Renormalization of pinned elastic systems: how does it work beyond one loop ?

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We study the field theories for pinned elastic systems at equilibrium and at depinning. Their β-functions differ to two loops by novel "anomalous" terms. At equilibrium we find a roughness $\zeta = 0.20829804\epsilon + 0.006858\epsilon^2$ (random bond), $\zeta = \epsilon/3$ (random field). At depinning we prove two-loop renormalizability and that random field attracts shorter range disorder. We find $\zeta = \frac{\epsilon}{3}(1+0.14331\epsilon)$, $\epsilon = 4-d$, in violation of the conjecture $\zeta = \epsilon/3$, solving the discrepancy with simulations. For long range elasticity $\zeta = \frac{\epsilon}{3}(1+0.39735\epsilon)$, $\epsilon = 2-d$, much closer to the experimental value (≈ 0.5 both for liquid helium contact line depinning and slow crack fronts) than the standard prediction 1/3.

The aim of this Letter is to report progress on a conceptual issue and, as a byproduct, to resolve a longstanding discrepancy between theory and numerical simulations or experiments. The issue is whether it is possible to construct a field theory of disordered elastic systems, at equilibrium and at depinning, renormalizable beyond one loop as for standard critical phenomena. A discrepancy exists at present between the value for the roughness exponent ζ predicted by theory ($\zeta = \epsilon/3$ exactly) and simulations as well as experiments on wetting and on cracks.

Numerous experimental systems can indeed be modelled as elastic objects pinned by random impurities, with specific features. Interfaces in magnets [1] experience either random bond RB (i.e. short range) disorder or random field RF (i.e. long range) disorder. Charge density waves (CDW) or the Bragg glass in superconductors [2] are periodic objects. The contact line of liquid helium meniscus on a rough substrate is governed by long range elasticity and so are slowly propagating cracks [3–6]. They can all be parametrized by a height (or displacement) field $u(x)$ (x being the d-dimensional internal coordinate of the elastic object), with in some cases $N > 1$ components. The roughness exponent ζ:

$$|u(x) - u(x')|^2 \sim |x - x'|^{2\zeta} \tag{1}$$

is measured in experiments for systems at equilibrium ($\zeta_{\rm eq}$) or driven by a force f. Other exponents describe the velocity near the depinning threshold f_c, $v \sim (f - f_c)^\beta$, the scaling of the dynamical response, $t \sim x^z$, and the local velocity correlation length $\xi \sim (f - f_c)^{-\nu}$.

The study of pinned elastic systems, among a broader class of disordered models (e.g. random field spin models), is notably difficult due to dimensional reduction DR which renders naive perturbation theory useless [1,7]. Indeed to *any* order in the disorder at zero temperature $T = 0$, any physical observable is found to be *identical* to its (trivial) average in a Gaussian random force (Larkin) model. A bold way out of this puzzle was proposed by Fisher [8] within a one-loop renormalization group analysis of the interface problem in $d = 4 - \epsilon$. He noted that the coarse grained disorder correlator becomes *non-analytic* beyond the Larkin scale L_c, yielding large scale results distinct from naive perturbation theory. An infinite set of operators become relevant in $d < 4$, parameterized by the second cumulant $R(u)$ of the random potential, i.e. $\overline{V(x,u)V(x',u')} = \delta_{x-x'}R(u-u')$. Explicit solution of the one-loop Functional RG equation (FRG) for $R(u)$ gives several non trivial attractive fixed points (FP) to $\mathcal{O}(\epsilon)$ proposed in [8] to describe RB, RF disorder and in [2], periodic systems (RP) such as CDW or vortex lattices. All these FP exhibit a "cusp" singularity as $R^{*\prime\prime}(u) - R^{*\prime\prime}(0) \sim |u|$ at small $|u|$. Large N and variational methods [11,2] confirmed the picture and the cusp was interpreted in terms of shocks in the renormalized force [12]. A FRG was also developed to one loop [9,10] to describe the *driven dynamics* just above depinning $f = f_c^+$, the cusp being linked to the threshold $f_c \sim |\Delta'(0^+)|$. Surprisingly, the flow equation for the correlator $\Delta(u)$ of the force $F(x,u)$ is, to one loop, *identical* to the one of the statics (with $\Delta(u) = -R''(u)$). Extension to temperature $T > 0$ yielded rounding of the cusp in a layer $u \sim T$ and the celebrated creep law [13].

Despite these successes, serious difficulties remain. First, in the last fifteen years since [8], no study has addressed whether the FRG yields, beyond one loop, a renormalizable field theory able to predict universal results [14]. Doubts were even raised [15] about the validity of the ϵ-expansion beyond the order $\mathcal{O}(\epsilon)$. Second, numerous simulations near depinning [9,16–18] seem to exclude $\zeta = \epsilon/3$ argued in [10] to be exact. In the case of long range elasticity, the prediction $\zeta = (2-d)/3$ [4] *disagrees* with the systematically larger value $\zeta \approx 0.55$ ($d = 1$) measured for liquid Helium contact line depinning [3] and for the in plane roughness of slow crack fronts [6] (see also simulations [19]).

In this Letter, we address these issues both for dynamics and statics. The main difficulty is the non-analytic nature of the theory (i.e. the fixed point action) at $T = 0$, which makes it a priori quite different from conventional critical phenomena. For *depinning*, we overcome the problem and show renormalizability at two-loop order. As a result we resolve several questions left unclear in previous works. We find that (i) quasi-static driven dynamics differs from statics at two loops (ii) shorter range disorder is within the RF universality class and (iii) the conjecture $\zeta = \epsilon/3$ is violated. This last result resolves the longstanding discrepancy with simulations. In the case of long range elasticity it yields $\zeta \approx 0.5$ for $d = 1$ and may thus explain the high value of ζ found in experiments on cracks and wetting. For the *statics* we find ambiguities at $T = 0$ which we lift using a renormalizability condition, yielding fixed points and $\zeta_{\rm eq}$ to $\mathcal{O}(\epsilon^2)$. This result is also obtained within an independent exact FRG study [20]. The FRG equation for the disorder contains new anomalous terms both for statics and dynamics, which are absent in an analytic theory. Our predictions for all exponents are shown in Tables I,II.

	d	ϵ	ϵ^2	estimate	simulation
	3	0.33	0.38	0.38±0.02	0.34±0.01 [9]
ζ	2	0.67	0.86	0.82±0.1	0.75±0.02 [16]
	1	1.00	1.43	1.2±0.2	1.25±0.05 [16]
	3	0.89	0.85	0.84±0.01	0.84±0.02 [9]
β	2	0.78	0.62	0.53±0.15	0.64±0.02 [9]
	1	0.67	0.31	0.2±0.2	$\approx$ 0.3 [16,18]
	3	0.58	0.61	0.62±0.01	
ν	2	0.67	0.77	0.85±0.1	0.77±0.04 [17]
	1	0.75	0.98	1.25±0.3	1±0.05 [18]
	3	0.208	0.215	0.215 ± 0.003	0.22 ± 0.01 [27]
$\zeta_{\rm eq}$	2	0.417	0.444	0.438 ± 0.007	0.41 ± 0.01 [27]
	1	0.625	0.687	2/3	2/3

Table I: exponents for depinning and statics ($\zeta_{\rm eq}$) as obtained, respectively: from setting $\epsilon = 4 - d$ in the one loop and two loop result, from Padé estimates together with scaling relations and from numerical works. For $\zeta_{\rm eq}$ we have improved the estimate using the exact result $\zeta_{\rm eq}(d = 1) = 2/3$.

	ϵ	ϵ^2	estimate		ϵ	ϵ^2	estimate
ζ	0.33	0.47	0.5 ± 0.1	β	0.78	0.59	0.4 ± 0.2
z	0.78	0.66	0.7 ± 0.1	ν	1.33	1.58	$2. \pm 0.4$

Table II: depinning exponents for long range elasticity in $d = 1$: ζ is consistent with experiments on contact line depinning ($\zeta \approx 0.5$ [3]) and cracks ($\zeta \approx 0.55 \pm 0.05$ [6]).

The starting point is the equation of motion:

$$\eta \partial_t u_{xt} = \partial_x^2 u_{xt} + F(x, u_{xt}) \tag{2}$$

with friction η and in the case of long range elasticity we replace (in Fourier) $q^2 u_q$ by $|q| u_q$ in the elastic force. Disorder averaged correlations $\overline{\langle A[u_{xt}] \rangle} = \langle A[u_{xt}] \rangle_S$ and responses $\overline{\delta \langle A[u] \rangle / \delta h_{xt}} = \langle \hat{u}_{xt} A[u] \rangle_S$ can be computed from the standard averaged dynamical action:

$$S = \int_{xt} \hat{u}_{xt} (\eta \partial_t - \partial_x^2) u_{xt} - \frac{1}{2} \int_{xtt'} \hat{u}_{xt} \hat{u}_{xt'} \Delta(u_{xt} - u_{xt'})$$

Finite temperature is studied adding $-\eta T \int_{xt} \hat{u}_{xt}^2$, driven dynamics adding $-f \int_{xt} \hat{u}_{xt}$ and shifting $u \to u + vt$ in S. We study the quasi-static limit $v = 0^+$, as well as equilibrium dynamics $f = 0$ where, via fluctuation dissipation relations, static quantities can be equivalently computed using S or the replicated hamiltonian [21].

It is useful to first study naive perturbation theory, in an *analytic* $\Delta(u)$ i.e. in its derivatives $\Delta^{(n)}(0)$, using the diagrammatic rules of Fig. 1. Since at each vertex there are one conservation rule for momentum and two for frequency we consider both unsplitted (local x) and splitted (bilocal t, t') vertices (and splitted a, b vertices in the statics). $T = 0$ power counting yields $\int_t \hat{u} u \sim x^{d-2}$ and $u \sim x^{\zeta}$, where $\zeta = \mathcal{O}(\epsilon = 4 - d)$ has to be determined. For an analytic $\Delta(u)$ the perturbation expansion of any (analytic) observable yields identical results [22] as setting $\Delta(u) \equiv \Delta(0)$ and one obtains the incorrect DR roughness $\zeta = \epsilon/2$. Temperature is formally irrelevant and must be scaled [23] as $T = \tilde{T} \Lambda^{-2+\epsilon-2\zeta}$ with the UV cutoff Λ (and fixed dimensionless $\tilde{T}$). By power counting the only superficially UV divergent irreducible vertex functions (IVF) are found to involve only one or two response fields $\hat{u}$ (at $T > 0$ each $\tilde{T}$ comes with a required Λ^{2-d} factor to compensate the divergence [23]). The statistical tilt symmetry $u_{xt} \to u_{xt} + \text{const.}$ (see e.g. [9,10]) further restricts the needed counterterms at $f = f_c$ to only one for η and one for the full function $\Delta(u)$. The one loop (D) and two loops (A,B,C) diagrams which correct the disorder at $T = 0$ are shown in Fig. 1 (unsplitted). The splitted graphs corresponding to A in the statics (and which do not vanish or cancel in what follows) are shown in Fig. 2. The dynamical diagrams are obtained from the static ones by adding one external $\hat{u}$ on each connected component (e.g. b generates $b_1, \ldots, b_6$). To escape triviality at $T = 0$ we must now develop perturbation theory in a non-analytic interaction $\Delta(u)$ (or $R(u)$), a non trivial extension of conventional field theory. Let us illustrate the new rules. Derivation

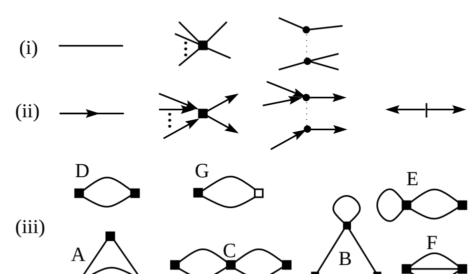

FIG. 1. (i) diagrammatic rules for the statics: replica propagator $\langle u_a u_b \rangle_0 \equiv T \delta_{ab}/q^2$, unsplitted vertex, equivalent splitted vertex $-\sum_{ab} \frac{1}{2T^2} R(u_a - u_b)$. (ii) dynamics: response propagator $\langle \hat{u} u \rangle_0 \equiv R_{q,t-t'}$, unsplitted vertex, splitted vertex $-\frac{1}{2} \hat{u}_{xt} \hat{u}_{xt'} \Delta(u_{xt} - u_{xt'})$ and temperature vertex. Arrows are along increasing time. An arbitrary number of lines can enter these functional vertices. (iii) unsplitted diagrams to one loop D, with inserted counterterm G and two loop A,B,C,E,F.

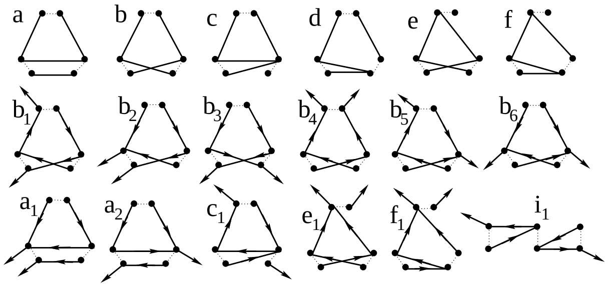

FIG. 2. (a-f): the six splitted (static) diagrams corresponding to two loop A diagram. Below: the corresponding non vanishing diagrams in the dynamics. The last one is the only non trivial C diagram (see text).

by extracting a leg from a vertex can be done as usual only for a vertex evaluated at a generic u (e.g. graphs b_i in Fig 2). If it is evaluated at $u = 0$ (e.g. graph e_1), one must expand $\Delta(u)$ in powers of $|u|$, i.e. $\Delta(u) = \Delta(0) + \Delta'(0^+)|u| + \Delta''(0^+)u^2/2 + \ldots$ and carefully apply Wick's rules. The result is that the above diagrammatic rules (Fig. 1,2) can still be used except that the values of the diagrams are *different*. The graphs of Fig. 2 correspond to performing four Wick contractions and some end up in evaluating non trivial averages of e.g. sign or delta functions. For instance e_1, which vanishes in the analytic theory since $\Delta'(0) = 0$, now reads:

$$e_1 = \Delta'(0^+)^2\Delta''(u)\int_{t_i>0,r_i} R_{r_1,t_1}R_{r_1,t_2}R_{r_3-r_1,t_3}R_{r_3,t_4}F_{r_i,t_i} \ ,$$

where $F_{r_i,t_i} = \langle \mathrm{sgn}(X)\mathrm{sgn}(Y)\rangle$, $X = u_{r_1,-t_3} - u_{r_1,-t_4-t_1}$, $Y = u_{0,-t_4} - u_{0,-t_3-t_2}$, computed with Gaussian averages. The limit $T \to 0$ at $v = 0$ yields $\langle \mathrm{sgn}(X)\mathrm{sgn}(Y)\rangle = \frac{2}{\pi}\mathrm{asin}(\langle XY\rangle/\sqrt{\langle X^2\rangle\langle Y^2\rangle})$, and a complicated $T = 0$ expression for e_1 in the statics [24]. The opposite limit $v \to 0$ at $T = 0$ corresponds to depinning, with $\langle \mathrm{sgn}(X)\mathrm{sgn}(Y)\rangle \to \mathrm{sgn}(t_4 + t_1 - t_3)\mathrm{sgn}(t_3 + t_2 - t_4)$, and more generally to $\Delta^{(n)}(u_t - u_{t'}) \to \Delta^{(n)}(v(t - t'))$ in any vertex evaluated at $u = 0$.

We now focus on depinning at $T = 0$. Using these rules we compute in perturbation of $\Delta \equiv \Delta(u)$ the contributions to the disorder IVF to one and two loops:

$$\delta^1\Delta = -(\Delta'^2 + (\Delta - \Delta(0))\Delta'')I \tag{3}$$

$$\delta^2\Delta = ((\Delta - \Delta(0))\Delta'^2)''I_A \tag{4}$$

$$+\frac{1}{2}((\Delta - \Delta(0))^2\Delta'')''I^2 + \Delta'(0^+)^2\Delta''(I_A - I^2) \tag{5}$$

with $I = \int_q 1/q^4$ and $I_A = \int_{q_1,q_2} 1/q_1^2 q_2^4 (q_1 + q_2)^2$ [25], whose divergent parts $\delta^1_{div}\Delta$, $\delta^2_{div}\Delta$ yield the one loop and two loop counterterms respectively. These are computed here adding a mass $q^2 \to q^2 + m^2$, using dimensional regularization $Im^\epsilon = N_d(\frac{1}{\epsilon} + \mathcal{O}(\epsilon))$, $I_A m^{2\epsilon} = N_d(\frac{1}{2\epsilon^2} + \frac{1}{4\epsilon})$ and absorbing $N_d = (d-2)/(4\pi)^{d/2}\Gamma(\frac{d}{2})$ in Δ. (4) comes from $a_1 + a_2 + \sum_i b_i$, the first term in (5) from all graphs C (not detailed) *except* graph i_1 (shown) which contributes to the last (anomalous) term in (5), together with e_1, f_1, c_1 (the B contribution vanishes). Inverting the relation between bare and renormalized disorders yields the β function $\beta_\Delta = \partial\Delta = \epsilon\Delta + \epsilon\delta^1_{div}\Delta + \epsilon(2\delta^2_{div}\Delta - \delta^{1,1}\Delta)$ where the $1/\epsilon$ terms cancel nicely, the hallmark of a renormalizable theory ($\delta^{1,1}\Delta$ is the counterterm to graph G in Fig.1 and $\partial \equiv -m\partial_m$). We obtain the 2-loop FRG equation:

$$\partial\Delta(u) = (\epsilon - 2\zeta)\Delta(u) + \zeta u\Delta'(u) - \frac{1}{2}\left[(\Delta(u) - \Delta(0))^2\right]''$$
$$+\frac{1}{2}\left[(\Delta(u) - \Delta(0))\Delta'(u)^2\right]'' + \frac{1}{2}\Delta'(0^+)^2\Delta''(u). \tag{6}$$

Computing the other needed counterterm, i.e. the renormalized friction $\eta_R = Z^{-1}\eta_0$, we obtain the dynamical exponent $z = 2 - \partial \ln Z$. The $1/\epsilon$ divergences again cancel yielding the finite result $z = 2 - \Delta''(0^+) + \Delta''(0^+)^2 + \Delta'''(0^+)\Delta'(0^+)(\frac{3}{2} - \ln 2)$. We stress that (6) cannot be read at $u = 0$ [28]. Indeed, it (and the cancellation of divergent parts) was established only for $u \neq 0$. To complete two loop renormalizability we checked that IVF which are $u = 0$ quantities are also rendered finite by the above counterterms. We found that the time dependence in diagrams cancels by subsets as in [22], i.e. correlations (already rendered finite by the above procedure) are thus *static* for $v = 0^+$ at variance with previous works [9].

For periodic $\Delta(u)$ (CDW depinning [10,26]) we find a fixed point of (6) with $\zeta = 0$ reading (for a period 1) $\Delta^*(u) = \frac{\epsilon}{36} + \frac{\epsilon^2}{108} - (\frac{\epsilon}{6} + \frac{\epsilon^2}{9})u(1-u)$ $(0 < u < 1)$. This yields the correlations $\overline{(u_x - u_0)^2} = A_d \ln|x|$ with $A_d = \epsilon/18 + 5\epsilon^2/108$, the RP dynamical exponent $z = 2 - \frac{1}{3}\epsilon - \frac{1}{9}\epsilon^2$ and $\beta = z/2$ from the scaling relation [9,10] $\beta = (z - \zeta)/(2 - \zeta)$. $\int_0^1 \Delta^*$ becomes non zero to two loops, a signature of *nonequilibrium effects*.

Another single FP is found to describe both random field and *all shorter range disorder*, including RB, demonstrating the instability of the apparent one loop short range fixed points. It is determined numerically [24] but ζ is obtained analytically. Integrating (6) over $u > 0$ yields $\partial D = (\epsilon - 3\zeta)D - \Delta'(0^+)^3$ where $D = \int_0^{+\infty}\Delta$ (only assuming $\Delta(+\infty) = 0$). The FP condition then implies [28] (both for RB and RF):

$$\zeta = \frac{1}{3}\epsilon + \zeta_2\epsilon^2 = \frac{\epsilon}{3}(1 + \frac{\epsilon}{9\gamma\sqrt{2}}) = \frac{\epsilon}{3}(1 + 0.14331\epsilon) \ , \tag{7}$$

where we used that at one loop $D^* = \sqrt{6}\epsilon\gamma\Delta^*(0)^{3/2}$ with $\gamma = \int_0^1 dy\sqrt{y - 1 - \ln y} = 0.54822$ [13]. This demonstrates a violation of the conjecture of [10]. It reconciles theory and numerical results as shown in Table I where the dynamical exponent $z = 2 - \frac{2}{9}\epsilon + \epsilon^2(\frac{\zeta_2}{3} - \frac{\ln 2}{54} - \frac{5}{108}) = 2 - \frac{2}{9}\epsilon - 0.04321\epsilon^2$ as well as β obtained via the scaling relation, $\beta = 1 - \frac{1}{9}\epsilon - 0.040123\epsilon^2$, are also given.

The case of long range elasticity is obtained changing $q^2 + m^2 \to \sqrt{q^2 + m^2}$ in all propagators, shifting the upper critical dimension to $d_{\mathrm{uc}} = 2$. It yields a renormalizable theory, with $\epsilon = 2 - d$ and a two loop beta function

[24] obtained by multiplying all $\mathcal{O}(\Delta^3)$ terms in (6) by $4\ln 2$. This yields $\zeta = \frac{\epsilon}{3}(1 + \frac{4\ln 2}{9\gamma\sqrt{2}}\epsilon) = \frac{\epsilon}{3}(1 + 0.39735\epsilon)$, i.e. a strong deviation from $\epsilon/3$ (see Table II), and $z = 1 - \frac{2}{9}\epsilon + \epsilon^2(\frac{4\ln 2}{27\gamma\sqrt{2}} - \frac{\pi+20\ln 2}{108}) = 1 - \frac{2}{9}\epsilon - 0.1133\epsilon^2$.

We now turn to the *statics*, using replicas. In the $T = 0$ limit, the FRG beta function at which we arrive [24]:

$$\partial R = (\epsilon - 4\zeta_{\rm eq})R + \zeta_{\rm eq} u R' + \frac{1}{2}R''^2 - R''(0)R'' + \frac{1}{2}(R'' - R''(0))R'''^2 - \lambda R'''(0^+)^2 R'' \tag{8}$$

has a new "anomalous" term $\propto \lambda$. The other part, i.e. (8) with $\lambda = 0$ (from graphs a, b and repeated one loop counterterm - B graphs cancel in the sum) could as well be obtained for an analytic $R(u)$, as in [14], which by itself would be *inconsistent* since the FP is non-analytic. Ambiguities arise only at two loops (not at one loop since $R''(0) = R''(0^+)$), in the graphs e, f in Fig. 2 which correct $R(u)$ determining λ, since some vertices are evaluated at $u = 0$. However we have shown that the theory can be renormalizable in the usual sense only if:

$$\lambda = 1/2 \ . \tag{9}$$

Indeed, the form of the repeated one loop counterterm (i.e. to G in Fig.1) $\delta^{1,1}R = [(R'' - R''(0))R'''^2 + (R'' - R''(0))^2 R'''' - R'''(0^+)^2 R'']I^2$ which is *non ambiguous* because $\delta^1 R(u)$ is twice differentiable at $u = 0$, *imposes* the coefficient of the ambiguous term $e + f$ of $\delta^2 R$ implying (9). Interestingly, this value of λ is also *the only one* which prevents the occurrence of a further problem in the two loop FRG, the *supercusp* [29]. Indeed, e.g. in the periodic case, the FP of (8) is $R^*(u) = \text{const.} - (\frac{\epsilon}{72} + \frac{\epsilon^2}{108})u^2(1-u)^2 + \frac{\epsilon^2}{432}(2\lambda - 1)u(1-u)$ and possesses a stronger singularity than at one loop, since $R^{*\prime}$ is discontinuous. Thus, unless $\lambda = 1/2$, one has $\int_0^1 R'' = 2R'(0^+) \neq 0$, i.e. a violation of potentiality (as naturally occurs above in the driven dynamics). The $\lambda = 1/2$ theory yields $A_d = \frac{\epsilon}{18} + \frac{7\epsilon^2}{108}$ for one component Bragg glass (and $\int_0^1 \Delta^*$=0 as natural), $\zeta_{\rm eq} = \epsilon/3$ for RF disorder and, via numerics, $\zeta_{\rm eq} = 0.20829804\epsilon + 0.006858\epsilon^2$ for RB disorder. The corresponding extrapolations (Table I) improve the predictions compared to the one loop result.

Methods aiming at deriving a FRG equation, i.e. computing λ, beyond (physical) renormalizability or potentiality requirements, are explored in [24]. An alternative exact FRG method, based on multilocal expansion, also provides [20] a procedure to lift the $u = 0$ vertex ambiguities at $T = 0$, and yields (8) with $\lambda = 1/2$ and universal coefficients. $\lambda = 1/2$ is also recovered [20] at $T > 0$ where it is easy to see how, at large scale where the running temperature $\tilde{T}_l$ flows to 0, anomalous terms as in (8) are generated, e.g. from a graph E of Fig. 1 (proportional to $\tilde{T}_l R''''(0)R''(u)$) since the thermal boundary layer analysis at one loop [13] yields $\tilde{T}_l R''''(0) \to R'''(0^+)^2$.

In summary, by finding a way to cope with the difficulties related to non-analyticity at $T = 0$ in the FRG, we obtained the exponents characterizing depinning and statics of pinned elastic systems to next order in $\epsilon = 4 - d$. We predict that similar anomalous terms arise in other disordered systems where dimensional reduction fails, e.g. random field spin models.

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[29] Fractional powers of disorder then appear in equilibrium expectation values.