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Shock statistics in higher-dimensional Burgers turbulence

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Abstract – We conjecture the exact shock statistics in the inviscid decaying Burgers equation in $D>1$ dimensions, with a special class of correlated initial velocities, which reduce to Brownian for $D=1$. The prediction is based on a field theory argument, and receives support from our numerical calculations. We find that, along any given direction, shock sizes and locations are uncorrelated.

Decaying turbulence is characterized by the existence of an inertial range in the inviscid limit (small viscosity limit) with scaling and multi-scaling. These features are shared by simpler models as passive advection [1–3] and the Burgers equation [4–6] where the velocity field presents manifolds of discontinuities called *shocks*. How the velocity field evolves from a prescribed random initial condition $\vec{v}(\vec{r}, t=0)$, $\vec{r} \in R^D$, and the statistics of these shocks, are of high interest. Unfortunately, exact results are restricted to few solvable cases in space dimension $D=1$ [4,7–15] or in the limit $D=\infty$ [16,17]. In this letter we present a solution for generic D, for a non-trivial class of random initial conditions. These appear naturally in related works in the context of elastic interfaces in $D=1$ [18–21] and more recently in higher D [21]. At this stage the solution is a conjecture, based on a field theory argument. Here, we state the conjecture and provide an accurate numerical test.

The decaying Burgers equation describing a potential flow velocity field $\vec{v}(\vec{r},t)=\vec{\nabla}\hat{V}(\vec{r},t)$ reads

$$\partial_t \vec{v} = \nu \nabla^2 \vec{v} - \frac{1}{2}\vec{\nabla}\vec{v}^2. \qquad (1)$$

The inviscid limit corresponds to $\nu = 0^+$ (see footnote[1]). Our model is defined by choosing the distribution of the initial velocity field $\vec{v}(\vec{r},0)$ as a centered Gaussian with increments $\delta\vec{v}(\vec{r},\vec{r}\,')=\vec{v}(\vec{r},t=0)-\vec{v}(\vec{r}\,',t=0)$ of correlations:

$$\frac{1}{2}\overline{\delta v_i(\vec{r}_0,\vec{r}_0+\vec{r})\delta v_j(\vec{r}_0,\vec{r}_0+\vec{r})} = \frac{B}{2}|\vec{r}\,|(\delta_{ij}+\hat{r}_i\hat{r}_j). \qquad (2)$$

$\hat{r}=\vec{r}/|r|$, B is a constant and $\overline{\cdots}$ denotes the average over initial conditions. The increments are stationary because the correlations are independent of r_0 (statistical translational invariance). In $D=1$ this reduces to a Brownian initial velocity [8–10]. In general D there is no obvious Markov property, except that the velocity along any given direction is a Brownian.

We study, without loss of generality, the velocity along the x-axis. For a given initial condition the velocity profile, at finite time, can be computed using the Cole-Hopf transformation (see below). A sketch of the solution in the inviscid limit is shown in fig. 1 for the case $D=2$. Let us denote by x_α the discrete set of points where shocks are locates along the x-axis, and by $\vec{S}_\alpha$ the shock size defined as

$$\vec{S}_\alpha = t\left[\vec{v}(x_\alpha - 0^+) - \vec{v}(x_\alpha + 0^+)\right], \qquad (3)$$

where α labels the shocks. We note from fig. 1 that the longitudinal shock component S_x is always positive, while the component of the shock orthogonal to x, $\vec{S}_y$, can be positive or negative. We also remark that, except for the linear uniform slope of v_x, the velocity increments, $[\vec{v}(x,t)-\vec{v}(0,t)\ ]$, are encoded in shocks, so that the joint distribution of the size and location of all shocks gives the complete characterization of the statistical properties of the velocity profiles.

The shock-size density is defined as $\rho(\vec{S}) = \overline{\sum_\alpha \delta(x-x_\alpha)\delta(\vec{S}-\vec{S}_\alpha)}$ and displays a power law

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[1] A small $\nu>0$ gives a small width to shocks, which here scales to zero in reduced units, and is thus irrelevant.

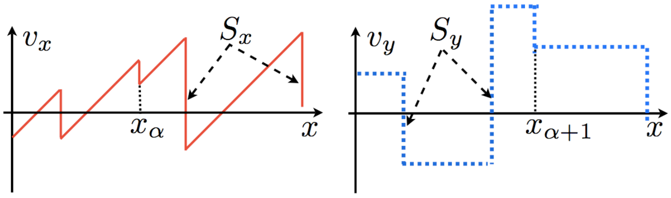

Fig. 1: (Color online) Sketch of the velocity profile along the x-axis, for $D=2$ Left: v_x-component. Right: v_y-component. The profile is given by the Cole-Hopf transformation [4] (see eq. (13)). Shocks correspond to the discontinuities of the velocity profile and are located in a discrete set of points x_α, $x_{\alpha+1},\ldots$; the size of the shocks is defined in eq. (3).

behavior: typical shocks are very small, but the velocity increments are dominated by larger and rarer shocks (see [18] for a detailed discussion). The characteristic size, S_m, of these large shocks grows with time and is defined from the first two moments of S_x as

$$S_m=\frac{\langle S_x^2\rangle}{2\langle S_x\rangle}, \tag{4}$$

where $\langle\cdots\rangle$ denotes averages over the shock density $\rho(\vec{S})$ which takes the form

$$\rho(\vec{S})=\frac{1}{S_m^2}p\left(\frac{\vec{S}}{S_m}\right). \tag{5}$$

$p(\vec{s})$ is a function of the reduced shock-size $\vec{s}:=\vec{S}/S_m$. By construction $\langle s_x^2\rangle=2$. A second identity (see below),

$$\int \mathrm{d}\vec{S}\,\rho(\vec{S})S_x=1, \tag{6}$$

implies a further normalization condition $\langle s_x\rangle=1$, for the function $p(\vec{s})$. Note that here and below $\langle\ldots\rangle$ denote moments either over $\rho(S)$ or $p(s)$. It is useful to introduce the generating function for the distribution of reduced shock sizes,

$$\tilde{Z}(\vec{\lambda}):=\langle e^{\vec{\lambda}\vec{s}}-1\rangle:=\int \mathrm{d}\vec{s}\,(e^{\vec{\lambda}\cdot\vec{s}}-1)p(\vec{s}). \tag{7}$$

We now state our prediction valid for the initial conditions (2) and obtained from field-theoretical considerations: i) along a given direction, the locations of shocks are independent (*i.e.* Poisson distributed); ii) shock sizes are mutually uncorrelated and independent of locations; iii) the characteristic shock size is $S_m=Bt^2$, the function $\tilde{Z}(\vec{\lambda})$ is given below in some special directions, its full expression, not reproduced here, is computed in [21]. Its expansion is

$$\begin{aligned}\tilde{Z}(\vec{\lambda}) &= \lambda_x+\frac{1}{2}\lambda_x^2+\frac{1}{2}\vec{\lambda}^2+2\lambda_x\vec{\lambda}^2+\frac{3}{2}(\vec{\lambda}^2)^2\\ &\quad+\frac{9}{2}\vec{\lambda}^2\lambda_x^2-\lambda_x^4+\ldots .\end{aligned} \tag{8}$$

It implies universal moment ratios, in particular

$$\frac{\langle S_x^2\rangle}{\langle S_\perp^2\rangle}=\frac{2}{\langle s_\perp^2\rangle}=\frac{2}{D-1}. \tag{9}$$

While the set of shocks along x are uncorrelated both in position and size, by contrast, longitudinal and transverse components of a given shock are correlated, as from (8) one can calculate higher moments, *e.g.*

$$4\frac{\langle S_xS_\perp^2\rangle\langle S_x\rangle}{\langle S_x^2\rangle^2}=\langle s_xs_\perp^2\rangle=4(D-1). \tag{10}$$

This conjecture characterizes the statistics of velocity increments, $[\vec{v}(x,t)-\vec{v}(0,t)]$. It is possible to show (and the converse is also true) [22] that for uncorrelated and independent shocks, the characteristic function of the velocity increments can be written as

$$\overline{e^{-\vec{\lambda}\cdot[\vec{v}(x,t)-\vec{v}(0,t)]}}=e^{x[Z_t(\frac{\vec{\lambda}}{t})-\frac{\lambda_x}{t}]}, \tag{11}$$

where $Z_t(\vec{\lambda})$ is the generating function for the shock density $\rho(\vec{S})$. On the other hand, from dimensional analysis, we know that $Z_t(\vec{\lambda})=\tilde{Z}(S_m\vec{\lambda})/S_m$.

We now indicate the origin of our conjecture, by recalling the connection to disordered systems. Equation (1) is solved by the Cole-Hopf transformation [4] in the limit $\nu\to 0$:

$$\hat{V}(\vec{r},t)=\min_{\vec{r}\,'}\left[\frac{1}{2t}(\vec{r}\,'-\vec{r})^2+V(\vec{r}\,')\right], \tag{12}$$

where $V(\vec{r}\,')$ is the potential associated with the initial condition, *i.e.* $\vec{v}(\vec{r},t=0)=\vec{\nabla}V(\vec{r})$. Hence for a random initial condition the problem is equivalent to finding the minimum energy position of a particle in a random potential, plus a harmonic well. Denoting by $\vec{u}(\vec{r})$ the position of the minimum in (12), the velocity field is $\vec{v}(\vec{r},t)=[\vec{r}-\vec{u}(\vec{r})]/t$. If we are interested in the velocity profile along the x-axis we have

$$v_x(x,t)=[x-u_x(x)]/t,\quad \vec{v}_\perp(x,t)=-\vec{u}_\perp(x)/t. \tag{13}$$

Equation (13) allows us to justify the sketch of fig. 1 with all the properties we derived. In particular, the uniform slope of v_x (fig. 1, left) can now be identified with the term x/t in eq. (13). At the shocks, the minimum changes its location, and the shock size is $\vec{S}=\vec{u}(x_\alpha+0^+)-\vec{u}(x_\alpha-0^+)$. Moving the position of the parabolic well along the x-axis we now understand that the location of the new minimum always increases along u_x (thus S_x is always positive), but not along $\vec{u}_\perp$ ($\vec{S}_\perp$ can be either positive or negative). Finally noting that $\overline{\vec{u}(\vec{x})}=\vec{x}$ we prove $\langle S_x\rangle=1$ as stated in eq. (6).

The random potential $V(\vec{r})$ corresponding to the present model (2) is a generalization of the 1D random acceleration process [23,24] to D dimensions. To define it one needs a large-scale regularization; we choose periodic boundary conditions of period L in all D directions,

$$V(\vec{r})=L^{-\frac{D}{2}}\sum_{\vec{q}\neq 0}V_{\vec{q}}e^{i\vec{q}\cdot\vec{r}},\qquad \overline{V_{\vec{q}}V_{\vec{q}\,'}}=\frac{\sigma^2\delta_{\vec{q},-\vec{q}\,'}}{(q^2)^{\frac{D}{2}+H}}, \tag{14}$$

where $\vec{q} = \frac{2\pi}{L}\vec{n}$, $\vec{n} \in \{-L/2+1, \ldots, L/2-1, L/2\}^D$, in the limit $L \to \infty$, and $H = 3/2$. In real space this leads to a non-analytic *cubic* potential correlator $\overline{V(\vec{r})V(\vec{r}\,')} = R_0(\vec{r} - \vec{r}\,')$ with

$$R_0(\vec{r}) - R_0(0) = -\frac{1}{2}Ar^2\,L + \frac{B}{6}|r|^3 + O(1/L), \qquad (15)$$

where the constants A and B can be computed from (14): $A = \sigma^2 0.0182\ldots + O(1/L)$ and $B = \sigma^2/(3\pi) + O(1/L)$. The initial velocity correlator is $\overline{v_i(\vec{r}, t=0)v_j(0, t=0)} = -\partial_i\partial_j R_0(\vec{r})$ with the stationary increments $\delta\vec{v}$ distributed as in eq. (2).

In a nutshell, the basis for our conjecture is as follows: the present model is the $d=0$ limit of a model of an elastic manifold (of internal dimension d) in a quadratic well of curvature $1/t$ and a random potential[2]. In this model, the analogous variable to $\vec{u}(\vec{r})$ is the location of the center of mass of the manifold for a given well position $\vec{r}$. Note that both the center-of-mass location and the well position are points in the $D-d$ dimensional space. At time t, the energy of the optimal configuration as a function of well position $\vec{r}$ is $\hat{V}(\vec{r}, t)$. Its second cumulant defines a renormalized potential disorder correlator $R(\vec{r})$, which obeys a functional RG equation as t is varied. This equation can be solved perturbatively in R in a $d = 4-\epsilon$ expansion. It turns out that the initial correlator $R_0(\vec{r})$ corresponding to (15) solves the FRG equation *to all orders* in ϵ, *i.e.* there are no loop corrections. The explicit computation for $D-d=1$ with $R_0(r) \sim Br^3/6$ has been performed in [18], the conjecture we present here is valid for any D (and also for any d although we need only $d=0$ (Burgers)). The remarkable property of the initial condition defined in (15) under the action of the functional RG allows to compute the *exact* correlation functions to tree level either by recursion or from a saddle-point method. This leads to the main statements of our conjecture: i) $S_m = Bt^2$, ii) eq. (11), and iii) the explicit form of $\tilde{Z}(\vec{\lambda})$. The detailed calculations will be presented elsewhere [21]. A further result, proved to lowest order in $\epsilon = 4-d$ [21], but which we expect to hold for any d, is that (2) is an *attractive* fixed point of the RG, hence for velocity correlations which differ from (2) only at small r, the behaviour at large t again follows (11) (see footnote[1]).

Of course our analysis of the functional RG equations cannot exclude the presence of non-perturbative corrections, hence our prediction is, strictly speaking, a conjecture. However for $D=1$ and $d=0$ our predictions are in perfect agreement with results rigorously proved in [10]. To check it in $D=2$ and $d=0$ we now turn to numerics.

A powerful algorithm allows to solve this problem for a slightly modified version of eq. (12), with a discretized variable $\vec{r}\,' = (i, j)$ and a continuous variable $\vec{r} = (x, 0)$,

$$\hat{V}(x,t) = \min_{1 \leqslant i,j \leqslant L}\left[\frac{(i-x)^2}{2t} + \frac{j^2}{2t} + V(i,j)\right], \qquad (16)$$

[2]Think, as an example, of a single vortex in a superconductor: its behavior can be modeled by an elastic line ($d=1$) in the 3-dimensional space ($D=3$).

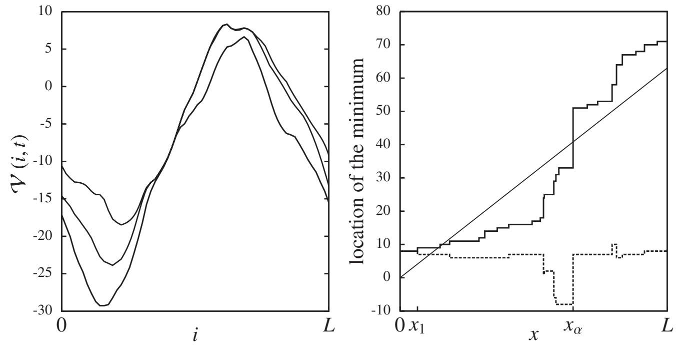

Fig. 2: Left: Step 1: *reduction to one dimension*. Effective 1-dimensional potential $\mathcal{V}(i,t)$ after the minimization over j, as given in eq. (18), at different times (from top to bottom $t = 0$, 2, 8). The potential becomes deeper and deeper as time increases. Right: Step 2: *location of the minimum*. The solid stair-case line is $i_{\min}(x)$, the dashed line is $j_{\min}(x)$ for $t=8$. The drift x is indicated. Shocks are only forward in x-direction.

for any x in the interval $(0, L)$. Let us now discuss how the algorithm finds the site $\vec{u}(x) = (i_{\min}(x), j_{\min}(x))$ which satisfies the minimization condition (16):

Step 1: *Reduction to a 1-dimensional problem.* For each value of i we perform a minimization over the transverse coordinate j, keeping in memory the location of the minimum, $j^*_{\min}(i)$. Since this operation does not involve x, the effective dimension of the problem is reduced to 1, and eq. (16) becomes

$$\hat{V}(x,t) = \min_{1 \leqslant i \leqslant L}\left[\frac{(i-x)^2}{2t} + \mathcal{V}(i,t)\right], \qquad (17)$$

$$\mathcal{V}(i,t) = \min_{1 \leqslant j \leqslant L}\left[\frac{j^2}{2t} + V(i,j)\right]. \qquad (18)$$

The reduced potential $\mathcal{V}(i,t)$ is plotted in fig. 2 (left).

Step 2: *Determination of* $i_{\min}(x)$. The latter is an increasing piecewise constant function of x. The minimum location in the original $D=2$ lattice is given by $j_{\min}(x) = j^*_{\min}(i_{\min}(x))$. For $x=0$ the minimum position is found from eq. (17). Increasing x, the minimum remains in $i_{\min}(x=0)$ up to a threshold x_1, above which the minimum takes a new value $i_{\min}(x_1) > i_{\min}(0)$. For all $i > i_{\min}(x=0)$ we find the value of x satisfying

$$\frac{(i-x)^2}{2t} + \mathcal{V}(i,t) = \frac{(i_{\min}(0)-x)^2}{2t} + \mathcal{V}(i_{\min}(0),t). \qquad (19)$$

x_1 is the smallest value of x for which this condition is satisfied:

$$x_1 = \min_{i_{\min} < i \leqslant L} \frac{1}{2}(i + i_{\min}(0)) + \frac{t(\mathcal{V}(i_{\min}(0),t) - \mathcal{V}(i,t))}{i_{\min}(0) - i}. \qquad (20)$$

One then searches for the next minimum and the procedure is iterated up to $x = L$, see fig. 1 (right).

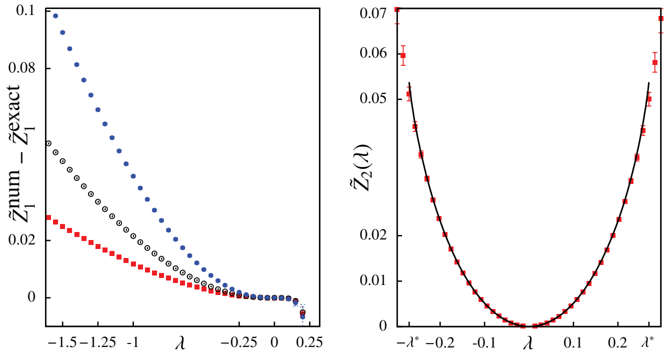

Fig. 3: (Color online) Left: convergence of the measured $\tilde{Z}_1^{\rm num}(\lambda)$ to the analytical prediction (21) for different times: $t=2.04$ (filled circles), $t=2.78$ (open circles), $t=4$ (squares). Right: measured $\tilde{Z}_2(\lambda)$ for $t=4$ (squares) compared to the prediction (22) (solid line).

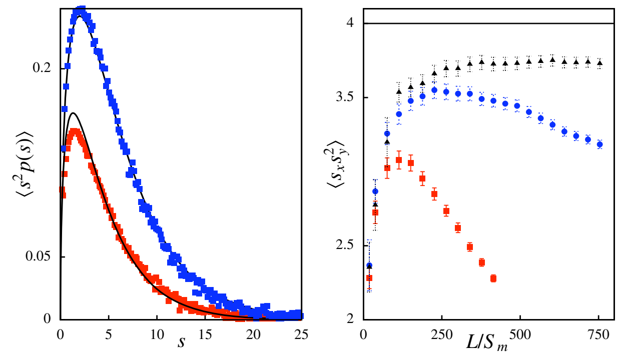

Fig. 4: (Color online) Left: plot of $s_x^2 p_1(s_x)$ (top curve) and $s_\perp^2[p_2(s_\perp)+p_2(-s_\perp)]$ (bottom curve). Shocks at $t=8$ ($S_m=7\pm0.1$). The solid line represents the analytical predictions. Right: test of the ratio (10). Squares are for $L=2^8$, circles for $L=2^{10}$, and triangles for $L=2^{12}$. Numerical data approach the predicted value when L is large. For $L/S_m\to0$ the particle feels the periodic potential, *i.e.* $S_x=L$ and $S_\perp=0$. For $t\to0$, lattice spacing is important: $S_x=1$ and $S_\perp=0$. The plateau value is consistent with $4-c_1L^{-1/2}$.

Step 3: *Shock sizes.* Given the sequence of minima locations $\vec{u}(x)=(i_{\rm min}(x),j_{\rm min}(x))$, the shocks sizes $\vec{S}$ are the discontinuities in these piecewise functions of x. The velocity profile is $v_x(x,t)=(x-i_{\rm min}(x))/t$, $v_y(x,t)=-j_{\rm min}(x)/t$.

Note that our algorithm is different from the algorithms proposed in the literature, where the numerical study of eq. (12) is performed on models which are discrete both in $\vec{r}\,'$ and $\vec{r}$. In that case minimization is efficiently achieved using a fast Legendre transform [5]. Here, thanks to the simplification of Step 1, we can keep the variable x continuous and determine the exact location of the shock, the only discretization comes from the *size* of the shock that is constrained to be integer. The minimal shock has thus size $S_0=1$, the maximal shock cannot be larger than the system size L. We conclude that self-affine scaling is expected to hold in the continuum limit when $S_0\ll S_m\ll L$ or equivalently $S_0/S_m=1/(Bt^2)\ll s\ll L/(Bt^2)$.

Step 4: *Numerical implementation.* In practice, we consider a $D=2$ square latice (usually of size $L=2^{12}$), the correlated random potential $V(i,j)$ is constructed from L^2 independently distributed Gaussian random numbers via a "fast Fourier transform" of eq. (14). Note that the sum over the components of $\vec{n}$ is now running over integers from $-L/2+1$ to $L/2$. The zero mode $\vec{n}=0$ is set to zero, $V_0=0$. We choose $\sigma^2=1$, which implies that $B=1/(3\pi)$ in formula (2). We collected a large number of shocks ($\sim10^6$–10^7) using many samples, from which we computed S_m and verified the prediction $S_m=t^2/(3\pi)$. From the reduced sizes $\vec{s}_\alpha:=\vec{S}_\alpha/S_m$, we measured $\tilde{Z}(\lambda_x,\lambda_\perp)=\frac{1}{N}\sum_\alpha(e^{\vec{\lambda}\cdot\vec{s}_\alpha}-1)$, specifically $\tilde{Z}_1(\lambda):=\tilde{Z}(\lambda,0)$ and $\tilde{Z}_2(\lambda):=\tilde{Z}(0,\lambda)$. The conjecture states that for the longitudinal component of the shock

$$\tilde{Z}_1(\lambda)=\frac{1}{2}(1-\sqrt{1-4\lambda}),\quad p_1(s)=\frac{1}{2\sqrt{\pi}s^{3/2}}e^{-s/4} \tag{21}$$

with $p_1(s_x):=\int{\rm d}s_\perp p(s_x,s_\perp)$, *i.e.* the same as obtained for $D=1$ [10,14] and for the related Galton process [20,25]. This is verified in fig. 3 (left) and fig. 4 (left). Since the agreement is very good, we have plotted in fig. 3 (left) the difference with the analytical prediction to emphasize the small deviations. These deviations are more important for large negative $\lambda\sim-1/s_0$, sensitive to the small lattice cutoff $s_0=S_0/S_m$ for the reduced shock sizes. Increasing the time, s_0 decreases as $s_0\sim1/t^2$. The prediction for the characteristic function of the y-component of the shock sizes, $\tilde{Z}_2(\lambda)$, is obtained by elimination of θ in the system of equations

$$\lambda(\theta)=\sin\theta\frac{\sqrt{5-\cos(4\theta)}+2}{\left[1-\cos(2\theta)+\sqrt{5-\cos(4\theta)}\right]^2}, \tag{22}$$

$$\tilde{Z}_2(\theta)=\frac{\cos\theta}{2}\,\frac{\sqrt{5-\cos(4\theta)}-2}{1-\cos(2\theta)+\sqrt{5-\cos(4\theta)}}. \tag{23}$$

Numerically, the Laplace inversion can be performed to determine $p_2(s_\perp)=\int{\rm d}s_x\,p(s_x,s_\perp)$ with high precision. (It is an integral over a segment of θ in the complex plane.) $p_2(s)$ is plotted in fig. 4 (left). For large s, $p_2(s)\approx1.7304|s|^{-5/2}e-0.2698|s|$; while for small s $p_2(|s|)=0.12375|s|^{-3/2}$. We have plotted the measured and calculated $\tilde{Z}_2(\lambda)$ in fig. 3 (right). Since $p_2(s)$ is symmetric in s, the same holds true for $\tilde{Z}_2(\lambda)$. The left and right edges of the analytic curve are at $|\lambda^*|=0.2698\ldots$, the constant in the exponential decay of $p_2(s)$. The agreement is excellent up to this point, where the size L cuts the divergence for $|\lambda|>\lambda^*$.

We now discuss shock correlations: *First*, the universal ratio (9) was measured to be 2.034 ± 0.015, very close to its analytical prediction. *Second*, correlations of jumps in the different directions are measured by the ratio (10), plotted in fig. 4 (right). In both cases, the deviations can be attributed to finite-size corrections, see the caption

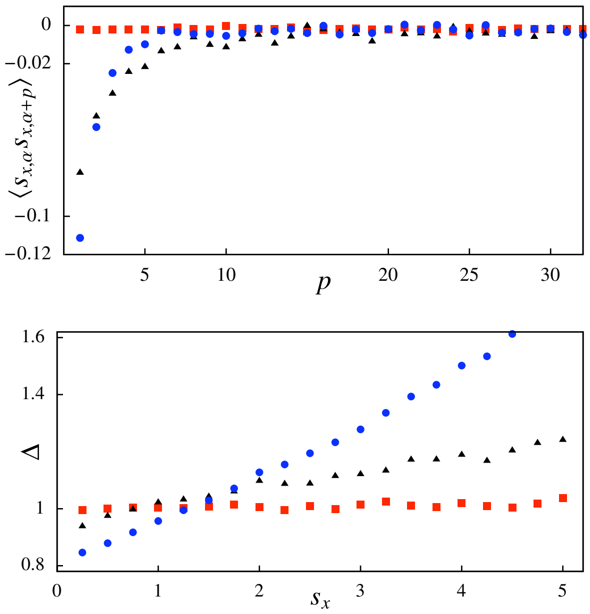

Fig. 5: (Color online) Circles correspond to $H=1/2$, triangles to $H=1$ and squares to $H=3/2$. Top: connected size correlations of subsequent shocks. Correlation decay is slower for $H=1$, for $H=3/2$ no correlation has been detected. Bottom: normalized shock distance.

of fig. 3. *Third*, we studied the correlations between *subsequent shocks*. To emphasize the remarkable nature of the present model ($H=3/2$) we compare with two other ones, with potential given by (14) with $H=0.5$ and $H=1$. In fig. 5 (top) we show the connected correlation $\langle s_{x,\alpha} s_{x,\alpha+p}\rangle^c$ of the longitudinal size $s_{x,\alpha}$ of a shock with the p-th subsequent shock. Figure 5 (bottom) focuses on the correlations between the location and the size of shocks. For example, are large shocks more isolated with respect to small shocks? To check this we compute the average distance $\langle x_{\alpha+1} - x_\alpha\rangle$ between consecutive shocks, normalized by its averaged and called $\Delta(s_x)$, as a function of the longitudinal size s_x of the shock α. Figure 5 (bottom) shows strong correlations for $H=0.5$ and $H=1$. For $H=3/2$ no effect was detected. This remarkable statistical independence of the shocks is essential for the main formula (11).

To conclude, we proposed a conjecture for the D-dimensional decaying Burgers equation with initial conditions which generalize the Brownian for $D=1$. We tested it numerically and checked that the shocks and the velocity increments along an axis are statistically independent at any time t. The conjecture is based on vanishing loop corrections in the field theory for the disordered problem and its generalization to elastic manifolds. Although confirmed within our numerical accuracy, any deviation would have important consequences for the —probably non-perturbative— corrections to the field theory. We hope this motivates efforts to prove or falsify our conjecture on a rigorous basis.

* * *

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