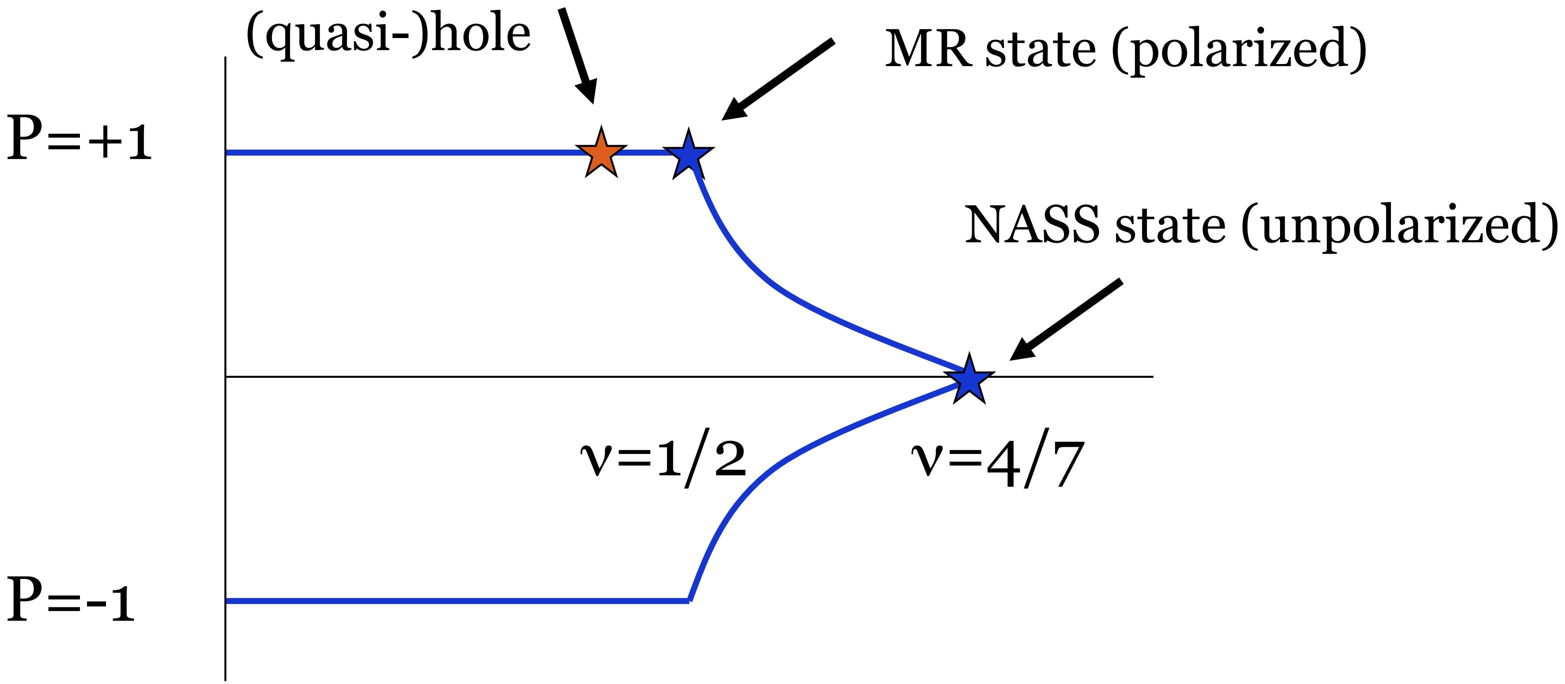
Spin @ ν=5/2

- polarization vs. filling in LLL for both spins, subject to MR pairing condition



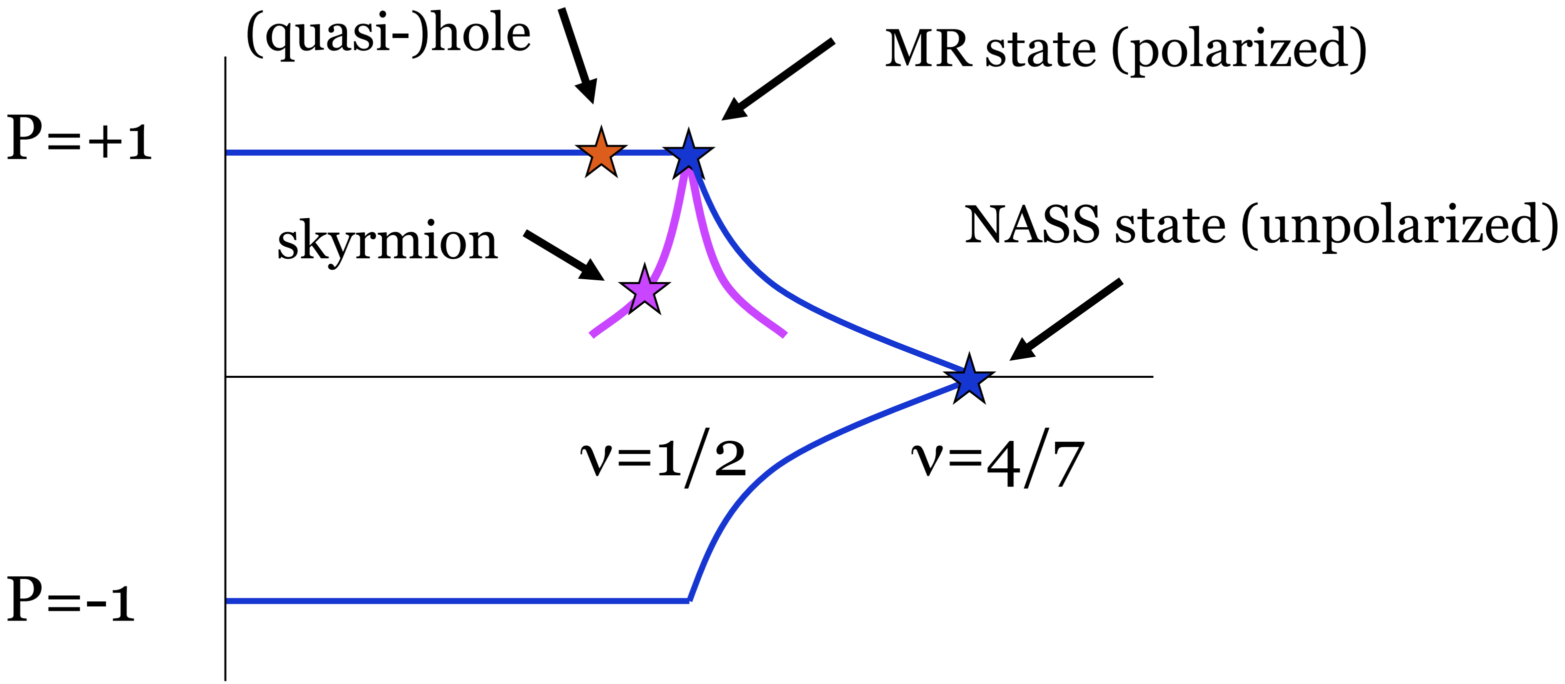
Spin @ ν=5/2

- polarization vs. filling in LLL for both spins, subject to MR pairing condition



Spin @ ν=5/2

- polarization vs. filling in LLL for both spins, subject to MR pairing condition



Spherical geometry: state counting

- NASS: unique paired state w/t *(L=0, S=0)* at

$$N_\phi^{\mathrm{NASS}} = \frac{7}{4} N - 3$$

- excess flux gives rise to qh over NASS state, their numbers are set by

$$n_\uparrow + n_\downarrow = 4(N_\phi - N_\phi^{\mathrm{NASS}})$$
$$n_\uparrow + N_\uparrow = n_\downarrow + N_\downarrow$$

- partition sum at excess flux

$$Z_{\mathrm{qh}}(q) = \sum_{\substack{F_1, F_2 \\ N_\uparrow + N_\downarrow = N \\ n_\uparrow + n_\downarrow = n}} q^{(F_1^2 + F_1^2 - F_1 F_2)/2} \begin{pmatrix} \frac{n_\uparrow + F_2}{2} \\ F_1 \end{pmatrix}_q \begin{pmatrix} \frac{n_\downarrow + F_1}{2} \\ F_2 \end{pmatrix}_q \begin{pmatrix} \frac{N_\uparrow - F_1}{2} + n_\uparrow \\ n_\uparrow \end{pmatrix}_q \begin{pmatrix} \frac{N_\downarrow - F_2}{2} + n_\downarrow \\ n_\downarrow \end{pmatrix}_q$$

Ardonne, Read, Rezayi, KjS, NPB 1999

State counting @ N=4

	S=0
L=0	1

NASS (arrow to L=0, S=0)

$N_\phi=4$

	S=0	S=1	S=2
L=0	1	0	1
L=1	0	1	0
L=2	1	0	0

MR (arrow to L=0, S=2)

$N_\phi=5$

	S=0	S=1	S=2
L=0	1	0	1
L=1	0	2	0
L=2	2	1	1
L=3	0	1	0
L=4	1	0	0

MR+ 2 e/4 qh (arrow to L=0, S=2)

MR+ e/2 qh (arrow to L=2, S=2)

$N_\phi=6$

Skyrmions in spherical geometry

Strategy for analytical study

✔ • restrict analysis to LLL states with MR pairing condition

✔ • simplify further by analyzing bosonic case with MR at filling ν= 1 and NASS at ν= 4/3

• use Ansatz wavefunctions of form

$$\Psi = \underset{\{I,II\}}{\mathrm{Symm}}\left[\Psi_I \Psi_{II}\right]$$

• obtain $\Psi_{I,II}$ via group theoretical analysis at $N_\phi = 1$ and successive lift into the LLL

Bosonic Moore-Read state at ν =1

Bosonic MR state written in form

$$\Psi = \underset{\{I,II\}}{\mathrm{Symm}}\left[\Psi_I \Psi_{II}\right]$$

$$\Psi_{MR}^{\nu=1}(z_1,\ldots,z_N) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)$$

$$= \underset{\{I,II\}}{\mathrm{Symm}}\left[\prod_{i<j\in I}(z_i - z_j)^2 \prod_{k<l\in II}(z_k - z_l)^2\right]$$

rationale: combinatorics or bosonization of underlying Ising CFT

2-group expression MR from bosonization

starting from CFT expression $\psi_{\rm e} = \psi\, {\rm e}^{i\varphi_c}$

$$\Psi_{MR}^{\nu=1}(z_1,..,z_N) = \left\langle \psi_{\rm e}(z_1)\psi_{\rm e}(z_2)\ldots\psi_{\rm e}(z_N)\psi_{\rm BG}(z_\infty)\right\rangle_{\rm CFT}$$

bosonizing $\psi \leftrightarrow {\rm e}^{i\varphi} + {\rm e}^{-i\varphi}$

leads to

$$\Psi_{MR}^{\nu=1}(z_1,..,z_N) = \left\langle (e^{i\varphi}+e^{-i\varphi})(z_1)\ldots(e^{i\varphi}+e^{-i\varphi})(z_N)\right\rangle \prod_{i<j}(z_i - z_j)$$

$$\prod_{I}(z_i - z_j)\prod_{II}(z_k - z_l)\prod_{I,II}(z_i - z_k)^{-1}$$

Bosonic Moore-Read state at ν =1

Bosonic MR state written in form $\Psi = \underset{\{I,II\}}{\mathrm{Symm}}\left[\Psi_I \Psi_{II}\right]$

$$\Psi_{MR}^{\nu=1}(z_1,\ldots,z_N) = \mathrm{Pf}\,\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)$$

$$= \underset{\{I,II\}}{\mathrm{Symm}}\left[\prod_{i<j\in I}(z_i - z_j)^2 \prod_{k<l\in II}(z_k - z_l)^2\right]$$

Idea: at $N_\phi = N_\phi^{MR}+1$ introduce separate spin structures in groups *I, II*

N=2+2 : SU(4) group theory

For constructing CST wavefunctions over $N=4$ MR state, split as $N=2+2$, total no. of states becomes $10 \times_s 10 = 55$

$(N=2)_I$ $\times_s$ $(N=2)_{II}$ =

	S=0	S=1
L=0	1	0
L=1	0	1

$\times_s$

	S=0	S=1
L=0	1	0
L=1	0	1

=

	S=0	S=1	S=2
L=0	1,1	0	1
L=1	0	1,1	0
L=2	1	0	1

no. states = 35 + 20

SU(4):

$$(200) \otimes_s (200) = (400) \oplus (020)$$

General N

SU(4): $(\frac{N}{2}00)\otimes_s(\frac{N}{2}00)=(N00)\oplus(N-4\;20)\oplus\ldots(0\frac{N}{2}0)$

Interpretation: denoting orbitals as *(L_z=-1/2, L_z=1/2)*

and spins in *I* as ↑,↓ and in *II* as ⇑,⇓

charge *e* qh

HWS for (*N 0 0*) : (0, $\uparrow_{N/2}\Uparrow_{N/2}$)

HWS for (*0 N/2 0*) : $(\uparrow_{N/2},\Uparrow_{N/2})+(\Uparrow_{N/2},\uparrow_{N/2})$

two *e*/2 qh

MR: LLL lift

LLL lift from $N_\phi = 1$ to $N_\phi = N_\phi^{MR}+1$

$$\Psi_{\mathrm{MR}}^{\mathrm{spin}} = \underset{\{I,II\}}{\mathrm{Symm}}\left[\Psi_B^I \prod_{i<j\in I}(z_i - z_j)^2 \; \Psi_B^{II} \prod_{k<l\in II}(z_k - z_l)^2\right]$$

LLL lift preserves the L and S quantum numbers!

N=4: skyrmion and 2CST

MR+ 2 *e*/2 qh

	S=0	S=1	S=2
L=0	1	0	1
L=1	0	2	0
L=2	2	1	1
L=3	0	1	0
L=4	1	0	0

MR+ *e* qh

N=4: skyrmion and 2CST

$$\Psi = \underset{\{I,II\}}{\mathrm{Symm}}\left[\Psi_I \Psi_{II}\right]$$

MR+ 2 e/2 qh

	S=0	S=1	S=2
L=0	1	0	1
L=1	0	2	0
L=2	2	1	1
L=3	0	1	0
L=4	1	0	0

MR+ e qh

	S=0	S=1	S=2
L=0	1=1	0	1
L=1	0	1,1	0
L=2	1	0	1

Red: LLL lift of su(4) irrep (400)=35, skyrmion

Blue: LLL lift of su(4) irrep (020)=20, two separated CST

Skyrmions in spherical geometry

Strategy for analytical study

✔ • restrict analysis to LLL states with MR pairing condition

✔ • simplify further by analyzing bosonic case with MR at filling ν= 1 and NASS at ν= 4/3

✔ • use Ansatz wavefunctions of form

$$\Psi = \underset{\{I,II\}}{\mathrm{Symm}}\left[\Psi_I \Psi_{II}\right]$$

✔ • obtain $\Psi_{I,II}$ via group theoretical analysis at $N_\phi = 1$ and successive lift into the LLL

Explicit expressions

$(N\ 0\ 0)$: $(0, \uparrow_{N/2} \Uparrow_{N/2})$

charge e CST (skyrmion)

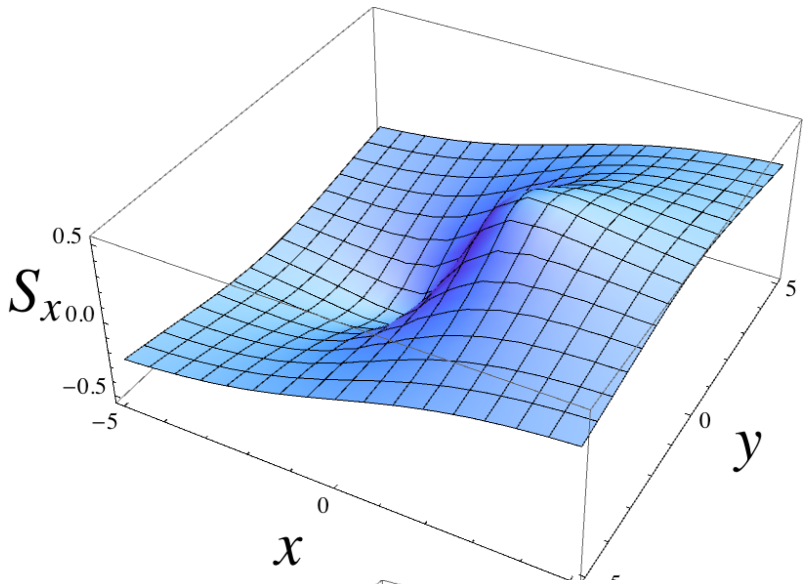
$$\left[\underset{\{I,II\}}{\text{Symm}}\right][\text{LLL}-\text{lift}]\sum_{K_I,K_{II}}\lambda_I^{K_I}\lambda_{II}^{K_{II}}\left|\downarrow_{K_I}\Downarrow_{K_{II}},\uparrow_{N/2-K_I}\Uparrow_{N/2-K_{II}}\right\rangle$$

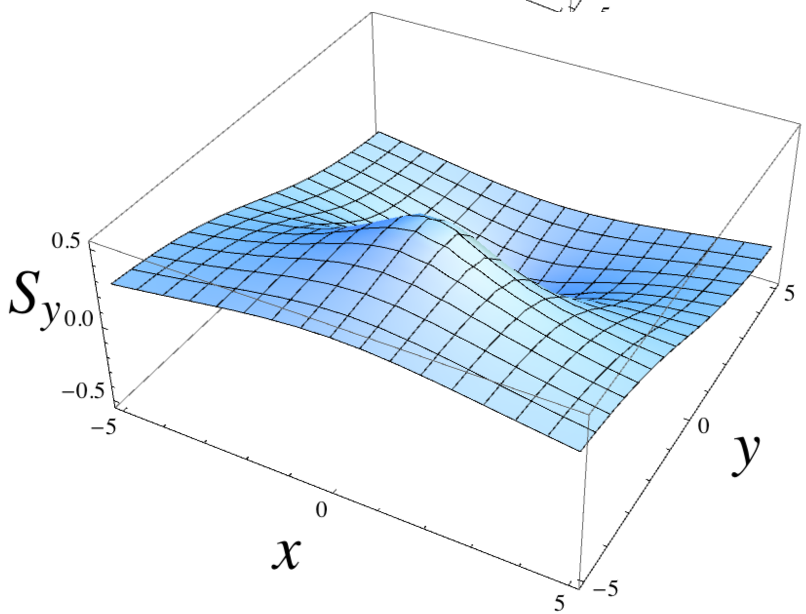
$(0\ N/2\ 0)$: $(\uparrow_{N/2}, \Uparrow_{N/2}) + (\Uparrow_{N/2}, \uparrow_{N/2})$

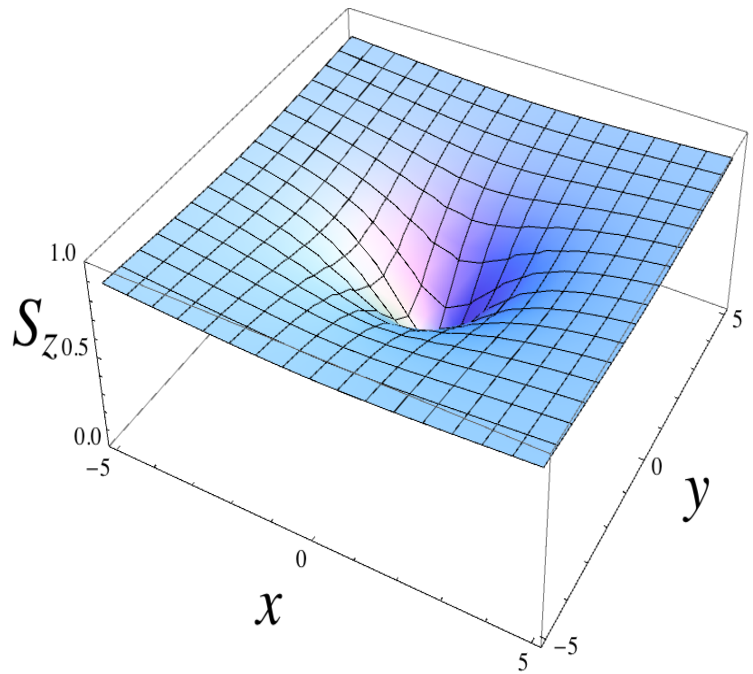
left and right
charge $e/2$ CST

$$\left[\underset{\{I,II\}}{\text{Symm}}\right][\text{LLL}-\text{lift}]\sum_{K_I,K_{II}}\lambda_I^{K_I}\lambda_{II}^{K_{II}}\left|\uparrow_{N/2-K_I}\Downarrow_{K_{II}},\downarrow_{K_I}\Uparrow_{N/2-K_{II}}\right\rangle$$

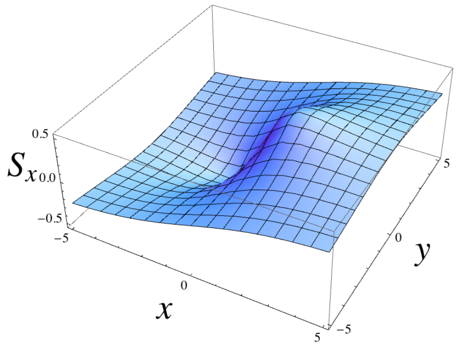
charge e/2 CST: spin vector





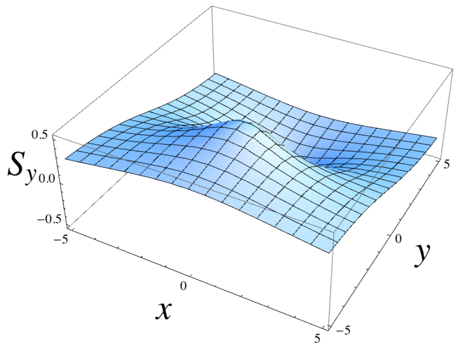


charge e/2 CST



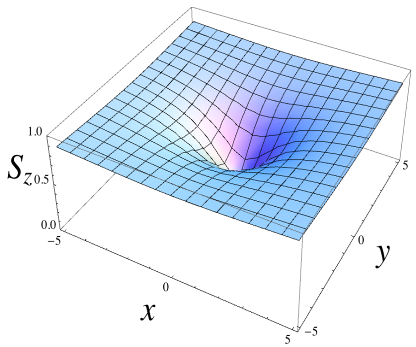
Continuum limit is spin texture with vanishing spin vector in the core

➔ **polar core vortex**



$$\vec{S}(r,\phi) = (\sqrt{2\rho(1-\rho)}\cos\phi, -\sqrt{2\rho(1-\rho)}\sin\phi, \rho)$$

Rationale: MR pairs carry spin-1, spin-1 spinor condensate (such as BEC) has polar and polarized phases. Polar core vortex captured by CP2 spinor



$$\xi(z) = \begin{pmatrix} z \\ 1 \\ 0 \end{pmatrix}$$

fusing elementary CST

setting

- total excess flux ΔN_ϕ leads to $n=2\Delta N_\phi$ elementary CST at positions $w_1 \ldots w_n$
- the fusion channel of the underlying MR quasi-holes can be characterized by the number F of unpaired fermions (Read-Rezayi 1996)
- upon sending all $w_i \rightarrow 0$ the elementary CST merge into a composite charge $ne/2$ CST

- what will be the associated spin texture?

2-group expression from bosonization

starting from Read Rezayi expression

$$\Psi^{(F)}_{m_1 m_2 \ldots m_F}(w_1,\ldots w_n; z_1,\ldots,z_N)$$

sending w_i ➔ 0, taking maximal L_z

$$\Psi^{(F)}(z_1,\ldots,z_N) \approx \left\langle \psi \partial\psi \ldots \partial^{F-1}\psi(0)\, \psi(z_1)\psi(z_2)\ldots\psi(z_N) \right\rangle_{\mathrm{CFT}}$$

fusion product of σ(0)σ(0)…σ(0)

bosonizing

$$\Psi^{(F)} \approx \left\langle (e^{iF\varphi} + e^{-iF\varphi})(0)\, (e^{i\varphi} + e^{-i\varphi})(z_1)\ldots(e^{i\varphi} + e^{-i\varphi})(z_N) \right\rangle$$

need *(N-F)/2* particles in group *I* and *(N+F)/2* in group *II*

General construction: CST[w_I,w_{II}]

$$\Psi^{(\mathrm{F})} \approx \underset{\{I,II\}}{\mathrm{Symm}}\left[\prod_{i\in I} z_i^{\Delta N_\phi + F} \prod_{i<j\in I}(z_i - z_j)^2 \prod_{k\in II} z_k^{\Delta N_\phi - F} \prod_{k<l\in II}(z_k - z_l)^2\right]$$

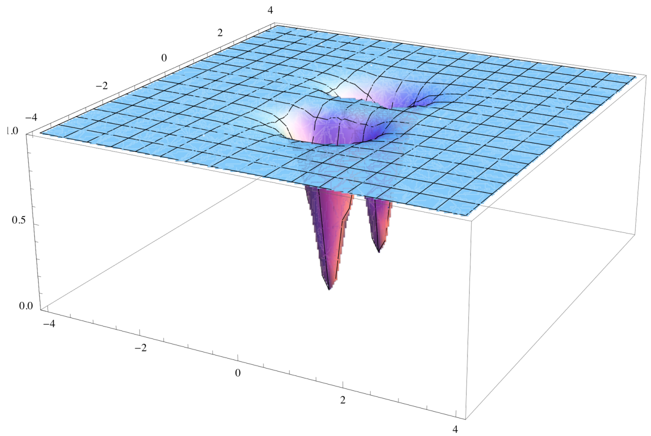
leads to spin texture

$$\Psi_{\mathrm{CST}}^{(\mathrm{N}_\phi, F)} = \underset{\{I,II\}}{\mathrm{Symm}}\left[\Psi_{\mathrm{B}}^{I} \prod_{i<j\in I}(z_i - z_j)^2 \, \Psi_{\mathrm{B}}^{II} \prod_{k<l\in II}(z_k - z_l)^2\right]$$

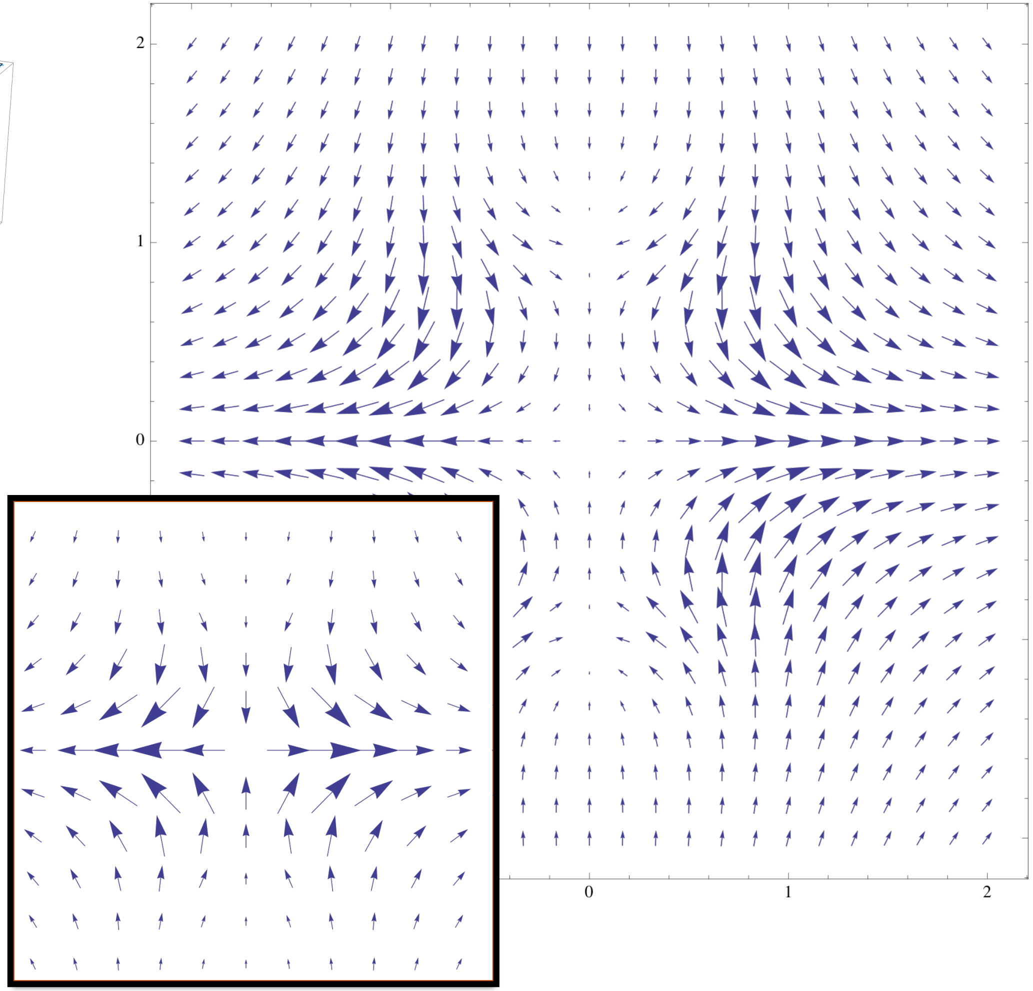
Group *I,II*:
texture (skyrmion) with
winding numbers
$w_I = \Delta N_\phi + F$, $w_{II} = \Delta N_\phi - F$

$$\Psi_{\mathrm{B}}^{(w)} = \prod_{m=0}^{\infty}\left[\lambda\left|\downarrow_m\right\rangle + z_m^w\left|\uparrow_m\right\rangle\right]$$

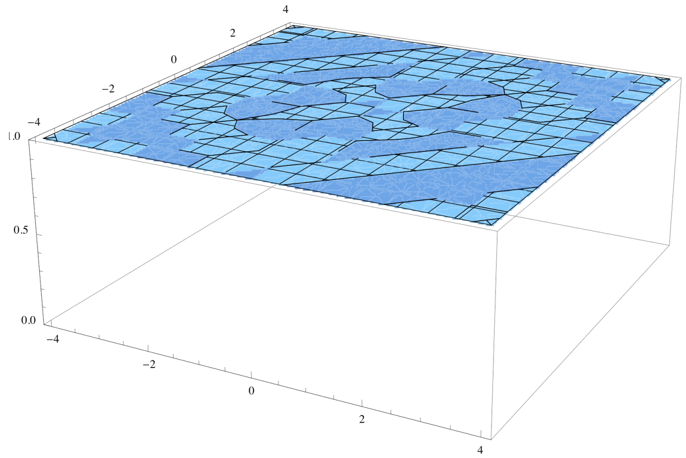
Ex: *N* odd, *n*=*4*: CST[3,1]



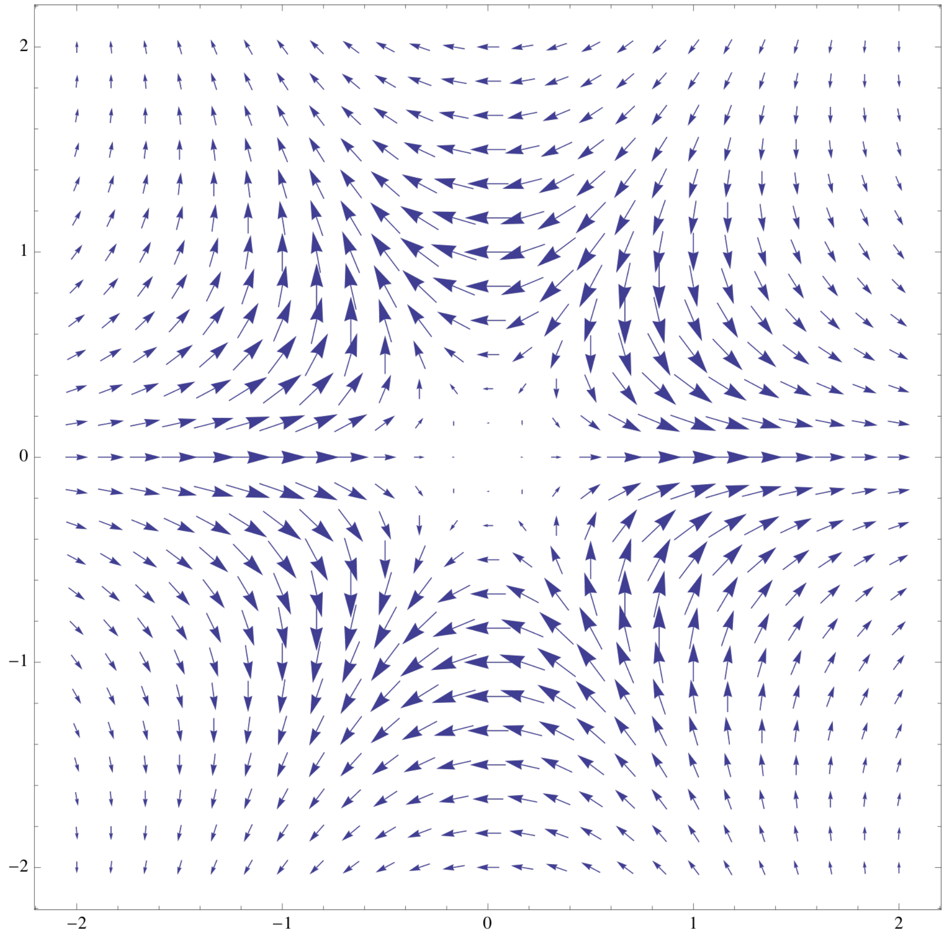
$$\xi(z) = \begin{pmatrix} z^4 \\ \frac{1}{\sqrt{2}}\left(z + z^3\right) \\ 1 \end{pmatrix}$$



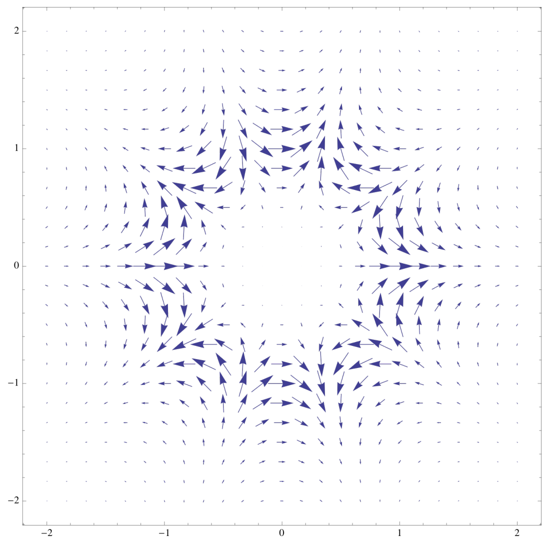
Ex: *N* even, *n*=4: CST[2,2]

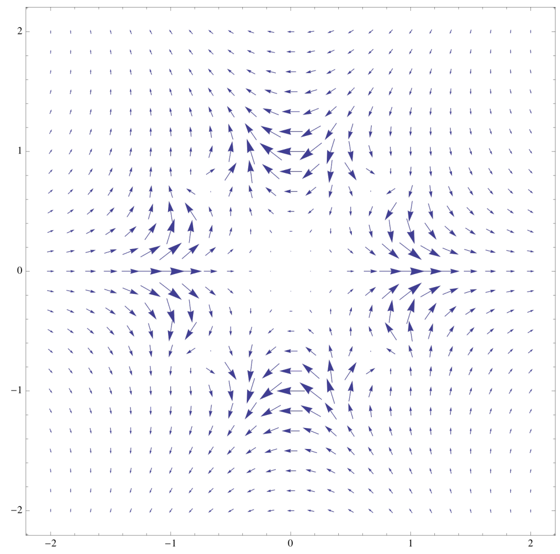


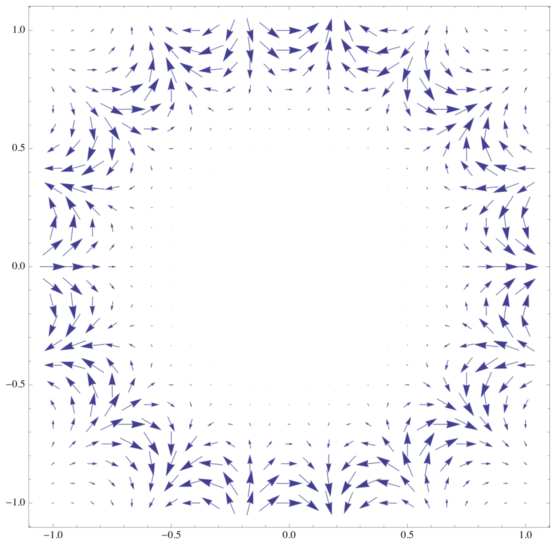
$$\xi(z) = \begin{pmatrix} z^4 \\ \sqrt{2}\, z^2 \\ 1 \end{pmatrix}$$



Ex: *N* even, *n=8*: CST[4,4], CST[6,2], CST[8,0]

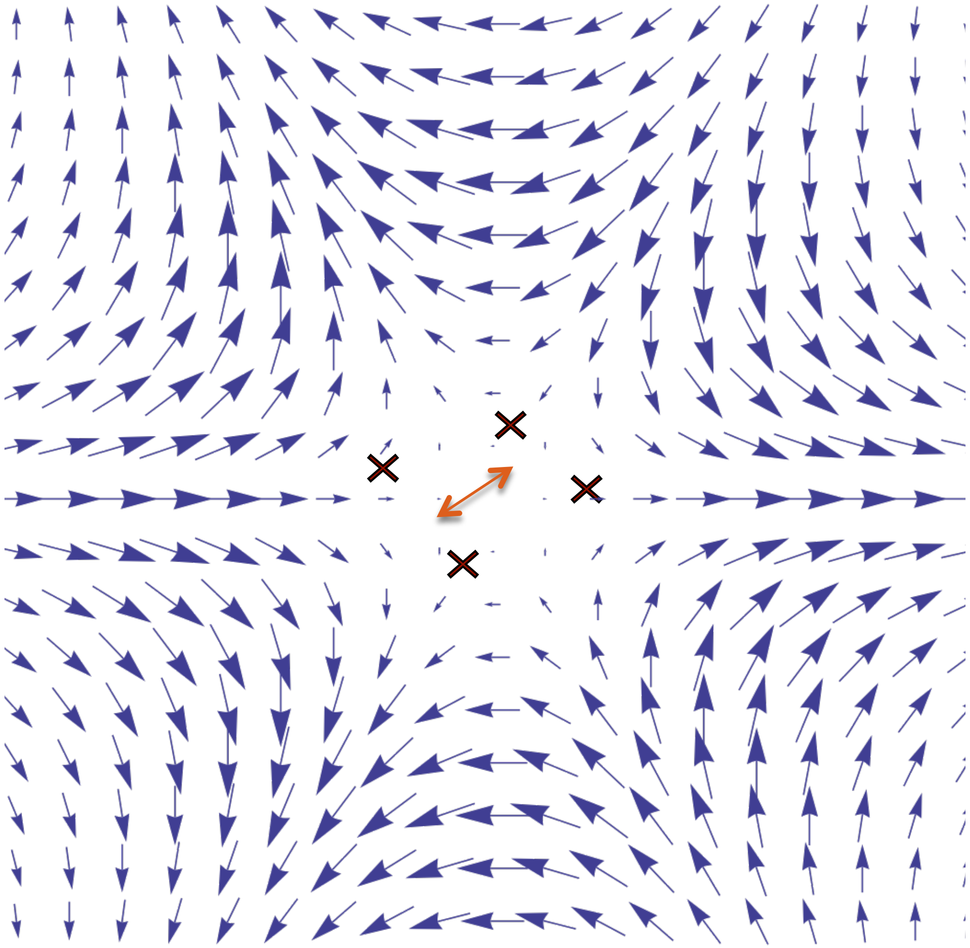






length scales

for Coulomb rather than ultra-local interactions:



relevant regime:

$d << l_{CST}$

"spin texture read out of MR topological quantum register"

Conclusion

• elementary CST, going with charge e/4 quasi-hole over the fermionic MR state, identified with polar core vortex

• the spin-texture going with the fusion product of *n* elementary CST, in fusion sector with *F* unpaired fermions, is *CST[n/2+F,n/2-F]*