Topological order and quantum criticality

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Physics in the plane:
From **condensed matter** to **string theory**

news & views

TOPOLOGICAL PHASES

Wormholes in quantum matter

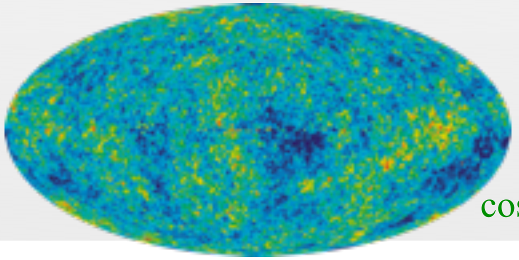
Proliferation of so-called anyonic defects in a topological phase of quantum matter leads to a critical state that can be visualized as a ‘quantum foam’, with topology-changing fluctuations on all length scales.

Kareljan Schoutens

Topological quantum liquids

Spontaneous symmetry breaking

- ground state has **less** symmetry than high-T phase
- Landau-Ginzburg-Wilson theory
- **local** order parameter



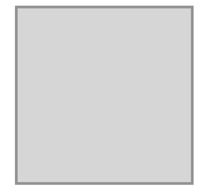
cosmic microwave background

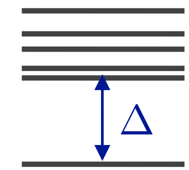
Topological order

- ground state has **more** symmetry than high-T phase
- degenerate ground states
- **non-local** order parameter
- quasiparticles have fractional statistics = **anyons**

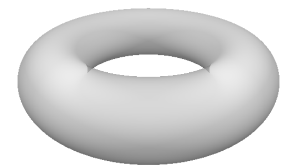
Topological quantum liquids

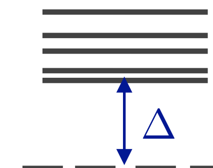
- Gapped spectrum
- **No** broken symmetry



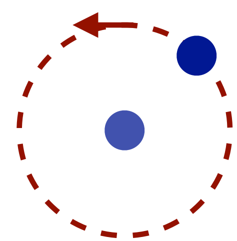


- Degenerate ground state on torus



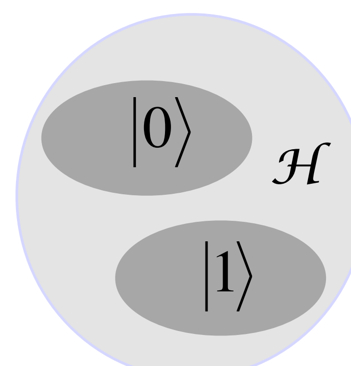


- Fractional statistics of excitations



$$e^{i\theta}$$

- Hilbert space split into topological sectors
  No **local** matrix element mixes the sectors



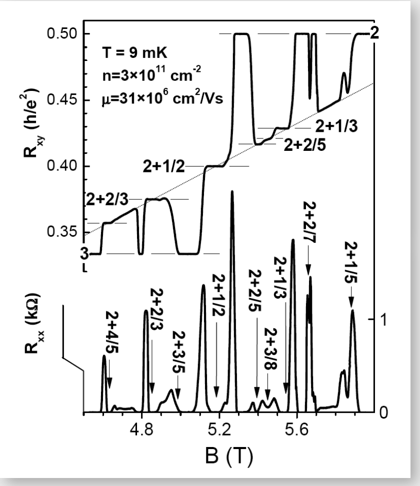
Topological order

time reversal symmetry

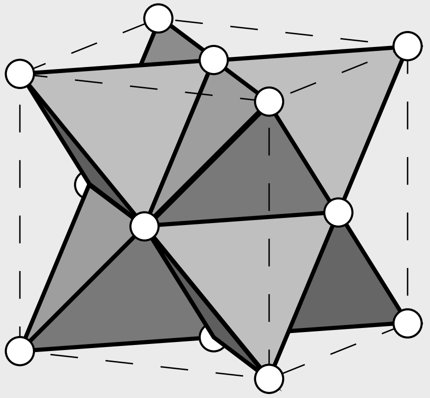
broken

invariant

quantum Hall liquids



magnetic materials



Quantum phase transitions

time-reversal invariant systems

How can we describe these phase transitions?

no local order parameter

↓

no Landau-Ginzburg-Wilson theory

complex field theoretical framework

↓

doubled (non-Abelian) Chern-Simons theories

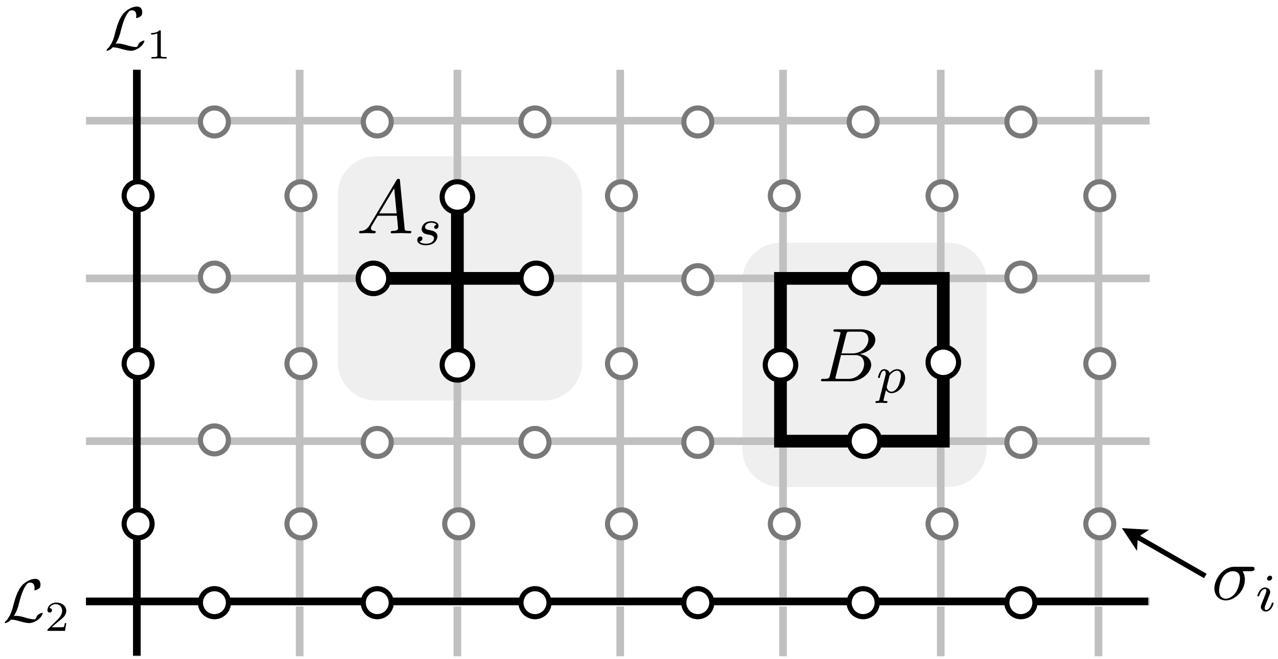
Liquids on surfaces can provide a “topological framework”.

The toric code in a magnetic field

A first example

The toric code

A. Kitaev, Ann. Phys. **303**, 2 (2003).



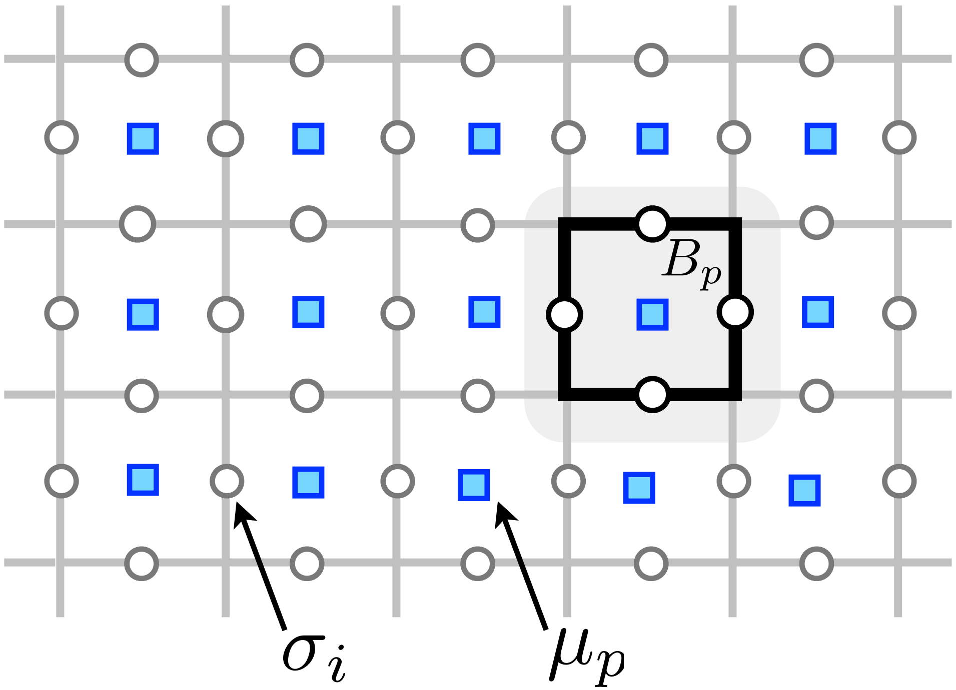
$$A_s = \prod_{j \in \mathrm{star}(s)} \sigma_j^x$$

$$B_p = \prod_{j \in \partial p} \sigma_j^z$$

$$\mathcal{H}_{\mathrm{TC}} = -J_e \sum_s A_s - J_m \sum_p B_p + \sum_i \left(h_x \sigma_i^x + h_z \sigma_i^z \right)$$

topological phase **paramagnet**

Toric code and the transverse field Ising model



$$B_p = \prod_{j \in \partial p} \sigma_j^z$$

$$B_p = 2\mu_p^z$$

$$\sigma_i^x = \mu_p^x \mu_q^x$$

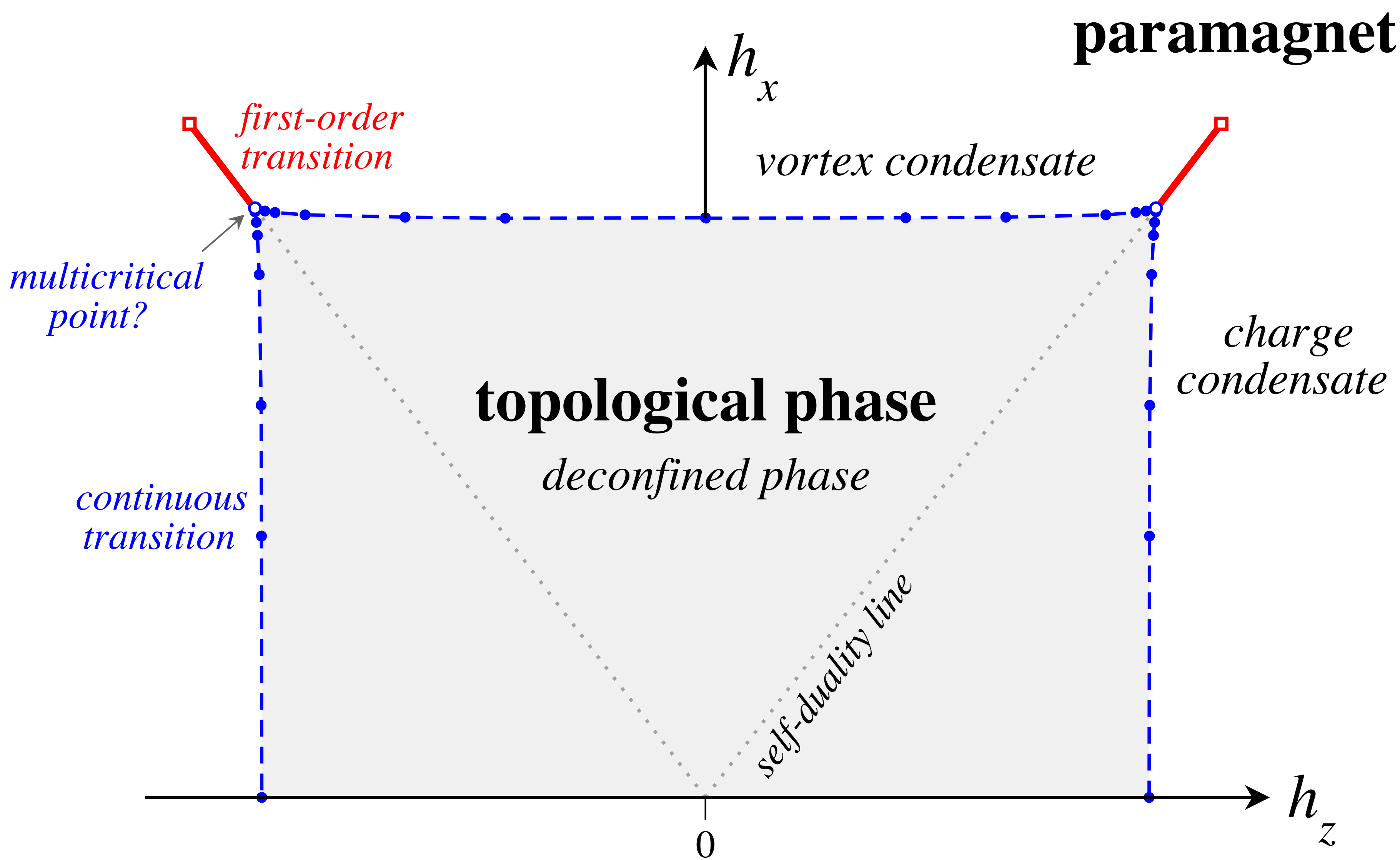
$$\mathcal{H}_{\text{TC}} = -J_m \sum_p B_p + h_x \sum_i \sigma_i^x$$

⇓

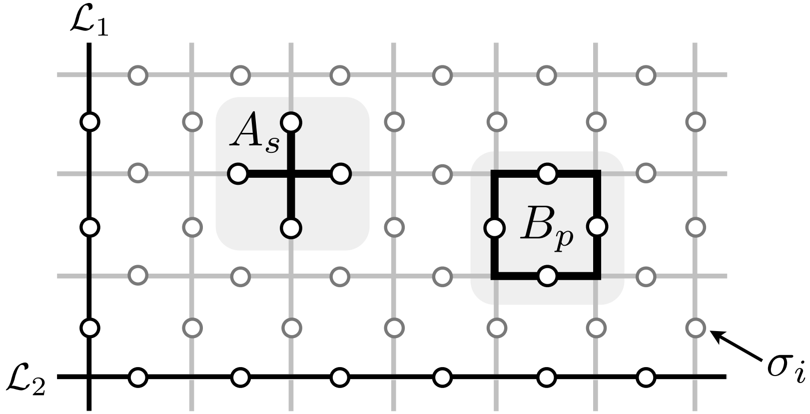
$$\mathcal{H}_{\text{TFIM}} = -2J_m \sum_p \mu_p^z + h_x \sum_{\langle p,q \rangle} \mu_p^x \mu_q^x$$

Phase diagram

I.S. Tupitsyn *et al.*, arXiv:0804.3175



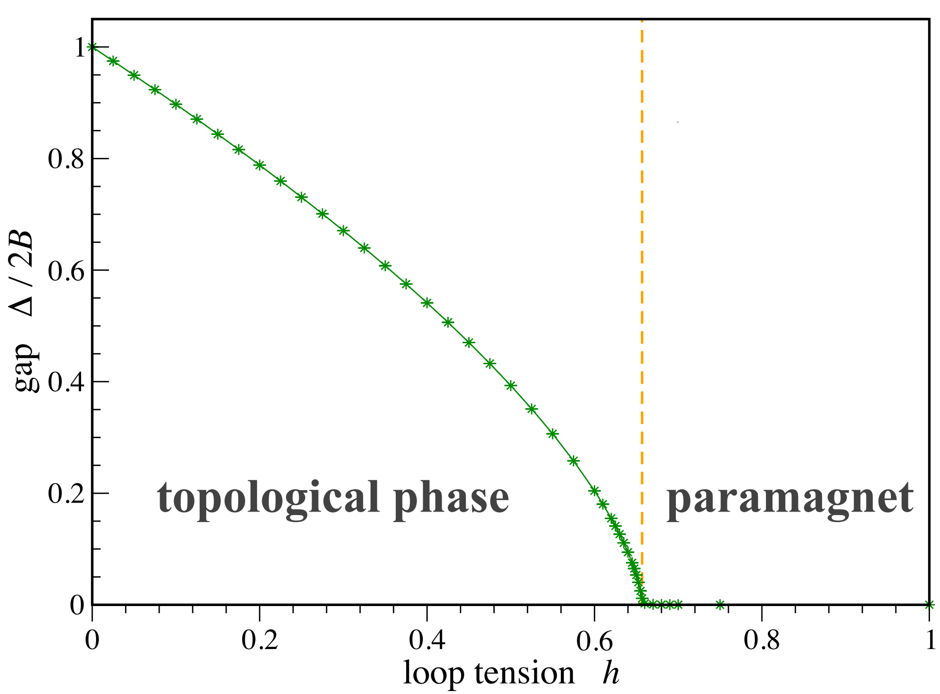
Excitations: condensation vs. confinement



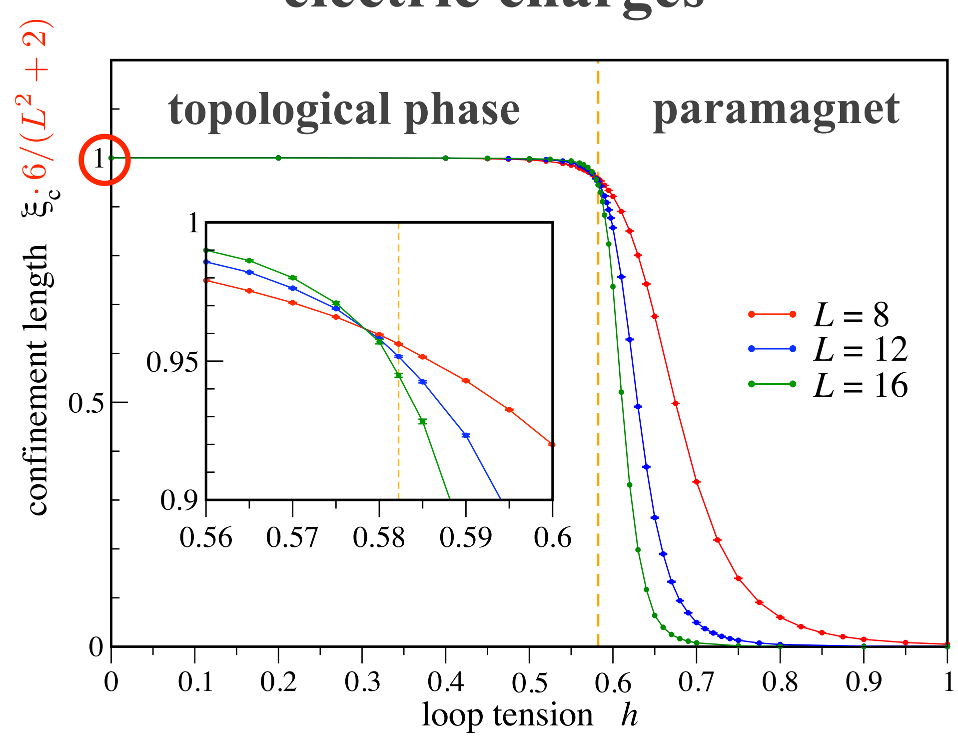
vertex excitations
“electric charges”

plaquette excitations
“magnetic vortices”

magnetic vortices

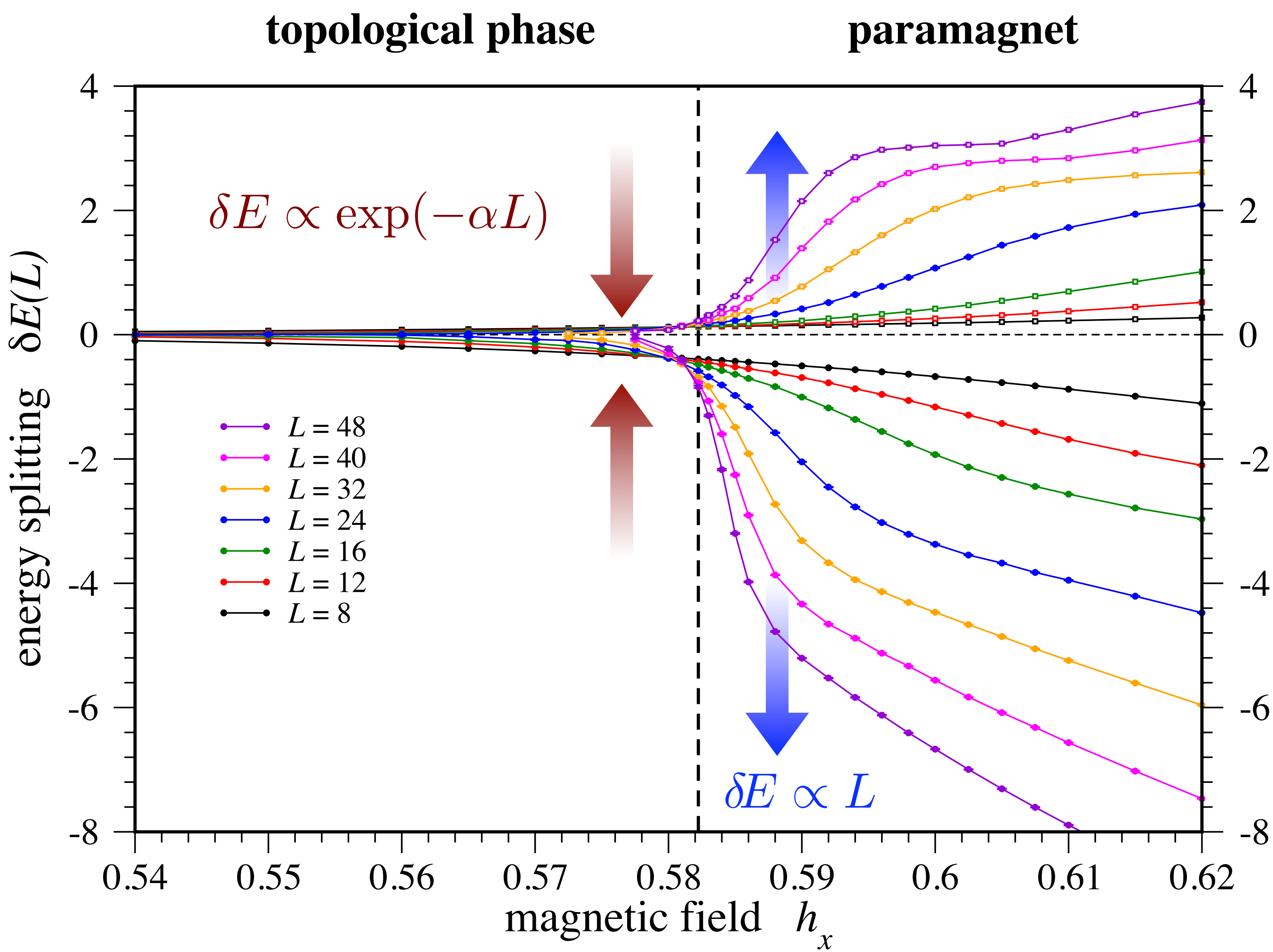


electric charges



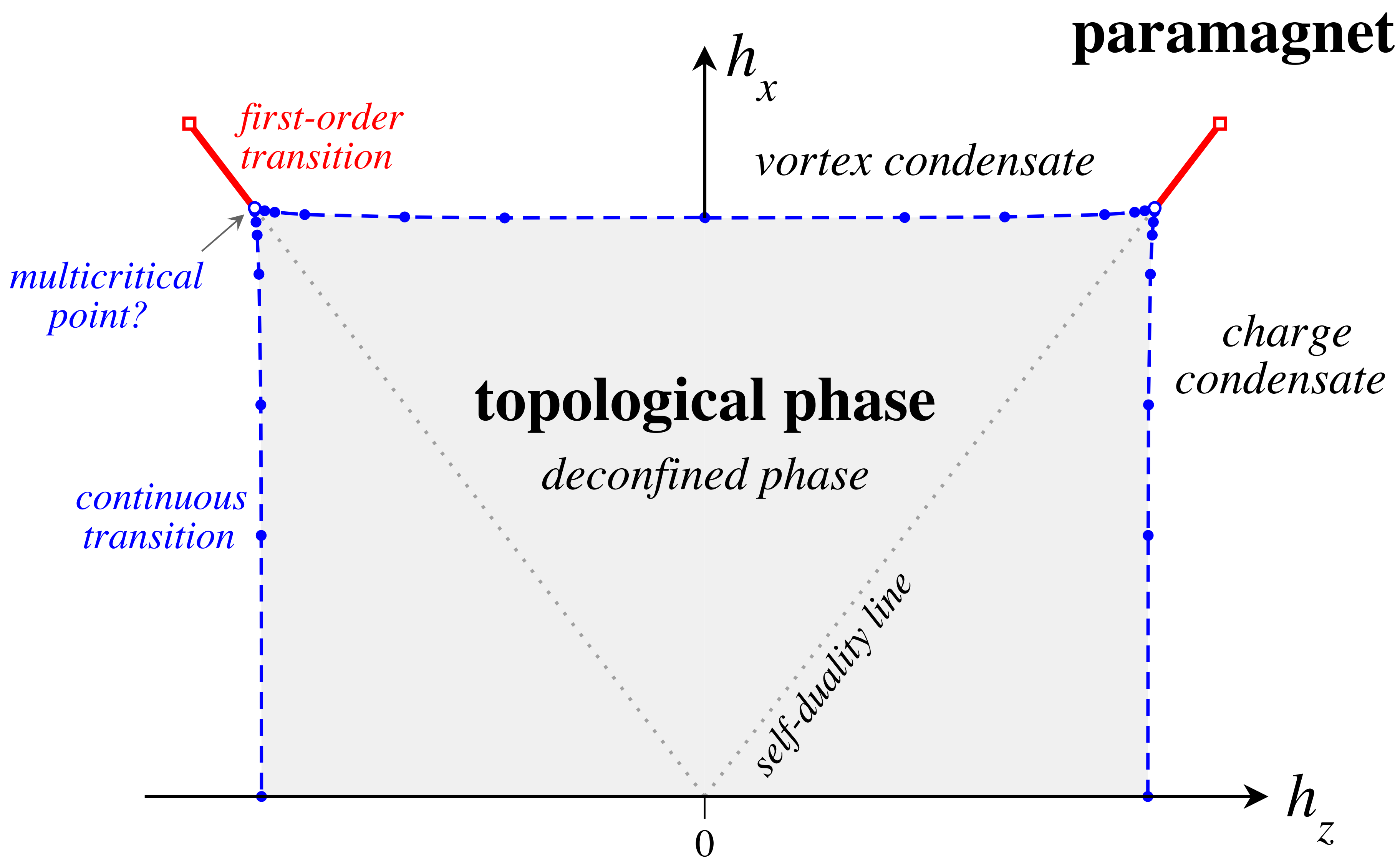
Splitting the topological degeneracy

ST *et al.*, Phys. Rev. Lett. **98**, 070602 (2007).

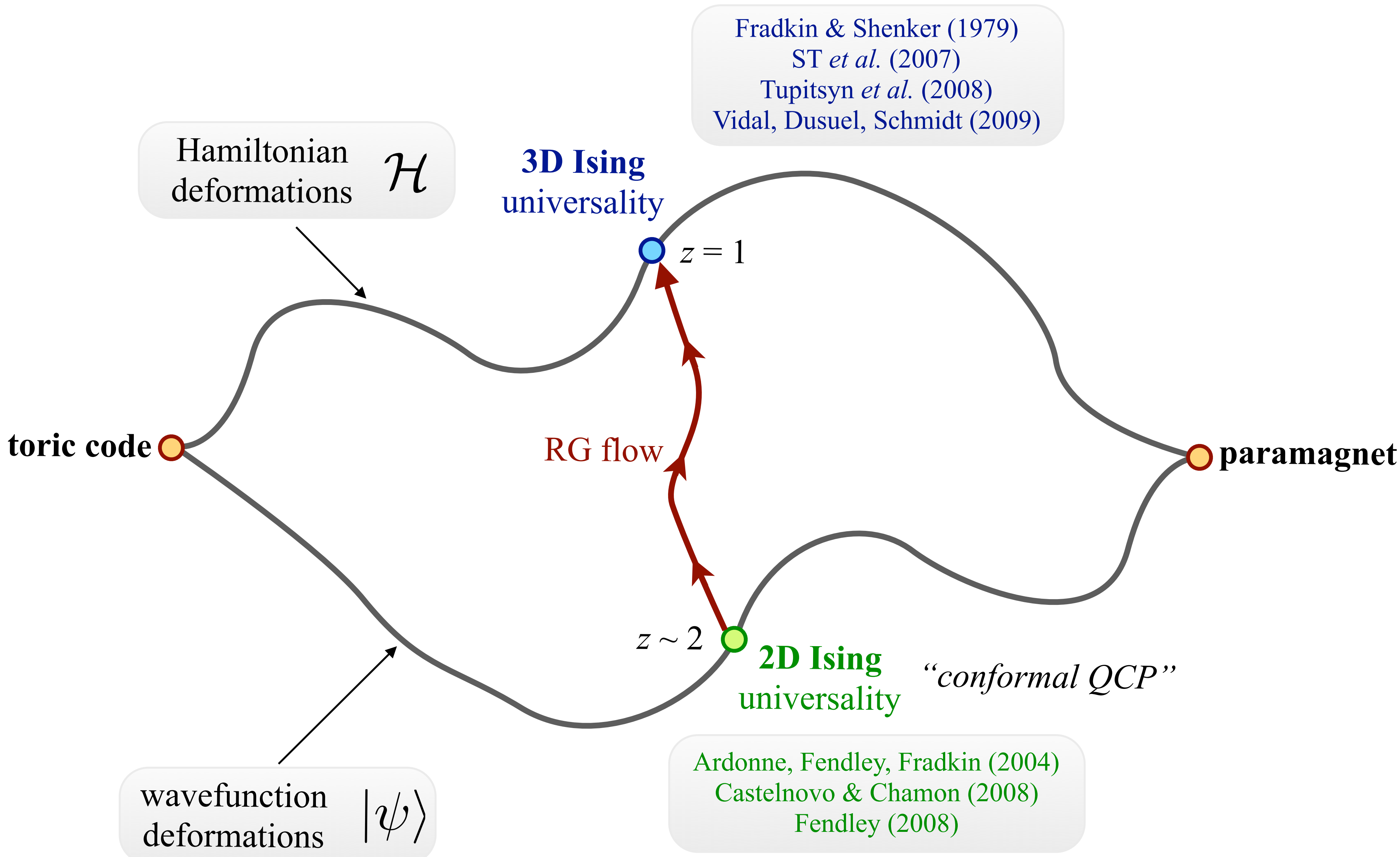


Phase diagram

I.S. Tupitsyn *et al.*, arXiv:0804.3175



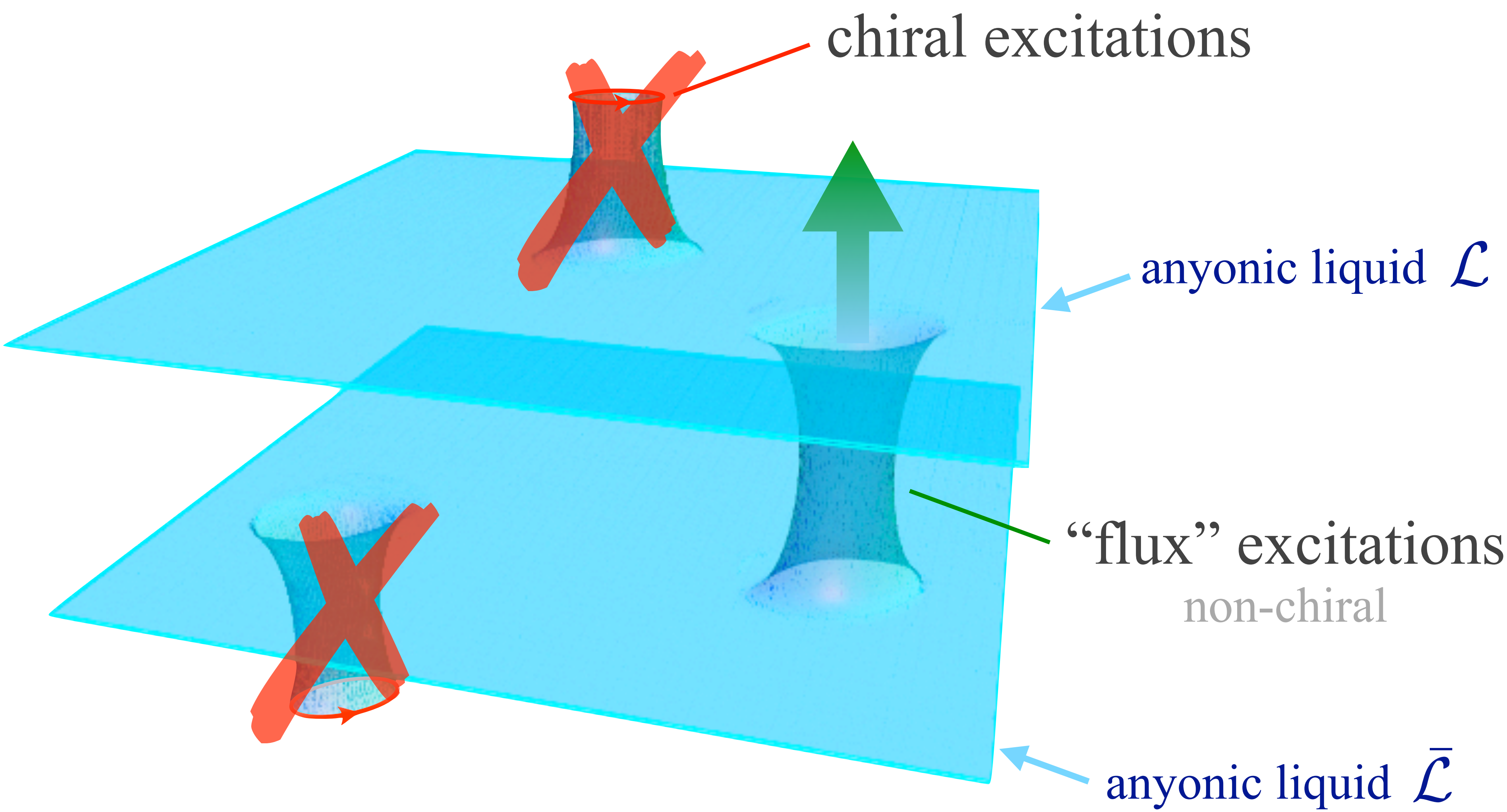
Toric code: phase transitions



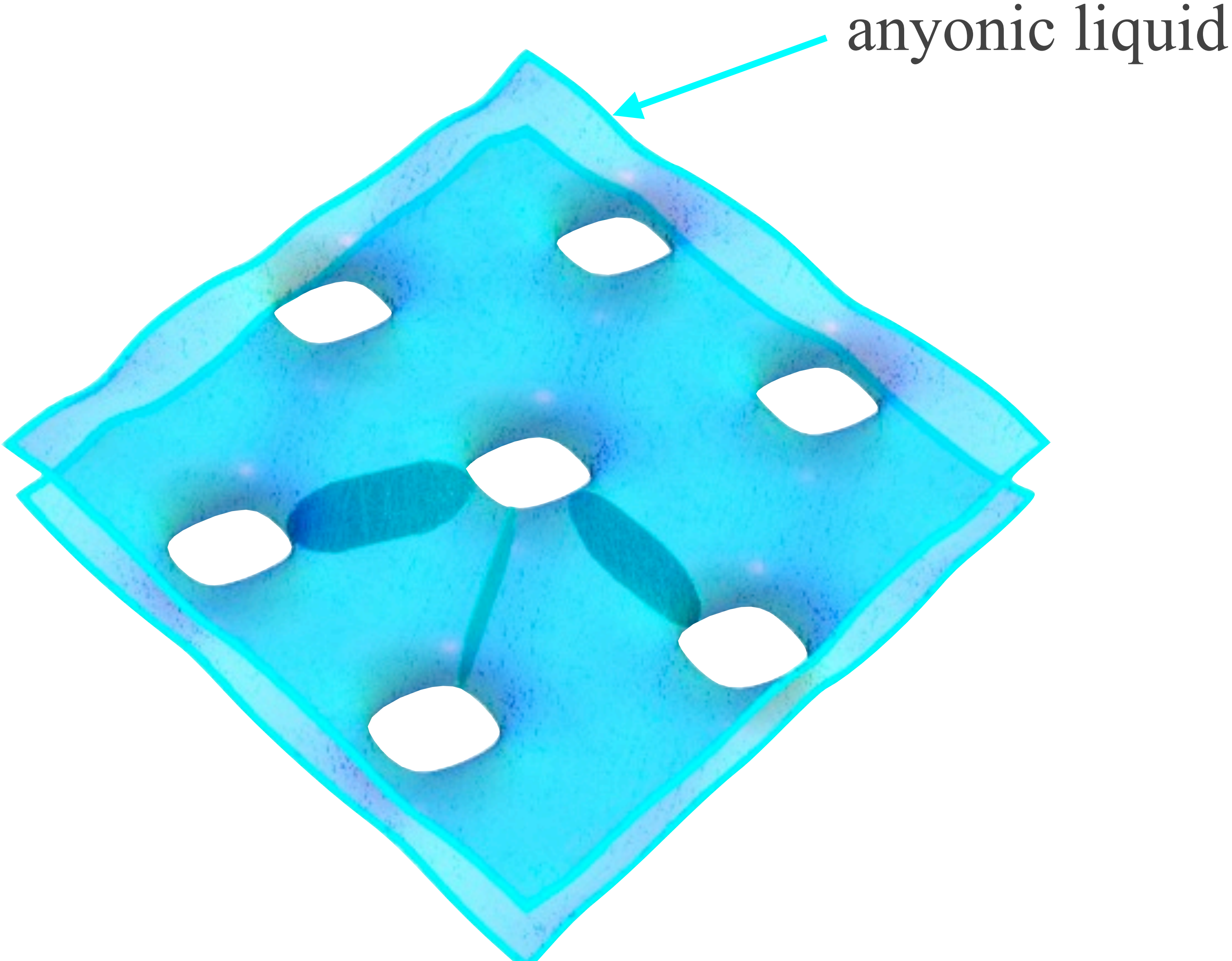
Quantum double models

for time-reversal invariant systems

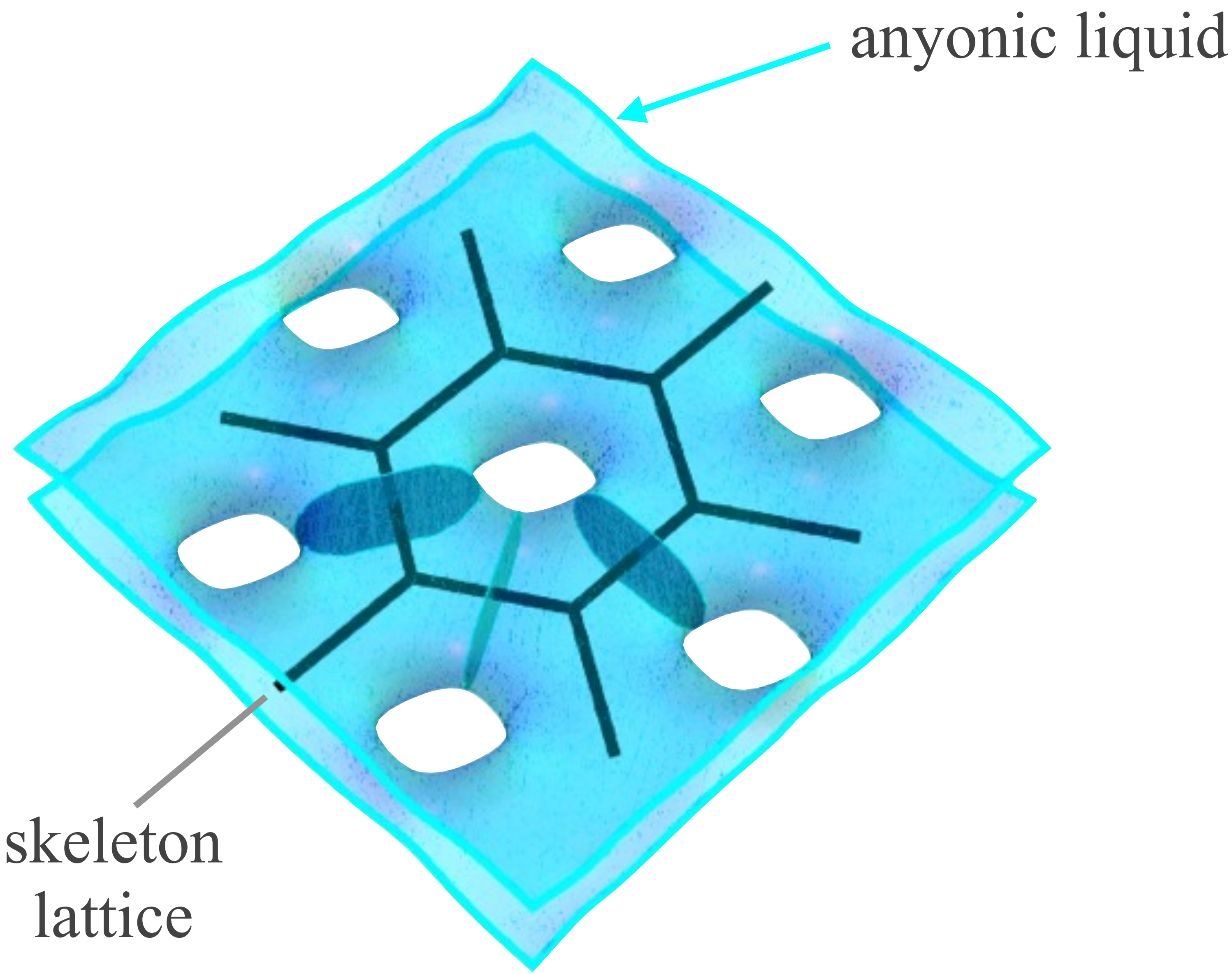
Time-reversal invariant liquids



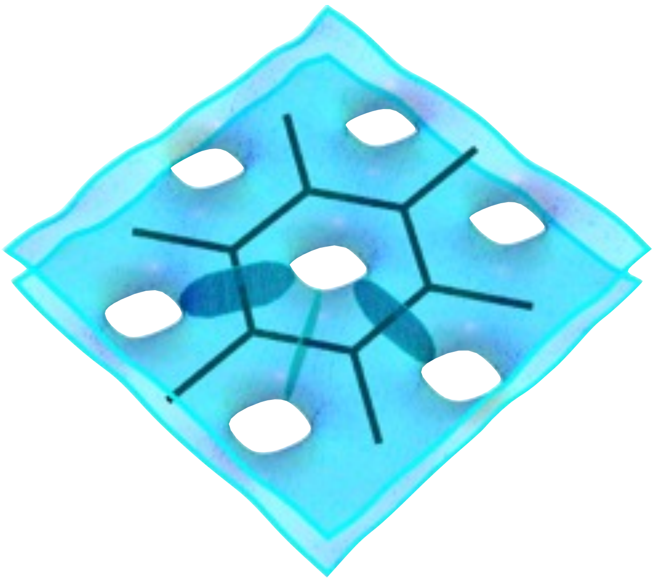
The skeleton lattice

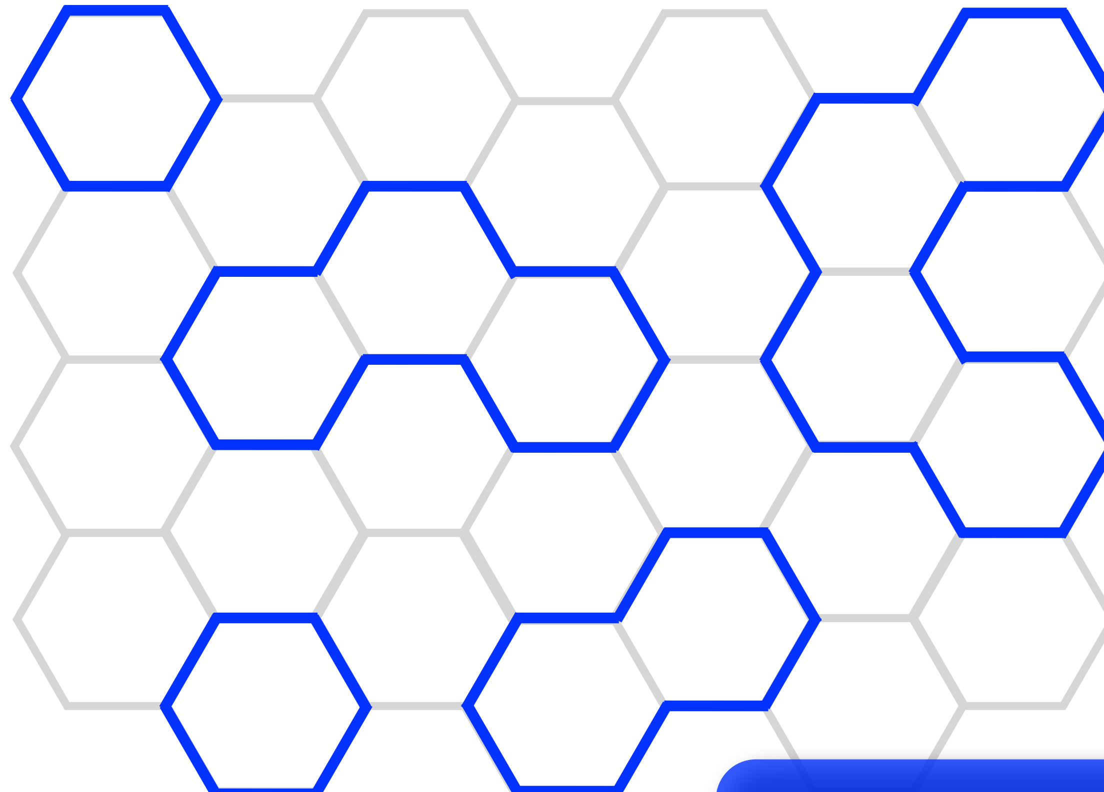


The skeleton lattice



Abelian liquids → loop gas / toric code

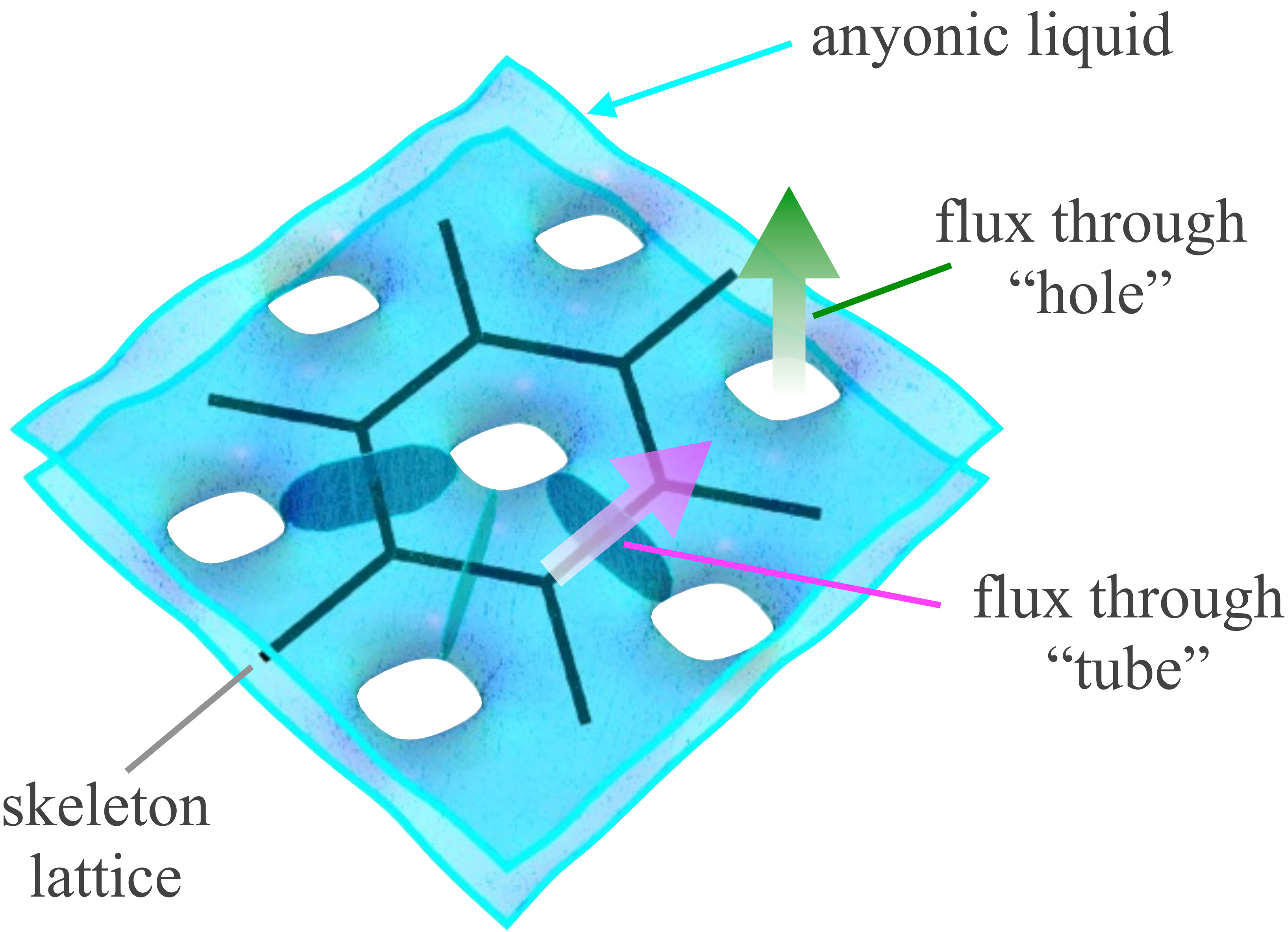




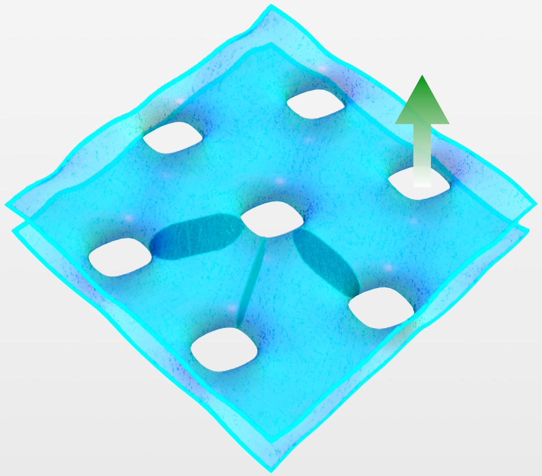
loop gas

Semion fusion rules $s \times s = 1$

Flux excitations



Extreme limits



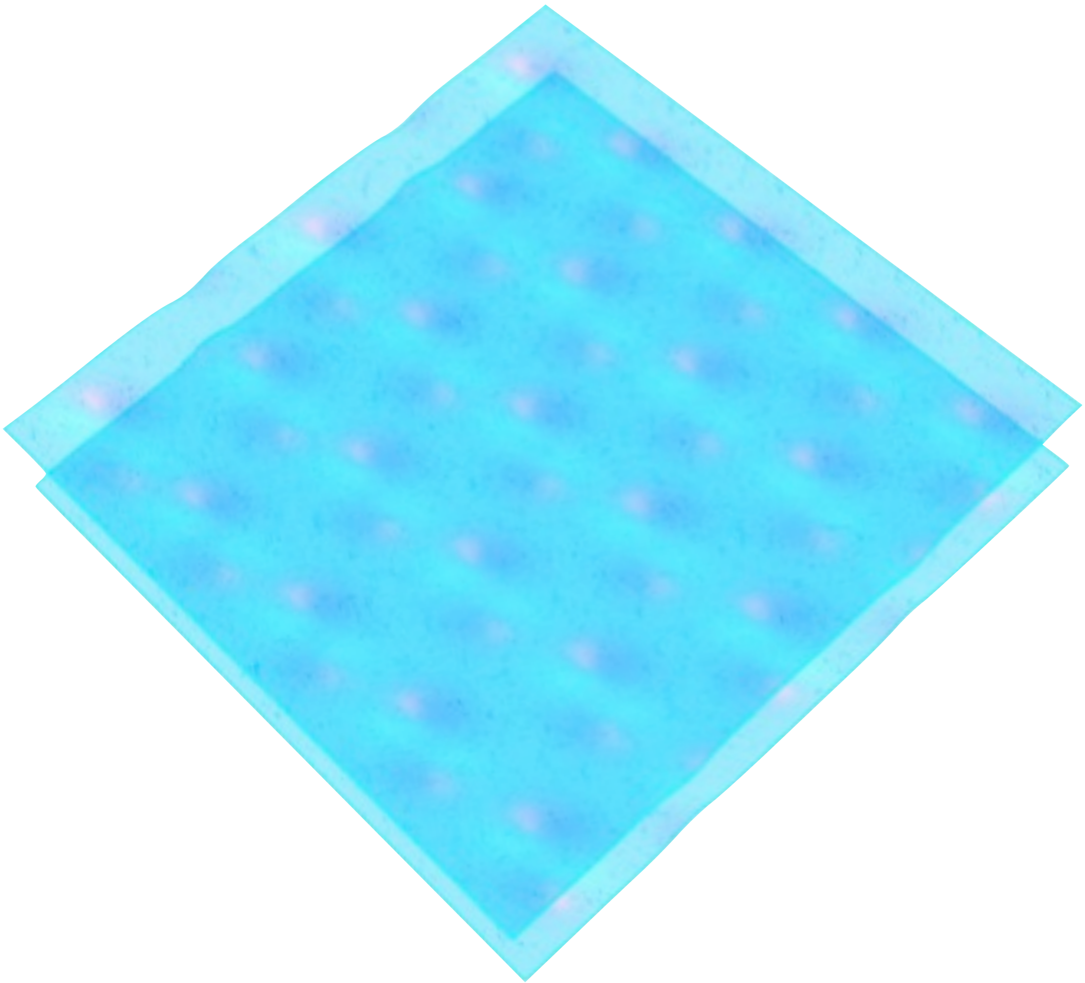
no flux through "holes"

↓

close holes

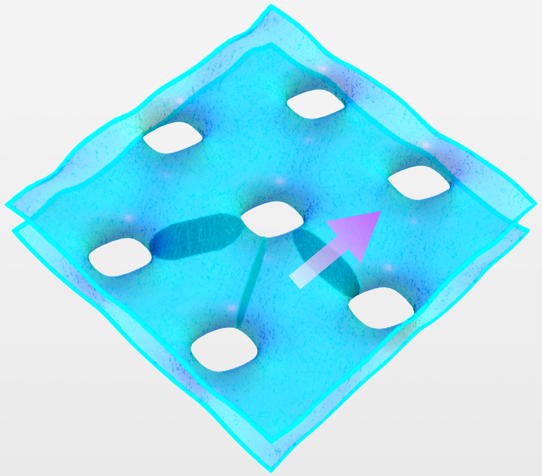
↓

"two sheets"

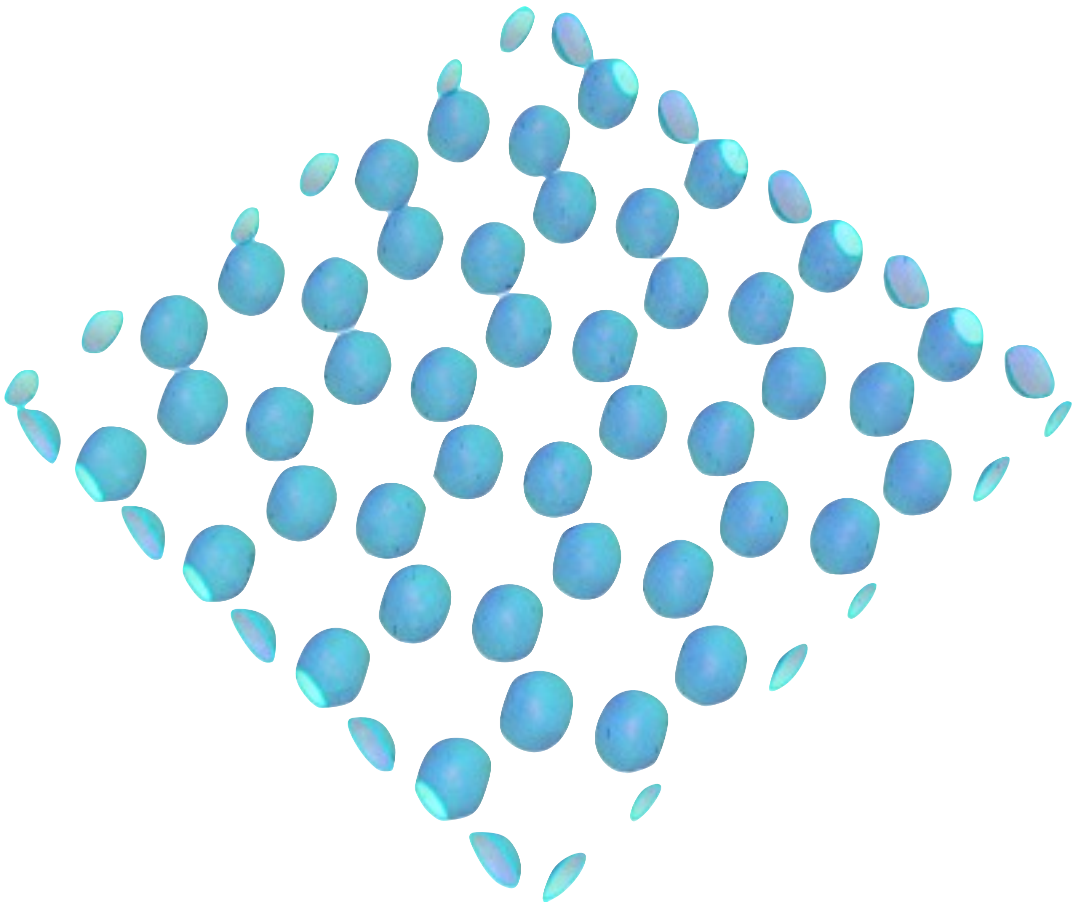


The "two sheets" ground state exhibits **topological order**.
In the toric code model this is the flux-free, loop gas ground state

Extreme limits

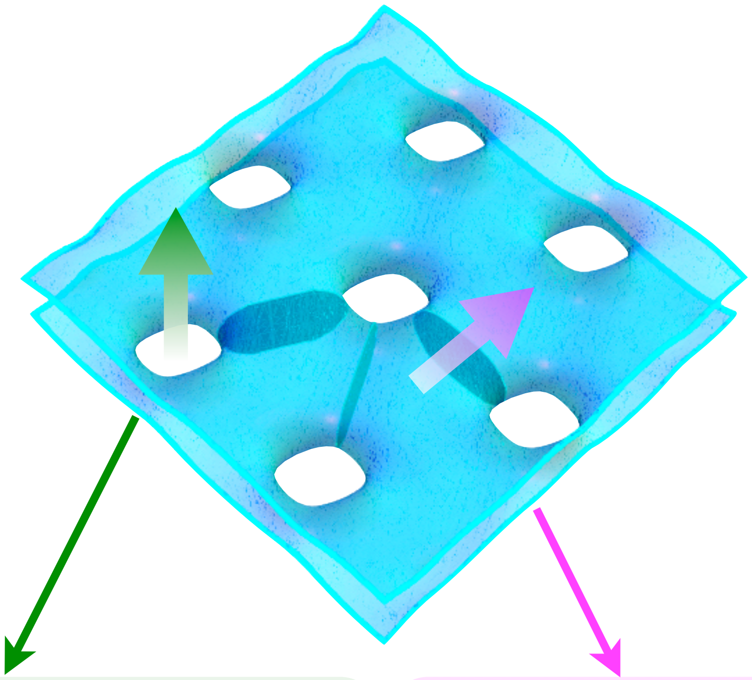


no flux through “tubes”
↓
pinch off tubes
↓
“decoupled spheres”



The “decoupled spheres” ground state exhibits **no topological order**.
In the toric code model this is the paramagnetic state.

Connecting the limits: a microscopic model

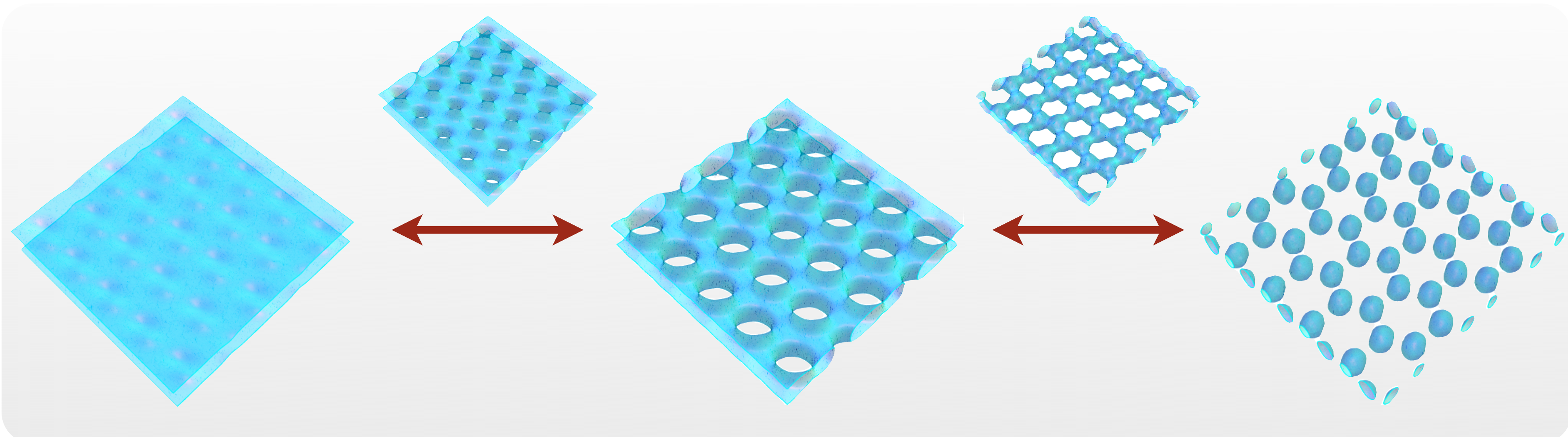


$$\mathcal{H} = -J_p \sum_{\text{plaquettes } p} \delta_{\phi(p),1} - J_e \sum_{\text{edges } e} \delta_{\ell(e),1}$$

Varying the couplings J_e/J_p we can connect the two extreme limits.

Semions (Abelian):
Toric code in a magnetic field / loop gas with loop tension J_e/J_p.

The quantum phase transition

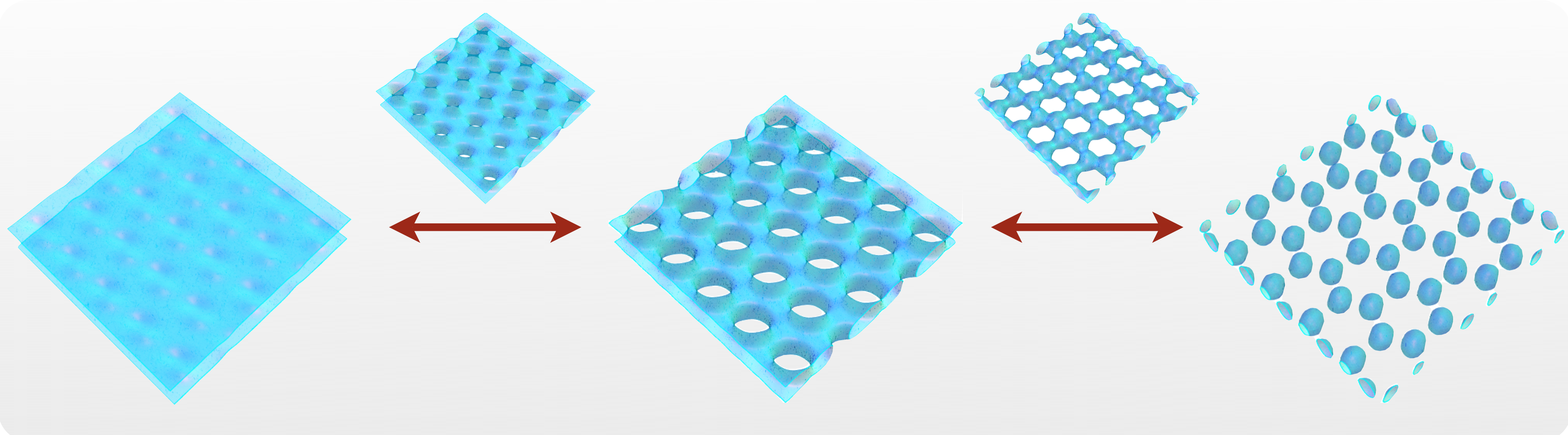


“two sheets”
topological order

“decoupled spheres”
no topological order

vary **loop tension** J_e/J_p

The quantum phase transition



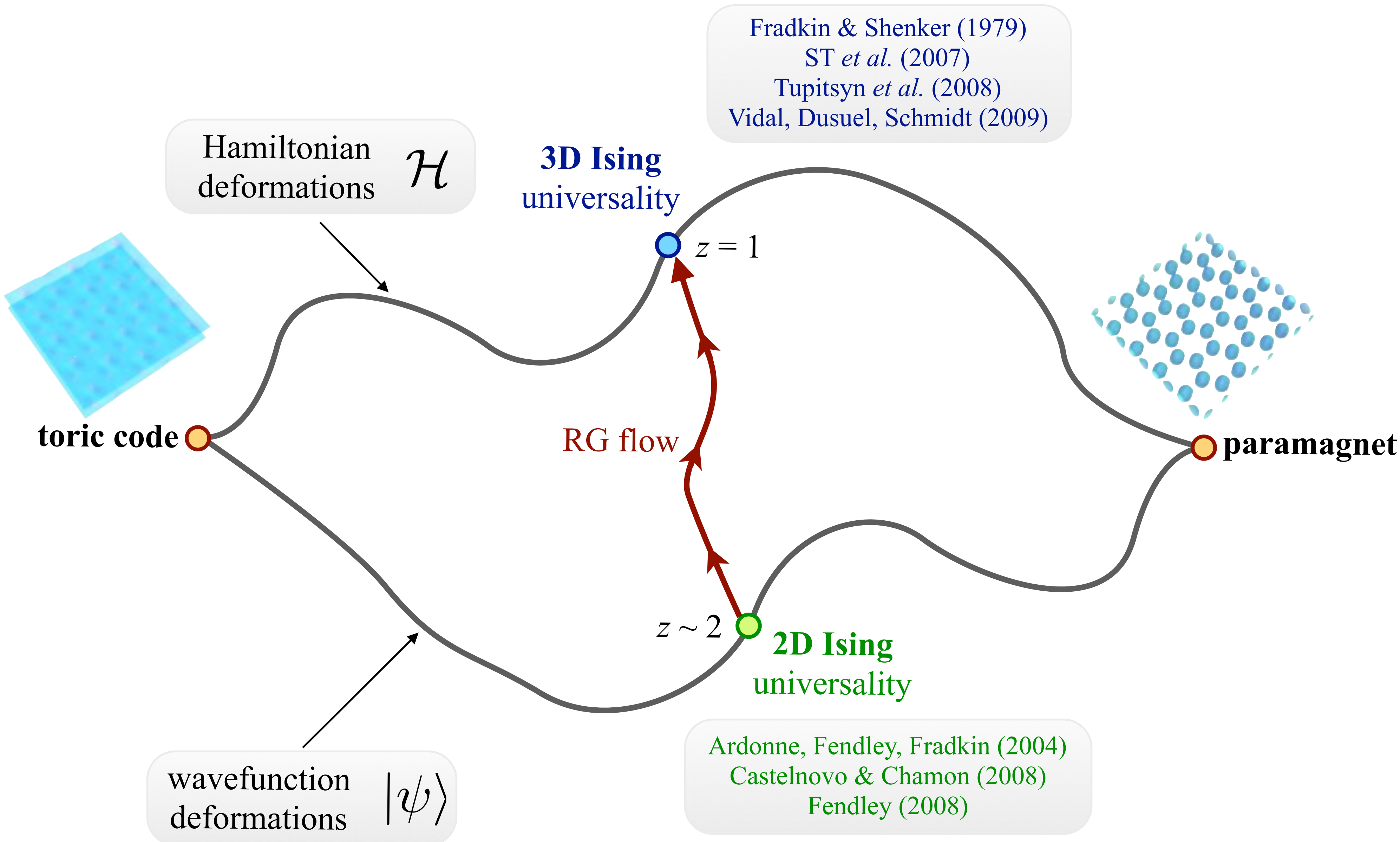
“two sheets”
topological order

“decoupled spheres”
no topological order

“quantum foam”
topology fluctuations on all length scales

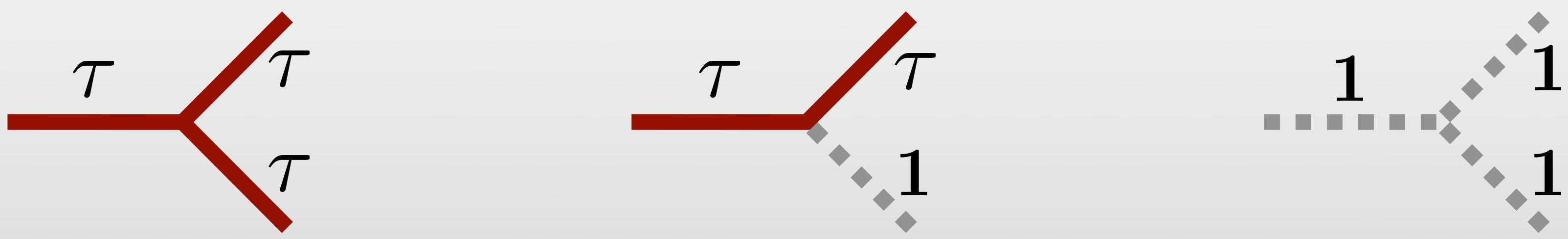
continuous transition

Universality classes (semions)

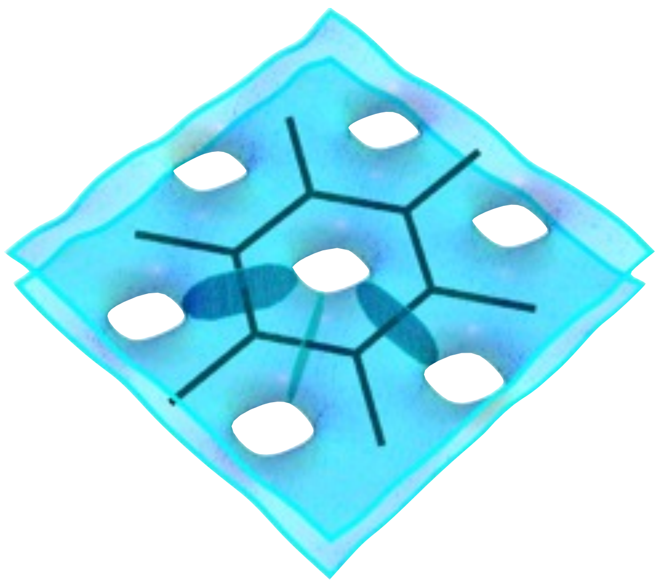


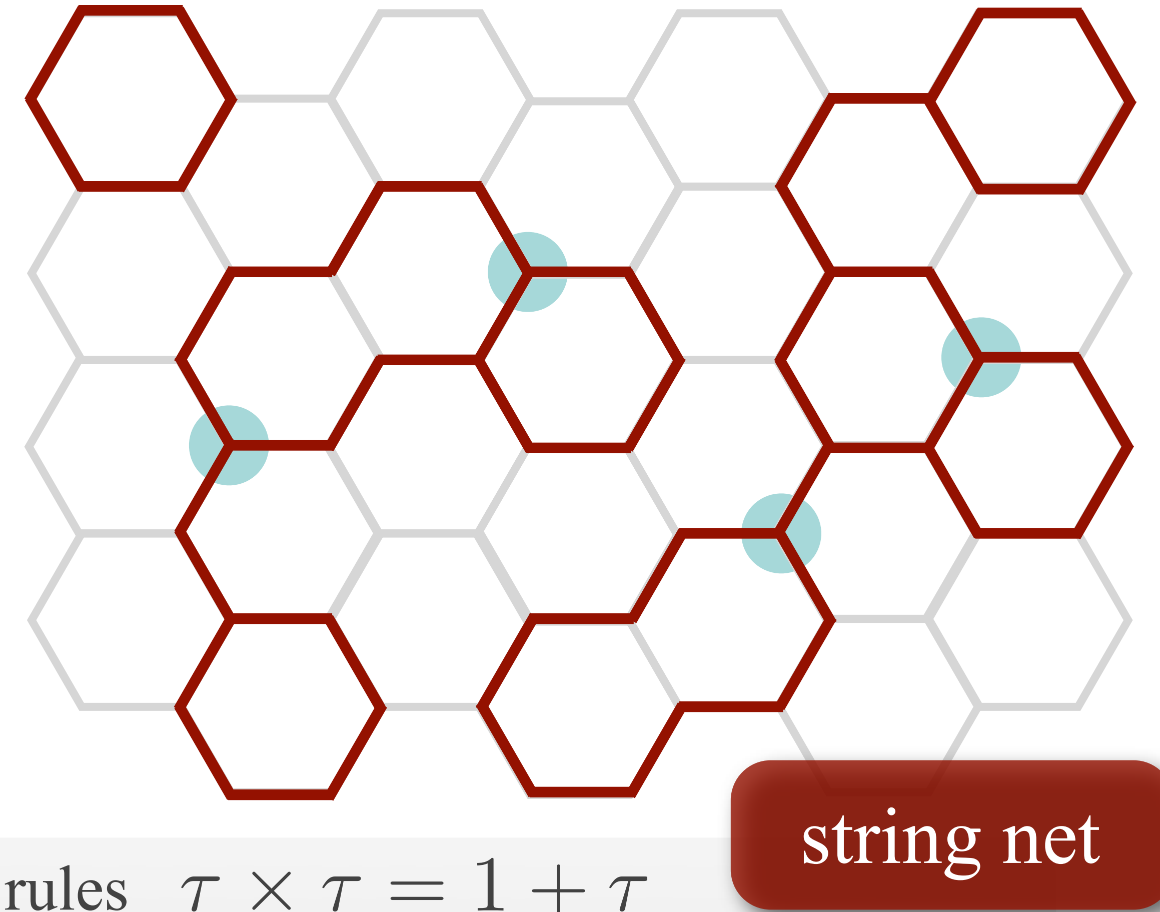
The **non-Abelian** case

Fibonacci anyon fusion rules $\tau \times \tau = 1 + \tau$

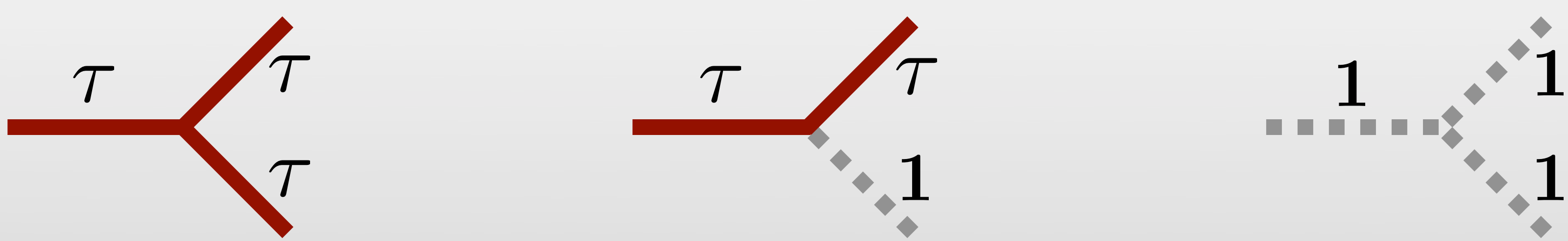


Non-abelian liquids → string net

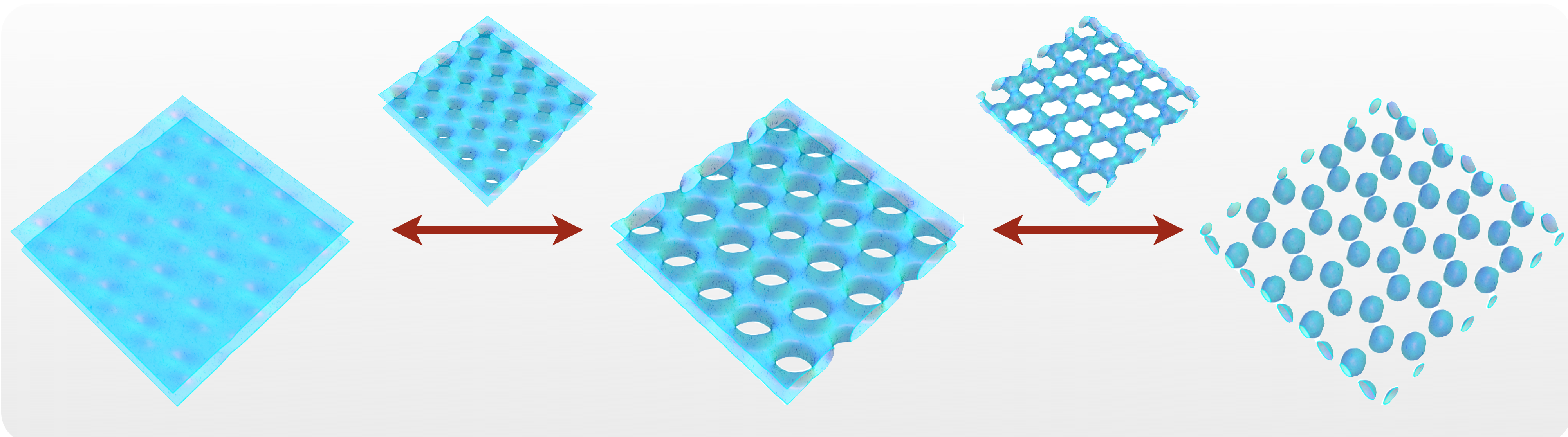




Fibonacci anyon fusion rules $\tau \times \tau = 1 + \tau$



The quantum phase transition

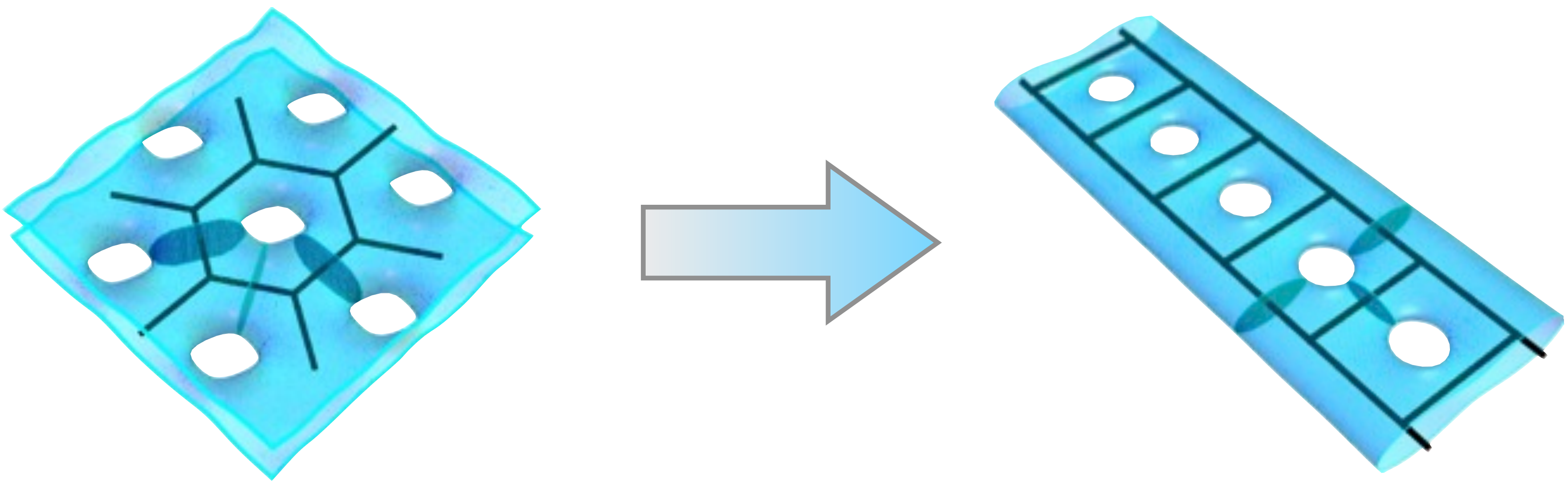


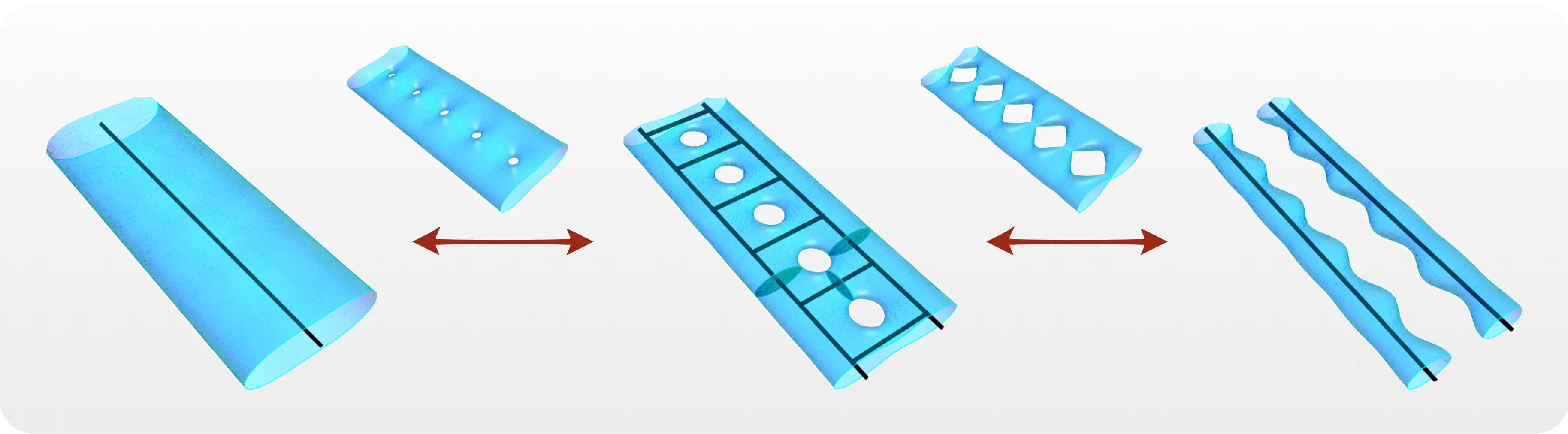
“two sheets”
topological order

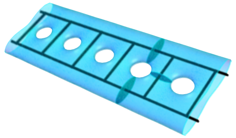
“decoupled spheres”
no topological order

vary **string tension** J_e/J_p

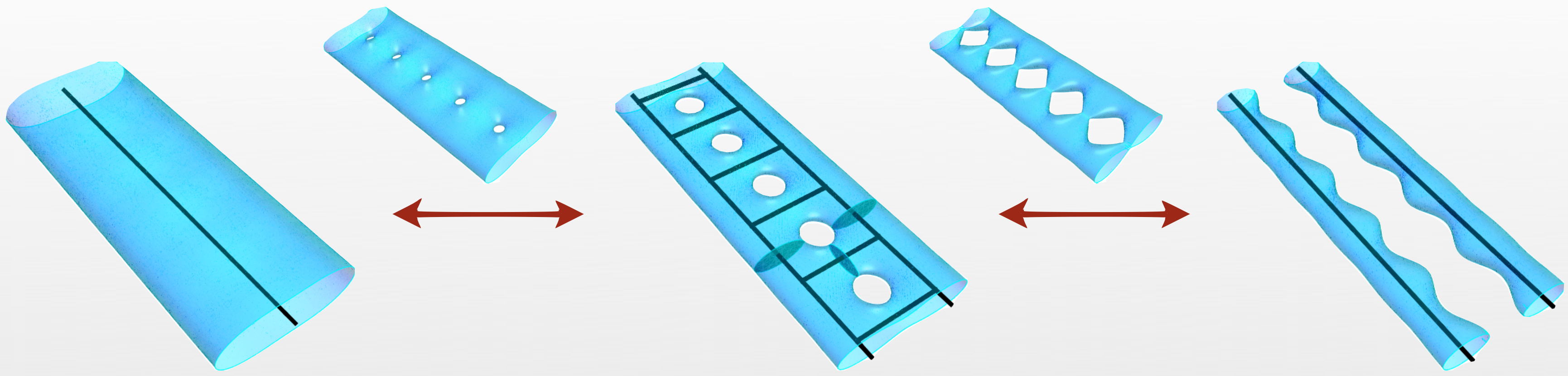
One-dimensional analog





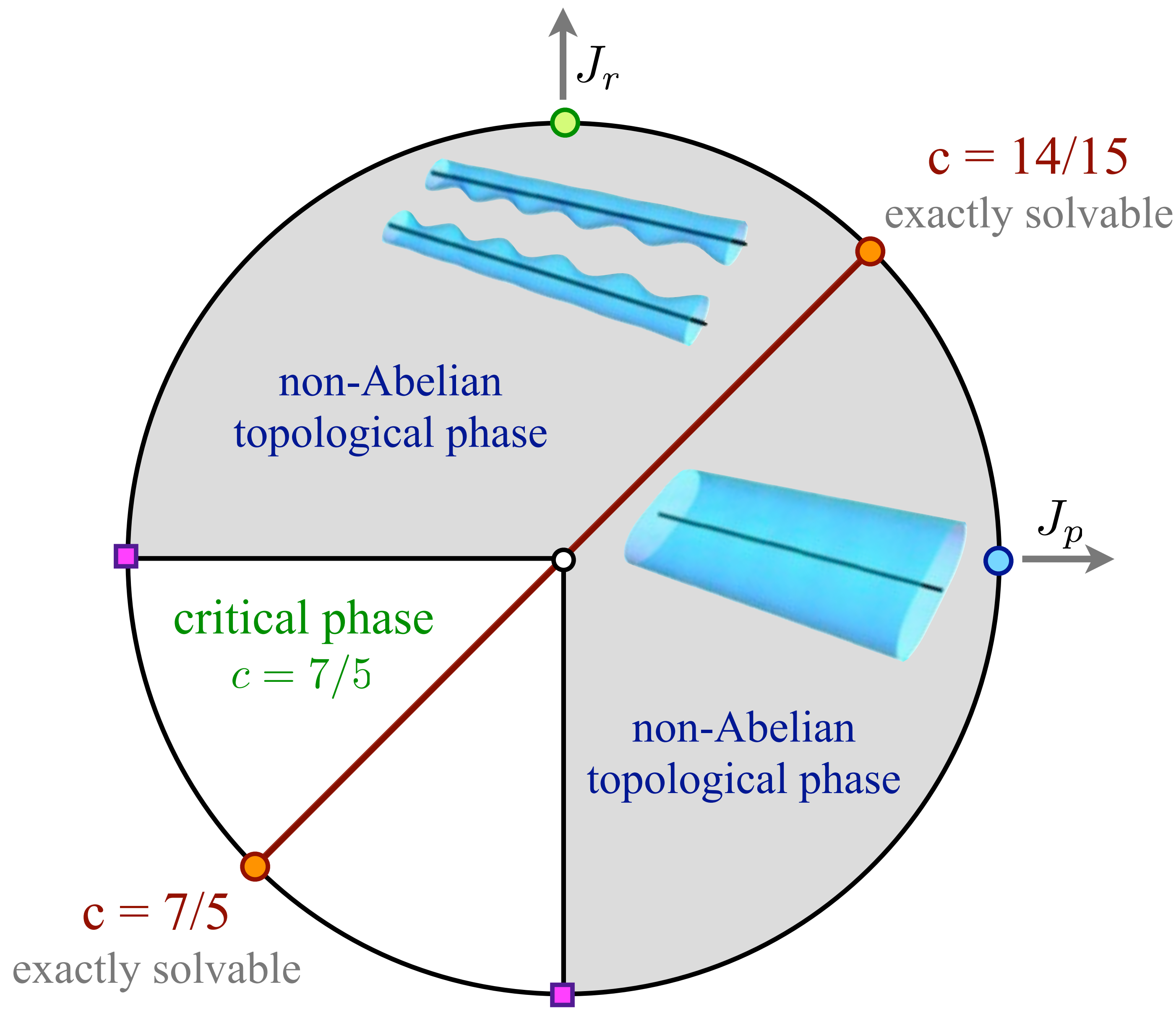


A **continuous** transition



- A **continuous** quantum phase transition connects the two extremal topological states.
- The transition is driven by **topology fluctuations** on all length scales.
- The gapless theory is **exactly sovable**.

Phase diagram



Gapless theory & exact solution

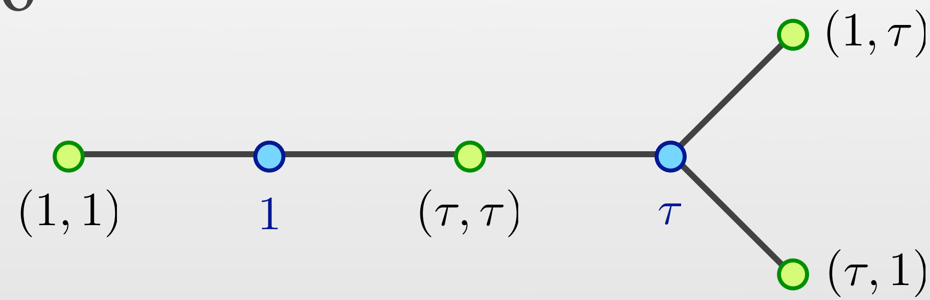
The operators in the Hamiltonian form a
Temperley-Lieb algebra

$$(X_i)^2 = d \cdot X_i \qquad X_i X_{i\pm1} X_i = X_i \qquad [X_i, X_j] = 0$$

for $|i-j| \geq 2$

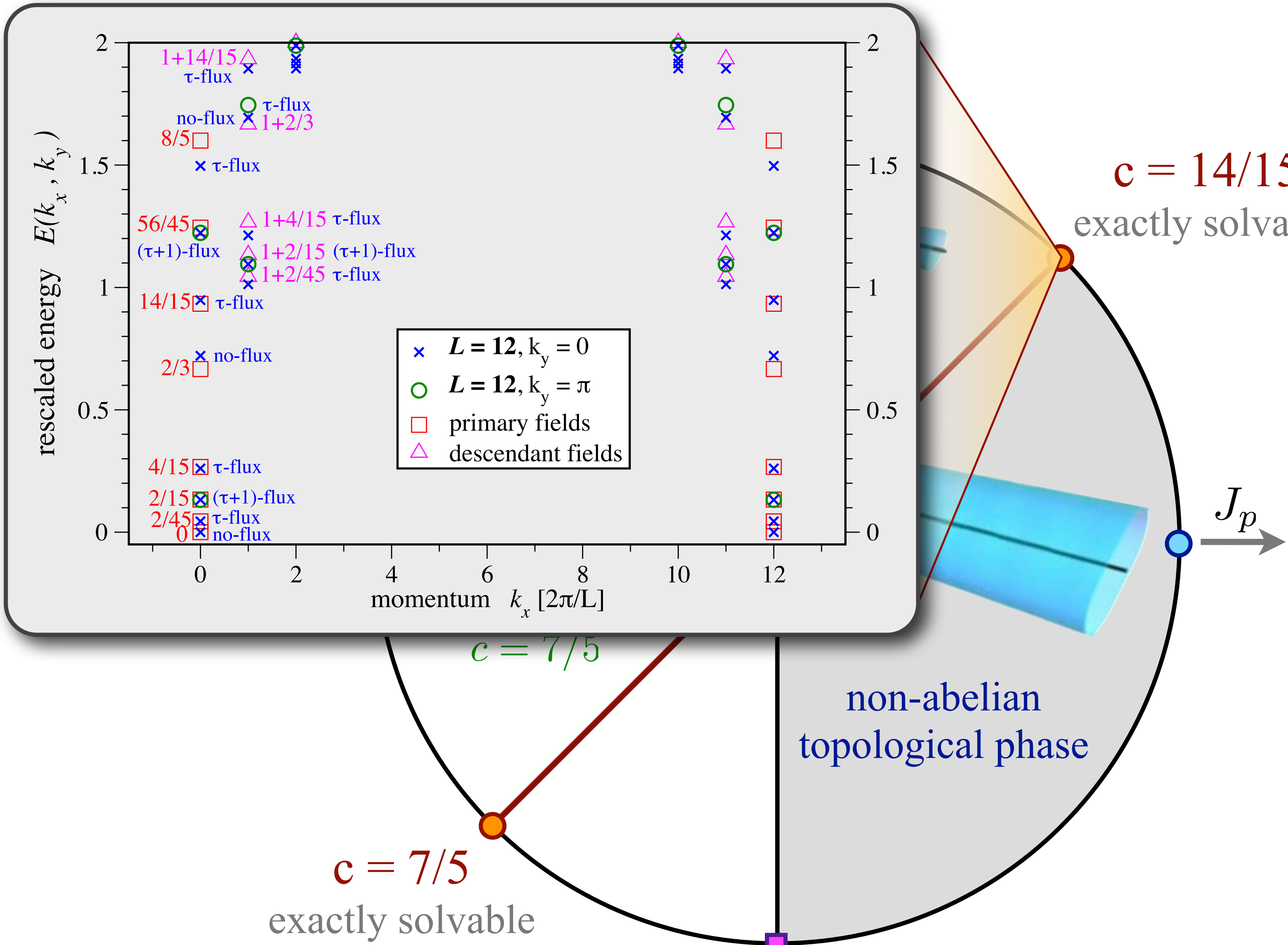
$$d = \sqrt{2+\phi}$$

The Hamiltonian maps onto the Dynkin diagram **D₆**

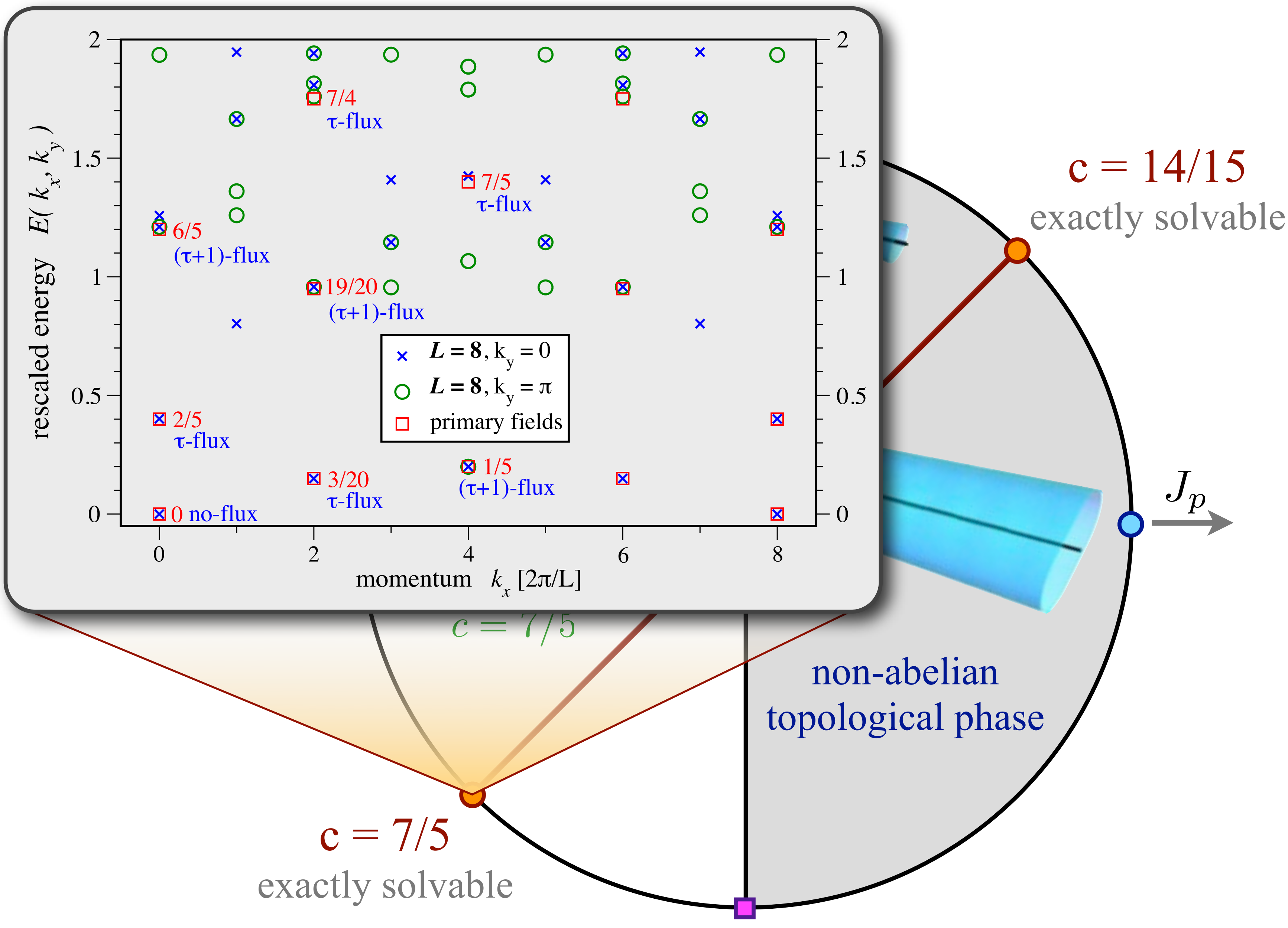


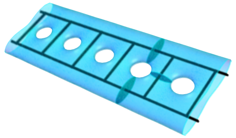
The gapless theory is a CFT with $c = 14/15$.

Gapless theory & exact solution

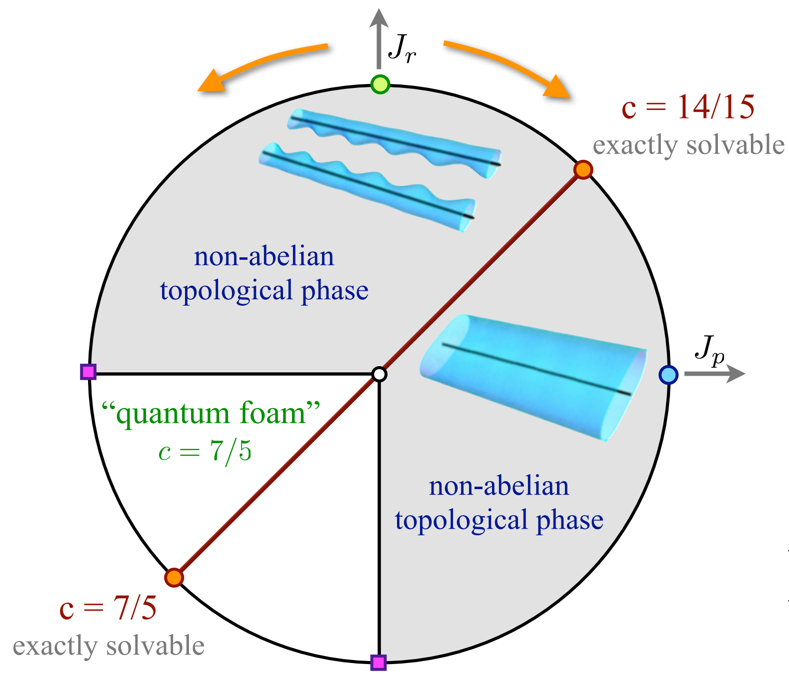


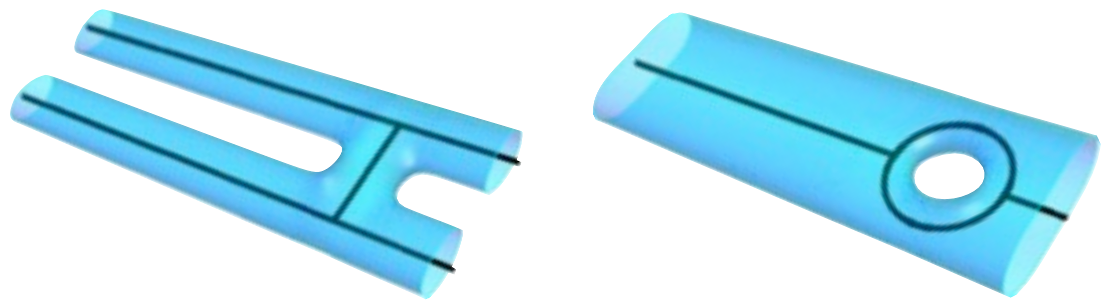
Gapless theory & exact solution

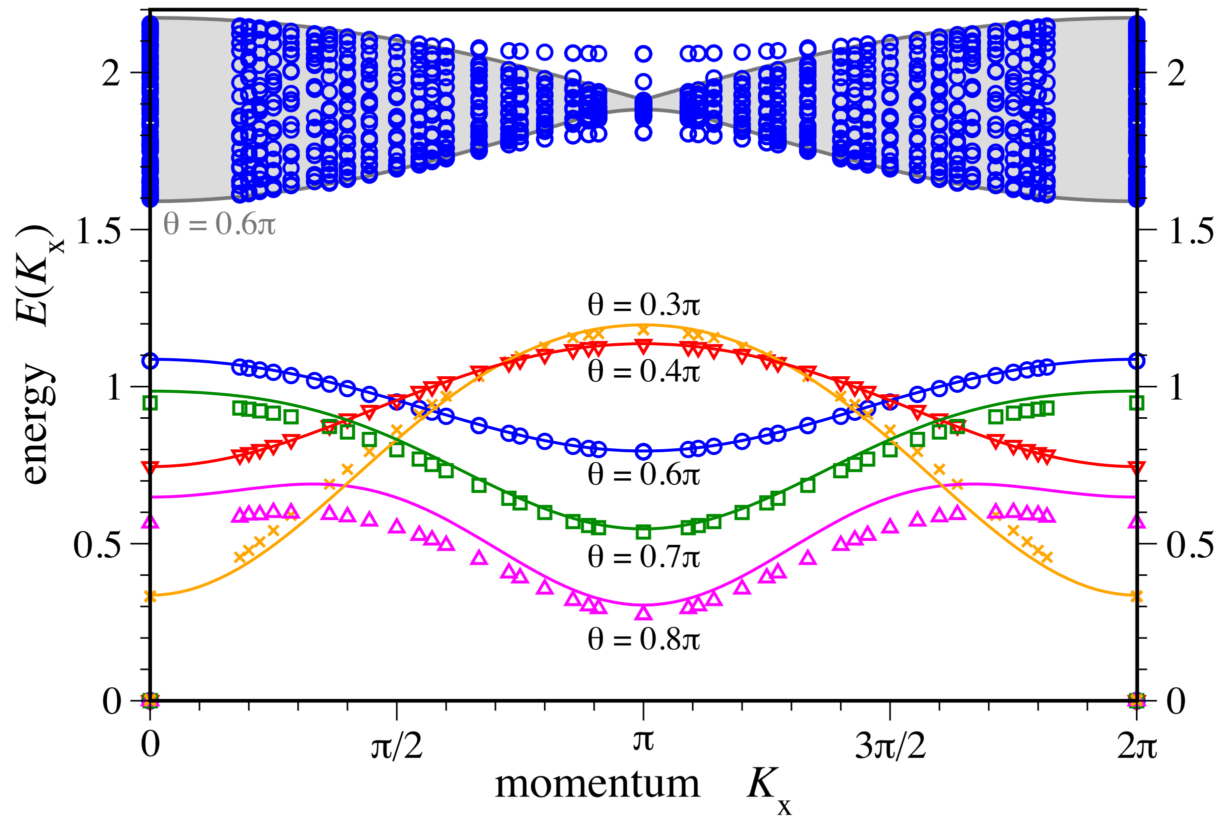




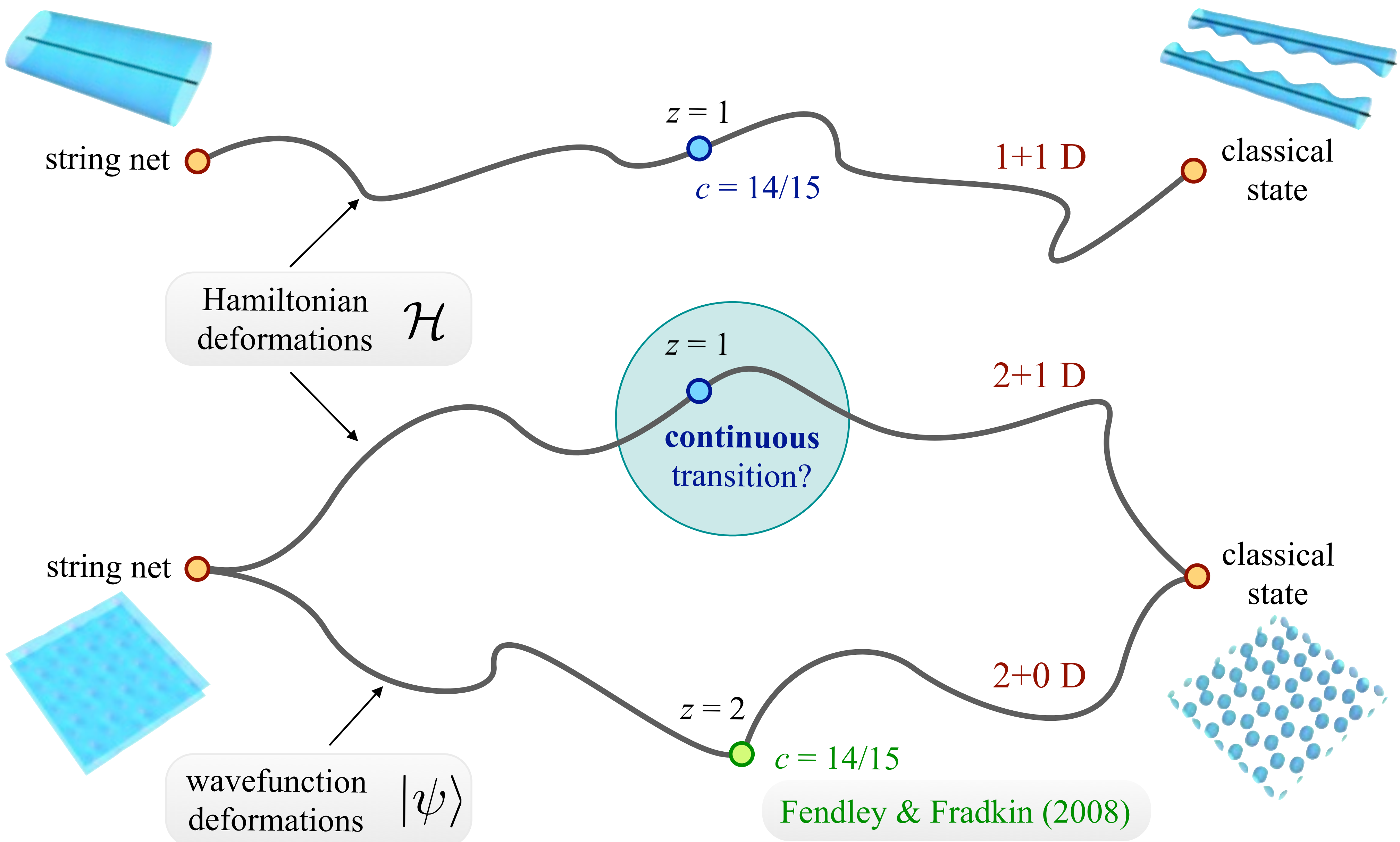
Dressed flux excitations







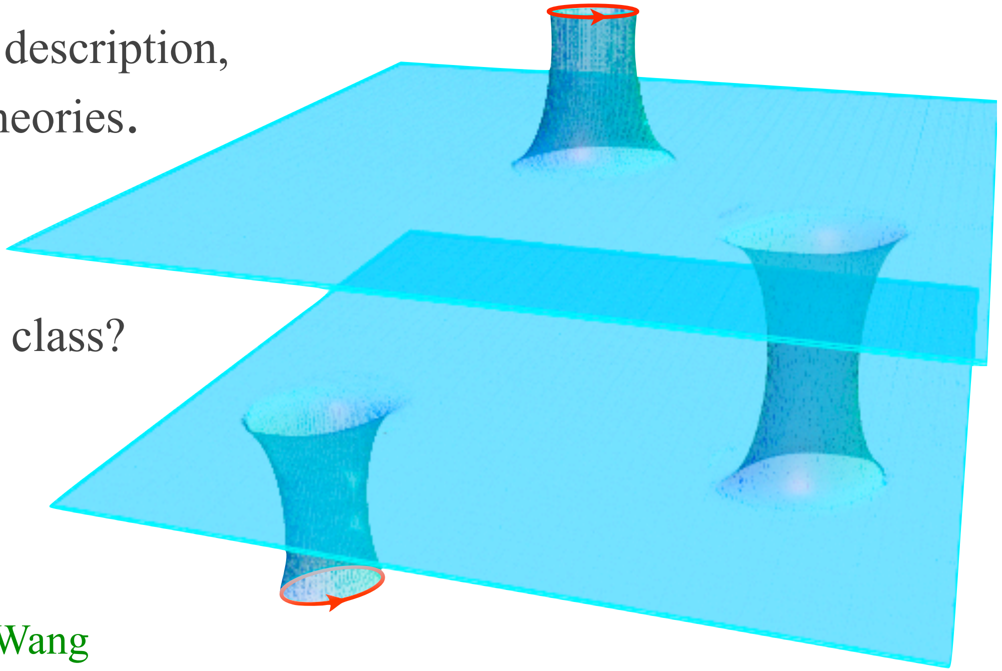
Back to two dimensions



Summary

- A “topological” framework for the description of topological phases and their phase transitions.
- Unifying description of loop gases and string nets.
- Quantum phase transition is driven by fluctuations of topology.

- Visualization of a more abstract mathematical description, namely doubled non-Abelian Chern-Simons theories.
- The 2D quantum phase transition out of a non-Abelian phase is still an open issue: A continuous transition in a novel universality class?



C. Gils, ST, A. Kitaev, A. Ludwig, M. Troyer, and Z. Wang
arXiv:0906.1579 → Nature Physics **5**, 834 (2009).