Random field spin models beyond one loop: a mechanism for decreasing the lower critical dimension

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(Dated: October 28, 2005)

The functional RG for the random field and random anisotropy $O(N)$ sigma-models is studied to two loop. The ferromagnetic/disordered (F/D) transition fixed point is found to next order in $d = 4 + \epsilon$ for $N > N_c$ ($N_c = 2.8347408$ for random field, $N_c = 9.44121$ for random anisotropy). For $N < N_c$ the lower critical dimension $d = d_{\rm lc}$ plunges below $d_{\rm lc} = 4$: we find *two* fixed points, one describing the quasi-ordered phase, the other is novel and describes the F/D transition. $d_{\rm lc}$ can be obtained in an $(N_c - N)$-expansion. The theory is also analyzed at large N and a glassy regime is found.

It is important for numerous experiments to understand how the spontaneous ordering in a pure system is changed by quenched substrate impurities. One class of systems are modeled by elastic objects in random potentials (so-called random manifolds, RM). Another class are $O(N)$ classical spin models with ferromagnetic (Ferro) couplings in presence of random fields (RF) or anisotropies (RA). The latter describe amorphous magnets [1]. Examples of RF are liquid crystals in porous media [2], He-3 in aerogels [3], nematic elastomers [4], and ferroelectrics [5]. The XY random field case $N = 2$ is common to both classes and describes periodic RM such as charge density waves, Wigner crystals and vortex lattices [6]. Larkin showed [7] that the well-understood pure fixed points (FP) of both classes are perturbatively unstable to weak disorder for $d < d_{\rm uc}$ ($d_{\rm uc} = 4$ in the generic case). For a continuous symmetry (i.e. the RF Heisenberg model) it was proven [8] that order is destroyed below $d = 4$. This does not settle the difficult question of the lower critical dimension $d_{\rm lc}$ as a weak-disorder phase can survive below $d_{\rm uc}$, if associated to a non-trivial FP, as predicted in $d = 3$ for the Bragg-glass phase with quasi long-range order (QLRO) [9]. For the random field Ising model $N = 1$ (RFIM) it was argued [10], then proven [11] that the ferromagnetic phase survives in $d = 3$. Developing a field theory to predict $d_{\rm lc}$, and the exponents of the weak-disorder phase and the Ferro/Disordered (F/D) transition, has been a long-standing challenge. Both extensive numerics and experiments have not yet produced an unambiguous picture. Among the debated issues are the critical region of the 3D RFIM [12] and the possibility of a QLRO phase in amorphous magnets [5, 13, 14].

A peculiar property shared by both classes is that observables are identical to all orders to the corresponding ones in a $d - 2$ thermal model [15]. This dimensional reduction (DR) naively predicts $d_{\rm lc} = 4$ for the weak-disorder phase in a RF with a continuum symmetry [16] and no Ferro order for the $d = 3$ RFIM, which is proven wrong [11]. It also predicts $d^{\rm t}_{\rm uc} = 6$ for the F/D *transition* FP. While there is agreement that multiple local minima are responsible for DR failure, constructing the field theory beyond DR is a formidable challenge. Recent attempts include a reexamination of the ϕ^4 theory (i.e. soft spins) for the F/D transition near $d = 6$ [17]. Previous large-N approaches failed to find a non-trivial FP, but a self-consistent resummation including the $1/N$ corrections hinted at exponents different from DR (without succeeding in computing them) from a solution breaking replica symmetry [18].

As for the pure $O(N)$ model, an alternative to the soft-spin version (near $d = 6$) is the sigma model near the lower critical dimension (here presumed to be $d = 4$). In 1985 D.S. Fisher [19] noticed that an infinite set of operators become relevant near $d = 4$ in the RF $O(N)$ model. These were encoded in a single function $R(\phi)$ for which Functional RG equations (FRG) were derived to one loop, but no new FP was found. For a RM problem [20] it was found that a cusp develops in the function $R(\phi)$ (the disorder correlator), a crucial feature which allows to obtain non-trivial exponents and evade DR. A fixed point for the RF model was later found [9] in $d = 4 - \epsilon$ for $N = 2$. It was noticed only very recently [21] that the 1-loop FRG equations of Ref. [19] possess fixed points in $d = 4 + \epsilon$ for $N \geq 3$, providing a description of the long-sought critical exponents of the F/D transition.

In spite of these advances, many questions remain. Constructing FRG beyond one loop (and checking its internal consistency) is highly non-trivial. Progress was made for RM [22, 23], and one hopes for extension to RF. Some questions necessitate a 2-loop treatment, e.g. for the depinning transition, as shown in [24]. In RF and RA models the 1-loop analysis predicted some repulsive FP in $d = 4 + \epsilon$ (for larger values of N), and some attractive ones [9, 25] in $d = 4 - \epsilon$. The overall picture thus suggests a lowering of the critical dimension, but how it occurs remains unclear. Finally the situation at large N is also puzzling. Recently, via a truncation of exact RG [26] it was claimed that DR is recovered for N large.

Our aim in this Letter is twofold. We reexamine the overall scenario for the fixed points and phases of the $O(N)$ model using FRG. This requires the FRG to two loop. Here we present selected results, details are presented elsewhere [27]. We find a novel mechanism for how the lower critical dimension is decreased below $d = 4$ for $N < N_c$ at some critical value N_c. We obtain a description of the bifurcation which occurs at N_c, and below N_c we find *two* perturbative FPs. Thanks to 2-loop terms $d_{\rm lc}$ can be computed in an expansion in $N_c - N$, and the Ferro/Para FP *below* $d = 4$ is found. A study at large N indicates that some glassy behavior survives there.

Let us consider $O(N)$ classical spins $\vec{n}(x)$ of unit norm $\vec{n}^2 = 1$. To describe disorder-averaged correlations one introduces replicas $\vec{n}_a(x)$, $a = 1, \ldots, k$, the limit $k = 0$ being implicit everywhere. The starting model is a non-linear sigma model, of partition function $\mathcal{Z} = \int \mathcal{D}[\pi]\, \mathrm{e}^{-\mathcal{S}[\pi]}$ and action:

$$\mathcal{S}[\pi] = \int \mathrm{d}^d x \, \Big[\frac{1}{2T_0} \sum_a [(\nabla \vec{\pi}_a)^2 + (\nabla \sigma_a)^2] - \frac{1}{T_0} \sum_a M_0 \sigma_a - \frac{1}{2T_0^2} \sum_{ab} \hat{R}_0(\vec{n}_a \vec{n}_b) \Big] \,, \tag{1}$$

where $\vec{n}_a = (\sigma_a, \vec{\pi}_a)$ with $\sigma_a(x) = \sqrt{1 - \vec{\pi}_a(x)^2}$. A small uniform external field $\sim M_0(1, \vec{0})$ acts as an infrared cutoff. Fluctuations around its direction are parameterized by $(N-1)$ π-modes. The ferromagnetic exchange produces the 1-replica part, while the random field yields the 2-replica term $\hat{R}_0(z) = z$ for a bare Gaussian RF. RA corresponds to $\hat{R}_0(z) = z^2$. As shown in [19] one must include a full function $\hat{R}_0(z)$, as it is generated under RG. It is marginal in $d = 4$.

To obtain physics at large scales, one computes perturbatively the effective action $\Gamma[n_a(x)]$. It can be expanded in gradients near a uniform background configuration n_a^0, and splitted in 1-, 2- and higher-replica terms. From rotational invariance it is natural to look for Γ in the form (1) with $\vec{n}_a \to \vec{n}_a^R = (\sigma_a^R, \vec{\pi}_a^R)$, $\sigma_a \to \sigma_a^R = \sqrt{1 - (\pi_a^R)^2}$, $\pi_a \to \pi_a^R = Z^{-1/2}\pi_a$, $T_0 \to T_R = T_0/Z_T$, $M_0 \to M_R = M_0\sqrt{Z}/Z_T$, $m = \sqrt{M_R}$ the renormalized mass of the $\vec{\pi}_a$ modes, and $\hat{R}_0(\vec{n}_a\vec{n}_b) \to m^\epsilon \hat{R}(\vec{n}_a^R \vec{n}_b^R)$. Higher vertices generated under RG are irrelevant by power-counting, hence discarded. Renormalization of T contributes to the flow of $\hat{R}$, and one sets $T = 0$ at the end.

One computes Z, Z_T and $\hat{R}$ perturbatively in $\hat{R}_0$ and extracts β and γ functions $\beta[\hat{R}](z) = -m\partial_m \hat{R}(z)$, $\gamma = -m\partial_m \ln Z$ and $\gamma_{\mathrm{T}} = -m\partial_m \ln Z_T$, derivatives taken at fixed $\hat{R}_0, T_0, M_0$. Although calculation of the Z-factors is simplified due to DR, anomalous contributions appear from the non-analyticity of $\hat{R}(z)$. To compute $\hat{R}(z)$, one chooses a pair of uniform background fields (n_a^0, n_b^0) for each (a, b). We use a basis for the fluctuating fields (to be integrated over) such that $\vec{n}_a = (\sigma_a, \eta_a, \vec{\rho}_a)$, $\vec{n}_b = (\sigma_b, \eta_b, \vec{\rho}_b)$, where η lies in the plane common to $(\vec{n}_a^0, \vec{n}_b^0)$, and $\vec{\rho}_a$ along the perpendicular $N-2$ directions; both have diagonal propagators. Denoting $\vec{n}_a^0 \vec{n}_b^0 = \cos\phi_{ab}$, one has $\vec{n}_a\vec{n}_b = \cos\phi_{ab}\,(\sigma_a\sigma_b + \eta_a\eta_b) + \sin\phi_{ab}\,(\sigma_a\eta_b - \sigma_b\eta_a) + \vec{\rho}_a\vec{\rho}_b$. One gets factors of $(N-2)$ from the contraction of $\vec{\rho}$. Our calculation to 2 loops results in the flow-equation for the function $R(\phi) = \hat{R}(z = \cos\phi)$, and $\epsilon = 4 - d$:

$$\begin{aligned}
\partial_\ell R(\phi) = {} & \epsilon R(\phi) + \frac{1}{2} R''(\phi)^2 - R''(0)R''(\phi) + (N-2)\left[\frac{1}{2}\frac{R'(\phi)^2}{\sin^2\phi} - \cot\phi R'(\phi)R''(0)\right] \\
& + \frac{1}{2}(R''(\phi) - R''(0))R'''(\phi)^2 + (N-2)\Big[\frac{\cot\phi}{\sin^4\phi}R'(\phi)^3 - \frac{5+\cos 2\phi}{4\sin^4\phi}R'(\phi)^2 R''(\phi) + \frac{1}{2\sin^2\phi}R''(\phi)^3 \\
& - \frac{1}{4\sin^4\phi}R''(0)\Big(2(2+\cos 2\phi)R'(\phi)^2 - 6\sin 2\phi R'(\phi)R''(\phi) + (5+\cos 2\phi)\sin^2\phi R''(\phi)^2\Big)\Big] \\
& - \frac{N+2}{8}R'''(0^+)^2 R''(\phi) - \frac{N-2}{4}\cot\phi R'''(0^+)^2 R'(\phi) - 2(N-2)\Big[R''(0) - R''(0)^2 + \gamma_a R'''(0^+)^2\Big]R(\phi)
\end{aligned} \tag{2}$$

with $\partial_l := -m\partial_m$, and the last factor proportional to $R(\phi)$ is $-2\gamma_{\mathrm{T}}$ and takes into account the renormalization of temperature. Thanks to the anomalous terms, arising from a non-analytic $R(\phi)$, this β-function preserves a (at most) linear cusp (i.e. finite $R'''(0^+)$), and reproduces for $N = 2$ the previous 2-loop results for the periodic RM [22]. For $N > 2$, anomalous contributions are determined following [28]. γ is found as

$$\gamma = (N-1)R''(0) + \tfrac{3N-2}{8} R'''(0^+)^2 \,, \tag{3}$$

either via a calculation of $\langle \sigma_a \rangle$ [30], or of the mass corrections, a result consistent with the β function (2) [31]. The determination of γ_{T} is more delicate [32], and we have allowed for an amalous contribution γ_a, whose effect is minor, and discussed below. The correlation exponents (standard definition [21]) are obtained as $\bar{\eta} = \epsilon - \gamma$, $\eta = \gamma_{\mathrm{T}} - \gamma$ at the FP. (2) has the form:

$$\partial_\ell R = \epsilon R + \mathrm{B}(R,R) + \mathrm{C}(R,R,R) + O(R^4) \tag{4}$$

We now discuss its solution, first in the RF case, and setting $\gamma_a = 0$. The 1-loop flow-equation (setting $C = 0$) admits, in dimensions larger than 4, a fixed point $R^*_{\mathrm{F/D}}$ with a single repulsive direction, argued by Feldman to describe the F/D zero temperature transition. This is true only for $N > N_c$. For $N < N_c$ this fixed point *disappears* and instead an *attractive*

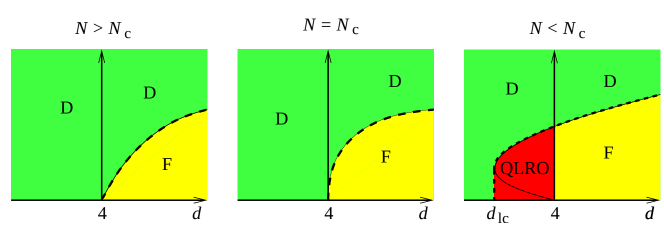

FIG. 1: (Color online) Phase diagram. D = disordered, F = ferromagnetic, QLRO = quasi long-range order.

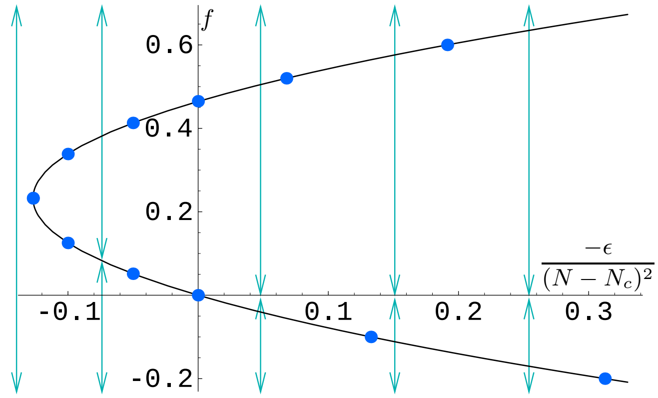

FIG. 2: (Color online) Parametric plot for solutions of (5) for $N < N_c$ (dots) for RF, equivalent to (6) (solid line, parabola) and flow (arrows). f parameterizes disorder and only $f \geq 0$ is physical. Cmp. with fig. 1, right.

fixed point $R^*_{\rm QLRO}$ appears which describes the Bragg glass for $N = 2$. We have determined $N_c = 2.8347408$ and the solution $R_c(u)$ which satisfies $\mathrm{B}(R_c, R_c)|_{N=N_c} = 0$. It is formally the solution at $\epsilon = 0$. Since the FRG flow vanishes to one loop along the direction of R_c, examination of the 2-loop terms is needed to understand what happens at $N = N_c$. In particular the F/D transition should still exist for $N < N_c$, though it cannot be found at one loop. It is not even clear a priori whether it remains perturbative.

The scenario found is perturbative, accessible within a double expansion in $\sqrt{|\epsilon|}$ and $N - N_c$. To this aim, we write the leading terms in $N - N_c$ and ϵ of (4), namely

$$\partial_\ell R = \epsilon R + \mathrm{B}_c(R,R) + \mathrm{C}_c(R,R,R) + (N - N_c)\,\mathrm{B}_N(R,R)$$
$$\mathrm{B}_c(\ldots) = \mathrm{B}(\ldots)|_{N=N_c}\,, \qquad \mathrm{C}_c(\ldots) = \mathrm{C}(\ldots)|_{N=N_c} \qquad (5)$$

One looks for a fixed-point solution of the form $R(u) = gR_c(u) + g^2\delta R(u)$, with $g > 0$, $R_c''(0) = -1$, and its flow. This analysis is done numerically and leads to the flow shown schematically on Fig. 1. The RG-flow projected onto the direction of g is equivalent to

$$\partial_l g = \epsilon g + 1.092(N - N_c)g^2 + 2.352g^3\,. \qquad (6)$$

As a solution of the functional flow near N_c, its simplicity is surprising. Setting $g = (N_c - N)f$, there are three FP:

$$\frac{\epsilon}{(N - N_c)^2} - 1.092f + 2.352f^2 = 0\,, \quad \text{or} \quad f = 0\,. \qquad (7)$$

For $N > N_c$ the physical branch is $f < 0$. As seen in Fig. 1, for $d > 4$ there is a ferro phase (i.e. $f = 0$ is attractive) and an unstable FP describing the F/D transition, given by the negative branch of (7). At $N = N_c$ one sees from (6) that the F/D fixed point is still perturbative, but in a $\sqrt{\epsilon}$ expansion for g (and for the critical exponents). For $N < N_c$ the physical side is $f > 0$ and there are two branches on Fig. 2 corresponding to *two* non-trivial fixed points. One is the infrared attractive FP for weak disorder which describes the Quasi-Ordered ferromagnetic phase; the second one is unstable and describes the transition to the disordered phase with a flow to strong coupling. These two fixed points exist only for $\epsilon < \epsilon_c$ and annihilate at ϵ_c. The lower critical dimension of the RF-model for $N < N_c$ is lowered from $d = 4$ to

$$d_{\rm lc}^{\rm RF} = 4 - \epsilon_c \approx 4 - 0.1268(N - N_c)^2 + O((N - N_c)^3)\,. \quad (8)$$

Note that the mechanism is different from the more conventional criterion $d - 4 + \eta(d) = 0$ at $d = d_{\rm lc}$.

The same analysis for the random anisotropy class yields $N_c = 9.44121$. The equivalent of (6) becomes $\partial_l g = \epsilon g + 0.549(N - N_c)g^2 + 47.6g^3$, leading to $d_{\rm lc}^{\rm RA} \approx 4 - 0.00158(N - N_c)^2$. Although it yields $d_{\rm lc}(N = 3) \approx 3.93$ and no QLRO phase in $d = 3$, naive extrapolation should be taken with caution given the high value of N_c. Numerical values for $d_{\rm lc}$ are changed for a $\gamma_a \neq 0$, but the scenario is robust for $\gamma_a < \gamma_c$ [33].

We now discuss the FRG flow-equations for N large. From a truncated exact RG Tarjus and Tissier (TT) [26] found: that the linear cusp of the F/D fixed point for $d > 4$ vanishes for $N > N^*(d)$, i.e. $R'''(0^+) = 0$; and that the non-analyticity becomes weaker as N increases (as $|\phi|^n$ with $n \sim N$). Analytical study of the derivatives of (2) confirms the existence of this peculiar FP to two loop and predicts $N^*(d, 2p)$ beyond which the set of $\{R^{(2k)}(0)\}$ for $k \leq p$ admits a stable FP, with $R^{(2k-1)}(0^+) = 0$ for $k \leq p$ and $R^{(2k-1)}(0^+) \neq 0$ for $k > p$. We find:

$$N^*(d) = N^*(d, 4) = 18 + 49\epsilon/5 + \ldots \qquad (9)$$

which yields a slope roughly twice the one of Fig. 1 of [26]. This remarkable FP raises some puzzles. Although weaker than a cusp its non-analyticity should imply some (weaker) metastability in the system. It is thus unclear whether DR is fully restored: to prove it one should rule out feedback from anomalous higher-loop terms in exponents or the β-function. Finally, one also wonders about its basin of attraction. As shown in Fig. 3, the FRG flow for $R''''(0)$ is still to large values if its bare value is large enough, indicating some tendency to glassy behaviour.

To explore these effects we now study the F/D phase transition at large N and $d > 4$. We obtain, both at large N and fixed d (extending Ref. [23]), and to one loop, the flow equation for the rescaled $\tilde{R}(z = \cos\phi) = NR(\phi)/|\epsilon|$:

$$\partial_l \tilde{R} = -\tilde{R} + 2\tilde{R}_1'\tilde{R} - \tilde{R}_1'\tilde{R}'z + \tfrac{1}{2}\tilde{R}'^2 = 0\,. \qquad (10)$$

We denote $y(z) = \tilde{R}'(z)$, $y_0 = \tilde{R}'(1) = -NR''(0)/|\epsilon|$ and $r_4 = NR''''(0)/|\epsilon|$. There are two analytic FPs $\tilde{R}(z) = z - 1/2$ and $\tilde{R}(z) = z^2/2$, corresponding both to $y_0 = 1$ and to $r_4 = 1$ and $r_4 = 4$ respectively. This agrees with the flow of thed erivatives for analytic $R(\phi)$: $\partial_l y_0 = y_0(y_0 - 1)$, and at $y_0 = 1$: $\partial_l r_4 = \frac{1}{3}(r_4 - 1)(r_4 - 4)$. The first FP is the large-N limit of the TT fixed point, the second is repulsive and divides the region where $r_4 \to \infty$ (non-analytic $R(\phi)$) in a finite RG time l_c (Larkin scale). For $y_0 > 1$, we find a family of NA fixed points with a linear cusp, parameterized by an

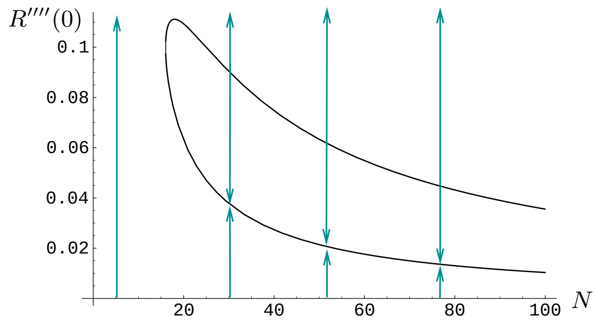

FIG. 3: Flow for $R''''(0)/|\epsilon|$ for $d = 4 - \epsilon > 4$ as a function of N. The two branches behave as $1/N$ and $4/N$ at large N.

integer $n \geq 2$, s.t. $y_0 = n/(n-1)$, $z = y - (y_0 - 1)(y/y_0)^n$. The solutions with n (i.e. $z(y)$) odd correspond to random anisotropy ($R'(\phi) = R'(\phi + \pi)$). The $n = 2$ RF fixed point is $R(\phi) = 2\cos(\phi) + \frac{8\sqrt{2}}{3}\sin^3(\phi/2) - \frac{4}{3}$. To elucidate their role, we obtained the exact solution for the flow both below l_c, i.e. $z = \frac{y}{y_0} + (y_0 - 1)\Phi(\frac{y}{y_0})$ ($\Phi(x)$ parameterizes the bare disorder, $\Phi(1) = 0$), and above l_c, with an anomalous flow for y_0. Matching at l_c yields the critical manifold for RF disorder, defined from the conditions that $\Phi'(w) = \Phi(w)/w = 1$ has a root $0 \leq w \leq 1$. It is different from the naive DR condition $y_0 = 1$, valid for small r_4. The $n = 2$ FP corresponds to bare disorder such that the root $w = 0$. Hence it is multicritical [34]. Generic initial conditions within the critical F/P manifold flow back to the TT FP i.e. the linear cusp decreases to zero [35]. This however occurs only at an infinite scale, hence we expect a long crossover within a glassy region, characterized by a cusp, and metastability on finite scales [36]. The large-N limit here is subtle. Taking $N \to \infty$ at fixed volume on a bare model with $\hat{R}_0(z) = z$ yields only the analytic FP, equivalent to a replica-symmetric saddle point. Higher monomials z^p are generated in perturbation theory, at higher order in $1/N$. Thus, for N large but fixed and infinite size, one must first coarse grain to generate a non-trivial function $\hat{R}_0(z)$, before taking the limit of $N \to \infty$.

In conclusion we obtained the 2-loop FRG functions for the random field and anisotropy σ-models. We found a new fixed point and a scenario for the decrease of the lower critical dimension. This rules out the scenario left open at one loop that the bifurcation close to $d = 4$ simply occurs within the (quasi-) ordered phase.

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[30] For $N = 2$, one can also use $\eta = \sin\phi$ and a RM calculation with $\langle\sigma_a\rangle \approx e^{-\frac{1}{2}\langle\phi_a^2\rangle}$, since the field correlator is gaussian up to $O(\epsilon^4)$ [29].
[31] Reexpressing (2) in $\hat{R}$, (3) is the rescaling term $\gamma z \hat{R}'(z)$.
[32] 1-loop corrections to correlations at non-zero momentum are anisotropic $\sim \mu_{ij}(v)$ (see (13) in [28]) in presence of a background $\hat{n}_a^0$ here, v_a there. This yields formally $\gamma_a^\eta = \frac{1}{4}$, $\gamma_a^\rho = (3N-4)/(8(N-2))$.
[33] The coefficient in (6) and (7) becomes $2.35(1 - \gamma_a/\gamma_c)$ with $\gamma_c = 2.04$, and similarly for RA $47.6(1 - \gamma_a/\gamma_c)$, $\gamma_c = 1.23$, with a corresponding shift $4 - d_{\rm lc}(\gamma_a) = (4 - d_{\rm lc}(\gamma_a = 0))/(1 - \gamma_a/\gamma_c)$. For $\gamma_a > \gamma_c$ the scenario reverses (fig. 2 is flipped w.r.t. the f-axis). The FP, for $\epsilon < 0$ and $N < N_c$ are at $\epsilon > 0$ and $N > N_c$. The bifurcation occurs entirely within the Ferro phase, and the QLRO branch survives above $d = 4$. This scenario would imply a F/D fixed point inaccessible to FRG, contradicting [21]. It is unlikely, since [32] suggests $\frac{1}{4} \leq \gamma_a \leq \frac{1}{2}$.
[34] We thank G. Tarjus and M. Tissier for pointing out this important fact.
[35] This is true only on one side of the multicritical FP. The other side, if accessible from physically realizable bare disorder, would correspond to a strong disorder regime.
[36] The physics associated to a similar reentrant crossover for RM for $d > 4$ is discussed in Appendix H of: L. Balents and P. Le Doussal, Annals of Physics **315** (2005) 213.