Glassy Trapping of Manifolds in Nonpotential Random Flows

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We study the dynamics of polymers and elastic manifolds in nonpotential static random flows. Barriers are found to be generated from combined effects of elasticity, disorder, and thermal fluctuations. This leads to glassy trapping even in pure barrier-free divergenceless flows, with anomalously small response to an applied force. We find a new renormalization-group fixed point at finite temperature and compute the roughness, dynamical, and response exponents for directed and isotropic manifolds. [S0031-9007(98)05374-5]

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There is a lot of current interest in nonequilibrium dynamics, ranging from the growth of domains of an ordered phase to driven dynamics of systems with broken symmetries [1]. Additional quenched disorder usually leads to much slower dynamics [2]. The competition between the elasticity arising from the internal order, and the quenched disorder gives rise to complex phenomena such as pinning and glassy behavior with ultraslow dynamics and an anomalously small response to external perturbations. This happens in many experimental systems such as vortex lattices in superconductors, random magnets, and charge density waves [3]. Up to now the study of pinning and glassy dynamics has focused on *potential* systems [4,5], typically an elastic manifold in a random potential [3,6]. However, in many situations it is important to study *nonpotential* dynamics in the presence of disorder, such as driven systems on disordered substrates [7] or domain growth in the presence of shear [8]. One then expects interesting new physics from the competition between, on one hand, disorder and elasticity, which tend to create pinned or frozen states, and, on the other hand, the energy pumped into the system which tends to destroy glassy properties. Indeed, the existence of pinning and barriers in nonpotential systems was proposed recently in the context of driven vortex lattices [7]. Related observations were made in (mostly mean field) models [9,10]. Since little is known about nonpotential systems with disorder [11], it is of great interest to study other examples. An outstanding question is whether the effective temperature generated from the constant dissipation does overwhelm static disorder.

In this Letter, we study a model of a polymer in a nonpotential static random flow (a "randomly driven polymer") and its generalization to an elastic manifold. We work directly in finite dimension using a dynamical renormalization group (RG). We study two cases: either the polymer is directed (each monomer sees a different flow) or it is isotropic (all monomers are in the same flow); see Fig. 1. Our main finding is that the physics is described by new RG fixed points with both $T > 0$ and finite disorder, leading to anomalous roughness ζ and glassy trapping by the flow [with sublinear $v(f) \sim f^{\phi}$]. Compared to the single particle studies [12], new features emerge such as the crucial role of internal elasticity in generating barriers. While being consistent with the Hartree results ($d \to \infty$) for long range (LR) disorder [10], our present study yields new universal fixed points for short range (SR) disorder. It also generalizes the dynamics of self-avoiding manifolds [13] to a quenched disorder situation, although self-avoidance is perturbatively less relevant in most regimes studied here. For real polymers, or gels, in realistic flows ours is mostly a toy model which could be improved by including hydrodynamic forces. Nevertheless, some of the physics unveiled here will be present in more realistic situations, such as the existence of preferred regions in the flows. Inhomogeneities in polymer distribution and nonlinearities in $v(f)$ can be investigated experimentally.

Let us illustrate how elasticity leads to a dynamical generation of barriers in a divergenceless flow. It is well known that for a single particle convected in such a flow, the stationary measure at $T > 0$ is spatially uniform and the drift velocity under an applied force is $v = f$. Remarkably as soon as one considers two coupled particles, preferred regions appear, e.g., in the $2d$ flow,

$$\begin{aligned} \dot{z}_1 &= c(z_2 - z_1) + i(\omega + \delta)z_1 + \eta_1 , \\ \dot{z}_2 &= c(z_1 - z_2) + i(\omega - \delta)z_2 + \eta_2 , \end{aligned} \tag{1}$$

$z_k = x_k + iy_k$ is the complex position of particle k, c is the elastic coupling, and $\langle \eta_i \eta_j^* \rangle = 4T\delta_{ij}$ is the thermal noise. Without disorder, $\delta = 0$, the motion is just

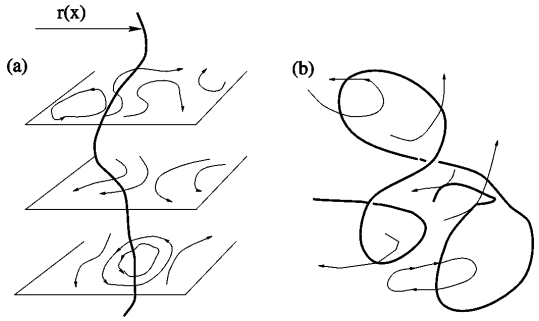

FIG. 1. (a) Directed polymer; (b) isotropic chain.

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a collective rotation around the center (the matrix eigenvalues are $i\omega$ and $-2c + i\omega$). At $T > 0$ there is free collective diffusion and $v = f$ as before. As soon as $\delta > 0$ the zero mode disappears and the two particles converge towards the center (the eigenvalues are $\lambda_\pm = -c + i\omega \pm \sqrt{c^2 - \delta^2}$). This effect persists at $T > 0$ since one finds $\langle |z_k|^2 \rangle = 2T(c^2 + \delta^2)/(c\delta^2)$ $(k = 1, 2)$, and thus there is a genuine bound state despite the divergenceless nature of the flow. The stationary conformation is twisted since $\langle z_1 z_2^* \rangle = 2T(c + i\delta)/\delta^2$. Extending (1) to a directed chain, one finds similar results (the decay towards the center at $T = 0$ is exactly the problem of spin depolarization [14]). Thus a directed polymer in a flow will be attracted to regions where elastic energy and dissipation are smaller (though the precise balance remains to be understood). Presumably polymers in realistic flows will be repulsed by high vorticity regions. Nonlinear extensions of (1) will thus show the generation of barriers. Similar effects exist if all monomers see the same flow (isotropic chain), reminiscent of the effect discovered by Thual and Fauve [15]; mapping their Landau-Ginsburg function onto the complex position $z(x)$ of an elastic chain in a $2d$ nonpotential nonlinear flow, one sees that (even at $T = 0$) it may converge to stable localized (rotating and twisted) conformations $z(x, t) \to z_0(x) e^{i\omega t}$.

We now turn to our full disordered model where these effects can be studied quantitatively. We consider a manifold of internal dimension D parametrized by a d-component field $r^\alpha(x)$. The polymer corresponds to $D = 1$ (x labels the monomers), and a single particle to $D = 0$. We study the Langevin dynamics

$$\eta \partial_t r^\alpha_{xt} = c\nabla_x^2 r^\alpha_{xt} + F_\alpha[r_{xt}, x] + \zeta^\alpha_{xt} . \tag{2}$$

η is the friction, c the elastic coefficient, and the Gaussian thermal noise is $\langle \zeta^\alpha_{xt} \zeta^\beta_{x't'} \rangle = 2\eta T \delta_{\alpha\beta} \delta(t - t') \delta^D(x - x')$. $\langle \cdots \rangle$ denote thermal and the overbar denotes disorder averages. $F_\alpha[r, x]$ is a Gaussian quenched random force field of correlations

$$\overline{F_\alpha[r, x] F_\beta[r', x']} = \Delta_{\alpha\beta}(r - r') h_{x-x'} . \tag{3}$$

There are two main cases of interest. If the manifold is *directed* (e.g., a polymer oriented by an external field), then $h_{x-x'} = \delta^D(x - x')$. If the manifold is *isotropic* (e.g., a Gaussian chain in a static flow), the force field does not depend on the internal coordinate $F_\alpha[r, x] = F_\alpha[r]$ and $h_{x-x'} = 1$. We consider a statistically rotationally invariant force field with both a potential (L) ("electric") and a divergence-free (T) part ("magnetic"). We thus study correlators $\Delta_{\alpha\beta}(r) = \int_K \Delta_K^{\alpha\beta} e^{iK\cdot r}$ with, at small K

$$\Delta_K^{\alpha\beta} \sim K^{a-d} \left[g_L \frac{K_\alpha K_\beta}{K^2} + g_T \left(\delta_{\alpha\beta} - \frac{K_\alpha K_\beta}{K^2} \right) \right], \tag{4}$$

with $\int_K = \int \frac{d^d K}{(2\pi)^d}$. We are mostly interested in *SR correlated forces* $a = d$ (force correlator decaying faster than r^{-d}). We also give results for *LR correlated forces* (force correlator decaying as r^{-a} with $a < d$). Note that *SR random potentials* $\Delta_{\alpha\beta}(K) \sim \tilde{g}_L K_\alpha K_\beta + \tilde{g}_T(\delta_{\alpha\beta} K^2 - K_\alpha K_\beta)$ (formally $a > d$) renormalize to the random force case, except in the potential case $g_T = 0$, much studied previously [3,6] which is preserved by RG.

We study the time translational invariant steady state. One defines the radius of gyration (or roughness) exponent $\overline{\langle (r_{xt} - r_{x't})^2 \rangle} \sim |x - x'|^{2\zeta}$, the single monomer diffusion exponent $\overline{\langle (r_{xt} - r_{xt'})^2 \rangle} \sim |t - t'|^{2\nu}$, and assumes scaling behavior $\overline{\langle (r_{xt} - r_{00})^2 \rangle} \sim |x|^{2\zeta} \overline{B}(t/x^z)$ with $\zeta = z\nu$. The drift velocity under a small additional applied force f in (2) is $v \sim \overline{\langle r_{xt} \rangle}/t$. We find $v \sim f^\phi$ at small f, with $\phi > 1$, and thus the polymer is trapped by the flow. Without disorder one has $\zeta_0 = (2 - D)/2$, $\nu_0 = (2 - D)/4$ for $D < 2$, and bounded fluctuations for $D > 2$.

We start with simple Flory (i.e., dimensional) estimates. Balancing all terms in (2) (e.g., for directed manifolds it reads $r/t \sim r/x^2 \sim r^{-d/2} x^{-D/2}$) yields

$$\zeta_F^{\rm dir} = 2\nu_F^{\rm dir} = \frac{4 - D}{2 + d} \tag{5}$$

for directed and isotropic manifolds, respectively (with $d \to a$ for LR disorder). This suggests that the manifold will be stretched by disorder [16]. We now study (2) and (3) using the dynamical generating functional $Z = \int Dr\, D\hat{r}\, e^{-S_0[r,\hat{r}] - S_{\rm int}[r,\hat{r}] + if\hat{r}}$ with

$$\begin{aligned} S_0 &= \int d^D x\, dt\, i\hat{r}^\alpha_{xt} (\eta \partial_t - c\nabla_x^2) r^\alpha_{xt} - \eta T i\hat{r}^\alpha_{xt} i\hat{r}^\alpha_{xt} , \\ S_{\rm int} &= -\frac{1}{2} \int_{x,x',t,t'} (i\hat{r}^\alpha_{xt})(i\hat{r}^\beta_{x't'}) \Delta_{\alpha\beta}(r_{xt} - r_{x't'}) h_{x-x'} . \end{aligned} \tag{6}$$

We used dynamical RG methods, both via a Wilson RG scheme (presented here) as in [7,17] and via multilocal operator product expansion as in [13,18] (detailed in [19]). The free propagator is $B(x, t) = \frac{1}{2d} \langle (r_{xt} - r_{00})^2 \rangle_0$, and the free response function is $R(x, t) = \frac{1}{d} \langle i\hat{r}_{00} \cdot r_{xt} \rangle_0$. Equations (6) and (4) is invariant under the rescaling $x = e^l x'$, $t = e^{zl} t'$, $r = e^{\zeta l} r'$, $\hat{r} = e^{(2 - z - \zeta - D + \beta)l} \hat{r}'$, provided $\eta \to \eta e^{(2 - z + \beta)l}$, $T \to T e^{(2 - D - 2\zeta + \beta)l}$, $c \to c e^{\beta l}$. Also $g_{T,L} \to g_{T,L} e^{[4 - D + 2\beta - (a+2)\zeta]l}$ (directed case) and $g_{T,L} \to g_{T,L} e^{[4 + 2\beta - (a+2)\zeta]l}$ (isotropic case) with $a = d$ everywhere for SR forces. There are generically *three* independent exponents (z, ζ, β) and from them one gets

$$\nu = \zeta/z, \qquad \phi = (z - \zeta)/(2 - \zeta + \beta), \tag{7}$$

using $f \to f e^{(D + z + \alpha)l}$, $v \to v e^{(\zeta - z)l}$. Power counting at the Gaussian fixed point with no disorder [$z = 2$, $\beta = 0$, $\zeta = (2 - D)/2$] shows that disorder is relevant when (Fig. 2) $d < d_c^{\rm dir} = 4/(2 - D)$ for directed manifolds and $d < d_c^{\rm iso} = (4 + 2D)/(2 - D)$ for isotropic manifolds ($a < d_c$ for LR disorder). Thus for directed polymers disorder is relevant for $d < 4$ and for isotropic polymers for $d < 6$. Power counting and symmetries show that the only relevant terms generated in perturbation are those in (6). To lowest order T, c, η are corrected as

$$\begin{aligned} \delta T &= \frac{(d - 1) g_T}{\eta d} \int_{\tau > 0, y, K} h_y K^{a-d} \\ &\quad \times [e^{-K^2 B(y,\tau)} - e^{-K^2 B_\infty}], \\ \delta c &= \frac{g_L}{2dD} \int_{y,K} y^2 h_y K^{a-d} [e^{-K^2 B(y,0)} - e^{-K^2 B_\infty}], \end{aligned} \tag{8}$$

and $(d-1)\delta\eta = (g_L/g_T)\eta\delta T$ [20]. These expressions were simplified using the fluctuation dissipation theorem (FDT) relation $\theta(t)\frac{d}{dt}B(x,t) = TR(x,t)$. A useful feature of the present model is that, although the FDT relation is not obeyed by the exact Green's functions (since disorder is nonpotential), it is within each scale of the RG. The FDT violation simply appears as a nontrivial renormalization of T. The relevant corrections to disorder read

$$\delta\Delta_P^{\alpha\beta} = \int_{x,y,\tau,\tau'}\int_{K,K'=P-K}[\Delta_K^{\alpha\beta}(K\cdot\Delta_{K'}\cdot K) + (K'\cdot\Delta_K^{\alpha})(K\cdot\Delta_{K'}^{\beta})]R_{x,\tau}R_{y,\tau'}h_{x-y}e^{K\cdot K'(B_{x,\tau}+B_{y,\tau'})}. \quad (9)$$

The small P expansion in (9) is well behaved and because of analyticity the only divergent terms generated are proportional to $\delta_{\alpha\beta}$ yielding $\delta g_L = \delta g_T$. One can thus take the limit $P \to 0$ [$K' = -K$ in (9)]. One checks explicitly from (9) that (i) starting with $\Delta_{\alpha\beta}(K) \sim \tilde{g}_L K_\alpha K_\beta + \tilde{g}_T(\delta_{\alpha\beta}K^2 - K_\alpha K_\beta)$ with both $\tilde{g}_T > 0$ and $\tilde{g}_L > 0$ one generates SR random forces, and that (ii) a divergence-free random force field does not remain so. The RG equations (specified to various cases below and in Table I) read

$$\begin{aligned}
\frac{dc}{cdl} &= \beta + A_c\overline{g}_L, \qquad \frac{d\eta}{\eta dl} = 2 - z + \beta + A_\eta\overline{g}_L,\\
\frac{dT}{Tdl} &= 2 - D - 2\zeta + \beta + A_T\overline{g}_T,\\
\frac{d\overline{g}_L}{dl} &= \epsilon\overline{g}_L - (B-A)\,\overline{g}_L\overline{g}_T + E\overline{g}_L^2,\\
\frac{d\overline{g}_T}{dl} &= \epsilon\overline{g}_T + (A+E)\,\overline{g}_L\overline{g}_T - B\overline{g}_T^2.
\end{aligned} \quad (10)$$

(i) Directed manifolds, SR disorder.—$\epsilon = 2 - (2-D)\frac{d}{2}$. Then $h_y = \delta^D(y)$, and there is no correction to the elastic coefficient $\delta c = 0$ (guaranteed by the statistical tilt symmetry [7] for SR and LR disorder). Thus $\beta = 0$ and there are only two independent exponents. The dimensionless coupling [21] constant is $\overline{g}_{T,L} = C\Lambda^\epsilon g_{T,L}/T^{(d+2)/2}$. The RG flow is depicted in Fig. 3. There is a globally attractive isotropic fixed point I^* at $\overline{g}_L = \overline{g}_T = \epsilon/(B-A)$ (note that $B > A$) and the line $\overline{g}_L = \overline{g}_T$ is preserved. The potential line $g_T = 0$ is preserved with a flow to strong coupling. There is another *apparent* fixed point at $g_L = 0$, $g_T = \epsilon/B$, attractive along the line $g_L = 0$ (divergenceless flows). However, for $D > 0$, this line is not in the physical domain since, as discussed above, a finite g_L term is generated and one flows to I^*. At I^* one gets

$$z = 2 + \epsilon/a_{D^*}, \qquad \zeta = \frac{2-D}{2} + \epsilon/(2b_{D^*}),$$
$$b_D = \frac{4-D}{2-D} - \frac{4}{D}\Big(\frac{1}{2\Gamma[D/2]}\Big)^{2/(2-D)}, \quad (11)$$

$a_D = \frac{2+D}{2-D}\,b_D$, and ν and ϕ using (7) [22]. The expansion can be carried from any point of the line $d^* = 4/(2-D^*)$. Optimizing over D^* as in [18] yields the estimates $\zeta = 0.625 \pm 0.02$, $z = 2.085 \pm 0.02$, $\phi = 1.061 \pm 0.015$, $(d=3)$ and $\zeta = 0.9 \pm 0.1$, $z = 2.2 \pm 0.1$, $\phi = 1.182 \pm 0.08$ $(d=2)$. For small ϵ, $\zeta > \zeta_F$ and Flory is thus likely to be a lower bound [23].

(ii) Directed manifolds, LR disorder.—$\epsilon = 2 - \frac{(2-D)a}{2}$ with $a < d$. The novelty is $\delta g_{L,R} = 0$ since the LR part of the disorder is not renormalized [24]. The dimensionless coupling [21] $\overline{g}_{L,T} = Cg_{L,T}/T^{(a+2)/2}$ will experience nontrivial renormalization only because T renormalizes. Thus the ratio $\frac{\overline{g}_L(l)}{\overline{g}_T(l)} = \frac{g_L}{g_T}$ is preserved which leads to a *line of fixed points.* Thus $\overline{g}_T(l) \to g_T^* = \epsilon/B$ and $\overline{g}_L(l) \to \frac{g_L}{g_T}g_T^*$. This yields the continuously varying exponent $z = 2 + 2\frac{g_L}{g_T}\frac{\epsilon}{(2+a)(d-1)}$, and $\zeta = \zeta_F$ holds *to all orders* in ϵ in the LR case since neither the vertex nor c renormalize. As conjectured in [10], ν is found identical to the Hartree approximation [formula (12) [25] of [10]].

(iii) Isotropic manifolds (SR and LR disorder).—Then $\epsilon = D + 2 - a\frac{2-D}{2}$ ($a = d$ for SR disorder). The novelty is a renormalization of the elasticity of the manifold (it becomes stiffer $\delta c > 0$) and thus a third nontrivial exponent β. The dimensionless constant is $\overline{g}_{T,L} = C\frac{g_{T,L}}{T^{(2+a)/2}c^{(2-a)/2}}$. For SR disorder the RG flow is similar to the directed case. There is a fully attractive (isotropic

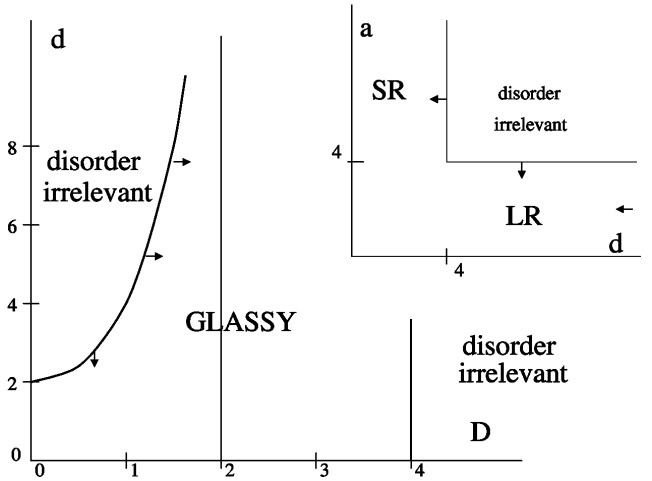

FIG. 2. Regions in the (d,D) plane where SR disorder is relevant for a directed manifold. Inset: Regions in (a,d) where SR or LR disorder is relevant for directed polymers $D = 1$.

TABLE I. Coefficients of (10). One must set $d = d^*$, $D = D^*$. One denotes $I_a = \int_0^{+\infty} dt\,\tilde{B}(1,t)^{-a/2}$ with $\tilde{B}(1,t) = \frac{1}{\Gamma[D/2]}\{(4t)^{(2-D)/2}e^{-1/(4t)} + \Gamma[\frac{D}{2},\frac{1}{4t}]\}$ with $\tilde{I}_{d=6} \approx 1.7935$.

	A_c	A_η	A_T	B	E	A
dirSR	0	$\frac{2}{d}$	$\frac{2(d-1)}{d}$	$\frac{d+2}{2}A_T$	0	$\frac{16\pi(d-1)}{d(d-2)(2-D)}\,(\frac{S_D}{4\pi})^{d/2}$
dirLR	0	$\frac{2}{d}$	$\frac{2(d-1)}{d}$	$\frac{a+2}{2}A_T$	0	0
isoSR	$\frac{1}{2dD}$	$\frac{I_d}{d}$	$\frac{(d-1)I_d}{d}$	$\frac{d+2}{2}A_T$	$\frac{d-2}{2}A_c$	$\frac{2(d-1)\Gamma[\frac{D}{2-D}]^2}{d(d-2)(2-D)^2\Gamma[\frac{2D}{2-D}]}$
isoLR	$\frac{1}{2dD}$	$\frac{I_a}{d}$	$\frac{(d-1)I_a}{d}$	$\frac{a+2}{2}A_T$	$\frac{a-2}{2}A_c$	0

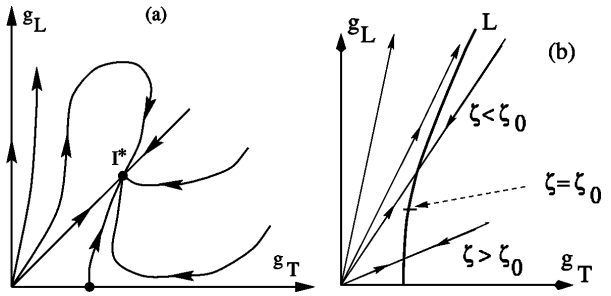

FIG. 3. RG flows (a) directed manifolds with SR disorder. The physics is controlled by a fixed point I^* at $g_T = g_L$. (b) Isotropic manifolds LR disorder. The flow is along straight lines, with a line of fixed points L and an apparent separatrix. $\zeta - \zeta_0$ changes sign suggesting progressive localization.

disorder) fixed point I^* at $\overline{g}_T = \overline{g}_L = 1/(B - E - A)$. As before, LR disorder is not renormalized, the flow lines are $\frac{\overline{g}_L(l)}{\overline{g}_T(l)} = \frac{g_L}{g_T} = k$, and there is a line of fixed points $g_T = \epsilon/(B - kE)$, $g_L = kg_T$ parametrized by k. The general formula for the exponent is $\zeta = (2 - D)/2 + \epsilon(A_T - kA_c)/2(B - kE - A)$, $z = 2 + k\epsilon(A_\eta - A_c)/(B - kE - A)$, and $\beta = -k\epsilon A_c/(B - kE - A)$, with the values given in Table I (and set $k = 1$ for SR). For SR disorder this yields for $D = 1$, $\zeta = 0.5 + 0.13\epsilon$, $z = 2 + 0.04\epsilon$, $\beta = -0.015\epsilon$. Extrapolations as in [18] suggest $\zeta = 0.85 \pm 0.1$, $z = 2.1 \pm 0.1$, $\beta = -0.025 \pm 0.01$ for polymers in $d = 3$. For LR disorder the line of fixed points is not parallel to the axis any more (Fig. 3), and there is *no fixed point* for $k > k_c = B/E$. There is even a value $k^* = \frac{a-2}{a+2}k_c < k_c$, where $\zeta - \zeta_0$ changes sign. For $k^* < k < k_c$ the localization effects dominate. Together with the apparent separatrix in the flow, this suggests two phases (a localized one and one with a continuously varying ζ), which would be interesting to check by nonperturbative methods (as in [10]).

Thus elastic manifolds in static random flows are described by new RG fixed points at finite disorder. There are similarities with the glass phase of manifolds in random *potentials* [3,6], such as sublinear $v(f)$ response and anomalous roughness. The main difference is that here a temperature is generated, which weakens disorder [26] [and renders the fixed point perturbatively accessible—the coupling is $g/T^{(d+2)/2}$]. Numerical simulations would also help to check present results and further investigate this new type of glassy behavior [27] and its consequences for experimental systems such as polymers or gels in static flows. A competition between localization and driving was found for isotropic manifolds (correlated disorder).

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[16] However, $\zeta = \zeta_F$ is exact *only* for directed manifolds with LR correlations with also $\nu = \nu_F$ $g_L = 0$ and $d \to \infty$ (for $(D = 0, g_L = 0)$ $\nu = \nu_F$ is *always* exact [10]).
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[20] Where $B_\infty = \lim_{\tau\to\infty} B(0,\tau)$ with both UV (Λ) and IR (L) cutoffs (important to recover the $T \to 0$ limit, or study $D > 4$ where disorder is again perturbatively irrelevant [7]). Here we are mostly interested in $D < 2$, d near d_c, $\Lambda L \to \infty$, and we can set $\exp(-K^2 B_\infty) = 0$ and use $B(x,0) = \frac{T|x|^{2-D}}{(2-D)S_D c}$, $B(0,t) = \frac{2T|\mu t|^{2-D/2}}{(2-D)(4\pi c)^{D/2}}$ with $S_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$ and $\int_K e^{-K^2 B} K^{a-d} = (4\pi)^{-d/2}\frac{\Gamma[a/2]}{\Gamma[d/2]}B^{-a/2}$.
[21] We use $C = \frac{\Gamma[a/2]}{\Gamma[d/2]}(4\pi)^{a-d/2-1}(\frac{2-D}{2})^{a/2}$ and $C' = (2 - D)^{a/2}\frac{\Gamma[a/2]}{\Gamma[d/2]}S_D^{1+a/2}(4\pi)^{-d/2}$ with $a = d$ for SR disorder.
[22] Continuation to $D = 0$ yields $\zeta = 1 + \frac{\epsilon}{2}$, $z = 2 + \epsilon$ (with $\epsilon = 2 - d$). This yields $\nu = \zeta/z = 1/2 + O(\epsilon^2)$ and $\phi = 1 + \epsilon$ at I^* which is the correct known result for the particle, obtained via a different RG [12].
[23] Elasticity slows diffusion compared to a single particle.
[24] Equation (9) renormalizes only the SR part of disorder and becomes important in the region of the (a, d) plane where the crossover [12] LR-SR occurs (not studied here).
[25] Setting $g_2 \to \frac{g_L}{d}$, $g_1 - g_2 \to (1 - \frac{1}{d})g_T$, and $\epsilon \to \frac{\epsilon}{2-D}$. We have extended the calculation of [10] and obtained the ϕ exponent for Hartree in agreement with (7).
[26] Reminiscent of the "marginal glass" in the statics [17,28] with effectively logarithmically growing barriers.
[27] Nonpotential forces could possibly interrupt the hierarchy of wells within wells of a static glass at a random level, resulting in broad distributions of release time, or generate Levy walk-type jumps between trapped configurations.
[28] J. Cardy and S. Ostlund, Phys. Rev. B **25**, 6899 (1982).