Conformal loop models and beyond

J.Dubail, J.L. Jacobsen & H.Saleur

IPhT, Saclay - LPT ENS, Paris

"Physics in the Plane" - Les Houches - March 1st, 2010

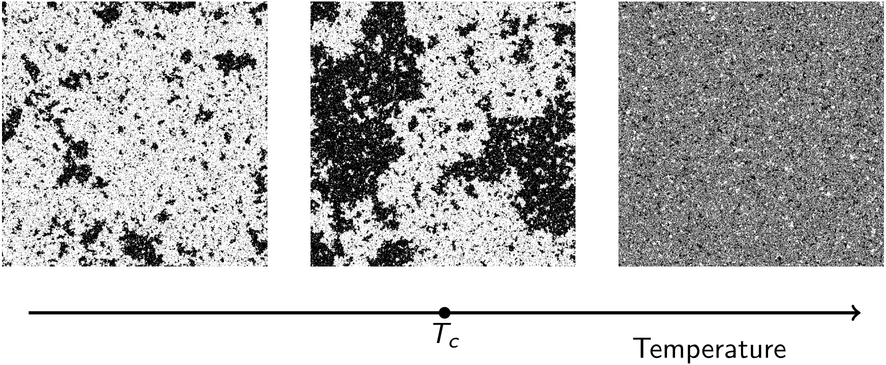
Conformal Field Theory in Statistical Physics

and geometry at criticality

CFT: basic ideas

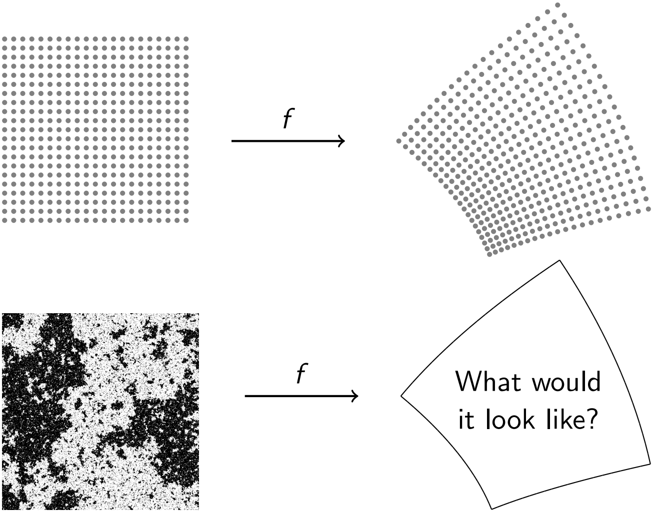
Starting point: Renormalization Group picture of a statistical model.

Example: Ising model in zero magnetic field.



Scaling symmetry at a critical point (RG fixed point).

CFT: basic ideas



The CFT toolbox

- The rich algebraic structure of CFT (after Belavin, Polyakov, Zamolodchikov, Kac, Fuchs, Feigin, Rocha-Caridi, . . .) puts strong constraints on the possible scaling limits of statistical models.

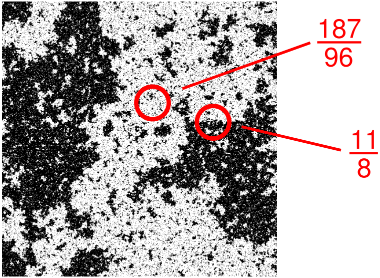
The CFT toolbox

- The rich algebraic structure of CFT (after Belavin, Polyakov, Zamolodchikov, Kac, Fuchs, Feigin, Rocha-Caridi, . . .) puts strong constraints on the possible scaling limits of statistical models.
- Coulomb gas allows to cook up arguments that relate lattice models to field theory.

The CFT toolbox

Nice geometrical results have been derived.

- exact fractal dimensions of clusters, or cluster boundaries, in critical percolation, Ising model, SAW,... (after Duplantier, Saleur, ...)



- exact formulae for geometrical observables: Cardy's formula, Schramm's formula, ...

Some famous achievements of CFT

These geometrical results are exact.

However, the link between field theory and geometrical objects (such as cluster boundaries) is not straightforward.

The SLE approach

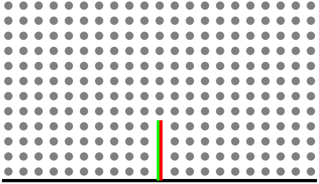
A new approach initiated by Oded Schramm in 2000:

(Stochastic) Schramm-Löwner Evolution

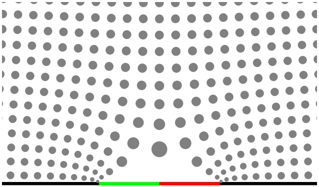
Further developed by Lawler, Schramm & Werner.

What is SLE?

- The Löwner part (Löwner chains)



$$z \mapsto g(z)$$

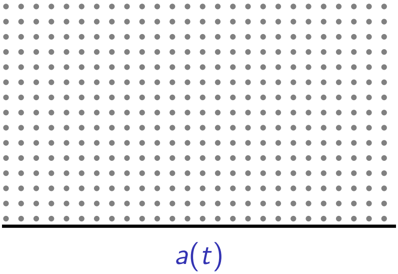


$g(z) = \sqrt{z^2 + 1}$ is a conformal mapping

What is SLE?

- The Löwner part (Löwner chains)

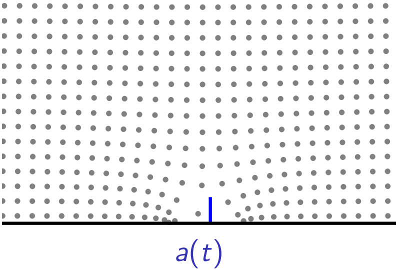
Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



What is SLE?

- The Löwner part (Löwner chains)

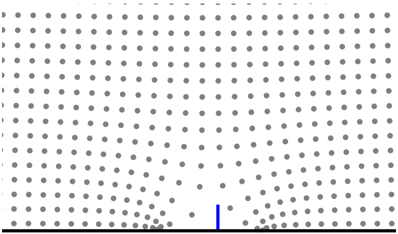
Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$

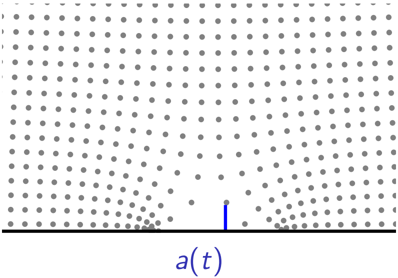


$a(t)$

What is SLE?

- The Löwner part (Löwner chains)

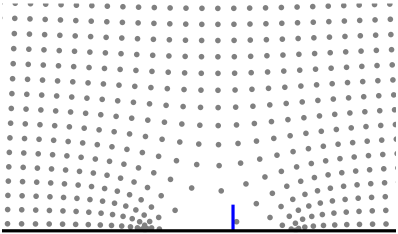
Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$

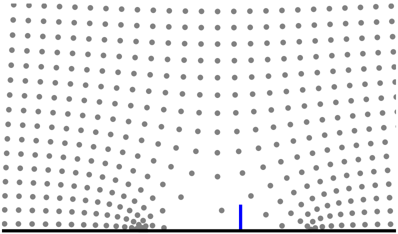


$a(t)$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$

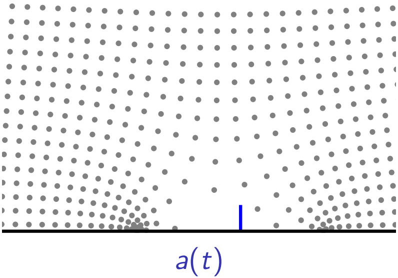


$a(t)$

What is SLE?

- The Löwner part (Löwner chains)

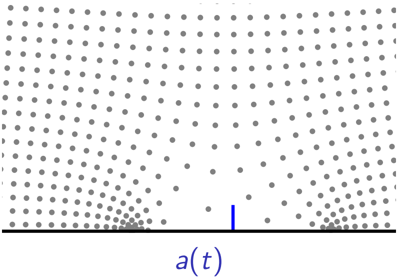
Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



What is SLE?

- The Löwner part (Löwner chains)

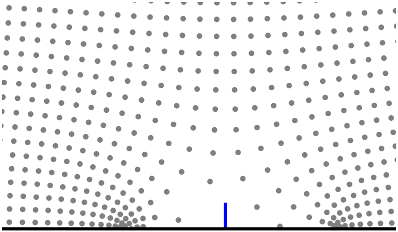
Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$

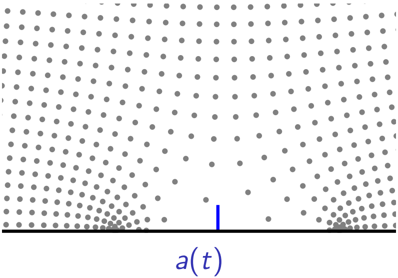


$a(t)$

What is SLE?

- The Löwner part (Löwner chains)

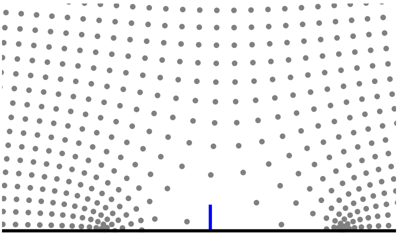
Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$

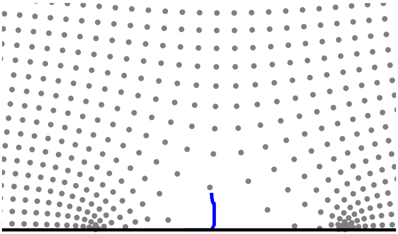


$a(t)$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

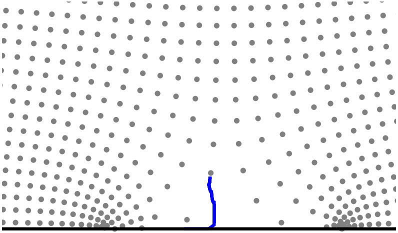
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

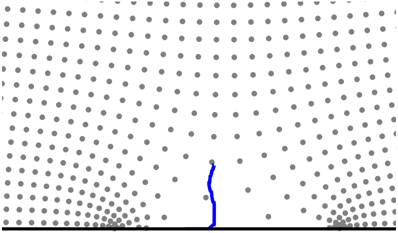
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

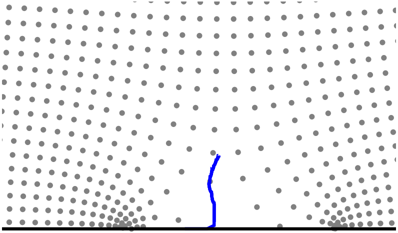
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

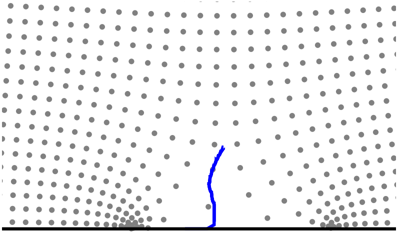
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

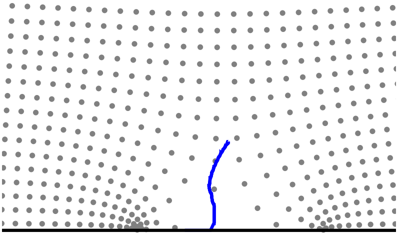
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

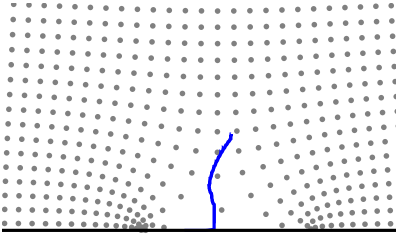
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

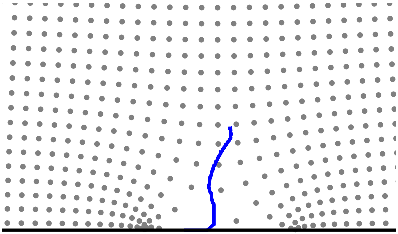
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

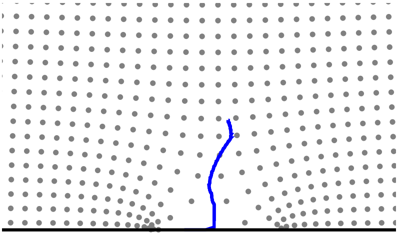
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

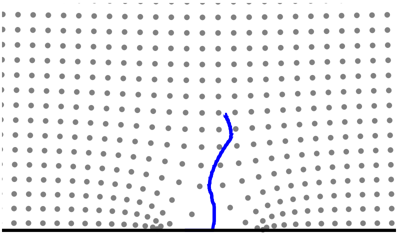
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

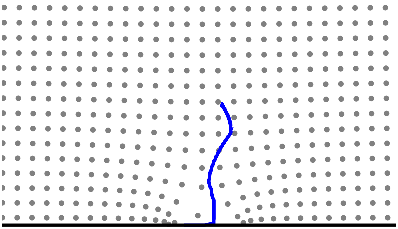
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

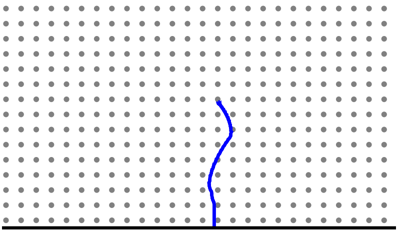
One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Löwner part (Löwner chains)

Now introduce an infinitesimal version $dg(z)$ and apply it inductively. The slit is allowed to move continuously: $a(t)$



$a(t)$

One can describe a curve by the Löwner chain

$$g_t = dg \circ dg \circ dg \circ \cdots \circ dg$$

What is SLE?

- The Schramm part

  if the Löwner trace is an interface in a statistical model at a critical point then

  a(t) should be a Brownian motion

$$a(t) = \sqrt{\kappa} B_t$$

- This defines SLE_κ.

What is SLE?

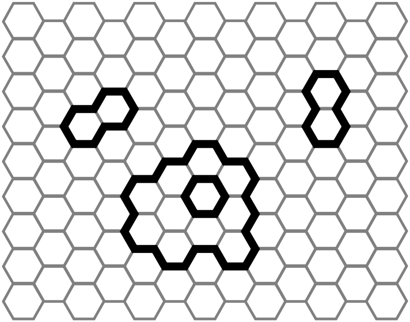
- SLE is designed to describe geometrical objects in the scaling limit.
- It does not involve field theory: computations with SLE boil down to stochastic calculus.
- SLE allows mathematical proofs of many results obtained previously by physicists . . .

What is SLE?

- SLE is designed to describe geometrical objects in the scaling limit.
- It does not involve field theory: computations with SLE boil down to stochastic calculus.
- SLE allows mathematical proofs of many results obtained previously by physicists . . .
  when the scaling limit of the lattice model is proved to be conformally invariant (work of Smirnov & al.)

Lattice Loop Models

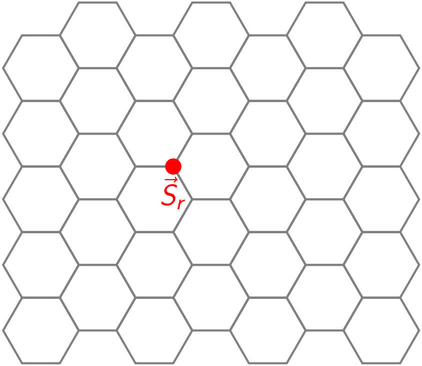
What is a lattice loop model?



Boltzmann weight of a configuration

$$x^{\#\ \mathrm{links}}\ n^{\#\ \mathrm{loops}}$$

Loops in the $O(n)$ model



$O(n)$ spins

$\vec{S}_r \in \mathcal{S}^{n-1}$ (n-dimensional)

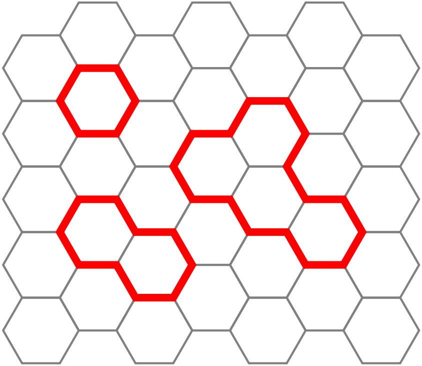
Normalization :

- $\mathrm{Tr} 1 = 1$
- $\mathrm{Tr} S_r^a S_r^b = \delta_{ab}$
- $\mathrm{Tr} S_r^a = \mathrm{Tr} S_r^a S_r^b S_r^c = 0$

Partition function

$$Z = \mathrm{Tr} \prod_{<rr'>} \left(1 + x \vec{S}_r . \vec{S}_{r'}\right)$$

Loops in the $O(n)$ model



$O(n)$ spins

$\vec{S}_r \in \mathcal{S}^{n-1}$ (n-dimensional)

Normalization :

- $\mathrm{Tr} 1 = 1$
- $\mathrm{Tr} S_r^a S_r^b = \delta_{ab}$
- $\mathrm{Tr} S_r^a = \mathrm{Tr} S_r^a S_r^b S_r^c = 0$

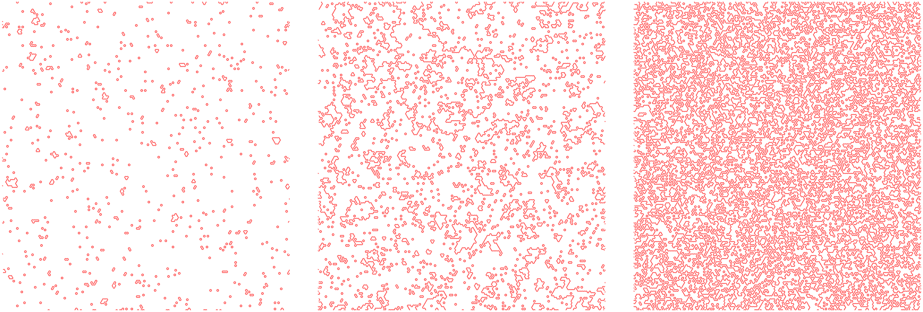
Partition function

$$
\begin{aligned}
Z &= \mathrm{Tr} \prod_{<rr'>} \left(1 + x \vec{S}_r . \vec{S}_{r'}\right) \\
&= \sum_{\text{loop conf.}} x^{\#\text{bonds}} n^{\#\text{loops}}
\end{aligned}
$$

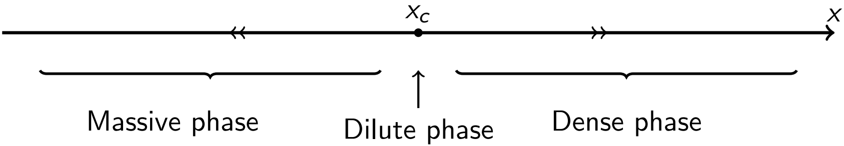
Scaling Limits of Loop Models

The O(n) loop model $\quad -2 < n \leq 2$

Massive | Dilute | Dense



RG flow:



The O(n) loop model $-2 < n \leq 2$

- The dilute point is described by a (non-unitary) conformal field theory with central charge

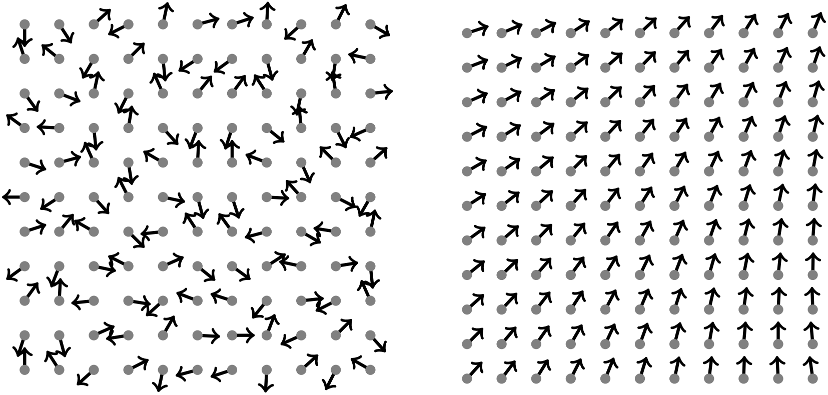
$$c = 1 - 6\frac{(g-1)^2}{g} \qquad 1 \leq g < 2$$

  if the loop fugacity is $n = -2\cos\pi g$.
  The loops are geometric objects that are conjectured to be described by SLE_κ with $\kappa = 4/g \leq 4$.
- The dense phase: same relations but $0 < g \leq 1$.

What is the surface critical behaviour of loop models?

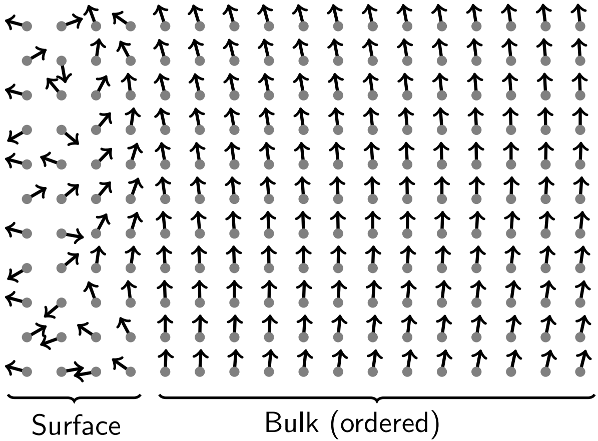
Let us take some ideas from the (spin) O(n) model in d$>$2.

$O(n)$ model = a classical lattice spin model for the para/ferro-magnetic transition with $O(n)$ symmetry.



Surface critical behaviour

Spins at the surface have less neighbours $\Rightarrow$ harder to order.
If x is just above the critical coupling x_c:

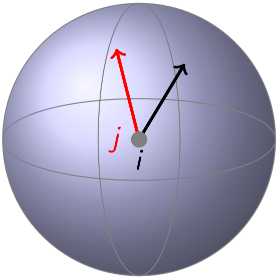


Surface effects are important only within a thin region of width ξ (bulk correlation length).

Boundary critical behaviour

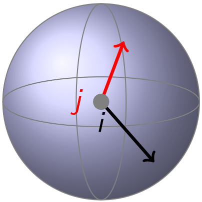
RG treatment of that model $\Rightarrow$ introduce a surface coupling $y \neq x$

Bulk



coupling $E_{<ij>} = -x\vec{S}_i.\vec{S}_j$

Surface

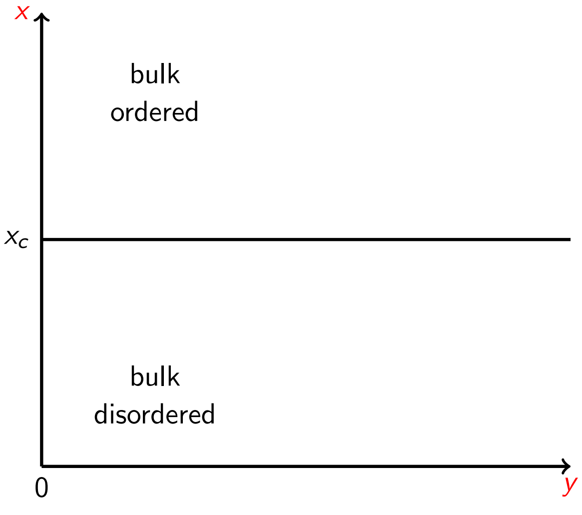


coupling $E_{<ij>} = -y\vec{S}_i.\vec{S}_j$

Boundary critical behaviour

RG treatment of that model $\Rightarrow$ introduce a surface coupling $y \neq x$

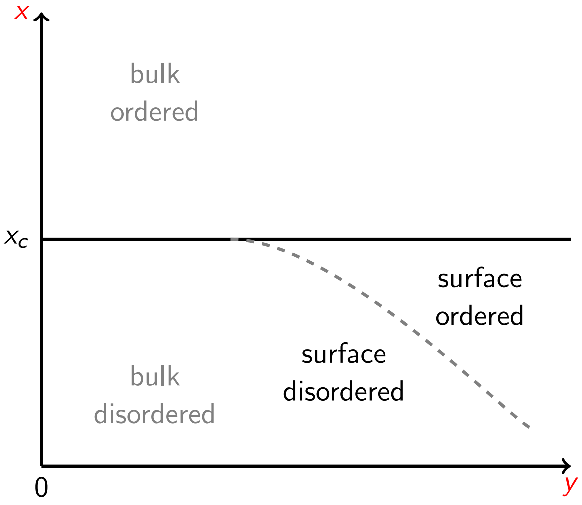
Phase diagram:



Boundary critical behaviour

RG treatment of that model ⇒ introduce a surface coupling $y \neq x$

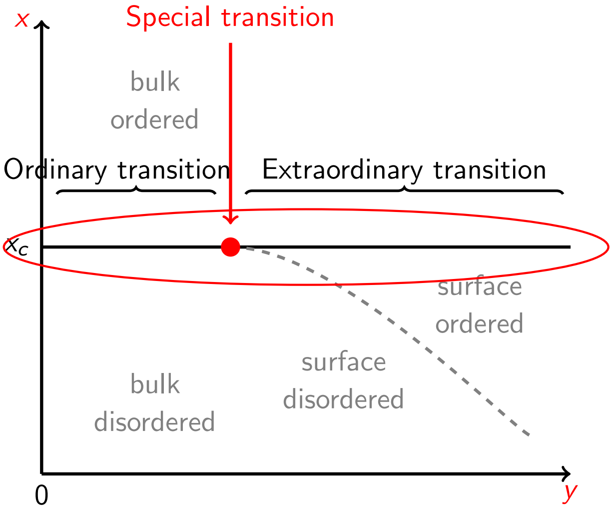
Phase diagram:



Boundary critical behaviour

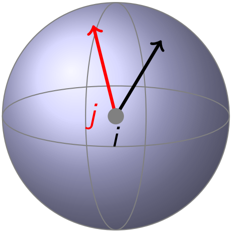
RG treatment of that model ⇒ introduce a surface coupling $y \neq x$

Phase diagram:



Anisotropic boundary interaction

Assume that, for some reason, the nearest-neighbour coupling at the surface is anisotropic.



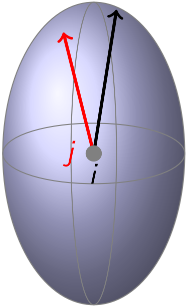
$n = 3 \qquad n_1 = 1 \qquad \Delta = 0$

$$E_{<ij>} = -y \sum_{\alpha=1}^{n} S_i^\alpha S_j^\alpha - \Delta \sum_{\beta=1}^{n_1} S_i^\beta S_j^\beta$$

Broken symmetry $O(n) \rightarrow O(n_1) \times O(n - n_1)$.

Anisotropic boundary interaction

Assume that, for some reason, the nearest-neighbour coupling at the surface is anisotropic.



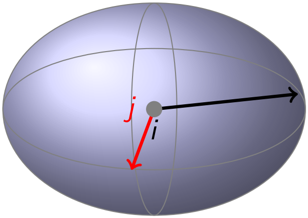
$$E_{<ij>} = -y \sum_{\alpha=1}^{n} S_i^\alpha S_j^\alpha - \Delta \sum_{\beta=1}^{n_1} S_i^\beta S_j^\beta$$

$n = 3 \qquad n_1 = 1 \qquad \Delta > 0$

Broken symmetry $O(n) \to O(n_1) \times O(n - n_1)$.

Anisotropic boundary interaction

Assume that, for some reason, the nearest-neighbour coupling at the surface is anisotropic.



$$E_{<ij>} = -y \sum_{\alpha=1}^{n} S_i^\alpha S_j^\alpha - \Delta \sum_{\beta=1}^{n_1} S_i^\beta S_j^\beta$$

$n = 3 \qquad n_1 = 1 \qquad \Delta < 0$

Broken symmetry $O(n) \to O(n_1) \times O(n - n_1)$.

Is the anisotropy a relevant boundary perturbation?

Is the anisotropy a relevant boundary perturbation?

No, not around the ordinary transition.

Is the anisotropy a relevant boundary perturbation?

No, not around the ordinary transition.

Yes, around the special and extraordinary transitions [Diehl & Eisenriegler, 1987].

Is the anisotropy a relevant boundary perturbation?

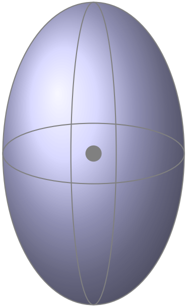
No, not around the ordinary transition.

Yes, around the special and extraordinary transitions [Diehl & Eisenriegler, 1987].

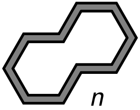
What about loops in 2D?

Loops and surface anisotropy

How do we translate the anisotropic surface coupling in terms of the loops?

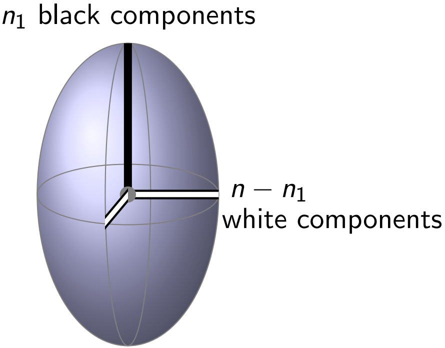


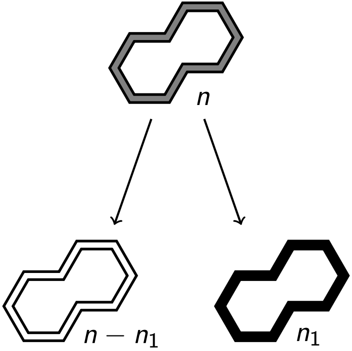
?



Loops and surface anisotropy

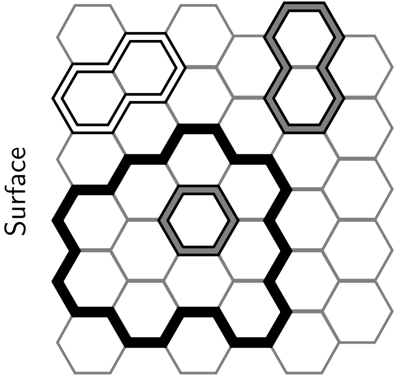
How do we translate the anisotropic surface coupling in terms of the loops?





Loops and surface anisotropy

How do we translate the anisotropic surface coupling in terms of the loops?



Boltzmann weights

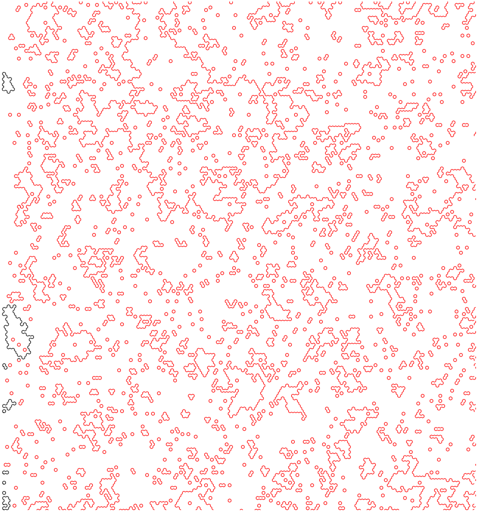
- Bulk loop weight
  $n\ x^{\text{bulk length}}$
- Surface black loop
  $n\ x^{\text{bulk length}}\, w_{\bullet}^{\text{surface length}}$
- Surface white loop
  $n\ x^{\text{bulk length}}\, w_{\circ}^{\text{surface length}}$

Surface critical behaviour of the loops

$w_\bullet \ll 1$

$w_\circ \ll 1$

Ordinary

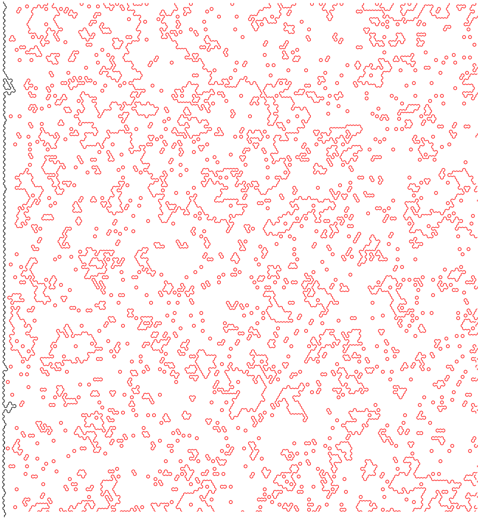


Surface critical behaviour of the loops

$w_\bullet > w_\circ$

$w_\bullet \gg 1$

Extraordinary (black)

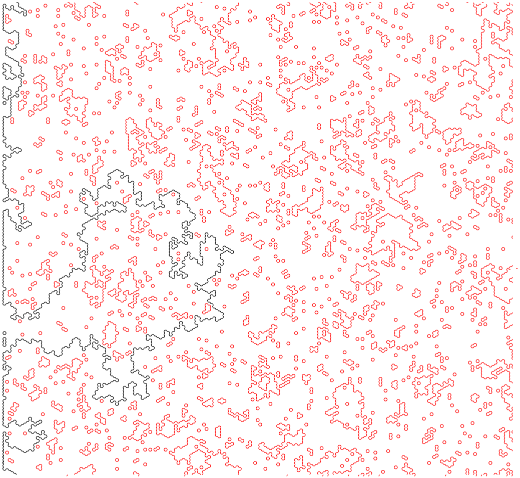


Surface critical behaviour of the loops

$w_\bullet > w_\circ$

$w_\bullet$ and $w_\circ$ are fine tuned

Anisotropic special (black)



What is the CFT of the loop anisotropic special (AS) transition?

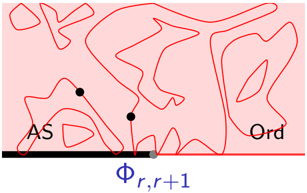
CFT of the anisotropic special transition

- recall that the critical O(n) model is a CFT with

$$c = 1 - 6\frac{(g-1)^2}{g} \qquad n = -2\cos \pi g \qquad g > 1$$

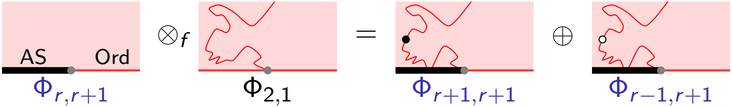
- boundary-condition-changing operators: B.C.C. (Ord/AS) is $\Phi_{r,r+1}$ with scaling dimension

$$h_{r,r+1} = \frac{[(g-1)r-1]^2 - (g-1)^2}{4g} \qquad n_1 = \frac{\sin\left((r+1)(1-g)\pi\right)}{\sin\left(r(1-g)\pi\right)}$$



CFT of the anisotropic special transition

- the whole spectrum of the theory can be generated by fusion with operators starting a piece of loop at a boundary point: $\Phi_{2,1}$

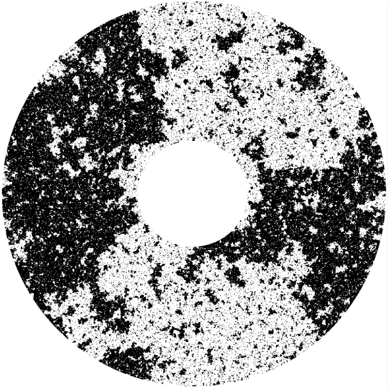


one can choose the set of components (black $O(n_1)$ or white $O(n-n_1)$) that correspond to the piece of loop

CFT of the anisotropic special transition

Once this CFT framework is set up, what kind of quantities can we compute?

- A funny result: crossing probability of Ising clusters on an annulus.



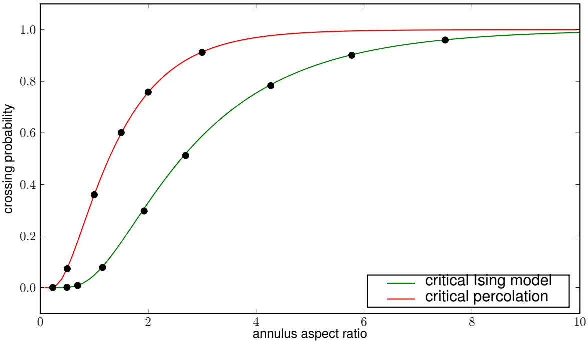
$$P_c^{\mathrm{Ising}}(\tau) = \frac{\eta(i\tau)\eta(i\tau/12)^2}{\eta(i\tau/2)^2\eta(i\tau/6)} \qquad P_c^{\mathrm{perc}}(\tau) = \frac{\eta(i\tau)\eta(i\tau/6)^2}{\eta(i\tau/2)^2\eta(i\tau/3)}$$

(η stands for Dedekind's eta function, $\tau = R_1/R_2$ is the aspect ratio)

CFT of the anisotropic special transition

Once this CFT framework is set up, what kind of quantities can we compute?

- A funny result: crossing probability of Ising clusters on an annulus. Monte Carlo results:

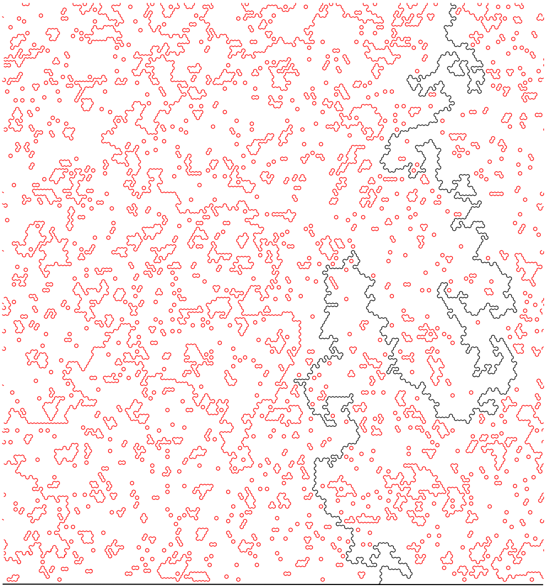


$$P_c^{\mathrm{Ising}}(\tau) = \frac{\eta(i\tau)\eta(i\tau/12)^2}{\eta(i\tau/2)^2\eta(i\tau/6)} \qquad P_c^{\mathrm{perc}}(\tau) = \frac{\eta(i\tau)\eta(i\tau/6)^2}{\eta(i\tau/2)^2\eta(i\tau/3)}$$

(η stands for Dedekind's eta function, $\tau = R_1/R_2$ is the aspect ratio)

We have a consistant CFT framework for the loop AS transition.

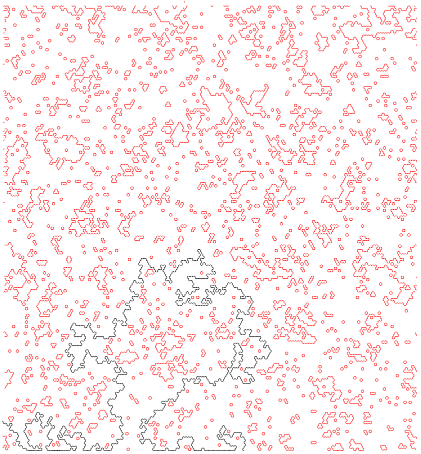
Now what about SLE?



recall that
SLE describes
this loop

Anisotropic special transition and $\mathrm{SLE}_{\kappa,\rho}$

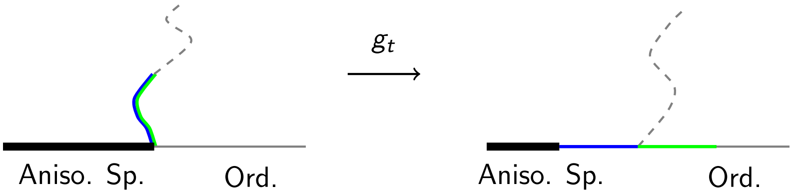
Now, what about the loops at the anisotropic special transition?



Anisotropic special　　Ordinary

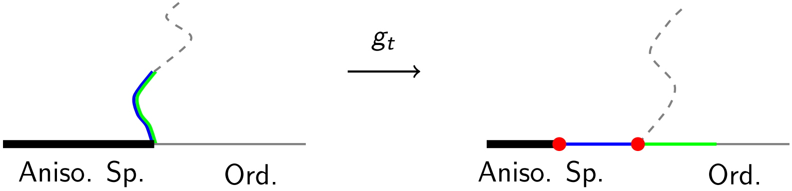
Anisotropic special transition and SLE$_{\kappa,\rho}$

Now, what about the loops at the anisotropic special transition?



Anisotropic special transition and $\mathrm{SLE}_{\kappa,\rho}$

Now, what about the loops at the anisotropic special transition?

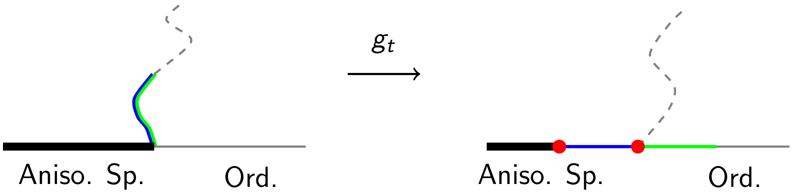


Two distinguished points moving on the boundary

$\longrightarrow$ affects the motion of the driving function $a(t)$.

Anisotropic special transition and $\mathrm{SLE}_{\kappa,\rho}$

Now, what about the loops at the anisotropic special transition?



Two distinguished points moving on the boundary

$\longrightarrow$ affects the motion of the driving function $a(t)$.

This is $\mathrm{SLE}_{\kappa,\rho}$ [Lawler, Schramm, Werner, 2003].

Anisotropic special transition and $\mathrm{SLE}_{\kappa,\rho}$

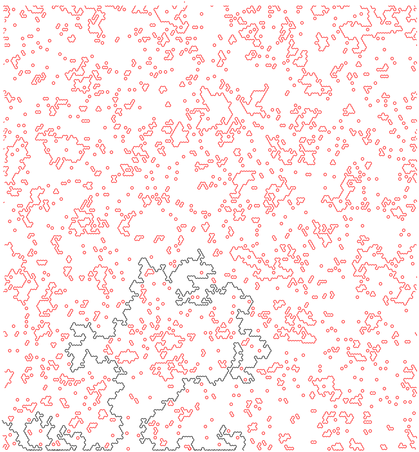
$\mathrm{SLE}_{\kappa,\rho}$ is a stochastic process defined by

$$
\begin{aligned}
dg_t(z) &= \frac{2dt}{g_t(z) - a(t)} \\
da(t) &= \sqrt{\kappa}dB_t + \frac{\rho dt}{a(t) - g_t(x)}
\end{aligned}
$$

κ and ρ correspond to n and n_1 in the $O(n)$ model

$$
\begin{aligned}
n &= -2\cos\frac{4\pi}{\kappa} \\
n_1 &= \frac{\sin\left(\frac{2\rho+8-\kappa}{\kappa}\pi\right)}{\sin\left(\frac{2\rho+4}{\kappa}\pi\right)}
\end{aligned}
$$

Some conjecture for the boundary fractal dimension



Bulk (proved) $d_f = 1 + \frac{\kappa}{8}$

Surface $d_f = \left(1 + \frac{\rho}{4}\right)\left(2 - \frac{8}{\kappa} - \frac{4\rho}{\kappa}\right)$?

Boundaries in conformal loop models: a summary

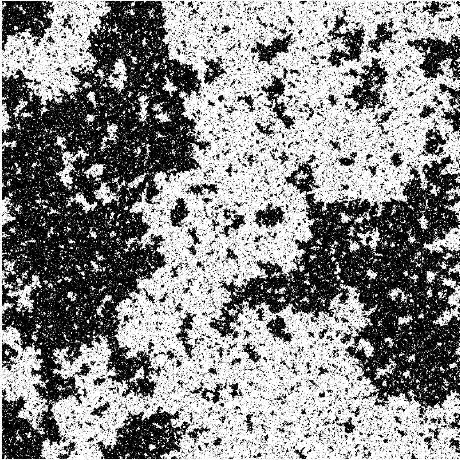
- Continuous set of conformal boundary conditions in loop models
- Consistency of standard CFT concepts (B.C.C operators, fusion, ...) with geometric objects (loops touching the surface)
- 2D version of the physics of the (anisotropic) special transition, and exact solution.
- Lattice (integrable) models with scaling limits conjectured to be $\mathrm{SLE}_{\kappa,\rho}$
- Exact relations between the parametrizations

$$\text{lattice model } (n, n_1) \leftrightarrow \text{CFT } (c, h_{r,r+1}) \leftrightarrow \text{SLE } (\kappa, \rho)$$

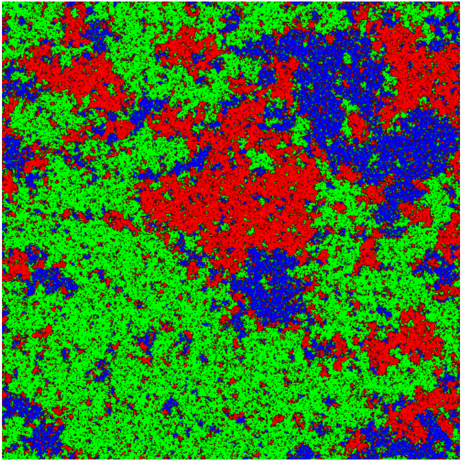
Many results have been rederived in the context of random surfaces and matrix models (J.-E. Bourgine, I. Kostov & K. Hosomichi).

$\longrightarrow$ Jean-Emile's talk this afternoon

Conformal loop models . . . and beyond?

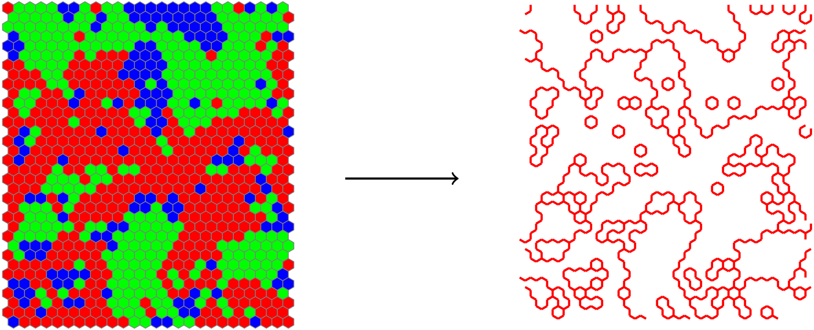


- It is great to be able to describe the domain-walls in the Ising model.



- It is great to be able to describe the domain-walls in the Ising model.
- What about the 3-states Potts model?

Branching domain walls on the lattice



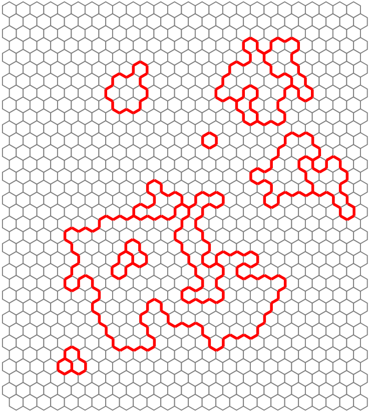
The domain-walls are branching. It is no longer a loop model.

Branching domain-walls on the lattice

- The success of loop models comes from the fact that the loop fugacity n is a real parameter (corresponding to a continuum of CFTs).
- The Q-states Potts model can be defined for continuous Q (Fortuin & Kasteleyn). What are the domain-walls then?

Branching domain-walls on the lattice

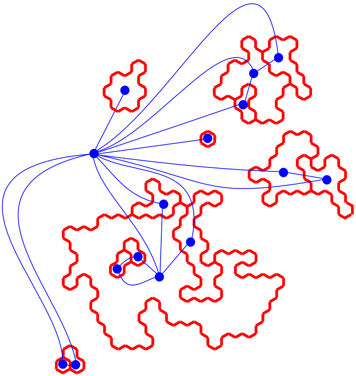
The Q-states Potts model can be defined in terms of its domain-walls (after Fendley, Read, ...):



- Pick up any configuration G of domain-walls on the lattice

Branching domain-walls on the lattice

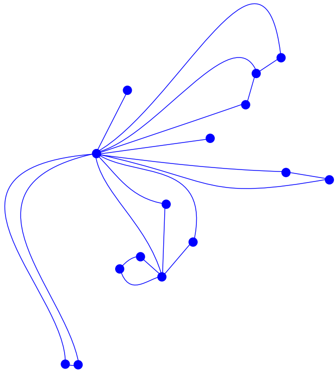
The Q-states Potts model can be defined in terms of its domain-walls (after Fendley, Read, ...):



- Pick up any configuration G of domain-walls on the lattice
- Look at its dual graph $\hat{G}$

Branching domain-walls on the lattice

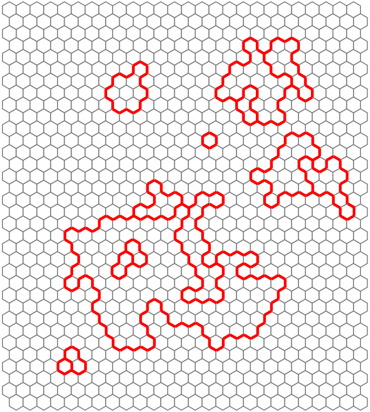
The Q-states Potts model can be defined in terms of its domain-walls (after Fendley, Read, ...):



- Pick up any configuration G of domain-walls on the lattice
- Look at its dual graph $\hat{G}$
- Compute the chrompatic polynomial $\chi_{\hat{G}}(Q)$

Branching domain-walls on the lattice

The Q-states Potts model can be defined in terms of its domain-walls (after Fendley, Read, ...):



- Pick up any configuration G of domain-walls on the lattice
- Look at its dual graph $\hat{G}$
- Compute the chrompatic polynomial $\chi_{\hat{G}}(Q)$

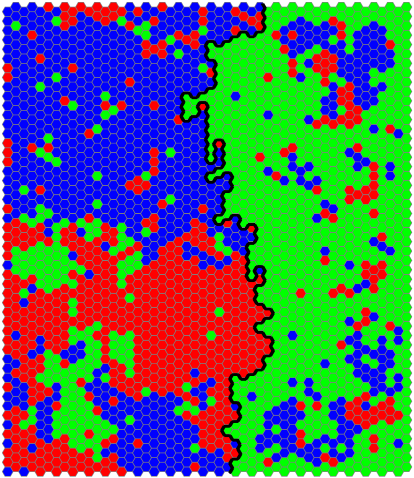
The Boltzmann weight of the configuration G is

$$e^{-K \times (\textit{total length of } G)} \chi_{\hat{G}}(Q)$$

Question: is there a scaling limit? Is it conformally invariant?

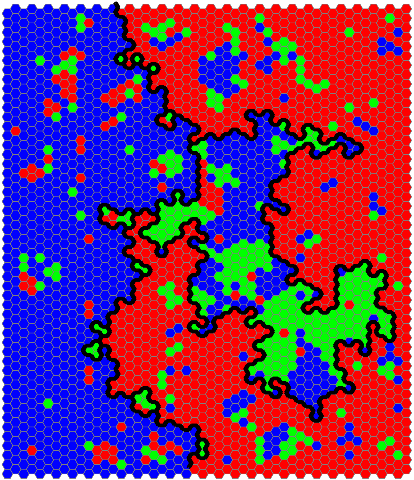
Previous results

Previous results (Cardy & Gamsa, Santachiara) suggest that $\mathrm{SLE}_{\frac{10}{3}}$ appears in the 3-states Potts model.



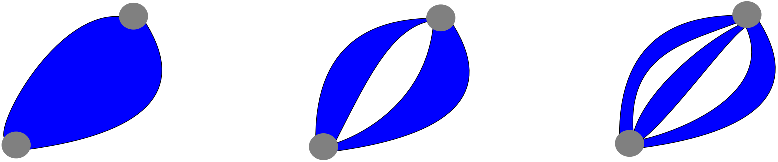
Branching domain-walls

What about branching interfaces?



New exponents in the Q-states Potts model

For any Q one can compute (numerically) the L_1-clusters exponents:

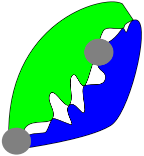


$h_{1,0}$ $h_{2,0}$ $h_{3,0}$

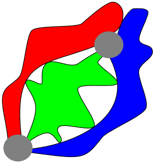
Here the clusters have the same color. In general one finds the $O(n)$ model $2L_1$-legs exponent $h_{L_1,0}$.

New exponents in the Q-states Potts model

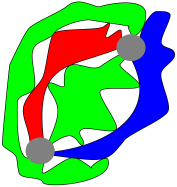
For any Q one can compute (numerically) the L_2-clusters exponents:



$h_{2,4}$



$h_{3,6}$



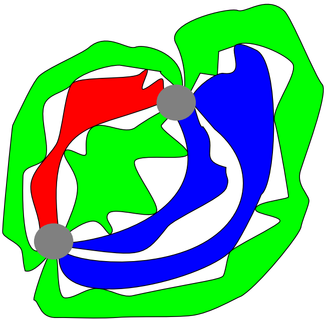
$h_{4,8}$

Here the successive clusters have different colors. In general the exponent is $h_{L_2,2L_2}$.

New exponents in the Q-states Potts model

For L_1 successive clusters with the same color and L_2 successive ones with different colors the exponent is

$$h_{L_2-L_1,2L_2}$$



(here $L_1 = 2$ and $L_2 = 4$)

Conclusion

- Boundary conditions in loop models, from the lattice model to CFT, and their link with $\mathrm{SLE}_{\kappa,\rho}$.
- Domain-walls in the Potts model are not loops. However, numerical results suggest that some geometrical observables are conformally invariant. Can we describe branching processes in CFT/SLE?

Thank you.