Solving the AdS/CFT Y-system

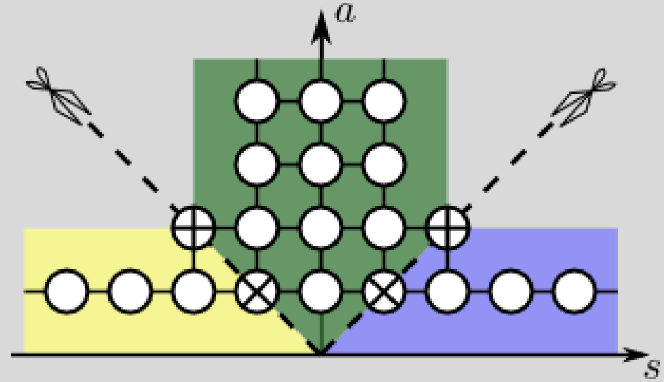
Dmytro Volin

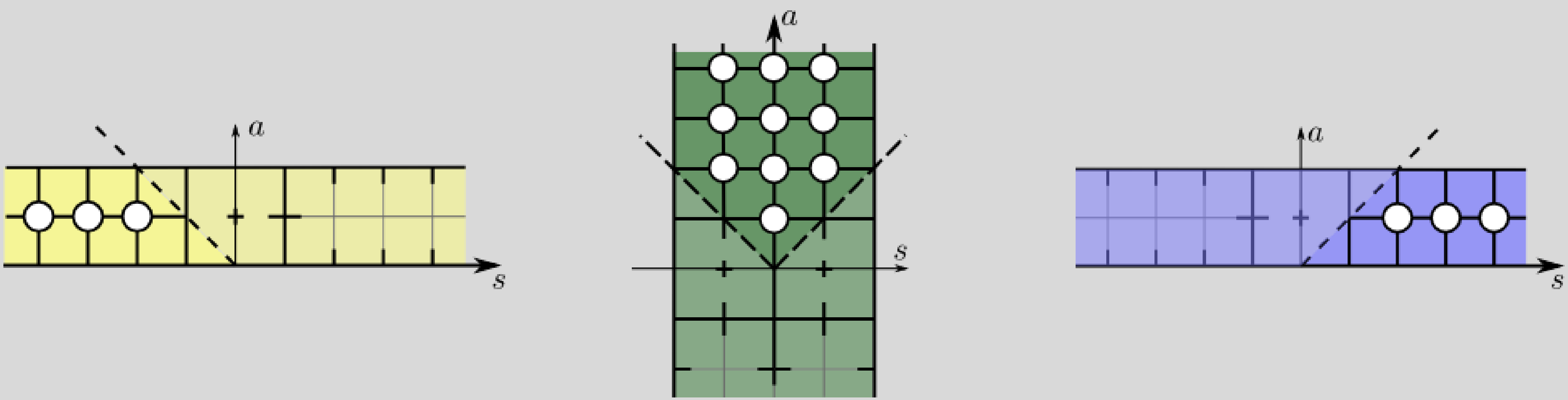
Nordita, Stockholm

IHP, 2011

1110.0562

N.Gromov,
V.Kazakov,
S.Leurent.
D.V.





Setup:

$\mathcal{N}=4$ SYM = IIB, $AdS_5 x S^5$

0 → ∞

$$g^2 = \frac{g_{YM}^2 N_c}{16\pi^2} = \frac{\lambda}{16\pi^2}$$

This is a talk about the spectral problem:

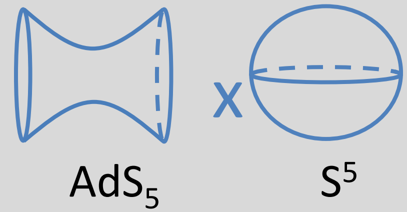
Conformal dimension of local operators = Energy of string states = ?

Can treat this problem at any coupling, because of integrability

Nature of integrability:

From the point of view of string we are dealing with the conventional integrability in 2d QFT.

(Free) String is a sigma-model on $\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}$



PSU(2,2|4) has $\mathbb{Z}_4$ grading,

$$h = h^0 \oplus h^1 \oplus h^2 \oplus h^3$$
$$[h^i, h^j] \subset h^{i+j}$$

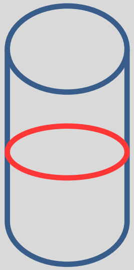
coset respects it

$$h^0 = so(1,5) \oplus so(5)$$

This allows us to construct the Lax connection -> classical integrability

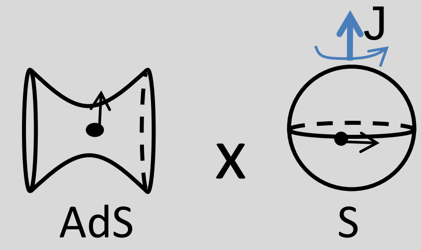
$$dA(u) + A(u) \wedge A(u) = 0$$

$$T = Tr\, Pe^{\int A(u)}$$



[Zakharov, Mikhailov]
[Bena, Polchinski, Roiban `03]

SU(2) PCM	AdS_5xS^5 String
• sigma model on: $\frac{SU(2)\times SU(2)}{SU(2)}$	string sigma model on: $\frac{PSU(2,2\|4)}{SO(1,4)\times SO(5)}$
• Excitations are over the vacuum	Excitations over BMN “vacuum”.



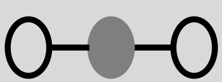
Symmetry

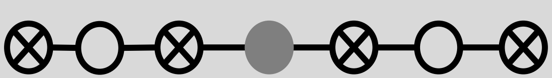
• SU(2) x SU(2) x Poincare • (unbroken)	$(PSU(2\|2)\times PSU(2\|2))\ltimes\mathbb{R}^3$ (PSU(2,2|4) is broken by choice of BMN vacuum)

Large volume description:

• Massive particles, • Factorized scattering • Mass is dynamically generated	• Massive particles, • Factorized scattering • Mass is present classically (cf. centrifugal force)

Asymptotic Bethe Ansatz solution:





SU(2) PCM	AdS_5xS^5 String

Symmetry

SU(2) x SU(2) x Poincare	$(PSU(2\|2) \times PSU(2\|2)) \ltimes \mathbb{R}^3$

Small volume description:

CFT	N/A

Weak coupling description:

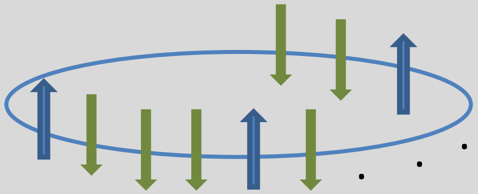
N/A

PSU(2,2|4) version of Heisenberg spin chain.

- It can be solved by the algebraic Bethe Ansatz which explicitly preserves the whole PSU(2,2|4) symmetry.
- Weak coupling means in particular **large** volume

$$X = \Phi_1 + i\Phi_2$$
$$Z = \Phi_3 + i\Phi_4$$

$$\mathrm{Tr}\, XZZZXZ\ldots XZZ$$



SU(2) PCM

$AdS_5 x S^5$ String

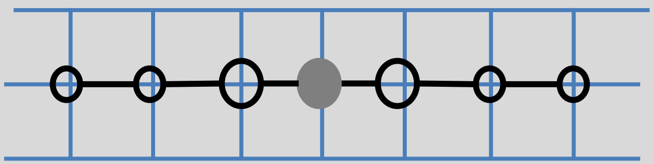
Symmetry

SU(2) x SU(2) x Poincare

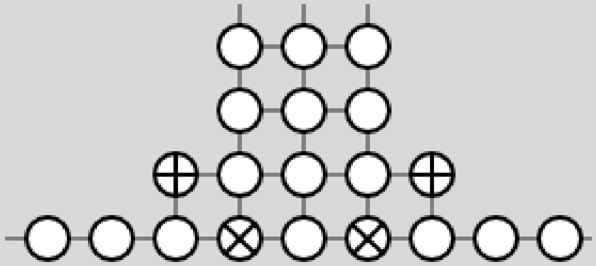
$$(PSU(2|2) \times PSU(2|2)) \ltimes \mathbb{R}^3$$

Finite volume description:

Y-system

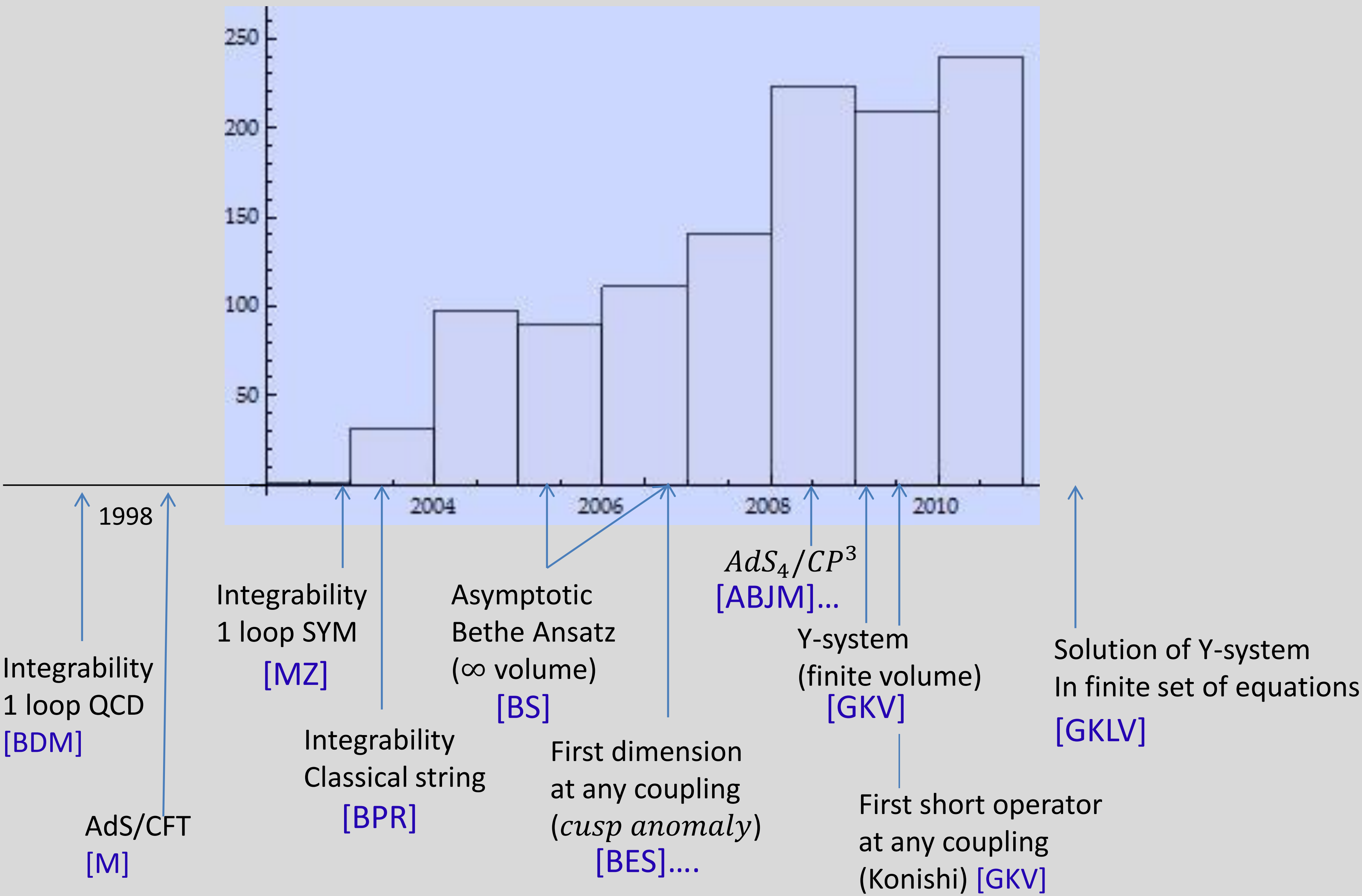


Y-system



Remarkably, PSU(2,2|4) symmetry reappears.

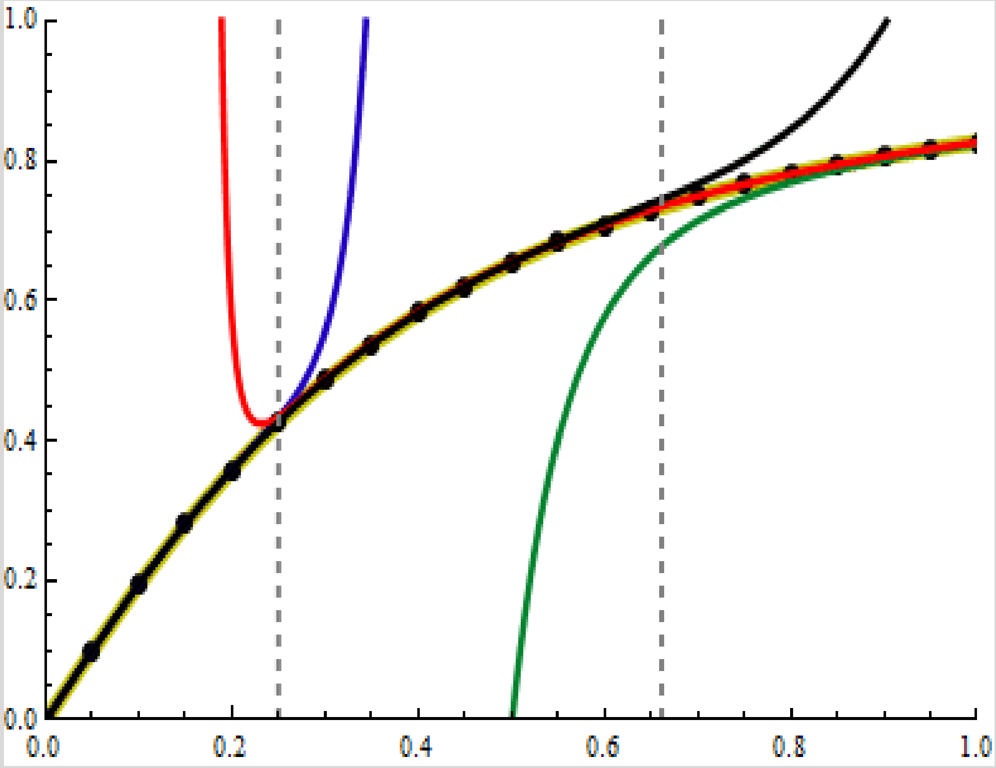
Scientific output: about 1000-2000 papers (1306 on the plot):



State of art explicit results for **infinite** volume case.

Cusp anomalous dimension $Tr\, Z\, D^S Z$

$$\Delta - S = f[g]\log S + \ldots, \quad S \to \infty$$



Weak coupling:

[Moch, Vermaseren, Vogt, 04]
[Lipatov et al., 04]
[Bern et al., 06]
[Cachazo et al., 06]

$$f[g] = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 - \left(\frac{584}{315}\pi^6 + 64\zeta(3)^2\right) g^8 + \ldots$$

[Beisert, Eden, Staudacher, 06]

Numerics: [Benna, Benvenuti, Klebanov, Scardicchio, 06]

Strong coupling:

[Gubser, Klebanov, Polyakov, 02] [Frolov, Tseytlin, 02] [Roiban, Tseytlin, 07]

$$f[g] = 4g - \frac{3\log 2}{\pi} - \frac{1}{4g}\frac{K}{\pi^2} - \ldots$$

[Klebanov et al, 06]
[Kotikov,Lipatov, 06]
[Alday et al, 07]
[Kostov, Serban, D.V., 07]
[Beccaria, Angelis, Forini, 07]

[Casteill, Kristjansen, 07]
[Belitsky, 07]

[Basso, Korchemsky, Kotanski, 07]
[Kostov, Serban, D.V., 08]

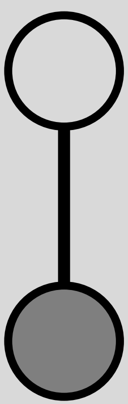
Nonperturbative corrections: [Basso, Korchemsky, 09]

Can we do the same for the finite volume case?

Simple example: SU(2) chiral Gross-Neveu model at finite volume

$$S_{GN} = \frac{1}{\lambda}\int d^2x\, \overline{\psi}_a i\partial\!\!\!/\psi^a + \frac{1}{2}\left(\left(\overline{\psi}_a\psi^a\right)^2 - \left(\overline{\psi}_a\gamma^5\psi^a\right)^2\right), \quad a = 1,2$$

- Asymptotically free theory
- Dynamically generated mass scale m
- At infinite volume: described by factorized scattering theory
- Particle content: fundamental representation of SU(2)
- Asymptotic Bethe Ansatz solution

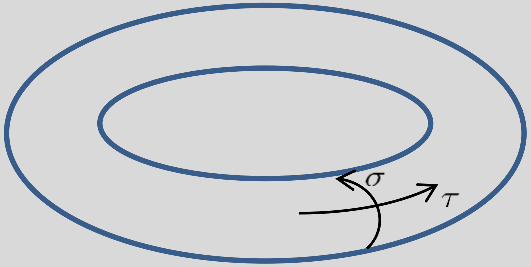


$$\prod_{k=1}^{M}\frac{\lambda_i - \theta_k + i/2}{\lambda_i - \theta_k - i/2} = -\prod_{j=1}^{K}\frac{\lambda_i - \lambda_j + i}{\lambda_i - \lambda_j - i}, \qquad i \in \overline{1,K}$$

$$e^{-imL sinh(\pi\theta_r)} = -\prod_{k=1}^{M} S_0(\theta_r - \theta_k)\prod_{j=1}^{K}\frac{\theta_r - \lambda_j + i/2}{\theta_r - \lambda_j - i/2}, \qquad r \in \overline{1,M}$$

For finite volume one can use TBA & exchange time and space coordinates

[Zamolodchikov]



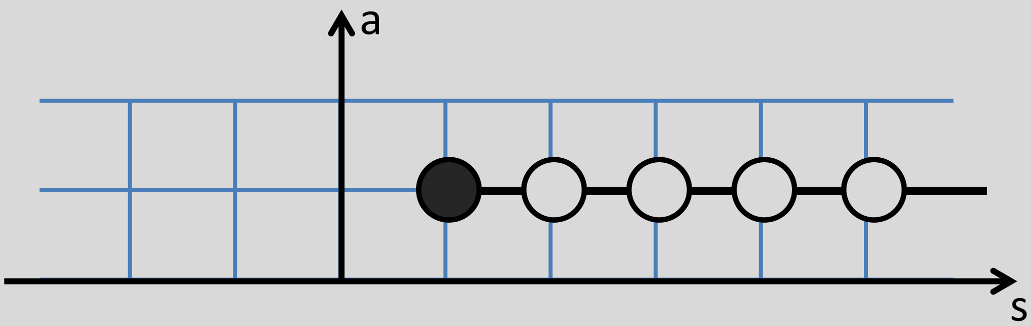
$$\sigma \simeq \sigma + L$$
$$\tau \simeq \tau + R$$

$$E_0[L] = -\lim_{R\to\infty} \frac{\log Z}{R} = \mathcal{F}[T = 1/L]$$

Minimization of free energy lead to TBA equations:

$$\log Y_{1,s} = \frac{1}{2\pi}\frac{1}{\cosh(\pi\theta)} * \log(1+Y_{1,s+1})(1+Y_{1,s-1}), \quad s>1$$

$$\log Y_{1,1} = -mL\cosh(\pi\theta) + \frac{1}{2\pi}\frac{1}{\cosh(\pi\theta)} * \log(1+Y_{1,2})$$

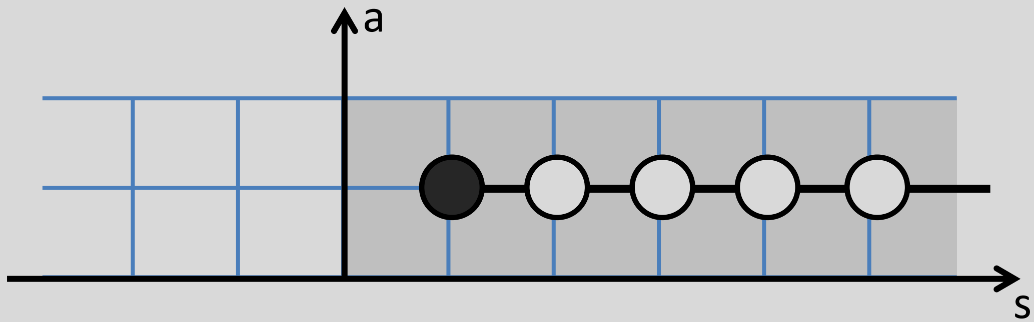


- This is an infinite set of nonlinear integral equations.
- From these equations we can derive Y-system:

$$Y_{1,s}^{+}Y_{1,s}^{-} = (1+Y_{1,s+1})(1+Y_{1,s-1})$$

- Reverse derivation requires additional input

$$f^{\pm} = f(\theta \pm \tfrac{i}{2})$$



$$Y^+_{1,s}Y^-_{1,s} = (1+Y_{1,s+1})(1+Y_{1,s-1})$$

- Y-system is equivalent to Hirota system:

$$Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}$$

$$T^+_{a,s}T^-_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s}$$

Gauge freedom:

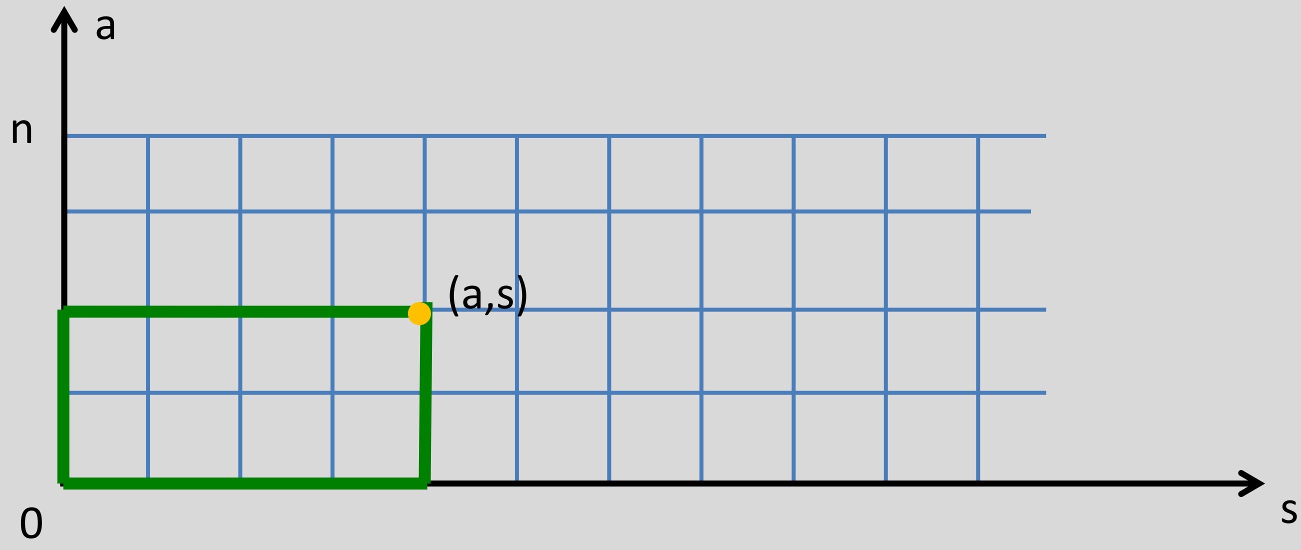
$$T_{a,s} \to g_1^{[a+s]} g_2^{[a-s]} g_3^{[-a+s]} g_4^{[-a-s]} T_{a,s}$$

$$g^{[\pm a]} \equiv g[u \pm ia/2]$$

- T-functions satisfy exactly the same system as the transfer matrices of SU(2) XXX spin chain!
- Indeed, chiral GN is a special limit of inhomogeneous SU(2) XXX.

Character identities vs Hirota equation:

SU(n)



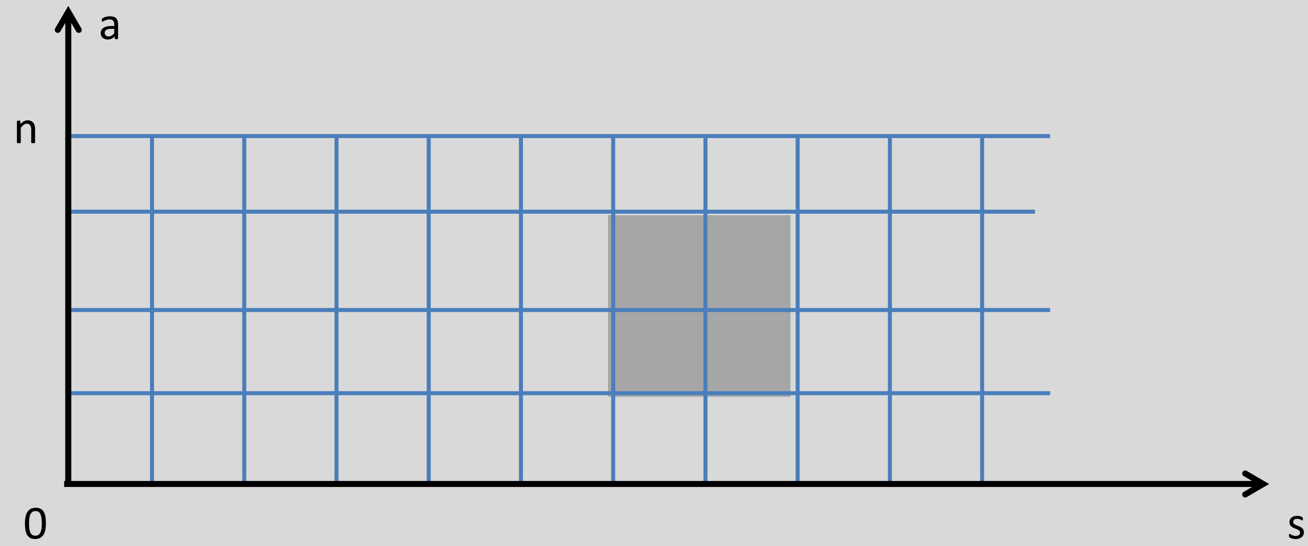
$$\chi_{a,s}\chi_{a,s} = \chi_{a,s+1}\chi_{a,s-1} + \chi_{a+1,s}\chi_{a-1,s}$$

Hirota equation:

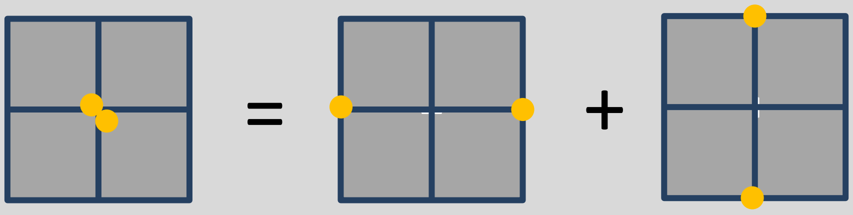
$$T^{+}_{a,s}T^{-}_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s} \qquad f^{\pm} = f(u \pm \tfrac{i}{2})$$

Character identities vs Hirota equation:

SU(n)



$$\chi_{a,s}\chi_{a,s} = \chi_{a,s+1}\chi_{a,s-1} + \chi_{a+1,s}\chi_{a-1,s}$$



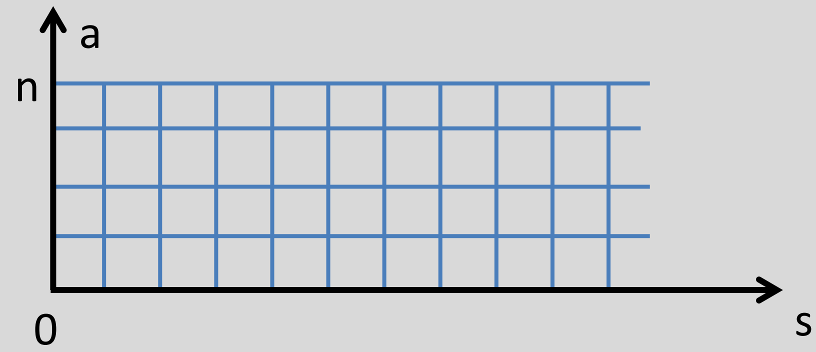
Hirota equation:

$$T^{+}_{a,s}T^{-}_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s}$$

$$f^{\pm} = f(u \pm \tfrac{i}{2})$$

Wronskian solution:

$$T^+_{a,s}T^-_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s}$$



Generically need 2n+2 functions, 4 can be fixed by gauge freedom

$$Q_1,\ Q_2,\ \ldots Q_n$$
$$P_1,\ P_2,\ \ldots P_n$$
$$Q_\emptyset,\ P_\emptyset$$

$$Q \equiv \sum_{i=1}^{n} Q_i e^i \qquad P \equiv \sum_{i=1}^{n} P_i e^i$$

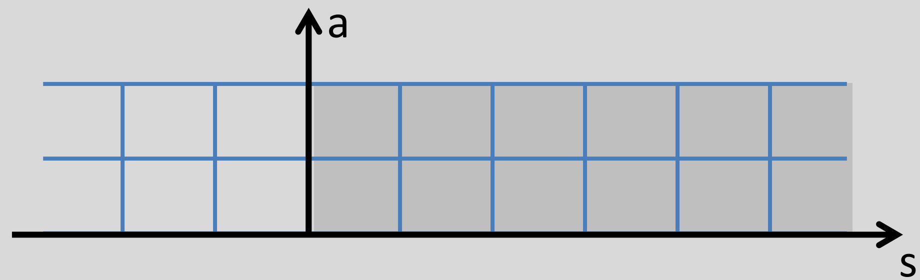
Define fused products: $$Q_{(k)} = \frac{Q^{[k-1]} \wedge Q^{[k-3]} \wedge \ldots Q^{[1-k]}}{Q_\emptyset^{[k-2]} Q_\emptyset^{[k-4]} \ldots Q_\emptyset^{[2-k]}} \qquad Q_{(0)} \equiv Q_\emptyset$$

The most general solution
Of Hirota equation is:

$$T_{a,s} = Q_{(a)}^{[+n/2+s]} \wedge P_{(n-a)}^{[-n/2-s]}$$

Back to SU(2) XXX

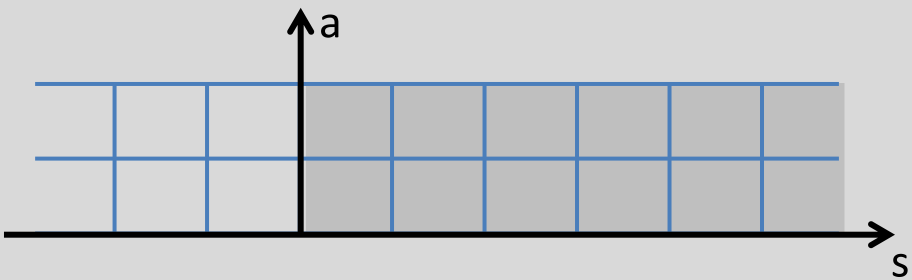
$$T_{a,s} = Q_{(a)}^{[+n/2+s]} \wedge P_{(n-a)}^{[-n/2-s]}$$



- Those are identity transfer matrices: $Q_\emptyset = P_\emptyset = 1$
- $T_{1,s} = Q^{[+1+s]} \wedge P^{[-1-s]}$, $T_{1,-1} = 0$ → $Q = P$
- $$\begin{aligned} T_{0,s} &= Q_{(2)}^{[-1-s]} \\ T_{1,s} &= Q^{[s+1]} \wedge Q^{[-s-1]} \\ T_{2,s} &= Q_{(2)}^{[1+s]} \end{aligned}$$
- Transfer matrix in trivial representation: $T_{1,0} = \theta^L = Q_1^+ Q_2^- - Q_1^- Q_2^+$
- Transfer matrix in fundamental representation (enters Baxter equation): $T_{1,1} \equiv T$

Baxter equation as Plucker identity

$$T_{0,s} = Q_{(2)}^{[-1-s]}$$
$$T_{1,s} = Q^{[s+1]} \wedge Q^{[-s-1]}$$
$$T_{2,s} = Q_{(2)}^{[1+s]}$$



$$T_{1,-1} = 0 \qquad T_{1,0} = \theta^L = Q^+ \wedge Q^- \qquad T_{1,1} \equiv T = Q^{++} \wedge Q^{--}$$

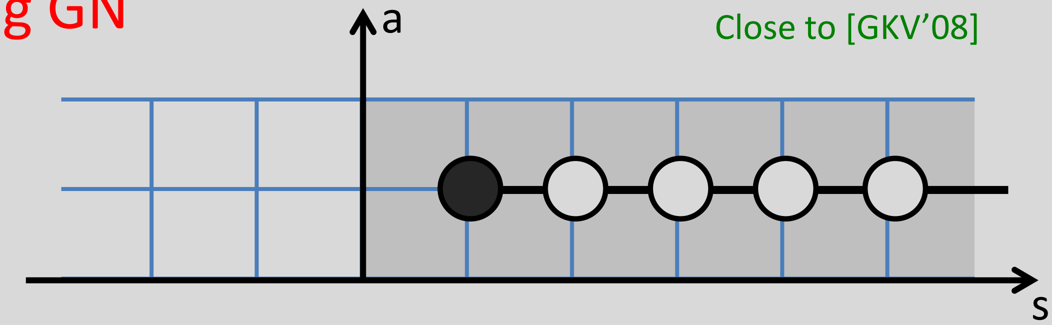
Plucker identity: $(Q^{[+2]} \wedge Q^{[-2]}) \cdot Q = (Q^{[+2]} \wedge Q) \cdot Q^{[-2]} + (Q \wedge Q^{[-2]}) \cdot Q^{[+2]}$

Baxter equation: $T\,Q = (\theta^L)^+ Q^{[-2]} + (\theta^L)^- Q^{[+2]}$

- From regularity of T we derive Bethe equations -> Fixes Q -> Solve spectrum of XXX
- Analyticity input: It was important in our considerations that all functions are polynomials and that we know $T_{1,0} = \theta^L$

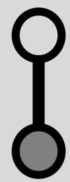
Single integral equation solving GN

Presented derivation is Close to [GKV'08]



Suppose that

- Hirota is satisfied
- $Y_{1,1} \simeq e^{-mL\cosh(\pi\theta)}$
- At large volume Bethe Ansatz is reproduced



- Consider large volume first, black node drops out, we get XXX spin chain again, but:

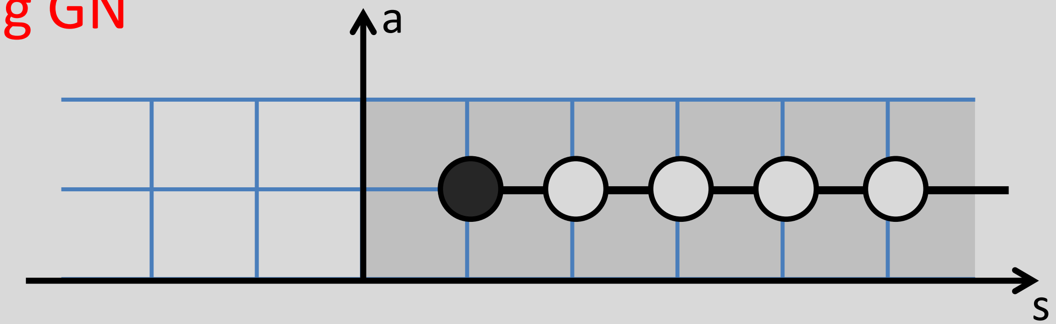
$$
\begin{aligned}
T_{0,s+1} &= Q_{(2)}^{[-1-s]} \\
T_{1,s+1} &= Q^{[s+1]} \wedge Q^{[-s-1]} \\
T_{2,s+1} &= Q_{(2)}^{[+1+s]}
\end{aligned}
\qquad
T_{1,1} = \prod_{r=1}^{M} (\theta - \theta_r) = Q_1^+ Q_2^- - Q_1^- Q_2^+
$$

- For simplicity consider only vacuum, then M=0, $T_{1,1} = 1, Q_2 = Q = 1, Q_1 = -iu$.

- Finite volume is considered as deformation. We can choose a gauge such that $Q_2 = 1$ always.

Single integral equation solving GN

$$
\begin{aligned}
T_{0,s} &= \bar{Q}_1^{[-s-1]} - \bar{Q}_1^{[-s+1]} \\
T_{1,s} &= Q_1^{[s]} + \bar{Q}_1^{[-s]} \\
T_{2,s} &= Q_1^{[+s+1]} - Q_1^{[+s-1]}
\end{aligned}
$$



- We can parameterize Q_1 as

$$Q_1 = -iu + \int_{-\infty}^{+\infty} \frac{dv}{2\pi i} \frac{\rho(v)}{v-u}, \qquad Im(u) > 0$$

$$\bar{Q}_1 = +iu - \int_{-\infty}^{+\infty} \frac{dv}{2\pi i} \frac{\rho(v)}{v-u}, \qquad Im(u) > 0$$

- $T_{1,0} = \rho$

$$Y_{1,1}^{+} Y_{1,1}^{-} = (1 + Y_{1,2}) = \frac{T_{1,2}^{+} T_{1,2}^{-}}{T_{2,2} T_{0,2}} \qquad Y_{1,1} = \frac{\rho T_{1,2}}{T_{2,1} T_{0,1}}$$

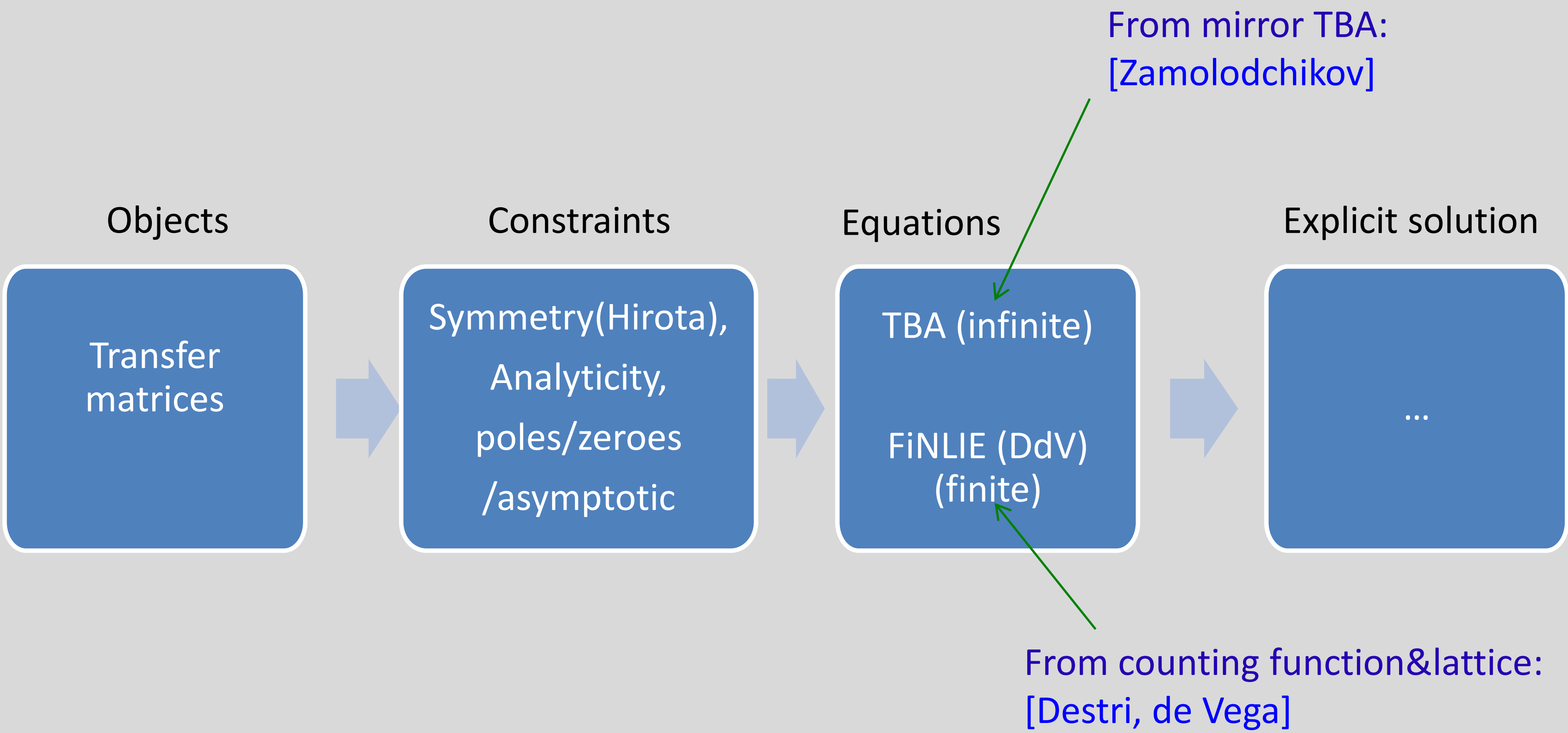
$$\rho = e^{-mL\cosh(\pi\theta)} T_{2,1} T_{0,1} \exp\left[-\frac{1}{2\pi} \frac{1}{\cosh(\pi\theta)} * \log T_{2,2} T_{0,2} \right]$$

Conclusion

In the chiral GN case it is enough to know:

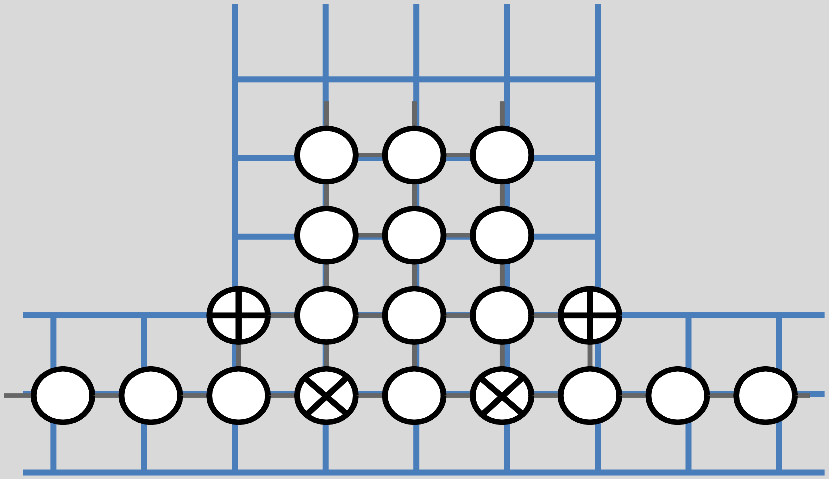
- Hirota equation,
- Analytical structure of T's,
- Asymptotical behavior at large volume.

Bootstrap for finite volume, relativistic case:



Observation: in AdS/CFT case, by applying approach of Zamolodchikov, Hirota equation was derived!

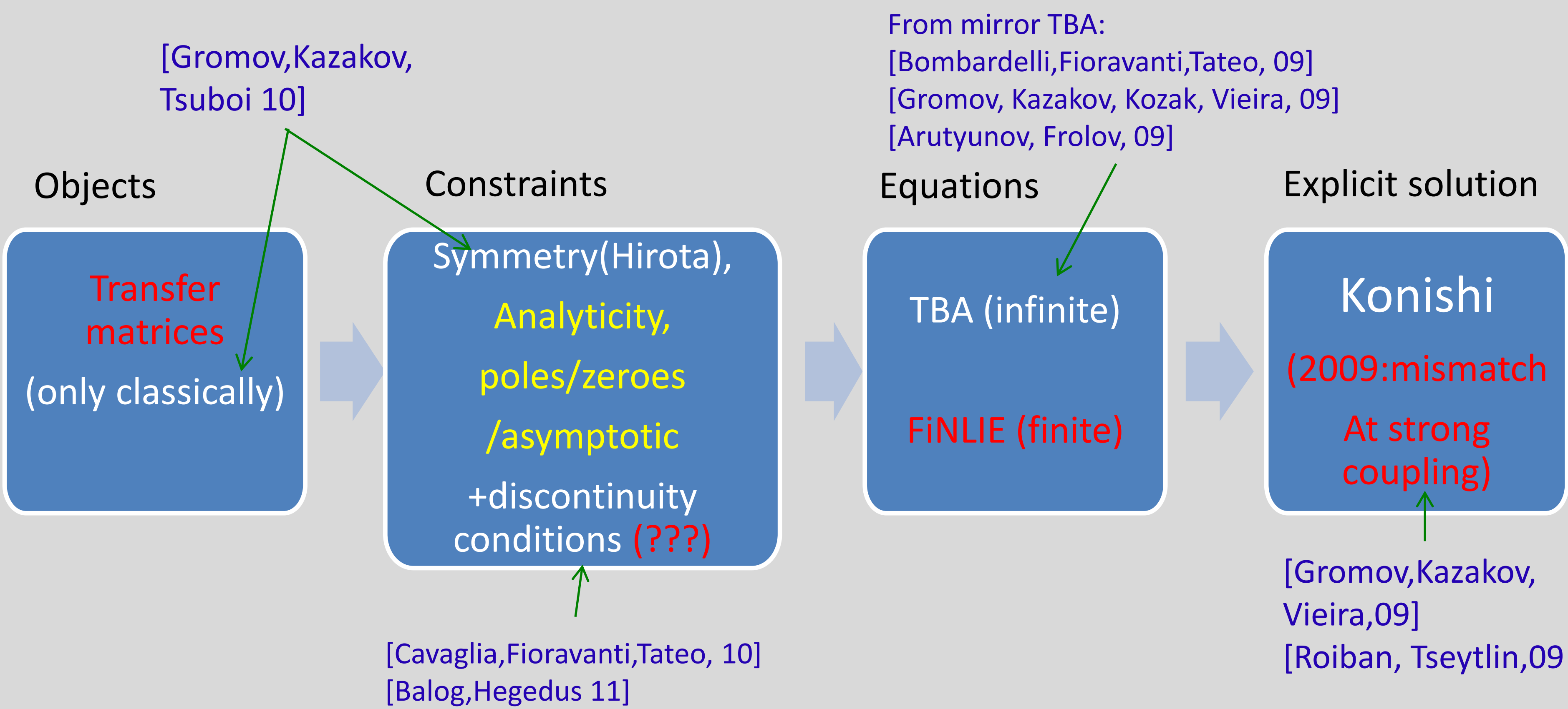
From mirror TBA:
[Bombardelli,Fioravanti,Tateo, 09]
[Gromov, Kazakov, Kozak, Vieira, 09]
[Arutyunov, Frolov, 09]



$$T^+_{a,s}T^-_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s}$$

We believe therefore that there is a kind of bootstrap program which will solve spectral problem, more directly, elegantly and rigorously than mirror TBA approach.

Finite volume bootstrap programm, for AdS/CFT (status prior 2011):



Strong coupling of the sl(2) sector (Konishi et al):

Konishi

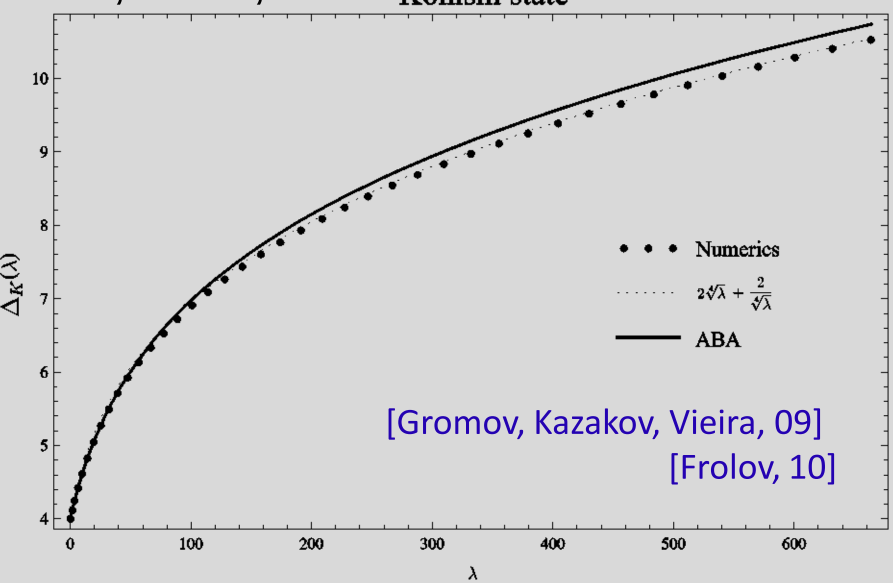
(2011:agreement)

2009: $\Delta[g] = 2\sqrt{4\pi g} + \frac{2 \text{ or } 1}{\sqrt{4\pi g}}$ [Gromov, Kazakov, Vieira, 09] [Roiban, Tseytlin, 09]

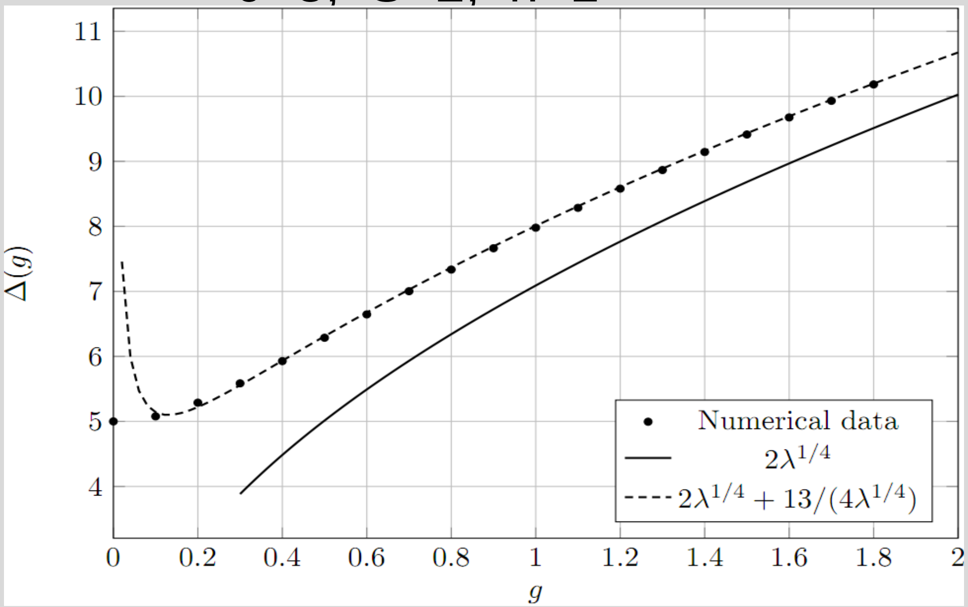
02/2011: Analytical derivations (using yet to be proved assumptions):
[Gromov,Shenderovich,Serban, D.V.]
[Roiban,Tseytlin]
[Masuccato,Valilio]

$$\Delta - J - S = \lambda^{1/4}\sqrt{2S} + \frac{1}{\lambda^{1/4}}\frac{2J^2+S(3S-2)}{4\sqrt{2S}} + \dots$$

J=2, S=2, n=1 Konishi state

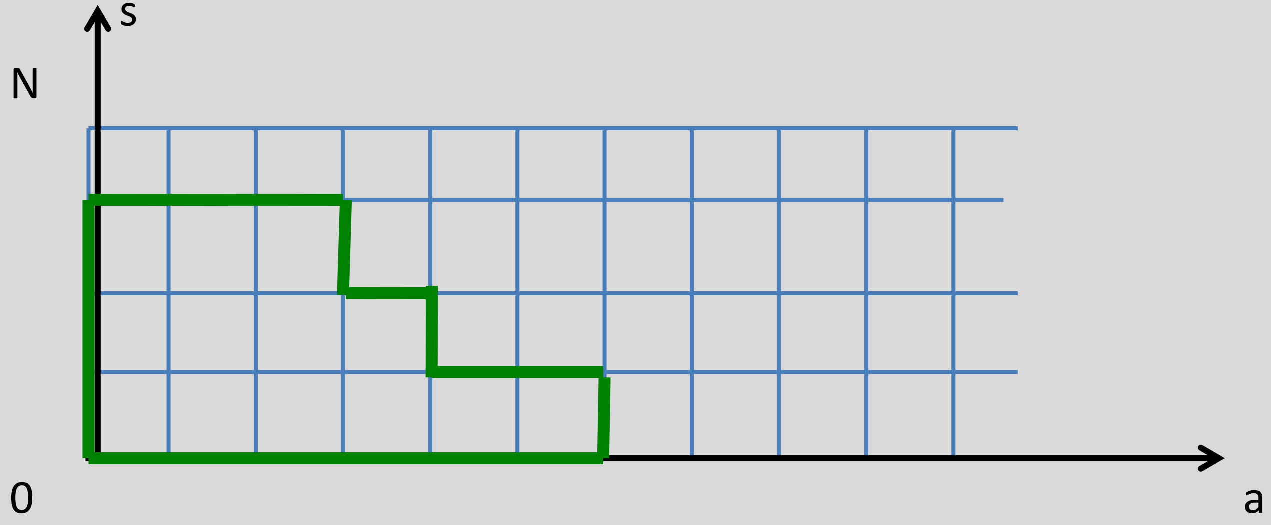


J=3, S=2, n=1



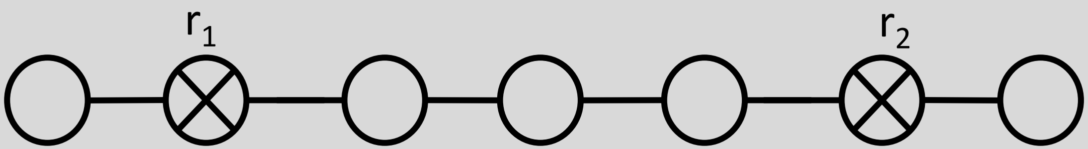
Implementation of symmetry:

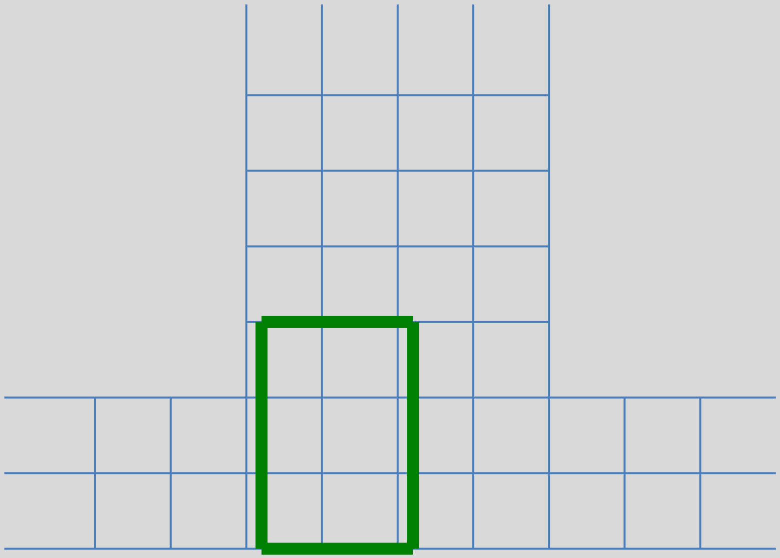
SU(N)

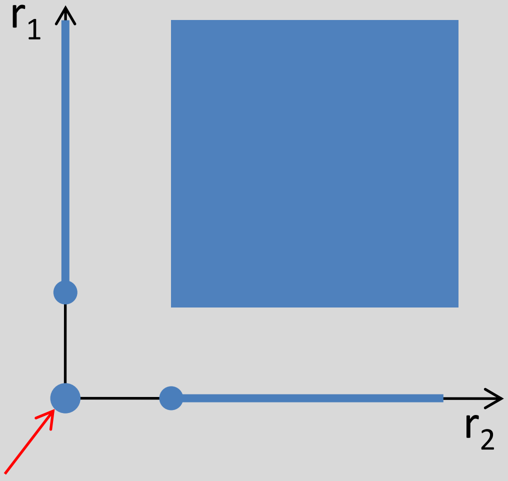


[Gromov, Kazakov, Tsuboi,'10] : mapping Young tableaux inside T-hook to highest weight irreps.

$$SU(2,2|4) \supset SU(2) \oplus SU(4) \oplus SU(2) \oplus U(1) \oplus U(1)$$

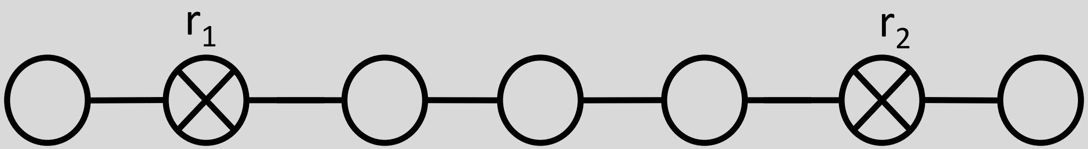


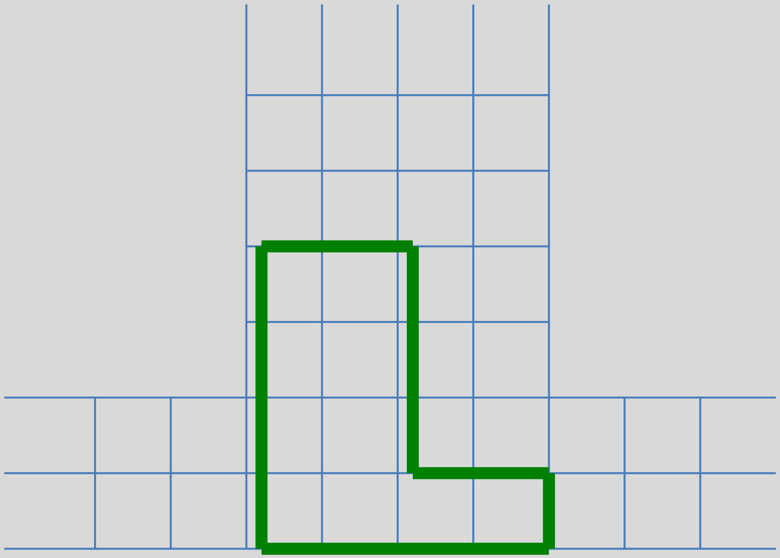


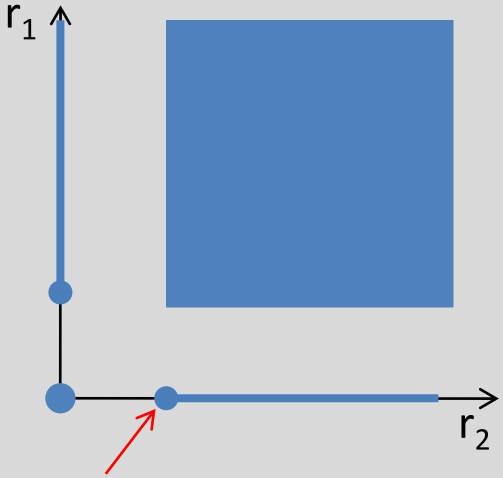


[Gromov, Kazakov, Tsuboi,'10] : mapping Young tableaux inside T-hook to highest weight irreps.

$$SU(2,2|4) \supset SU(2) \oplus SU(4) \oplus SU(2) \oplus U(1) \oplus U(1)$$







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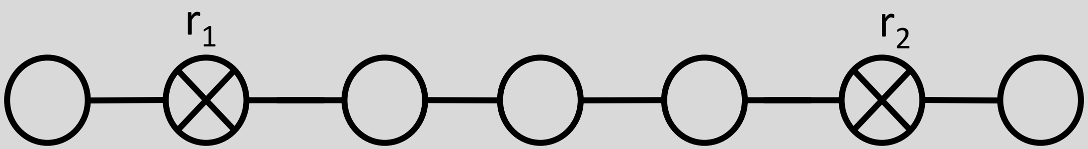
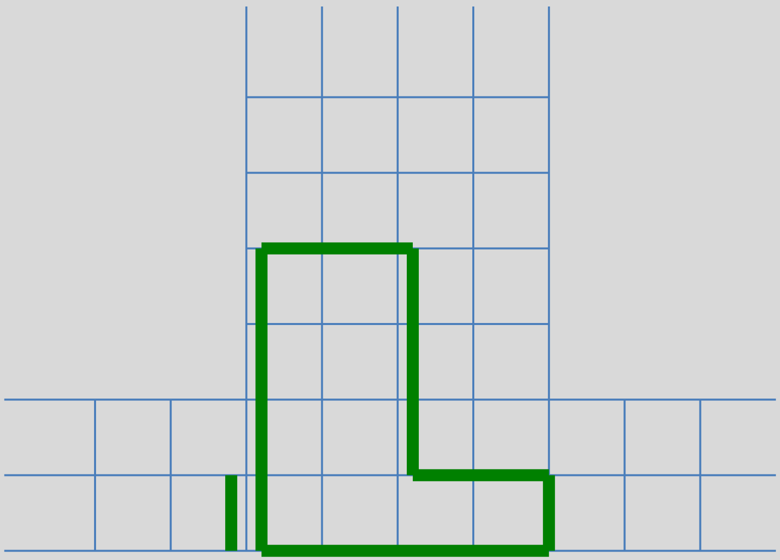
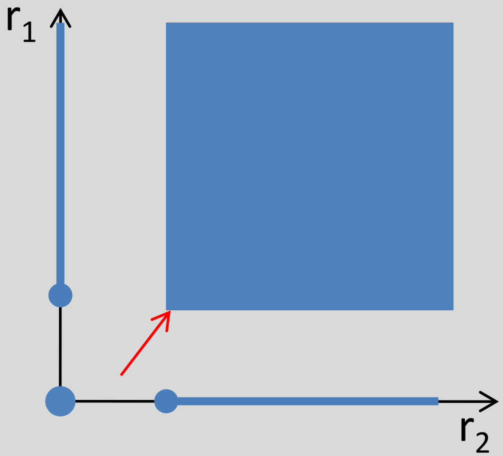




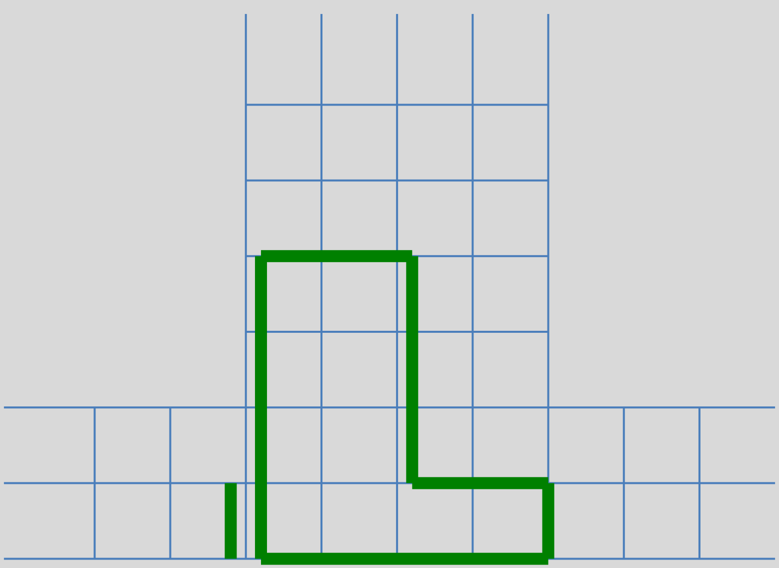


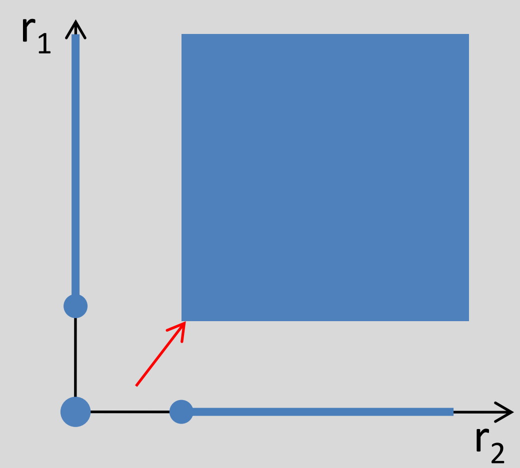
T-hook and classification of Unitary highest weight representations:

[Gromov, Kazakov, Tsuboi,’10] : mapping T-hook Young tableaux to unitary highest weight irreps.

Conjecture [D.V.’10] : T-hook classifies all unitary highest weight representations of SU(n,m|k)

- Proved for all subcases (m=0 or k=0)
- Agreees with Dobrev-Petkova for SU(2,2|4)

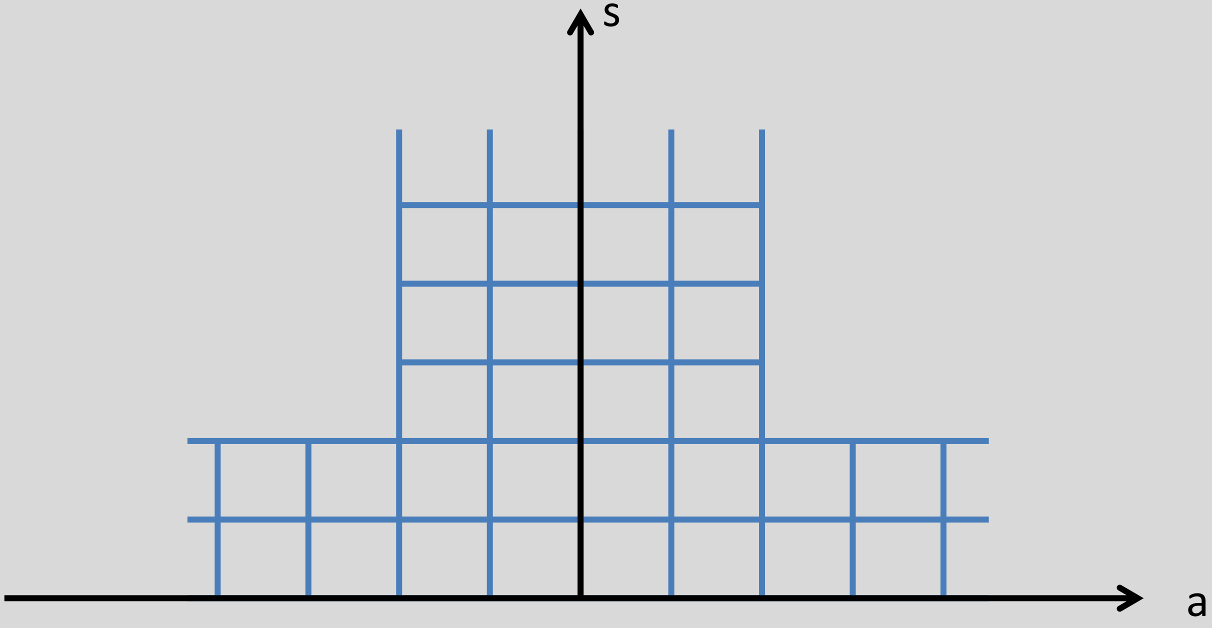




- Transfer matrix interpretation of T-functions exist only at strong coupling
  - classically (T=character):
  - first nontrivial quantum correction (new!)

Assuming that T's are transfer matrices,
i.e. physical objects – quantum version of PSU(2,2|4) characters,
what constraints apart Hirota can we put on them?

$PSU(2,2|4)$



$$\mathrm{T}_{n,2} = \mathrm{T}_{2,n},\ \mathrm{T}_{n,-2} = \mathrm{T}_{2,-n},\ n \geq 2$$

Unimodularity:

T is a physical gauge

$$\text{sdet} = \frac{Q_{\emptyset}^{+} Q_{\bar{\emptyset}}^{-}}{Q_{\emptyset}^{-} Q_{\bar{\emptyset}}^{+}}$$

[Gromov, Kazakov, Leurent, Tsuboi, '10]

Want to impose sdet=1

$$Q_{\emptyset} = 1 \longrightarrow Q_{\bar{\emptyset}}^{+} = Q_{\bar{\emptyset}}^{-}$$

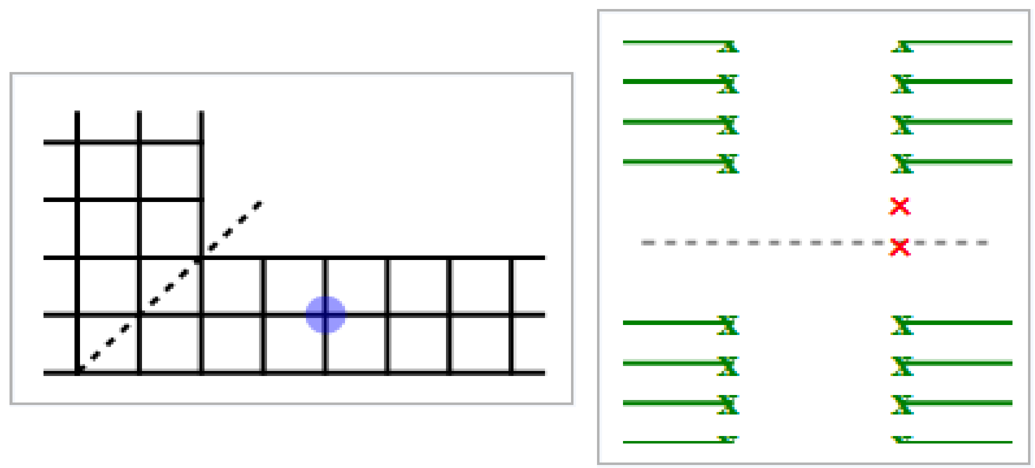
$$\mathbf{T}_{0,0} = Q_{\emptyset} Q_{\bar{\emptyset}} \longrightarrow \boxed{\mathbf{T}_{0,0}^{+} = \mathbf{T}_{0,0}^{-}}$$

$\mathbb{Z}_4$ Symmetry:

Hirota and branch points $$T^+_{a,s}T^-_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s}$$

$T_{a,s}$

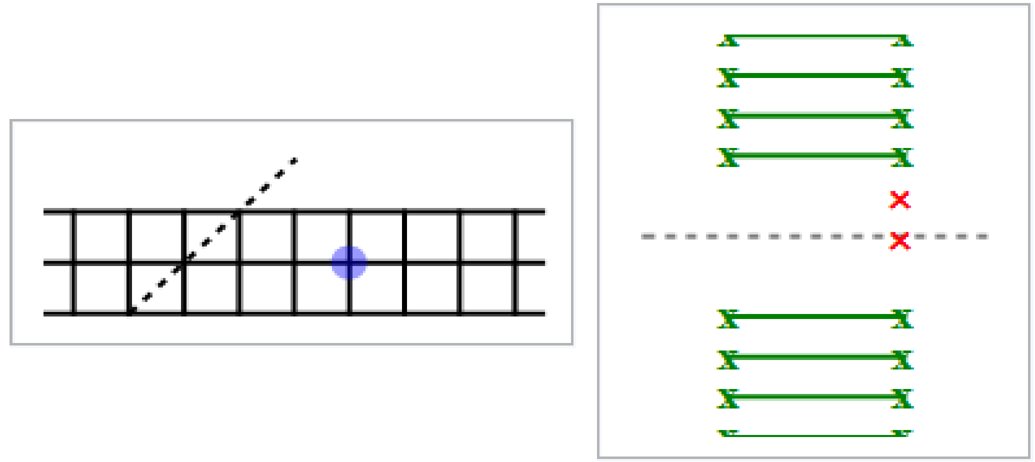
Mirror:



$\hat{T}_{a,s}$

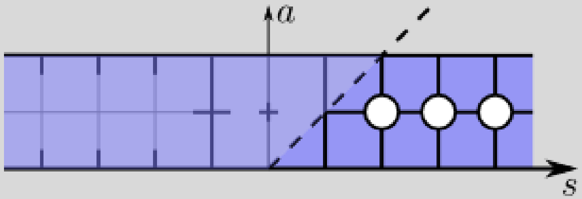
$\hat{T}_{1,0} = 0$

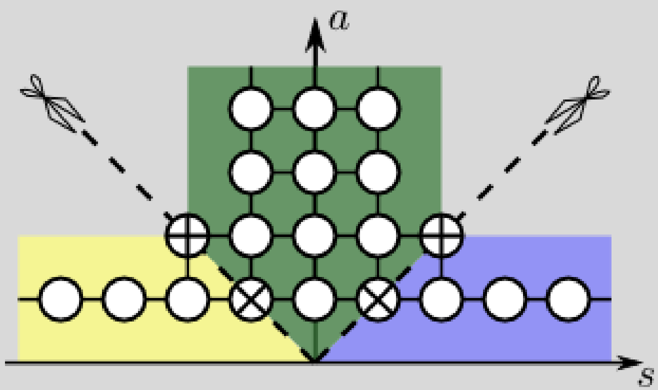
Magic:

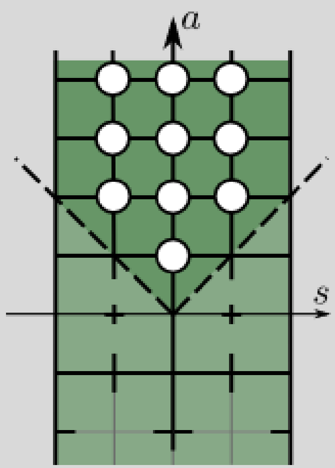


$\mathbb{Z}_4$ Symmetry:

Right band: $$\hat{\mathbb{T}}_{a,s} = (-1)^a \hat{\mathbb{T}}_{a,-s}$$







Upper band: $$\hat{\mathbf{T}}_{a,s} = (-1)^s \hat{\mathbf{T}}_{-a,s}$$

Complete set of properties of $\mathbf{T}$ and $\mathbb{T}$ gauges:

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V.Kazakov,
S.Leurent.
D.V.

Symmetry

$$\mathrm{T}_{n,2} = \mathrm{T}_{2,n},\ \mathrm{T}_{n,-2} = \mathrm{T}_{2,-n},\ n \geq 2$$

$$\mathbf{T}^{+}_{0,0} = \mathbf{T}^{-}_{0,0}$$ (Unimodularity)

$$\hat{\mathbf{T}}_{a,s} = (-1)^s \hat{\mathbf{T}}_{-a,s} \qquad (\mathbb{Z}_4)$$

$$\mathbb{T}_{a,s} = \mathbf{T}_{a,s} (\mathcal{F}^{[a+s]})^{a-2},\ \mathcal{F} \equiv \sqrt{\mathbf{T}_{0,0}}$$

$$\hat{\mathbb{T}}_{a,s} = (-1)^a \hat{\mathbb{T}}_{a,-s} \qquad (\mathbb{Z}_4)$$

Analyticity

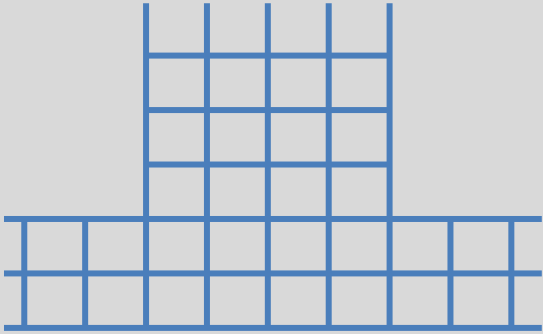
$$\mathbf{T}_{a,0} \in \mathcal{A}_{a+1}$$

$$\mathbf{T}_{a,\pm 1} \in \mathcal{A}_{a}$$

$$\mathbf{T}_{a,\pm 2} \in \mathcal{A}_{a-1}$$

No poles

Minimal # of zeroes



$$\mathbb{T}_{0,\pm s} = 1$$

$$\mathbb{T}_{1,\pm s} \in \mathcal{A}_{s}$$

$$\mathbb{T}_{2,\pm s} \in \mathcal{A}_{s-1}$$

Two cuts for $\mathbb{T}_{1,\pm s}$

No poles

1110.0562
N.Gromov,
V.Kazakov,
S.Leurent.
D.V.

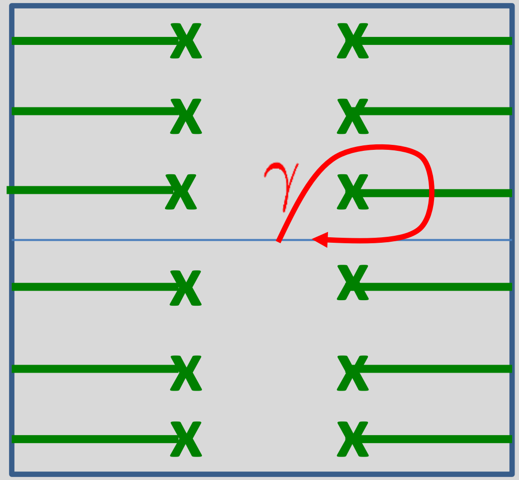
- Listed above properties (symmetry+analyticity) + correct large volume (asymptotic Bethe Ansatz) behavior uniquely fix solution of the Y-system = Hirota system

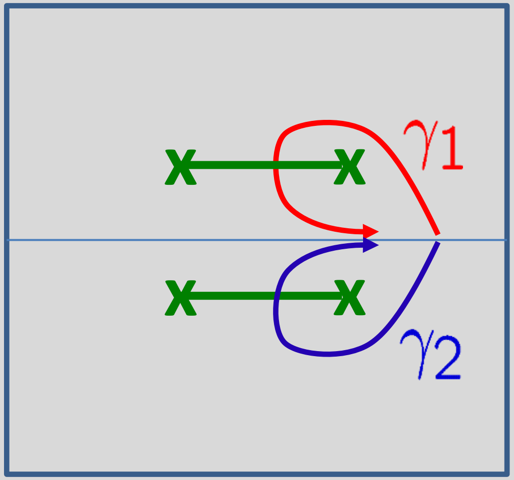
- In particular, these properties are equivalent to the TBA equations

Symmetry
(Hirota, det=1, Z4)
+Analyticity
+Poles/zeroes/asymptotics

- We also get a new way to extract energy from the T's….

Exact Bethe equations





$$Y_{1,0}^{\gamma}(u_j) = -1$$

$$\frac{\mathbb{T}_{1,1}^{\gamma_1}(u_j)}{\mathbb{T}_{1,1}^{\gamma_2}(u_j)} = -1$$

This is a condition for absence of singularities

In the physical **T**-gauge

New formula for the energy

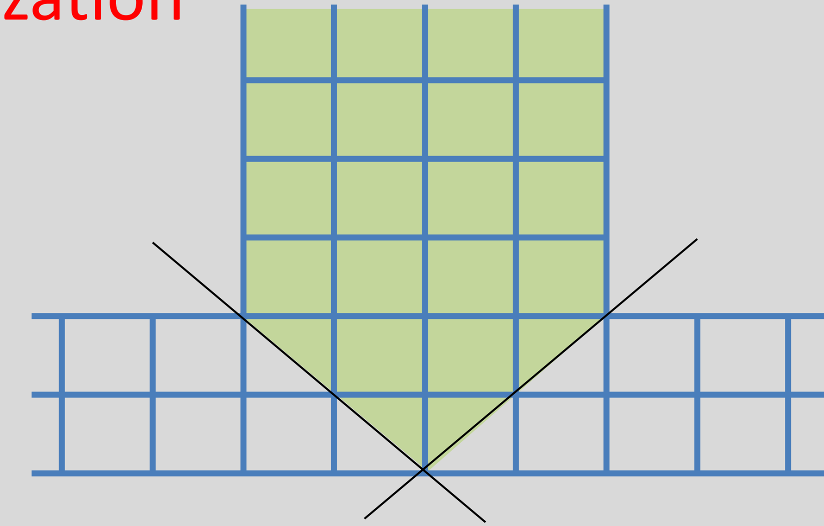
$$\partial_u \log \mathbf{T}_{1,0} \simeq \frac{2E}{u}, \quad u \to \infty$$

1110.0562
N.Gromov,
V.Kazakov,
S.Leurent.
D.V.

FiNLIE !

- Instead of infinite set of TBA equations we propose a FiNLIE

Upper band, Wronskian paremeterization



$$q_\emptyset \quad q_1 \quad q_2 \quad q_3 \quad q_4$$

$$p_\emptyset \quad p_1 \quad p_2 \quad p_3 \quad p_4$$

$$q \equiv \sum_{i=1}^{4} q_i e^i \qquad p \equiv \sum_{i=1}^{4} p_i e^i$$

$$q_{(2)} \equiv \frac{q^{+} \wedge q^{-}}{q_\emptyset} \qquad q_{(3)} \equiv \frac{q^{++} \wedge q \wedge q^{--}}{q_\emptyset^{+} q_\emptyset^{-}} \qquad q_{(4)} \equiv \frac{q^{+++} \wedge q^{+} \wedge q^{-} \wedge q^{---}}{q_\emptyset^{++} q_\emptyset q_\emptyset^{--}}$$

$$T_{a,s} = q_{(2-s)}^{[+a]} \wedge p_{(2+s)}^{[-a]}$$

- Due to reality of T-functions p-s can be expressed through complex conjugation of q-s
- Need to define q-s
- Initial basis:

$q_{\emptyset}$

q_1 q_2 q_3 q_4

q_{12} q_{13} q_{14} q_{23} q_{24} q_{34}

q_{123} q_{124} q_{134} q_{234}

q_{1234}

- Due to reality of T-functions p-s can be expressed through complex conjugation of q-s
- Need to define q-s
- Alternative basis:

$$q_{\emptyset}$$

$$q_1 \quad q_2 \quad q_3 \quad q_4$$

$$q_{12} \quad q_{13} \quad q_{14} \quad q_{23} \quad q_{24} \quad q_{34}$$

$$q_{123} \quad q_{124} \quad q_{134} \quad q_{234}$$

$$q_{1234}$$

- Due to reality of T-functions p-s can be expressed through complex conjugation of q-s
- Need to define q-s
- Alternative basis, use gauge freedom and LR symmetry to simplify:

$$\prod_{k=1}^{M}(u-\tilde{u}_k)$$

$q_\emptyset$

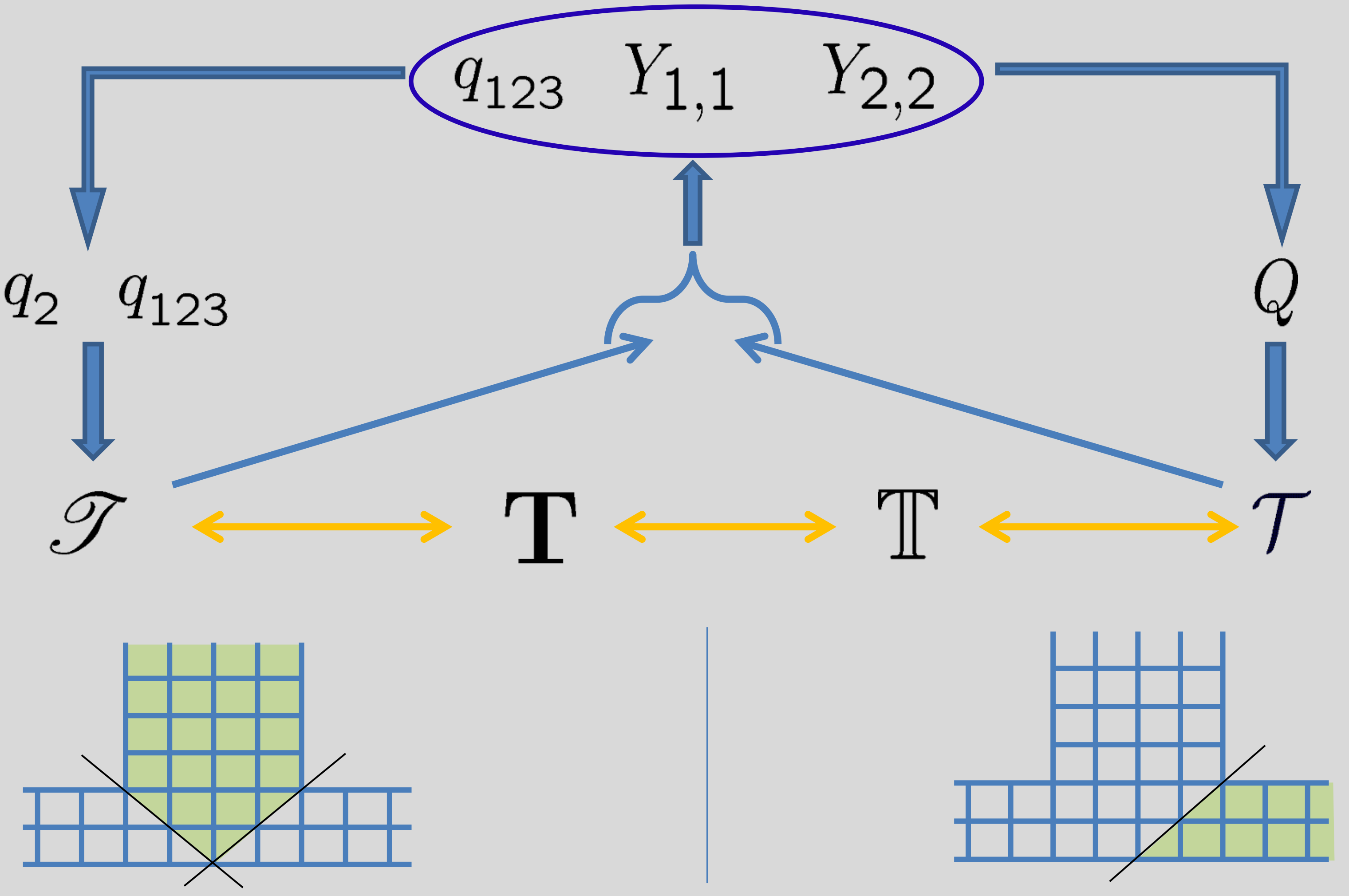
1 $\quad q_2 \quad q_3 \quad q_4$

$q_{13} \quad q_{14} \quad q_{23} \quad q_{24} \quad q_{34}$

$q_{123} \quad q_2 q_{123} \quad q_{134} \quad q_{234}$

q_{1234}

Closing system of equations



Complete system of equations:

Right band:

$$Q(u) = -iu + \int_{-2g}^{2g} \frac{dv}{2\pi i} \frac{\rho(v)}{v-u}$$

$$\mathcal{T}_{0,s} = 1 \quad \mathcal{T}_{1,s} = Q^{[+s]} - Q^{[-s]}$$

$$\mathcal{T}_{2,s} = (Q^{[+s+1]} - Q^{[+s-1]})(Q^{[-s+1]} - Q^{[-s-1]})$$

$$\frac{1+Y_{1,1}}{1+\frac{1}{Y_{2,2}}} = \frac{\mathcal{T}_{1,1}^{+}\mathcal{T}_{1,1}^{-}\mathcal{T}_{2,3}}{\mathcal{T}_{2,2}^{+}\mathcal{T}_{2,2}^{-}}$$

Upper band:

$$T_{a,s} = q_{(2-s)}^{[+a]} \wedge p_{(2+s)}^{[-a]}$$

$$q_1 = 1 \qquad -q_2 = P_{M-1} + \int_{-2g}^{2g} \frac{dv}{2\pi i} \frac{\rho_2(v)}{(v-u)} + \int_{-\infty}^{\infty} dv \left(q_3^{[+0]} \bar{q}_4^{[-0]} + q_4^{[+0]} \bar{q}_3^{[-0]} \right) \qquad q_{12} = \prod_{k=1}^{M} (u - \tilde{u}_k)$$

$$q_\emptyset q_{ij} = q_i^+ q_j^- - q_j^+ q_i^-$$

$$q_i q_{ijk} = q_{ij}^+ q_{ik}^- - q_{ik}^+ q_{ij}^-$$

$$\frac{1+Y_{2,2}}{1+\frac{1}{Y_{1,1}}} = \frac{\mathscr{T}_{2,2}^{+}\mathscr{T}_{2,2}^{-}\mathscr{T}_{1,0}}{\mathscr{T}_{1,1}^{+}\mathscr{T}_{1,1}^{-}\mathscr{T}_{3,2}}$$

Gluing equaitons:

$$\log Y_{1,1} = \log\left(-\frac{R^{(+)}\mathcal{T}_{1,2}}{R^{(-)}\mathscr{T}_{2,1}}\right) + \mathcal{Z} *_{\mathcal{Z}} \log\frac{\mathscr{T}_{1,0}}{\mathcal{Q}^+\mathcal{Q}^-} - \frac{1}{2}(\mathcal{Z}_1 + \mathcal{K}_1) * \log\frac{\mathscr{T}_{0,0}}{\mathcal{Q}^2} - \mathcal{K}_1 * \log\frac{\mathcal{T}_{1,1}}{\mathscr{T}_{1,1}}$$

$$\log q_{123} = \log\Lambda + \log\frac{\hat{h}}{f^+} + \frac{1}{2}\Psi * \rho_c \quad \log\hat{h} = -\frac{L+2}{2}\log\hat{x}(u) + \mathcal{Z}\hat{*}\log\left(\frac{\left(f\bar{f}\sqrt{\mathscr{T}_{0,0}}\right)^{+}(Y_{1,1}Y_{2,2}-1)}{\rho}\right) \quad \log f^2 = \Psi^+ * \rho_b$$

$$\rho_b(v) = \begin{cases} \log\dfrac{\mathscr{T}_{1,0}^2}{\mathscr{T}_{0,0}^{+}\mathscr{T}_{0,0}^{-}Y_{1,1}^2Y_{2,2}^2} & , \ |v| < 2g \\ \log\dfrac{\mathscr{T}_{1,0}^2}{\hat{\mathscr{T}}_{0,0}^{+}\hat{\mathscr{T}}_{0,0}^{-}} & , \ |v| > 2g \end{cases} \qquad \rho_c = \log\frac{\mathscr{T}_{0,0}^{-}}{\mathscr{T}_{0,0}^{+}}\left(\frac{\mathscr{T}_{1,1}^{+}}{\mathcal{T}_{1,1}^{+}}\frac{\mathcal{T}_{1,1}^{-}}{\mathscr{T}_{1,1}^{-}}\right)^2$$

- We explicitly checked equation numerically for the Konishi state and got a complete agreement with the TBA approach.

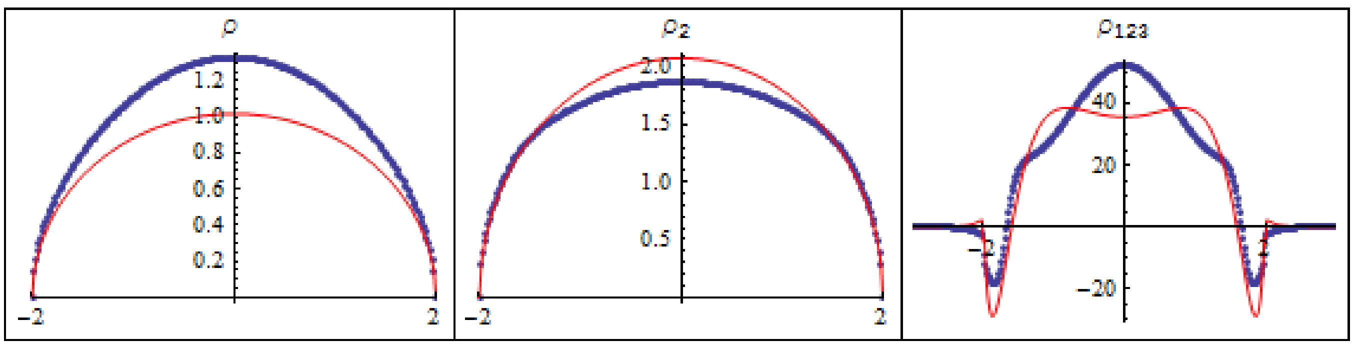
Data from Black box output

Presentation

In[93]:= `SolutionForKonishi[] // FullSimplify`

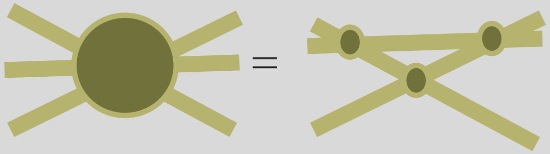
g=1.

Out[93]=

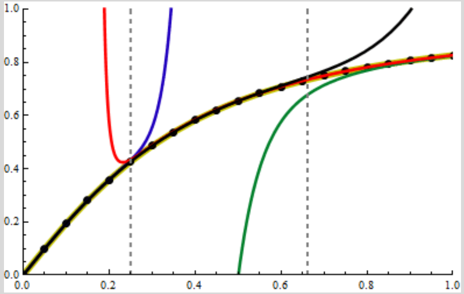


Discussion

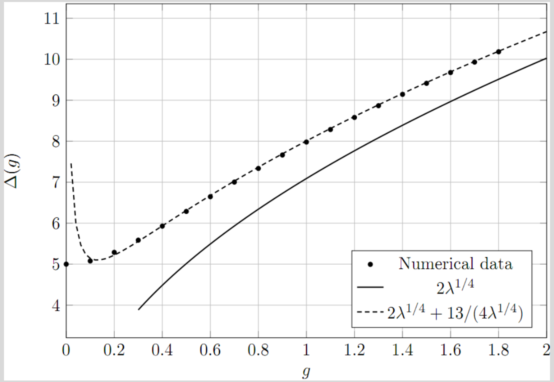
- Infinite volume case was solved by infinite volume bootstrap, based on old idea about factorization of the transfer matrix.



- Cusp anomalous dimension can be efficiently computed at any coupling

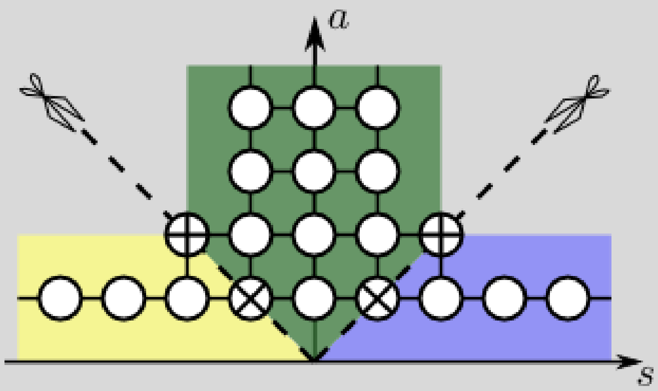


- First computations available for finite volume (e.g. Konishi), though not most efficient and not systematic.



Discussion

- An important advance in bootstrap program for finite volume. Based on Hirota dynamics.



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N.Gromov,
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D.V.

$$T^{+}_{a,s}T^{-}_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s}$$

Mirror

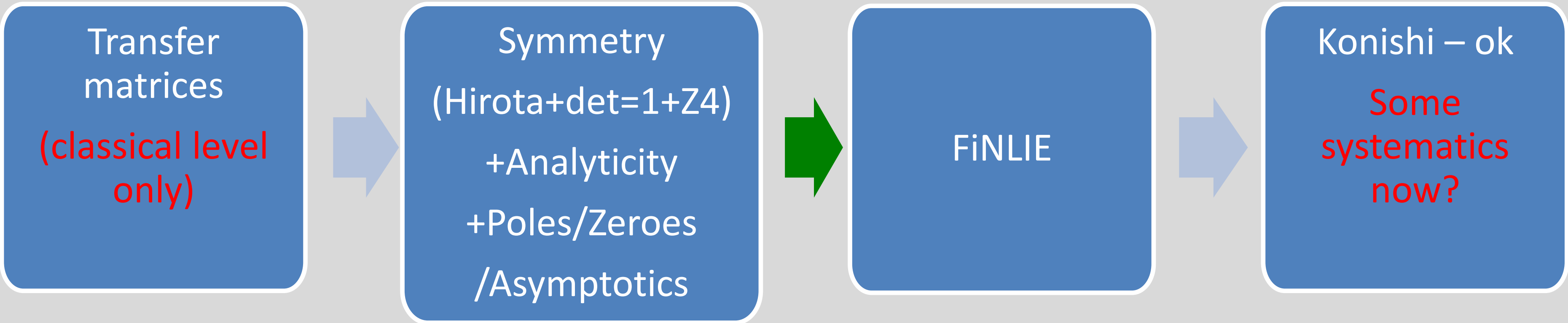
Magic

U(2,2|4) + Det=1 + $\mathbb{Z}_4$

+ analyticity + large volume explicitly = solution of the spectral problem

Discussion

- An important advance in bootstrap program for finite volume. Based on Hirota dynamics.



Approaching now to the systematic study:

- Weak coupling (e.g transcendentality structure)
- Strong copuling (asymptotic? Borel summable?)
- BFKL?

Need to define transfer matrices and Q-operators at weak coupling!

Need to quantize transfer matrices, need to define Q-operators at strong coupling

Z4 symmetry is not used to its full power, can simplify more FiNLIE
(reduce to only finite support densities)