Two-factor synaptic consolidation reconciles robust memory with pruning and homeostatic scaling

bioRxiv
THE PREPRINT SERVER FOR BIOLOGY

Georgios Iatropoulos, Wulfram Gerstner, Johanni Brea

24 October 2024

Maturation and plasticity in biological and artificial networks
Cargèse, Corsica

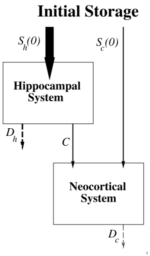

Georgios Iatropoulos

EPFL

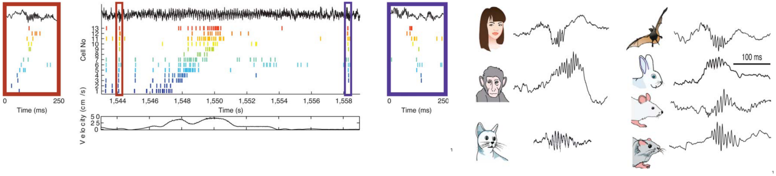
Swiss National Science Foundation

Blue Brain Project

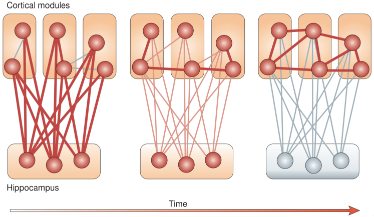
Memory consolidation and replay



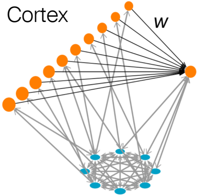
McClelland, McNaughton, O'Reilly (1995)



Buzsàki (2015)



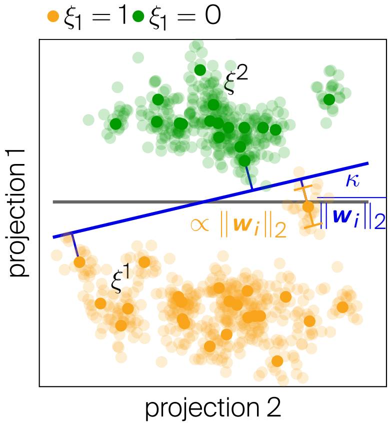
Frankland, Bontempi (2005)



adapted from Sun, Advani, Spruston, Saxe, Fitzgerald (2022)

How to store memory patterns.

patterns		$x_i, \xi_i^\mu \in \{0,1\}$
activity		$x_i(t+1) = H\left(\sum_j W_{ij} x_j(t) - I_i\right)$
		$= H(\boldsymbol{w}_i \cdot \boldsymbol{x}(t) - I_i)$
pattern $\boldsymbol{\xi}^\mu$ is stored	if	$\xi_i^\mu = H(\boldsymbol{w}_i \cdot \boldsymbol{\xi}^\mu - I_i)$
neural noise		random bit flip
Amari, Hopfield		$W_{ij} = \frac{1}{P}\sum_{\mu=1}^{P}(2\xi_i^\mu - 1)(2\xi_j^\mu - 1)$
max margin		maximize $\frac{\kappa}{\\|\boldsymbol{w}_i\\|_2}$ such that
		$(2\xi_i^\mu - 1)(\boldsymbol{w}_i \cdot \boldsymbol{\xi}^\mu - I_i) > \kappa$
maximize SNR_2		$\frac{\lvert \boldsymbol{w}_i \cdot \boldsymbol{\xi}^\star - I_i \rvert}{\\|\boldsymbol{w}_i\\|_2} = \frac{\text{signal}}{\text{neural noise}}$

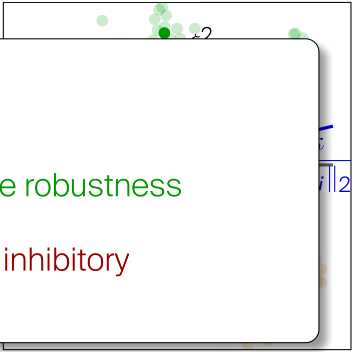


$$\|\boldsymbol{w}_i\|_2 = \sqrt{\sum_j W_{ij}^2}$$

How to store memory patterns.

patterns $x_i, \xi_i^\mu \in \{0,1\}$

activity $x_i(t+1) = H\left(\sum_j W_{ij} x_j(t) - I_i\right)$



Hopfield $\to$ max SNR$_2$

consolidation with optimal neural noise robustness

dense connectivity

neurons are neither excitatory nor inhibitory

$(2\xi_i^\mu - 1)(\boldsymbol{w}_i \cdot \boldsymbol{\xi}^\mu - I_i) > \kappa$

projection 2

maximize SNR$_2$ $\frac{|\boldsymbol{w}_i \cdot \boldsymbol{\xi}^\star - I_i|}{\|\boldsymbol{w}_i\|_2} = \frac{\text{signal}}{\text{neural noise}}$

$\|\boldsymbol{w}_i\|_2 = \sqrt{\sum_j W_{ij}^2}$

EPFL

Is cortical connectivity optimized for storing information?

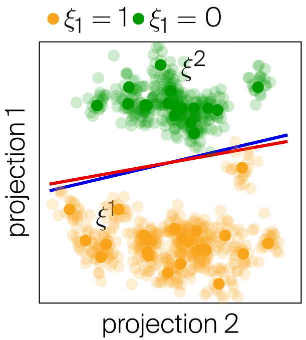
N. Brunel, 2016:
maximize number M of patterns such that
$(2\xi_i^\mu - 1)(\boldsymbol{w}_i \cdot \boldsymbol{\xi}^\mu - I_0) > \kappa_0$ and $W_{ij} \geq 0$.

approximately the same as

maximizing $\mathrm{SNR}_1 = \frac{|\boldsymbol{w}_i \cdot \boldsymbol{\xi}^\star - I_i|}{\|\boldsymbol{w}\|_1}$

$\|w\|_1 = \sum_j |W_{ij}|$

$$\frac{1}{N}\sum_j W_{ij}\xi_j^\mu \approx \frac{f}{N}\sum_j W_{ij} = \frac{f}{N}\|\boldsymbol{w}_i\|_1.$$



Is cortical connectivity optimized for storing information?

N. Brunel 2016:

Hopfield → max SNR$_1$

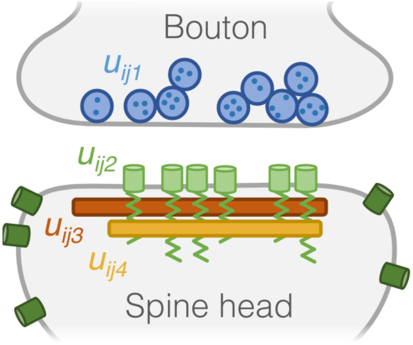
consolidation without optimal neural noise robustness

sparse connectivity

neurons are excitatory or inhibitory

Which synaptic plasticity rule achieves this result?

Is the synaptic connection strength a product of z factors?



$$w_{ij} \propto u_{ij1} \cdot u_{ij2} \cdot u_{ij3} \cdot u_{ij4} \cdots u_{ijz}$$

- Awake: one-shot attractor formation with e.g. standard Hopfield learning rule
- Asleep: batch-perceptron (Krauth & Mézard, 1987)
  - replay patterns
  - tag "weakest pattern"
    $\mu^* = \arg\min_\mu (2\xi_i^\mu - 1)(\boldsymbol{w}_i \cdot \boldsymbol{\xi}^\mu - I_i)$
  - update factors
    $\Delta u_{ijk} = \eta(2\xi_i^{\mu^*} - 1)\xi_j^{\mu^*} \prod_{l \neq k} u_{ijl}$
  - multiplicative scaling of factors
    $u_{ijk} \to u_{ijk} / \sum_j u_{ijk}^2$
  - update of inhibition
    $I_i \to I_i - \eta_{\text{inh}}(2\xi_i^{\mu^*} - 1)$

Results in maximization of SNR_1 for $z = 2$.

This learning rule maximizes

$$\text{SNR}_{\frac{2}{z}} = \frac{|\boldsymbol{w}_i \cdot \xi^\star - l_i|}{\|\boldsymbol{w}_i\|_{\frac{2}{z}}}$$

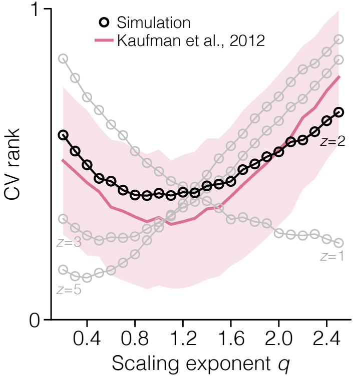
Peter D. Hoff, 2017

If $(\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_z)$ is a minimizer of
$f(\boldsymbol{u}_1 \circ \boldsymbol{u}_2 \circ \cdots \circ \boldsymbol{u}_z) + \frac{\lambda}{z} \sum \|\boldsymbol{u}_k\|_2^2$
then $\boldsymbol{w} = \boldsymbol{u}_1 \circ \boldsymbol{u}_2 \circ \cdots \circ \boldsymbol{u}_z$ is a minimizer of
$f(\boldsymbol{w}) + \lambda \|\boldsymbol{w}\|_q^q$ with $q = \frac{2}{z}$.

EPFL

More evidence for $z = 2$

Coefficient of variation over time (CV) of $\|\boldsymbol{w}\|_q$ is smallest for $q = \frac{2}{z} \approx 1$.

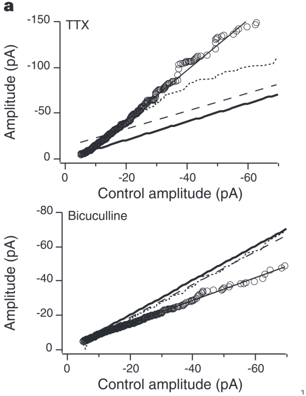


Experimental data: proxy of synaptic strength (PSD-95:EGFP fluorescence) measured over days in cortical neurons grown in dishes.

Multiplicative homeostatic scaling

G. Turrigiano et al., 1998

Homeostatic changes affect each synapse in proportion to its initial strength.



Multiplicative homeostatic scaling of factors u_{ijk} implies multiplicative scaling of weights w_{ij}.

EPFL

Multiplicative homeostatic scaling

G. Turrigiano et al., 1998

Hopfield → max SNR_1 with two-factor synapses

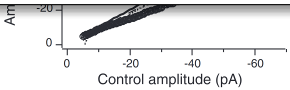
consolidation without optimal neural noise robustness

sparse connectivity

neurons are excitatory or inhibitory

simple learning rule

multiplicative homeostatic scaling



Is synaptic noise driven by one volatile factor?

Assumptions

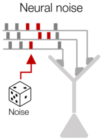
- Factors co-vary (in the long term).
- Fluctuations in synaptic strength are dominated by a single volatile factor.

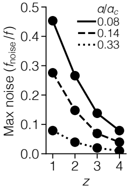
Consequences

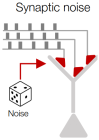
- Neural input under this synaptic noise varies with

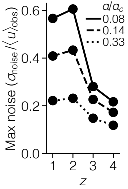
$$\sum_j W_{ij}^{2-\frac{2}{z}}$$

for $z = 2$ we have $\frac{2}{z} = 2 - \frac{2}{z} = 1$ and noise matches optimized SNR_1.









Is synaptic noise driven by one volatile factor?

Consequences

Hopfield → max SNR_1 with two-factor synapses

consolidation with optimal synaptic noise robustness

sparse connectivity

neurons are excitatory or inhibitory

simple learning rule

multiplicative homeostatic scaling

nd

Is synaptic noise driven by one volatile factor?

Assumptions

- Factors co-vary (in the long term).
- Fluctuations in synaptic strength are dominated by a single volatile factor.

Consequences

- Neural input under this synaptic noise varies with

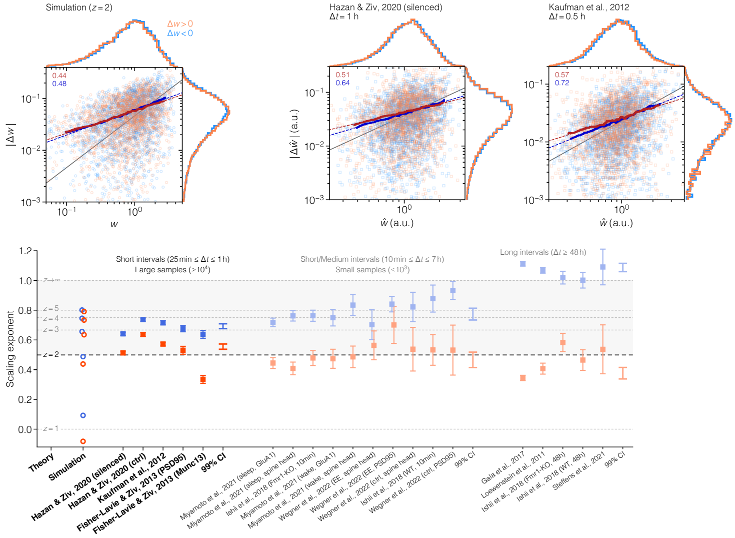
$$\sum_j W_{ij}^{2-\frac{2}{z}}$$

for $z = 2$ we have $\frac{2}{z} = 2 - \frac{2}{z} = 1$ and noise matches optimized SNR_1.

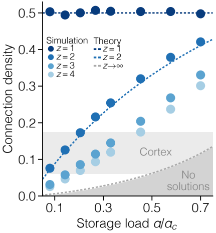
- Fluctuations in individual synaptic strength are proportional to $W_{ij}^{1-\frac{1}{z}}$ for $z = 2$: $\sqrt{W_{ij}}$.

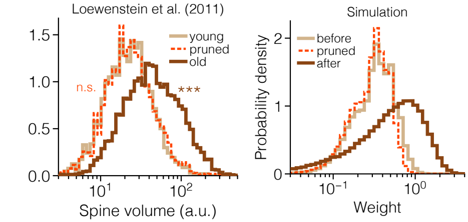
EPFL

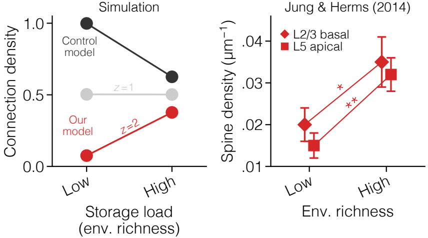
Data is consistent with synaptic noise driven by one volatile factor.

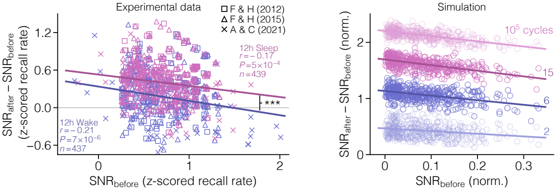


Other results









Conclusions

bioRxiv
THE PREPRINT SERVER FOR BIOLOGY

The model consists of

- two-factor excitatory synapses.
- updates during replay with the weakest patterns.
- multiplicative scaling of the synaptic factors.
- synaptic noise due to volatility of one factor.

It is consistent with

- maximization of signal-to-synaptic-noise ratio SNR_1.
- observed minimal coefficient of variation of $\|\boldsymbol{w}\|_1$.
- fluctuations $\propto \sqrt{W_{ij}}$ of individual synapses.
- multiplicative homeostatic scaling of synaptic strengths.
- qualitative recall rate behavior before and after sleep.
- connection density estimates.

Georgios Iatropoulos

EPFL Swiss National Science Foundation Blue Brain Project