Le Doussal and Wiese Reply: The authors of [1] correctly point out that the Schwartz-Soffer (SS) inequalities [2] put useful constraints on the phase diagram of the *random-field* (RF) $O(N)$ model and its (subtle) dependence in N. In our Letter [3] we have studied the functional renormalization group (FRG) at large N and obtained a series of fixed points indexed by $n = 2, 3, \ldots$ where the disorder correlator $\hat{R}(z)$ (notations of [3]) has a nonanalyticity at $z = 1$. The $n = 2$ fixed point (FP) has RF symmetry and $n = 3$ has random anisotropy (RA) symmetry [$\hat{R}(z)$ even in z]. In addition we found two infinite-N analytic fixed points which obey dimensional reduction. One of them [$\hat{R}(z) = z - 1/2$] is the large-N limit of the Tarjus-Tissier (TT) FP [4] which exists for $N > N^*$ (at two loop we found $N^* = 18 - \frac{49}{5}\bar{\epsilon}$, $\bar{\epsilon} = d - 4 \geq 0$) and has a weaker and weaker "subcusp" nonanalyticity as N increases. The question is which of these FPs describes the ferromagnetic or disordered (FD) transition at large N for $d \geq 4$.

First, one should carefully distinguish (i) strictly infinite $N = \infty$ from large but finite N, (ii) RF symmetry versus RA. We have shown [3,5] that for RF at $N = \infty$ physical initial conditions on the critical FD manifold converge to the $n = 2$ FP if the bare disorder is strong enough ($r_4 > 4$ in [3]). Hence for $N = \infty$ all these nonanalytic (NA) FPs are consistent. They have a positive probability distribution of the disorder since all $\hat{R}^{(n)}(0)$, the variances of the corresponding random fields and anisotropies, are positive—a condition referred to as physical. Further, the SS inequality does not yield any useful constraint at $N = \infty$ because it contains an amplitude itself proportional to $\sqrt{N}$.

Next, each of the above FPs can be followed to finite N [5,6]. It yields to $O(\bar{\epsilon} = d - 4)$ the critical exponents $\bar{\eta}(n, N)$ and $\eta(n, N)$ to high orders in $1/N$. One finds that the $n = 2$ FP acquires a *negative* $\hat{R}'(0)$ at order $1/N$, $\hat{R}'(0) = -\frac{3}{4}(\bar{\epsilon}/N^2) + O(1/N^3)$; hence, it becomes unphysical at finite N, consistent with the violation of the SS inequality $\bar{\eta} \leq 2\eta$ pointed out in [1]. A natural scenario for RF, as indicated in [3], is that the FRG flows to the TT FP for any *finite* $N > N^*$. However, as discussed there, if bare disorder is strong enough, it may approach the TT FP along a NA direction, since these arguments relied only on blowing up of $R''''(0)$, with $R(\phi) = \hat{R}[z = \cos(\phi)]$.

A very interesting point, missed in the first version of [1], hereafter corrected taking into account this Reply, is that the SS inequalities do not constrain the 2-point function of the spin $S^i(x)$ for *random anisotropy*, but the 2-point function of $\chi_{ij}(x) = S^i(x)S^j(x)$ as disorder couples to the latter. We find [5,6] that the $n = 3$ RA FP, which reads $NR(\phi)/|\epsilon| = \frac{9}{8}[2\cos(\phi)\cos(\frac{\phi+\pi}{3}) + \cos(\frac{\pi-2\phi}{3}) - 1]$ in the $N = \infty$ limit, *remains physical* for finite N. Denoting $\hat{R}(z) = \bar{\epsilon}\mu\tilde{R}(z)$ with $\mu = \frac{1}{N-2}$ and $y_0 = \tilde{R}'(1)$, we obtain the following expansion to $O(\bar{\epsilon})$ for $\eta = y_0\bar{\epsilon}/(N-2)$, $\bar{\eta} = (\frac{N-1}{N-2}y_0 - 1)\bar{\epsilon}$, where

$$y_0 = \frac{3}{2} + 23\mu - \frac{1750\mu^2}{3} + \frac{2\,129\,692\mu^3}{27} - \frac{13\,386\,562\,376\mu^4}{1215} + \frac{2\,004\,388\,412\,086\,052\mu^5}{1\,148\,175} - \frac{107\,423\,933\,633\,514\,594\,598\mu^6}{361\,675\,125} \quad (1)$$

and all coefficients in the expansion of $\hat{R}(z)$ near $z = 0$ remain indeed positive; e.g.,

$$\tilde{R}'(z) = \left[\frac{70\mu}{9} + 1\right]z + \left[\frac{1192\mu}{243} + \frac{4}{27}\right]z^3 + \left[\frac{4384\mu}{2187} + \frac{16}{243}\right]z^5 + \left[\frac{68\,608\mu}{59\,049} + \frac{256}{6561}\right]z^7 + \left[\frac{3\,735\,040\mu}{4\,782\,969} + \frac{14\,080}{531\,441}\right]z^9 + O(z^{11}).$$

Finally, for the $1/N$ expansion of the *analytic* (DR) FP corresponding to RA we obtain (with $y_0 = 1$): $\tilde{R}(z) = z^2/2 + (-\frac{3}{2} + 4z^2 - 2z^4)\mu + \cdots$; hence, it is *unphysical* at finite N. [$R''''(0)$ exists for $N > N^*$ and corresponds to the *unstable* line in Fig. 3 of [3].] The scenario is thus the opposite of RF: The NA FP $n = 3$ is the only one physical at large N (it exists for $N > N_c = 9.441\,21$) and has precisely one unstable eigenvector (within the RA symmetry) as expected for the FD transition. Using our 2-loop result [3] we obtained, up to $O(\mu^2)$: $y_0 = \frac{3}{2} + 23\mu + (9\gamma_a - \frac{97}{4})\mu\bar{\epsilon}$, $\eta = \mu[\frac{3}{2}\bar{\epsilon} + \bar{\epsilon}^2(3\gamma_a - \frac{27}{8})]$, and $\bar{\eta} = \frac{\epsilon}{2} + \mu[\frac{49}{2}\bar{\epsilon} + \bar{\epsilon}^2(9\gamma_a - \frac{203}{8})]$, where γ_a was defined in [3].

Our conclusion is thus that the random anisotropy FP smoothly matches our solution $n = 3$ at $N = \infty$ and remains nonanalytic for all N, breaking dimensional reduction. It does not exhibit the TT phenomenon which seems a peculiarity of the RF class. It is further studied in [5,6].

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