Functional Renormalization Group at Large N for Disordered Systems

Pierre Le Doussal[1] and Kay Jörg Wiese[2]
[1]*CNRS-Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris, France*
[2]*KITP, University of California at Santa Barbara, Santa Barbara, California 93106-4030*
(Received 12 September 2001; published 27 August 2002)

We introduce a method, based on an exact calculation of the effective action at large N, to bridge the gap between mean-field theory and renormalization in complex systems. We apply it to a d-dimensional manifold in a random potential for large embedding space dimension N. This yields a functional renormalization group equation valid for any d, which contains both the $O(\epsilon = 4 - d)$ results of Balents-Fisher and some of the nontrivial results of the Mezard-Parisi solution, thus shedding light on both. Corrections are computed at order $O(1/N)$. Applications to the Kardar-Parisi-Zhang growth model, random field, and mode coupling in glasses are mentioned.

DOI: 10.1103/PhysRevLett.89.125702 PACS numbers: 64.60.Ak, 05.40.–a

The random manifold problem, i.e., the behavior of an elastic interface in a random potential, is important for many experimental systems and still offers a considerable theoretical challenge [1]. It is the simplest example of a class of disordered systems, including random field magnets, where the so-called dimensional reduction [2] renders conventional perturbation theory trivial and useless. It also provides powerful analogies, via mode-coupling theory, to complex systems such as structural glasses [1]. Two analytical approaches have been devised so far, in limits where the problem *appears* to simplify while remaining nontrivial: the functional renormalization group (FRG) [3–5] and the (mean-field) replica Gaussian variational method (GVM) [6], together with their dynamical versions [7–9]. The FRG is hoped to be controlled for small $\epsilon = 4 - d$, where d is the internal dimension of the interface [parametrized by a N-component vector $\vec{u}(x)$ in the embedding space]. It follows the second cumulant of the random potential $R(u)$ under coarse graining, which becomes nonanalytic at $T = 0$ beyond the Larkin scale. The GVM approximates the replica measure by a replica symmetry broken (RSB) Gaussian, equivalently, the Gibbs measure for u as a random superposition of Gaussians [6], and is claimed to be exact for $N = \infty$. Computing next order corrections is fraught with difficulties [10], and it is still unclear in which sense both methods describe small but *finite* ϵ or $1/N$. The GVM, for instance, predicts a transition for $d = 0$ which must disappear for any finite N. We found recently [11,12] that higher loop FRG equations for $R(u)$ at $u \neq 0$ contain nontrivial, potentially ambiguous "anomalous terms" involving the nonanalytic structure of $R(u)$ at $u = 0$. Although a solution was found to two loops [11,12], the many loop structure remains mysterious. While both methods circumvent dimensional reduction by providing a nonperturbative mechanism, the GVM via replica symmetry breaking and the FRG via the generation of a cusplike nonanalyticity in $R''(u)$, they are disconcertingly different in spirit. Physically, however, both capture the metastable states beyond the Larkin scale R_c and should thus be related: this is not only a technical but also a deep physical problem [13]. A useful, quantitative, and more general method, where this connection appears naturally, is still lacking.

In this Letter, we introduce such a method, encompassing both the FRG and the GVM. Via an exact calculation of the effective action $\Gamma[u]$ at large N, we obtain the FRG β function in any d at large N. Its detailed analysis at dominant order, $N = \infty$, reveals that the FRG *exactly* reproduces (without invoking spontaneous RSB) the nontrivial result of the GVM for small overlap. The connections can be clarified using that $\Gamma[u]$ also gives the probability distribution of a given mode u_q. $O(1/N)$ corrections are computed, with the aim of understanding finite but large N. Further results, extensions, and discussions will appear in [14].

We start from the partition sum of an interface $\mathcal{Z}_V = \int \mathcal{D}[u] e^{-\mathcal{H}_V[u]/T}$ in a given sample, with energy

$$\mathcal{H}_V[u] = \int_q \frac{1}{2}(q^2 + m^2) u_{-q} \cdot u_q + \int_x V_x(u(x)), \quad (1)$$

where $\int_q \equiv \int \frac{d^d q}{(2\pi)^d}$, $\int_x \equiv \int d^d x$. The small confining mass m provides a scale. To obtain a nontrivial large N limit, one defines the scaled field $v = u/\sqrt{N}$ and chooses the distribution of the random potential $O(N)$ rotationally invariant, e.g., its second cumulant as

$$\overline{V_x(u) V_{x'}(u')} = R(u - u')\delta_{xx'} = NB((v - v')^2)\delta_{xx'}, \quad (2)$$

in terms of a function $B(z)$. Higher connected cumulants are scaled as $\overline{V_{x_1}(u_1) \cdots V_{x_p}(u_p)}^{\mathrm{conn}} = N\delta_{x_1,\ldots,x_p} S^{(p)}(v_1, \ldots, v_p)$. Physical observables can be obtained for any N from the replicated action at $n = 0$ with a source $\mathcal{Z}[j] = \int \mathcal{D}[u]\mathcal{D}[\chi]\mathcal{D}[\lambda] e^{-NS[u,\chi,\lambda,j]}$

$$S[u, \chi, \lambda, j] = \frac{1}{2}\int_q (q^2 + m^2) v^a_{-q} \cdot v^a_q + \int_x \left[U(\chi_x) - \frac{1}{2} i\lambda^{ab}_x (\chi^{ab}_x - v^a_x \cdot v^b_x) - j^a_x \cdot v^a_x \right], \quad (3)$$

 0031-9007/02/89(12)/125702(4)$20.00

where the replica matrix field $\chi_x \equiv \chi_x^{ab}$ has been introduced through a Lagrange multiplier. The bare interaction matrix potential $U(\chi) = \frac{-1}{2T^2}\sum_{ab} B(\tilde{\chi}_{ab}) - \frac{1}{3!T^3}\sum_{abc} S(\tilde{\chi}_{ab}, \tilde{\chi}_{bc}, \tilde{\chi}_{ca}) + \ldots$ depends only on $\tilde{\chi}_{ab} = \chi_{aa} + \chi_{bb} - \chi_{ab} - \chi_{ba}$ and has a cumulant expansion in terms of sums with higher numbers of replicas.

The effective action functional is defined as Legendre transform $\Gamma[u] + \mathcal{W}[J] = \int J \cdot u$, with $\mathcal{W}[J] = \ln Z[j]$, $J = \sqrt{N} j$. Its full calculation is given in [14]. Since $\Gamma[u]$ defines the renormalized vertices, its zero momentum limit defines the *renormalized disorder*. Thus we need only the result (per unit volume) for a *uniform* configuration of the replica field $u_x^a = u^a = \sqrt{N} v^a$:

$$\tilde{\Gamma}(v) = \frac{1}{L^d N}\Gamma(u) = \frac{1}{2T} m^2 v_a^2 + \tilde{U}(vv), \qquad (4)$$

where vv stands for the matrix $v_a \cdot v_b$. We have computed the two first coefficients of the renormalized disorder in the $1/N$ expansion $\tilde{U} = \tilde{U}^0 + \frac{1}{N}\tilde{U}^1 + \ldots$. Defining the notation $\partial_{ab} U(\phi) \equiv \partial_{\phi_{ab}} U(\phi)$ for any matrix ϕ with components ϕ_{ab}, we find at dominant order

$$\partial_{ab} \tilde{U}^0(vv) = \partial_{ab} U(\chi(v)), \qquad (5)$$

$$\chi(v) = vv + T \int_q \{(q^2 + m^2)\delta + 2T\partial U(\chi(v))\}^{-1}, \quad (6)$$

i.e., a self-consistent equation for $\partial \tilde{U}^0(vv)$, which, as we now show, contains both the GVM and the FRG.

For simplicity, we now set all bare cumulants except B to zero. The above equations contain a huge amount of information, since they encode the full distribution (i.e., all cumulants) of the renormalized disorder, and are thus quite nontrivial to analyze. One limit where they "simplify" is when v is set to zero, since they then reproduce the Mezard-Parisi (MP) equations [6] with $\chi(v = 0)_{ab} = \int_k G_{ab}(k)$. These exhibit spontaneous RSB (with multiple solutions) and are solved by a hierarchical Parisi ansatz for $\chi(v = 0)_{ab} = \chi(v = 0)(\mathsf{u})$, where $0 \le \mathsf{u} \le 1$ is the overlap between replicas a and b. In the opposite limit of a "strong" explicit symmetry breaking field (all $v_{ab} \equiv v_a - v_b \neq 0$), we expect that the renormalized disorder $\tilde{U}(vv)$ is given by a single saddle point and can be expanded in replica sums in terms of unambiguous renormalized cumulants, i.e., up to a constant:

$$\tilde{U}(vv) = \frac{-1}{2T^2}\sum_{ab} \tilde{B}(v_{ab}^2) - \frac{1}{3!T^3}\sum_{abc} \tilde{S}(v_{ab}^2, v_{bc}^2, v_{ca}^2) + \ldots.$$

This is the limit solved here, which we show is the one natural in the FRG, and amounts (in the RSB picture) to forcing the manifold into distant states. Work is in progress to analyze the rich crossover to RSB contained in (6), when some of the v_{ab} are set to zero.

We now expand, as detailed in [14], any quantity in (6) in sums over an increasing number of free replica indices. Starting from $\chi_{ab}(v) = v_a v_b + T I_1 \delta_{ab} - 2I_2 \partial_{ab} U(\chi(v)) + 4I_3 \sum_c \partial_{ac} U(\chi(v)) \partial_{cb} U(\chi(v)) + \ldots$ [with $I_n := \int_k 1/(k^2 + m^2)^n$] and inserting this into (5), we expand the argument of U in those terms which contain an explicit sum over replicas. By counting the number of free summations, we obtain a cumulant expansion of $\tilde{U}^0$, whose leading term (no free sum) is the second cumulant and satisfies the *closed* equation ($a \neq b$)

$$\tilde{B}'(v_{ab}^2) = B'(v_{ab}^2 + 2TI_1 + 4I_2[\tilde{B}'(v_{ab}^2) - \tilde{B}'(0)]) \qquad (7)$$

with no other contributions from higher cumulants at any T. This is illustrated graphically in Fig. 1. The three replica term $\tilde{S}$ satisfies a closed equation involving only $\tilde{B}$, and all cumulants can be determined iteratively.

The self-consistent Eq. (7) for the renormalized disorder can either be inverted directly (done below) or, equivalently, turned into a FRG equation. We start with the solution $\tilde{B}(x) = B(x)$ for $m = \infty$ [in the presence of a fixed ultraviolet (UV) cutoff Λ] and then decrease m. Taking the derivative $m\partial_m$ and rearranging gives

$$m\partial_m \tilde{B}'(x) = \tilde{B}''(x)\{4(m\partial_m I_2)[\tilde{B}'(x) - \tilde{B}'(0)] + 2(m\partial_m T I_1)[1 + 4I_2 \tilde{B}''(0)]^{-1}\}, \quad (8)$$

valid for any d [15]. Since $-\frac{1}{2} m\partial_m I_1 = m^2 I_2$, (8) has a well defined $\Lambda \to \infty$ limit for $d < 4$. Then $I_2 = A_d \frac{m^{-\epsilon}}{\epsilon}$ with $A_d = 2(4\pi)^{-d/2}\Gamma(3 - d/2)$ and we can define the dimensionless function $b(x) = 4A_d m^{4\zeta - \epsilon} \tilde{B}(x m^{-2\zeta})$, ζ for now arbitrary, which satisfies:

$$-m\partial_m b(x) = (\epsilon - 4\zeta) b(x) + 2\zeta x b'(x) + \frac{1}{2} b'(x)^2 - b'(x) b'(0) + T_m b'(x)[1 + b''(0)/\epsilon]^{-1}, \qquad (9)$$

where $T_m = T\frac{4A_d}{\epsilon} m^\theta$, $\theta = d - 2 + 2\zeta$. The FRG equation that we have derived is valid, to dominant order in $1/N$, *in any dimension* $d < 4$ and at any temperature T. Restricted to $T = 0$, it correctly matches the one obtained by Balents and Fisher [4] at any N but to lowest order in $\epsilon = 4 - d$. Furthermore, due to the self-consistent Eq. (7), it is fully integrable (not noted in [4]). Indeed, (7) can be inverted into

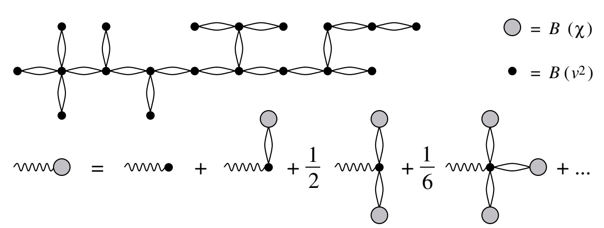

FIG. 1. Top: typical $T = 0$ contribution to $\tilde{B}(v_{ab}^2)$. Bottom: self-consistent equation at leading order for $\tilde{B}'(v_{ab}^2) = B'(\chi_{ab})$. The wiggly line denotes a derivative and is combinatorially equivalent to choosing one B. At finite T one can attach an additional arbitrary number of tadpoles to any B.

$$x = m^{2\zeta}\Phi\left(\frac{y}{4A_d m^{2\zeta-\epsilon}}\right) + \frac{1}{\epsilon}(y - y_0) - \tilde{T}_m, \tag{10}$$

where $y = -b'(x)$, $y_0 = -b'(0) = -4A_d m^{2\zeta-\epsilon}\tilde{B}'(0)$, Φ is the inverse function of $-B'(x)$, i.e., $(-B')(\Phi(y)) = y$ and $\tilde{T}_m = 2TI_1 m^{2\zeta}$ $[= T_m/(2-d)$ for $d < 2]$. That this is also the general solution of the FRG equation can be seen by noting that (9) can be transformed into a *linear* equation for the inverse function $x(y)$,

$$m\partial_m x = (\epsilon - 2\zeta)yx' + 2\zeta x - (y - y_0) + \frac{T_m \epsilon x'_0}{1 - \epsilon x'_0}, \tag{11}$$

with $x'_0 = x'(y_0)$. The general $T = 0$ solution of the homogeneous part reproduces the Φ term while a particular solution is $x = (y - y_0)/\epsilon$ using that $m\partial_m y_0 = (2\zeta - \epsilon)y_0$.

To analyze the solutions of the large N FRG equation (9), two approaches are legitimate, corresponding to different points of view. The first, natural in mean field, is exact integration. One discovers that (7) and (9) admit an analytic function as a solution, given by (10), only for $m > m_c$ where m_c is the Larkin mass (more generally a Larkin scale [15]). Indeed, its second derivative, $\tilde{B}''(0)^{-1} = B''(2TI_1)^{-1} - 4I_2$, diverges (always for $T = 0$, $d < 4$, in some cases for $T > 0$) when m is lowered down to m_c (which defines m_c). Since this expression is proportional to the replicon eigenvalue of the replica symmetric (RS) solution in the GVM, which exists for $m > m_c$, the generation of a cusp in the FRG exactly coincides at large N with the instability of the RS solution.

The second approach, natural in the RG [4], is to view the right-hand side (rhs) of (9) as the large-N limit of the true β function and to search for a zero. The general solution for $\theta > 0$, obtained from (11), is parametrized by ζ:

$$x = \frac{y}{\epsilon} - \frac{y_0}{2\zeta} + \frac{\epsilon - 2\zeta}{2\zeta\epsilon} y_0^{\epsilon/(\epsilon-2\zeta)} y^{-2\zeta/(\epsilon-2\zeta)} \tag{12}$$

for $\zeta > 0$, and $\epsilon x = y - y_0 - y_0 \ln(y/y_0)$ for $\zeta = 0$, with $y = -b'(x)$. Here $y_0 = -b'(0)$ is a fixed number. The value of the roughness exponent ζ (and of y_0 in the LR case) is selected by the decay of $R(u)$ in (2) at large u, argued to be identical for B and $\tilde{B}$; i.e., if $B'(z) \sim z^{-\gamma}$, one finds $\zeta = \zeta(\gamma) \equiv (4 - d)/2(1 + \gamma)$ or $\zeta = 0$ for shorter range correlations, to this order in $1/N$. All the fixed points (12) have a cusp $x'(y_0) = 0$ (for $\zeta \neq \epsilon/2$) and are expected to be the physically correct solutions at small m.

To show how to reconcile these two results, we study specific models, the long range (LR) correlations [6] $B(z) = \frac{\tilde{g}}{4(\gamma-1)A_d}(a^2 + z)^{1-\gamma}$ and the short range (SR) Gaussian correlator $B(z) = \frac{\tilde{g}}{4A_d}e^{-z}$. Choosing $\zeta = \zeta(\gamma)$ (10) yields:

$$x = (y/\tilde{g})^{-1/\gamma} + \epsilon^{-1}(y - y_0) - m^{2\zeta}a^2 - \tilde{T}_m, \tag{13}$$

[and $\ln(\tilde{g}m^{-\epsilon}/y)$ in the rhs for SR Gaussian with $\zeta = 0$, $a = 0$]. As one sees from Fig. 2, the rhs of (13) has a minimum and decreasing m the curve $x(y)$ cuts the axis $x = 0$ closer to the minimum. It reaches it at $m = m_c$, where the solution acquires a cusp $b'(0) - b'(x) \approx \sqrt{-2(\epsilon - 2\zeta)b'(0)x}$ and $m_c^{2\zeta}a^2 + \tilde{T}_{m_c} = (\tilde{g}\gamma/\epsilon)^{1/(1+\gamma)} \equiv \tilde{T}_c$, $b'(0) = -\tilde{g}^{1/(1+\gamma)}(\epsilon/\gamma)^{\gamma/(1+\gamma)}$ $(= -\epsilon$ for SR with $m_c^{\epsilon} = \tilde{g}/\epsilon)$. Although it is *a priori* unclear how to follow this solution for $m < m_c$, the following remarkable property indicates how one may proceed. If we compute the β function, i.e., the rhs of (9), using (13) at $m = m_c$ and $\zeta = \zeta(\gamma)$ we find that it *exactly vanishes* for all $x > 0$. Thus, for the potentials studied here, $b(x)$ evolves according to (13) until m_c where it reaches its fixed point, and does not evolve for $m < m_c$. This provides unambiguously a solution beyond the Larkin scale, reconciles the two approaches, and justifies the value obtained for ζ. The quantity $\tilde{B}''(v^2)^{-1} = B''(\chi(v))^{-1} - 4I_2$ plays the role of a replicon eigenvalue and remains frozen and positive for $v > 0$.

Here, for $N = \infty$, temperature plays only a minor role in the case where disorder is relevant, i.e., for $\theta(\gamma) > 0$ [i.e., $2 < d < 4$; $d < 2$ for $\gamma < \gamma_c = 2/(2 - d)$], which is described by the $T = 0$ fixed point (12). Contrarily to the one loop FRG, where $b(x)$ remains analytic in a boundary layer $x \sim T_m$ at $T > 0$, here the denominator in (9) (resumming all orders in ϵ) blows up and a cusp arises. Only in the marginal case ($\gamma = \gamma_c$; $d = 2$ for SR disorder) do we find a line of fixed points of (9) with $\zeta = (2 - d)/2$. For $T > T_c$ the disorder is analytic, given by (13) for all m ($\tilde{T}_m/\tilde{T}_c \equiv T/T_c$ does not flow, and $T_c = 2\pi$ in $d = 2$ for SR disorder). Below T_c, one recovers a cusp and the $T = 0$ fixed point. Finally, for $\gamma > \gamma_c$ ($d < 2$), no cusp is generated as $m \to 0$ and disorder is irrelevant, as in the corresponding RS solution of MP.

Since the FRG aims at obtaining the universal behavior at small m, k of the correlation function of the manifold $\langle v_k^a v_{-k}^b \rangle = m^{d-2\zeta}g(k/m)$, we can now compute its zero momentum limit $g(0)$, which is universal in the LR case, and compare with the GVM result $g_{\rm RSB}(0)$. Extending [6] in the presence of a mass, one shows that $m\partial_m \sigma_m(\mathsf{u}) = 0$ and thus the MP self-energy is $\sigma_m(\mathsf{u}) = C\mathsf{u}^{2/\theta-1}$ if $\mathsf{u} > \mathsf{u}_m$, $\sigma_m(\mathsf{u}) = \sigma_m(0)$ if $\mathsf{u} < \mathsf{u}_m$. Within the FRG, $g_{\rm FRG}(0) = -b'(0^+)/(2A_d)$ and substituting, we discover that $g_{\rm FRG}(0) = T\sigma_m(0)/m^4 < g_{\rm RSB}(0)$; i.e., the FRG gives only but exactly the contribution from the overlaps $\mathsf{u} < \mathsf{u}_m$ (the distant states). This nontrivial information, which

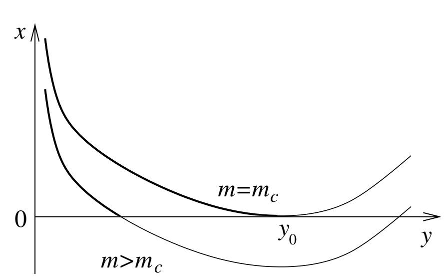

FIG. 2. The function $x(y)$ given by (13).

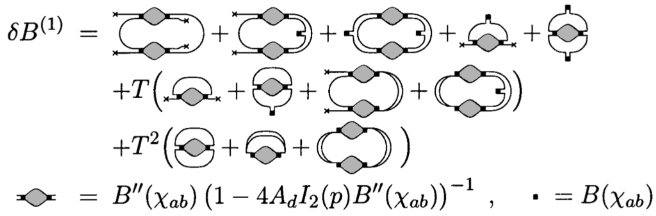

FIG. 3. Contribution to the second cumulant at order $1/N$.

within the MP approach *requires* a *full* RSB calculation, is obtained here *without any* RSB. Moreover, using $m\partial_m \sigma_m(\mathsf{u}) = 0$ one shows that contributions of larger overlaps in the MP result can be obtained by integrating the m dependent FRG result as $g_{\mathrm{RSB}}(0) = T[\sigma_m(0)/m^4 + \int_m^{m_c} d\sigma_{m'}(0)/m'^4 + m_c^{-2} - m^{-2}]$.

These results can be understood as follows. The effective action also describes the probability distribution $P_V(w)$ in a given environment V, of the center of mass of the interface $w = \frac{1}{N^{1/2}L^d}\int_x u_x$; i.e., one has $\tilde{\Gamma}[\{w_a\}] = -\lim_{L\to\infty}\frac{1}{NL^d}\ln\overline{P_V(w_1)\ldots P_V(w_n)}$. Extension of the FRG beyond the Larkin scale requires giving a meaning to the $u = 0^+$ limit. We find here that what the FRG actually computes [from $b'(0^+)$] is a second moment of w in the presence of a small extra field $\sqrt{N}j_a$ such that all $v_{ab} \neq 0$, i.e., an average such that, when there are several states, the different replicas are chosen in maximally separated states ($\mathsf{u} = 0$).

In a previous study aiming to connect the RSB solution to the FRG [13], $\overline{\ln P_V(w)\ln P_V(-w)}$ was computed and used to define a renormalized second cumulant of the disorder. This quantity is, however, *different* from the one in the FRG obtained here and does not reproduce the second moment of w, neither $g(0)_{\mathrm{RSB}}$, nor $g(0)_{\mathrm{FRG}}$. In addition, since [13] used the unperturbed MP saddle point, the two calculations focus on different regimes ($v_{ab}^2 \sim 1$ here, $v_{ab}^2 \sim 1/N$ there, $v_a \sim 1$ in both) [16]. Work is in progress to connect these regimes and obtain a more complete version of the FRG, using our Eqs. (6) and summing over RSB saddle points.

The FRG approach should allow a quantitative study of finite N beyond possible artifacts of $N = \infty$ and extension of [4] to any d. The calculation of the correction at order $1/N$ to $\tilde{B}$ is involved and was performed using two complementary methods, a graphical one (see Fig. 3) and the algebraic expansion in number of replica sums. The resulting expression for the β function at order $1/N$ is UV convergent and given at $T = 0$ in [17] and at $T > 0$ in [14]. We have checked that the two-loop FRG equation for the N-component model of [5,14] is reproduced.

In summary, via an exact calculation of the effective action at large N, we have derived the exact renormalization group equation, valid in any dimension, for the field theory of pinning. We show that it is consistent with the main results of the full and the marginal one-step RSB solutions of Mezard-Parisi. This is interesting since it has been widely debated [18] whether the RSB method captures the physics: our results raise no doubt for infinite N. The question at finite N is more complicated: despite large efforts [1,10], results beyond infinite N within RSB have not been calculated up to now. In contrast, the present method has allowed us to compute $1/N$ corrections to the effective action, resulting in a formidable expression which remains to be analyzed. This opens the way to quantitative progress in several outstanding problems. One example is the Kardar-Parisi-Zhang equation (which maps onto the $d = 1$ manifold) for which the upper critical dimension remains unknown [19]. Our method provides a unique candidate for a field theory of the strong-coupling phase. Another example is the random field problem, still under intense debate [20], for which we have also computed [14] the effective action at large N. Finally, the mode-coupling approximation in glasses [1,21] identifies with mean-field (large N) dynamics, exhibiting aging solutions. However, this leaves out thermally activated processes, and our $1/N$ method seems promising here.

K. J. Wiese thanks for support Deutsche Forschungsgemeinschaft (Heisenberg Grant No. Wi-1932/1-1) and NSF (Grant No. PHY99-07949).

[1] See reviews in *Spin Glasses and Random Fields*, edited by A. P. Young (World Scientific, Singapore, 1998).
[2] K. Efetov and A. Larkin, Sov. Phys. JETP **45**, 1236 (1977).
[3] D. S. Fisher, Phys. Rev. Lett. **56**, 1964 (1986).
[4] L. Balents and D. S. Fisher, Phys. Rev. B **48**, 5949 (1993).
[5] H. Bucheli *et al.*, Phys. Rev. B **57**, 7642 (1998).
[6] M. Mézard and G. Parisi, J. Phys. I (France) **1**, 809 (1991).
[7] L. Cugliandolo *et al.*, Phys. Rev. Lett. **76**, 2390 (1996).
[8] T. Nattermann *et al.*, J. Phys. II (France) **2**, 1483 (1992); O. Narayan and D. S. Fisher, Phys. Rev. B **46**, 11 520 (1992).
[9] P. Chauve *et al.*, Phys. Rev. B **62**, 6241 (2000).
[10] D. M. Carlucci *et al.*, J. Phys. I (France) **6**, 1031 (1996).
[11] P. Chauve, P. Le Doussal, and K. J. Wiese, Phys. Rev. Lett. **86**, 1785 (2000); P. Le Doussal, K. J. Wiese, and P. Chauve, cond-mat/0205108; (to be published).
[12] P. Chauve and P. Le Doussal, Phys. Rev. E **64**, 051102 (2001).
[13] L. Balents, J.-P. Bouchaud, and M. Mézard, J. Phys. I (France) **6**, 1007 (1996).
[14] P. Le Doussal and K. Wiese (to be published).
[15] One can also derive a FRG equation by varying the UV cutoff from 0 to Λ and a Larkin scale $\Lambda_c = R_c^{-1}$.
[16] With a $T > 0$ boundary layer there. We thank L. Balents for help in clarifying these issues.
[17] P. Le Doussal and K. J. Wiese, cond-mat/0109204v1.
[18] C. M. Newman and D. L. Stein, J. Stat. Phys. **106**, 213 (2002).
[19] E. Marinari *et al.*, J. Phys. A **33**, 8181 (2000).
[20] E. Brézin and C. De Dominicis, Europhys. Lett. **44**, 13 (1998).
[21] See T. R. Kirkpatrick and D. Thirulamai, Phys. Rev. B **36**, 5388 (1987).