Quantum Hall Interferometry: Status and Outlook

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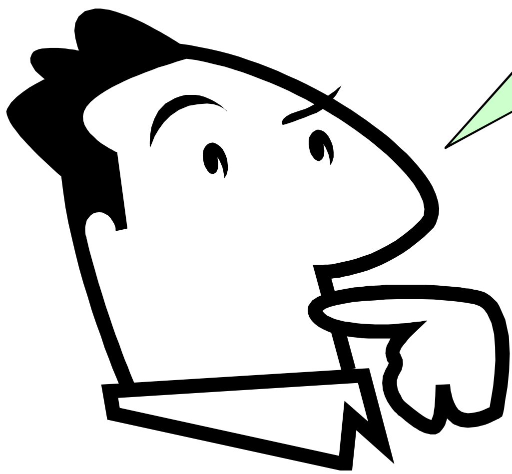
Collaborators: ***Woowon Kang***, ***Bernd Rosenow***

\+ Ady Stern, Bert Halperin, Eytan Grosfeld, Curt von Keyserlingk



Why are we interested in
Fractional Quantum Hall ?

Realization of interesting CFTs in condensed matter

(Moore-Read, Wen, …)

2+0 dimensional Bulk Wavefunction $\leftrightarrow$ CFT Correlator

1+1 dimensional Edge Theory $\leftrightarrow$ CFT

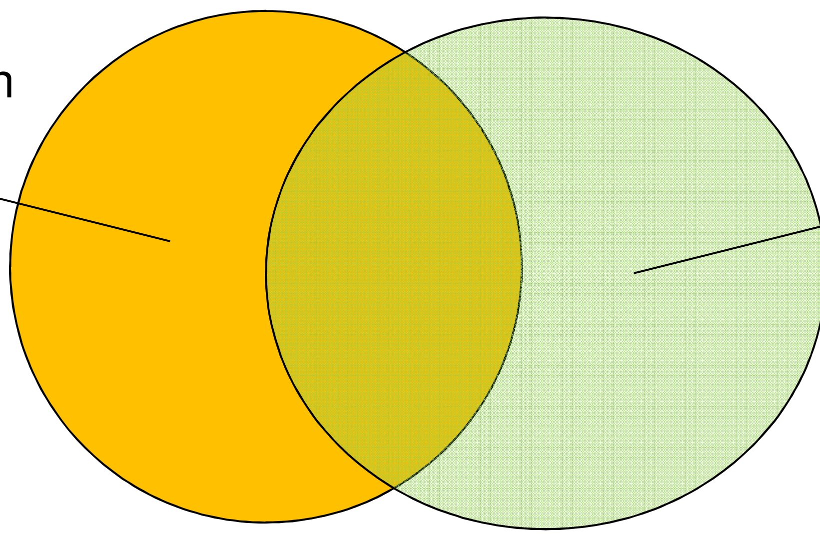
Many Theoretical Predictions (mainly regarding edge properties) Are based on CFT Calculations:

Kane and Fisher (Various Edge Transport Properties)
Ludwig, Fendley, Saleur (Noise at 1/3, Exact Calculation)
Chamon, et al (Interferometry at 1/3)
Stern Halperin; Shtengel, Bonderson, Kitaev (Interferometry at 5/2)
…. + many many more…

Experimental Situation:

Much less clear than one would hope

Experiments that seem to disagree with theory*



Experiments that are disputed by other experiment(alists)*

* = To a first approximation

Par Exemple :

1984: Prediction that quasiparticles of $\nu=1/3$ have fractional statistics

(Halperin; Arovas Schrieffer, Wilczek)

There is general agreement that as of today no published experiment has ever demonstrated fractional statistics.

How would you do it in principle?

The Quantum Hall Fabry-Perot Interferometer

Hoping to prove fractional statistics

Theory: Chamon, Wen, et al 1997 + many many others

Experiment: Goldman Group; Willett Group; Kang Group; Marcus Group; Heiblum Group

Beam Splitter

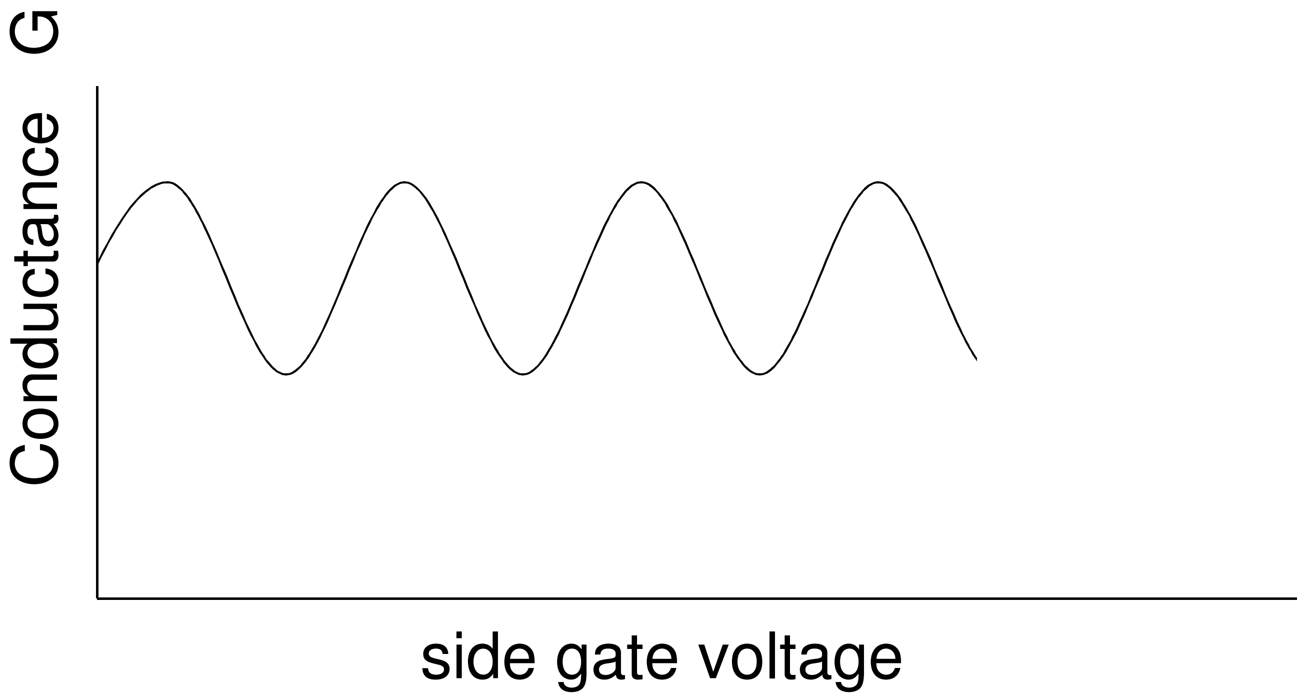
Mirror

interference of two partial waves

Conventional Quantum Hall States (ν=1/3)

Side gate changes phase $$B\Delta A = hc/e^*$$

Chamon, Wen, et al
$e^* = e/3$



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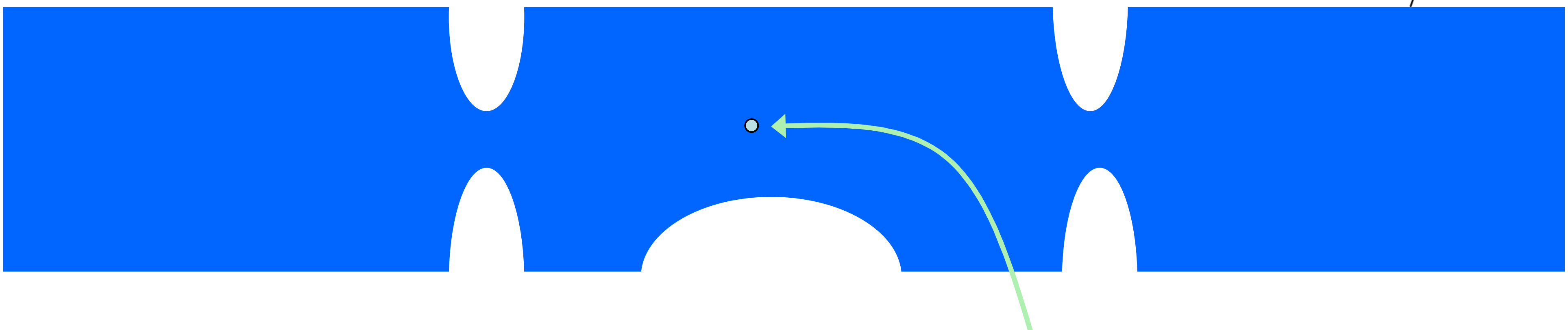
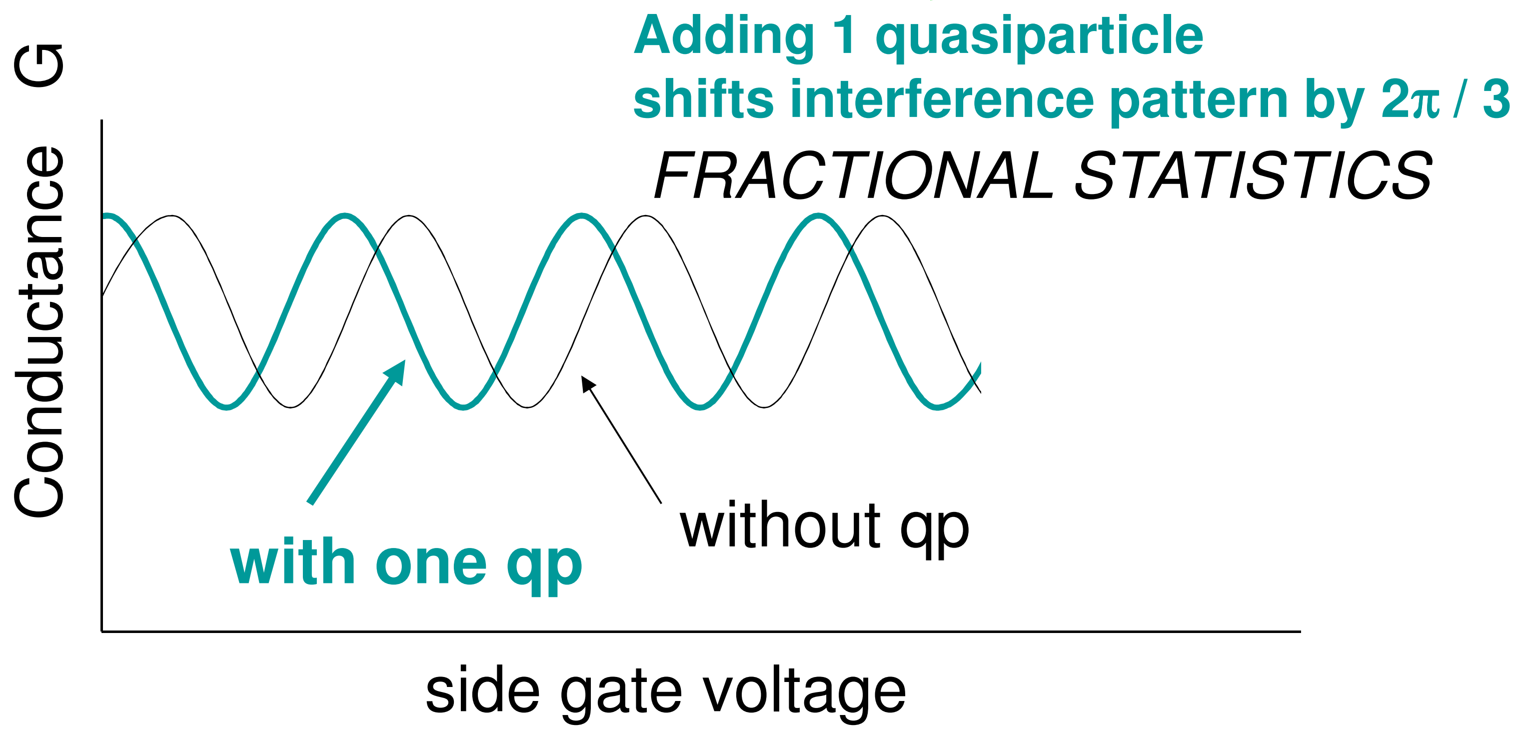
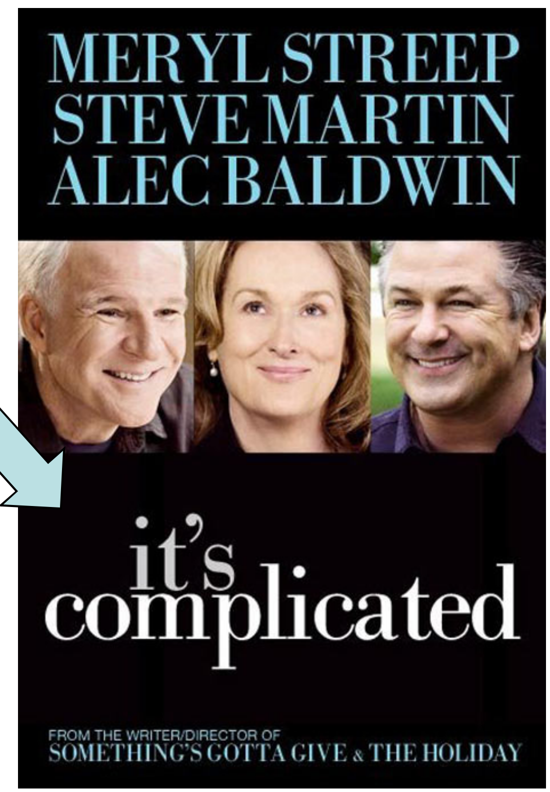




VERY LONG HISTORY TO EXPERIMENTAL EFFORTS TO DEMONSTRATE FRACTIONAL STATISTICS THIS WAY

Plagued with Complications and Confusion

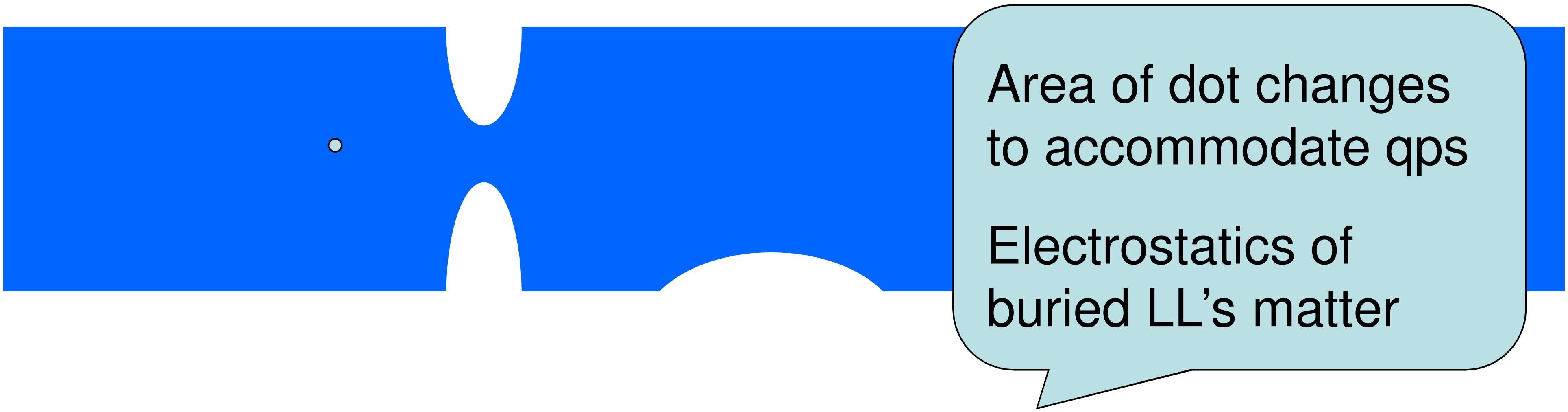


Complication #1: How do you know when you added a qp?

(a) Addition of flux

(b) Change of voltage

Complication #2: Can you add a qp without deforming the “dot”



Aharonov-Bohm Regime vs. *Coulomb Dominated Regime*

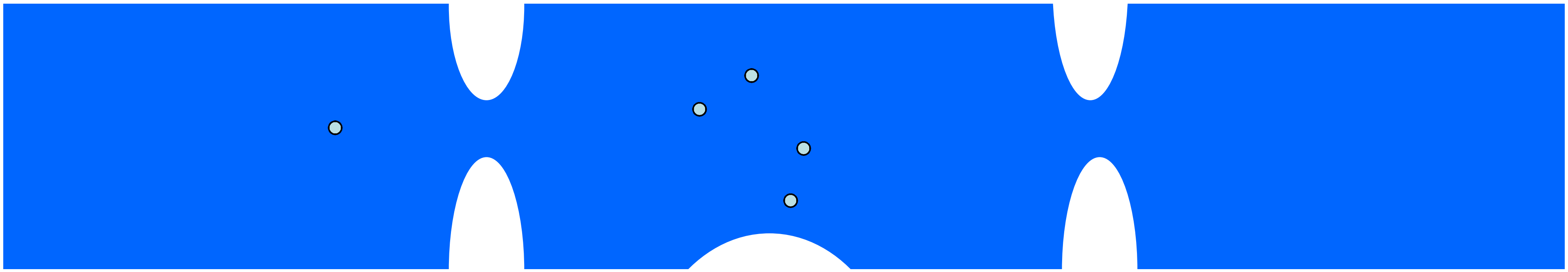
(Can be very complicated)

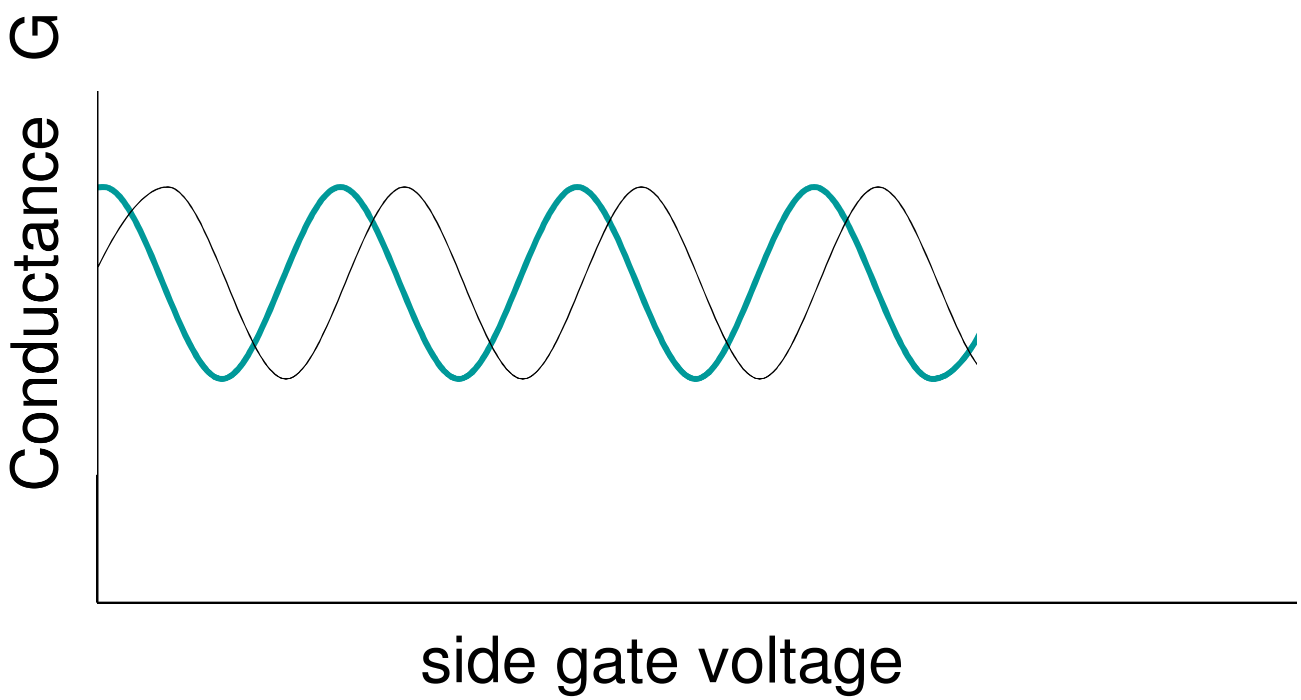
Theory: Rosenow, Halperin; Halperin, Stern, Neder, Rosenow

Exp: Y. Zhang et al (Marcus) ; N. Ofek et al (Heiblum); Godfrey et al (Kang)

Telegraph Noise

Slowish time scale = caused by glassy motion of dopant impurities





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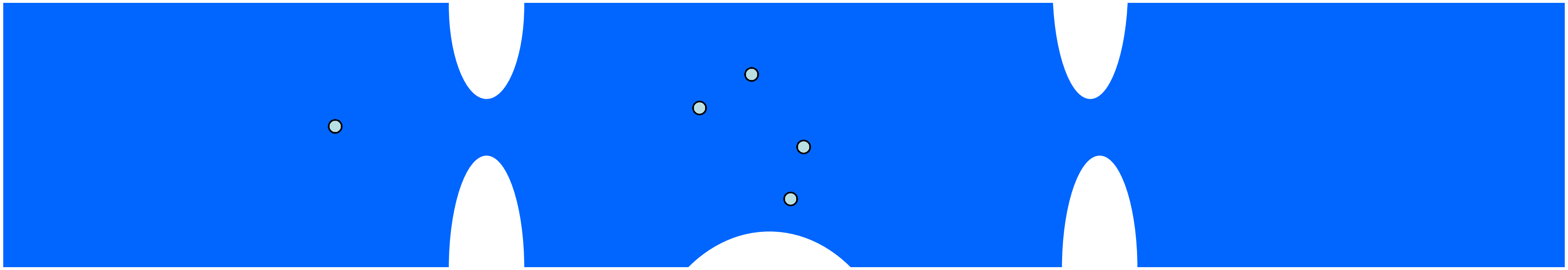
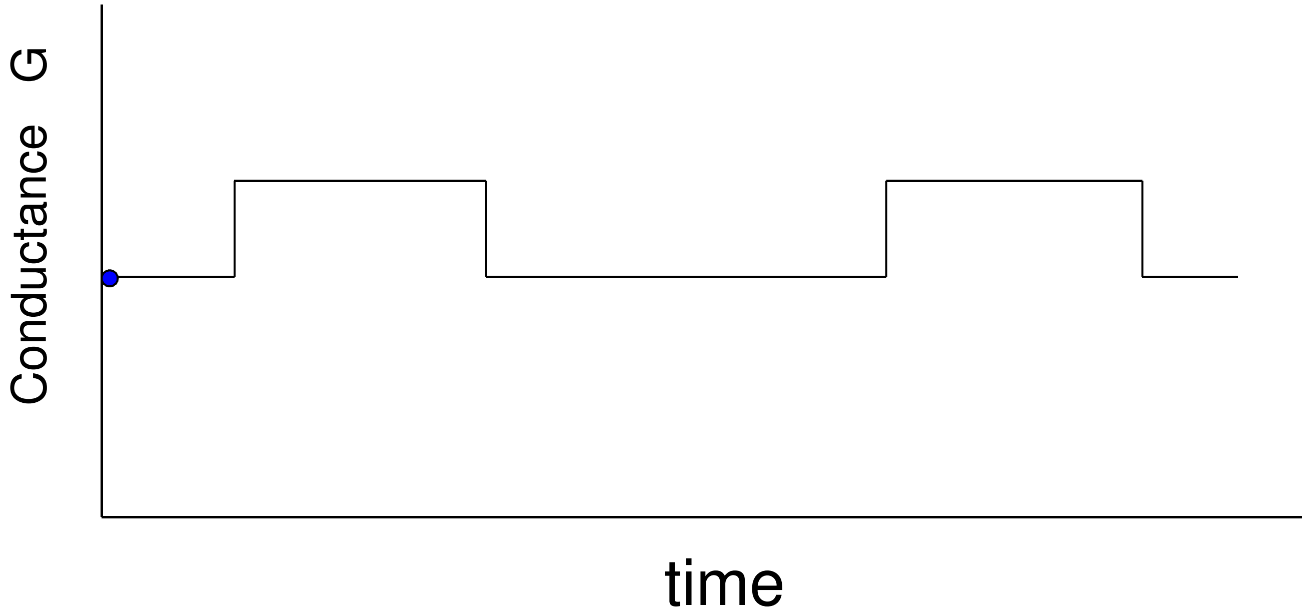


Fixed Side Gate Voltage



That is all I have to say about

Abelian Quantum Hall Effect

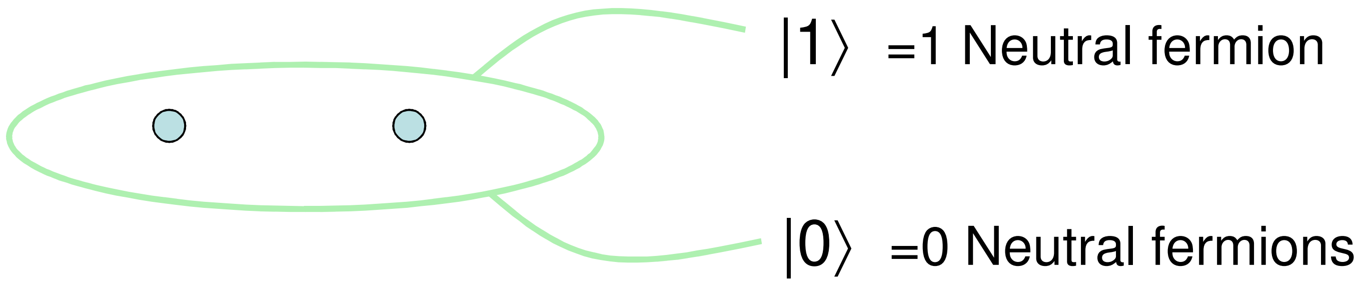
(take a deep breath)

… and on to the presumed

Non-Abelian ν=5/2

The Fundamental Principles of 5/2 Nonabelions

(Presumed Moore-Read or AntiPfaffian)



- For each *pair* of e/4 qps there is a single *two state system*.
  called: a "neutral (dirac) fermion" or a "qubit"

  (i.e, each qp associated with a majorana)

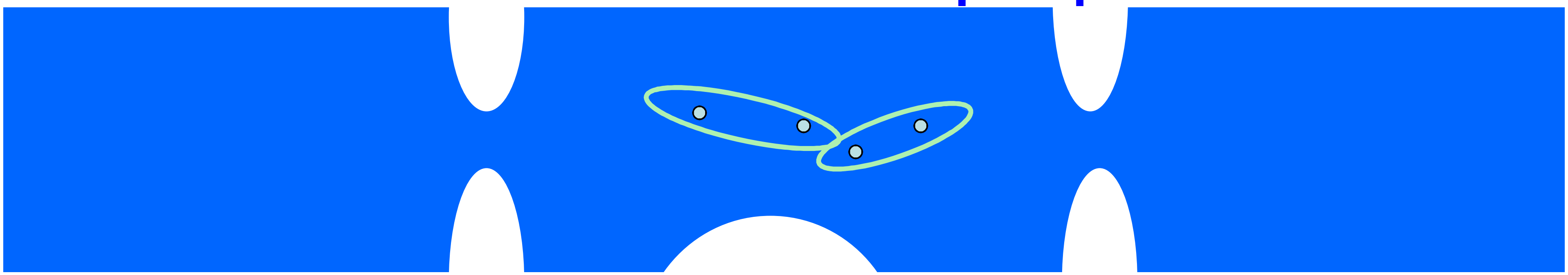
- Braiding a third qp through the two flips the state of the qubit

- A phase of π is accumulated going around a neutral fermion

5/2 state interference experiment

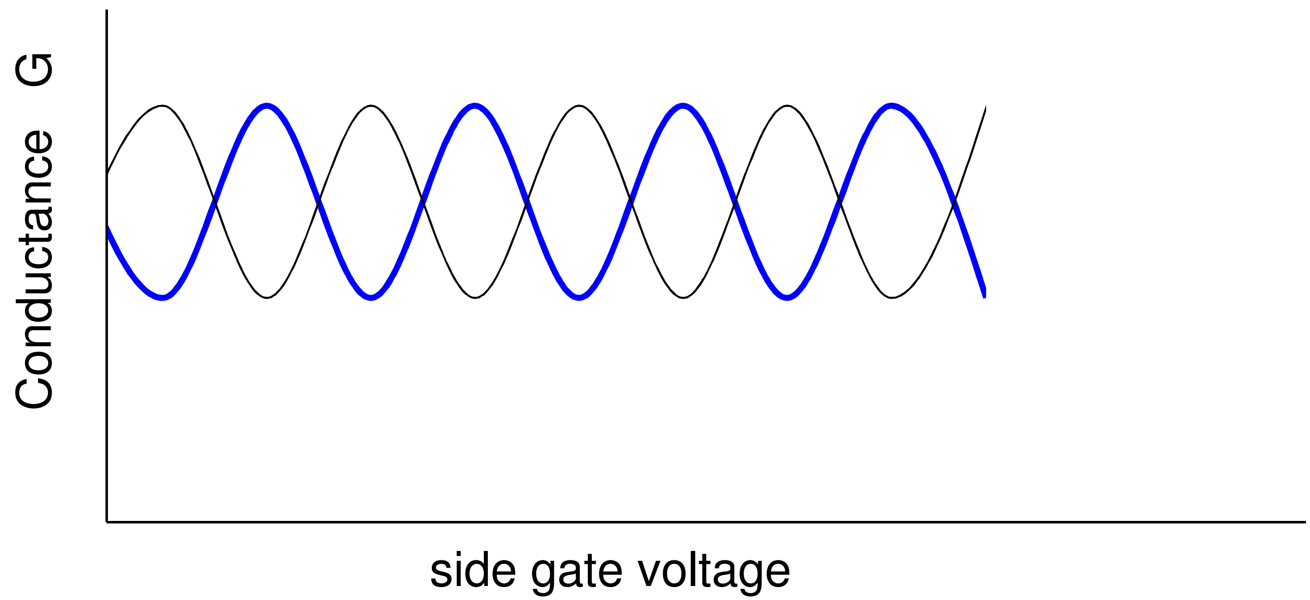
Nayak, Wilczek; Stern, Halperin; Bonderson, Shtengel, Kitaev; Das Sarma, Nayak, Freedman

With even number of quasiparticles



Can get π phase shift

Depending on even/odd neutral fermions



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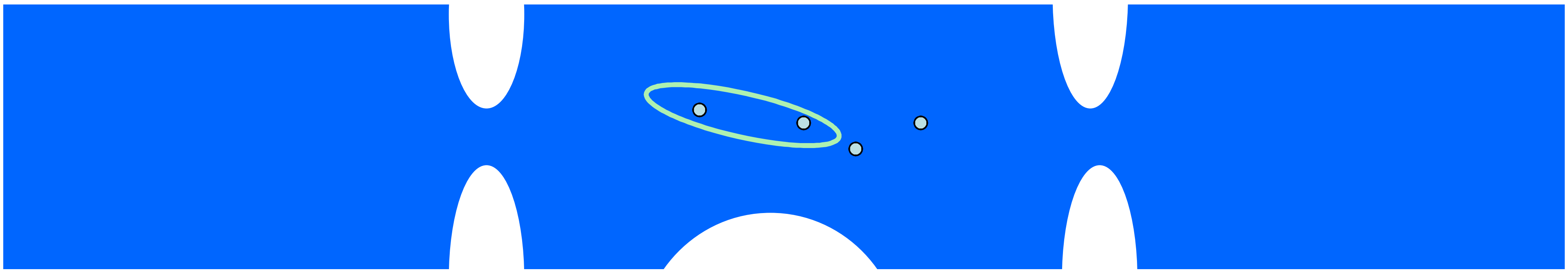

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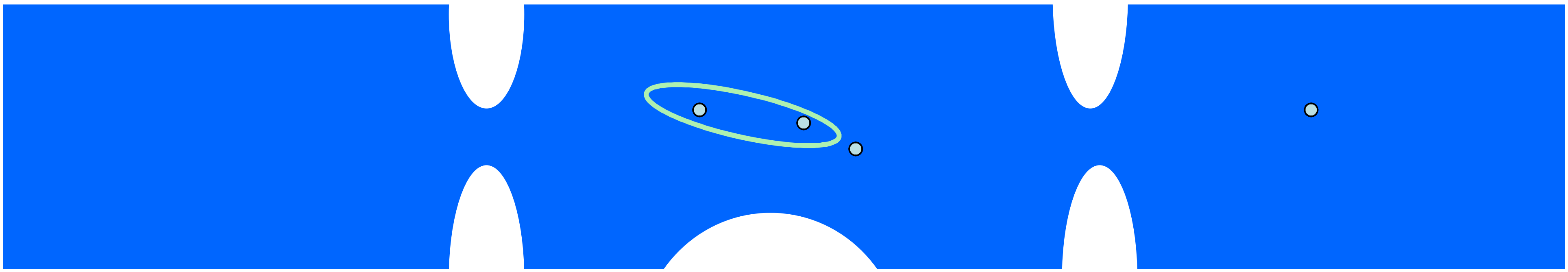
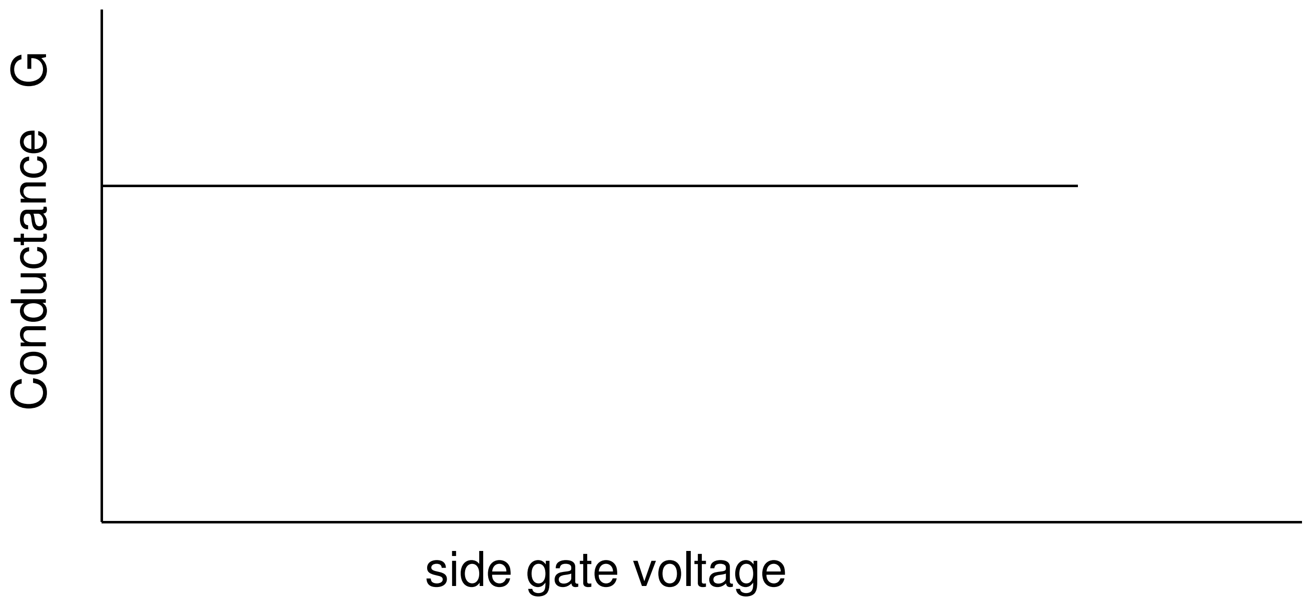


No Interference!



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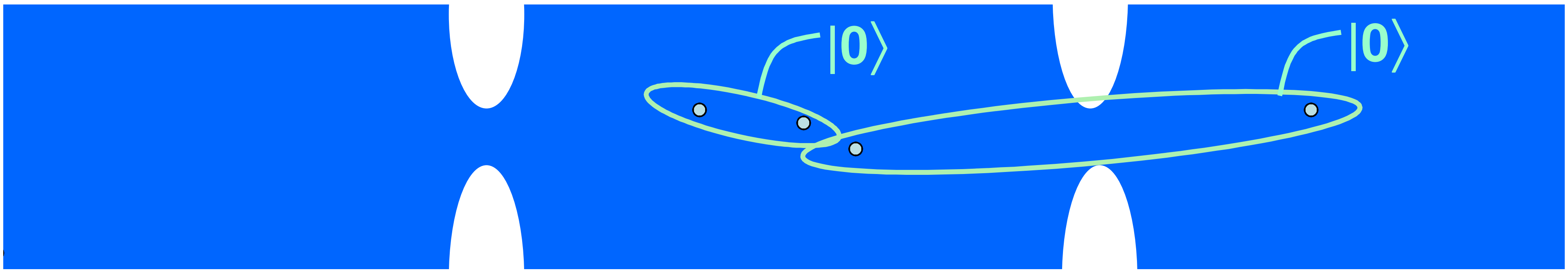
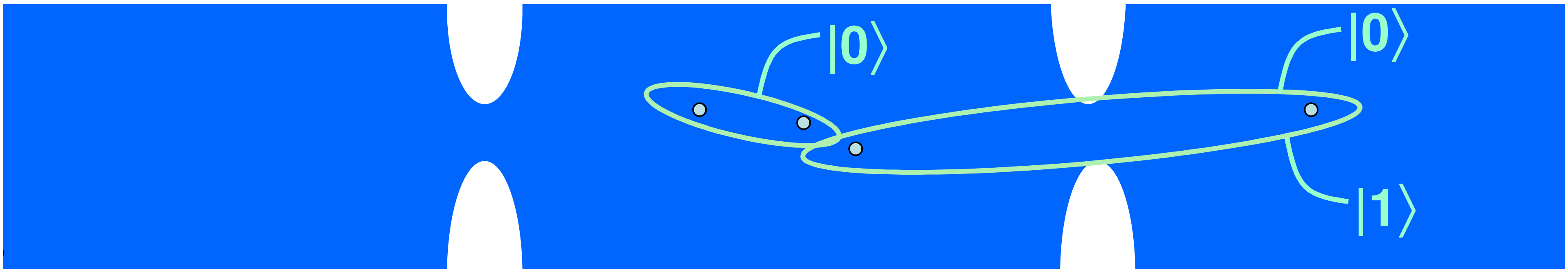


1st partial wave



2nd partial wave

Partial Waves are Orthogonal ⇒ No Interference!

5/2 state interference experiment

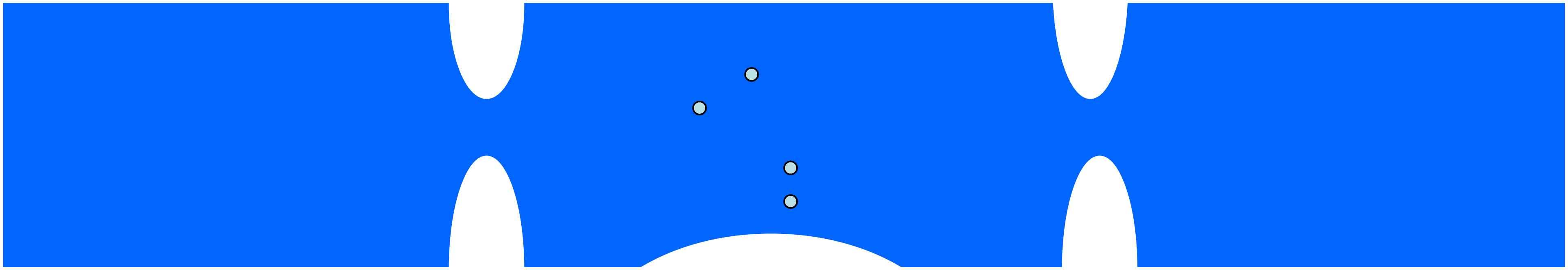
Summary of Orthodox Theory:

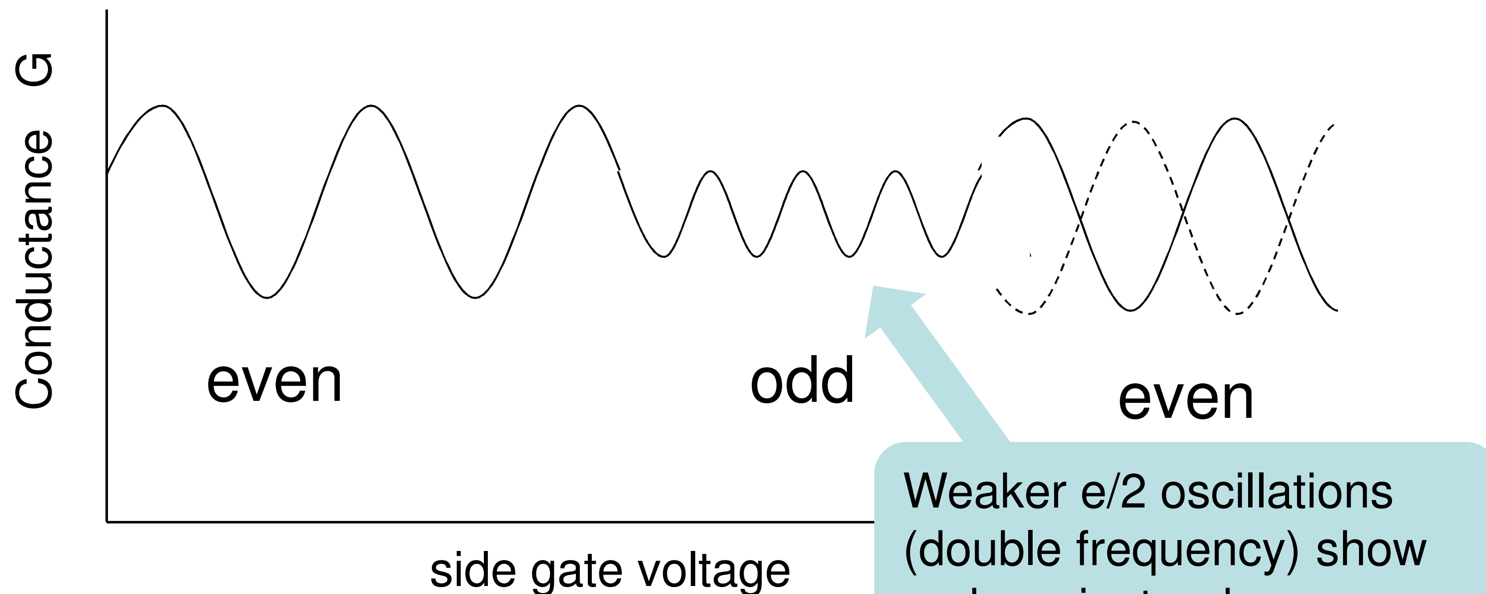
- If an odd # of qps are in the interferometer, no interference
- If an even # of qps are in the interferometer, yes interference

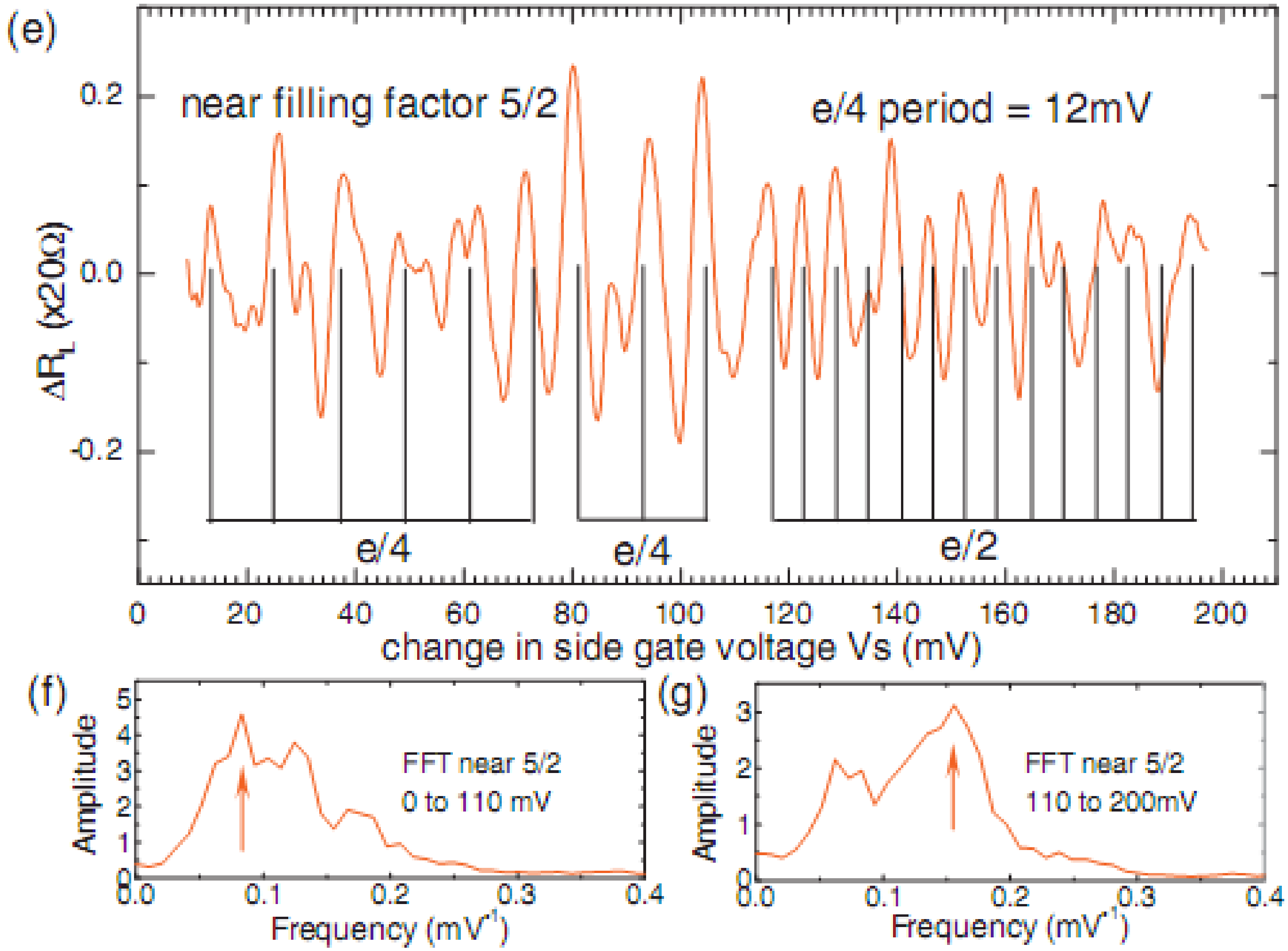
Phase = 0 if even # of neutral fermions
Phase = π if odd # of neutral fermions

plus interference of e/2 particles occurs all the time
- half gate-voltage period
- expect lower amplitude

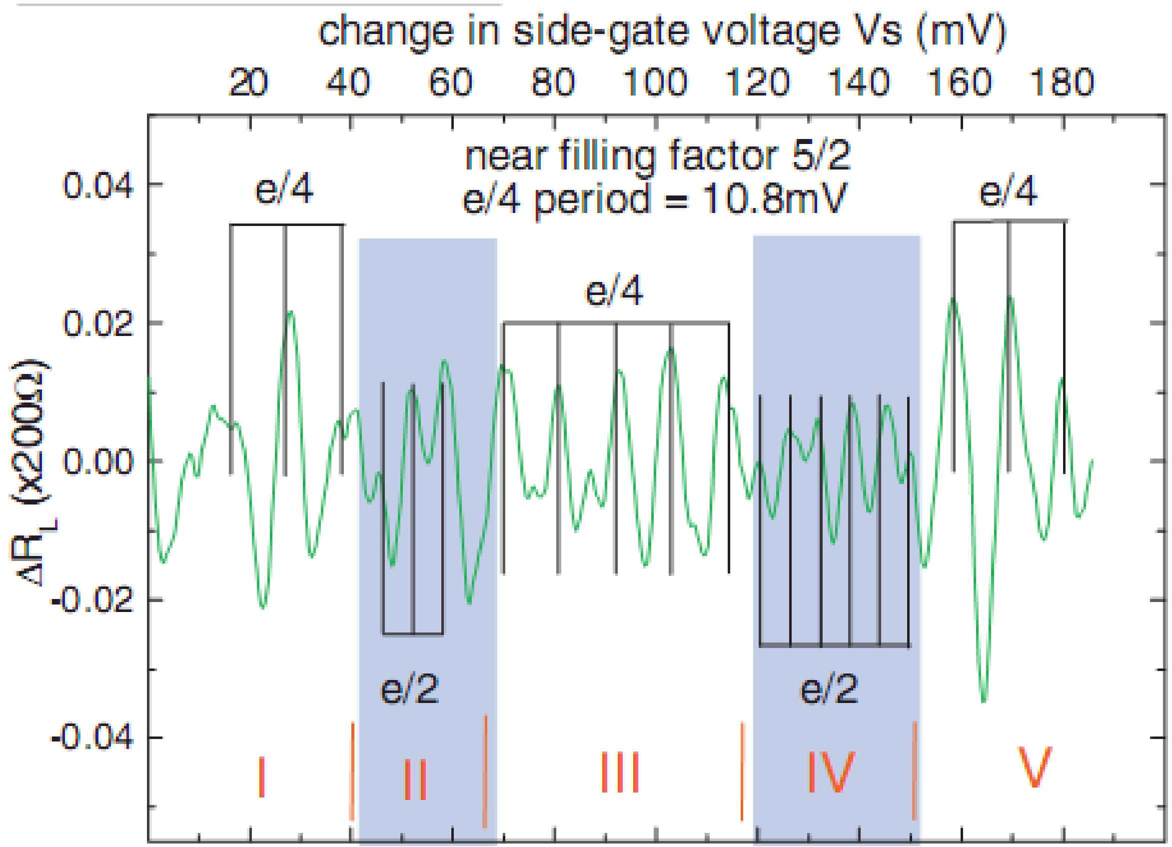
Willett's Picture (Halperin,Stern; Bonderson,Kitaev,Shtengel)







R. L. Willett, L. N. Pfeiffer, and K. W. West
Phys. Rev. B **82**, 205301 (2010)



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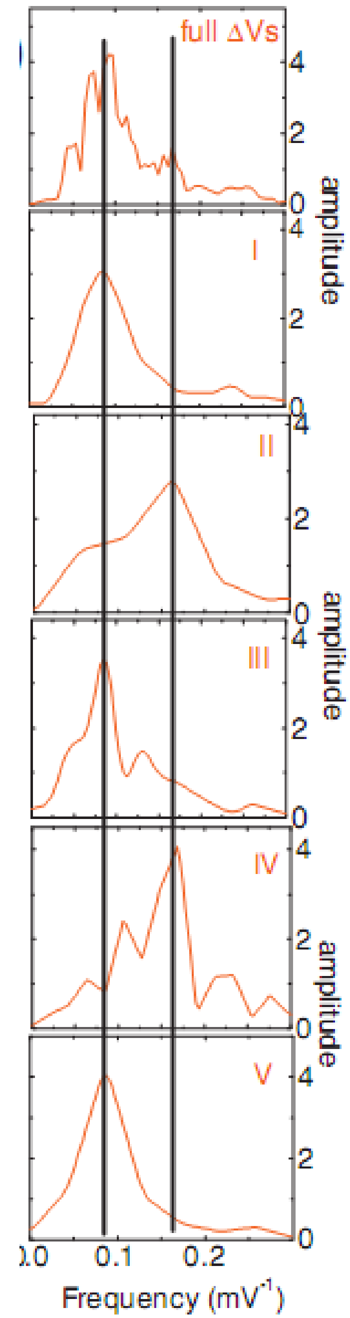
FFT



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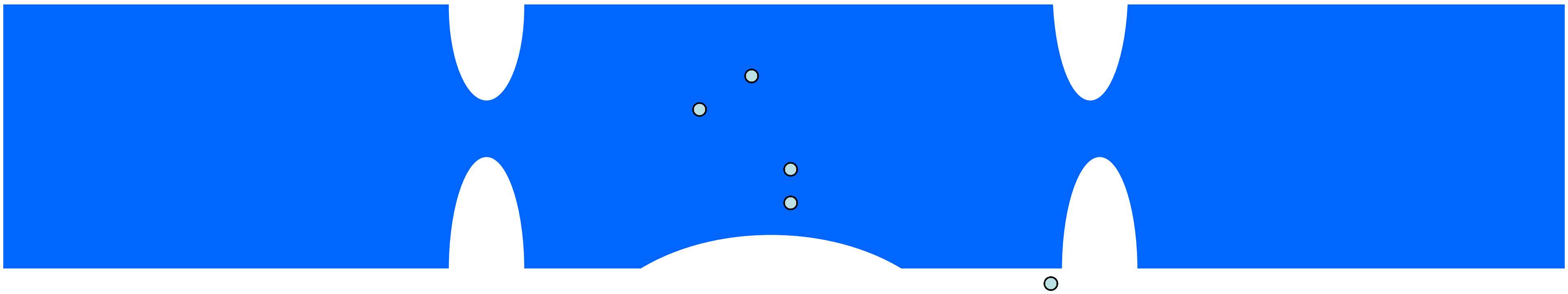
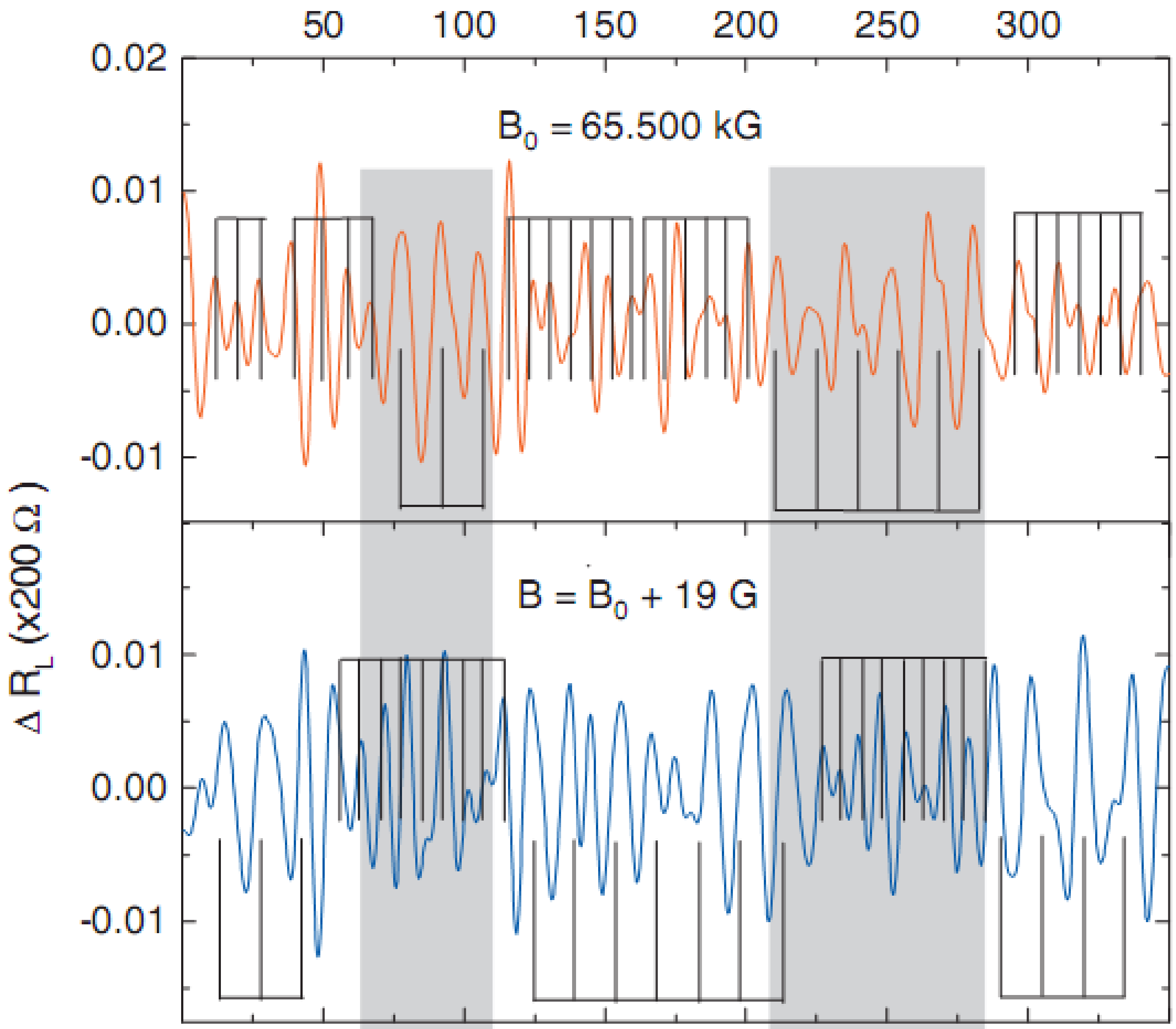


Adding 19 Gauss of Flux (presumed 1 qh) changes even to odd


50
100
150
200
250
300
0.02
0.01
0.00
-0.01
$B_0 = 65.500$ kG
$B = B_0 + 19$ G
ΔR_L (x200 Ω)

Is this validation of the “Orthodox” theory?

(1) Is the data convincing?

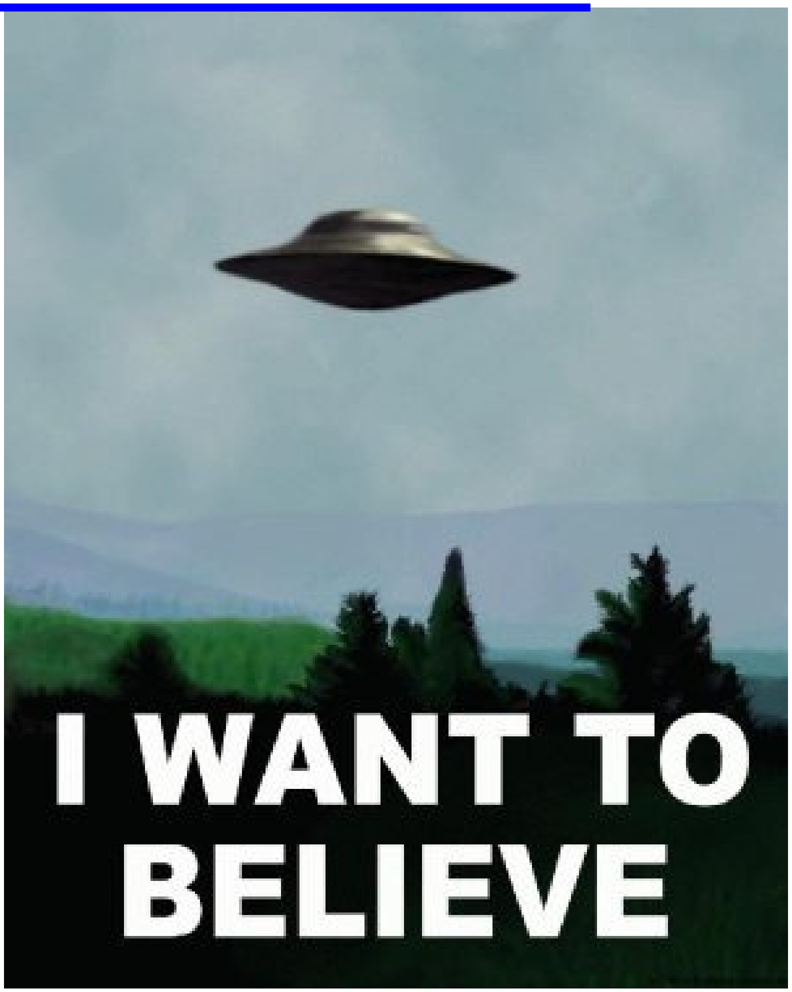
(2) Why is e/2 so strong?
Why does it come and go?

(3) Why does this appear to be Aharonov-Bohm and not Coulomb Dominated?

(4) How reproducible is it?

(5) Why does no other group observe this.

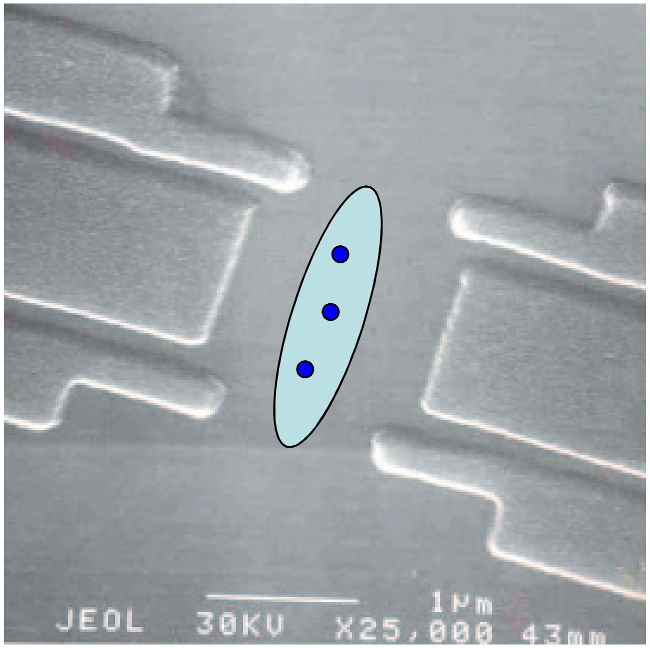
….



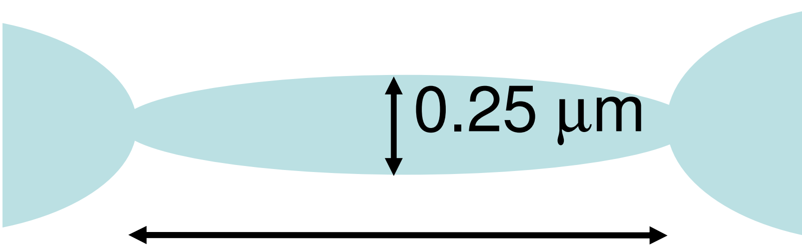
LET’ S BELIEVE THE EVEN-ODD EFFECT HAS BEEN SEEN

PROBLEM:

ORTHODOX THEORY SHOULD NOT HOLD!



Area Estimate = 0.2 μm^2



PROBLEM =
DEVICE IS SMALL….

qps (qubits) in the dot must be strongly coupled to each other, and to the edge…
By majorana hopping!

VERY UNLIKE
ORTHODOX THEORY

Energy Scales

(1) T ≈ V ≈ 10 mK ≈ 200 MHz

(2) qp-qp majorana coupling

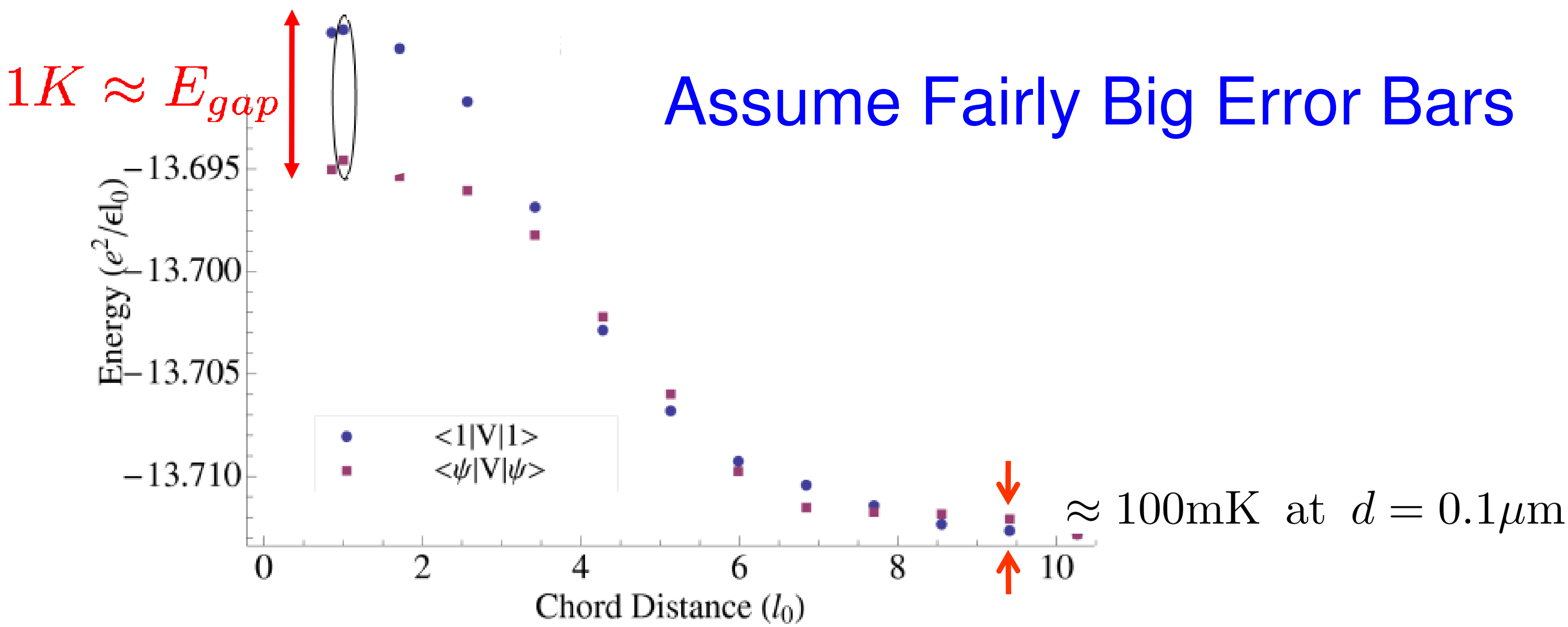
(3) qp-edge majorana coupling

All Potentially
Similar Order

Estimate from Trial Wavefunction Monte-Carlo for tunneling

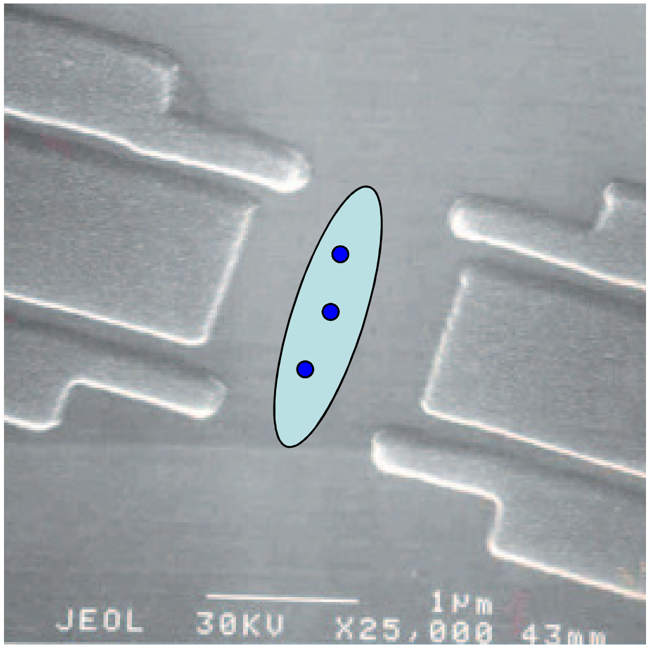
(Baraban, Zikos, Bonesteel, Simon):

Two qps a distance d apart (4 qps in the calculation=2 fusion channels)



$$E_{splitting} = \text{Oscillatory} \times e^{-d/\xi}$$

$$\xi \approx 2.3\ell_0 \approx 230\text{Å}$$



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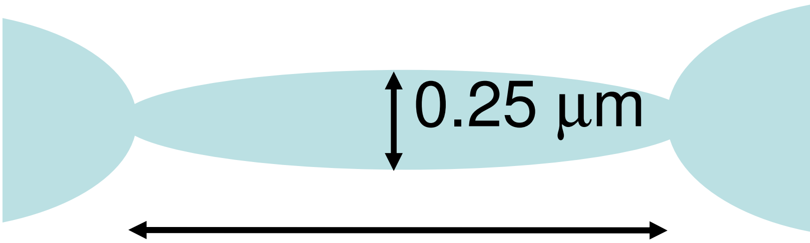

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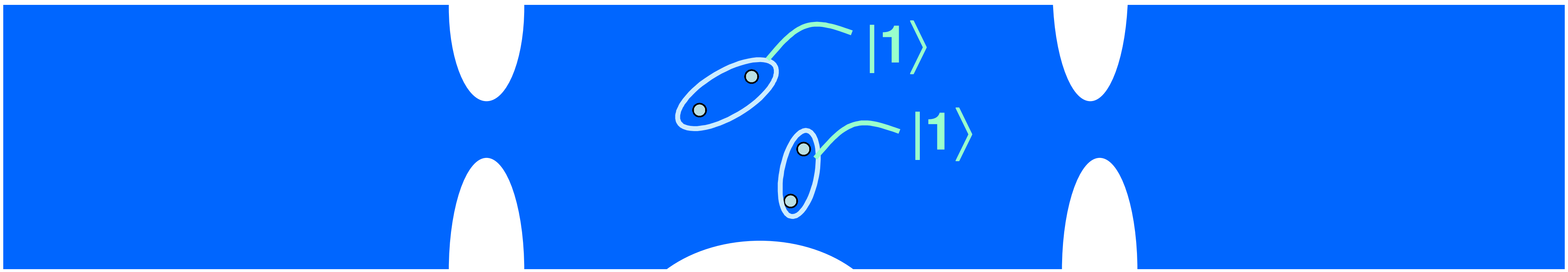
(4) 1/(Time of Experiment) = Hz = Tiny.

For orthodox interpretation to hold, need

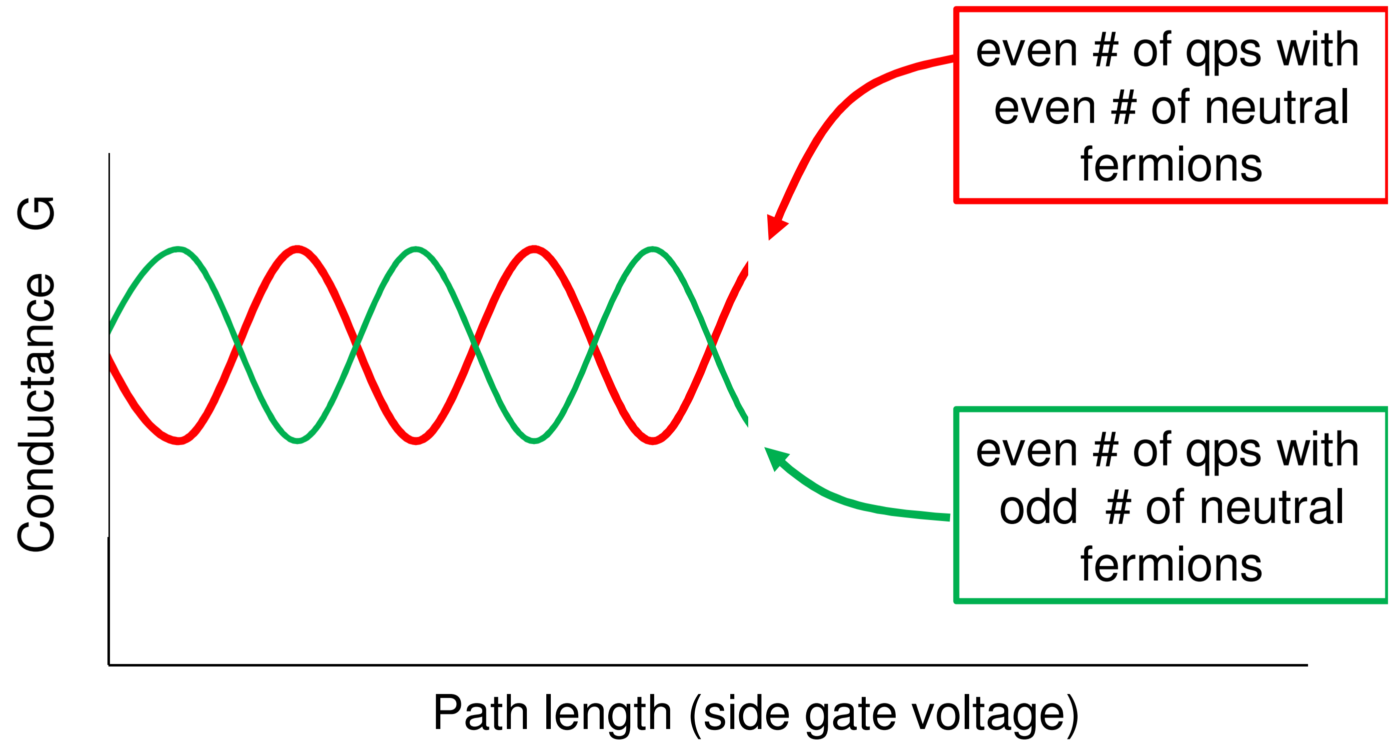
qp-edge coupling << 1 / (Time Scale of Experiment)

Why is edge-qp coupling a problem

Overbosch and Wen; Rosenow, Halperin, Simon, Stern ; Bishara and Nayak



“Fast” tunneling of neutral fermions to the edge kills intereference *!!*



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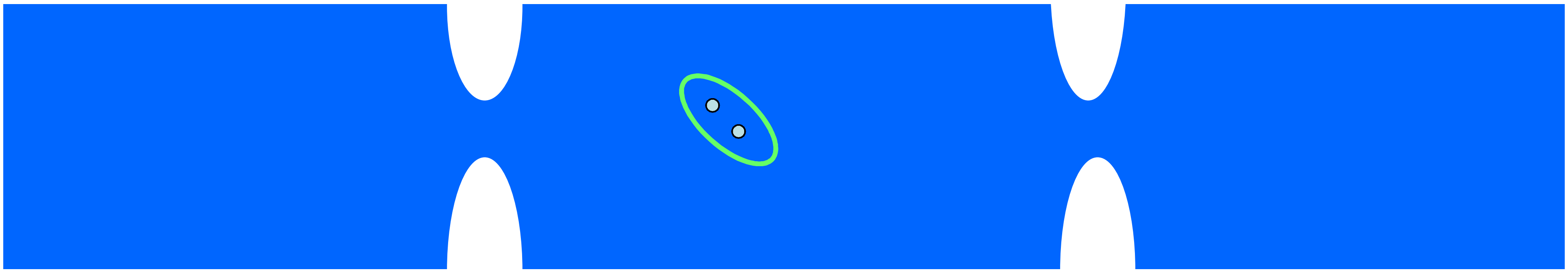
Modified (reform) interpretation, can save even-odd effect if

qp-qp coupling > T

Why does qp-qp coupling help?



Energy of $|0\rangle$ and $|1\rangle$ are split by E (qp-qp coupling)

If $T < E$, qubit freezes into a single state.
Does not fluctuate between two out of phase signals

Interference is then seen!...and even-odd effect!
… but not a good "qubit"

(actually need $T < E_{min}$ of band of majoranas)

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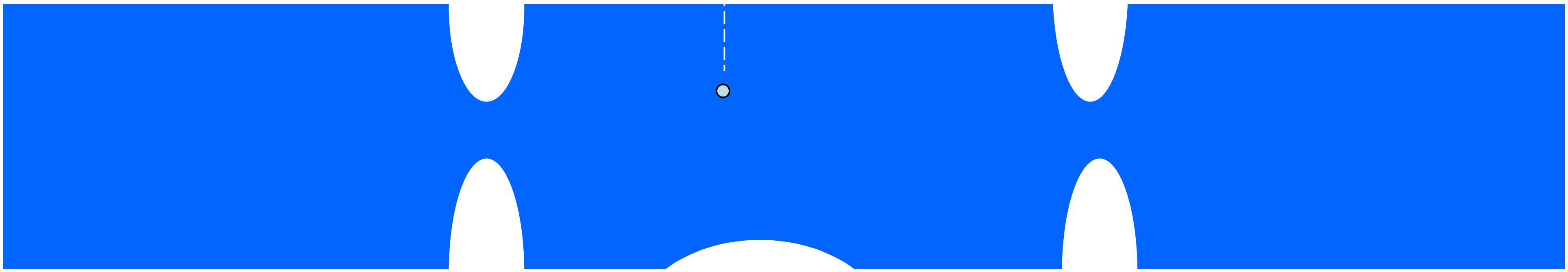
How can qp-edge coupling stop even-odd?

Overbosch and Wen;   Rosenow, Halperin, Simon, Stern ; Bishara and Nayak

For “odd” to kill interference, lone qp must be decoupled from edge

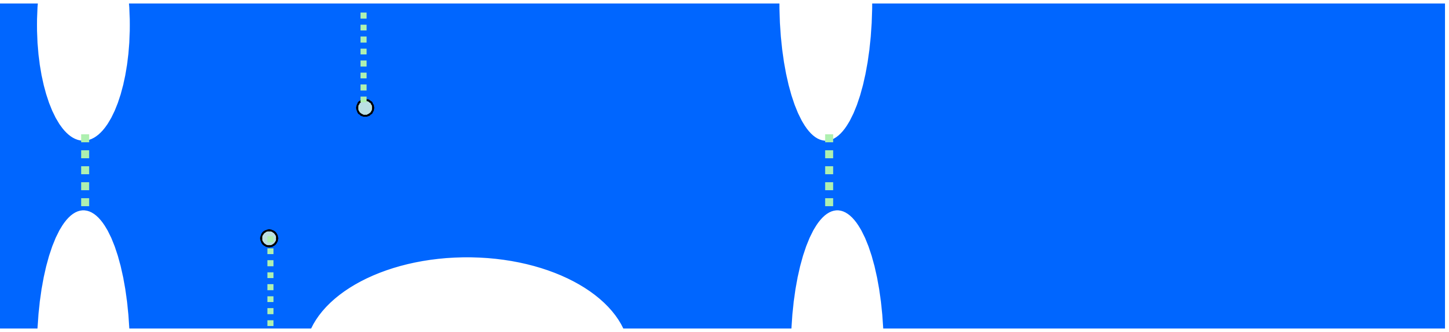


Need: $\boxed{\text{qp-edge coupling} \ll \ e^*V \ \approx T}$

If a qp is coupled strongly to the edge,

it becomes part of the edge $\Rightarrow$ *Nothing encircles it*

(ASIDE)



$$\mathcal{L}_{charge} = \frac{1}{4\pi\nu}\,\partial_x\varphi(v_c\partial_x \pm i\partial_\tau)\varphi$$

Edge charge mode (bosons)

$$\mathcal{L}_{neutral} = \psi(v_n\partial_x \pm \partial_t)\psi$$

Edge neutral (majorana) fermi mode

$$\mathcal{L}_{qps} = \Gamma_\alpha\,\partial_\tau\Gamma_\alpha$$

Vortex core (majorana) zero modes

$$\mathcal{L}_{edge-qp} = \lambda\Gamma_\alpha\psi(x_\alpha)$$

Edge to bulk coupling

$$\hat{T}(x) = t\;\underbrace{\sigma_u(x)\sigma_d(x)}_{\text{Edge neutral mode}}\;\underbrace{\left[e^{\frac{i}{\sqrt{8}}(\phi_u-\phi_d)} + h.c.\right]}_{\text{Moves charge across}}$$

Point Contacts

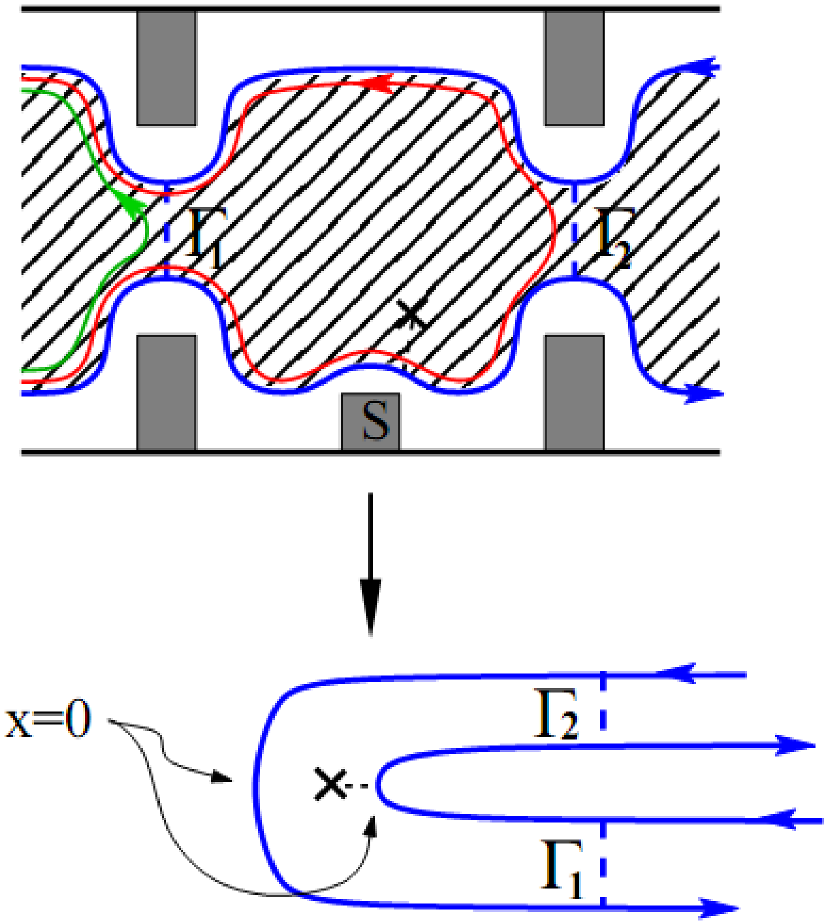
Moves charge across

Edge neutral mode

Interference Term = $\mathrm{Re}\int_\tau e^{iV\tau}\;\langle\hat{T}_1(\tau)\;\hat{T}_2(0)\rangle$

1. Perturbative
2. *Exact*

Bishara-Nayak



Ising
Model
With
Boundary

Chatterjee and
Zamolodchikov
1994

Rosenow, Halperin, Simon, Stern : Non-CFT Solution
(Majorana theories are quadratic Hamiltonians)

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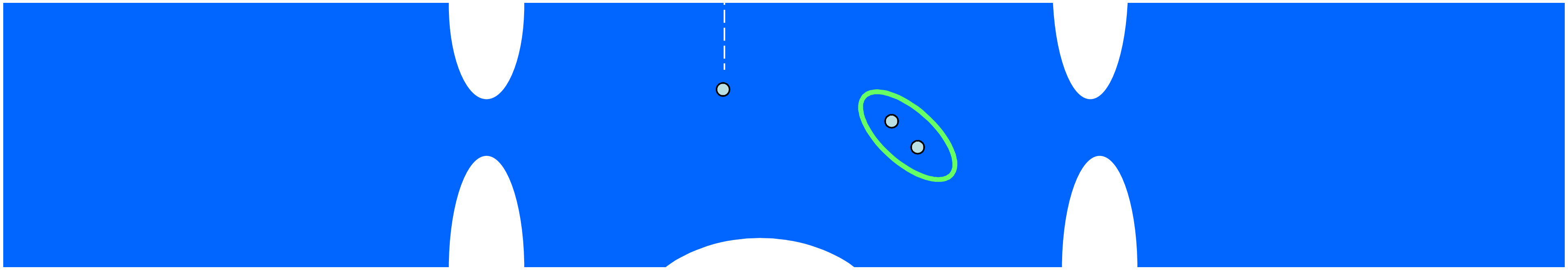


Need: qp-edge coupling << e*V ≈ T

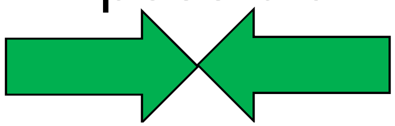
If a qp is coupled strongly to the edge,

it becomes part of the edge ⇒ *Nothing encircles it*

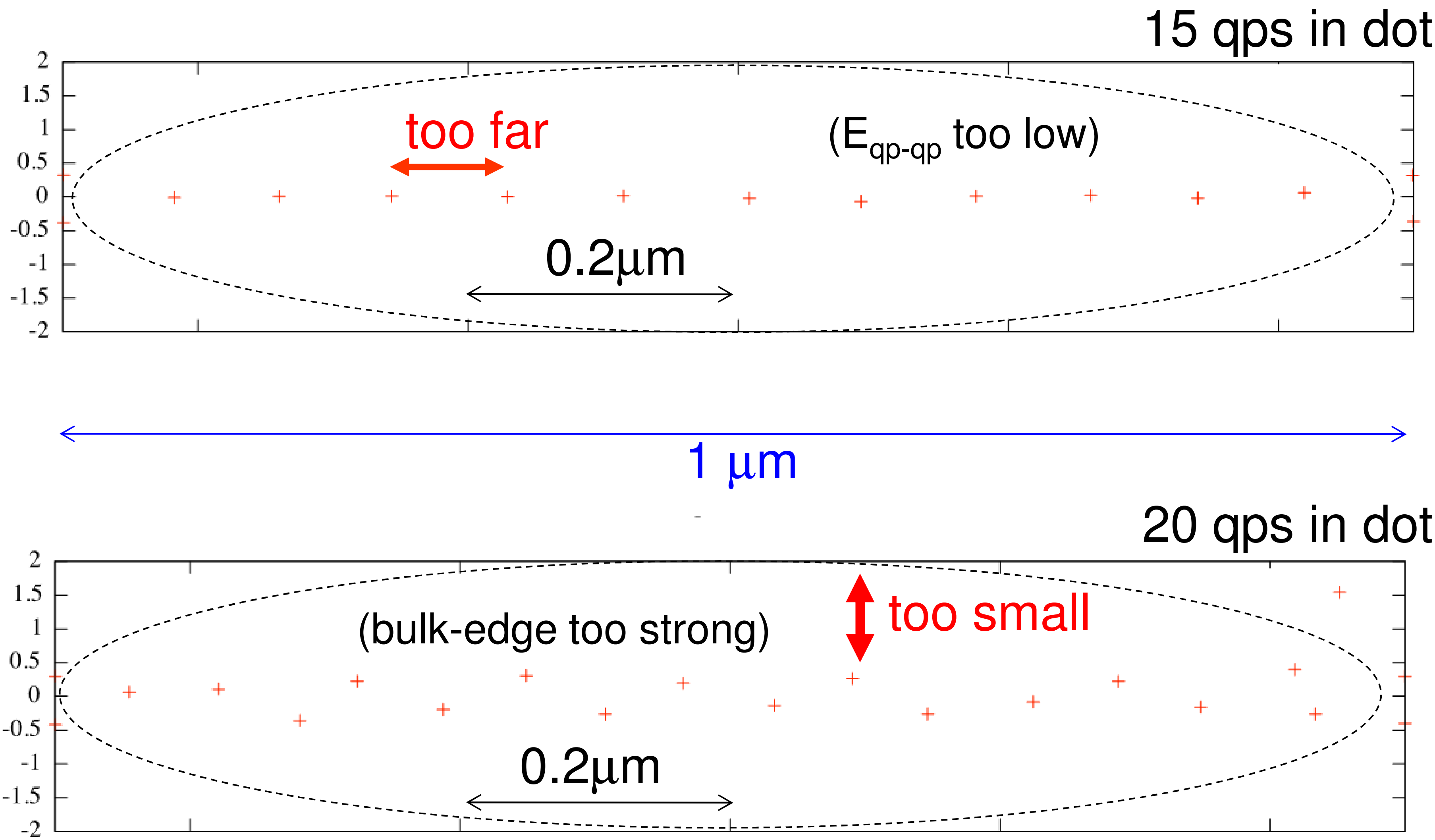
But also need to freeze qubit

T < qp-qp coupling

probably impossible



Detailed Electrostatic Simulation (w/ von Keyserlingk)



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Modified (reform) interpretation, can save even-odd effect if

qp-qp coupling > $T \approx V$
and qp-edge coupling << $T \approx V$

Energy Scales

(1) $T \approx V \approx 10$ mK ≈ 200 MHz

(2) qp-qp coupling

(3) qp-edge coupling

Prediction 1:

No Even-Odd Effect.

Always Have Interference

Never Mind the Experimental Data:

Assume couplings are strong (small dot limit)

qp-qp coupling $> \approx$ qp-edge coupling $\gg T \approx V$

Modified (reform) interpretation, can save even-odd effect if

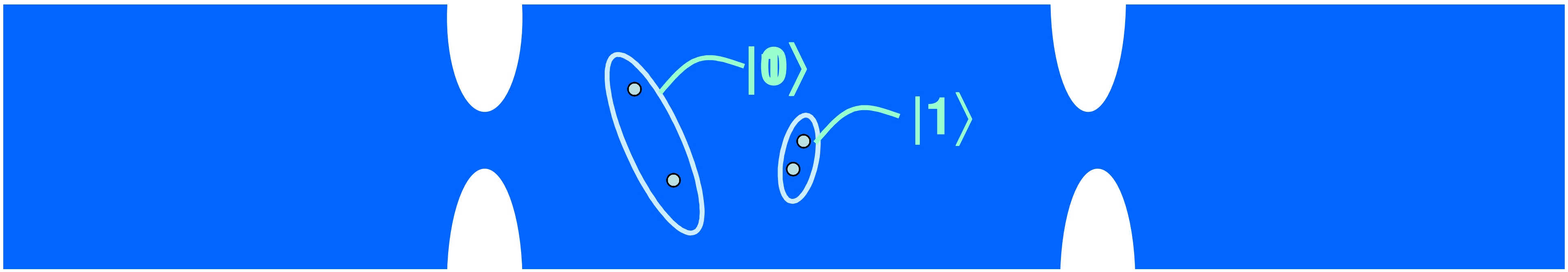
qp-qp coupling $> T \approx V$

and qp-edge coupling $\ll T \approx V$

Prediction 2

Expect $\boxed{\pi \text{ phase slips } !!}$

– can occur without adding a qp



Which is lower energy ($|0\rangle$ or $|1\rangle$) depends on the detailed configuration of qps in the dot.

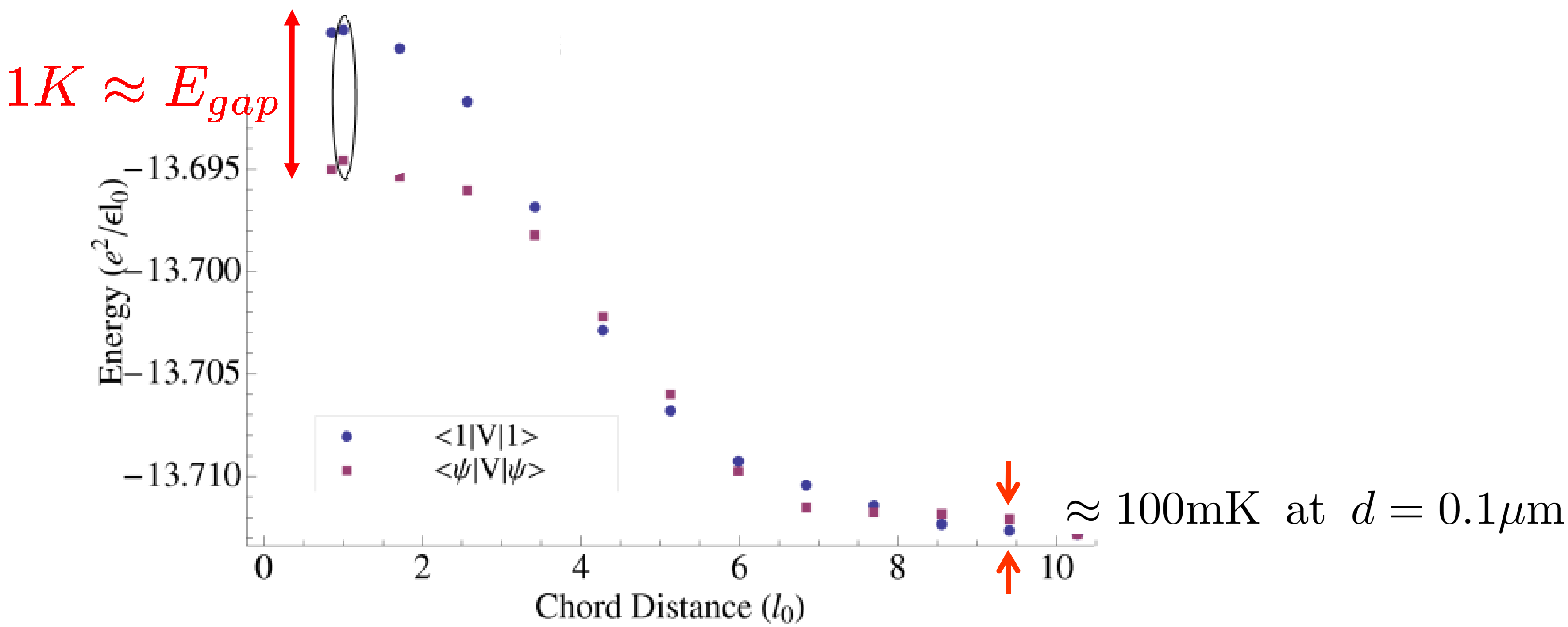
Interference signals can flip by π if a qp moves

"Friedel" oscillations in splitting as a function of distance

Estimate from Trial Wavefunction Monte-Carlo for tunneling

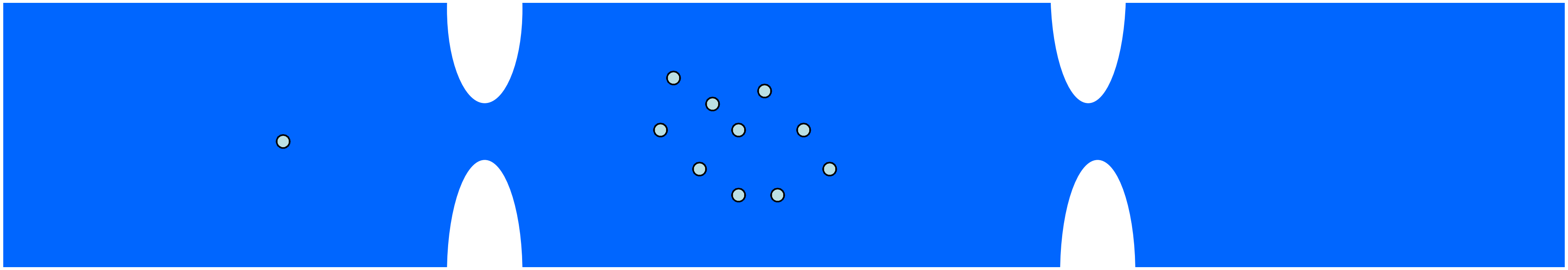
(Baraban, Zikos, Bonesteel, Simon):

Two qps a distance d apart (4 qps in the calculation=2 fusion channels)



Prediction 3

$\pm\,\pi/4$ phase slips – occur with qp/qh addition



Going from even to odd,

if $E_{edge\text{-}bulk} >> e^*V$, zero-mode majorana absorbed into edge

Only see phase slip ($\pm\,\pi/4$) from abelian piece of the qp.

$$\psi_{qh/qp} = \sigma e^{\pm i\phi/(2\sqrt{2})}$$

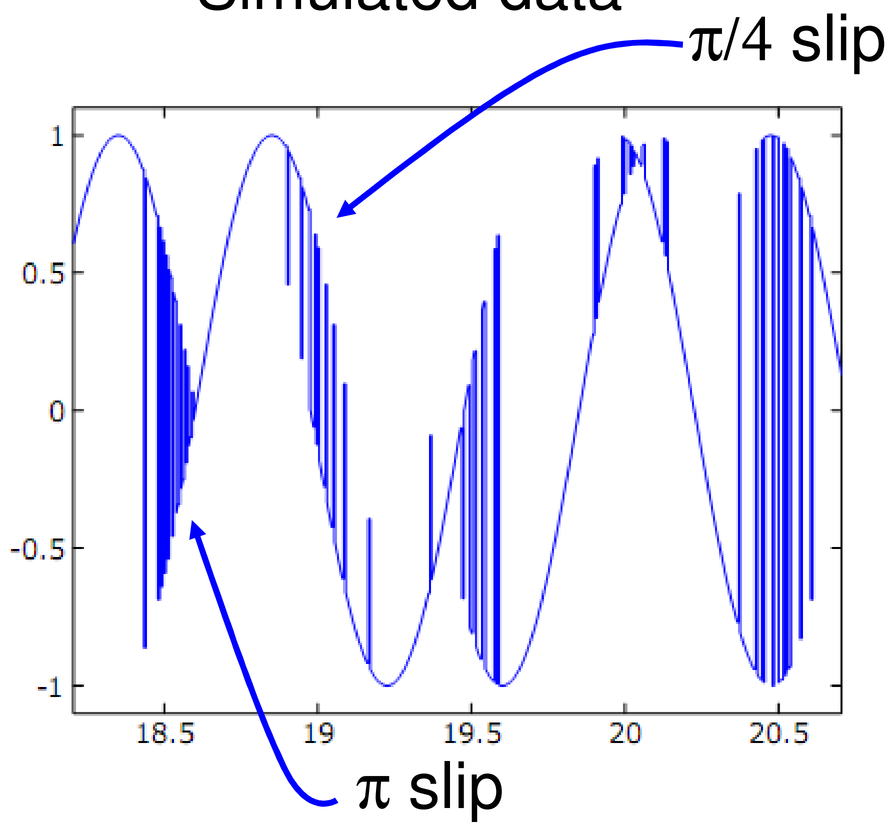
$E_{edge\text{-}bulk} \sim e^*V$, T gives not quite $\pi/4$
and less than full visibility of interference

Overbosch and Wen; Rosenow, Halperin, Simon, Stern ; Bishara and Nayak

Expected signatures:

Phase slips of π , $\pi/4$, $5\pi/4$,

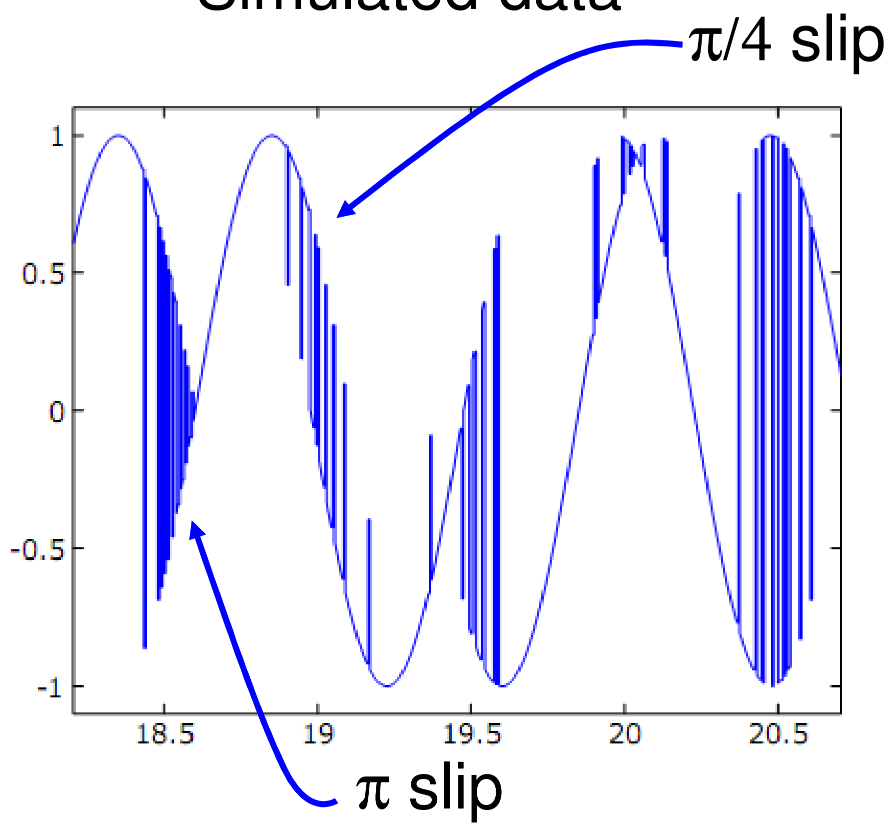
Simulated data



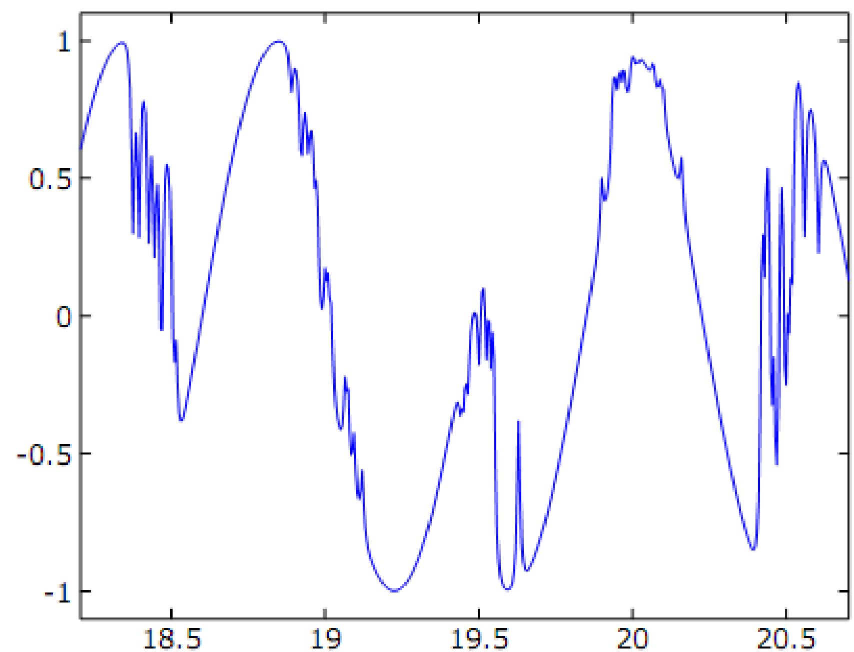
Expected signatures:

Phase slips of π , $\pi/4$, $5\pi/4$,

Simulated data

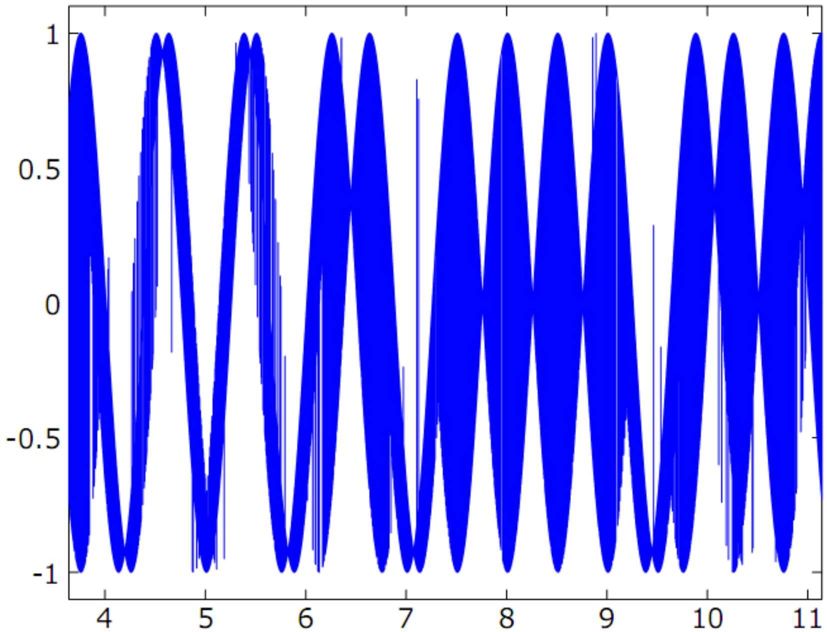


Simulated low-pass filter

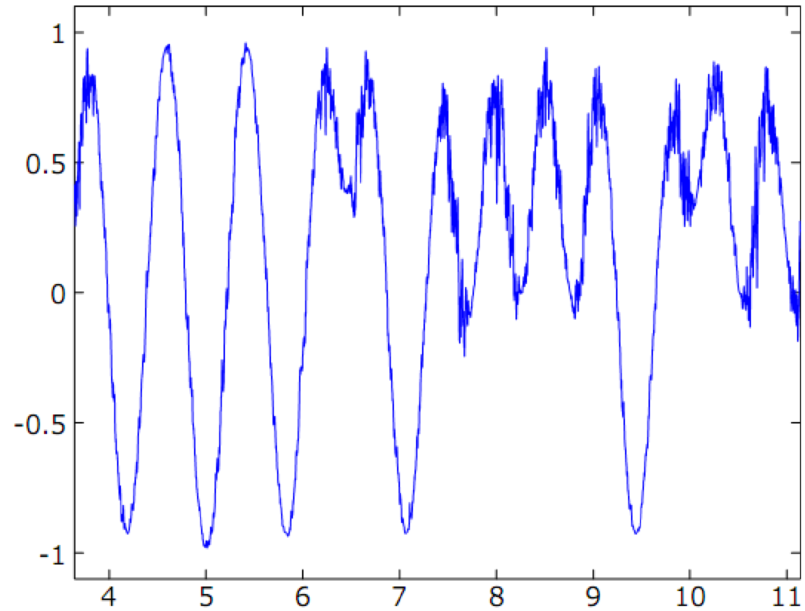


With (in-) appropriate filtering , one might obtain multiple periods that look a bit like e/2 and e/4.

Simulated Data



Simulated Low Pass Filter



Willett’s low pass filter has a time constant of 100 seconds

(This is not a complete explanation of Willett’s data)

Summary

- "Orthodox" explanation of the even-odd effect for 5/2 interferometer seems impossible

- "Reformed" theory (freezing qubit state) still looks unlikely – coupling to edge too strong.

- Likely in a regime where all couplings are large

  Expect to always see interference (no even-odd).
  Expect slips of π (qubit flips)
  Expect slips $\approx$ $\pm\pi/4$ ($\pm 5\,\pi/4$) for qp/qh addition

- Low pass filtering may obscure data

Phase slip measurements may be the cleanest way to demonstrate braiding statistics (7/3 and/or 5/2)

Quantum Hall Interferometry: Status and Outlook

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Thank You For Listening

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