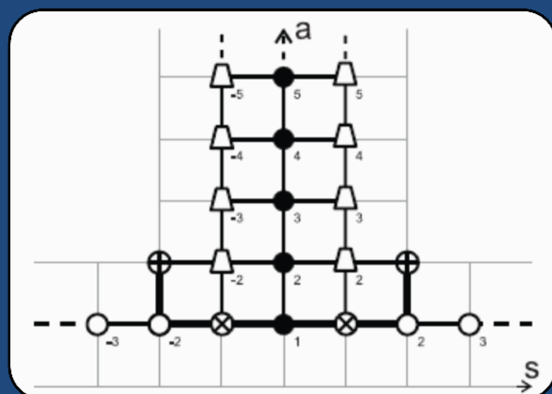
Nikolay Gromov
Based on work with V.Kazakov, Z.Tsuboi



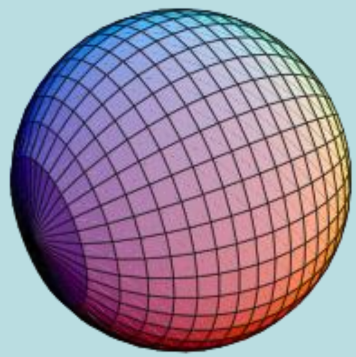
Y-SYSTEM AND QUASI-CLASSICAL STRINGS

PHYSICS IN THE PLANE: FROM CONDENSED MATTER TO STRING THEORY

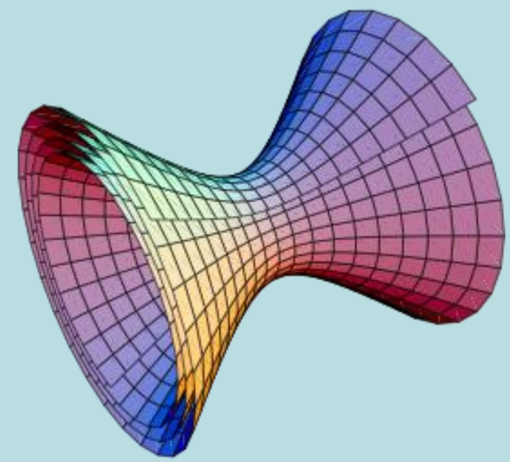
Introduction

AdS/CFT correspondence

(type IIB super) string theory in $AdS_5 x S^5$

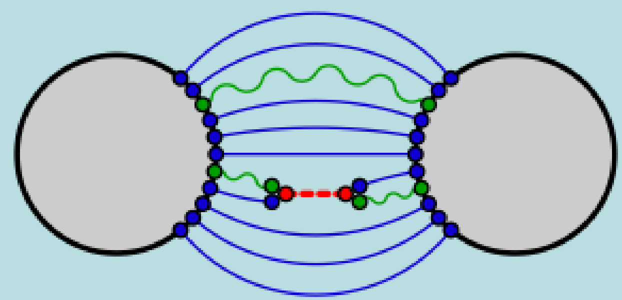


x



Maldacena

is dual to a 4 dimensional conformal field theory (N=4 SYM)



Local operators ⇔ String states

Introduction

AdS/CFT duality

SU(N) Super Yang-Mills :

$$S = \frac{1}{g_{YM}^2} \int d^4x \ \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_i)^2 - \frac{1}{2} [\Phi_i, \Phi_j]^2 + \mathrm{fermions} \right\}$$

$\lambda = g_{YM}^2 N$ `t Hooft coupling

Anomalous dimensions of YM = spectrum of 2D integrable field theories

Symmetry: $psu(2,2|4)$

Introduction

$AdS_5 \times S^5$ super string

10d super string action is a super coset model

$$\frac{PSU(2,2|4)}{SP(2,2) \times SP(4)}$$

Metsaev, Tseytlin

su(2,2|4) algebra is spanned by 8x8 matrix

$$M = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right)$$

A and D belong to u(2,2) and u(4) and
the fermionic fields B and C obey

$$C = B^\dagger \begin{pmatrix} \mathbb{I}_{2\times 2} & 0 \\ 0 & -\mathbb{I}_{2\times 2} \end{pmatrix}$$

Introduction

$AdS_5 \times S^5$ super string

su(2,2|4) algebra enjoys the $\mathbf{Z_4}$ automorphism

$$\Omega \circ M = \begin{pmatrix} EA^T E & -EC^T E \\ EB^T E & ED^T E \end{pmatrix} , \quad E = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

any algebra element can be split into

$$\sum_{i=0}^{3} M^{(i)}, \text{ where } \Omega \circ M^{(n)} = i^n M^{(n)}$$

then the action reads

$$S = \frac{\sqrt{\lambda}}{4\pi} \int \mathrm{str} \left(J^{(2)} \wedge * J^{(2)} - J^{(1)} \wedge J^{(3)} \right)$$

where $J = -g^{-1} dg$

Introduction

$AdS_5 \times S^5$ super string

In some particular parameterization the bosonic part of the action is

$$S_b = \frac{\sqrt{\lambda}}{4\pi} \int_0^{2\pi} d\sigma \int d\tau h^{\mu\nu} (\partial_\mu u \cdot \partial_\nu u - \partial_\mu v \cdot \partial_\nu v)$$

With a constraints

$$1 = u_6^2 + u_5^2 + u_4^2 + u_3^2 + u_2^2 + u_1^2$$

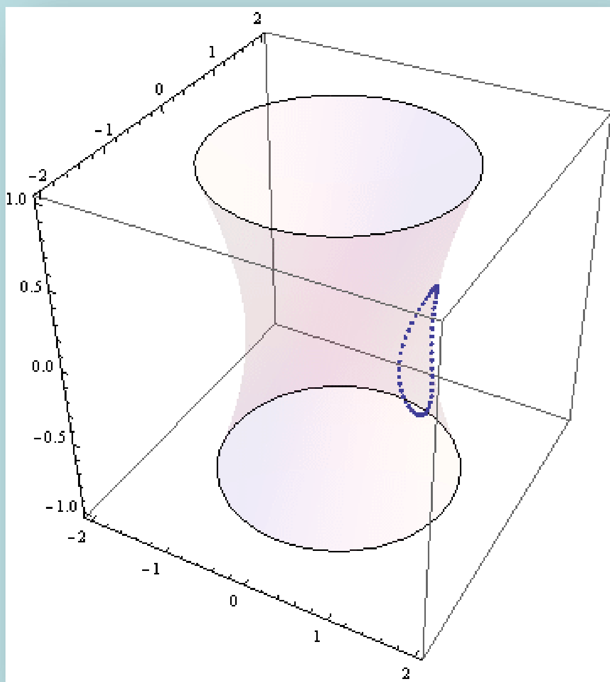
$$1 = v_6^2 + v_5^2 - v_4^2 - v_3^2 - v_2^2 - v_1^2$$

Fermionic part is more complicated

$$S_f = \frac{\sqrt{\lambda}}{8\pi} \int d^2\sigma \sqrt{h} h^{\mu\nu} \mathrm{tr}_4 \Big[V \partial_\mu \bar{V} \left(\theta \partial_\nu \bar{\theta} - \partial_\nu \theta \, \bar{\theta} \right) + U \partial_\mu \bar{U} \left(\partial_\nu \bar{\theta} \, \theta - \bar{\theta} \partial_\nu \theta \right) \Big]$$
$$\pm i \frac{\sqrt{\lambda}}{8\pi} \int d^2\sigma \, \epsilon^{\mu\nu} \, \mathrm{tr}_4 \left[V \partial_\mu \bar{\theta}^t \bar{U} \partial_\nu \bar{\theta} + \partial_\mu \theta U \partial_\nu \theta^t \bar{V} \right] + \mathcal{O}(\theta^4)$$

Introduction

Classical integrability



Motion of the string:

$$\partial^2 u_a + (\partial u_b \partial u^b) u_a = 0$$

Infinitely many
Integrals of motion:

$$\Omega(x,\tau) = \mathrm{Pexp} \oint_{\gamma(\tau)} A_\sigma(x) d\sigma \quad , \quad x \in C$$

Flat connection (on eq. of motion):

Bena, Polchinski, Roiban;
Kazakov, Marshakov, Minahan, Zarembo;

$$A(x) = J^{(0)} + \frac{x^2+1}{x^2-1} J^{(2)} - \frac{2x}{x^2-1} * J^{(2)}$$

Eigenvalues = integrals of motion

$$\Omega(x) \to (\lambda_1(x), \lambda_2(x), \lambda_3(x), \lambda_4(x) | \mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x))$$

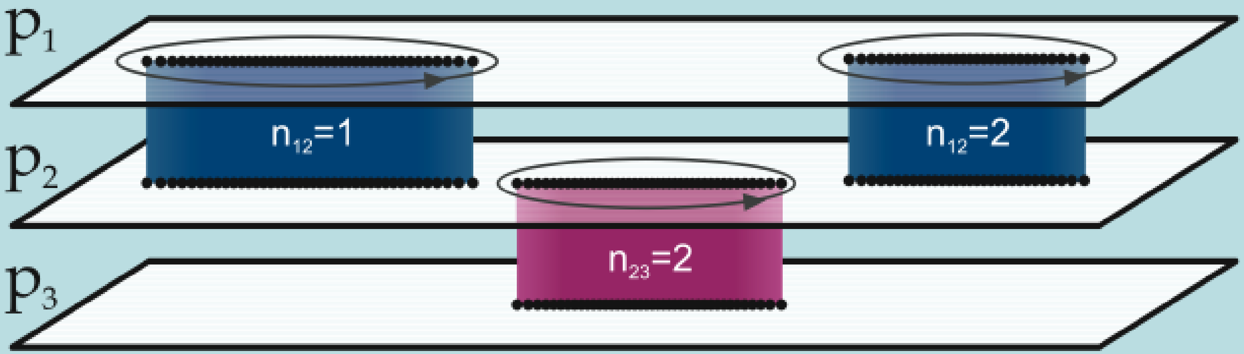
Introduction

Classical intgrability

According to Beisert, Kazakov, Sakai and Zarembo, we can map a classical string motion to an 8-sheet Riemann surface

$$\{e^{i\hat{p}_1}, e^{i\hat{p}_2}, e^{i\hat{p}_3}, e^{i\hat{p}_4} | e^{i\tilde{p}_1}, e^{i\tilde{p}_2}, e^{i\tilde{p}_3}, e^{i\tilde{p}_4}\}$$

Eigenvalues of a monodromy matrix



$$p_i^+ - p_j^- = 2\pi n_{ij} \ , \ x \in \mathcal{C}_n^{ij}$$

$$\oint_{\mathcal{C}_n^{ij}} p_i(z) dz = \frac{4\pi}{\sqrt{\lambda}} N_{ij}$$

Introduction

Integrability in 2D

Operator corresponding to an integral of motion $\widehat{C}_n$

$$\widehat{C}_n|k\rangle = \omega_n(k)|k\rangle$$

Between in and out states

$$_{out}\langle p_1, \ldots, p_m|\widehat{C}_n|k_1, \ldots, k_{m'}\rangle_{in}$$

The outgoing momenta are constrained:

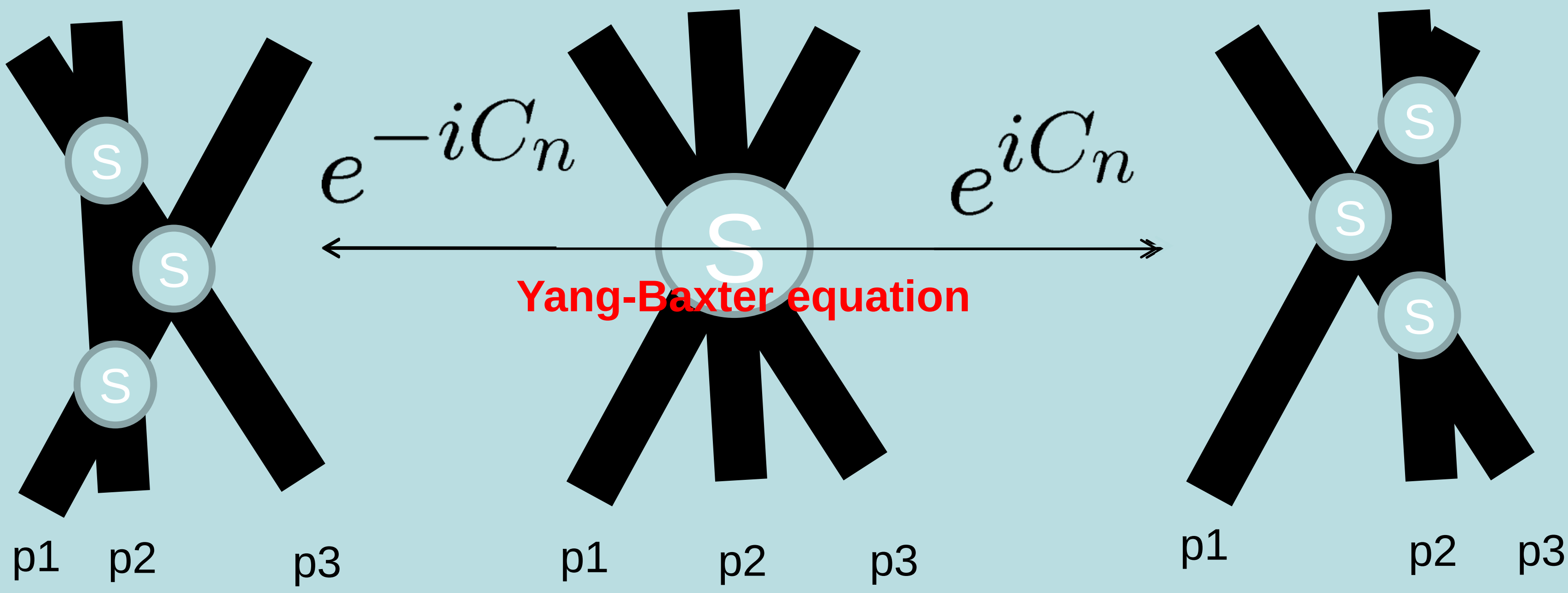
$$A_n = \sum_i \omega_n(k_i) = \sum_i \omega_n(p_i) \quad , \quad n = 1, \ldots$$

The only solution:

$$m = m' \qquad \{k_i\} = \{p_i\}$$

Introduction

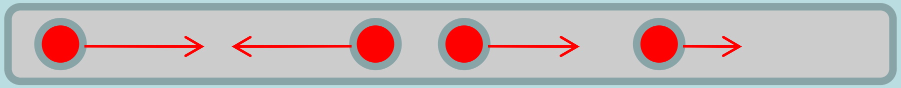
S-matrix factorization



Introduction

Asymptotic spectrum

- For spectral density we need finite volume



$$\Psi(x_1+L, x_2, \ldots) = e^{ip_1 L} S(p_1, p_2) \ldots S(p_1, p_n) \Psi(x_1, x_2, \ldots)$$

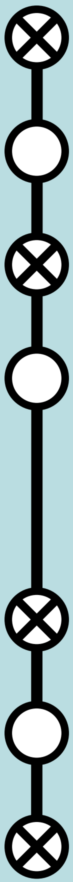
- From periodicity of the wave function

$$e^{ip_i L} = \prod_{j=1}^{M} S(p_i, p_j)$$

Introduction

Asymptotic spectrum

Beisert, Staudacher;
Beisert, Hernandez, Lopez;
Beisert,Eden,Staudacher



$$1 = \prod_{j=1}^{K_2} \frac{u_{1,k}-u_{2,j}+\frac{i}{2}}{u_{1,k}-u_{2,j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{1-1/x_{1,k}x^+_{4,j}}{1-1/x_{1,k}x^-_{4,j}},$$

$$1 = \prod_{j\neq k}^{K_2} \frac{u_{2,k}-u_{2,j}-i}{u_{2,k}-u_{2,j}+i} \prod_{j=1}^{K_3} \frac{u_{2,k}-u_{3,j}+\frac{i}{2}}{u_{2,k}-u_{3,j}-\frac{i}{2}} \prod_{j=1}^{K_1} \frac{u_{2,k}-u_{1,j}+\frac{i}{2}}{u_{2,k}-u_{1,j}-\frac{i}{2}},$$

$$1 = \prod_{j=1}^{K_2} \frac{u_{3,k}-u_{2,j}+\frac{i}{2}}{u_{3,k}-u_{2,j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{3,k}-x^+_{4,j}}{x_{3,k}-x^-_{4,j}},$$

$$1 = \left(\frac{x^-_{4,k}}{x^+_{4,k}}\right)^L \prod_{j\neq k}^{K_4} \frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i} \prod_{j}^{K_4} \left(\frac{1-1/x^+_{4,k}x^-_{4,j}}{1-1/x^-_{4,k}x^+_{4,j}}\right) \sigma^2(x_{4,k},x_{4,j})$$
$$\times \prod_{j=1}^{K_1} \frac{1-1/x^-_{4,k}x_{1,j}}{1-1/x^+_{4,k}x_{1,j}} \prod_{j=1}^{K_3} \frac{x^-_{4,k}-x_{3,j}}{x^+_{4,k}-x_{3,j}} \prod_{j=1}^{K_5} \frac{x^-_{4,k}-x_{5,j}}{x^+_{4,k}-x_{5,j}} \prod_{j=1}^{K_7} \frac{1-1/x^-_{4,k}x_{7,j}}{1-1/x^+_{4,k}x_{7,j}},$$

$$1 = \prod_{j=1}^{K_6} \frac{u_{5,k}-u_{6,j}+\frac{i}{2}}{u_{5,k}-u_{6,j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{5,k}-x^+_{4,j}}{x_{5,k}-x^-_{4,j}},$$

$$1 = \prod_{j\neq k}^{K_6} \frac{u_{6,k}-u_{6,j}-i}{u_{6,k}-u_{6,j}+i} \prod_{j=1}^{K_5} \frac{u_{6,k}-u_{5,j}+\frac{i}{2}}{u_{6,k}-u_{5,j}-\frac{i}{2}} \prod_{j=1}^{K_7} \frac{u_{6,k}-u_{7,j}+\frac{i}{2}}{u_{6,k}-u_{7,j}-\frac{i}{2}},$$

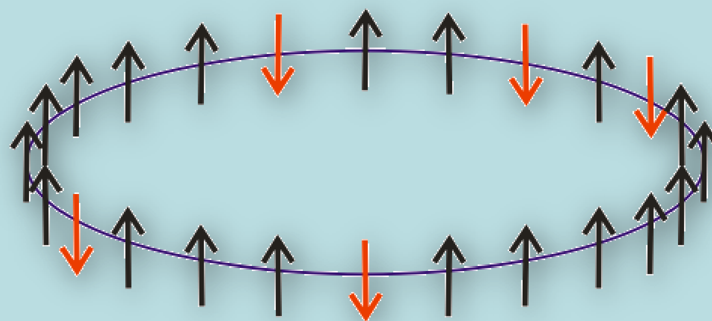
$$1 = \prod_{j=1}^{K_6} \frac{u_{7,k}-u_{6,j}+\frac{i}{2}}{u_{7,k}-u_{6,j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{1-1/x_{7,k}x^+_{4,j}}{1-1/x_{7,k}x^-_{4,j}}.$$

$$x + \frac{1}{x} = \frac{u}{g}\ , \quad x^{\pm} + \frac{1}{x^{\pm}} = \frac{u \pm i/2}{g} \qquad E = \sum_k \epsilon_k = \sum_k 2gi\left(\frac{1}{x^+_{4,k}} - \frac{1}{x^-_{4,k}}\right)$$

Introduction

Integrability in $\mathcal{N}=4$ SYM

$$\mathcal{O}_i(x) = \mathrm{tr}\Phi_1\Phi_2\Phi_1\Phi_1\Phi_1\Phi_2\Phi_2\Phi_1\Phi_1\Phi_1$$



$$\mathcal{O}_i^{\mathrm{ren}} = Z_{ij}(\Lambda)\mathcal{O}_j^{\mathrm{bare}} \qquad \Gamma = Z^{-1}\frac{dZ}{d\log\Lambda}$$

- Mixing matrix – integrable Hamiltonian

At one loop:

[Minahan, Zarembo 2002&2008]

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = -\prod_{k=1}^{M}\frac{u_j-u_k+i}{u_j-u_k-i}$$

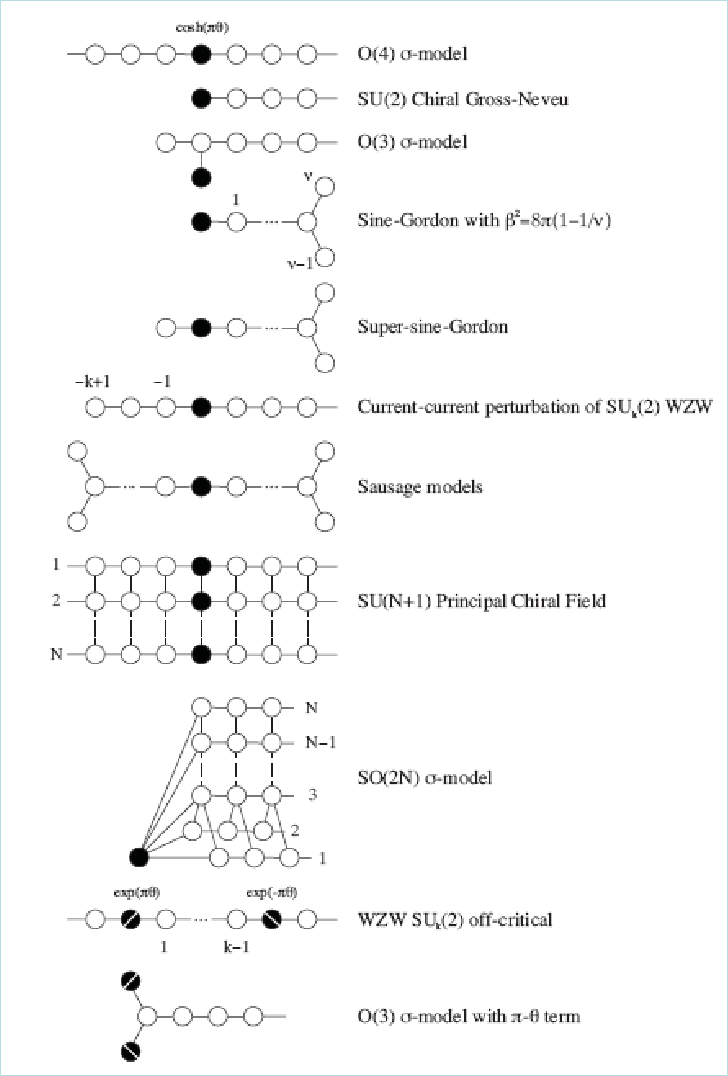
$$\gamma = \sum_{k=1}^{M}\frac{g}{u_k^2+1/4}$$

$$e^{ip_iL} = \prod_{j=1}^{M} S(p_i,p_j)$$

Finite size spectrum

Introduction

Some 2D Integrable models



$$\frac{Y^+_{a,s}Y^-_{a,s}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

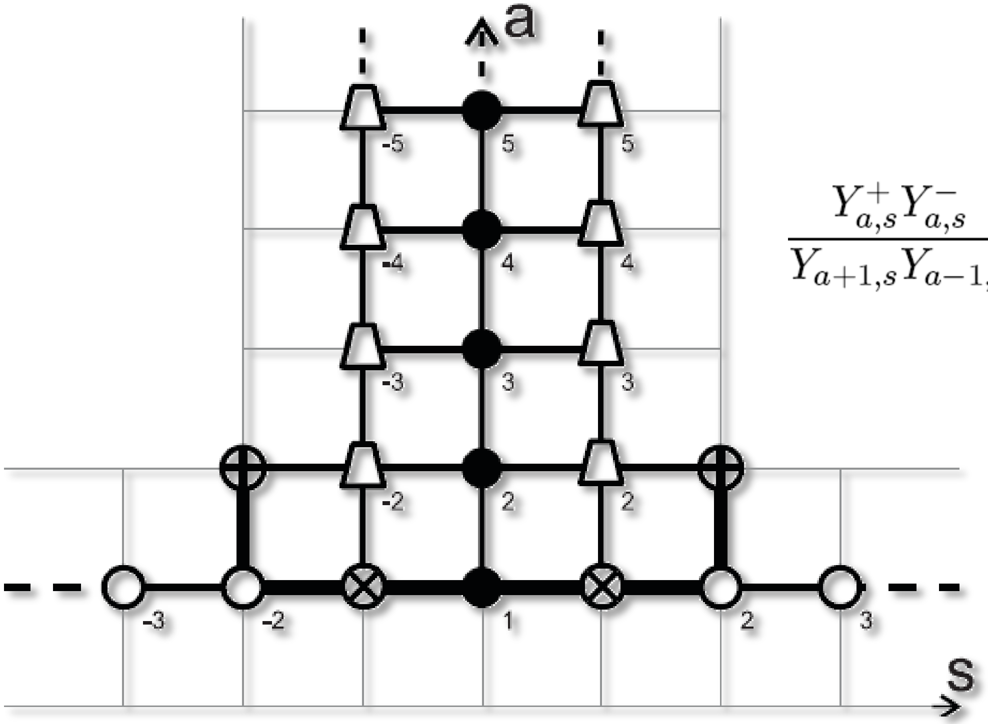
…Bazhanov, Lukyanov, Zamolodchikov, P.Dorey, Toteo…

…Destri de Vega, Bytsko ,Teschner….

Introduction

Y-system for AdS/CFT

N.G., Kazakov, Vieira



$$\frac{Y_{a,s}^{+}Y_{a,s}^{-}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

$$E = \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a}{\partial u} \log\Big(1 + Y_{a,0}(u)\Big) + \sum_{a} \epsilon_j(u_{4,j})$$

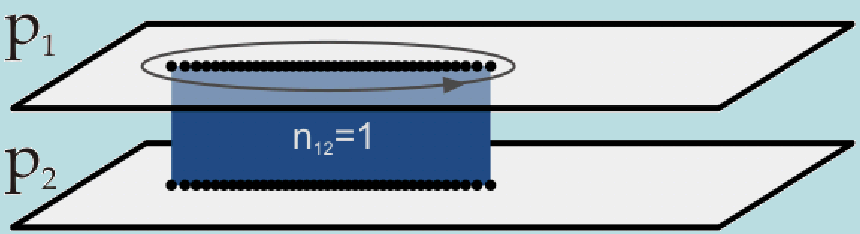
$$Y_{1,0}(u_{4,j}) = -1$$

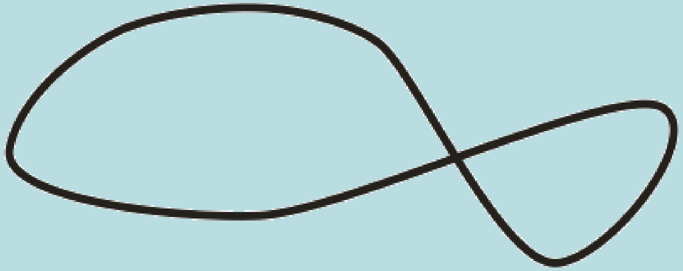
Quasiclassical quantization
From the algebraic curve

Quasi-classical quantization

Quantization of the curve

For any given classical solution





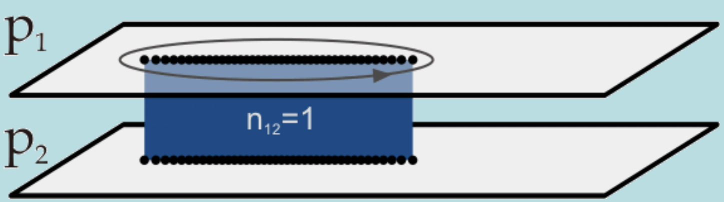
We can compute the first quantum correction to the energy
(and any other conserved charge) by deforming in an appropriate way
its algebraic curve!

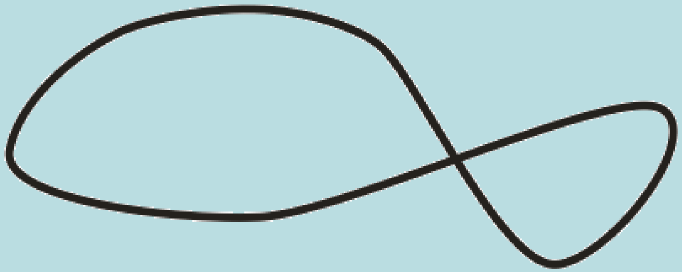
$$\mathcal{E} = \mathcal{E}_{cl} + \mathcal{E}_0 + \mathcal{O}(1/\sqrt{\lambda})$$

Quasi-classical quantization

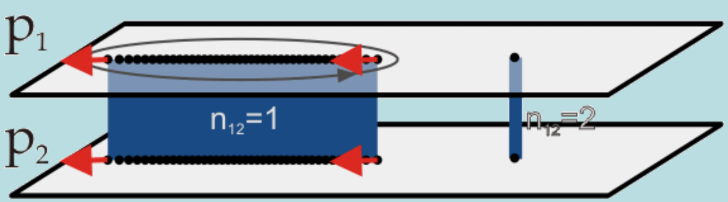
spectrum of fluctuations

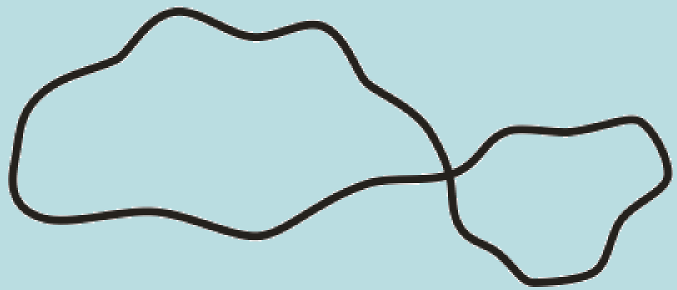
Consider some classical solution:





We can excite it by a small extra cut:





The action variables are known to be:

$$\oint_{\mathcal{C}_n^{ij}} p_i(z)dz = \frac{4\pi}{\sqrt{\lambda}} N_{ij} \quad \Rightarrow$$ Spectrum of excitations is quantized

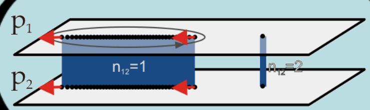
Quasi-classical quantization

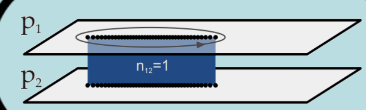
1-loop energy shift

For the harmonic oscillator we have

$$E = \hbar\omega\left(N + \frac{1}{2}\right)$$

So far we understood how to get

$$\mathcal{E}_n^{ij} = \mathcal{E}\Bigg($$


$$\Bigg) - \mathcal{E}\Bigg($$


$$\Bigg)$$

N.G. Pedro Vieira

Now we can compute shift of the energy level due to the zero point energies

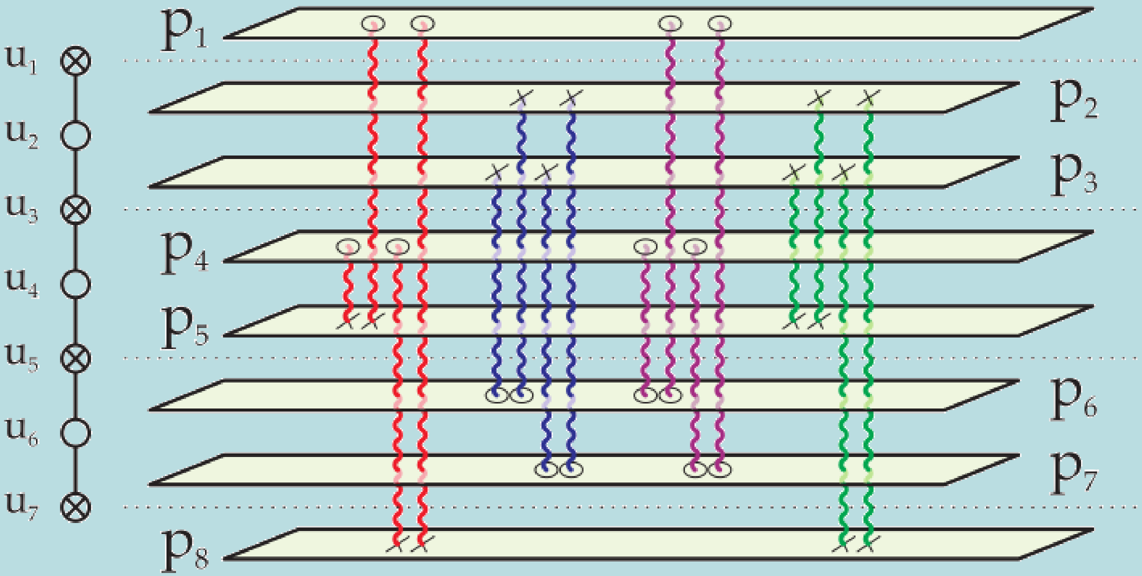
$$\mathcal{E}_0 = \frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{ij}(-1)^F \mathcal{E}_n^{ij}$$

Frolov, Tseytlin

Quasi-classical quantization

1-loop energy shift

For the harmonic oscillator we have



Poles can be only at the specia points of the curve

$$p_{\mathbf{i}}(x_n^{\mathbf{ij}}) - p_{\mathbf{j}}(x_n^{\mathbf{ij}}) = 2\pi n$$

For bosons

$$\lambda_{\mathbf{i}} = \lambda_{\mathbf{j}} \quad , \quad \mu_{\mathbf{i}} = \mu_{\mathbf{j}}$$

For fermions

$$\lambda_{\mathbf{i}} = \mu_{\mathbf{j}}$$

We can rewrite the sum as an integral

$$\frac{1}{2}\sum_{(\mathbf{ij})}\sum_{n}(-1)^{F_{\mathbf{ij}}}\omega(x_n^{(\mathbf{ij})}) = \int_{-1}^{1}\frac{dz}{2\pi}\frac{z}{\sqrt{1-z^2}}\partial_z\mathcal{N}_*$$

Where $\mathcal{N}_* \equiv \sum_{i=1,2}\sum_{j=3,4}\log\frac{(1-\mu_i/\lambda_j)(1-\lambda_i/\mu_j)}{(1-\mu_i/\mu_j)(1-\lambda_i/\lambda_j)}$

One-loop spectrum from Y-system

Quasi-classical limit of Y-system

Strong coupling solution

At strong coupling:

$$T_{a,s} = \begin{cases} (-1)^{(a+1)s} \left(\frac{x_3 x_4}{y_1 y_2 y_3 y_4}\right)^{s-a} \frac{\det\left(S_i^{\theta_{j,s+2}} y_i^{j-4-(a+2)\theta_{j,s+2}}\right)_{1\le i,j\le 4}}{\det\left(S_i^{\theta_{j,0+2}} y_i^{j-4-(0+2)\theta_{j,0+2}}\right)_{1\le i,j\le 4}} & , \quad a \ge |s| \\ \frac{\det\left(Z_i^{(1-\theta_{j,a})} x_i^{2-j+(s-2)(1-\theta_{ja})}\right)_{1\le i,j\le 2}}{\det\left(Z_i^{(1-\theta_{j,0})} x_i^{2-j+(0-2)(1-\theta_{j,0})}\right)_{1\le i,j\le 2}} & , \quad s \ge +a \end{cases}$$

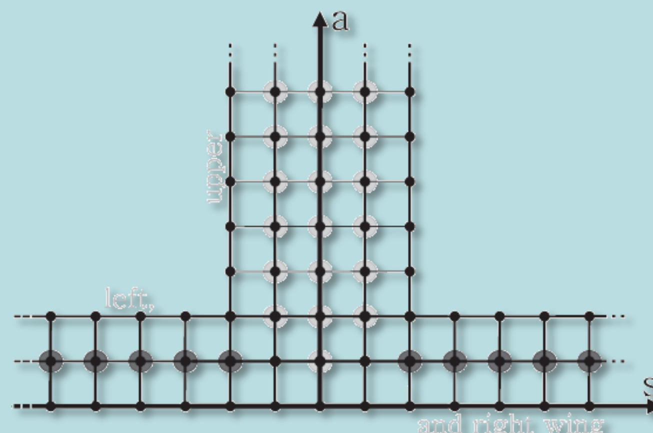
Hirota equation:

$$T_{a,s}^2 = T_{a+1,s} T_{a-1,s} + T_{a,s+1} T_{a,s-1}$$

$$Y_{a,s} = \frac{T_{a+1,s} T_{a-1,s}}{T_{a,s+1} T_{a,s-1}}$$

General solution is given by characters
With 8-paramiters (from V.Kazakov's talk)

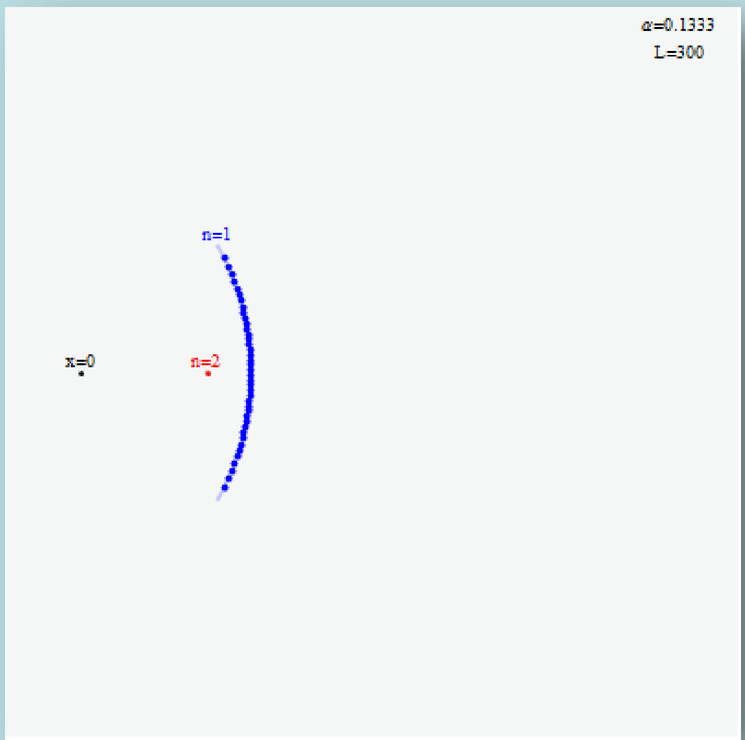
$$(x_1, x_2, x_3, x_4 | y_1, y_2, y_3, y_4)$$

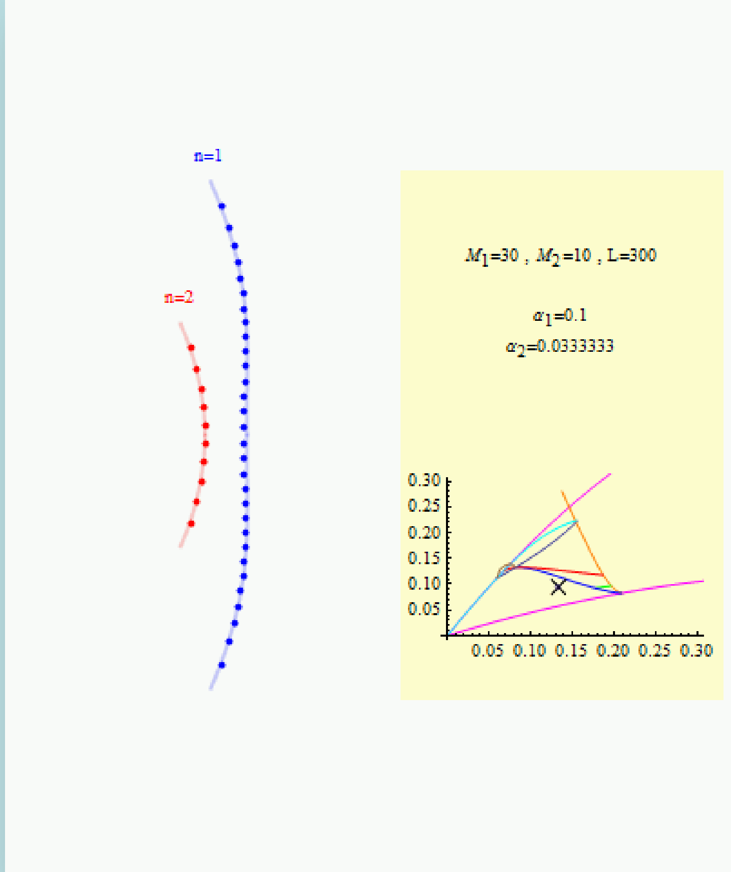


Quasi-classical limit of Y-system

Strong coupling solution

When number of Bethe roots scales as $\sqrt{\lambda}$ and $u_j \sim \sqrt{\lambda} \sim L$





Till Bargheer, Niklas Beisert, N. G.

Quasi-classical limit of Y-system

Strong coupling solution

Generating functional for the asymptotic solution (from V.Kazakov's talk)

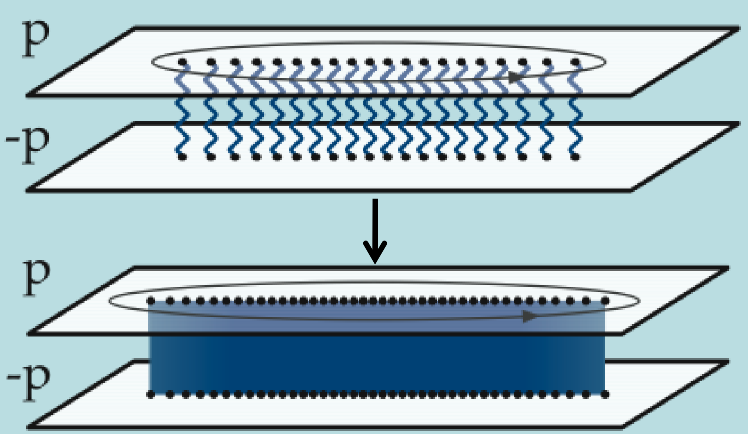
$$\sum_{s=0}^{\infty} \mathbf{T}^{\mathrm{R}}_{1,s}\left(u+i\tfrac{1-s}{2}\right) D^s = \left[1-\mathbf{T}^{\mathrm{R},1}_{1,1} D\right]\cdot\left[1-\mathbf{T}^{\mathrm{R},2}_{1,1} D\right]^{-1}\cdot\left[1-\mathbf{T}^{\mathrm{R},3}_{1,1} D\right]^{-1}\cdot\left[1-\mathbf{T}^{\mathrm{R},4}_{1,1} D\right], \qquad D = e^{-i\partial_u}$$

$$\mathbf{T}^{\mathrm{R},1}_{1,1}(u) = \frac{Q_1^-}{Q_1^+}\prod_{j=1}^{K_4}\frac{1-1/(x^+x^-_{4,j})}{1-1/(x^+x^+_{4,j})}\frac{x^- - x^-_{4,j}}{x^- - x^+_{4,j}}, \qquad \mathbf{T}^{\mathrm{R},2}_{1,1}(u) = \frac{Q_1^- Q_2^{++}}{Q_1^+ Q_2}\prod_{j=1}^{K_4}\frac{x^- - x^-_{4,j}}{x^- - x^+_{4,j}},$$

$$\mathbf{T}^{\mathrm{R},3}_{1,1}(u) = \frac{Q_2^{--} Q_3^+}{Q_2 Q_3^-}\prod_{j=1}^{K_4}\frac{x^- - x^-_{4,j}}{x^- - x^+_{4,j}}, \qquad \mathbf{T}^{\mathrm{R},4}_{1,1}(u) = \frac{Q_3^+}{Q_3^-}$$

In the scaling limit becomes:

$$\sum_{s=0}^{\infty} \mathbf{T}^{\mathrm{R}}_{1,s} D^s = \frac{(1-\lambda_1 D)(1-\lambda_2 D)}{(1-\mu_1 D)(1-\mu_2 D)}$$



Quasi-classical limit of Y-system

Strong coupling solution

N.G. 2009
N.G., V.Kazakov, Z.Tsuboi

$$\lim_{a\to+\infty}\frac{1}{a}\log\frac{T^g_{a,s}}{\mathbf{T}_{a,s}}=0\ ,\quad s=-2,-1,0,1,2$$

$$\lim_{s\to+\infty}\frac{1}{s}\log\frac{T^g_{a,s}}{\mathbf{T}_{a,s}}=0\ ,\quad a=1,2$$

$$\lim_{s\to-\infty}\frac{1}{s}\log\frac{T^g_{a,s}}{\mathbf{T}_{a,s}}=0\ ,\quad a=1,2$$

$$T^g_{a,s}=g_2^aT_{a,s}$$

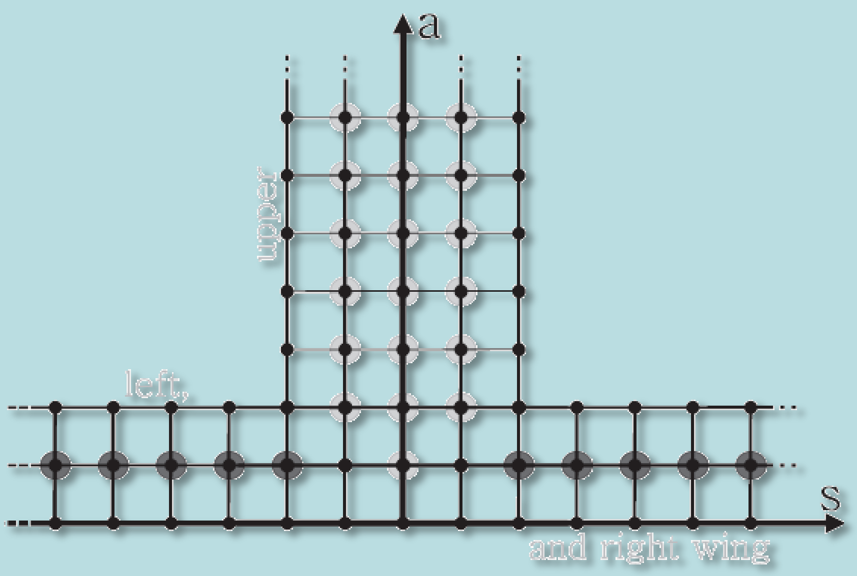
$$y_i=\lambda_i\ ,\quad x_i=\mu_i\ ,\quad i=1,\ldots,4$$

Thus very naturally we have:

$$T_{a,s}=\mathrm{Str}_{a,s}\Omega$$

Natural quantum generalization:

$$T_{a,s}=\langle\mathrm{state}|\mathrm{Str}_{a,s}\hat{\Omega}|\mathrm{state}\rangle$$



Quasi-classical limit of Y-system

Strong coupling solution

1) For a given classical solution we compute

$$\Omega(x,\tau) = \mathrm{Pexp} \oint_{\gamma(\tau)} A_\sigma(x) d\sigma \quad , \quad x \in C$$

2) Diagonalize it

$$\Omega(x) \to (\lambda_1(x), \lambda_2(x), \lambda_3(x), \lambda_4(x) | \mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x))$$

3) Compute super-character

$$\begin{aligned} T_{1,1} = \Big(& \lambda_1^3 (\lambda_2 - \lambda_3)(\lambda_2 - \lambda_4) + \lambda_2^3 \lambda_3 \lambda_4 - \lambda_3 \lambda_4 \mu_1 \mu_2 \mu_3 + \\ & \lambda_1^2 (\lambda_2 - \lambda_3)(\lambda_2 - \lambda_4)(\lambda_2 - \mu_1 - \mu_2 - \mu_3 - \mu_4) - \lambda_3 \lambda_4 \mu_1 \mu_2 \mu_4 - \lambda_3 \lambda_4 \mu_1 \mu_3 \mu_4 - \\ & \lambda_3 \lambda_4 \mu_2 \mu_3 \mu_4 + \lambda_3 \mu_1 \mu_2 \mu_3 \mu_4 + \lambda_4 \mu_1 \mu_2 \mu_3 \mu_4 - \lambda_2^2 \lambda_3 \lambda_4 (\mu_1 + \mu_2 + \mu_3 + \mu_4) + \\ & \lambda_2 (-\mu_1 \mu_2 \mu_3 \mu_4 + \lambda_3 \lambda_4 (\mu_3 \mu_4 + \mu_2 (\mu_3 + \mu_4) + \mu_1 (\mu_2 + \mu_3 + \mu_4))) + \lambda_1 \\ & \Big(-\lambda_2^3 (\lambda_3 + \lambda_4) - \mu_1 \mu_2 \mu_3 \mu_4 + \lambda_3 \lambda_4 (\mu_3 \mu_4 + \mu_2 (\mu_3 + \mu_4) + \mu_1 (\mu_2 + \mu_3 + \mu_4)) + \\ & \lambda_2^2 (\lambda_4 (\mu_1 + \mu_2 + \mu_3 + \mu_4) + \lambda_3 (\lambda_4 + \mu_1 + \mu_2 + \mu_3 + \mu_4)) - \\ & \lambda_2 (-\mu_1 \mu_2 \mu_3 - \mu_1 \mu_2 \mu_4 - \mu_1 \mu_3 \mu_4 - \mu_2 \mu_3 \mu_4 + \\ & \lambda_4 (\mu_3 \mu_4 + \mu_2 (\mu_3 + \mu_4) + \mu_1 (\mu_2 + \mu_3 + \mu_4)) + \lambda_3 \\ & (\mu_2 \mu_3 + \mu_2 \mu_4 + \mu_3 \mu_4 + \mu_1 (\mu_2 + \mu_3 + \mu_4) + \lambda_4 (\mu_1 + \mu_2 + \mu_3 + \mu_4)))\Big)\Big) / \\ & ((\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)(\lambda_1 - \lambda_4)(\lambda_2 - \lambda_4)) \end{aligned}$$

Quasi-classical limit of Y-system

Strong coupling solution

For some particular combinations the result is very simple:

$$\prod_{a=1}^{\infty}(1+Y_{a,0})^a=\prod_{i=1,2}\prod_{j=3,4}\frac{(1-\mu_i/\lambda_j)(1-\lambda_i/\mu_j)}{(1-\mu_i/\mu_j)(1-\lambda_i/\lambda_j)}$$

The expression for the energy becomes:

$$E=\sum_{i=1}^{M}\epsilon(u_{4,i})+\sum_{a=1}^{\infty}\int_{-\infty}^{\infty}\frac{du}{2\pi i}\frac{\partial\epsilon_a}{\partial u}\log\left(1+Y_{a,0}(u)\right)$$

Coinsides with the expression from the quasi-classical quantization!

$$\frac{1}{2}\sum_{(\mathbf{ij})}\sum_{n}(-1)^{F_{\mathbf{ij}}}\omega(x_n^{(\mathbf{ij})})=\int_{-1}^{1}\frac{dz}{2\pi}\frac{z}{\sqrt{1-z^2}}\partial_z\mathcal{N}_*$$

$$\mathcal{N}_*\equiv\sum_{i=1,2}\sum_{j=3,4}\log\frac{(1-\mu_i/\lambda_j)(1-\lambda_i/\mu_j)}{(1-\mu_i/\mu_j)(1-\lambda_i/\lambda_j)}$$

Numerical solution of Y-system

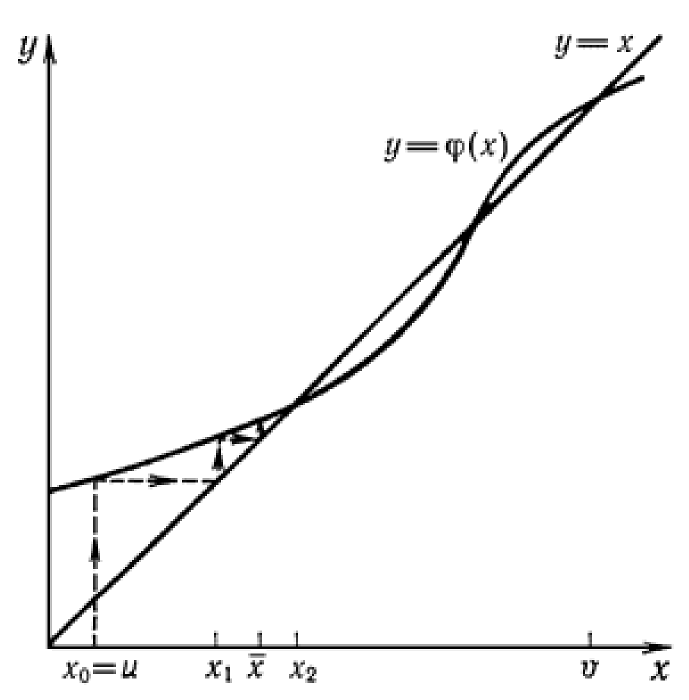
Numerical solution of Y-system

By iterations

$$\log Y_n(u) = \int K_{nm}(u,v)\log(1+Y_m(v))dv$$
$$+\Phi_n(u)$$

Bazhanov, Lukyanov, Zamolodchikov, P.Dorey, Totteo

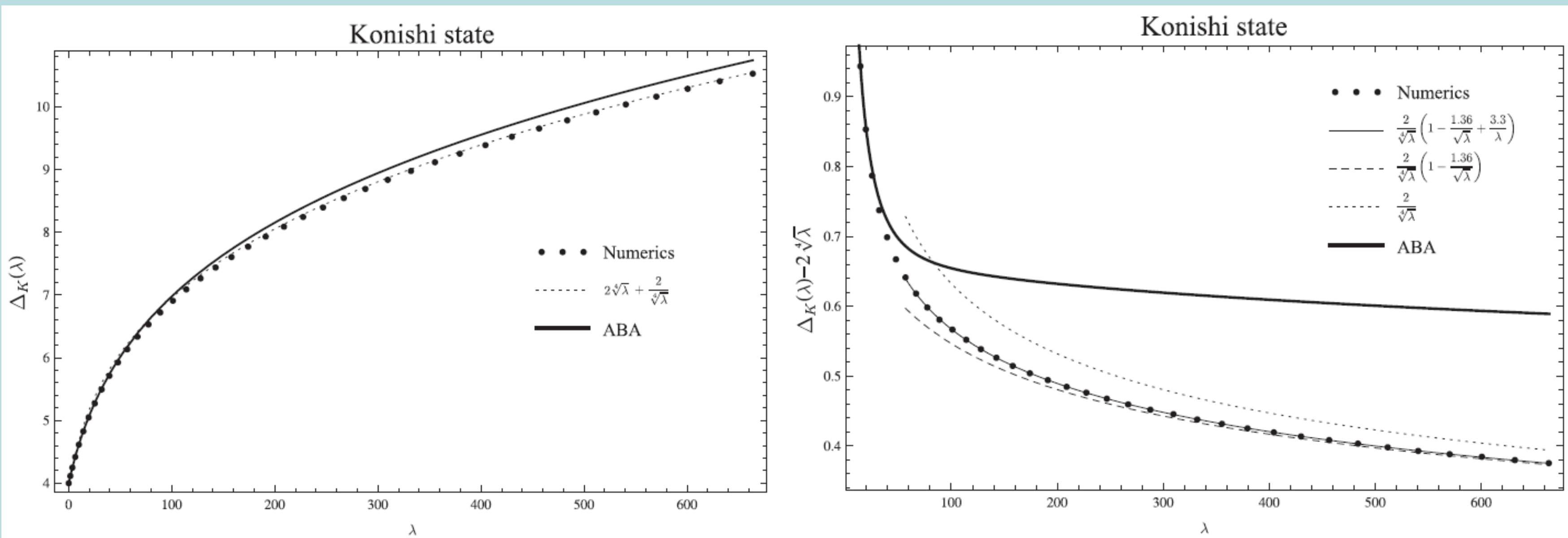
Bombardelli, Fioravanti, R.Tateo
N.G., Kazakov, Vieira
Arutynov, Frolov



Numerical solution of Y-system

Simplest (Konishi) operator

$$\mathcal{O} = tr(ZZWW) - tr(ZWZW)$$



Fit:

$$\Delta_K = 2\lambda^{1/4}\left(1.0002 + \frac{0.994}{\lambda^{1/2}} - \frac{1.30}{\lambda} + \frac{3.1}{\lambda^{3/2}} + \dots\right)$$

CONCLUSIONS

- We know the general strong coupling solution of Y-system
- Also large L and weak coupling solutions are known in general
- More tests should be done
- Application (BFKL, и т.п.…)
- YY? Hidden structures in the perturbation theory from the gauge side